Friction during a first-order phase transition

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1407.3132

INT, August 21, 2015

Aim

The aim is to derive a phenomenological model for friction for **sizable wall velocities**

$$\Box\phi + \frac{d\mathcal{F}(\phi,T)}{d\phi} = \eta\, u^\mu \partial_\mu \phi$$

Or integrated in the wall frame

$$\Delta\mathcal{F} = \gamma v_{wall} \eta \int dz \, (\partial_z \phi)^2$$

positive

depends on the scalar potential, latent heat

depends on particle content, W bsons, tops

Microscopic determination of η

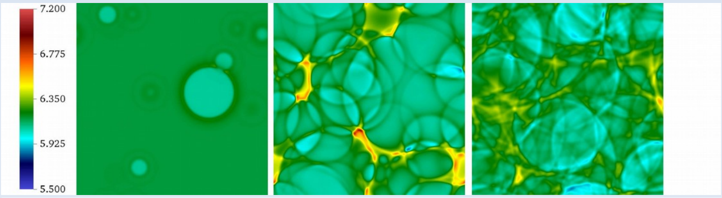
[Moore & Prokopec '95] $v_{wall} \ll 1$

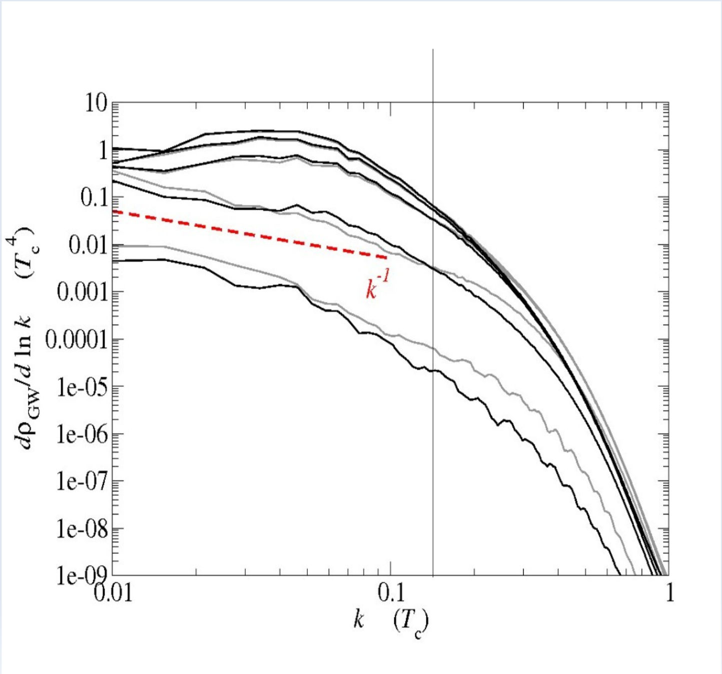
here: $v_{wall} \lesssim 1$

Motivation

$$v_{wall} \gtrsim c_s$$

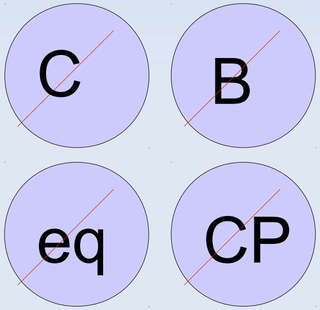
gravitational waves

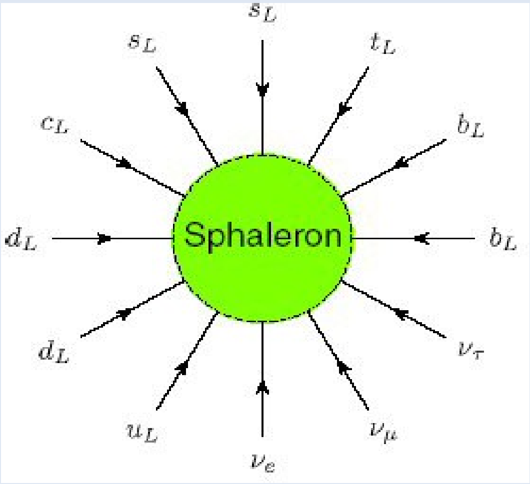




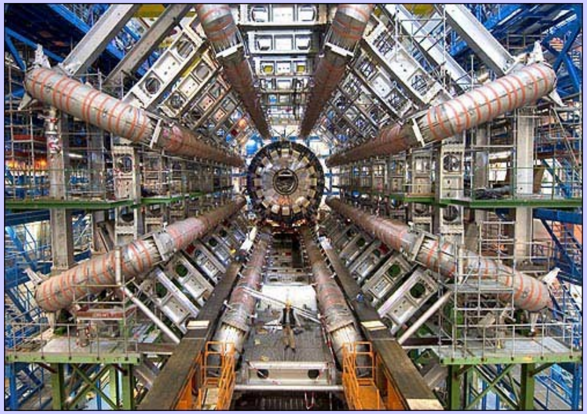
$$v_{wall} \lesssim c_s$$

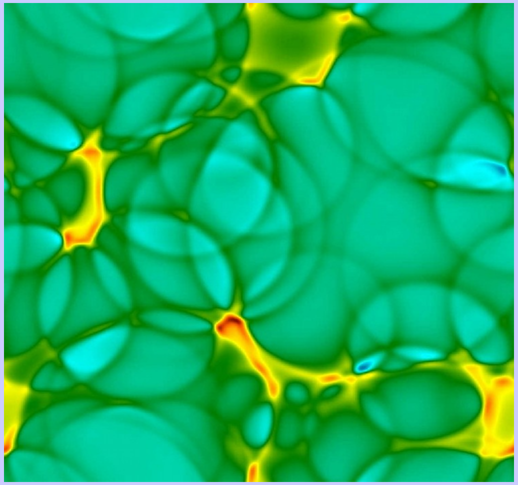
baryogenesis





Phase transition at T~100 GeV?





Possibly, the electroweak phase transition drove the Universe **out-of-equilibrium.**

Particle physics

	pheno	hierarchy problem	phase transition
SM	✓	✗	✗
MSSM	✗ ✓	✓	✗
nMSSM	✓	✓	✓

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$y_t \simeq 1$

$m_H \lesssim m_Z$

Outline

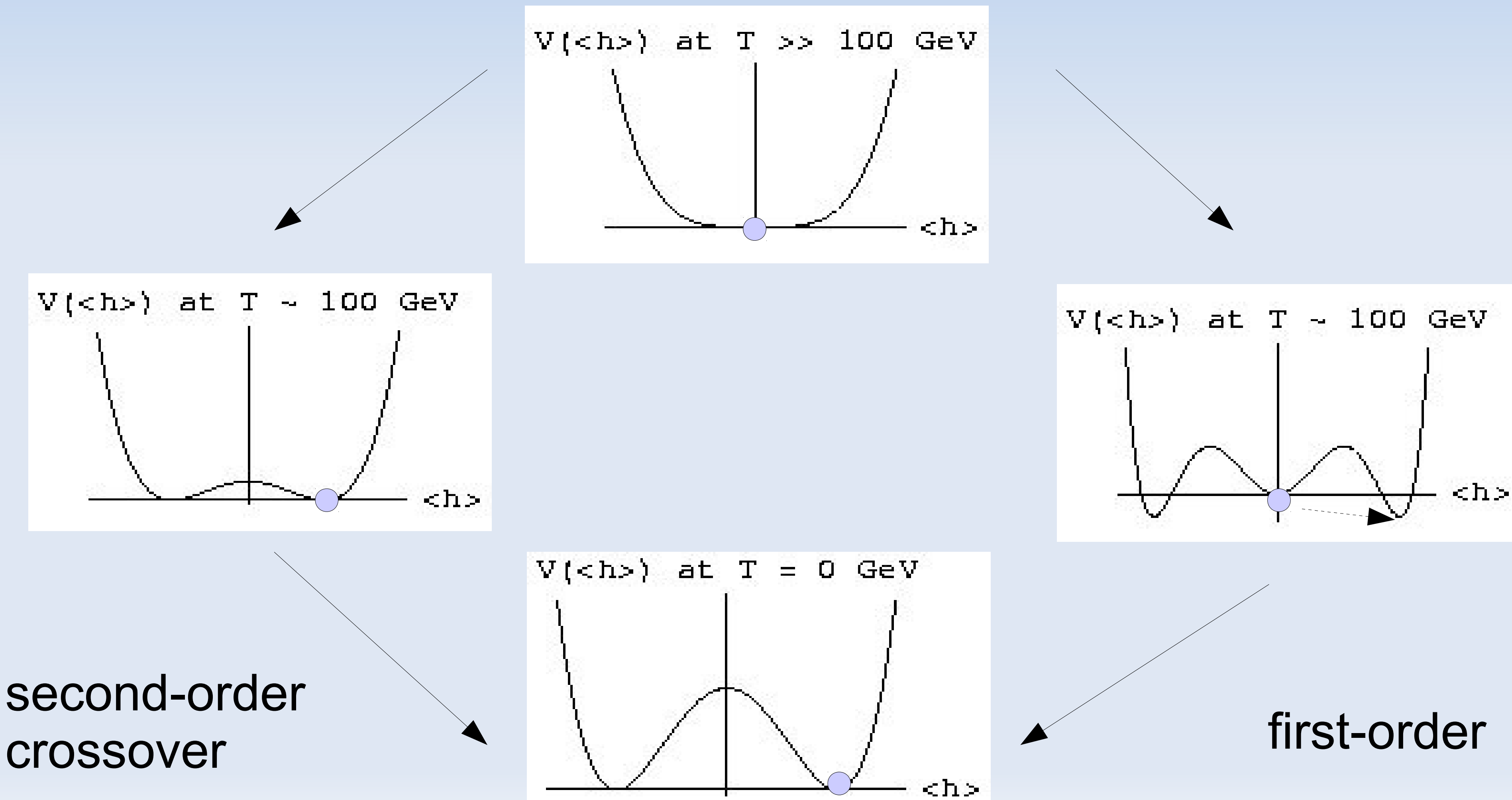
Phase transition

Hydrodynamics

Friction

Electroweak symmetry breaking

It can also be a strong phase transition if a **potential barrier** seperates the new phase from the old phase



Electroweak symmetry breaking

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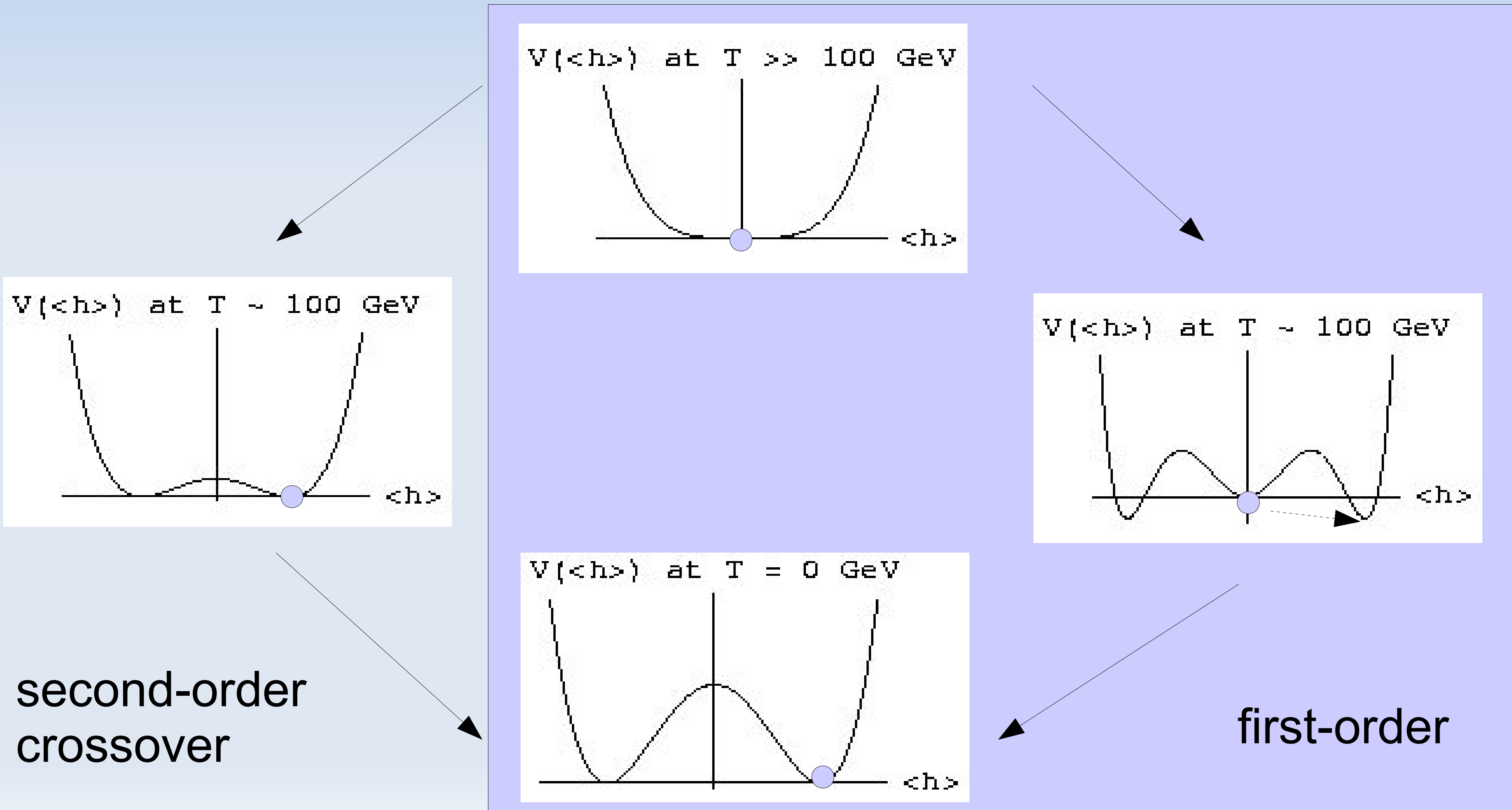

First-order phase transitions

- first-order phase transitions proceed by bubble nucleations

- in case of the electroweak phase transition, the ”Higgs bubble wall” separates the symmetric from the broken phase

- this is a violent process ($v_{wall} \simeq O(c)$) that drives the plasma out-of-equilibrium

Length scales

wall thickness

$L_{wall} \simeq \text{few}\, T^{-1}$

planar

out-of-equilibrium

friction

bubble size

$R_{bubble} \simeq H^{-1}$

$\simeq M_{pl}/T^2$

spherical

hydrodynamics

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Hydro setup

[Landau & Lifshitz]

Ideal fluid, gradients very small

$$T_{\mu\nu} = \omega u_\mu u_\nu - g_{\mu\nu}\, p \qquad\qquad u^\mu = \gamma(1, v)$$

Enthalpy and pressure depend ultimately on the Higgs vev and the temperature.

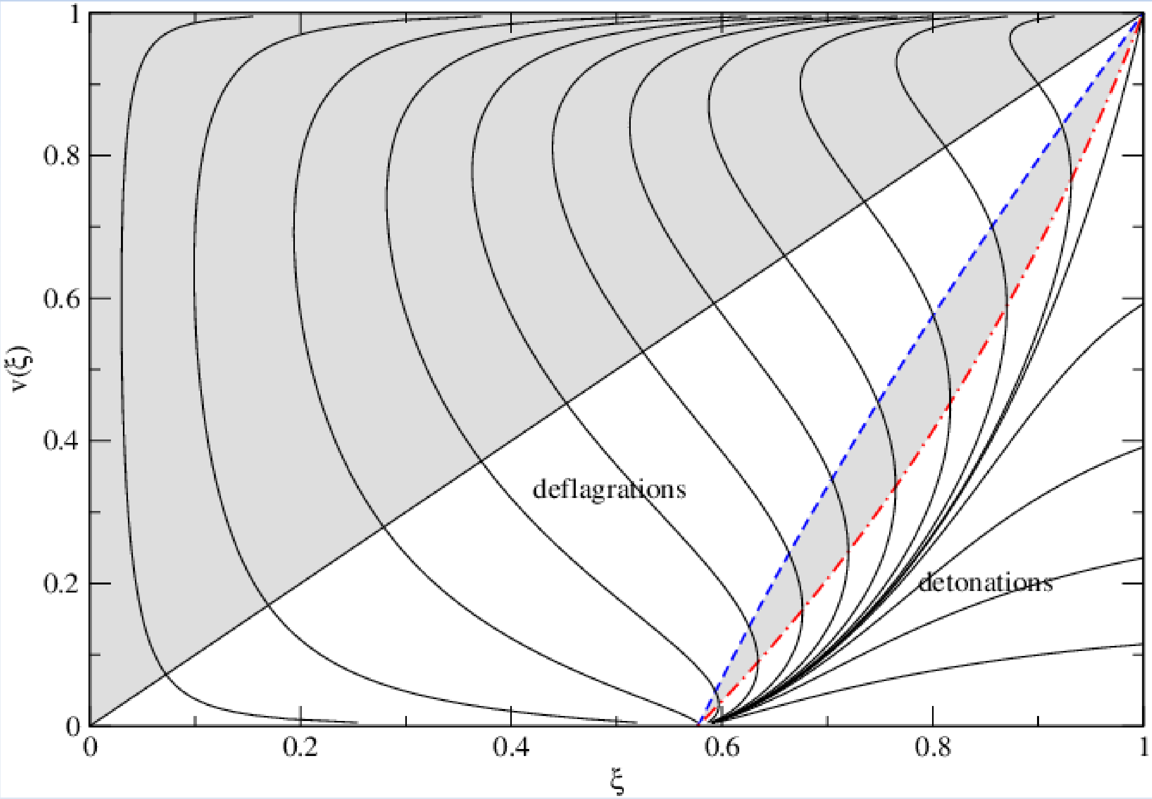
For simplicity: bag model

$$p_\pm = \frac{1}{3} a_\pm T^4 - \epsilon_\pm$$

Spherical symmetry and energy conservation demands self-similar solutions $\partial^\mu T_{\mu\nu}(\xi) = 0 \qquad \xi = r/t$

Solutions of the velocity profile

Full solutions are obtained by gluing together patches using the discontinuities



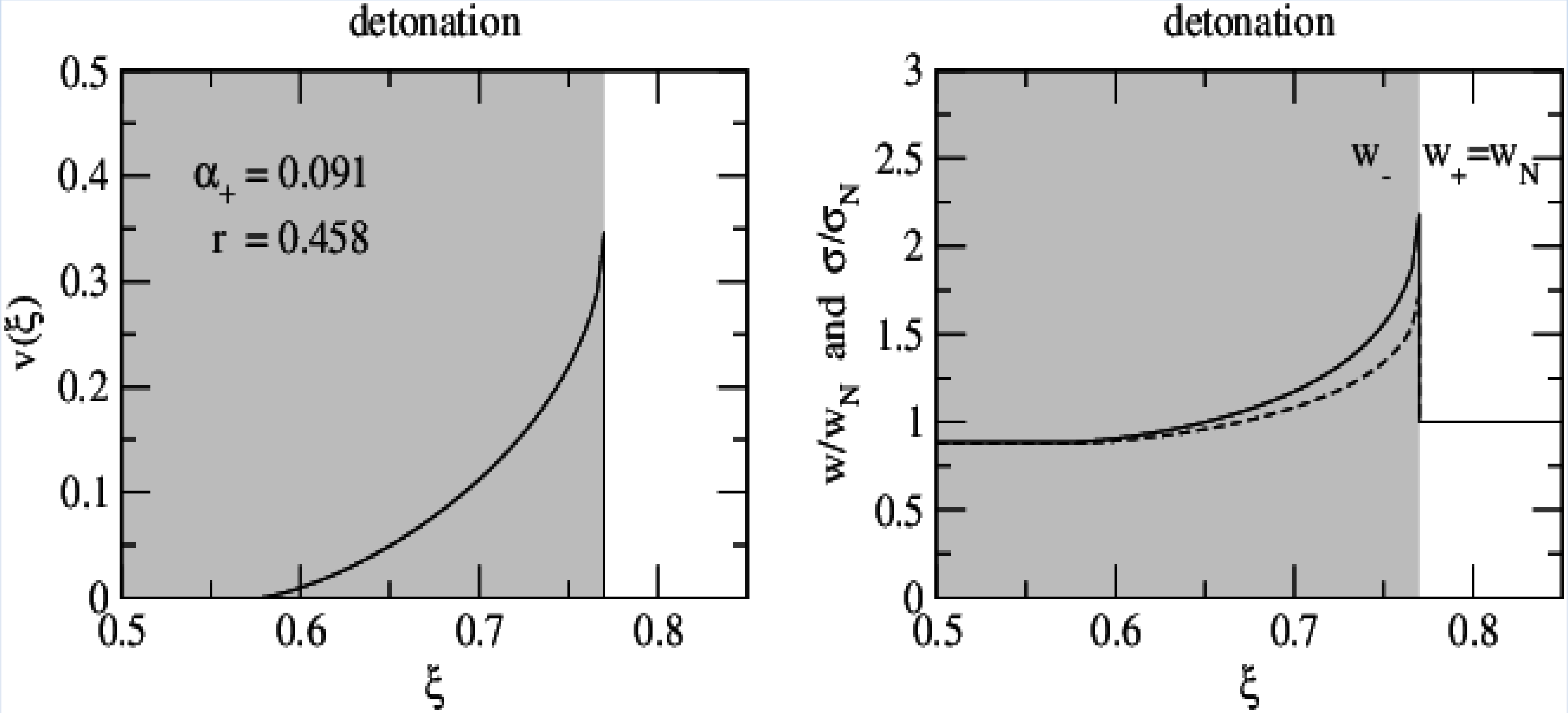
$$\xi = r/t$$

At the phase boundary $\partial^\mu T_{\mu\nu} = 0 \rightarrow \Delta T_{0z} = \Delta T_{zz} = 0$ in the wall frame

Discontinuity in ϵ $\rightarrow$ discontinuity in $v(\xi)\,\&\,T(\xi)$

Detonations

Detonations are **supersonic** expansion modes with a rarefaction wave **behind** the wall

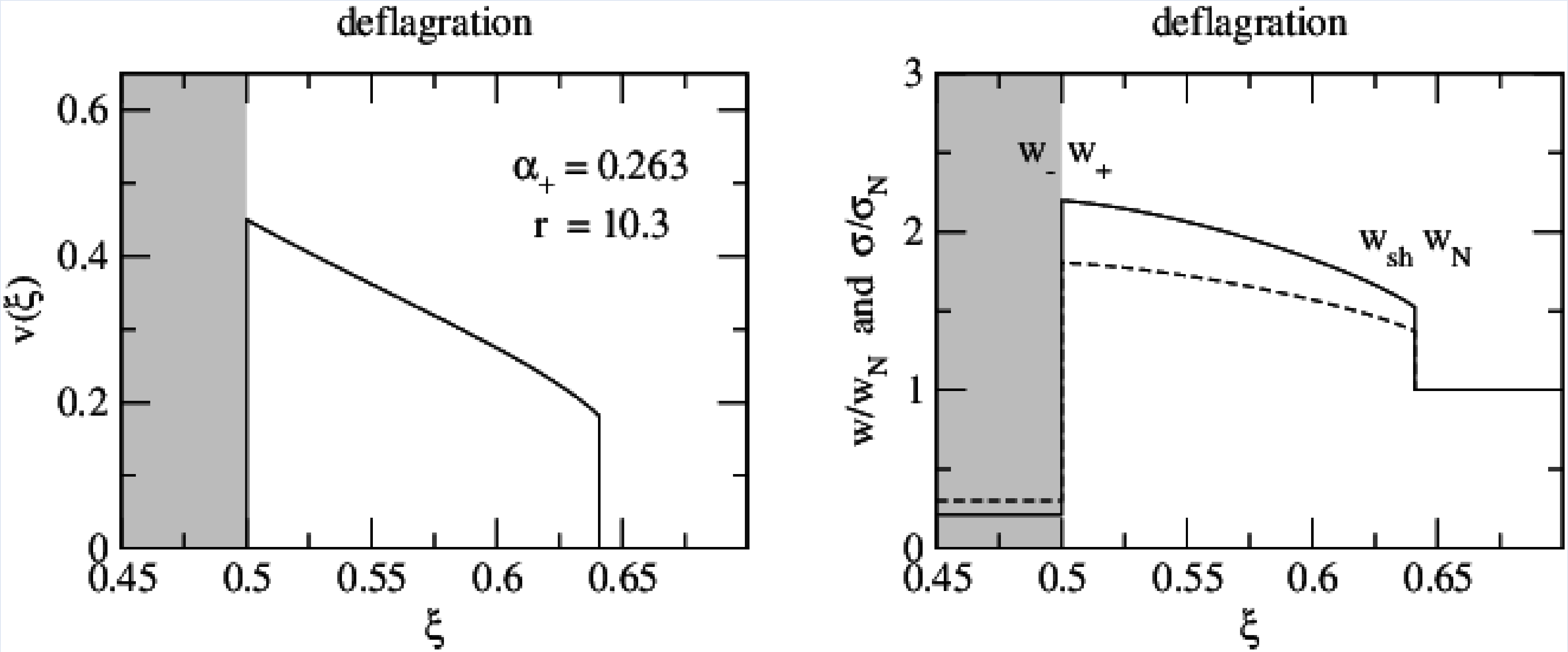


[Steinhardt '82] [Laine '93]
[Gyulassy, Kajantie, Kurki-Suonio, McLerran '84]
[J. Ignatius, K. Kajantie, H. Kurki-Suonio, M. Laine '93]

Deflagrations

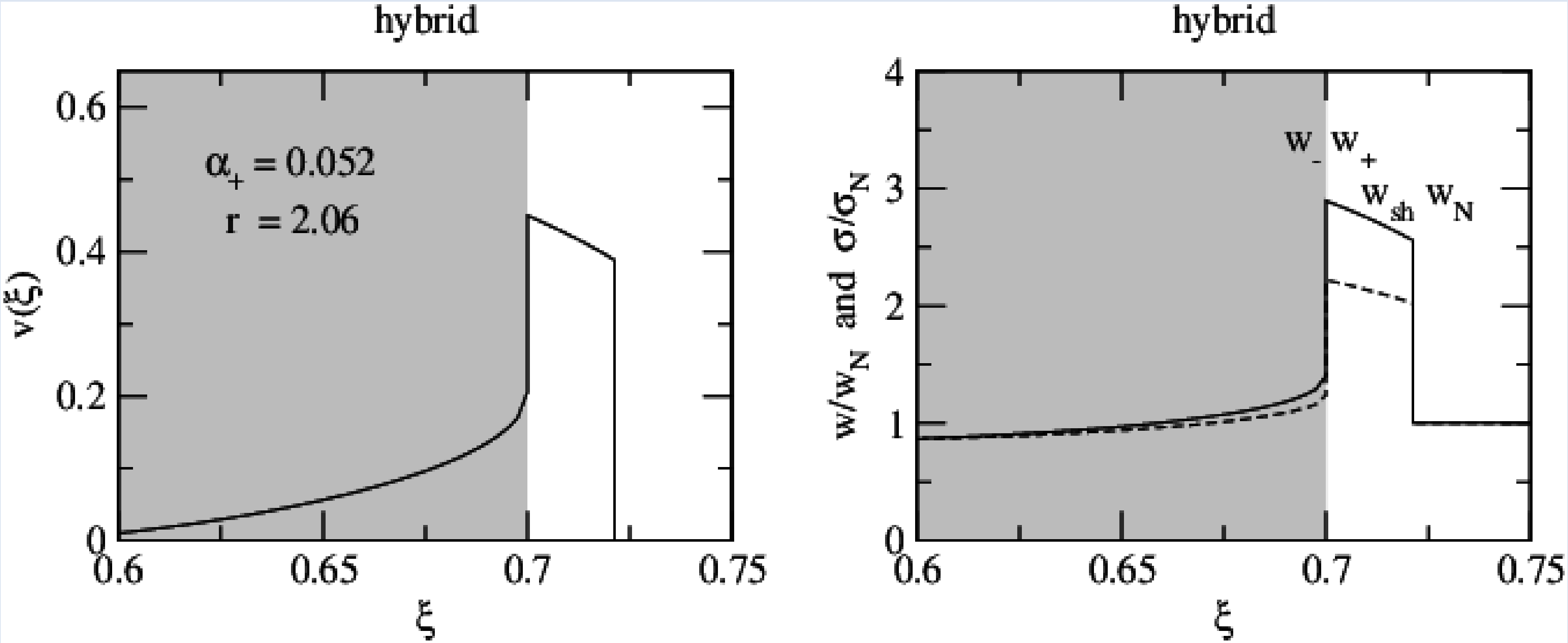
Deflagrations are **subsonic** expansion modes with a a shock wave and a shock front **in front** of the wall.

The shock front constitutes a discontinuity **without** phase change.



Hybrids

Supersonic expansion modes with **rarefaction wave and shock.**

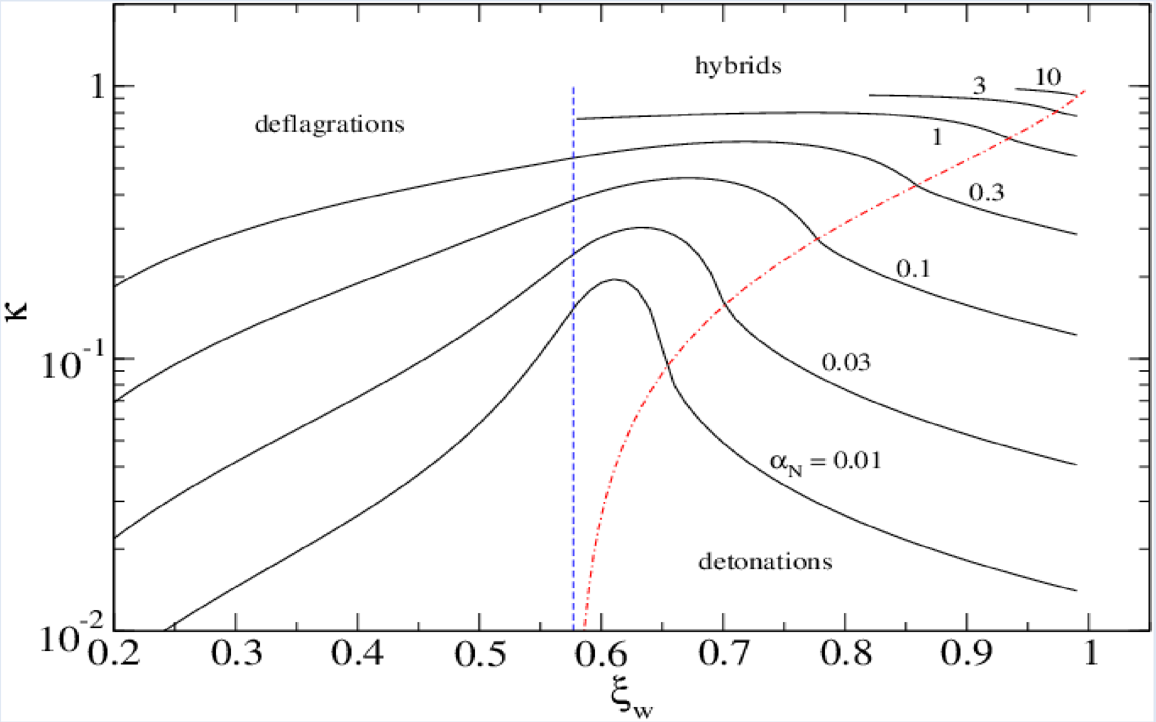


Energy budget

The hydro calculation is used for

> setting the boundary conditions of the friction calculation

> determining the energy budget: bulk kinetic motion vs. heating



$$\alpha = \epsilon / \rho_{thermal}$$

$$\kappa = \rho_{kinetic} / \epsilon$$

[Espinosa, TK, No, Servant '10]

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Or integrated in the wall frame

$$\Delta\mathcal{F} = \gamma v_{wall} \eta \int dz\, (\partial_z \phi)^2$$

depends on the scalar potential, latent heat

depends on particle content, W bsons, tops

But in a microscopic approach:

$$\text{friction} \quad \propto \quad \phi^4 \quad \not\propto \quad \gamma\, v_{wall} \quad \not\propto \quad 1/L_{wall}$$

Equation of Motion

The EoM of the plasma particles are the (relativistic) Kadanoff-Baym equations (neglecting the self-energy)

$$(p^2 - m^2)\, e^{i\diamond/2}\, G^<(p,x) = \text{coll.}$$

Moyal star product

expansion in gradients:

$$(p^2 - m^2)\, G^<(p,x) = 0$$

$$(p_\mu \partial^\mu + \frac{1}{2}\partial_\mu m^2 \partial_{p_\mu}) G^<(p,x) = \text{ coll}$$

Quasi particles

Using the quasi particle ansatz

$$G^{<}(p,x) = 2\pi\, f(\vec{p},x)\,\delta(p^2 - m^2)$$

leads to a Boltzmann type equation:

$$(p_\mu \partial^\mu + \frac{1}{2}\partial_\mu m^2 \partial_{p_\mu}) f(\vec{p},x) = \text{ coll}$$

$$m(\partial_t + \vec{v}\cdot\nabla) f$$

force term that will induce particle reflection at the wall → rarefaction/shock → latent heat

Notice that the mass is space-dependent through an implicit dependence from the Higgs vev

Higgs EoM

The Higgs equation of motion is obtained by conservation of the total energy-momentum tensor

$$\Box\phi + \frac{dV}{d\phi} + \sum_n \frac{dm_n^2}{d\phi} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E} f_n(\vec{p}, x) = 0$$

In equilibibrum this will reduce to

$$\Box\phi + \frac{d\mathcal{F}}{d\phi} = 0$$

which equilibrium?

and hence

$$\text{friction} = \sum_n \int d\phi \, \frac{dm_n^2}{d\phi} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E} \, \delta f_n(\vec{p}, x)$$

This agrees with the classical analysis of M&P.

Flow ansatz

To linearize the equations we use the flow ansatz

$$f = \frac{1}{e^X - 1} \qquad X = \frac{u_\mu p^\mu}{T}\left[1 - \frac{\delta T}{T} + \mu\right]$$

This should work if the plasma thermalizes locally. But we do not resort to small velocities.

We don't have to because e.g.

$$J^\mu = u^\mu \left(a + \frac{\delta T}{T}\, b + \mu\, c + O(\delta T/T, \mu)^2\right)$$

depend only on *m* and *T* but not the velocity

Particle content in the SM

EW gauge bosons $\mu_W \, , \, \delta T_W / T \, , \, v_W$

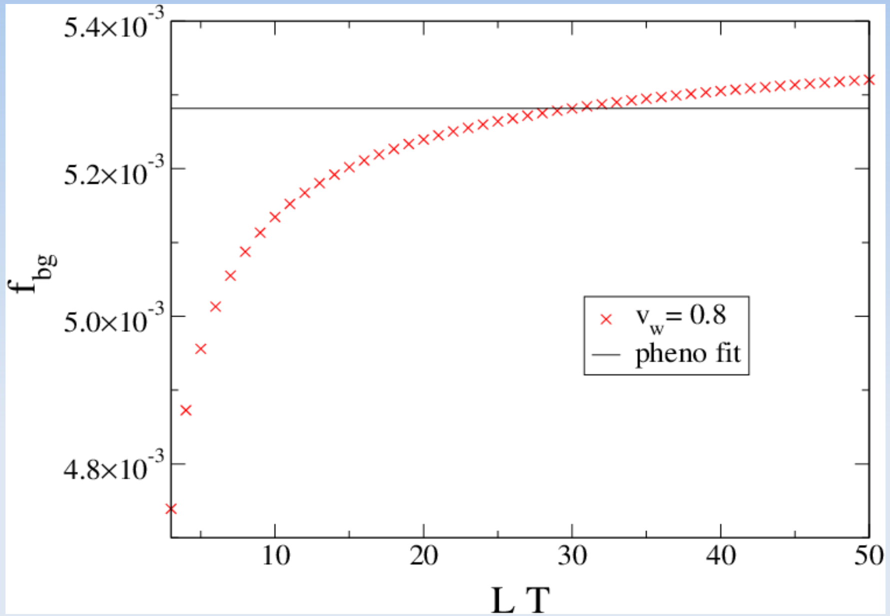
top quarks $\mu_t \, , \, \delta T_t / T \, , \, v_t$

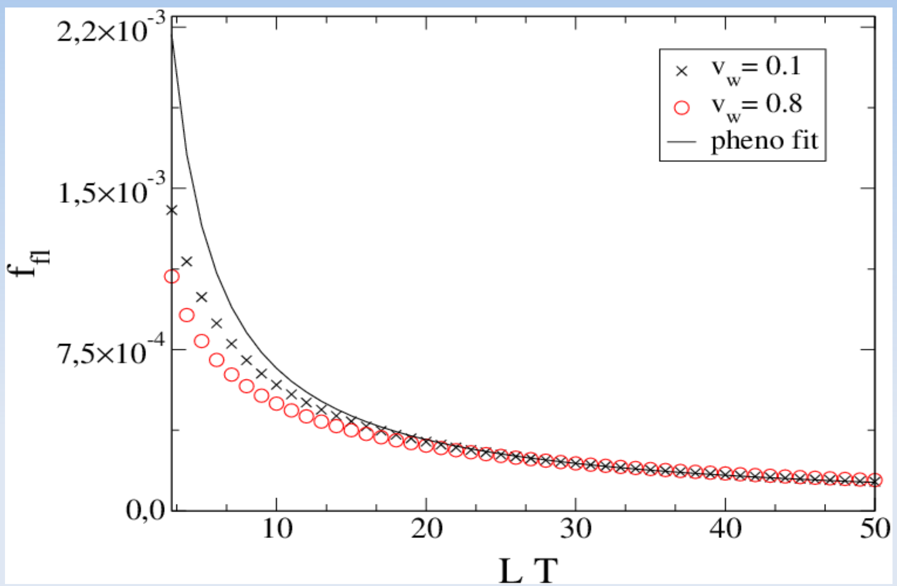
everything else
background $\mu_{bg} \, , \, \delta T_{bg} / T \, , \, v_{bg}$

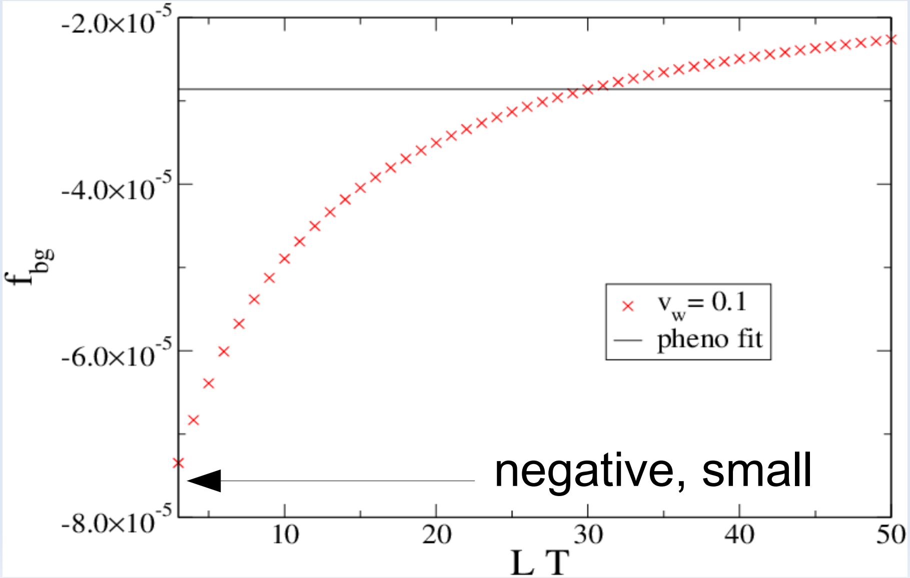
$$\Delta \mathcal{F} = \text{friction}_{W,t} + \text{friction}_{bg}$$

depends on ϕ, L_{wall}, v_{wall}

Numerical results







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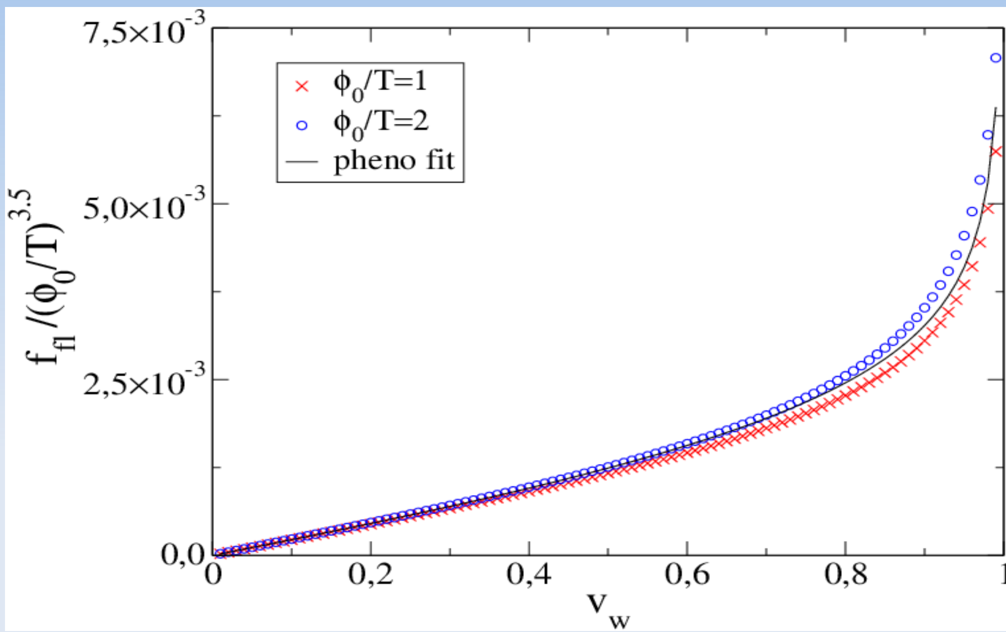
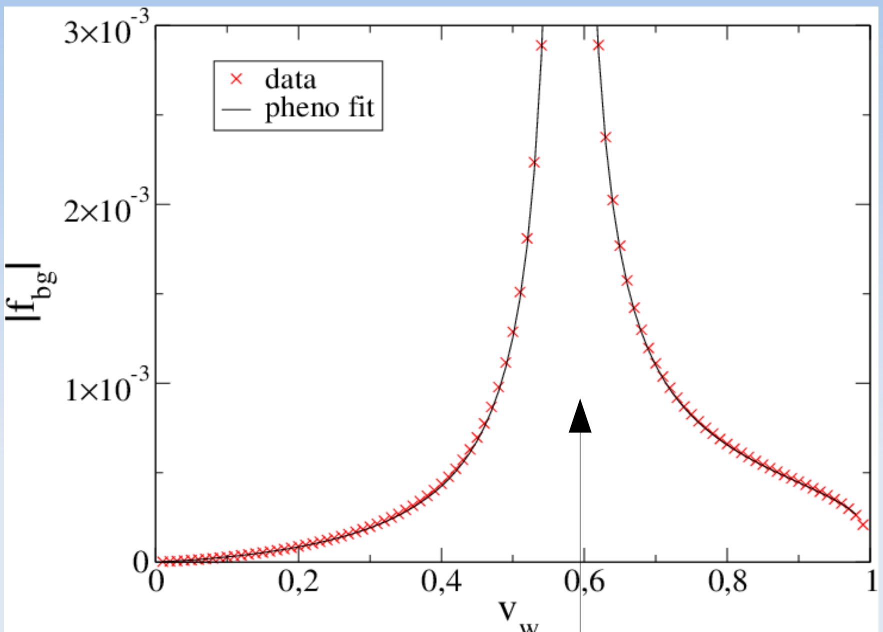




$\Gamma \gg 1/l_{wall}$ relaxation time approximation

$$\text{friction}_{W,t} \simeq \frac{\phi^{3.5}}{l_{wall}} \times \text{func}(v_{wall})$$

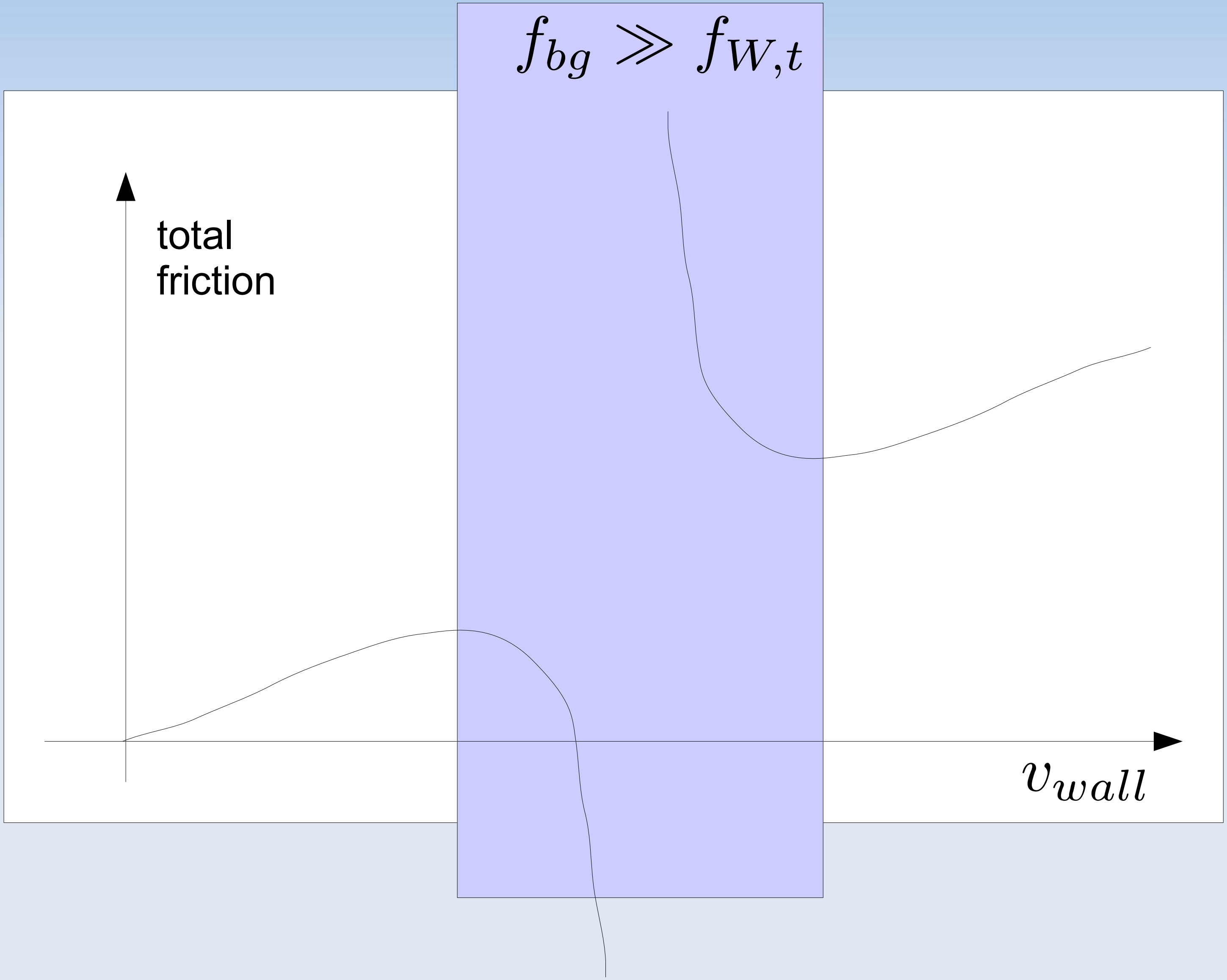
$$\text{friction}_{bg} \simeq \phi^4 \times \text{func}(v_{wall})$$

deflagrations → detonations

eigenvalues change sign

pole at
$v_{wall} \simeq c_s$

No hybrids



Conclusions

Microscopic friction calculations can be generalized to $v_{wall} \sim O(1) \not\sim c_s$

The dynamics of expanding bubble walls is **not very well** captured by phenomenological models for sizable velocities but fine for $v_{wall} \ll 1$

Hybrid solutions to hydrodynamics are disfavored by the friction analysis.

Moderately strong phase transitions $\alpha > 0.1$ proceed via detonations or even runaway.