Thermalization of mini-jets in a quark-gluon plasma

Edmond Iancu
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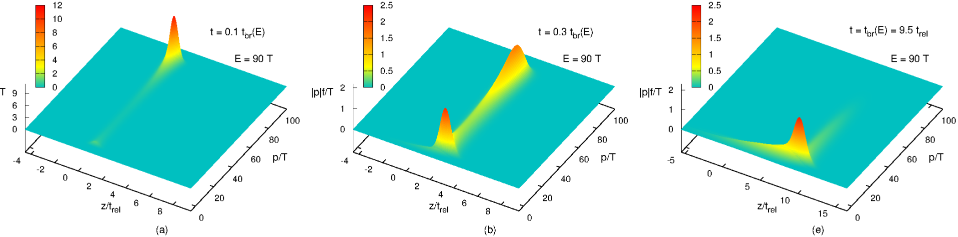
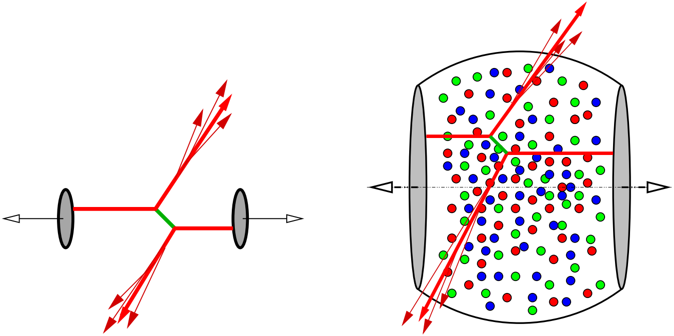
recent work with Bin Wu, arXiv:1506.07871 [hep-ph]

building upon previous work w/ J.-P. Blaizot, F. Dominguez, Y. Mehtar-Tani



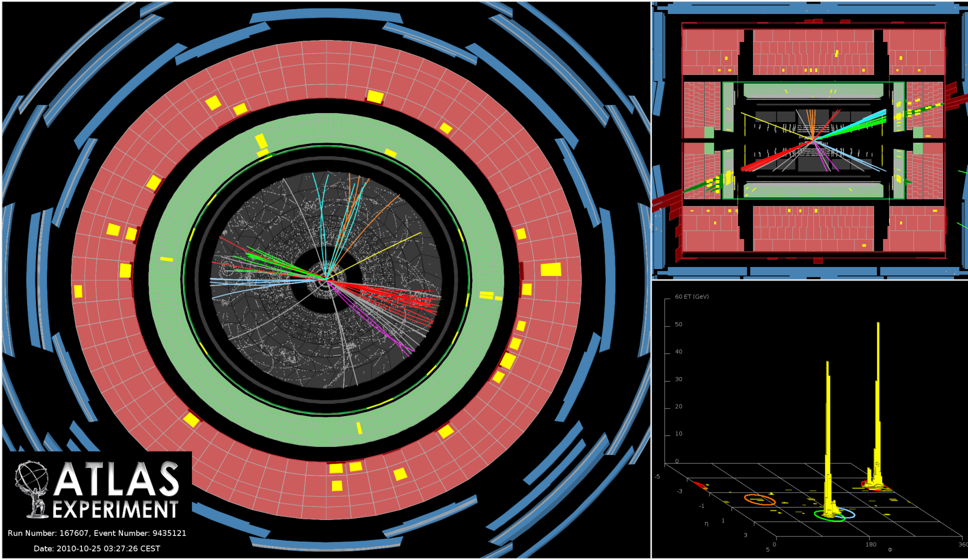
Jet quenching in heavy ion collisions

- Hard processes in QCD typically create pairs of partons which propagate back–to–back in the transverse plane
- In the vacuum, this leads to a pair of symmetric jets
- In a dense medium, the two jets can be differently affected by their interactions with the surrounding medium: 'di-jet asymmetry'

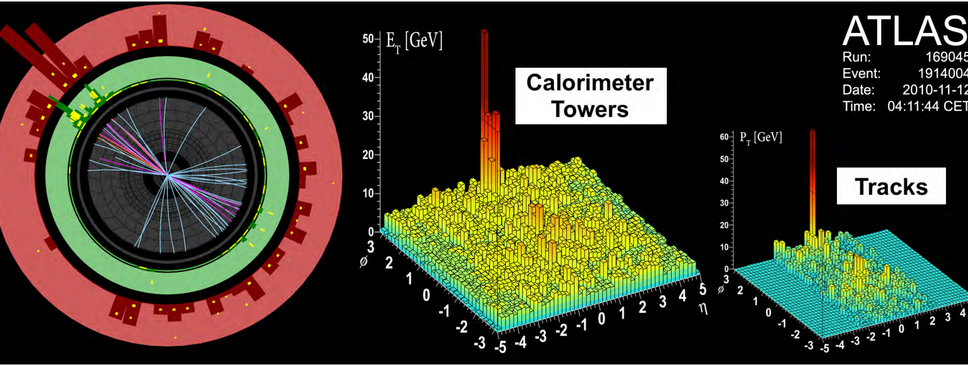


- The ensemble of medium-induced modifications: 'jet quenching'

Di–jets in p+p collisions at the LHC

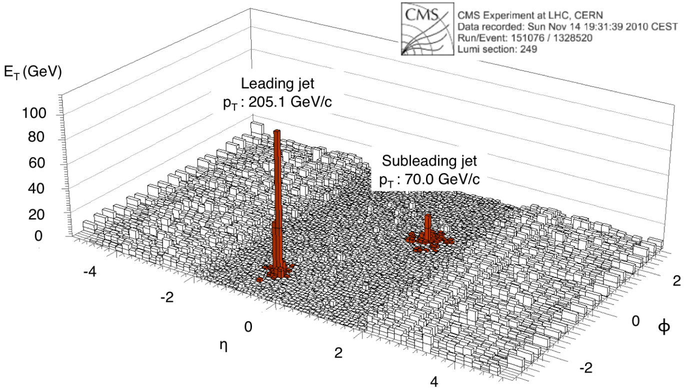


Di–jet asymmetry *(ATLAS)*



- Central Pb+Pb: 'mono–jet' events
- The secondary jet cannot be distinguished from the background: $E_{T1} \geq 100$ GeV, $E_{T2} > 25$ GeV

Di–jet asymmetry at the LHC *(CMS)*



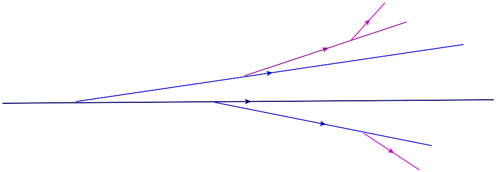
- Additional energy imbalance as compared to p+p : 20 to 30 GeV
- Compare to the typical scale in the medium: $T \sim 1$ GeV (average $p_\perp$)
- A remarkable pattern for the energy loss:

  many soft ($p_\perp < 2$ GeV) hadrons propagating at large angles

A challenge for the theorists

- Can one understand these data from first principles (QCD) ?
  - how gets the energy transmitted from the leading particle to these many soft quanta ?
  - do these soft quanta thermalize ? is the medium locally heated ?
  - is all that consistent with weak coupling ?

- Very different from the branching pattern for a jet in the vacuum



- quasi-collinear splittings
- pQCD: DGLAP equation
- energy carried by a few partons with large x
- energy remains within a narrow jet

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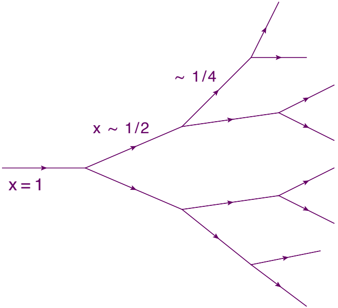
  - do these soft quanta thermalize ? is the medium locally heated ?
  - is all that consistent with weak coupling ?
- Medium-induced branching looks indeed very different !

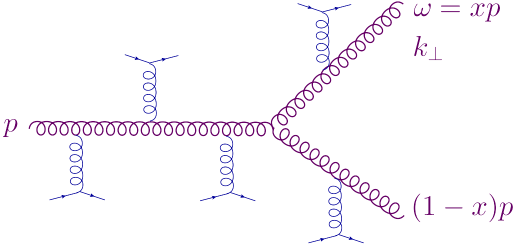


- multiple scattering $\Longrightarrow$ LPM effect
- quasi-democratic branchings
- wave turbulence: very efficient energy transmission from large x to small x
- efficient thermalization of the small-x quanta via collisions with the medium

Medium-induced gluon branching

Baier, Dokshitzer, Mueller, Peigné, and Schiff; Zakharov (96–97); Wiedemann (2000); Arnold, Moore, and Yaffe (2002–03); ...

- Incoming particle: an energetic gluon with $p = p_z \gg T$
- Multiple soft scattering $\Longrightarrow$ momentum broadening



$$\langle p_\perp^2 \rangle \simeq \hat{q}\Delta t$$

$$\langle \Delta p_z^2 \rangle \simeq \hat{q}_\ell \Delta t$$

$$\hat{q} \simeq \frac{m_D^2}{\lambda} \sim \alpha_s^2 T^3 \ln \frac{1}{\alpha_s}$$

- Transverse kicks provide acceleration and thus allow for radiation
- Gluon emissions require a formation time $t_\mathrm{f} \simeq \omega/k_\perp^2$
- During formation, the gluon acquires a momentum $k_\perp^2 \sim \hat{q} t_\mathrm{f}$

Formation time & emission angle

$$t_{\rm f}(\omega) \simeq \sqrt{\frac{\omega}{\hat q}} \qquad \& \qquad \theta_{\rm f}(\omega) \simeq \frac{\sqrt{\hat q t_{\rm f}}}{\omega} \sim \left(\frac{\hat q}{\omega^3}\right)^{1/4}$$

- This mechanism applies so long as

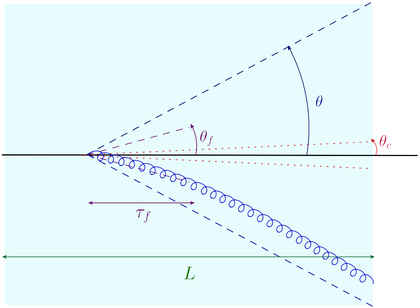
$$\lambda \ll t_{\rm f}(\omega) \le L \implies T \ll \omega \le \omega_c \equiv \hat q L^2$$

- Soft gluons : short formation times & large emission angles

$$\omega \ll \omega_c \implies t_{\rm f}(\omega) \ll L$$

$$\theta_{\rm f}(\omega) \gg \theta_c$$

$$\theta(\omega) \simeq \frac{\sqrt{\hat q L}}{\omega} \gg \theta_{\rm f}(\omega)$$



- The emission angle keeps increasing with time, via rescattering

Formation time & emission angle

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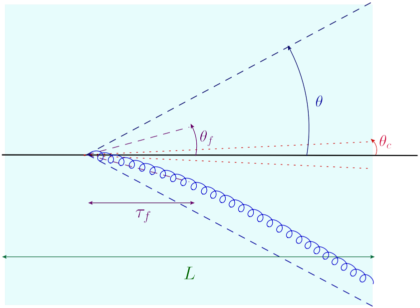
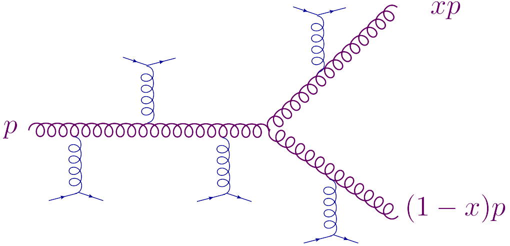


- Emissions can effectively be treated as collinear

Democratic branchings

- Probability for a branching $p \to \{xp,\ (1-x)p\}$ to occur during Δt



$$\Delta \mathcal{P} \sim \alpha_s \frac{\Delta t}{t_{\rm f}} \sim \alpha_s \sqrt{\frac{\hat{q}}{xp}}\,\Delta t$$

- LPM effect : the emission rate decreases with increasing $\omega = xp$
  - coherence: many collisions contribute to a single, hard, emission
- The probability becomes large for either ...
  - soft splittings : $x \ll 1$ (for generic p) ...
  - ... or democratic splittings ($x \sim 1/2$) but soft parent gluon (small p)

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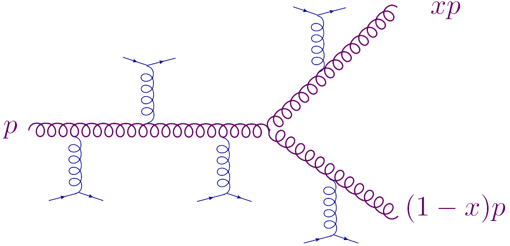
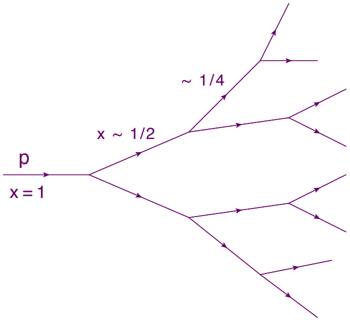


$$\Delta\mathcal{P} \sim \alpha_s \frac{\Delta t}{t_{\rm f}} \sim \alpha_s \sqrt{\frac{\hat{q}}{xp}}\, \Delta t$$

- Compare to bremsstrahlung in the vacuum : $\Delta\mathcal{P} \sim \alpha_s \ln(1/x)$
  - large probability for soft splittings alone
- Democratic branchings are known to be important at strong coupling *(Y. Hatta, E.I., Al Mueller '08)*
  - high density can mimic strong coupling

Democratic cascades

- When probability for one branching $\sim \mathcal{O}(1) \Rightarrow$ multiple branching
- Consider first quasi-democratic branchings : $x \sim 1/2$



$$\Delta\mathcal{P} \sim \alpha_s \, \Delta t \sqrt{\frac{\hat{q}}{p}}$$

$$\sim 1 \quad \text{for} \quad \Delta t \sim t_{\rm br}(p)$$

$$t_{\rm br}(p) = \frac{1}{\alpha_s}\sqrt{\frac{p}{\hat{q}}}$$

- $t_{\rm br}(p)$ = 'gluon lifetime' : the time it takes a gluon with energy p to disappear via a quasi-democratic branching
- Daughter gluons are softer, so they disappear even faster
  - energy transmitted to arbitrarily soft quanta during an interval $\sim t_{\rm br}(p)$

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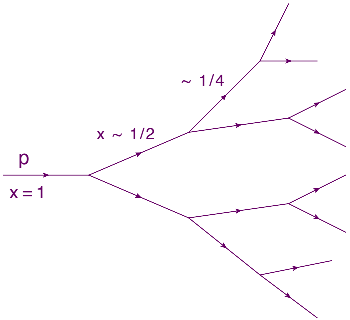

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- $t_{\rm br}(p) =$ the lifetime of the mini-jet initiated by p ('stopping time')

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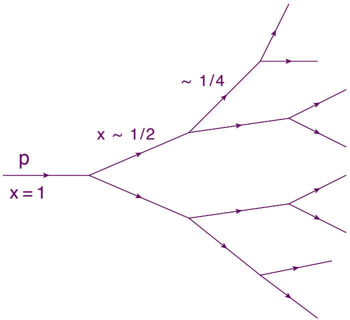

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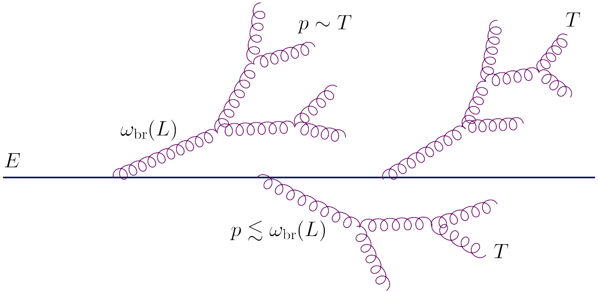
- $t_{\rm br}(p) =$ the lifetime of the mini-jet initiated by p ('stopping time')
- This requires $t_{\rm br}(p) \lesssim L \implies p \lesssim \omega_{\rm br}(L) \equiv \alpha_s^2 \hat{q} L^2$

Jet versus mini-jets

- At the LHC, the energy E of the leading particle is much higher

$$E \geq 100\,\text{GeV} \gg \omega_{\rm br} = \alpha_s^2 \hat{q} L^2 \simeq 10 \text{ GeV for } L = 5 \text{ fm}$$

  - the leading particle cannot suffer democratic branchings
  - it abundantly emits primary gluons with $p \lesssim \omega_{\rm br}(L)$



  - these primary gluons generate 'mini-jets' via democratic branchings
  - the energy carried by these mini-jets is deposited into the medium

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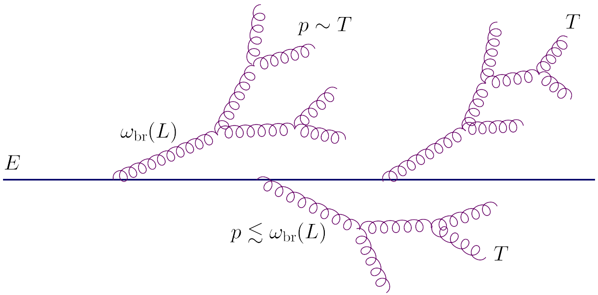
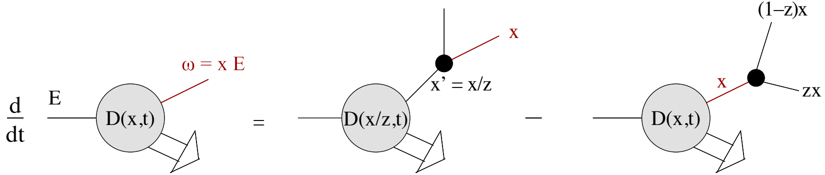


- How to transform this physical picture into explicit calculations ?

Kinetic theory: only branchings

- Multiple medium-induced gluon branching ≈ a Markovian process
  - non-trivial: potential interferences due to coherent emissions
  - coherence is efficiently washed out by scattering in the medium
    Mehtar-Tani, Salgado, Tywoniuk; Casalderrey-Solana, E. I. (10 –11)
    Blaizot, Dominguez, E.I., Mehtar-Tani (2012)
- Evolution equation for the gluon spectrum $D(x,t) \equiv x\frac{\mathrm{d}N}{\mathrm{d}x}$



$$\frac{\partial D(x,t)}{\partial t} = \frac{1}{t_{\rm br}(x)}\int \frac{\mathrm{d}z}{[z(1-z)]^{\frac{3}{2}}}\left[\sqrt{z}\,D\Big(\frac{x}{z},t\Big) - \frac{1}{2}\,D(x,t)\right]$$

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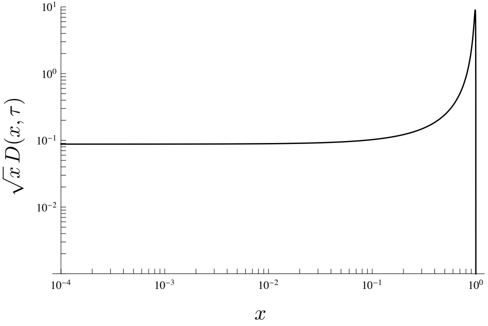
- Simplified version of the AMY equation *(Arnold, Moore, Yaffe, '03)*
  - previously used for studies of glasma thermalization
    Baier, Mueller, Schiff, Son '01 ('bottom-up'); Kurkela, Zhu, '15
  - ... and for the phenomenology of jet quenching:
    Schenke, Gale, Jeon, '09 (MARTINI)

Exact solution: the leading particle

J.-P. Blaizot, E. I., Y. Mehtar-Tani, PRL 111, 052001 (2013)

- Initial condition $D(x, t = 0) = \delta(x - 1)$ (that is, $\omega = E$)

$$D(x, \tau) = \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\, \mathrm{e}^{-\pi \frac{\tau^2}{1-x}}\,, \qquad \tau \equiv \frac{t}{t_{\rm br}(E)}$$



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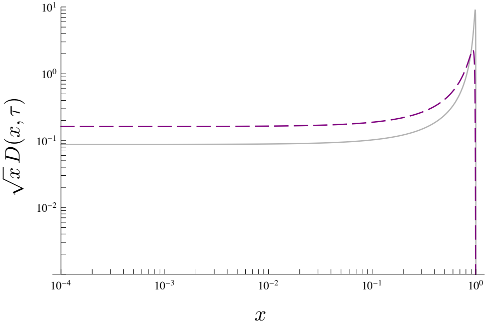
  - $x \simeq 1$ & $\tau \ll 1$: the broadening of the leading particle



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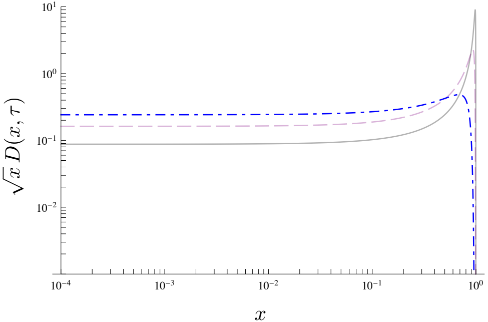
  - $x \simeq 1$ & $\tau \ll 1$: the broadening of the leading particle
  - when $\tau \sim 1$, the LP disappears via democratic branching



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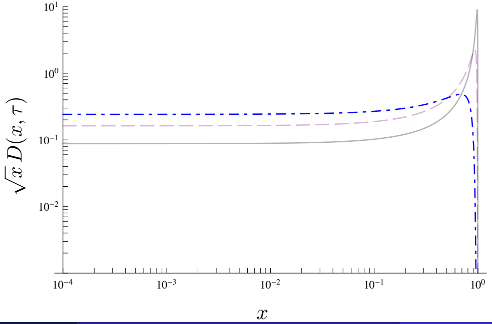
  - the case $\tau \ll 1$ is representative for the leading particle at the LHC
  - $\tau \gtrsim 1$ rather refers to a mini-jet



Exact solution: Wave turbulence

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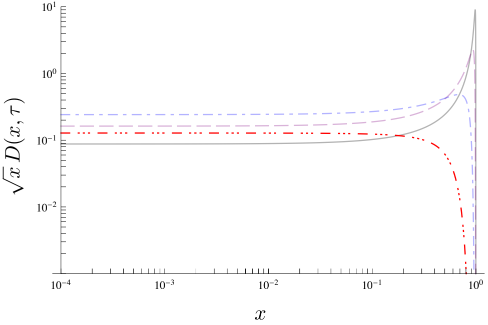
- $x \ll 1$: the same power spectrum as for a single emission

$$D(x,\tau) \simeq \frac{\tau}{\sqrt{x}}\, \mathrm{e}^{-\pi\tau^2} \qquad \text{for } x \ll 1$$

  - a fixed point of the branching term: Kolmogorov spectrum
  - with increasing τ, the spectrum is uniformly suppressed at any x



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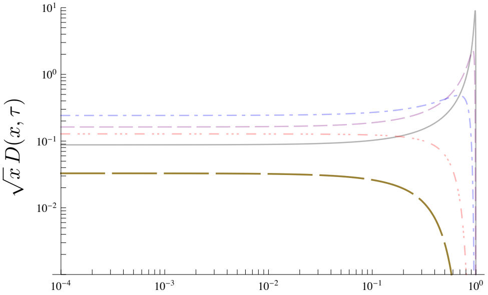
  - energy flows from large x to small x, w/o accumulating at any intermediate value of x : energy flux is independent of x
  - it (formally) accumulates into a condensate at $x = 0$



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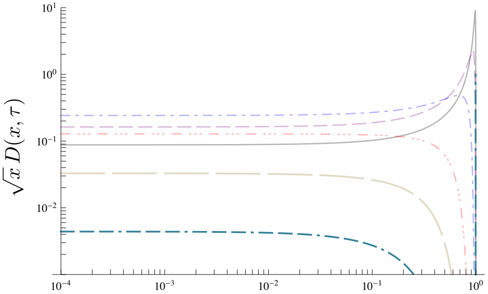


The thermalization problem

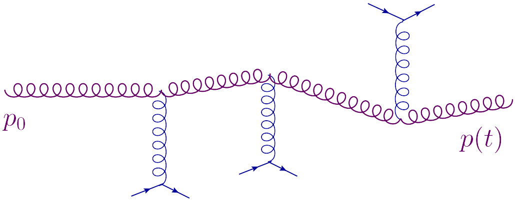
- This branching dynamics cannot extend all the way down to $x = 0$
  - when $p \sim T$, the formation time $t_{\mathrm{f}}(p)$ becomes comparable with the mean free path $\lambda \Longrightarrow$ no LPM suppression
  - gluons with $p \sim T$ can efficiently thermalize via collisions
  - recombination effects become important and stop the branching process
- Thermalization is not instantaneous: "medium is not a perfect sink"
  - the branching dynamics at $p > T$ can be modified as well
  - e.g. 'pile-up' around $p \sim T$ would destroy the power spectrum
- The full AMY equations (jet + medium) encode the relevant dynamics
  - ... but cumbersome to use in practice, even numerically
  - previous numerical applications assumed spatial homogeneity ...

    ... but the jet is highly localized to start with
- Understand the relevant time scales and write some simpler equations

  E.I. and Bin Wu, arXiv:1506.07871 [hep-ph]

Time scales: branching vs. thermalization

- Via elastic, $2 \to 2$ collisions, the particles loses energy ('drag') and broadens its momentum distribution ('diffusion')



$$\langle p_z(t) \rangle \simeq p_0 - \eta t$$

$$\langle \Delta p_z^2(t) \rangle \simeq \hat{q} t$$

$$\hat{q} = 4T\eta \quad \text{(thermal equil.)}$$

- Eventually, thermal equilibrium with the medium: $f_p \propto \mathrm{e}^{-p/T}$
- For hard gluons $(p_0 \gg T)$, collisional energy loss is not important

$$t_{\rm coll}(p_0) = \frac{p_0}{\eta} \gg t_{\rm br}(p_0) = \frac{1}{\alpha_s}\sqrt{\frac{p_0}{\hat{q}}}$$

  - hard gluons branch again before having the time to lose energy via drag

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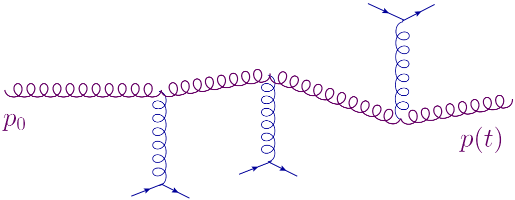

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- For soft gluons with $p \sim T$, the 2 processes compete with each other:

$$t_{\rm coll}(T) \sim t_{\rm br}(T) \sim t_{\rm rel} \equiv \frac{1}{\alpha_s^2 T \ln(1/\alpha_s)}$$

  - soft gluons thermalize within a typical time $\Delta t \sim t_{\rm rel}$

Kinetic theory: branchings + collisions

- When $p \gg T$, jet partons can be distinguished from medium ones
- Gluon system produced via branchings remains dilute: $f(t, \boldsymbol{x}, \boldsymbol{p}) \ll 1$
- The most interesting dynamics is the longitudinal one $(z,\, p_z)$
  - $p_z \gg p_\perp$ so long as $p \gg T \Longrightarrow p_z$ controls the branchings
  - the strongest inhomogeneity is in z: $f(t, z, p_z) \equiv \frac{\mathrm{d}N}{\mathrm{d}z\mathrm{d}p_z} \propto \delta(t-z)$
- Small angle scattering can be described by Fokker-Planck (leading-log)
- Linear equation for longitudinal gluon distribution produced by the jet

$$\left(\frac{\partial}{\partial t} + v_z \frac{\partial}{\partial z}\right) f(t, z, p_z) = \frac{\hat{q}}{4} \frac{\partial}{\partial p_z} \left[\left(\frac{\partial}{\partial p_z} + \frac{v_z}{T}\right) f\right]$$

$$+ \frac{1}{t_{\rm br}(p_z)} \int_{p_*} \mathrm{d}x\, \mathcal{K}(x) \left[\frac{1}{\sqrt{x}} f\left(\frac{p_z}{x}\right) - \frac{1}{2} f(p_z)\right]$$

  - drift, elastic collisions (drag + diffusion), collinear branchings

Kinetic theory: branchings + collisions

- When $p \gg T$, jet partons can be distinguished from medium ones
- Gluon system produced via branchings remains dilute: $f(t, \boldsymbol{x}, \boldsymbol{p}) \ll 1$
- The most interesting dynamics is the longitudinal one $(z,\, p_z)$
  - $p_z \gg p_\perp$ so long as $p \gg T \Longrightarrow p_z$ controls the branchings
  - the strongest inhomogeneity is in z: $f(t, z, p_z) \equiv \frac{\mathrm{d}N}{\mathrm{d}z\mathrm{d}p_z} \propto \delta(t-z)$
- Small angle scattering can be described by Fokker-Planck (leading-log)
- Linear equation for longitudinal gluon distribution produced by the jet

$$\left(\frac{\partial}{\partial t} + v_z \frac{\partial}{\partial z}\right) f(t, z, p_z) = \frac{\hat{q}}{4} \frac{\partial}{\partial p_z}\left[\left(\frac{\partial}{\partial p_z} + \frac{v_z}{T}\right) f\right]$$

$$+ \frac{1}{t_{\rm br}(p_z)} \int_{p_*} \mathrm{d}x\, \mathcal{K}(x) \left[\frac{1}{\sqrt{x}} f\left(\frac{p_z}{x}\right) - \frac{1}{2} f(p_z)\right]$$

  - branching process cut off by hand at a scale $p_* \sim T$

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  - strictly valid for $p_z \gg T$, parametric extrapolation down to $p_z \sim T$

The 'steady source' approximation

- The ultrarelativistic FP equation in $D = 1 + 1$ can be solved exactly

  (E.I. and Bin Wu, arXiv:1506.07871; not previously recognized)

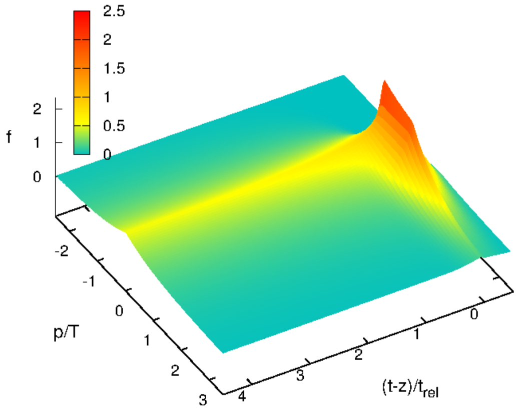
- The branching process $\approx$ a source of gluons with $p \sim T$
  - gluons with $p \gg T$ are not significantly affected by collisions
  - the medium is implicitly treated as a 'perfect sink'

- The steady source approximation : a realistic 'toy-model'

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z}\right) f = \frac{\hat{q}}{4}\frac{\partial}{\partial p}\left[\left(\frac{\partial}{\partial p} + \frac{v}{T}\right) f\right] + \delta(t-z)\delta(p-p_*)$$

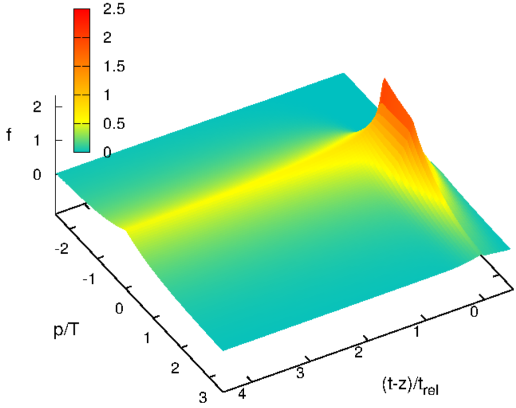
  - $p_* \sim T$: the lower end of the cascade
  - thermalization of the soft gluons with $p \sim T$ is much faster than the internal evolution of the cascade: $t_{\rm rel} \ll t_{\rm br}(E)$ for $E \gg T$
  - representative for a high-energy jet at the LHC : $L \ll t_{\rm br}(E)$

The front & the tail



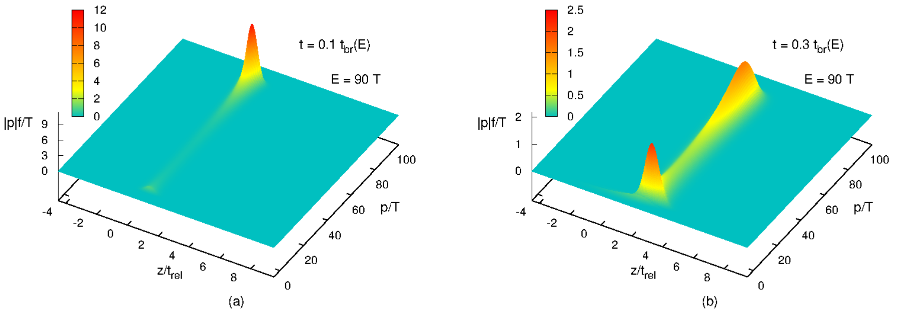
- The 'front' $\propto \delta(t-z)$:
  - ▷ gluons recently injected that had no time to thermalize
- The 'tail' at $z \lesssim t - t_{\rm rel}$: $f_p \propto {\rm e}^{-|p|/T}$
  - ▷ gluons in thermal equilibrium with the medium

The front & the tail



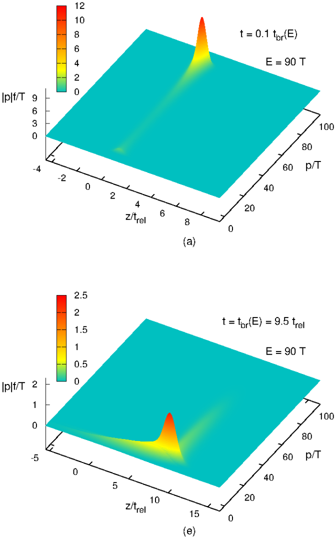
- After a sufficiently large time $t \gg t_{\rm rel}$, most of the energy injected by the source is found in the thermalized tail
- The energy lost by the jet is carried by soft, thermal, quanta

$$T = 0.5 \text{ GeV}\,,\ \ t_{\rm rel} = 1 \text{ fm}\,,\ \ E\ =\ 90\,T\,,\ \ t_{\rm br}(E) \simeq 9.5 \text{ fm}$$



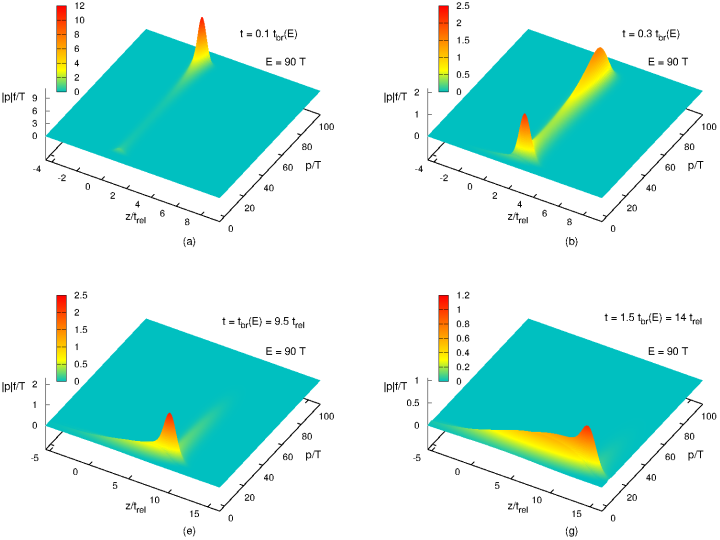
- $t = 0.1\, t_{\rm br}(E) \simeq 1$ fm is representative for the 'leading jet' at the LHC
- $t = 0.3\, t_{\rm br}(E) \simeq 3$ fm : the 'subleading jet' (partially quenched)
  - a second peak emerges near $p = T$ (branchings)
  - this second peak develops a thermalized tail at $z < t$ (collisions)
- The jet substructure is softening and broadening

Numerical studies of the full dynamics (2)



- $t = t_{\rm br}(E)$: the leading particle disappears (democratic branching)

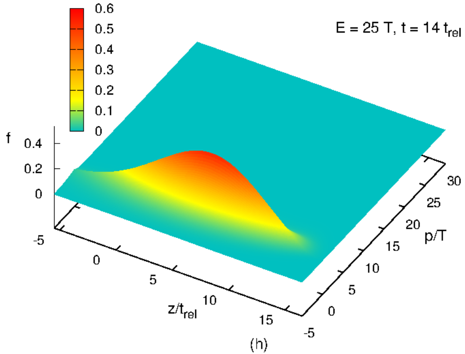
Numerical studies of the full dynamics (2)

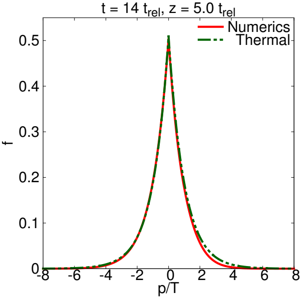


- $t = 1.5\, t_{\rm br}(E) \simeq 15$ fm : very large medium (almost fully quenched)

The late stages: thermalizing a mini-jet

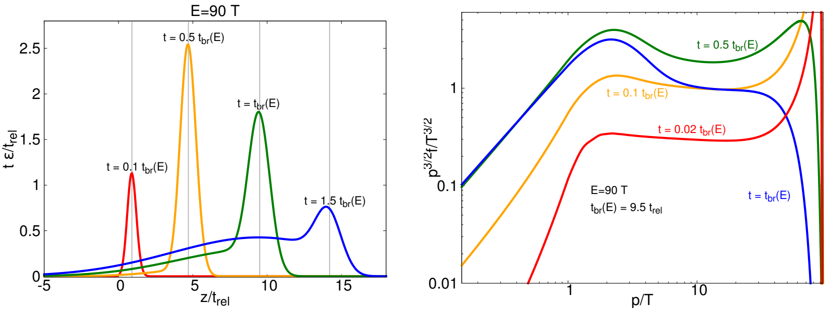
- At late times $t \gg t_{\rm br}(E)$, the jet is 'fully quenched'
  - no trace of the leading particle, just a thermalized tail
  - the typical situation for a mini-jet : $E \lesssim \omega_{\rm br}(L)$





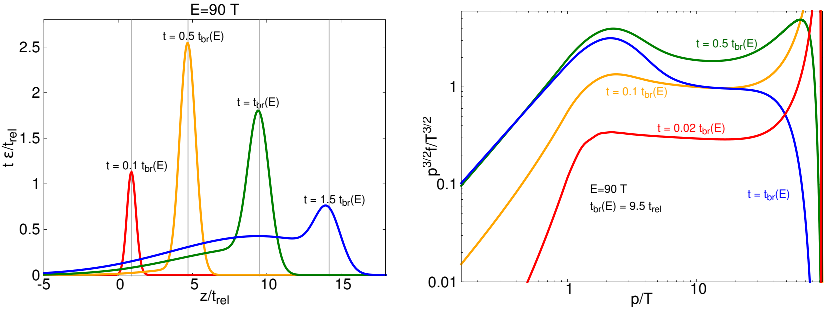
$$f(t,z,p) \simeq \mathrm{e}^{-\frac{|p|}{T}}\,\mathrm{e}^{-\frac{z^2}{4tt_{\rm rel}}}$$ (spatial diffusion $\Longrightarrow$ hydro)

Energy distribution



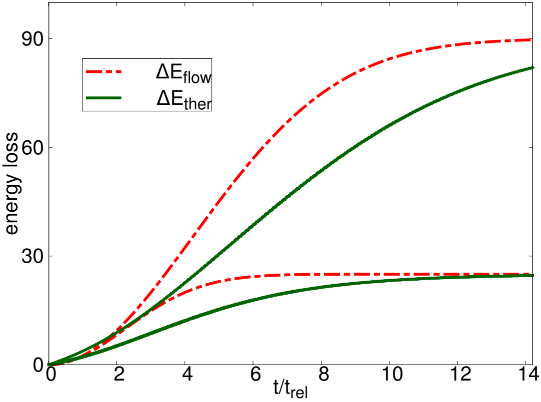
- Left: z-distribution of the energy density $\varepsilon(t, z) = \int \mathrm{d}p\,|p|\,f(t, z, p)$
  - even for $t = t_{\rm br}(E)$, most of the energy is still carried by the front ...
  - but the respective 'front' is mostly made with soft gluons $(p \sim T)$
  - branching products which did not yet have the time to thermalize
  - a thermalized tail is clearly visible at $z < t$

Energy distribution



- Left: z-distribution of the energy density $\varepsilon(t,z) = \int \mathrm{d}p\,|p|\,f(t,z,p)$
- Right: p-distribution near the front: $f(t,z,p)$ for $z=t$
  - scaling window at $t \ll t_{\rm br}(E)$: $p^{3/2} f \approx$ const. $\Longrightarrow$ wave turbulence
  - for $t \gtrsim 0.5\, t_{\rm br}(E)$, some pile-up is visible around $p=T$
  - thermalization is efficient but certainly not instantaneous

Energy loss towards the medium

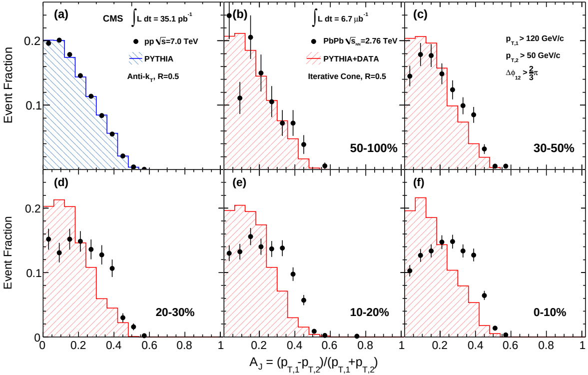


- Upper curves: $E = 90\,T$; lower curves: $E = 25\,T$
- The energy carried by the thermalized tail is ...
  - somewhat smaller than the 'flow' energy: medium is not a 'perfect sink'
  - ... but still substantial: $6 \div 20$ GeV for $L = 3 \div 6$ fm

Conclusions

- A simple, yet detailed, 2-step, picture for jet quenching from pQCD
  - copious production of 'mini-jets' via democratic branchings
  - thermalization of the soft branching products with $p \sim T$
- Characteristic, 'front + tail', structure of the (partially) quenched jet
  - 'front' : the leading particle, but also soft gluons radiated at late times
  - 'tail' : soft partons in local thermal equilibrium with the medium
- Energy loss by the jet towards the medium is identified as the energy carried by the thermalized tail.
- Very robust: based on separation of timescales between branching of hard partons and thermalization of their soft products
- Many approximations, need for more detailed/precise studies: coupled evolution of jet + medium in D= 1 + 3 + 3

Di–jet asymmetry : $A_{\rm J}$ (CMS)

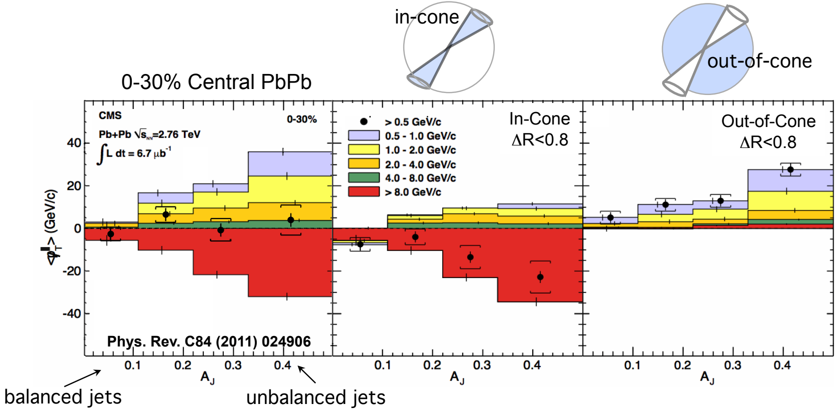


- Event fraction as a function of the di–jet energy imbalance in p+p (a) and Pb+Pb (b–f) collisions for different bins of centrality

$$A_{\rm J} = \frac{E_1 - E_2}{E_1 + E_2} \qquad (E_i \equiv p_{T,i} = \text{ jet energies})$$

- N.B. A pronounced asymmetry already in the p+p collisions !

Energy imbalance @ large angles: $R = 0.8$



- No missing energy : $E_{\rm Lead}^{\rm in+out} = E_{\rm SubLead}^{\rm in+out}$
- In-Cone : $E_{\rm Lead}^{\rm in} > E_{\rm SubLead}^{\rm in}$: di-jet asymmetry, hard particles
- Out-of-Cone : $E_{\rm Lead}^{\rm out} < E_{\rm SubLead}^{\rm out}$: soft hadrons @ large angles

An exact solution to the 1-D FP equation

- The Green's function for ultrarelativistic FP in D=1+1+1

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial z}\right) f = \frac{\hat{q}}{4}\,\frac{\partial}{\partial p}\left(\frac{\partial}{\partial p} + \frac{v}{T}\right) f\,, \qquad f(t=0,z,p) = \delta(z)\delta(p-p_0)$$

$$f(t,z,p>0) \;= \frac{\mathrm{e}^{-\frac{p_0-p}{2}-\frac{t}{4}}}{2\sqrt{\pi t}}\left[\mathrm{e}^{-\frac{(p-p_0)^2}{4t}} - \mathrm{e}^{-\frac{(p+p_0)^2}{4t}}\right]\delta(t-z)$$

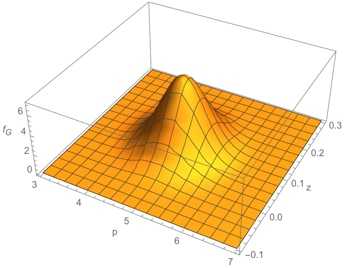
$$+\;\frac{\mathrm{e}^{-\frac{(p+p_0-z)^2}{4t}-p}}{8\sqrt{\pi}t^{5/2}}\left[t(t+2)-(p+p_0-z)^2\right]\times$$

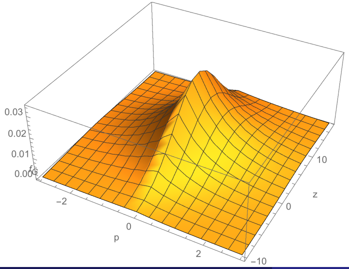
$$\times\;\mathrm{erfc}\left(\frac{1}{2}\sqrt{t-\frac{z^2}{t}}\left(\frac{p+p_0}{t+z}-1\right)\right)$$

$$+\;\frac{(t+z)(p+p_0+t-z)}{4\pi t^2\sqrt{t^2-z^2}}\,\mathrm{e}^{-\frac{(p+p_0)^2}{2(t+z)}+\frac{p_0-p}{2}-\frac{t}{4}}$$

Limiting behaviors

- Physics transparent at both small and large times: $t_{\rm coll}(p_0) = \frac{p_0}{T}\, t_{\rm rel}$





- small times: $t_{\rm rel} \lesssim t \ll t_{\rm coll}(p_0)$

$$f(t,z,p) \propto \mathrm{e}^{-\frac{(p-\langle p(t)\rangle)^2}{\hat{q}t}}\, \delta(t-z)$$

$$\langle p(t)\rangle = p_0 - \frac{t}{t_{\rm rel}}\, T$$

- energy loss & diffusion

- large times: $t > t_{\rm coll}(p_0) \gg t_{\rm rel}$

$$f \simeq \mathrm{e}^{-\frac{|p|}{T}}\, \exp\left\{-\frac{\left(z-(p_0/T)t_{\rm rel}\right)^2}{4tt_{\rm rel}}\right\}$$

- equilibrium & spatial diffusion

- plots: $p_0 = 5T$, $t_1 = t_{\rm rel}$, $t_2 = 20t_{\rm rel}$