Consistent kinetic equations for leptogenesis?

Andreas Hohenegger

University of Stavanger

Equilibration Mechanisms in Weakly and Strongly Coupled Quantum Field Theory
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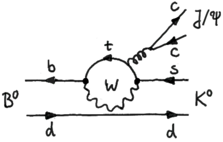
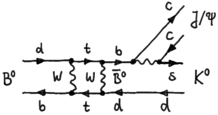
Baryon Asymmetry

$$\eta_B = \frac{n_b - n_{\bar{b}}}{n_\gamma} \simeq 10^{-9}$$

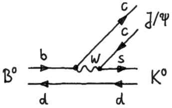
- CP-violation through



direct contribution (Penguin)



indirect contribution (Box)

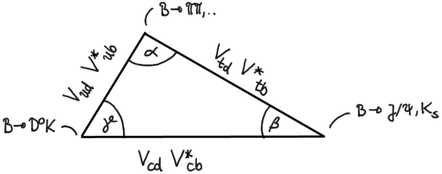


interference with Tree

- CP-violating observables

$$\frac{\Gamma_{P\to f} - \Gamma_{\bar{P}\to\bar{f}}}{\Gamma_{P\to f} + \Gamma_{\bar{P}\to\bar{f}}} \simeq \epsilon + \epsilon' \neq 0$$

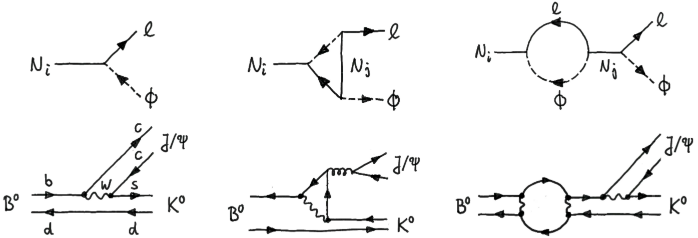
- Unitarity triangle and Jarlskog invariant (determinant)



$$J = \prod_{\substack{i>j \\ u,c,t;d,s,b}} (m_i^2 - m_j^2)\, \mathrm{Im}\{V_{ud} V_{cb} V_{ub}^* V_{cd}^*\}$$

CP-violation in Meson Decays

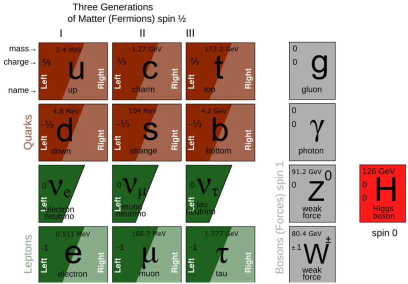
- Idea: Use it to explain matter-antimatter asymmetry
  - Meson decays do not work (no $\not{B}$; freeze-out too late)
- New heavy particles N_i, weakly interacting through Yukawa coupling
  - CP-violation in N_i decays at 1-loop level through interference with tree graph

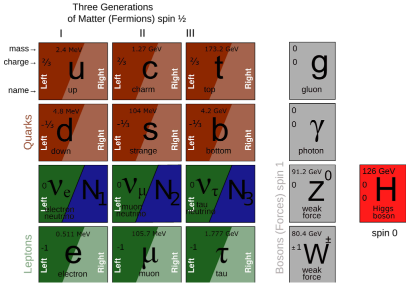


Penguin→Vertex

Box→Self-energy/Wave

RHN Extensions of Standard Model





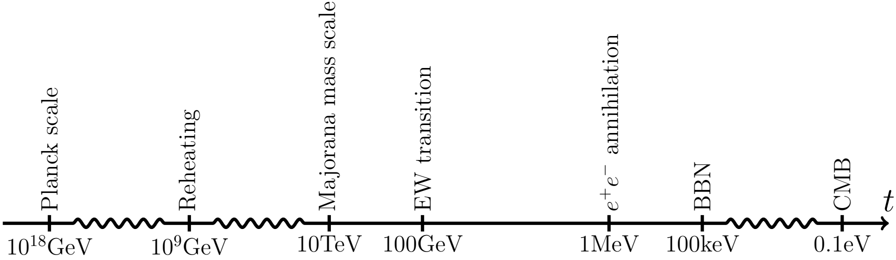
- Standard Model $\mathcal{L}_{SM}$ shortcomings
  - neutrino oscillations, dark matter, couple of 'anomalies', baryon asymmetry
  - hierarchy between weak and Planck scale, strong CP problem, flavor structure . . .
- Standard Model + 3RHN

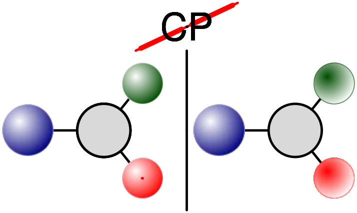
$$\mathcal{L}_{SM} + \frac{1}{2}\bar{N}_i\left(i\partial\!\!\!/ - M_i\right)N_i - h_{\alpha i}\bar{\ell}_\alpha\tilde{\phi}P_R N_i - h^\dagger_{i\alpha}\bar{N}_i\tilde{\phi}^\dagger P_L \ell_\alpha$$

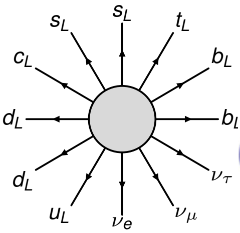
neutrino masses, dark matter possible, multiple *CP*-violating phases in weak sector characterized by invariants J

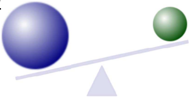
RHN Extensions of Standard Model

- possible thermal history of the universe





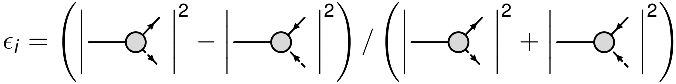




Baryon Asymmetry

$$\eta_B \simeq \frac{a_{Sphaleron}}{f} \frac{n_\ell - n_{\bar{\ell}}}{n_\gamma}$$

$$\frac{n_\ell - n_{\bar{\ell}}}{n_\gamma} \simeq \kappa\epsilon_1$$



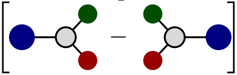
Consistent Kinetic Equations

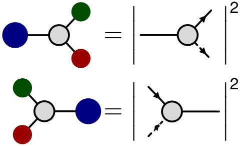
- Need
  - Non-negative number-densities/distribution functions
  - Conserved currents
  - Conserved energy-momentum
  - Approach to equilibrium (H-theorem)
  - No asymmetry without CP-violation in Lagrangian (Second Sakharov condition)
  - No asymmetry in equilibrium (Third Sakharov condition)
- Want/May need
  - *CP*-violating phases in amplitudes
  - Off-shell quantum effects
  - Mixing of heavy N_i
  - Flavour and their mixing

Textbook Kinetic Theory

$$(n_\ell - n_{\bar\ell}) = \int \frac{d^3k}{(2\pi)^3 E_k^\ell}\left[C^{\ell\phi\leftrightarrow N_i}(k) - C^{\bar\ell\bar\phi\leftrightarrow N_i}(k)\right]$$

$$C^{\ell\phi\leftrightarrow N_i}(k) = \frac{1}{2}\int d\Pi_p^{\phi}\, d\Pi_q^{N_i}(2\pi)^4\delta(k+p-q)$$

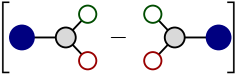




$$(1 - f_k^\ell)(1 + f_p^\phi) f_q^{N_i}$$

$$f_k^\ell f_p^\phi (1 - f_q^{N_i})$$

$$C^{\bar\ell\bar\phi\leftrightarrow N_i}(k) = \frac{1}{2}\int d\Pi_p^{\phi}\, d\Pi_q^{N_i}(2\pi)^4\delta(k+p-q)$$



Non-equilibrium Quantum Field Theory

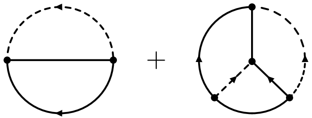
- Advances made in non-equilibrium QFT

W. Buchmüller and S. Fredenhagen, [hep-ph/0004145]; A. De Simone and A. Riotto, [hep-ph/0703175]; A. De Simone and A. Riotto, [hep-ph/0705.2183]; V. Cirigliano, A. De Simone, G. Isidori, I. Masina and A. Riotto, [hep-ph/0711.0778]; A. Anisimov, W. Buchmüller, M. Drewes and S. Mendizabal, [hep-th/0812.1934]; M. Garny, A. Hohenegger, A. Kartavtsev and M. Lindner, [0909.1559]; M. Garny, A. Hohenegger, A. Kartavtsev and M. Lindner, [hep-ph/0911.4122]; V. Cirigliano, C. Lee, M. J. Ramsey-Musolf and S. Tulin, [hep-ph/0912.3523]; A. Anisimov, W. Buchmüller, M. Drewes and S. Mendizabal, [hep-ph/1001.3856]; M. Garny, A. Hohenegger and A. Kartavtsev, [hep-ph/1002.0331]; M. Beneke, B. Garbrecht, M. Herranen and P. Schwaller, [hep-ph/1002.1326]; M. Beneke, B. Garbrecht, C. Fidler, M. Herranen and P. Schwaller, [hep-ph/1007.4783]; B. Garbrecht, [hep-ph/1011.3122]; A. Anisimov, W. Buchmüller, M. Drewes and S. Mendizabal, [hep-ph/1012.5821]; B. Garbrecht and M. Herranen, [hep-ph/1112.5954]; M. Garny, A. Kartavtsev and A. Hohenegger, [hep-ph/1112.6428]; M. Drewes and B. Garbrecht, [hep-ph/1206.5537]; B. Garbrecht, [hep-ph/1210.0553]; T. Frossard, M. Garny, A. Hohenegger, A. Kartavtsev and D. Mitrouskas, [hep-ph/1211.2140]; B. Garbrecht and M. J. Ramsey-Musolf, [hep-ph/1307.0524]; A. Hohenegger and A. Kartavtsev, [hep-ph/1309.1385]; S. Iso, K. Shimada and M. Yamanaka, [hep-ph/1312.7680]; S. Iso and K. Shimada, [hep-ph/1404.4816]; A. Hohenegger and A. Kartavtsev, [hep-ph/1404.5309]; B. Garbrecht, F. Gautier and J. Klaric, [hep-ph/1406.4190]; T. Frossard, A. Kartavtsev and D. Mitrouskas, [hep-ph/1304.1719].

$$\mathcal{D}^{\mu} j^{L}_{\mu}(x) = -\lim_{y\to x}(\mathcal{D}_x + \mathcal{D}_y) D_F(x,y)$$

$$D^{-1}(x,y) = D_0{}^{-1}(x,y) - \Sigma(x,y)$$

$$\Sigma(x,y) = 2i\frac{\delta\Gamma_2}{\delta D(y,x)}$$

$\Gamma_2 \quad =$ 

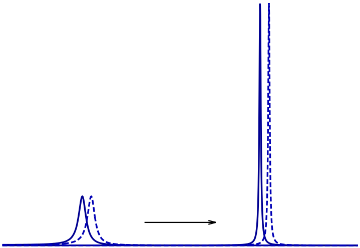
$$\hat{G}^{-1}(x,y) = \hat{G}_0{}^{-1}(x,y) - \hat{\Pi}(x,y)$$

(similar for ϕ)

- Gradient expansion (slow scales are H and Γ_i)
- Kadanoff-Baym ansatz and quasi-particle approximation for ℓ (and ϕ)

$$D_\rho(p) = (2\pi)\,\mathrm{sign}(p_0)\delta(p^2 - m^2)\,,$$
$$D_F(p) = [1 \pm f^\ell(p)]D_\rho(p)$$

- quasi-particle approximation for N_i problematic



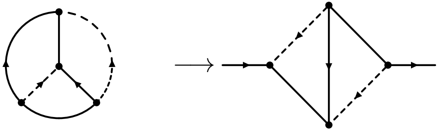
- diagonal approximation

$$G_\rho^{ij}(t,q) = \delta^{ij}\,(2\pi)\,\mathrm{sign}(p^0)\,\delta(p^2 - M_i^2)$$

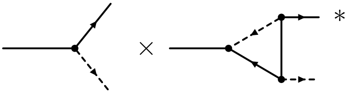
  neglects crucial cross-correlations
- overlap due to finite width neglected

Non-equilibrium Quantum Field Theory

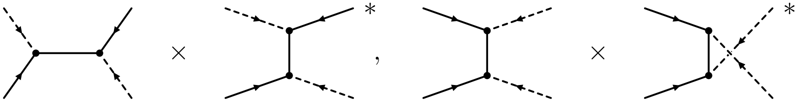
- 3-loop contributions



- vertex *CP*-violating parameter

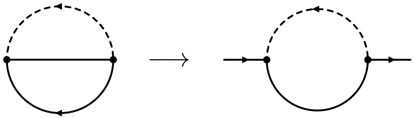


- ($s \times t$) and ($t \times u$) contributions to $\ell\phi \leftrightarrow \bar{\ell}\bar{\phi}$, $\ell\ell \leftrightarrow \bar{\phi}\bar{\phi}$ scattering

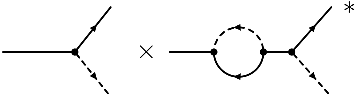


NON-EQUILIBRIUM QUANTUM FIELD THEORY

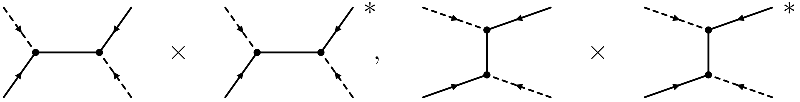
- 2-loop contributions



- self-energy *CP*-violating parameter (need off-diagonal elements G^{ij})



- ($s \times s$) and ($t \times t$) contributions to $\ell\phi \leftrightarrow \bar{\ell}\bar{\phi}$, $\ell\ell \leftrightarrow \bar{\phi}\bar{\phi}$ scattering (needs extended quasi-particle approximation of G^{ii})



RESONANT LEPTOGENESIS

- vertex contributions problematic since not $\propto J = 2M_1 M_2 (M_2^2 - M_1^2)\mathrm{Im}\{(h^\dagger h)_{12}^2\}$

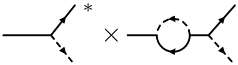
$$\epsilon_i^{V,vac} = \frac{1}{8\pi}\frac{\mathrm{Im}\left\{(h^\dagger h)_{ij}^2\right\}}{(h^\dagger h)_{ii}}\frac{M_j}{M_i}\left[1 - \left(1 + \frac{M_j^2}{M_i^2}\right)\ln\left(1 + \frac{M_i^2}{M_j^2}\right)\right]$$

- self-energy contributions

$$\epsilon_i^{S,vac} = -\frac{\mathrm{Im}\left\{(hh^\dagger)_{ij}^2\right\}}{(hh^\dagger)_{ii}(hh^\dagger)_{jj}}\frac{R}{R^2 + A^2}\,, \quad \text{with } R \equiv \frac{M_j^2 - M_i^2}{M_j \Gamma_j}$$

- form due to internal propagator of N_j

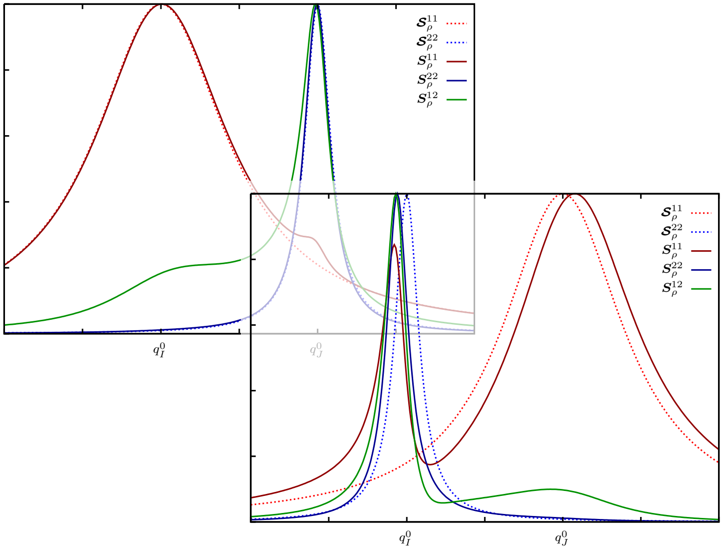
$$\epsilon_i^{S,vac} = 4\frac{\mathrm{Im}\left\{(hh^\dagger)_{ij}^2\right\}}{(hh^\dagger)_{ii} M_i}\mathrm{Im}\Big\{$$



$$\Big\}$$

- What happens if the spectral functions G^{ij} have a sizeable width?
- What happens if parameters in J are subject to changes due to medium effects?

- spectral functions for $R = 10$, $T = 0.1M_1$, $1M_1$



SIMPLIFICATIONS

- idea: attempt exact analytic solution for similar idealized problem
- simplifications
  - toy model (complex scalar field and two mixing real scalars)

$$\mathcal{L} = \frac{1}{2}\partial^\mu\psi_i\partial_\mu\psi_i + \partial^\mu\bar{b}\partial_\mu b - m^2\bar{b}b - \frac{1}{2}M^2_{ij}\psi_i\psi_j - \frac{h_i}{2!}\psi_i bb - \frac{h_i^*}{2!}\psi_i\bar{b}\bar{b}\,, \quad i,j = 1,2$$

$$\left(N_i \to \ell\phi,\; N_i \to \bar{\ell}\bar{\phi}\right) \Longleftrightarrow \left(\psi_i \to bb,\; \psi_i \to \bar{b}\bar{b}\right)$$

  - drop expansion of universe
  - light scalars b form thermal bath
  - 2-loop truncation of 2PI-functional
  - only source terms ($\equiv$ part that does not vanish for CP-symmetric state)
- CP-properties
  - transformation simplifies to $CPb(x)CP^{-1} = \bar{b}(-x)$
  - invariance if $CP\mathcal{L}CP^{-1} \sim \mathcal{L}$
  - CP-odd basis invariant ($H = hh^\dagger$)
    $J \equiv \mathrm{Im}\,\mathrm{Tr}(HM^3H^TM) = 2\,\mathrm{Im}\,H_{12}\mathrm{Re}\,H_{12}M_1M_2(M_2^2 - M_1^2)$

Non-equilibrium QFT

- divergence of current

$$j^b_\mu(x) = 2i\langle\left[\,\bar{b}(x)\partial_\mu b(x) - b(x)\partial_\mu \bar{b}(x)\right]\rangle$$

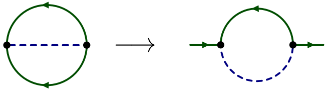
$$\longrightarrow \partial^\mu j^b_\mu(x) = \frac{dq^b}{dt} = i \lim_{y\to x}(\Box_x - \Box_y)D_F(x,y)$$

- propagators $D_F(x,y) \equiv \frac{1}{2}\langle\{b(x),\bar{b}(y)\}\rangle$ and $D_\rho(x,y) \equiv i\langle[b(x),\bar{b}(y)]\rangle$ governed by Kadanoff-Baym equation

$$[\Box_x + m^2]D_F(x,y) = \int_{t_0}^{y^0} d^4z\,\Sigma_F(x,z)D_\rho(z,y) - \int_{t_0}^{x^0} d^4z\,\Sigma_\rho(x,z)D_F(z,y)\,,$$

$$[\Box_x + m^2]D_\rho(x,y) = \int_{x^0}^{y^0} d^4z\,\Sigma_\rho(x,z)D_\rho(z,y)\,,$$

- self-energy Σ



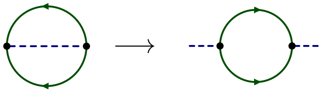
Non-equilibrium QFT

- $G_F^{ij}(x,y) = \frac{1}{2}\langle\{\psi_i(x),\psi_j(y)\}\rangle$ and $G_\rho^{ij}(x,y) = i\langle[\psi_i(x),\psi_j(y)]\rangle$ governed by

$$[\Box_x + M_{ik}^2]G_F^{kj}(x,y) = \int_{t_0}^{y^0} d^4z\, \Pi_F^{ik}(x,z)G_\rho^{kj}(z,y) - \int_{t_0}^{x^0} d^4z\, \Pi_\rho^{ik}(x,z)G_F^{kj}(z,y)\,,$$

$$[\Box_x + M_{ik}^2]G_\rho^{kj}(x,y) = \int_{x^0}^{y^0} d^4z\, \Pi_\rho^{ik}(x,z)G_\rho^{kj}(z,y)$$

- self-energy Π



Exact Solution

- equilibrium solution

$$G^{ij}_{F(\rho)}(x,y) = -\int_{t_0}^{\infty} d^4u \int_{t_0}^{\infty} d^4v\, G^{ik}_R(x,u)\Pi^{kl}_{F(\rho)}(u,v)G^{lj}_A(v,y)$$

$$G_{R(A)}(q) = \Omega^{-1}_{R(A)} \equiv (q^2 - M^2 - \Pi_{R(A)})^{-1}$$

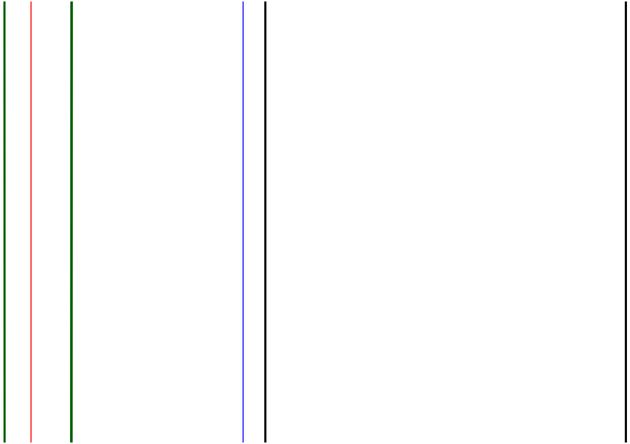
- deviation from equilibrium through weak solution

$$\Delta G^{ij}_\rho(x,y) = 0\,,$$

$$\Delta G^{ij}_F(x,y) = -\int d^3u \int d^3v\, G^{ik}_R(x^0, \mathbf{x}-\mathbf{u})\, \underbrace{\Delta^{kl}_F(\mathbf{u}-\mathbf{v})}_{\text{initial conditions}}\, G^{lj}_A(-y^0, \mathbf{v}-\mathbf{y})$$

- initial conditions $\Delta^{kl}_F = \delta^{kl}\Delta_F$ should respect *CP*-properties of Lagrangian

Exact Solution



Exact Solution

$$q_S^L(t) = \int \frac{d^3q}{(2\pi)^3} \Delta_F(\mathbf{q}) \, \mathrm{Tr}\eta(t,\mathbf{q})$$

$$\begin{aligned}\mathrm{Tr}\,\eta(t,\mathbf{q}) = & -\frac{2J}{\det M} \int_0^\infty \frac{dq_0}{2\pi} \int_0^\infty \frac{dp_0}{2\pi} \int_0^\infty \frac{dk_0}{2\pi} \, \Pi_\rho(q_0,\mathbf{q}) \\ & \times \mathrm{Im}\Bigg(\frac{\Pi_R(p_0,\mathbf{q})F(q_0,p_0,k_0,t)}{\det\Omega_R(p_0,\mathbf{q})\det\Omega_A(k_0,\mathbf{q})} - \frac{\Pi_R(p_0,\mathbf{q})F(-q_0,p_0,k_0,t)}{\det\Omega_R(p_0,\mathbf{q})\det\Omega_A(k_0,\mathbf{q})} \\ & + \frac{\Pi_R(p_0,\mathbf{q})F(q_0,p_0,-k_0,t)}{\det\Omega_R(p_0,\mathbf{q})\det\Omega_R(k_0,\mathbf{q})} - \frac{\Pi_R(p_0,\mathbf{q})F(-q_0,p_0,-k_0,t)}{\det\Omega_R(p_0,\mathbf{q})\det\Omega_R(k_0,\mathbf{q})} \Bigg)\end{aligned}$$

$$\det\Omega_R^{-1} = \det\left[q^2 - M^2 - \Pi_R\right], \quad \det\Omega_A = \det\Omega_R^*$$

$$F(q_0,p_0,k_0,t) = \frac{1-e^{i(q_0-p_0)t}}{q_0-p_0} \frac{1-e^{-i(q_0-k_0)t}}{q_0-k_0}$$

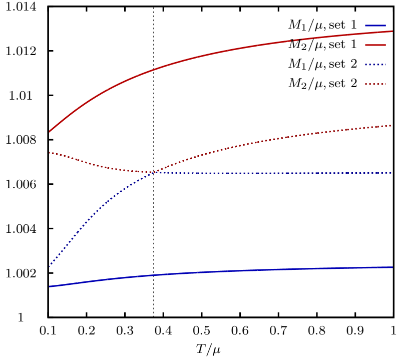
Exact Solution

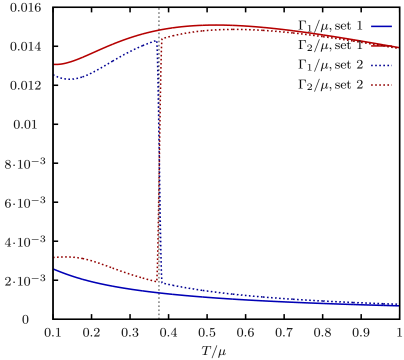
- effective masses and widths from analysis of poles of $\det \Omega_R^{-1}$
- approximation

$$\det \Omega_R^{-1}(q_0, \mathbf{q}) \approx \frac{Z}{(q_0^2 - q_{0,1}^2)(q_0^2 - q_{0,2}^2)}$$

with

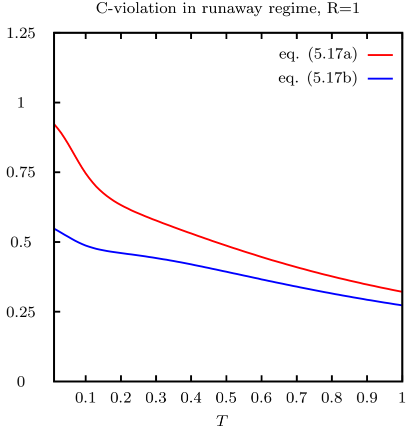
$$q_{0,I} = \pm\,\omega_I - \frac{i}{2}\Gamma_I\,, \quad \omega_I = (\mathbf{q}^2 + M_I^2)^{\frac{1}{2}}$$

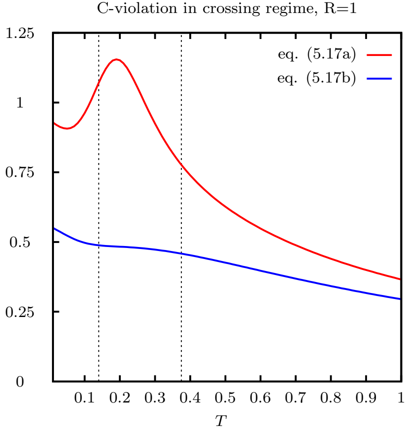




Exact Solution

- comparison between hierarchical Boltzmann (red) and quasi-degenerate approximation (blue)

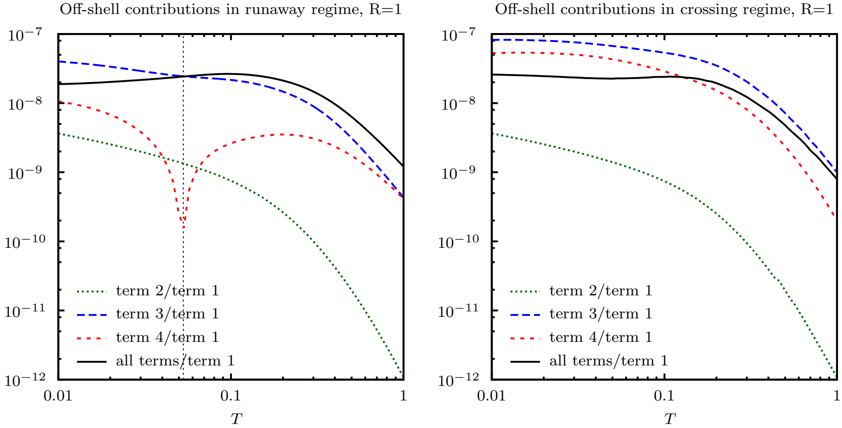




- spurious enhancement for Boltzmann approximation

Exact Solution

- relative importance of remaining terms for $t \to \infty$



- corrections due to Breit-Wigner shaped approximation also small

Conclusions

- NEQFT helps to understand leptogenesis
- quantum kinetic equations with off-shell dynamics in degenerate case
- kinetic equations reflect CP-properties of Lagrangian ($\propto J$)
- basis invariant treatment favourable
- many different effects enter phenomenological treatment
- elaborate quantitative (numerical) computations needed for precise statements (1% level)