Relativistic hydrodynamics at large gradients

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based on

1103.3452, **1302.0697**, **1409.5087** & **1503.07514**

with R. Janik, M. Spaliński and P. Witaszczyk

Hydrodynamization

M. P. Heller, R. A. Janik and P. Witaszczyk,

Phys. Rev. Lett. 108, 201602 (2012), **1103.3452**

Relativistic hydrodynamics

hydrodynamics is an EFT of the slow evolution of conserved currents in collective media close to equilibrium

DOFs: always local energy density ϵ and local flow velocity u^μ $(u_\nu u^\nu = -1)$

EOMs: conservation eqns $\nabla_\mu \langle T^{\mu\nu}\rangle = 0$ for $\langle T^{\mu\nu}\rangle$ systematically expanded in gradients

$$\langle T^{\mu\nu}\rangle = \epsilon\, u^\mu u^\nu + P(\epsilon)\{\, g^{\mu\nu} + u^\mu u^\nu \,\} - \eta(\epsilon)\, \sigma^{\mu\nu} - \zeta(\epsilon)\{\, g^{\mu\nu} + u^\mu u^\nu \,\}(\nabla \cdot u) + \ldots$$

microscopic input:

EoS ($P(\epsilon) = \frac{1}{3}\epsilon$ for CFTs)

shear viscosity

bulk viscosity (vanishes for CFTs)

Dissipation: $$\nabla_\mu \left\{ \frac{\epsilon + P(\epsilon)}{T} \cdot u^\mu + \ldots \right\} = \frac{\eta}{2T}\sigma_{\alpha\beta}\sigma^{\alpha\beta} + \frac{\xi}{T}(\nabla \cdot u)^2 + \ldots \geq 0$$

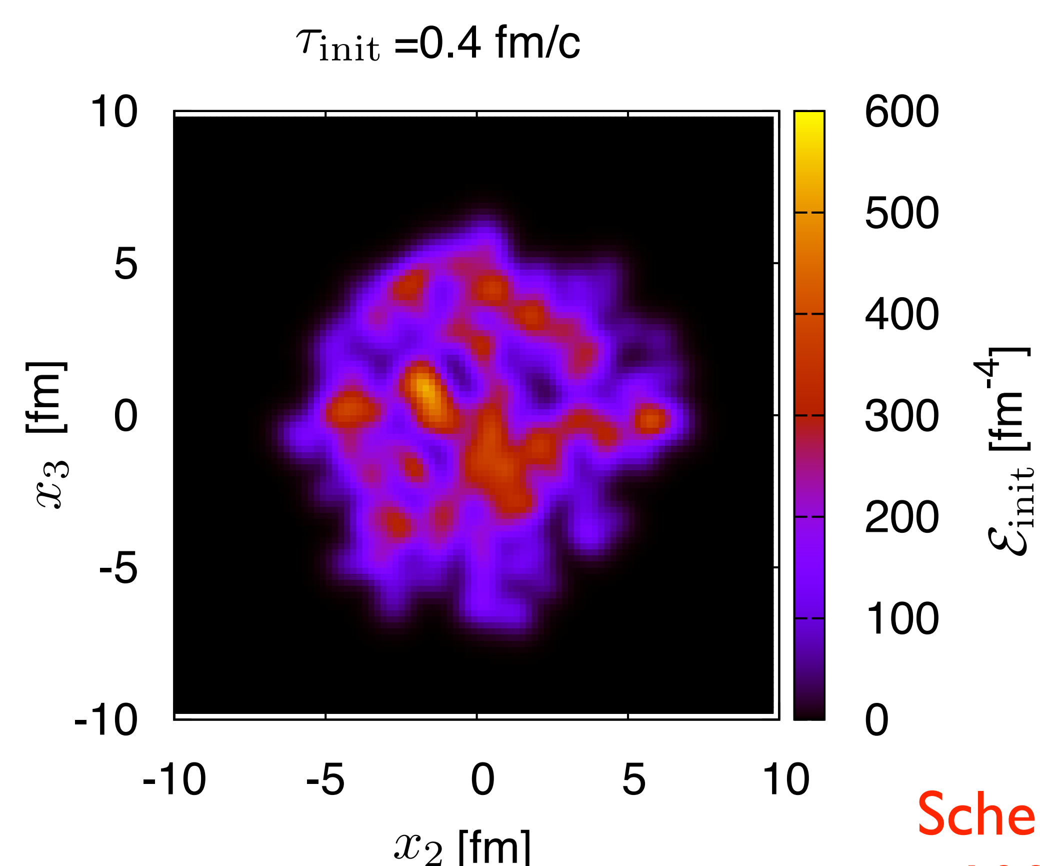
Early initialization of hydrodynamics in HIC

Hydrodynamic codes in HIC are initialized in < 1fm/c with temperature O(400MeV)

→ **large gradients:**

Longitudinal direction: $\sqrt{\sigma_{\mu\nu}\sigma^{\mu\nu}} \sim \frac{1}{\tau_{\text{init}}} = \mathcal{O}(T_{\text{init}})$

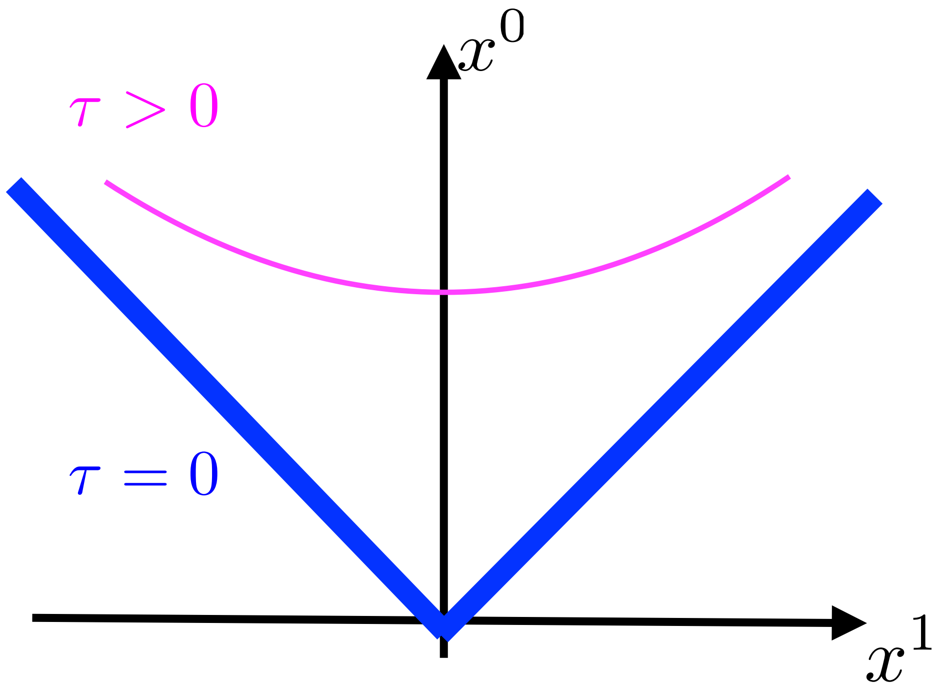
Transverse plane: $\frac{1}{\mathcal{E}_{\text{init}}}\partial_x\,\mathcal{E}_{\text{init}} = \mathcal{O}(T_{\text{init}})$



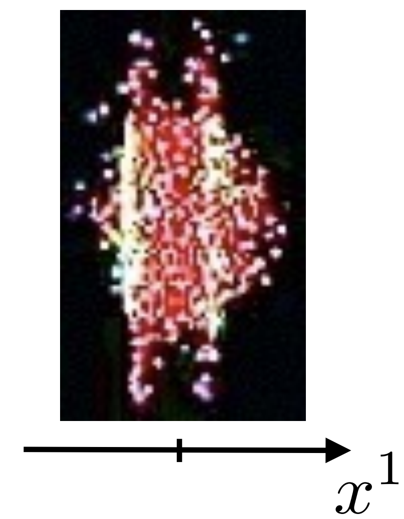
Schenke et al. 1009.3244

Can hydrodynamics work when gradients are large? If yes, why?

Boost-invariant flow [Bjorken 1982]



const x^0 slice:



Boost-invariance: in $\left(\tau \equiv \sqrt{x_0^2 - x_1^2}, \quad y \equiv \operatorname{arctanh}\frac{x_1}{x_0}, \, x_2, \, x_3\right)$ coords no y-dep

In a CFT: $\langle T^\mu_{\ \nu} \rangle = \mathrm{diag}\left\{ -\mathcal{E}(\tau), -\mathcal{E} - \tau\,\dot{\mathcal{E}}, \, \mathcal{E} + \frac{1}{2}\tau\,\dot{\mathcal{E}}, \, \mathcal{E} + \frac{1}{2}\tau\,\dot{\mathcal{E}} \right\}$

$\Delta\mathcal{P} \equiv \langle T^2_{\ 2} \rangle - \langle T^y_{\ y} \rangle$

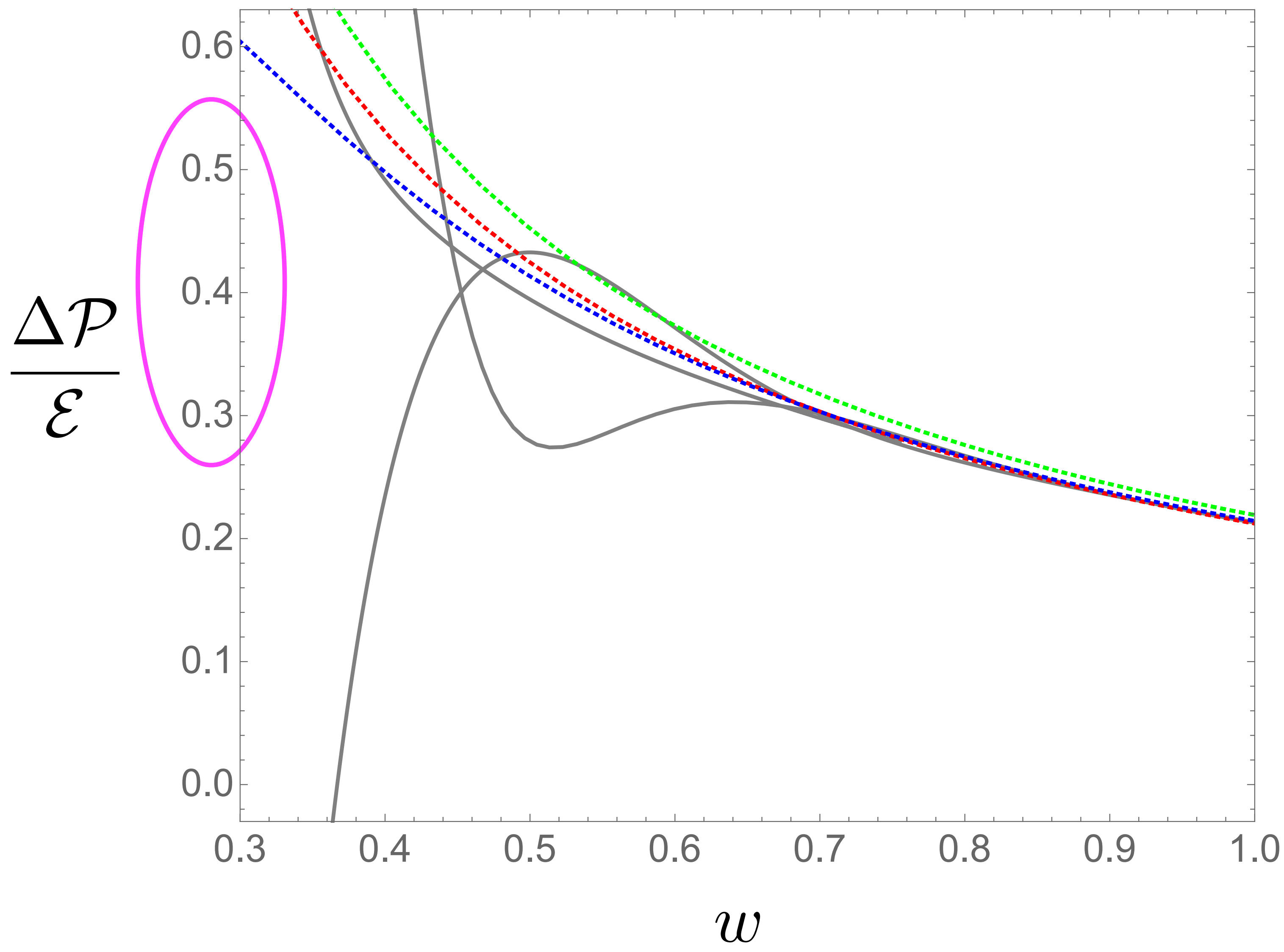
$T \equiv \left(\frac{\mathcal{E}(\tau)}{\frac{3}{8}\pi^2 N_c^2} \right)^{1/4}$

and via scale-invariance $\frac{\Delta\mathcal{P}}{\mathcal{E}}$ is a function of $w \equiv \tau\, T$

Gradient expansion: series in $\frac{1}{w}$.

Hydrodynamization 1103.3452 (see also Chesler & Yaffe 0906.4426, 1011.3562)

Ab initio calculation in N=4 SYM at strong coupling:



Hydrodynamics works despite huge anisotropy captured by $-\eta\,\sigma^{\mu\nu}$

$$\left.\frac{\Delta\mathcal{P}}{\mathcal{E}}\right|_{\text{hydro}} = \frac{0.21}{w} + \frac{0.0069}{w^2} - \frac{0.0049}{w^3} + \dots$$

Why hydrodynamization can occur?

M. P. Heller, R. A. Janik and P. Witaszczyk,

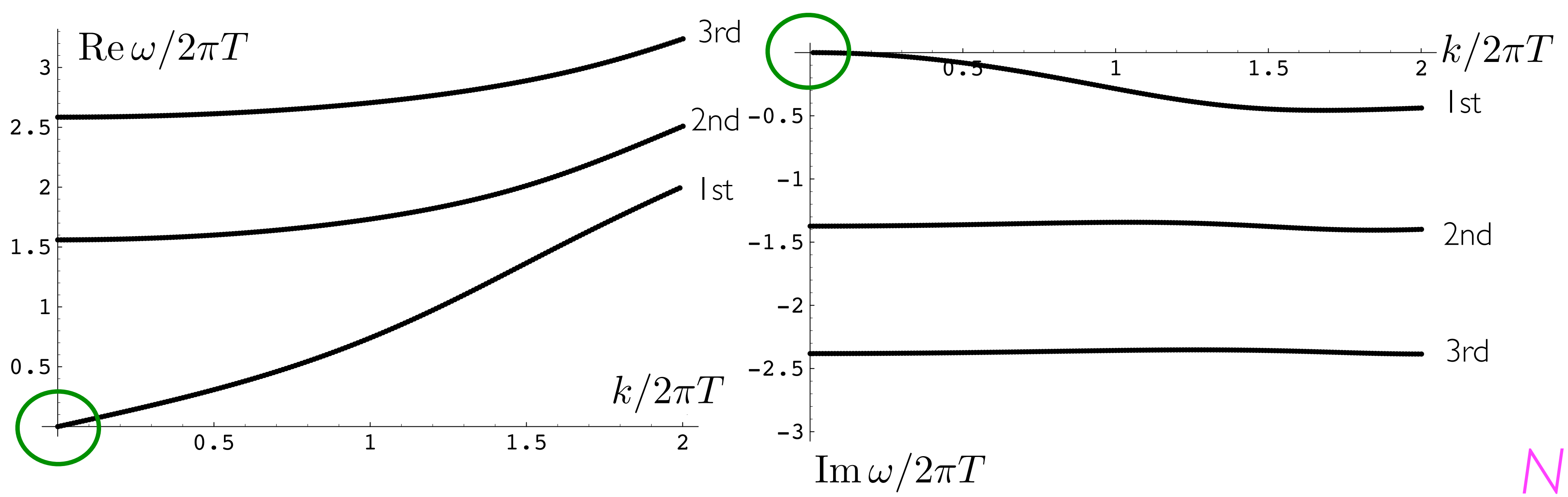
Phys. Rev. Lett. 110, 211602 (2013), **1302.0697**

Excitations in strongly-coupled plasmas

see, e.g. Kovtun & Starinets [hep-th/0506184]

see also Romatschke's week 1 talk

$$\frac{\partial \omega}{\partial k}\bigg|_{k\to 0} = c_{sound}$$



N=4 SYM

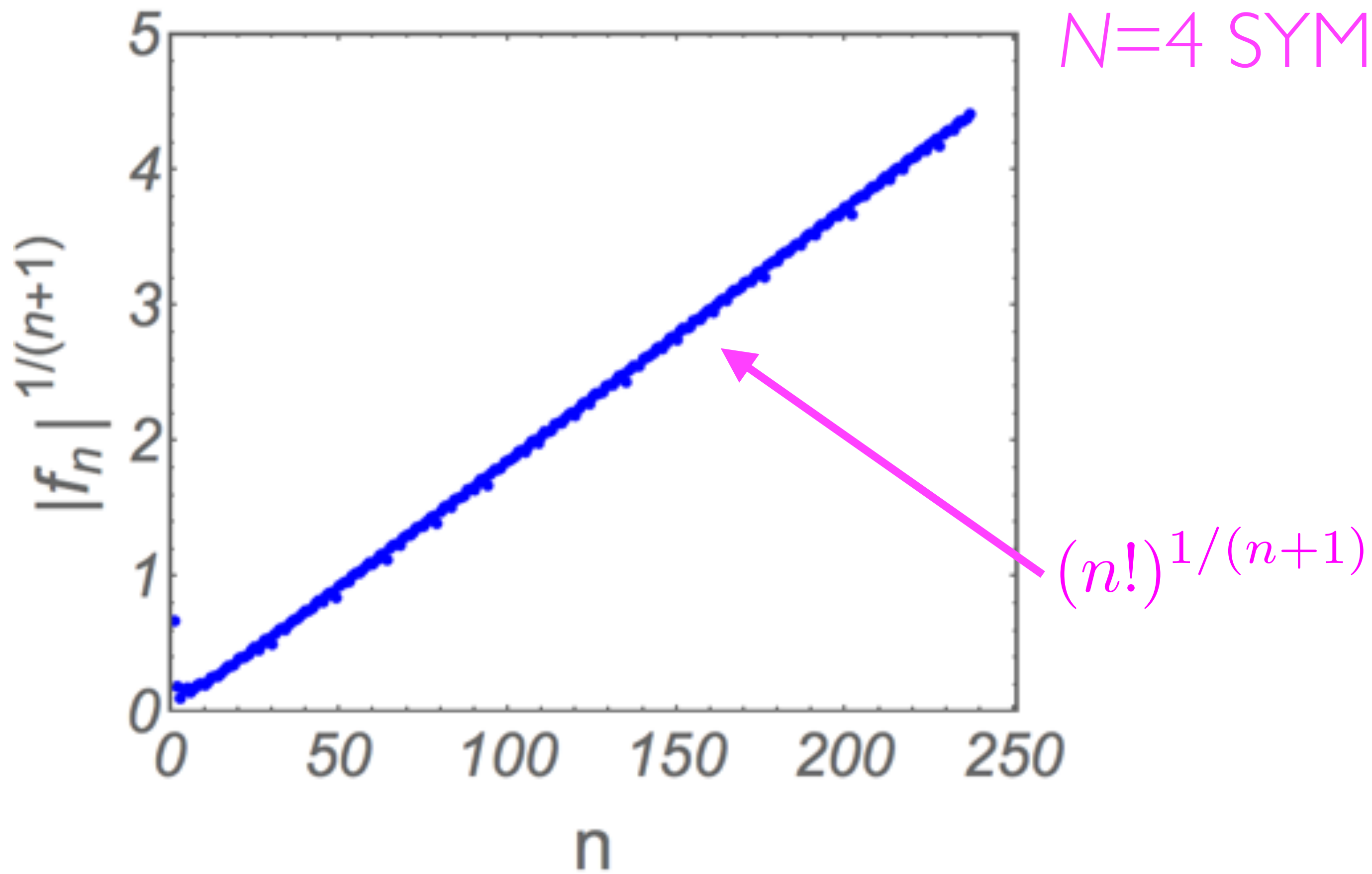
$\omega(k) \to 0$ as $k \to 0$: slowly dissipating modes (hydrodynamic sound waves)

all the rest: far from equilibrium (QNM) modes damped over $t_{therm} = \mathcal{O}(1)/T$

Hydrodynamic gradient expansion is divergent

In 1302.0697 we computed $f(w) \equiv \frac{2}{3} + \frac{1}{6}\frac{\Delta\mathcal{P}}{\mathcal{E}}$ up to $O(w^{-240})$:

$$f(w) = \sum_{n=0}^{\infty} f_n w^{-n} = \\ = \frac{2}{3} + \frac{1}{9\pi} w^{-1} + \ldots$$

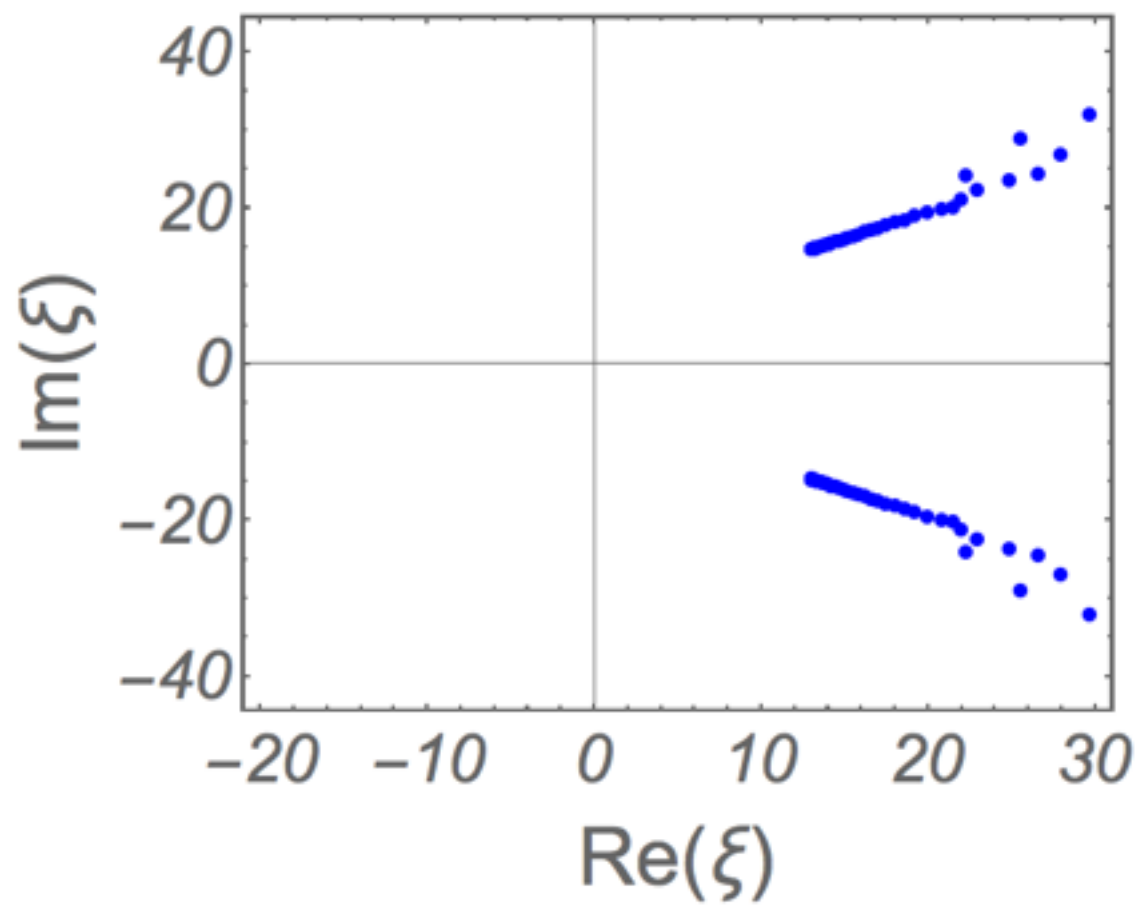


$(n!)^{1/(n+1)}\Big|_{n\to\infty} \approx \frac{1}{e} \cdot n$

Hydrodynamics and QNMs

1302.0697

Analytic continuation of $f_B(\xi) \approx \sum_{n=0}^{240} \frac{1}{n!} f_n \, \xi^n$ revealed the following singularities:



N=4 SYM

Branch cut singularities start at $\frac{3}{2} \, i \, \omega_{QNM_1}$!

EOMs for viscous hydro and quasinormal modes

M. P. Heller, R. A. Janik, M. Spaliński and P. Witaszczyk,

Phys. Rev. Lett. 113, 261601 (2014), **1409.5087**

Evolution equations for relativistic viscous fluids

$\nabla_\mu \{ \epsilon\, u^\mu u^\nu + P(\epsilon) \{ g^{\mu\nu} + u^\mu u^\nu \} - \eta(\epsilon)\, \sigma^{\mu\nu} \} = 0$ is acausal.

Remedy: make $\Pi^{\mu\nu} = \langle T^{\mu\nu} \rangle - (\epsilon\, u^\mu u^\nu + P(\epsilon)\{g^{\mu\nu} + u^\mu u^\nu\})$ a new DOF, e.g.

$$(\tau_\Pi \mathcal{D} + 1)\, \Pi^{\mu\nu} = -\eta \sigma^{\mu\nu}$$

Small perturbations obey Maxwell-Cattaneo equation

$$\partial_t^2 \delta u_z - \frac{\eta}{s} \frac{1}{\tau_\Pi T} \partial_x^2 \delta u_z + \frac{1}{\tau_\Pi} \partial_t \delta u_z = 0$$

Take it seriously:

$$\omega = -i \frac{\eta}{sT} k^2 + \ldots \quad \text{and} \quad \omega = -i \frac{1}{\tau_\Pi} + i \frac{\eta}{sT} k^2 + \ldots$$

hydrodynamics — purely imaginary "quasinormal mode"

Generalization that adds $\mathrm{Re}(\omega_{QNM})$: $\left((\frac{1}{T}\mathcal{D})^2 + 2\omega_I \frac{1}{T}\mathcal{D} + |\omega|^2 \right) \Pi^{\mu\nu} = -\, \eta |\omega|^2 \sigma^{\mu\nu}$

1409.5087

Resumming gradient expansion

M. P. Heller, M. Spaliński,

to appear in Phys. Rev. Lett. this Friday, **1503.07514**

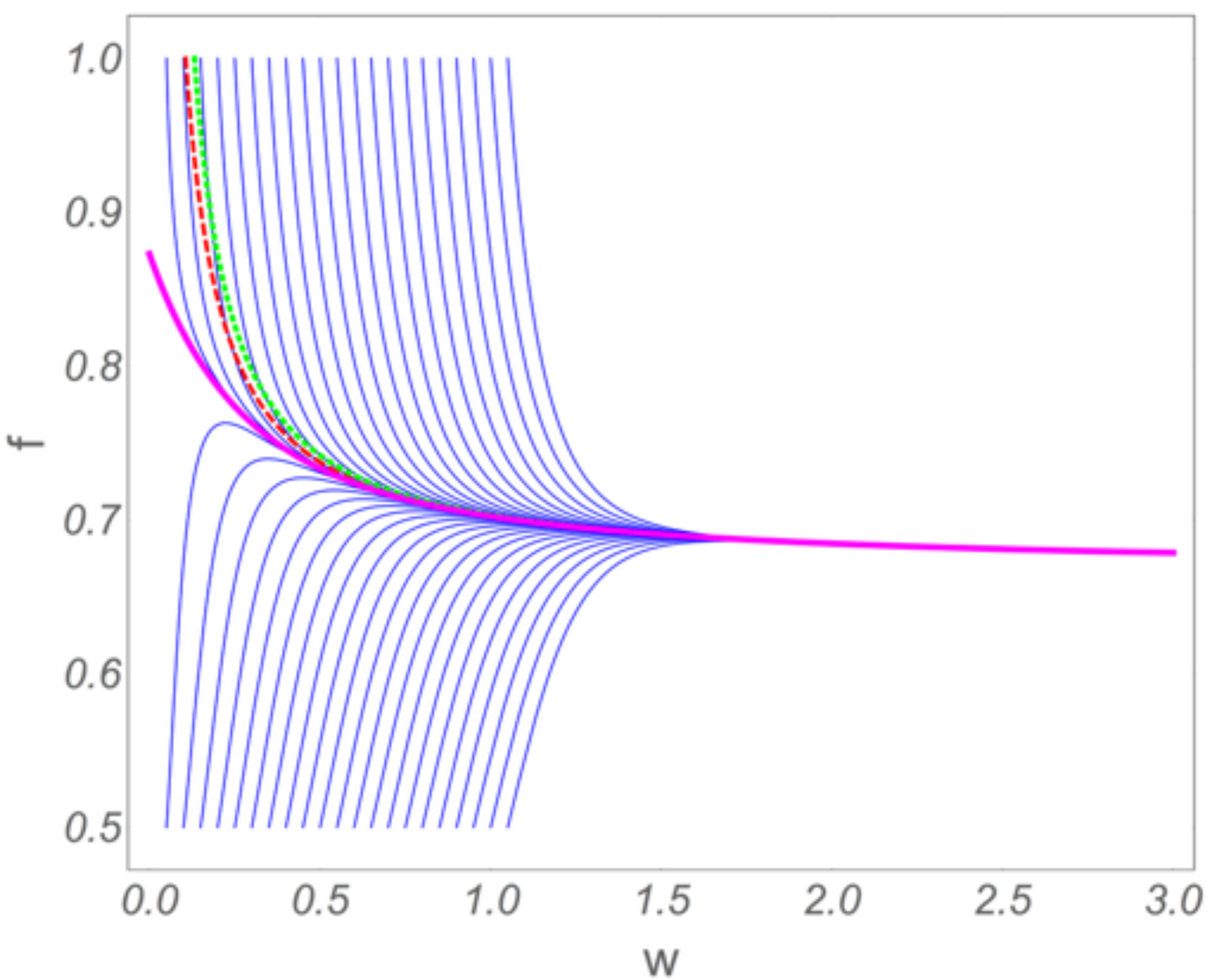
The boost-invariant attractor

1503.07514

$$(\tau_\Pi \mathcal{D} + 1)\, \Pi_{\mu\nu} = -\eta\sigma_{\mu\nu} + \dots$$

$C_\eta = \frac{\eta}{s}$ and $C_{\tau_\Pi} = \tau_\Pi\, T$

$$f' = -\frac{1}{C_{\tau_\Pi}} + \frac{2}{3\, C_{\tau_\Pi}\, f} + \frac{16}{3\, w} - \frac{16}{9\, w\, f} + \frac{4 C_\eta}{9\, C_{\tau_\Pi}\, w\, f} - \frac{4\, f}{w} + \dots$$



attractor

different solutions

$f(w) = f_0 + f_1 w^{-1}$

$f(w) = f_0 + f_1 w^{-1} + f_2 w^{-2}$

Gradient expansion

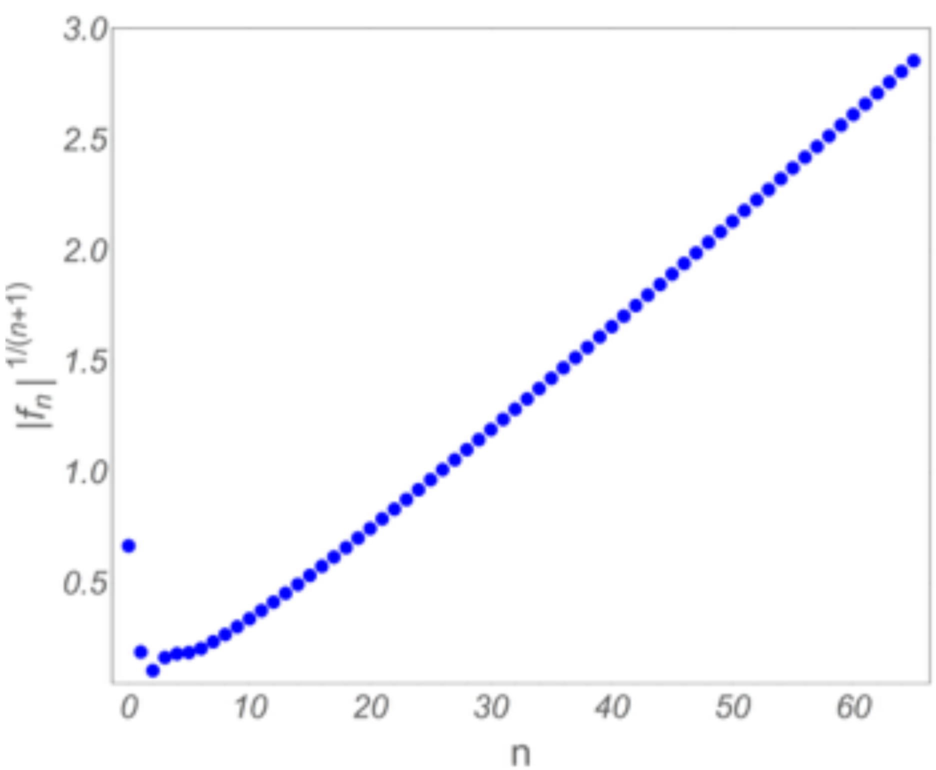
1503.07514

$$f' = -\frac{1}{C_{\tau_\Pi}} + \frac{2}{3\,C_{\tau_\Pi}\,f} + \frac{16}{3\,w} - \frac{16}{9\,w\,f} + \frac{4C_\eta}{9\,C_{\tau_\Pi}\,w\,f} - \frac{4\,f}{w} + \ldots$$

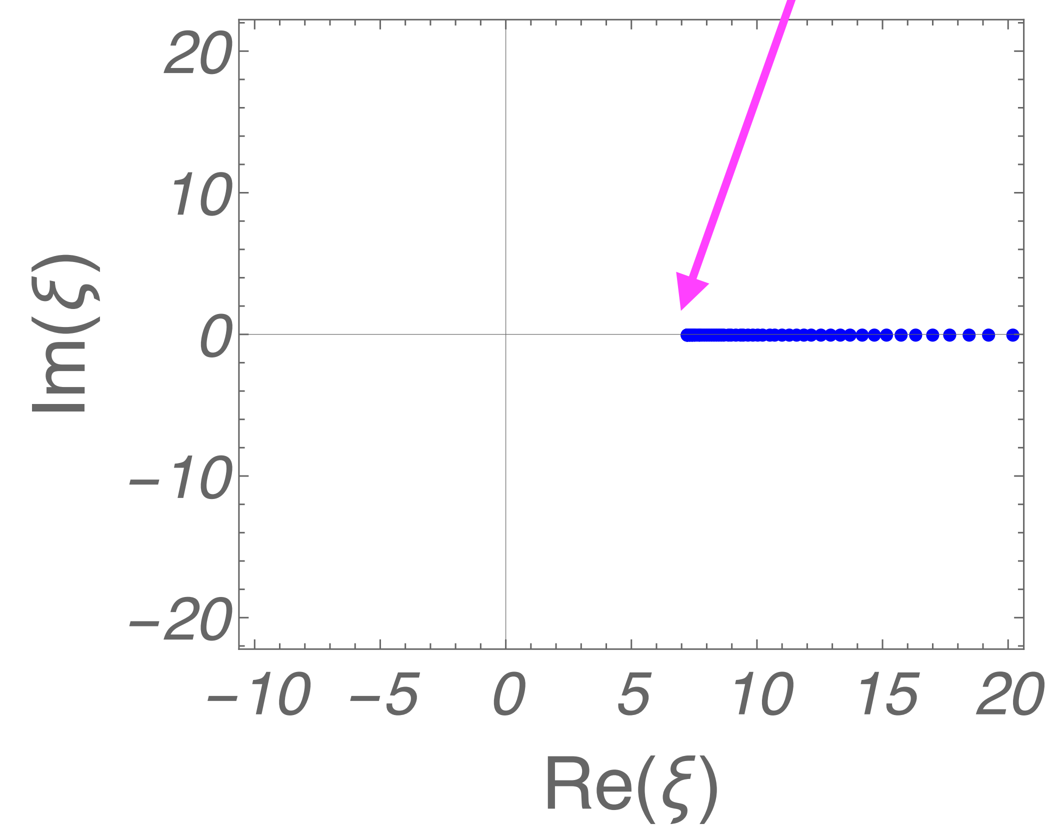
$$f = \sum_{n=0}^{\infty} \frac{f_n}{w^n} + \delta f + \mathcal{O}(\delta f^2)$$

$$\delta f \sim \exp\left(-\frac{3}{2C_{\tau_\Pi}} w\right) \times \ldots$$

$$\xi_{sing} = \frac{3}{2C_{\tau_\Pi}}$$



$$f_B(\xi) \approx \sum_{n=0}^{200} \frac{1}{n!} f_n\, \xi^n$$

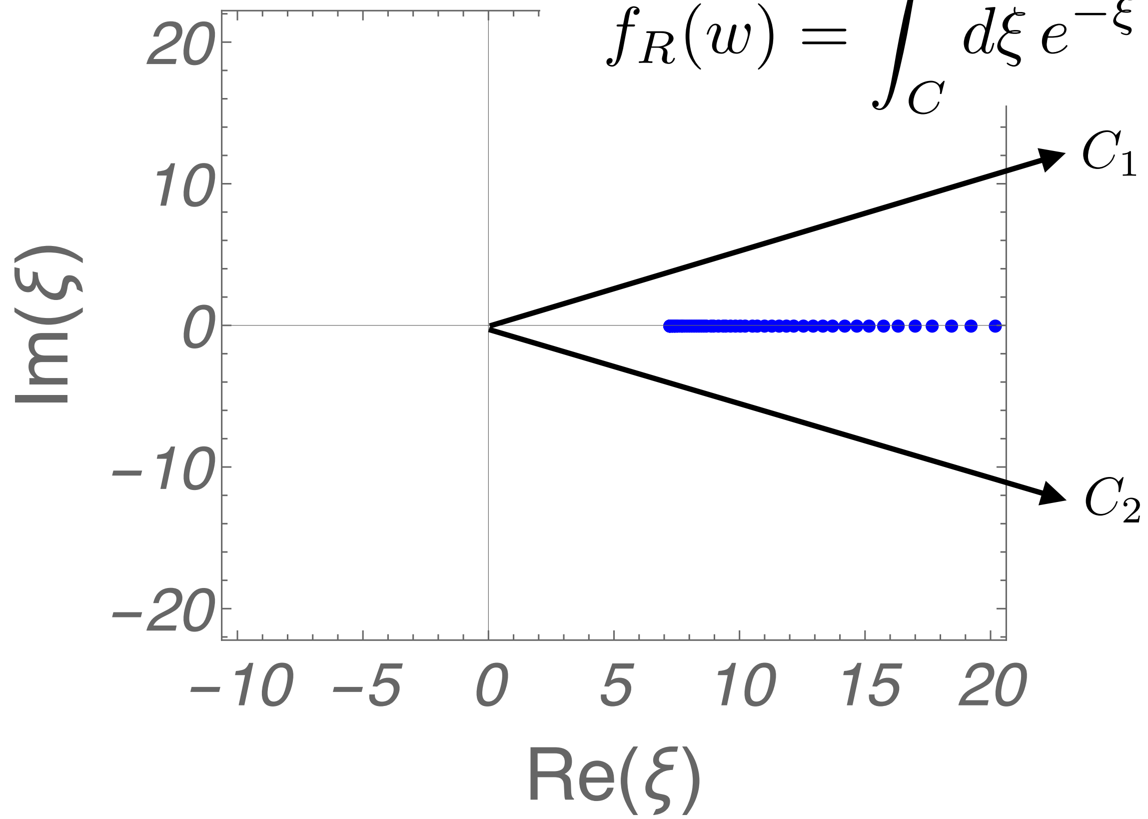


Transseries

1503.07514

Hydrodynamic gradient expansion is intrinsically ambiguous:

$$f_R(w) = \int_C d\xi \, e^{-\xi} \, f_B(\xi/w) = w \int_C d\xi \, e^{-w\xi} \, f_B(\xi)$$

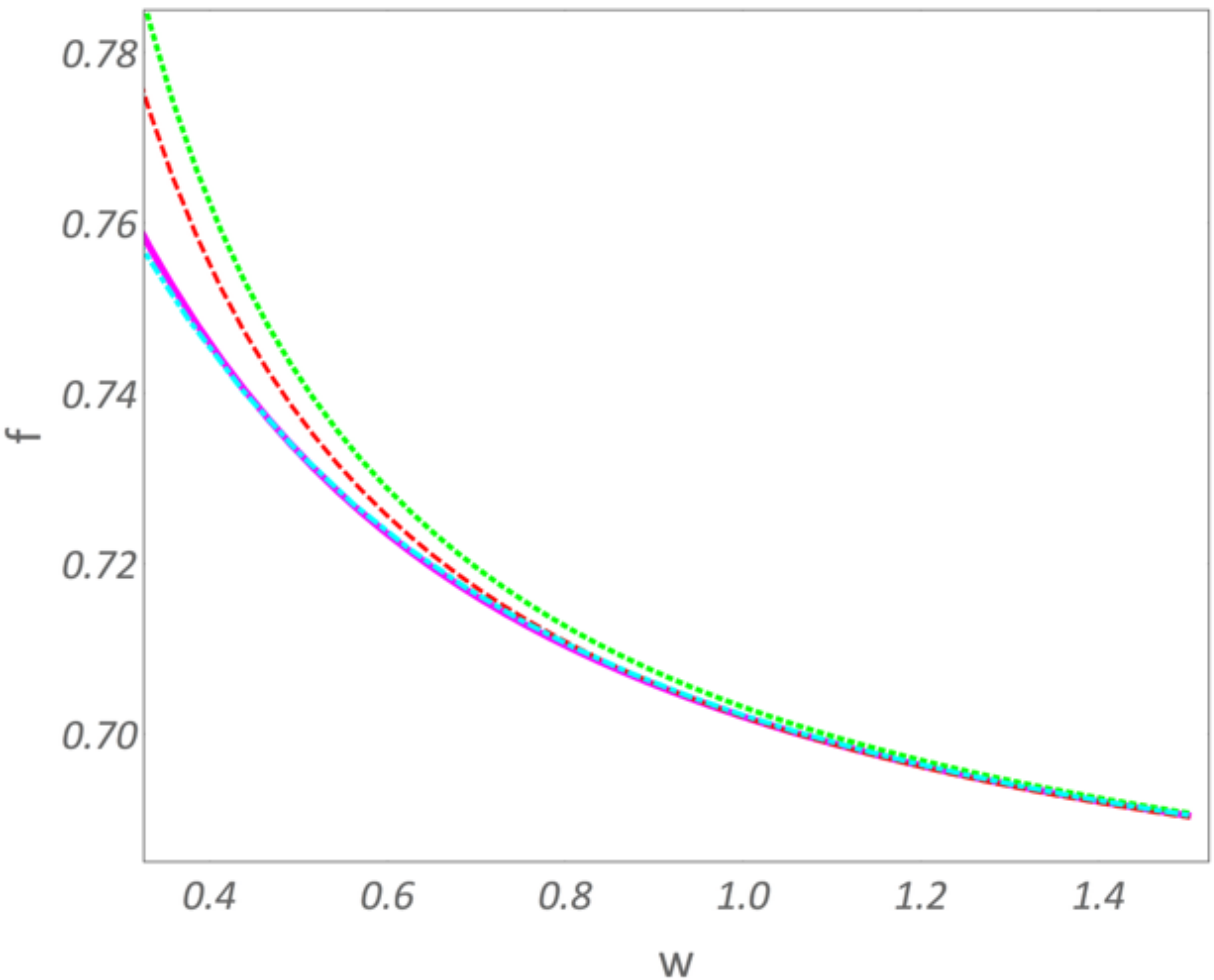


The ambiguity goes away upon including the quasinormal mode ($f_n = a_{0,n}$)

$$f = \sum_{m=0}^{\infty} (c_{amb} + r)^m \left\{ w^{\frac{C_\eta}{C_{\tau_\Pi}}} \exp\left(-\frac{3}{2C_{\tau_\Pi}} w \right) \right\}^m \sum_{n=0}^{\infty} a_{m,n} w^{-n}$$

Resummed hydrodynamics and the attractor

1503.07514



attractor

resummed transseries

$f(w) = f_0 + f_1 w^{-1}$

$f(w) = f_0 + f_1 w^{-1} + f_2 w^{-2}$

Note that matching to the attractor required choosing r = 0.049 (not 0 !).

Executive summary

1103.3452, **1302.0697**, **1409.5087** & **1503.07514**

with R. Janik, M. Spaliński and P. Witaszczyk

Ab initio calculations in holography show that early applicability of viscous hydrodynamics in the presence of large gradients / large anisotropies is not crazy.