NLO effective kinetic theory for jets and thermalization

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Overview

- Aim: extend the AMY effective kinetic theory to NLO Arnold Moore Yaffe 2002
- NLO means $O(g)$ effects from the medium
- Relies on cool new light-cone techniques (much more complicated for non-relativistic or mildly relativistic degrees of freedom)

Pedagogical review in JG Teaney **1502.03730**
Gritty details in JG Moore Teaney **150x.yyyy**

Overview

- Applications
  - Jet propagation and quenching in the QGP
    JG Moore Teaney, JG Schenke Teaney, Kurkela & co?
  - (Isotropic) thermalization (à la Kurkela Lu Moore York) at NLO JG Kurkela
  - In principle, transport coefficients (η,...) at NLO. In practice: severe roadblocks. Working on an estimate

Motivation

- How reliable is the perturbative treatment? Monday's discussion
  - For thermodynamical quantities (*p*, *s*, ...) either strict expansion in *g*, QCD (T) + EQCD (gT) + MQCD (g^2T) (Arnold-Zhai, Braaten Nieto, etc) or non-perturbative solution of EQCD (Kajantie Laine etc) or resummations (HTLpt, Andersen Braaten Strickland etc.)
  - For dynamical quantities? We now have 2 contrasting examples of $O(g)$ corrections

1) Heavy quark diffusion

- Defined through the noise-noise correlator in a Langevin formalism. In field theory it can be written as

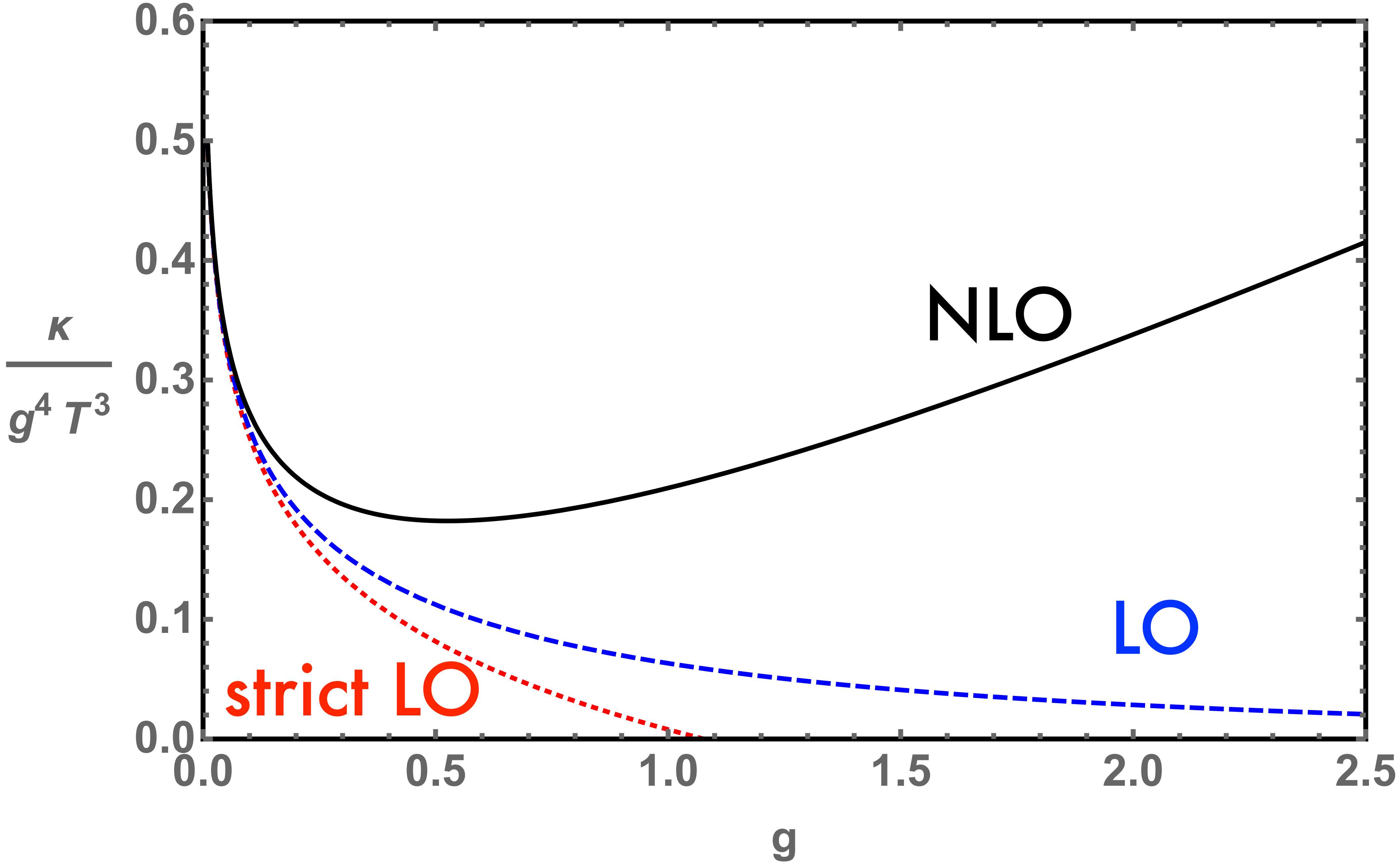
$$\kappa = \frac{g^2}{3N_c}\int_{-\infty}^{+\infty} dt \mathrm{Tr}\langle U(t,-\infty)^\dagger E_i(t) U(t,0) E_i(0) U(0,-\infty)\rangle$$

- The NLO computation factors in the coefficient C, which turns out to be sizeable

$$\kappa = \frac{C_H g^4 T^3}{18\pi}\left(\left[N_c + \frac{N_f}{2}\right]\left[\ln\frac{2T}{m_D} + \xi\right] + \frac{N_f \ln 2}{2} + \frac{N_c m_D}{T} C + \mathcal{O}(g^2)\right) \qquad \xi = \frac{1}{2} - \gamma_E + \frac{\zeta'(2)}{\zeta(2)}$$

Caron-Huot Moore **PRL100, JHEP0802** (2008)

1) Heavy quark diffusion



Caron-Huot Moore **PRL100, JHEP0802** (2008)

2) Thermal photon rate

- Defined through the current-current correlator

$$\frac{d\Gamma}{d^3k} = \frac{e^2}{(2\pi)^3 2k^0}\int d^4Y e^{-iK\cdot Y}\langle J^\mu(Y)J_\mu(0)\rangle$$

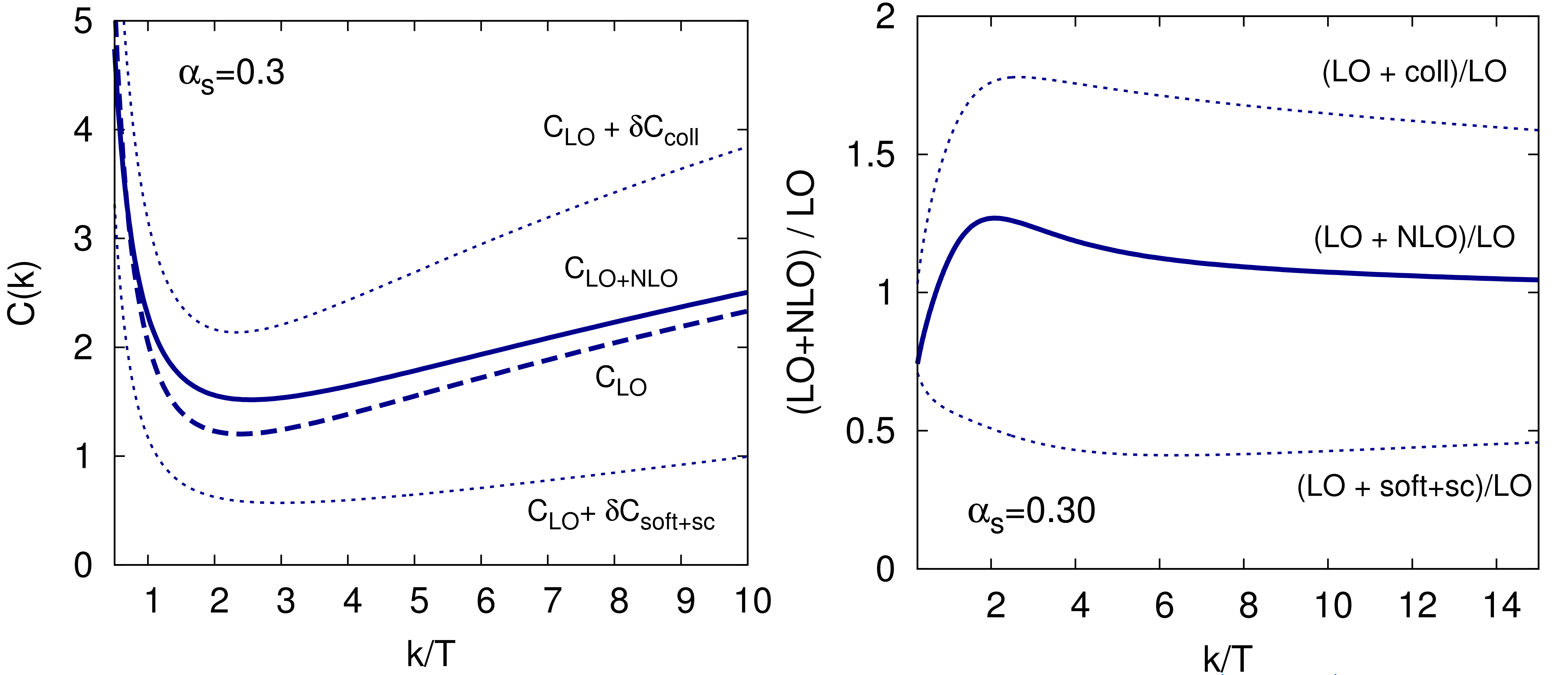
- At NLO one has two large, separate and largely cancelling contributions

$$(2\pi)^3\frac{d\Gamma}{d^3k}\bigg|_{\mathrm{LO}} = \mathcal{A}(k)\overbrace{\left[\log\frac{T}{m_\infty} + C_{2\to2}(k) + C_{\mathrm{coll}}(k)\right]}^{C_{\mathrm{LO}}(k)}$$

$$\mathcal{A}(k) = \alpha_{\mathrm{EM}}\, g^2\; C_F T^2\frac{n_{\mathrm{F}}(k)}{2k}\sum_f Q_f^2 d_f$$

$$(2\pi)^3\frac{d\delta\Gamma}{d^3k}\bigg|_{\mathrm{NLO}} = \mathcal{A}(k)\overbrace{\left[\underbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2}C_{\mathrm{soft+sc}}(k)}_{\delta C_{\mathrm{soft+sc}}(k)} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2}C_{\mathrm{coll}}^{\delta m}(k) + \frac{g^2C_AT}{m_D}C_{\mathrm{coll}}^{\delta\mathcal{C}}(k)}_{\delta C_{\mathrm{coll}}(k)}\right]}^{\delta C_{\mathrm{NLO}}(k)}$$

2) Thermal photon rate



- JG Hong Kurkela Lu Moore Teaney **JHEP1305** (2013)
- Similar picture for dileptons Laine (2013) Laine Ghisoiu (2014) JG Moore (2014)

Outline

- ✓ Introduction and motivation
- Overview of the effective kinetic theory at LO
- A useful reorganization leading to...
- the NLO extension, with
- effective descriptions in terms of Wilson line operators

Overview

The kinetic approach

- We evolve the distribution of a small number of high-energy particles in a thermalized medium

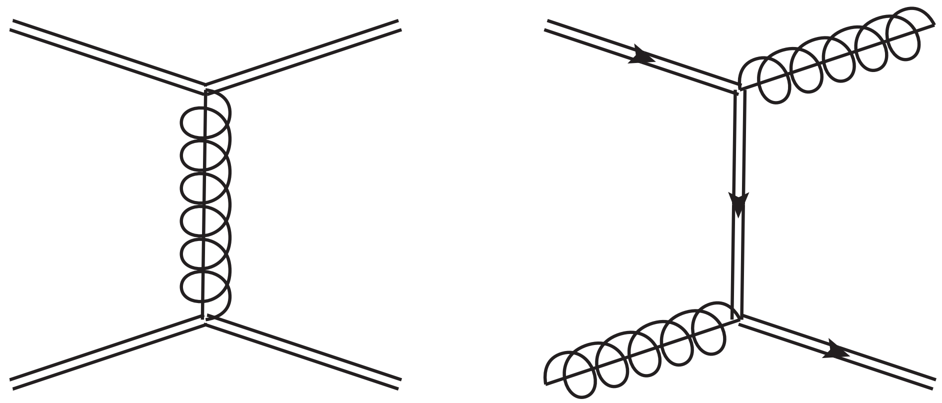
$$\left(\frac{\partial}{\partial t}+v_{\boldsymbol{x}}\cdot\boldsymbol{\nabla}_x\right)P^a(\boldsymbol{p},\boldsymbol{x},t)=-C_a^{\rm LO}[P]=-C_a^{2\leftrightarrow2}[P]-C_a^{1\leftrightarrow2}[P],$$

- At leading order*: elastic, number-preserving 2↔2 processes and collinear, number-changing 1↔2 processes

- D.o.f.s of the kinetic theory are hard, on-shell quarks and gluons ($P^2\lesssim g^2T^2,\ p^0\gtrsim T$). Questionable? Early stages and vacuum cascade?

* We do not consider T/E≪1, but only exp(-E/T)≪1

Elastic processes



Double line: hard (one component O(T) or larger)
Id. specified with curl or arrow when needed

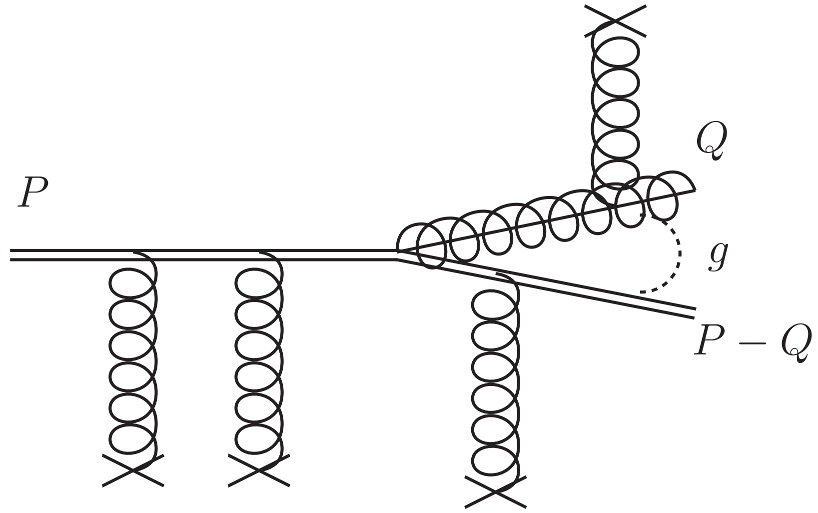
- Boltzmann picture, loss - gain terms

$$C_a^{2\leftrightarrow 2}[P](\boldsymbol{p}) = \frac{1}{4|\boldsymbol{p}|\nu_a}\sum_{bcd}\int_{\boldsymbol{k}\boldsymbol{p}'\boldsymbol{k}'} \left|\mathcal{M}^{ab}_{cd}\right|^2 \, (2\pi)^4\,\delta^{(4)}(P+K-P'-K') \times\left\{P^a(\boldsymbol{p})\,n^b(k)\,[1\pm n^c(p')]\,[1\pm n^d(k')] - \text{gain}\right\}$$

- Integration with bare matrix elements gives log divergences for soft intermediate states, cured by HTL resummation ⇒ nasty n-dimensional numerics?

Radiative processes

- Effective 1↔2: 1+n↔2+n with LPM suppression, collinear kinematics



$$C_a^{1\leftrightarrow 2}[P](\boldsymbol{p}) = \frac{(2\pi)^3}{|\boldsymbol{p}|^2\nu_a}\Bigg\{\sum_{bc}\int_0^{p/2} dq\ \gamma^a_{bc}(\boldsymbol{p};(p-q)\hat{\boldsymbol{p}},q\hat{\boldsymbol{p}})\Big\{P^a(\boldsymbol{p})\,[1\pm n^b(p-q)]\,[1\pm n^c(q)] - \text{gain}\Big\}$$
$$+\sum_{bc}\int_0^{\infty} dq\ \gamma^c_{ab}((p+q)\hat{\boldsymbol{p}};\boldsymbol{p},q\,\hat{\boldsymbol{p}})\Big\{P^a(\boldsymbol{p})\,n^b(q)[1\pm n^c(p+q)] - \text{gain}\Big\}\Bigg\}$$

- Rates (gain and loss terms) individually quadratically IR divergent for soft gluon emission/absorption, but gain-loss is finite

- Both processes are implemented in MARTINI Schenke Gale Jeon **PRC80** (2009)

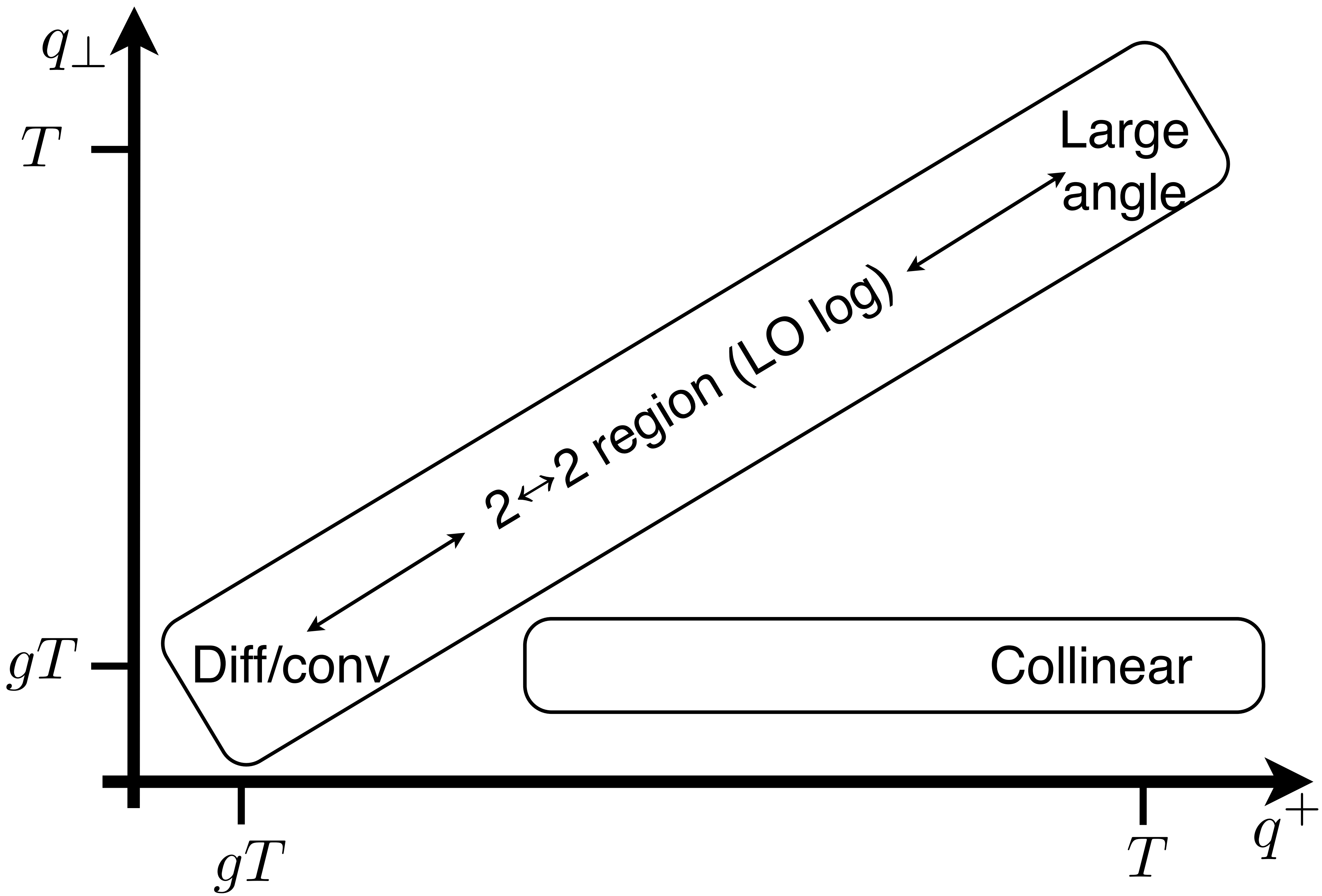
Reorganizing the kinetic theory

Basic principles

- The distinction between 1↔2 and 2↔2 processes gets blurred beyond LO
- Working with matrix elements becomes complicated when dealing with HTL resummation beyond LO
- Reorganize LO to isolate soft momentum exchanges (Q~gT) and introduce Wilson-line effective descriptions for these. Evaluate them with Euclidean and sum rule tech
- Particle identity is important
- Total is given by collinear, diffusion, conversion and large-angle scattering processes

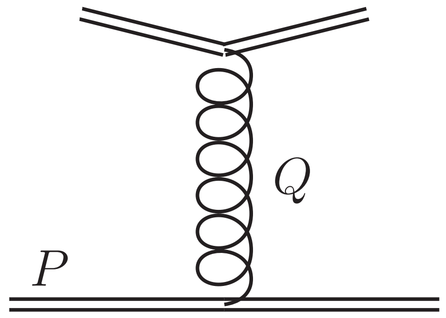
Basic principles

- If P=$(p^+,0,0)$ and Q is the largest momentum transfer that P undergoes

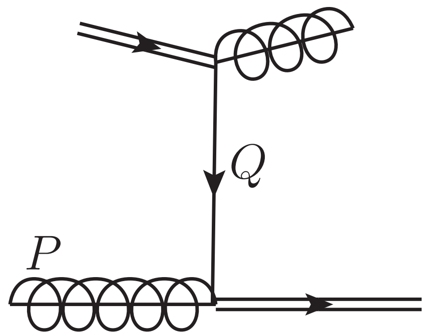


Basic principles

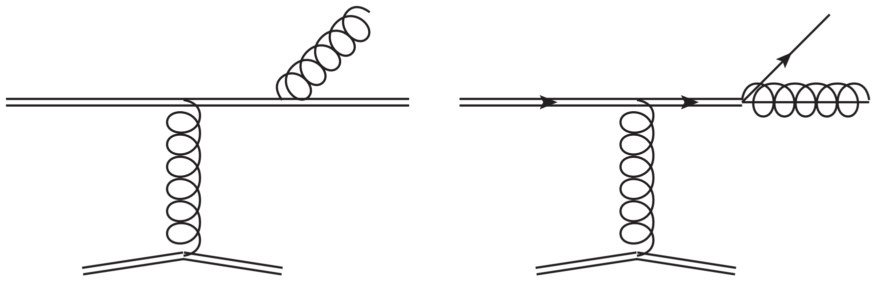
- Diffusion process at LO:



- Conversion process at LO:



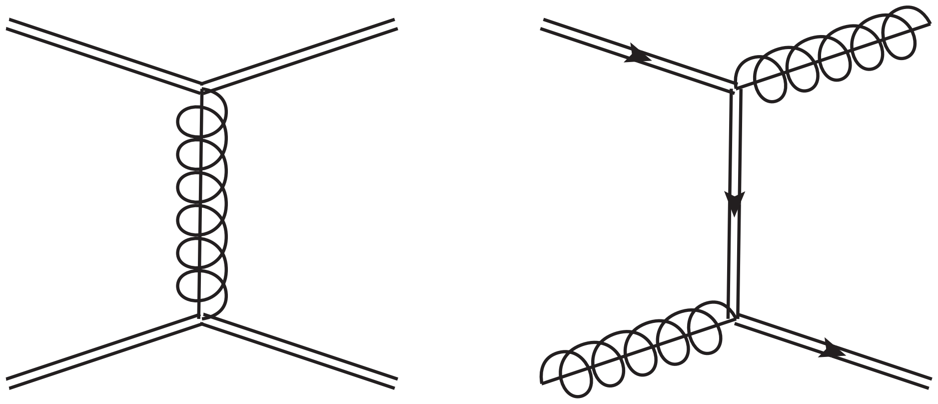
- At higher order: overlap with collinear



- Intermediate regulation or subtractions are necessary

The easy parts

- Large angle scattering: just take 2↔2 processes and stick in an IR regulator. No need for HTL resummation now (numerical good news)



- Collinear processes: the overlap region with diffusion and conversion is an O(g) region of the LO phase space. Might as well include it at LO (e.g. no change) and subtract it at NLO

Diffusion processes

- Landau expansion of C for small, identity-preserving momentum exchanges Svetitsky 1988

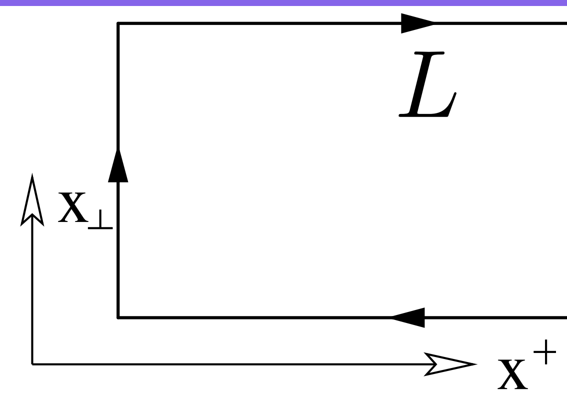
$$C_a^{\mathrm{diff}}[P] \equiv -\frac{\partial}{\partial p^i}\left[\eta_D(p) p^i P^a(\boldsymbol{p})\right] - \frac{1}{2}\frac{\partial^2}{\partial p^i \partial p^j}\left[\left(\hat{p}^i\hat{p}^j \hat{q}_L(p) + \frac{1}{2}(\delta^{ij} - \hat{p}^i\hat{p}^j)\hat{q}(p)\right)P^a(\boldsymbol{p})\right]$$

- Three coefficients: drag, longitudinal and transverse momentum diffusion.

$$\eta_D(p) = -\frac{1}{p_L}\frac{dp_L}{dt}, \qquad \hat{q}(p) \equiv \frac{d}{dt}\left\langle (\Delta p_\perp)^2 \right\rangle, \qquad \hat{q}_L(p) \equiv \frac{d}{dt}\left\langle (\Delta p_L)^2 \right\rangle$$

can forget p-dependence at LO and NLO. qhat is then the standard one, with well-defined Wilson loop definition and perturbative computation to NLO.

Transverse momentum diffusion



$\propto e^{\mathcal{C}(x_\perp)L}$

BDMPS-Z, Wiedemann, Casalderrey-Solana Salgado, D'Eramo Liu Rajagopal, Benzke Brambilla Escobedo Vairo

- All points at spacelike or lightlike separation, only preexisting correlations
- Soft contribution becomes Euclidean! Caron-Huot **PRD79** (2008)
  - Can be "easily" computed in perturbation theory
  - Possible lattice measurements Laine Rothkopf JHEP1307 (2013) Panero Rummukainen Schäfer **1307.5850**

Euclideanization of light-cone soft physics

- For $t/x_z = 0$: equal time Euclidean correlators.

$$G_{rr}(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\!\int_p G_E(\omega_n,p)e^{i\mathbf{p}\cdot\mathbf{x}}$$

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$$G_{rr}(t,\mathbf{x}) = \int dp^0 dp^z d^2p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp - p^0 x^0)} \left(\frac{1}{2} + n_{\mathrm{B}}(p^0)\right)(G_R(P) - G_A(P))$$

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- Change variables to $\tilde{p}^z = p^z - p^0(t/x^z)$

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- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => G_R analytical in p^0

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- Soft physics dominated by *n=0* (and *t*-independent) =>EQCD!

Caron-Huot **PRD79** (2009)

Euclideanization of light-cone soft physics

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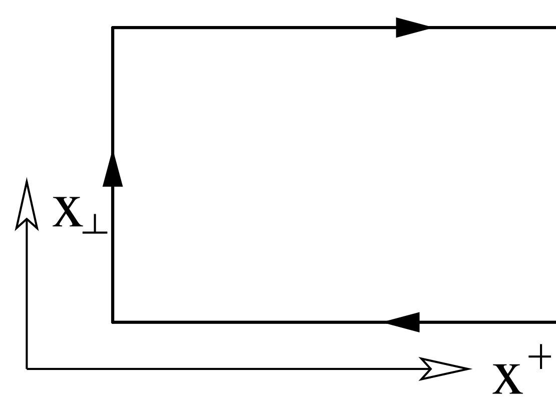
- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => G_R analytical in p^0

$$G_{rr}(t,\mathbf{x})_\mathrm{soft} = T\int d^3p\, e^{i\mathbf{p}\cdot\mathbf{x}}\, G_E(\omega_n = 0, \mathbf{p})$$

- Soft physics dominated by $n=0$ (and t-independent) =>EQCD!

Caron-Huot **PRD79** (2009)

Euclideanization of light-cone soft physics



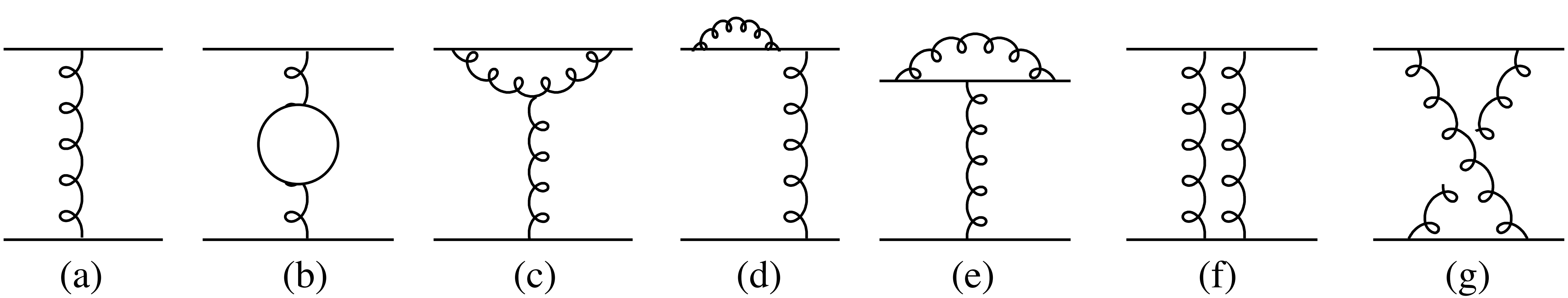
$$\propto e^{\mathcal{C}(x_\perp)L}$$

- At leading order

$$C(x_\perp) \propto T \int \frac{d^2q_\perp}{(2\pi)^2} \left(1 - e^{i\mathbf{x}_\perp \cdot \mathbf{q}_\perp}\right) G_E^{++}(\omega_n = 0, q_z = 0, q_\perp) = T \int \frac{d^2q_\perp}{(2\pi)^2} \left(1 - e^{i\mathbf{x}_\perp \cdot \mathbf{q}_\perp}\right) \left(\frac{1}{q_\perp^2} - \frac{1}{q_\perp^2 + m_D^2}\right)$$

- Agrees with the earlier sum rule in Aurenche Gelis Zaraket **JHEP0205** (2002)

- At NLO: Caron-Huot **PRD79** (2009)



Diffusion processes

$$C_a^{\rm diff}[P] \equiv -\frac{\partial}{\partial p^i}\left[\eta_D(p)p^i P^a(\boldsymbol{p})\right] - \frac{1}{2}\frac{\partial^2}{\partial p^i \partial p^j}\left[\left(\hat{p}^i\hat{p}^j\hat{q}_L(p) + \frac{1}{2}(\delta^{ij} - \hat{p}^i\hat{p}^j)\hat{q}(p)\right)P^a(\boldsymbol{p})\right]$$

- drag and longitudinal momentum diffusion: see Arnold hep-ph/991220{8,9}
  "[The solution] is something that, I believe, may be well known to the few people to whom it is well known. However, since there seems to be general confusion on this matter, it seems worthwhile to continue rather than simply ending here."
- This effective description must match to large-angle for intermediate Q *and* must lead to equilibration
- This leads to a relation between the three coefficients, which we use to fix the drag

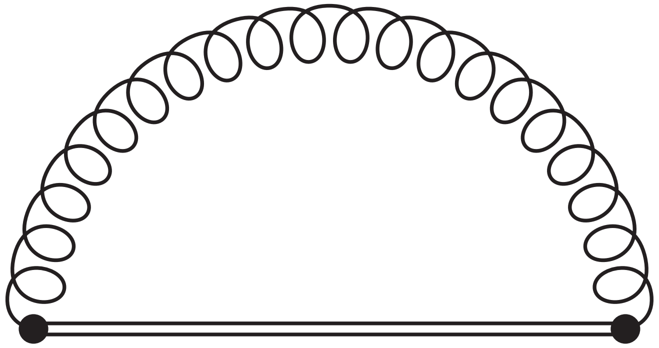
Longitudinal momentum diffusion

- Field-theoretical lightcone definition (justifiable with SCET)

$$\hat{q}_L \equiv \frac{g^2}{d_R}\int_{-\infty}^{+\infty} dx^+ \mathrm{Tr}\left\langle U(-\infty,x^+)F^{+-}(x^+)U(x^+,0)F^{+-}(0)U(0,-\infty)\right\rangle$$

$F^{+-}=E^z$, longitudinal Lorentz force correlator

- At leading order

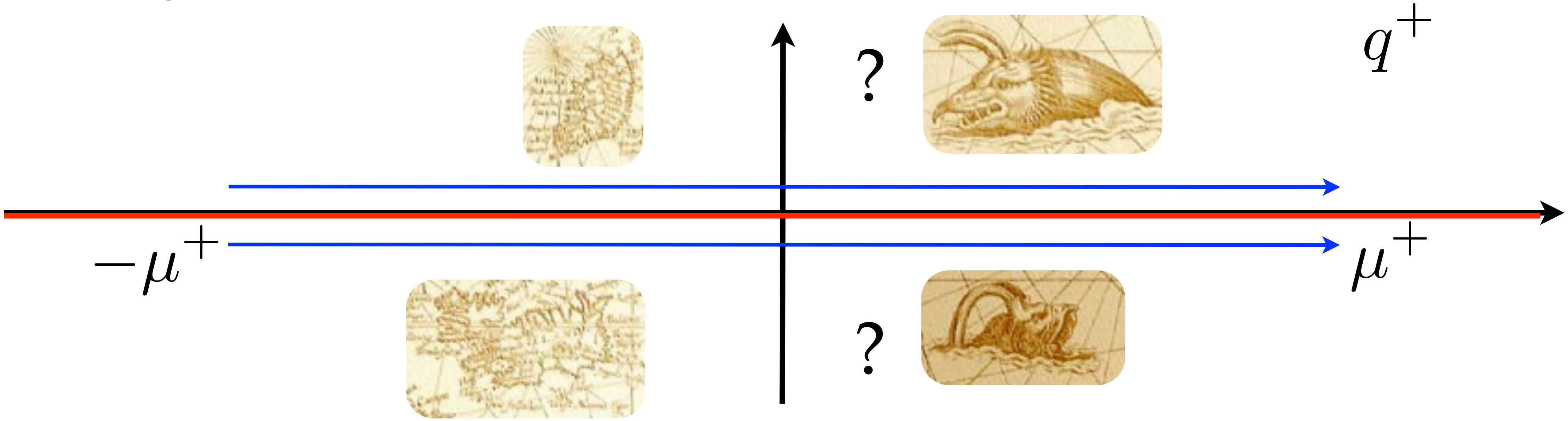


$$\hat{q}_L \propto \int \frac{dq^+ d^2q_\perp}{(2\pi)^3}(q^+)^2 G^>_{++}(q^+,q_\perp,0)$$

$$= \int \frac{dq^+ d^2q_\perp}{(2\pi)^3} T q^+ (G^R_{++}(q^+,q_\perp,0) - G^A)$$

Longitudinal momentum diffusion

$$\hat{q}_L\Big|_{\mathrm{LO}} = g^2 C_R \int \frac{dq^+ d^2 q_\perp}{(2\pi)^3} T q^+ \left(G_R^{--}(q^+, q_\perp) - G_A^{--}(q^+, q_\perp)\right)$$



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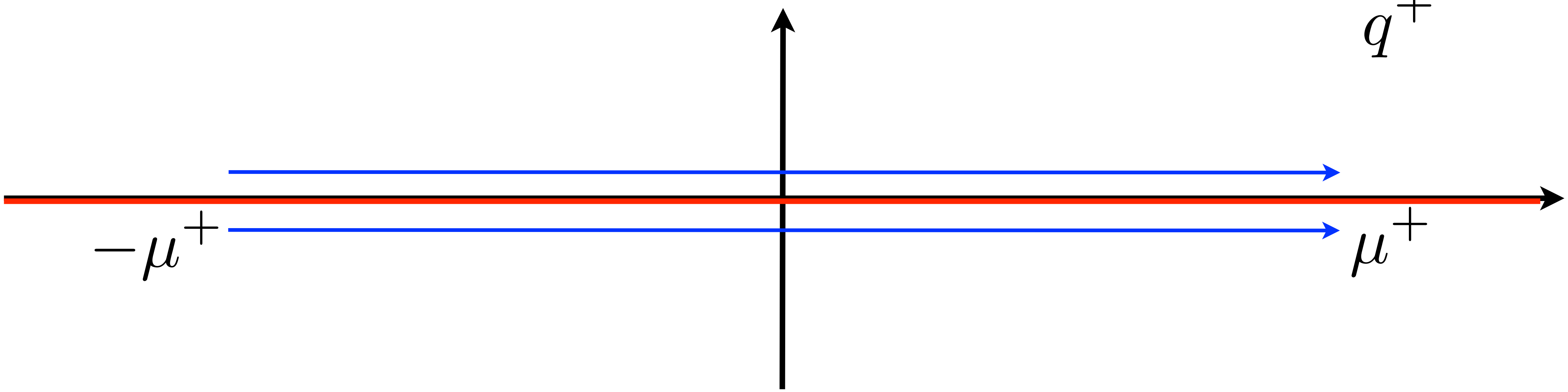

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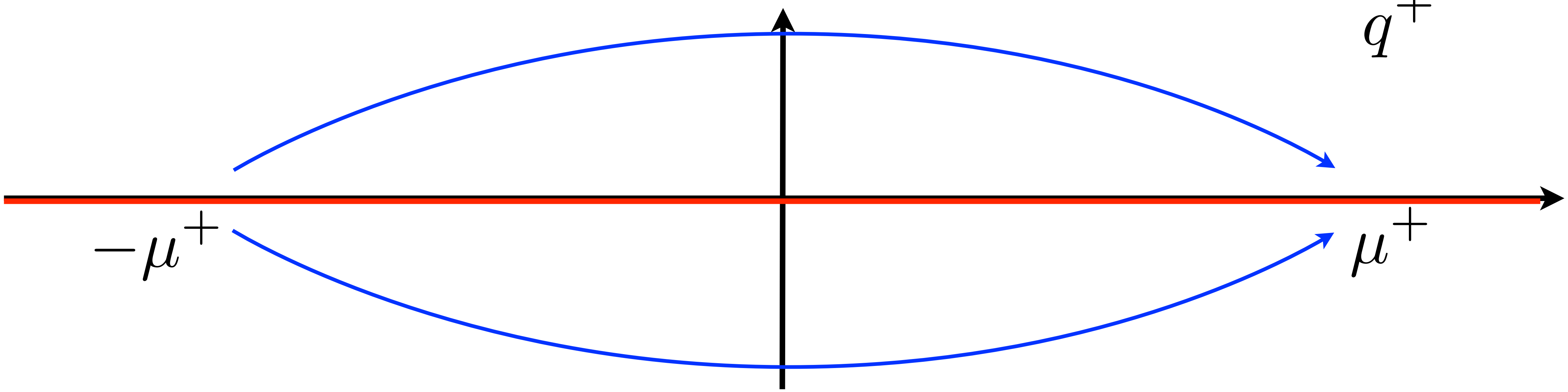

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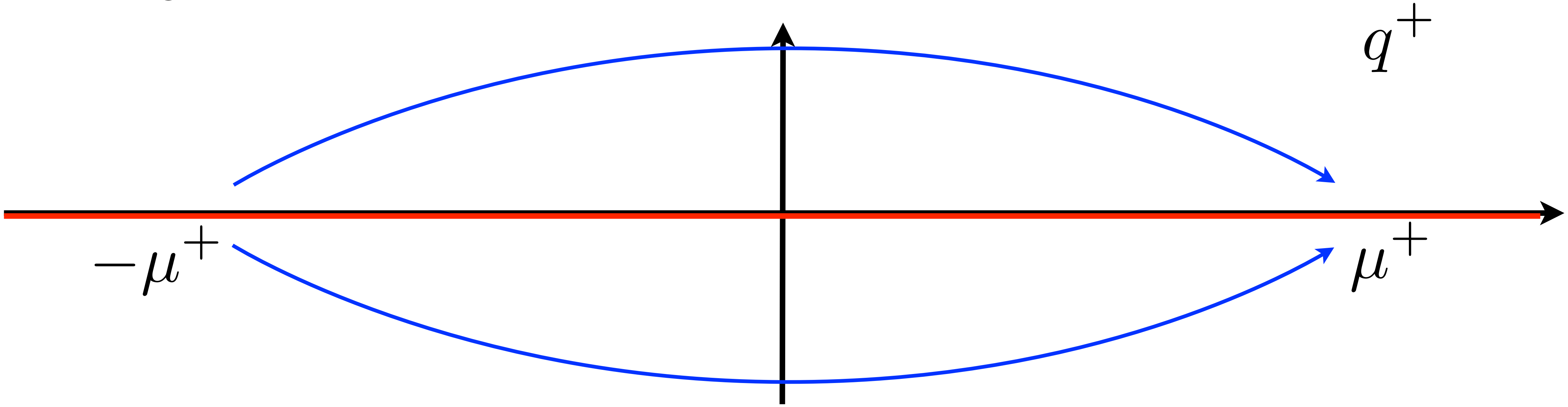


- Use analyticity to deform the contour away from the real axis and keep *1/q⁺* behaviour

$$\hat{q}_L\Big|_{\mathrm{LO}} = g^2 C_R T \int \frac{d^2q_\perp}{(2\pi)^2} \frac{M_\infty^2}{q_\perp^2 + M_\infty^2}$$

Conversion processes

- General structure:

$$
\begin{aligned}
C_{q_i}^{\rm conv}[P] &= P^{q_i}(\boldsymbol{p})\Gamma^{\rm conv}_{q\to g}(p) - P^g(\boldsymbol{p})\frac{d_A}{d_F}\Gamma^{\rm conv}_{g\to q}(p),\\
C_{\bar q_i}^{\rm conv}[P] &= P^{\bar q_i}(\boldsymbol{p})\Gamma^{\rm conv}_{\bar q\to g}(p) - P^g(\boldsymbol{p})\frac{d_A}{d_F}\Gamma^{\rm conv}_{g\to \bar q}(p),\\
C_{g}^{\rm conv}[P] &= \sum_{i=1}^{n_f}\Big\{P^g(\boldsymbol{p})\Big[\Gamma^{\rm conv}_{g\to q_i}(p)+\Gamma^{\rm conv}_{g\to \bar q_i}(p)\Big]\\
&\qquad -\frac{d_F}{d_A}\Big[P^{q_i}(\boldsymbol{p})\Gamma^{\rm conv}_{q\to g}(p)+P^{\bar q_i}(\boldsymbol{p})\Gamma^{\rm conv}_{\bar q\to g}(p)\Big]\Big\},
\end{aligned}
$$

- Momentum change is not relevant to LO and NLO

- Can get Wilson line definition

$$
\Gamma^{\rm conv}_{q\to g}(p) = -\frac{g^2}{8 d_F p}\int_{-\infty}^{+\infty} dx^+\,\big\langle {\rm Tr}\big[U_F(-\infty,x^+)T^a\bar\psi(x^+)\not{v}U_A(x^+,0)\psi(0)T^bU_F(0,-\infty)\big]\big\rangle
$$

- At LO: fermionic sum rule

$$
\Gamma^{\rm conv}_{q\to g}(p)\Big|_{\rm LO} = -\frac{g^2C_F}{8p}\int\frac{d^4Q}{(2\pi)^4}{\rm Tr}\left[\not{v}S^>(Q)\right]2\pi\delta(q^-) = \frac{g^2C_F}{4p}\int\frac{d^2q_\perp}{(2\pi)^2}\frac{m_\infty^2}{q_\perp^2+m_\infty^2}
$$

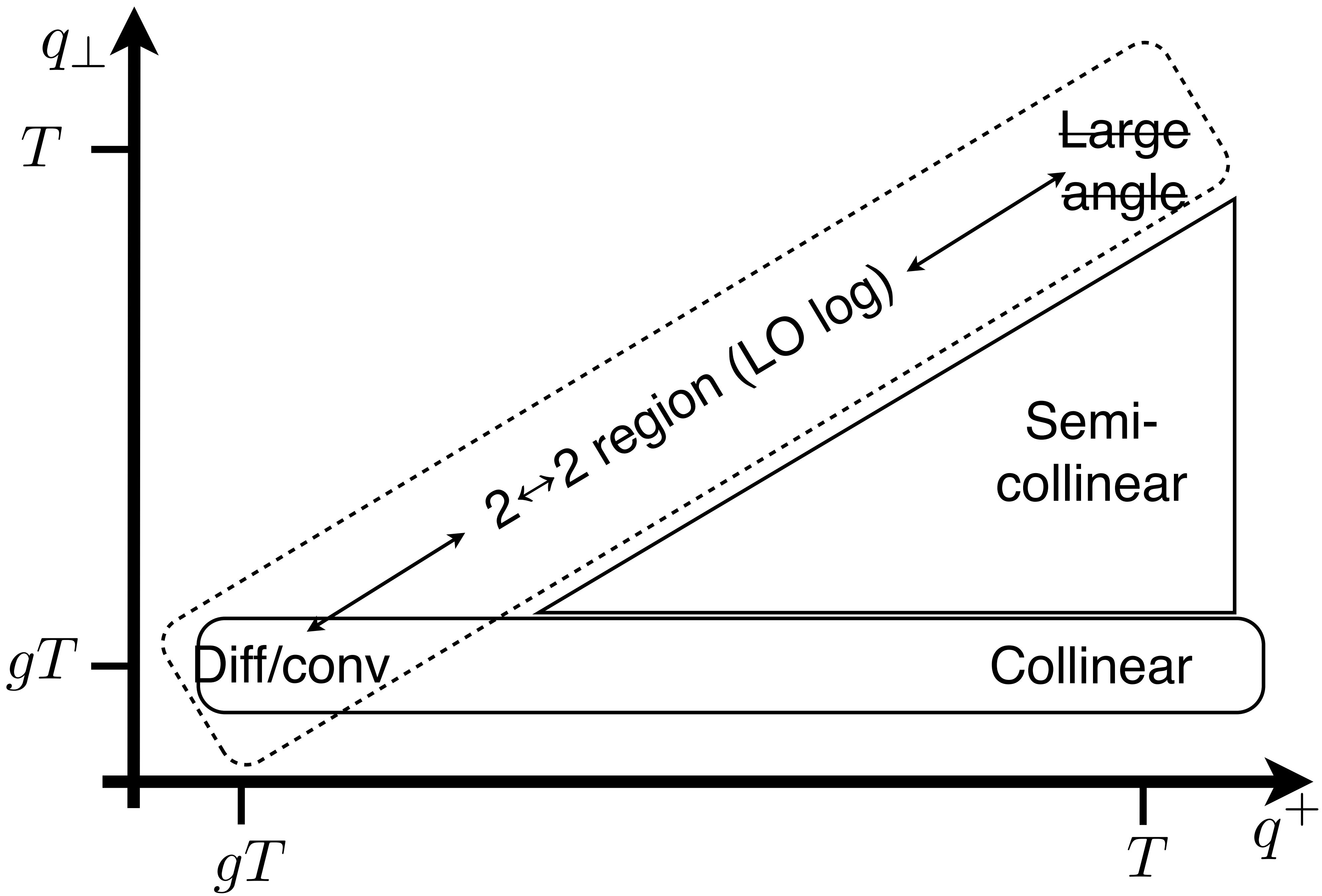
Besak Bödeker **JCAP1203** (2012) JG Hong Kurkela Lu Moore Teaney

Going to NLO

Sources of NLO corrections

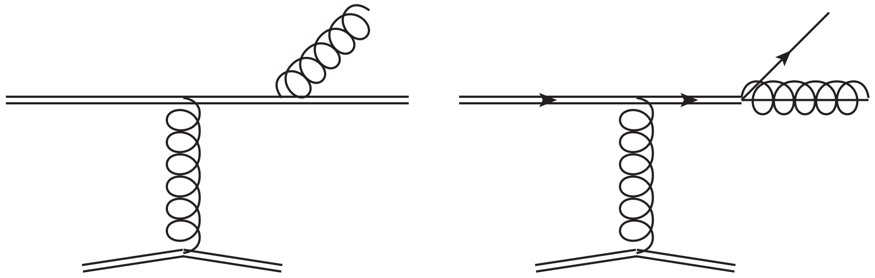
- As usual in thermal field theory, the soft scale gT introduces NLO $O(g)$ corrections
- The diffusion, conversion and the collinear regions receive $O(g)$ corrections
- There is a new semi-collinear region

Sources of NLO corrections



Collinear corrections

- Regions of overlap with the diffusion, conversion and semi-collinear regions need to be subtracted



- The differential eq. for LPM resummation gets correction from NLO $C(x_\perp)$ and from the thermal asymptotic mass at NLO (Caron-Huot 2009)

Collinear corrections

- In more detail, LO is

$$\left.\frac{d\Gamma(p,\omega)}{d\omega}\right|_{\rm coll} = \frac{g^2 C_R}{8\pi p^7}(1\pm n(\omega))(1\pm n(p-\omega)) \left\{ \begin{array}{cc} \frac{1+(1-x)^2}{x^3(1-x)^2} & q\to qg \\ \frac{d_F}{d_A}\frac{x^2+(1-x)^2}{x^2(1-x)^2} & g\to q\bar{q} \\ \frac{1+x^4+(1-x)^4}{x^3(1-x)^3} & g\to gg \end{array} \right\} \mathrm{Im}\,\boldsymbol{\nabla}_b\cdot\mathbf{F}(\mathbf{0})$$

Statistical functions x DGLAP splitting x transverse broadening

$$2\boldsymbol{\nabla}\delta^2(\mathbf{b}) = \frac{-1}{2p\omega(p-\omega)}\left(p(p-\omega)m^2_{\infty\,\omega} + p\,\omega\,m^2_{\infty\,p-\omega} - \omega(p-\omega)m^2_{\infty\,p} - \nabla^2_{\mathbf{b}}\right)\mathbf{F}(\mathbf{b})$$
$$+\,i\left(\mathcal{C}_R(|\omega|\,b) - \frac{\mathcal{C}_A(|\omega|\,b)}{2} + \frac{\mathcal{C}_A(|p|\,b)}{2} + \frac{\mathcal{C}_A(|p-\omega|b)}{2}\right)\mathbf{F}(\mathbf{b})$$

$$\mathcal{C}_R(|\omega|\,b) \equiv \int \frac{d^2k_\perp}{(2\pi)^2}\left(1-e^{i\omega\mathbf{b}\cdot\mathbf{k}_\perp}\right)\mathcal{C}_R(k_\perp)$$

- At NLO perturb **F** with δm_∞ and δC

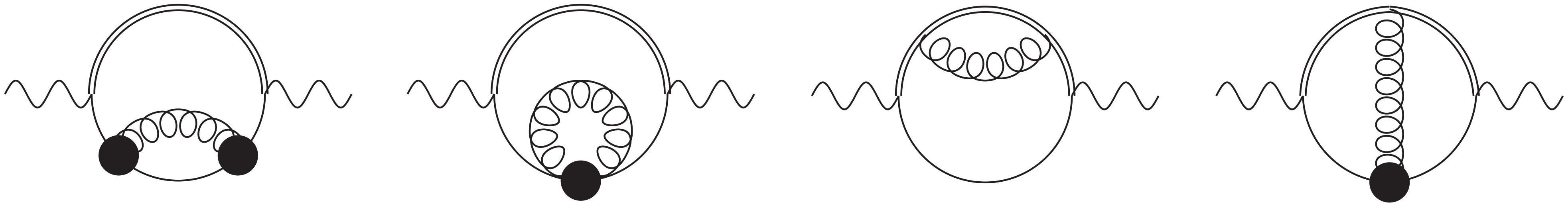
Conversion corrections

$$\Gamma^{\rm conv}_{q\to g}(p) = -\frac{g^2}{8d_F p}\int_{-\infty}^{+\infty} dx^+ \left\langle \mathrm{Tr}\left[U_F(-\infty,x^+)T^a\bar\psi(x^+)\not{v}U_A(x^+,0)\psi(0)T^bU_F(0,-\infty)\right]\right\rangle$$

- Operator ordering is not relevant at NLO: abelianization of Wilson line operator

$$\Gamma^{\rm conv}_{q\to g}(p) = -\frac{g^2C_F}{8d_F p}\int_{-\infty}^{+\infty} dx^+ \left\langle \mathrm{Tr}\left[\bar\psi(x^+)\not{v}U_F(0,x^+)\psi(0)\right]\right\rangle$$

- Exactly what was computed for the NLO photon rate:

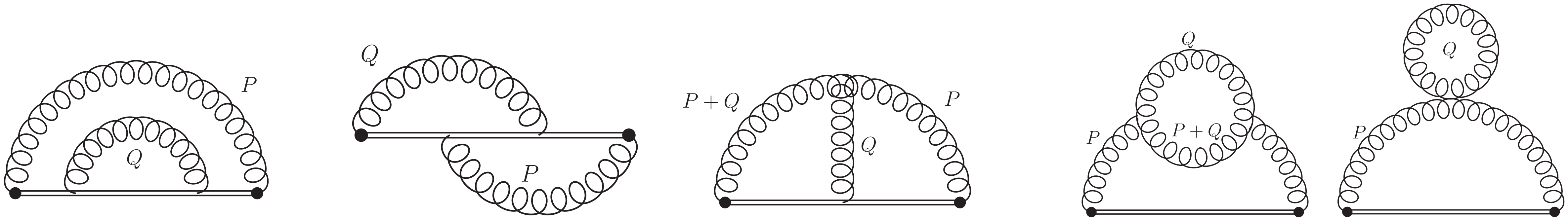


after collinear subtraction:

$$\Gamma^{\rm conv}_{q\to g}(p) \propto \int\frac{d^2q_\perp}{(2\pi)^2}\frac{m_\infty^2+\delta m_\infty^2}{q_\perp^2+m_\infty^2+\delta m_\infty^2} = \int\frac{d^2q_\perp}{(2\pi)^2}\left[\frac{m_\infty^2}{q_\perp^2+m_\infty^2}+\frac{q_\perp^2\,\delta m_\infty^2}{(q_\perp^2+m_\infty^2)^2}\right]$$

Diffusion corrections

- At NLO one has these diagrams



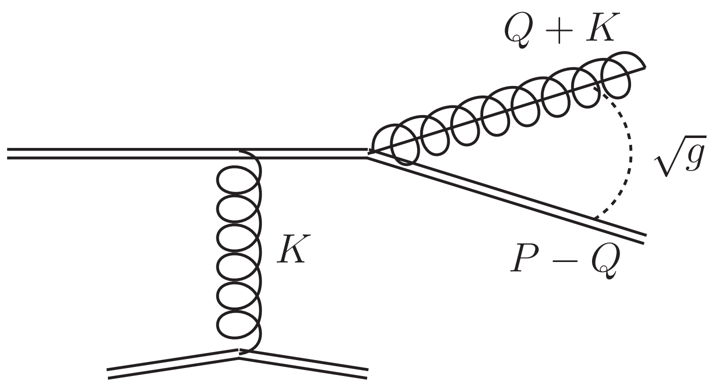
- Unsurprisingly?

$$\hat{q}_L \propto T \int \frac{d^2q_\perp}{(2\pi)^2} \frac{M_\infty^2 + \delta M_\infty^2}{q_\perp^2 + M_\infty^2 + \delta M_\infty^2} = T \int \frac{d^2q_\perp}{(2\pi)^2} \left[\frac{M_\infty^2}{q_\perp^2 + M_\infty^2} + \frac{q_\perp^2\, \delta M_\infty^2}{(q_\perp^2 + M_\infty^2)^2} \right]$$

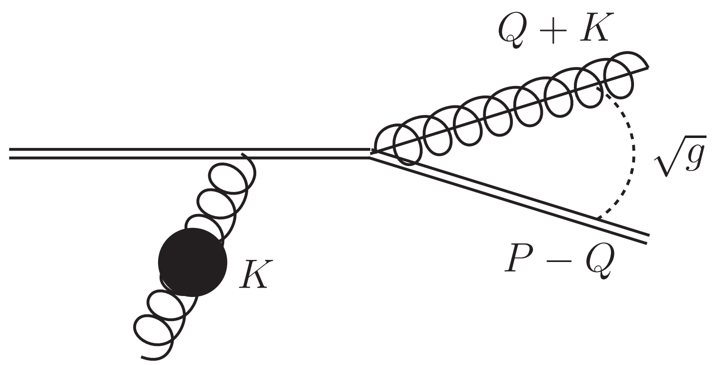
- Can a SCET-like EFT help understand these results?

Semi-collinear processes

- Seemingly different processes boiling down to wider-angle radiation



K soft cut, spacelike



K soft plasmon, timelike

- Evaluation: introduce "*modified* $\hat{q}$" that keep tracks of the changes in the small light-cone component p^- of the quarks

"*standard*" $$\frac{\hat{q}}{g^2 C_R} \equiv \frac{1}{g^2 C_R} \int \frac{d^2 q_\perp}{(2\pi)^2} q_\perp^2 \mathcal{C}(q_\perp) \propto \int d^4Q \langle F^{+\mu}(Q) F^{+}_{\mu}(-Q) \rangle_{q^-=0}$$

"*modified*" $$\frac{\hat{q}(\delta E)}{g^2 C_R} \propto \int d^4Q \langle F^{+\mu}(Q) F^{+}_{\mu}(-Q) \rangle_{q^-=\delta E}$$

- The "*modified* $\hat{q}$" can also be evaluated in EQCD

- The regulator dependence vanishes across all regions

Conclusions

- Useful reorganization of the kinetic theory that shows the appearance of gauge-invariant light-front operators that effectively describe soft momentum exchanges
- These operators can be evaluated using new techniques and are of two kinds
  - Euclidean ($C(x_\perp)$, $\hat{q}(\delta E)$): can also be evaluated on the (EQCD) lattice
  - "Collinear": are the same for bosons and fermions, include the effect of the modified dispersion relation at LO and NLO
- Applications are underway: implementation in MARTINI, thermalization studies and estimates of NLO corrections to η