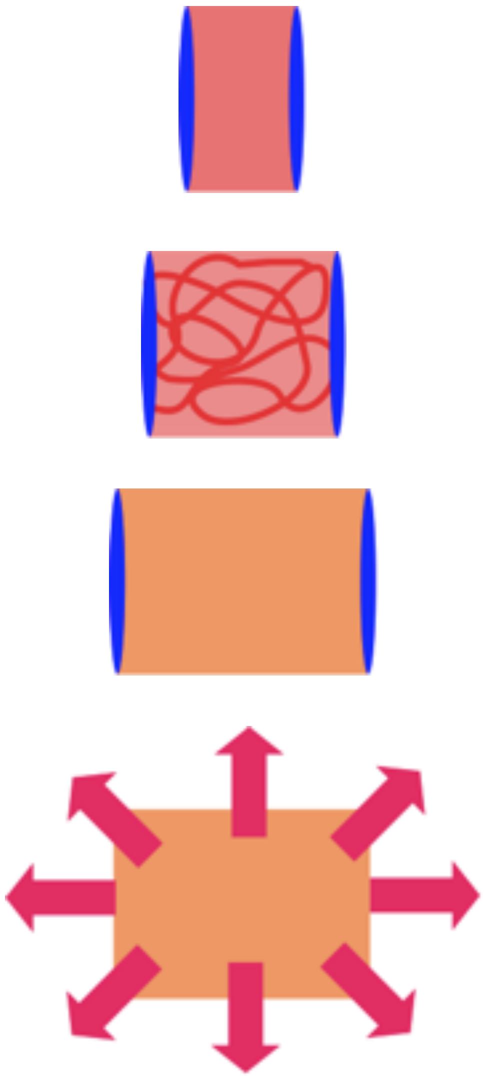
Quark Loops and Photons with CGC in pA

Kenji Fukushima
The University of Tokyo

ongoing work with Sanjin Benic

Four Steps in HIC



Color Glass Condensate (CGC)

$$\tau \lesssim 1/Q_s \sim 0.1\mathrm{fm/c}$$

Color Glass + Plasma = Glasma

$$\tau \lesssim \tau_0 \sim 1\mathrm{fm/c}$$

(s) Quark-Gluon Plasma

$$\tau \lesssim \tau_f \sim 10\mathrm{fm/c}$$

Hadronization (quarks → hadrons)

Lattice EoS ~ HRG

Three Keywords in Early Dynamics

Isotropization

Gelis, Epelbaum, Berges, Venugopalan, Schlichting

Complete isotropization is not necessary.
Stability of a certain isotropization (< 50%?) is required.

Hydronization

Chesler, Yaffe, Janik, Strickland, Heinz

Hydrodynamics would be a better description with more and more dissipative terms.
Anisotropic viscous hydro may work better?

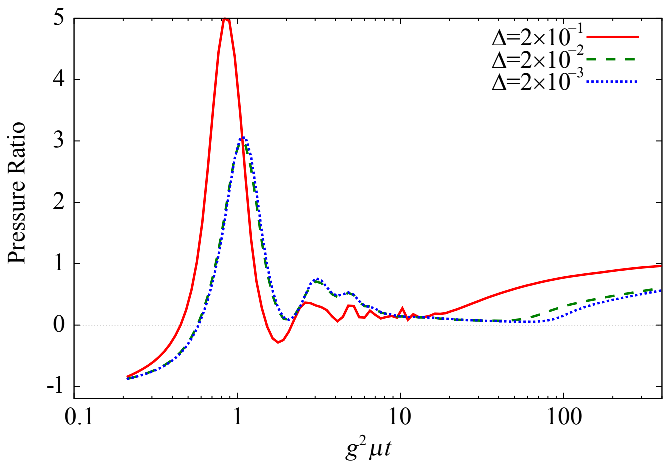
Thermalization

Blaizot, McLerran, Liao, Gelis, Berges, Kurkela, Moore

What is seen in experiment is a thermal p_t distribution of hadrons — thermal gluons? Turbulence? BEC? **Photons?**

Issues on Isotropization

Q: Is the CSA good to give fast isotropization if the system is NOT expanding?

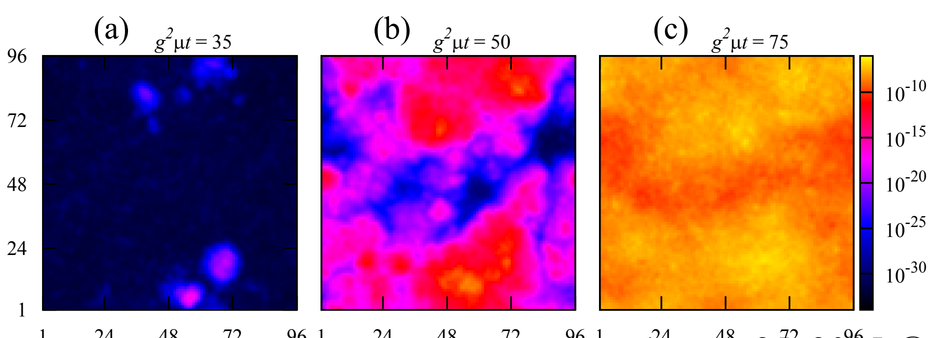


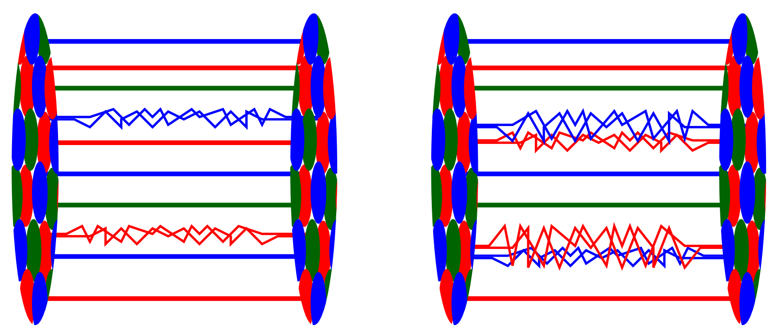
Fukushima (2013)

Maybe the full quantum fluct. cures?

yes: **Gelis, Epelbaum, Moore, Wu**

Free streaming fixed point?

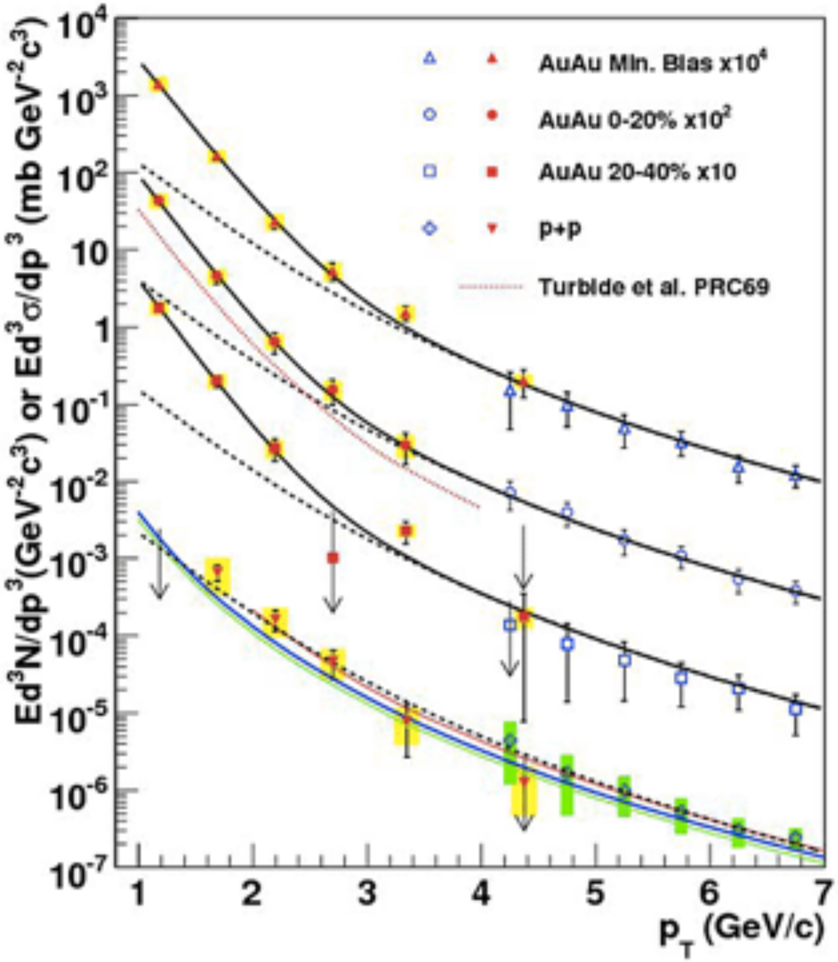




(a) Local Avalanche in Longitudinal

(b) Diffusion in Transverse

Issues on Thermalization

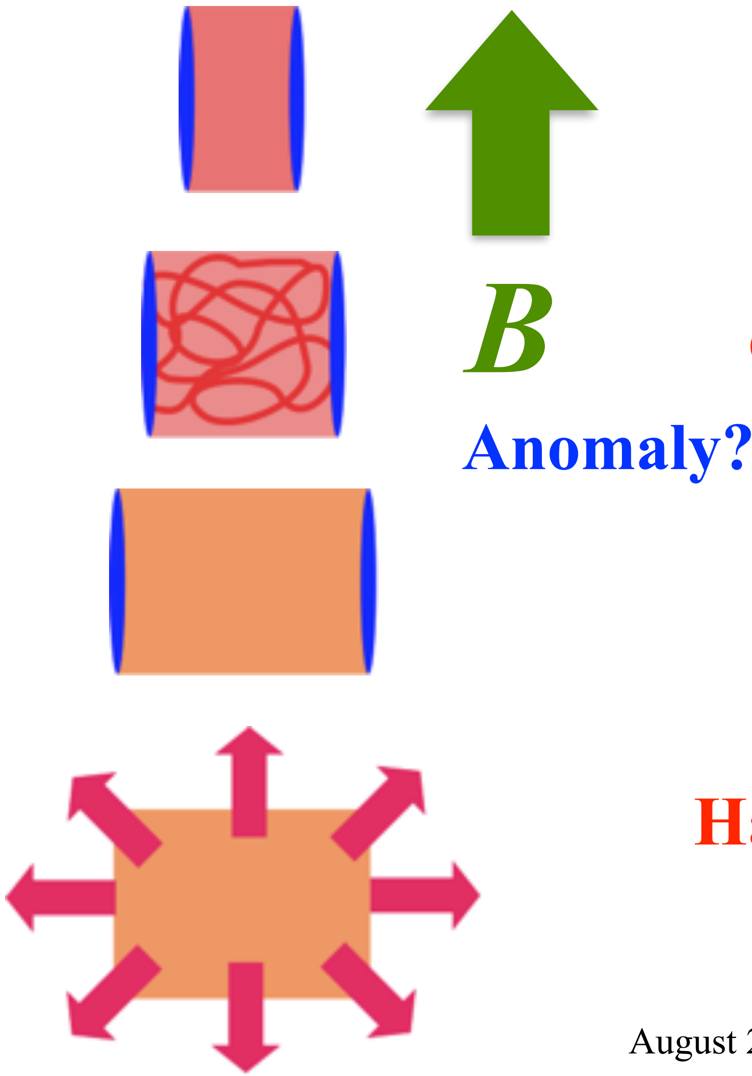


RHIC / PHENIX

* photon puzzle
  - baryons-antibaryons
  - pion Bremsstrahlung
    (Ralf Rapp)

* photon elliptic flow

Four Steps in HIC



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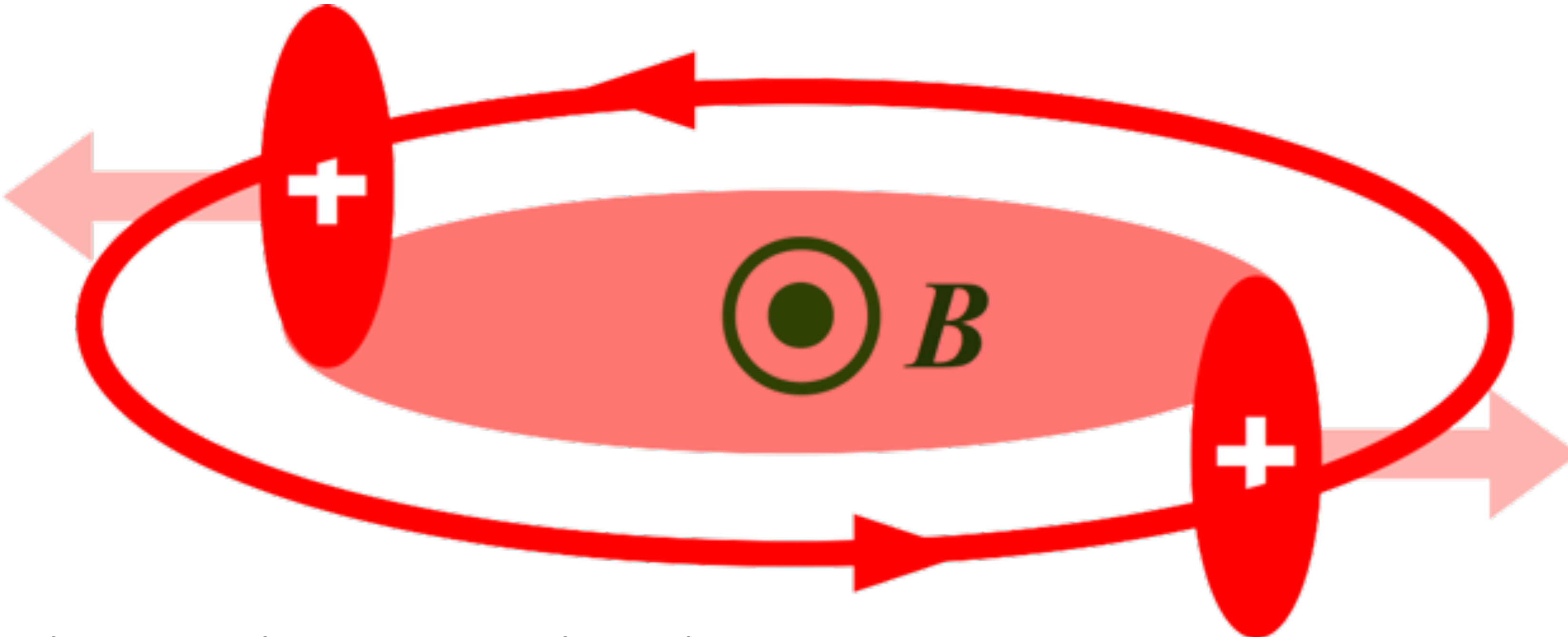
Hadronization (quarks → hadrons)

Classical Picture for B



Point-particle approximation:

$$eB_0 = (47.6\ \mathrm{MeV})^2 \left(\frac{1\ \mathrm{fm}}{b}\right)^2 Z \sinh Y \qquad t_0 = \frac{b}{2\sinh Y}$$

"strongest magnetic field in the Universe"

What I want to do...

Initial State in High-Energy AA Collisions

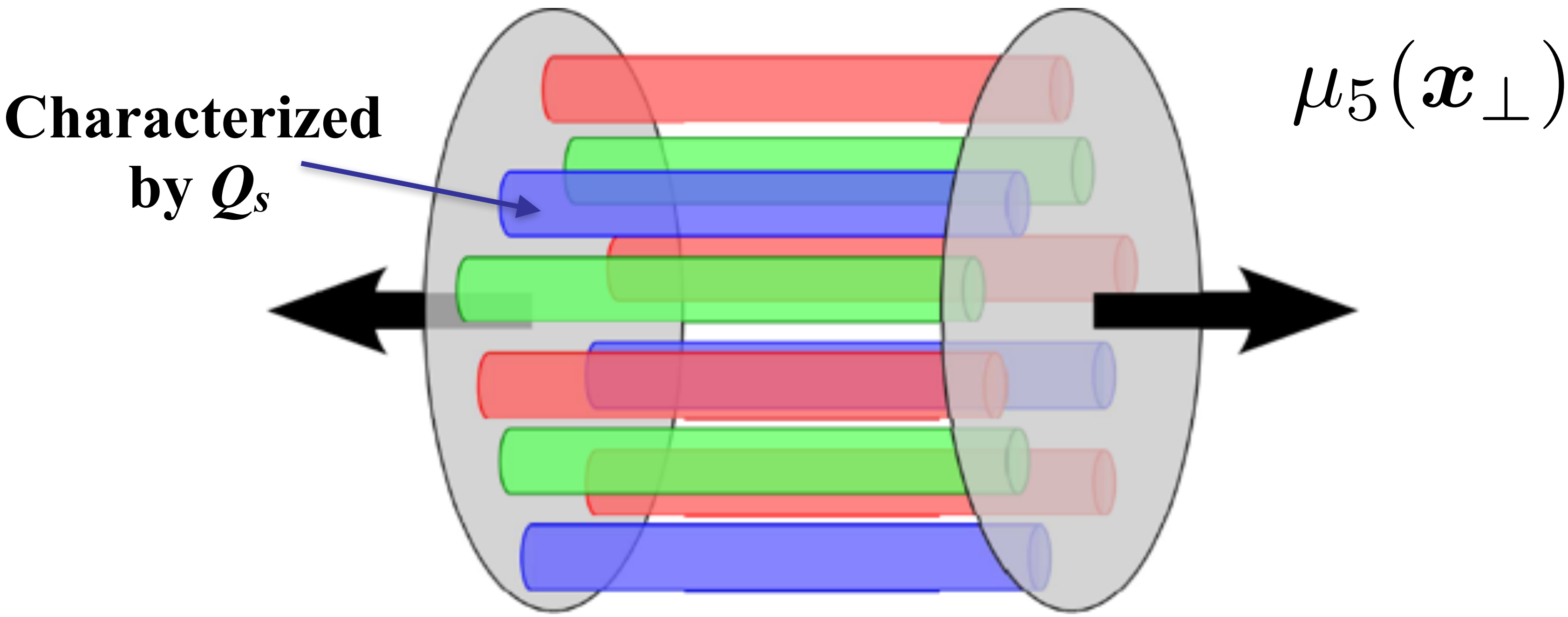
Magnetic Fields

Photon Production **Quark Pair Production**

Anomalous Transport
(more direct relevance than hydro/phase diagram)

Expanding CGC

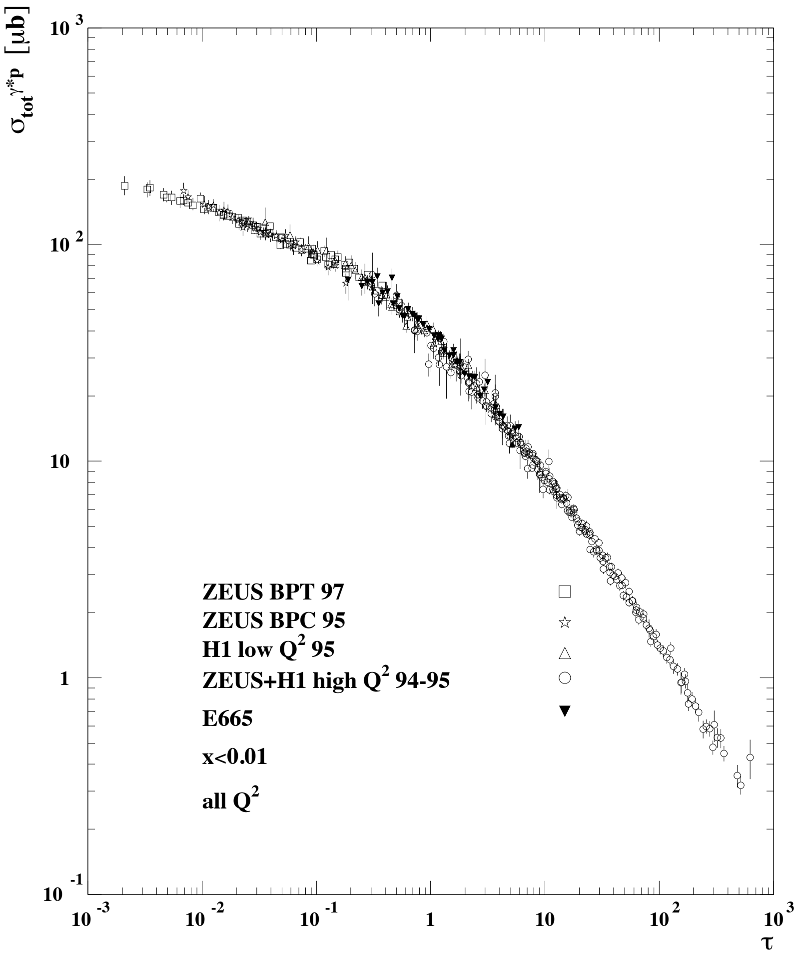
Longitudinal Fields (*Local Parity Violation*)



Simulation starts with "negative" pressure: $P_L < 0$

Remark on CGC

"Saturation" is not needed, but just "Scaling"



Scaling variable:

$$\tau = Q^2/Q_s^2(x)$$

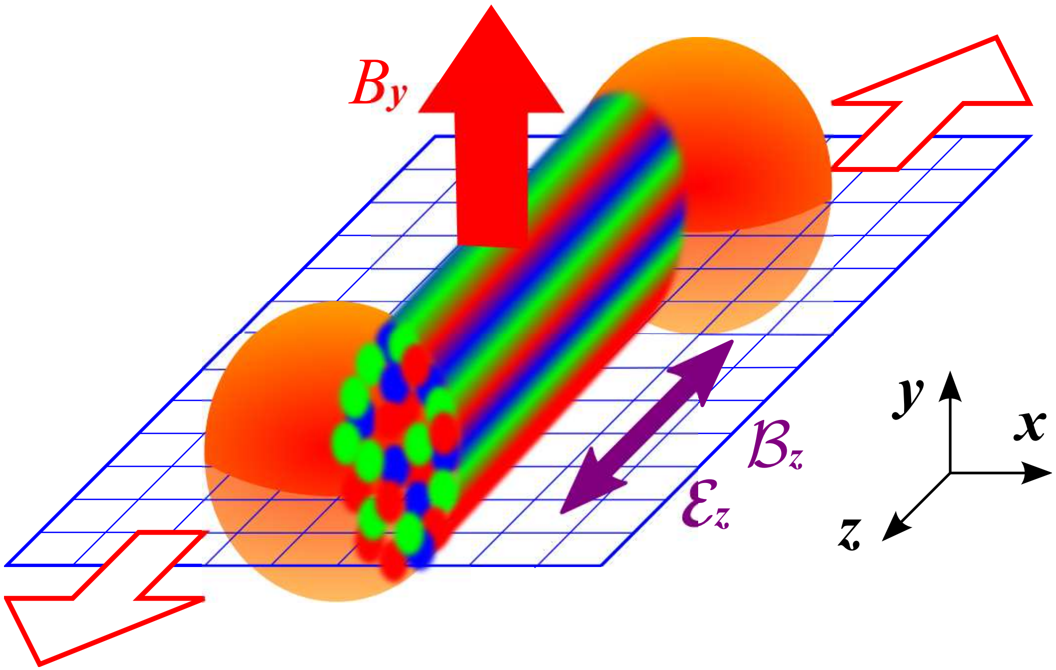
Saturation momentum:

$$Q_s^2(x) = Q_0^2(x/x_0)^{-\lambda}$$

Golec-Biernat, Kwiecinski, Stasto, Wuesthoff

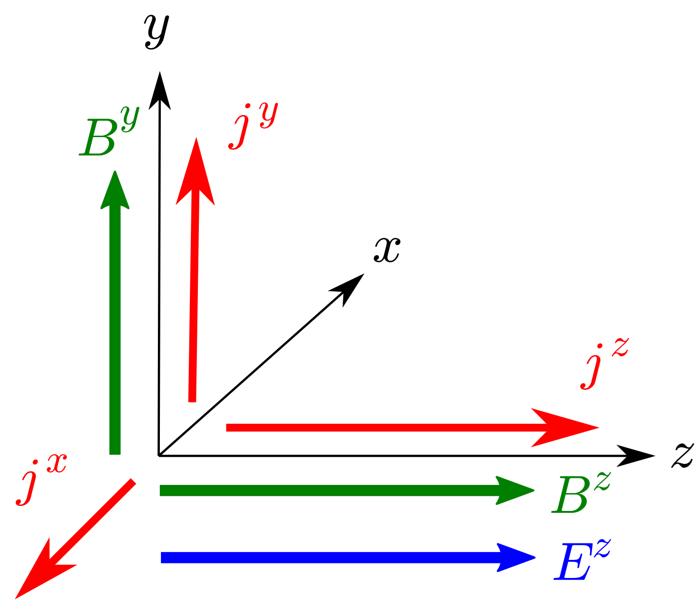
"Geometric Scaling"

B-CGC



$$j \sim (\mathcal{E}\mathcal{B})B$$

Schematically

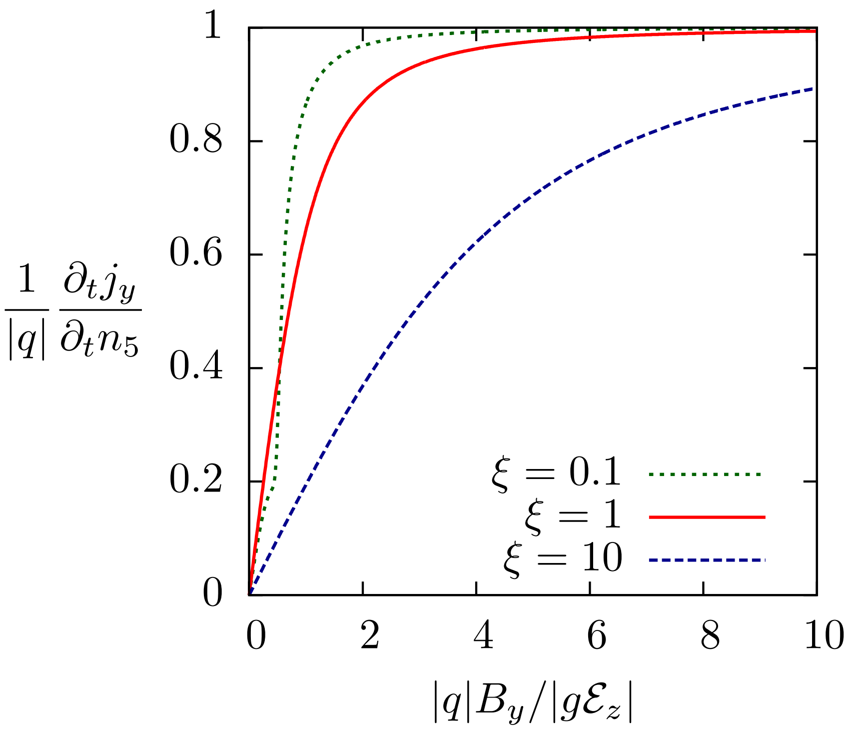


Real-time Chiral Magnetic Effect (CME)

Fukushima-Kharzeev-Warringa (2009)

B-CGC

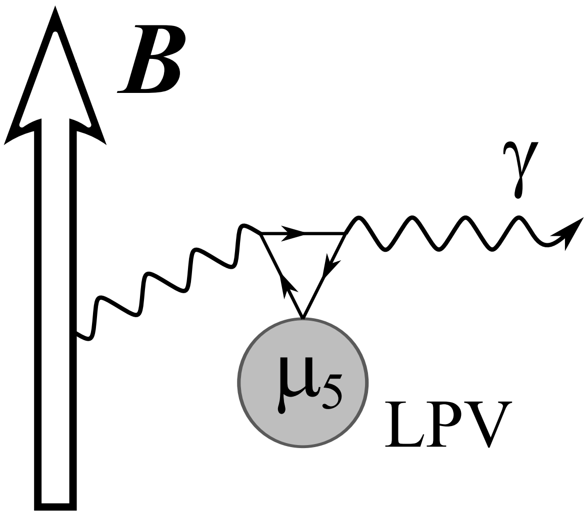
Analytical calculation for uniform fields
Current = CP-breaking Schwinger Mechanism



What is new with fields *inhomogeneous* in space/time?

B-induced Photons

Inhomogeneous *B* / μ_5 carries energy/momentum



$$\mathcal{L}_{\text{eff}} \sim \varepsilon^{\mu\nu\rho\sigma} A_\mu F_{\nu\rho} \partial_\sigma \theta$$

$$(j^z \sim \partial_t \theta F_{xy})$$

$$\frac{dN_\gamma}{d^3q} \propto \sum_i \epsilon^{(i)y}(\boldsymbol{q})\epsilon^{(i)y}(\boldsymbol{q})$$

$$= \frac{q_z^2 + q_x^2}{\boldsymbol{q}^2}$$

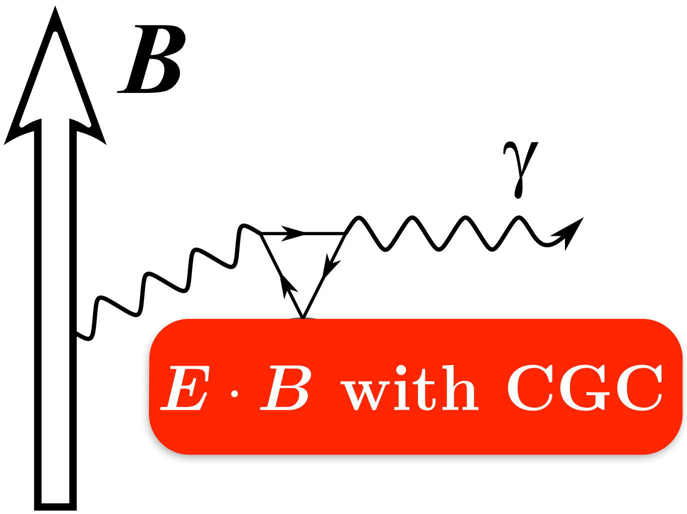
Reversed Primakoff Effect

Fukushima-Mameda (2012)

cf. Basar-Kharzeev-Skokov (2012)

B-induced Photons

Inhomogeneous *B* / μ_5 carries energy/momentum



$$\mathcal{L}_{\rm eff} \sim \varepsilon^{\mu\nu\rho\sigma} A_\mu F_{\nu\rho} \partial_\sigma \theta$$

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$$= \boxed{\frac{q_z^2 + q_x^2}{\boldsymbol{q}^2}}$$

Quark "one-loop" fluct.

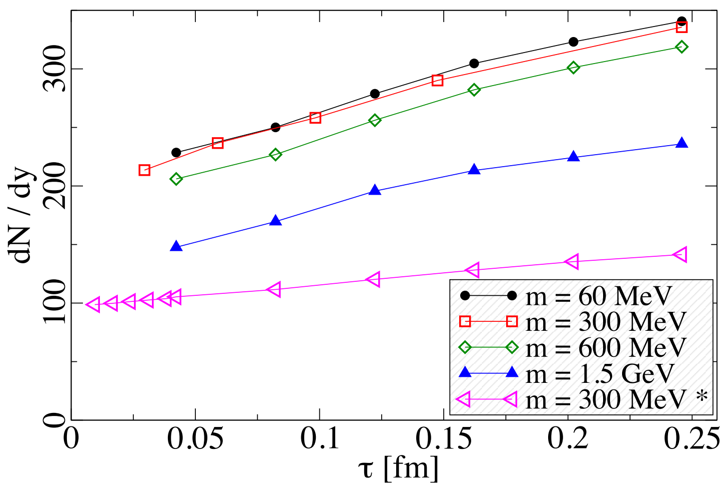
Non-perturbative Formulation

Gelis-Kajantie-Lappi (2005)

$$M_\tau(p,q) \equiv \int \frac{\tau \mathrm{d}z \mathrm{d}^2\mathbf{x}_T}{\sqrt{\tau^2+z^2}} \phi^\dagger_{\mathbf{p}}(\tau,\mathbf{x})\gamma^0\gamma^\tau\psi_{\mathbf{q}}(\tau,\mathbf{x})$$

Amplitude from *anti-particles* to *particles*

$$\frac{dN}{dy} = \int \frac{\mathrm{d}y_p \mathrm{d}^2\mathbf{p}_T}{2\left(2\pi\right)^3} \frac{\mathrm{d}y_q \mathrm{d}^2\mathbf{q}_T}{2\left(2\pi\right)^3} \delta\left(y-y_p\right)\left|M_\tau(p,q)\right|^2$$



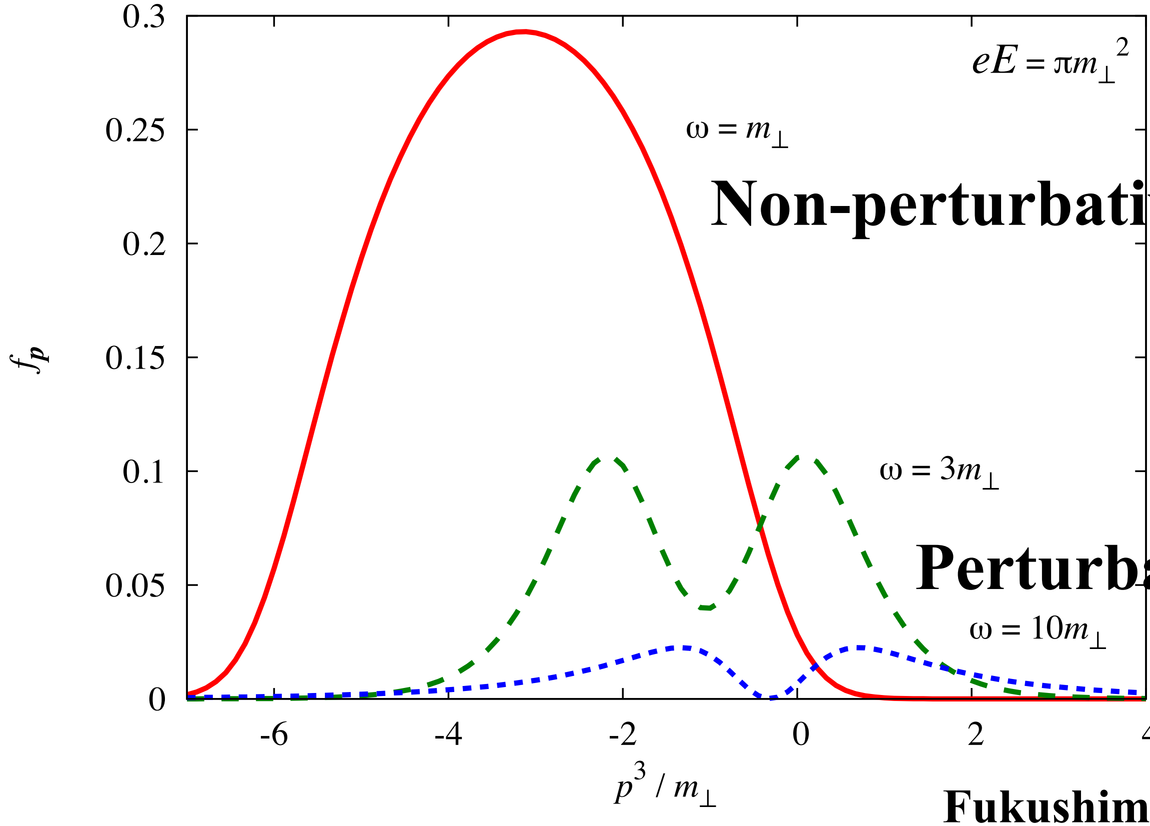
$$\psi_{\mathbf{q}}(t\to-\infty,\mathbf{x}) = e^{iq\cdot x}v(q)$$

$$\phi_{\mathbf{p}}(x) = e^{-ip\cdot x}u(p)$$

Very early ($\tau < 0.1$fm/c)

Simple Example

Schwinger mechanism in scalar QED



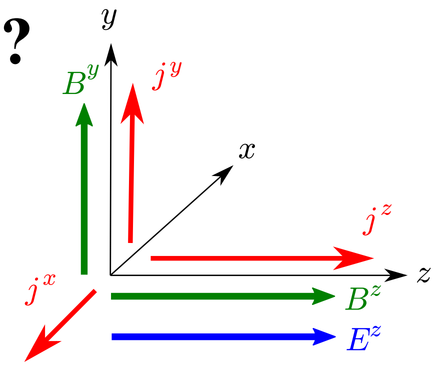
Non-perturbative (uniform)

Perturbative (pulse)

Fukushima-Gelis-Lappi (2009)

Consistency Check

$\mathcal{Q}$: Is it possible to reproduce the real-time CME for uniform fields using the GKL formalism?



$\mathcal{A}$: of course yes!

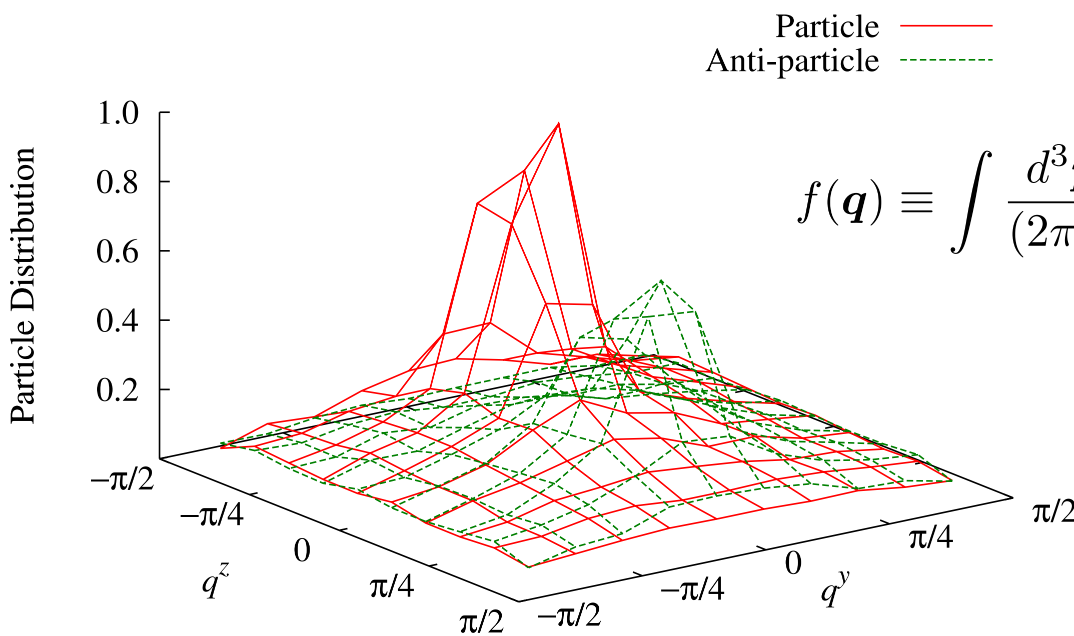
But, unclear whether it is technically feasible?

Numerical test with uniform fields without expansion

Consistency Check

$$\beta_{\boldsymbol{q},\boldsymbol{p}} = \int d^3\boldsymbol{x}\, \frac{u_{\rm R}^\dagger(\boldsymbol{q}_{A'}) e^{i|\boldsymbol{q}_{A'}|x^0 + i\boldsymbol{q}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{q}_{A'}|}}\, g_{-\boldsymbol{p}}(x^0,\boldsymbol{x})$$

Initial Cond. $g_{\boldsymbol{p}}(x) \longrightarrow \dfrac{v_{\rm R}(\boldsymbol{p}_{-A}) e^{i|\boldsymbol{p}_{-A}|x^0 - i\boldsymbol{p}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{p}_{-A}|}}$

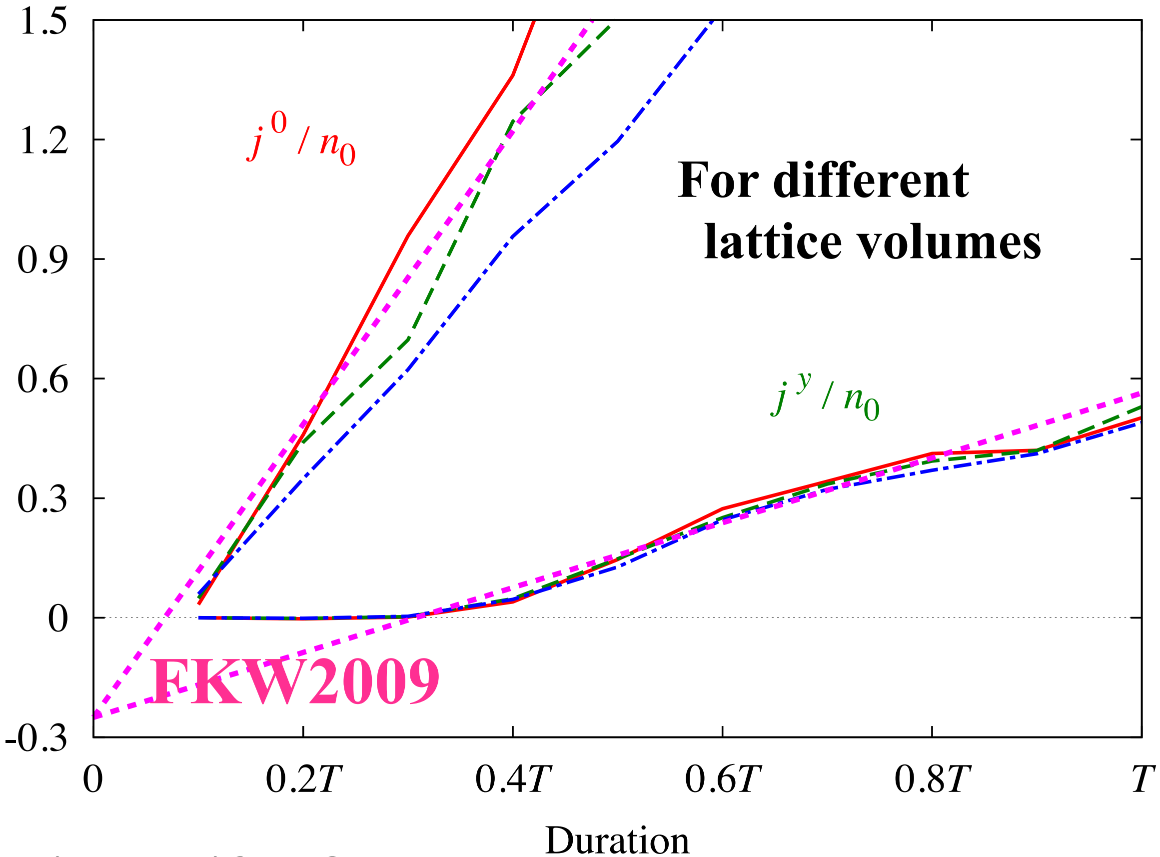


$$f(\boldsymbol{q}) \equiv \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\, |\beta_{\boldsymbol{q},\boldsymbol{p}}|^2$$

Asymmetric dist. in the *R*-sector

Fukushima (2015)

Consistency Check



Slopes look good

Offset???

UV contamin???

Semi-qualitatively Okay

lattice artifact?

Fukushima (2015)

Toward B-CGC Simulations

CGC Background Fields ✔
(Glasma instability is not needed for the moment)

Bogoliubov Coefficients (GKL formalism) ✔

Initial Conditions at $\tau = 0^+$?

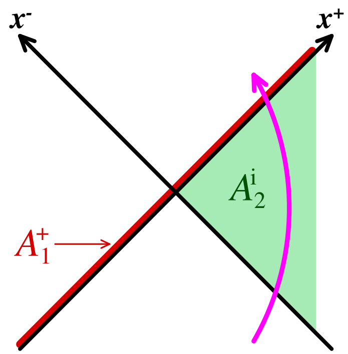
Toward B-CGC Simulations

CGC Background Fields ✔
(Glasma instability is not needed for the moment)

Bogoliubov Coefficients (GKL formalism) ✔

Initial Conditions at $\tau = 0^+$ ✔

Gelis-Tanji (2015)



What I want to do...

Initial State in High-Energy AA Collisions

Magnetic Fields

Photon Production **Quark Pair Production**

Anomalous Transport
(more direct relevance than hydro/phase diagram)

What I can do… so far…

Initial State in High-Energy ~~AA~~ Collisions

pA

~~Magnetic Fields~~

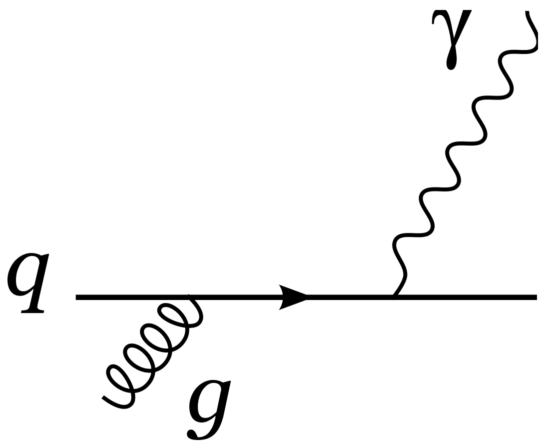
Photon Production

Quark Pair Production

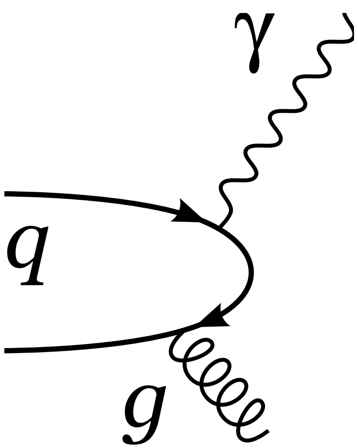
Photons from quark loops (for technical simplicity)

Conventional Photons

Compton Scattering



Annihilation



+ LPM

$$\propto \;\alpha_e \alpha_s \, n_q (1 - n_q) n_g$$

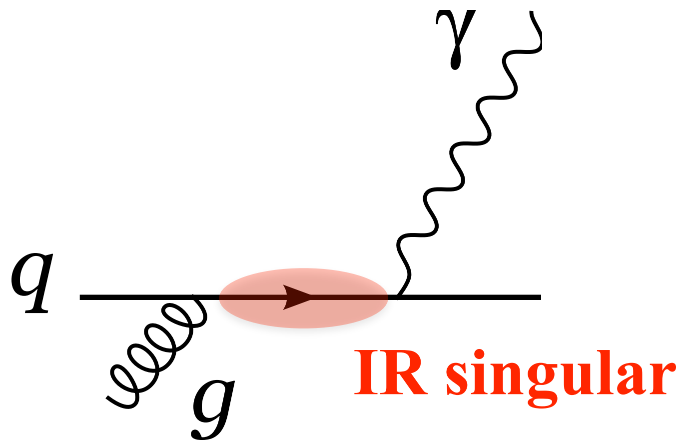
$$(qg \;\to\; q\gamma)$$

$$\propto \;\alpha_e \alpha_s \, n_q n_{\bar{q}} (1 + n_g)$$

$$(q\bar{q} \;\to\; g\gamma)$$

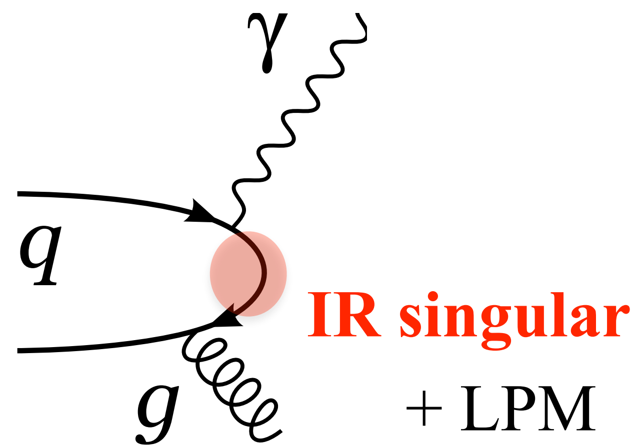
Conventional Photons

Compton Scattering



$$\propto \alpha_e \alpha_s n_q (1 - n_q) n_g$$
$$(qg \to q\gamma)$$

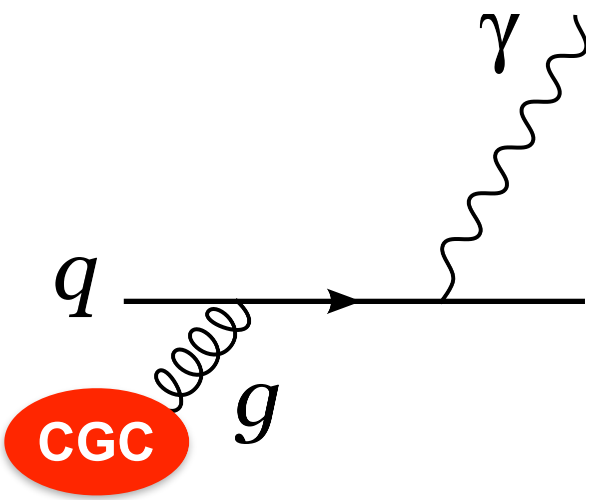
Annihilation



$$\propto \alpha_e \alpha_s n_q n_{\bar{q}} (1 + n_g)$$
$$(q\bar{q} \to g\gamma)$$

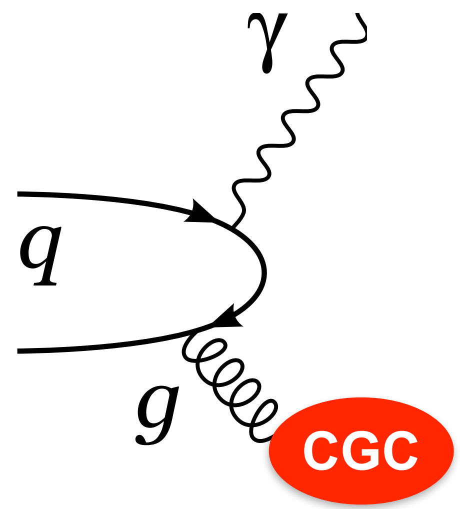
Diagrams involving CGC

Compton Scattering



$$\propto \alpha_e \alpha_s n_q (1 - n_q) \alpha_s^{-1}$$
$$\sim \alpha_e n_q (1 - n_q)$$

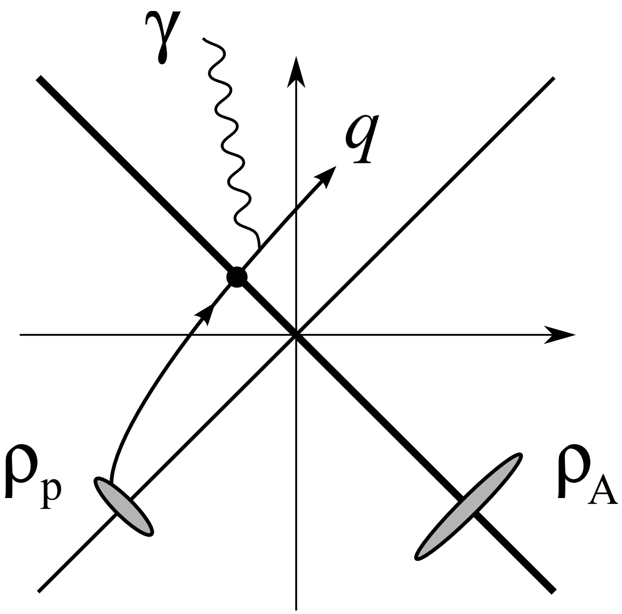
Annihilation



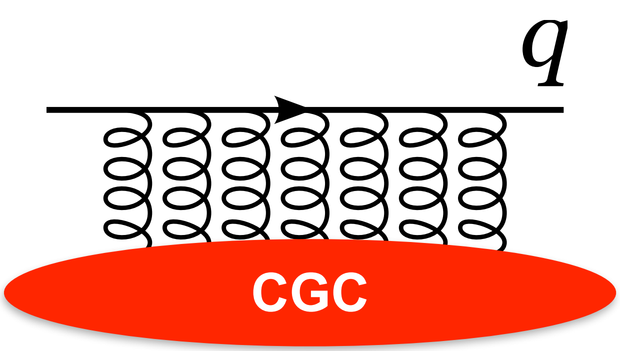
$$\propto \alpha_e \alpha_s n_q n_{\bar{q}} \alpha_s^{-1}$$
$$\sim \alpha_e n_q n_{\bar{q}}$$

Multiple Scattering

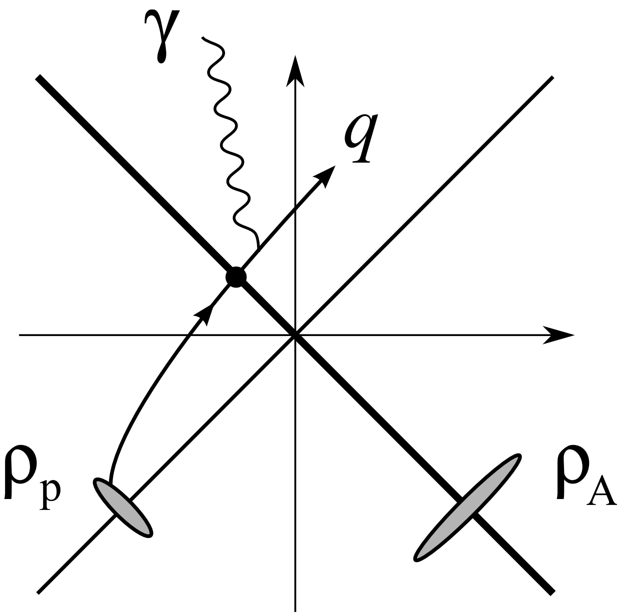
In a certain gauge: $A \sim \rho_{\rm A} \sim \delta(x^+)$



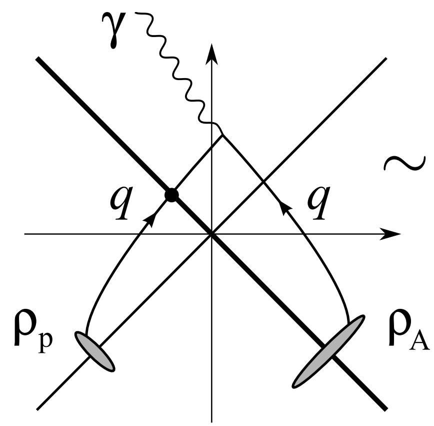
$\bullet \sim U \sim e^{igA} \sim 1 + igA + \frac{1}{2}(igA)^2 + \cdots$



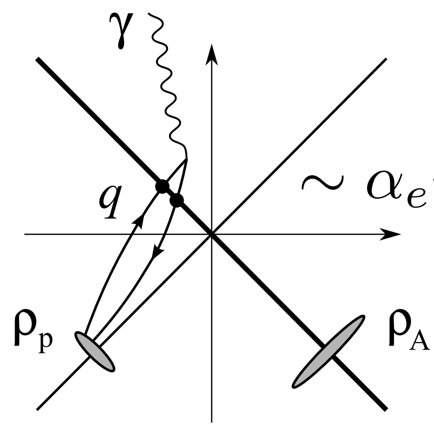
Leading-order Processes



$$\sim \alpha_e n_q^{(\mathrm{p})} \langle UU^\dagger \rangle$$



$$\sim \alpha_e n_q^{(\mathrm{p})} n_q^{(\mathrm{A})} \langle UU^\dagger \rangle$$

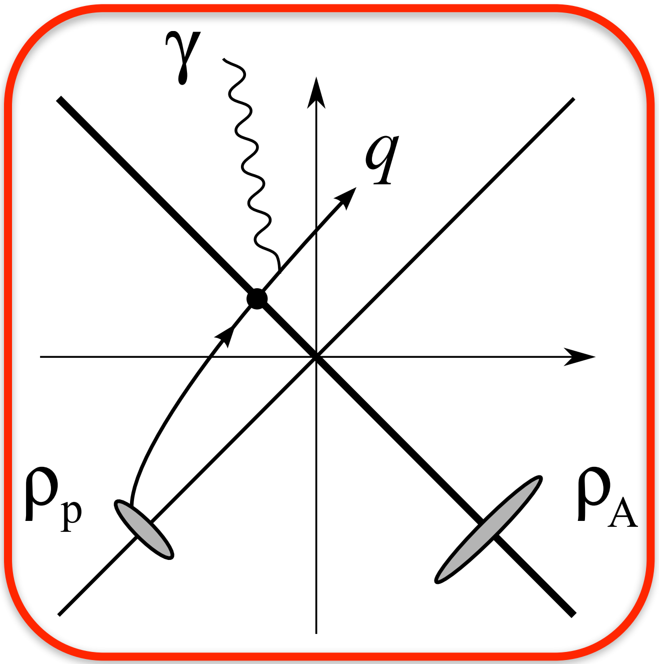


$$\sim \alpha_e n_q^{(\mathrm{p})} n_q^{(\mathrm{p})} \langle UU^\dagger UU^\dagger \rangle$$

Multiple Scattering with CGC

Leading-order Processes

Gelis-Jalilian-Marian (2002)



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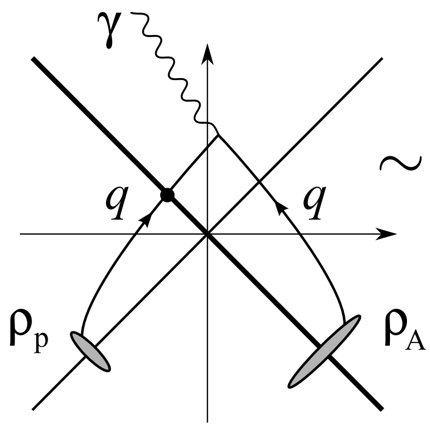
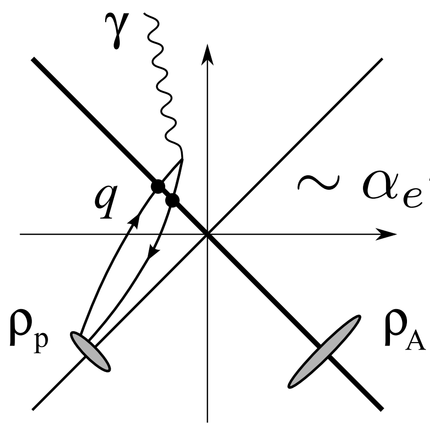


$$\sim \alpha_e n_q^{(\mathrm{p})} n_q^{(\mathrm{A})} \langle U U^\dagger \rangle$$



$$\sim \alpha_e n_q^{(\mathrm{p})} n_q^{(\mathrm{p})} \langle U U^\dagger U U^\dagger \rangle$$

Multiple Scattering with CGC

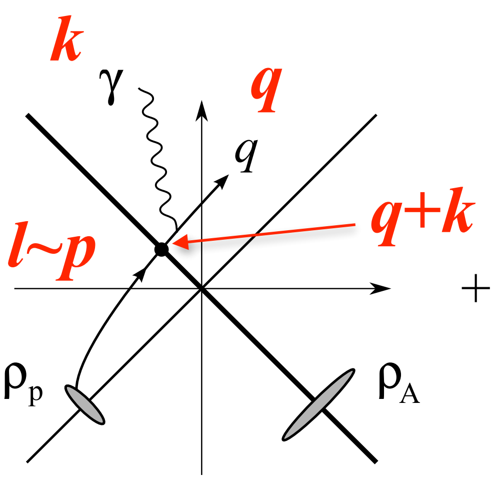
GJ Formula

$$\frac{1}{A_\perp}\frac{d\sigma^{q\to q\gamma}}{d^2\boldsymbol{k}_\perp} = \frac{2\alpha_e}{(2\pi)^4\boldsymbol{k}_\perp^2}\int_0^1 dz\,\frac{1+(1-z)^2}{z}\int d^2\boldsymbol{l}_\perp\,\frac{\boldsymbol{l}_\perp^2 C(\boldsymbol{l}_\perp)}{(\boldsymbol{l}_\perp-\boldsymbol{k}_\perp/z)^2}$$

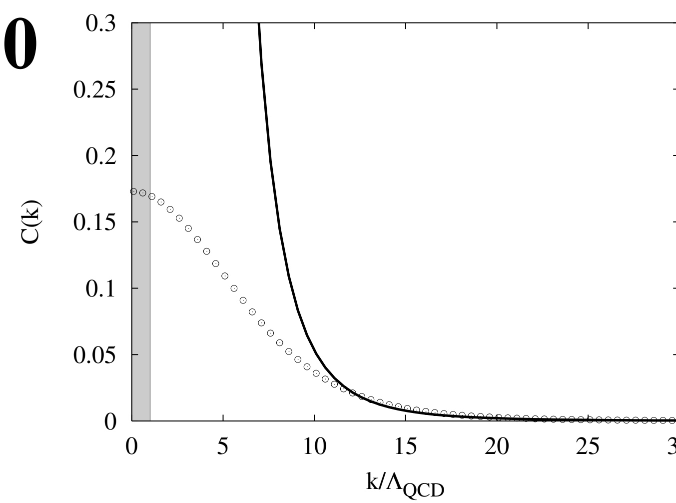
$$C(\boldsymbol{l}_\perp) \equiv \int d^2\boldsymbol{x}_\perp e^{i\boldsymbol{l}_\perp\cdot\boldsymbol{x}_\perp} e^{-B_2(\boldsymbol{x}_\perp)} = \int d^2\boldsymbol{x}_\perp e^{i\boldsymbol{l}_\perp\cdot\boldsymbol{x}_\perp}\left\langle U(0)U^\dagger(\boldsymbol{x}_\perp)\right\rangle_\rho$$

$$B_2(\boldsymbol{x}_\perp-\boldsymbol{y}_\perp) \equiv Q_s^2\int d^2\boldsymbol{z}_\perp[G_0(\boldsymbol{x}_\perp-\boldsymbol{z}_\perp)-G_0(\boldsymbol{y}_\perp-\boldsymbol{z}_\perp)]^2$$

Per one massless quark with *p* = 0

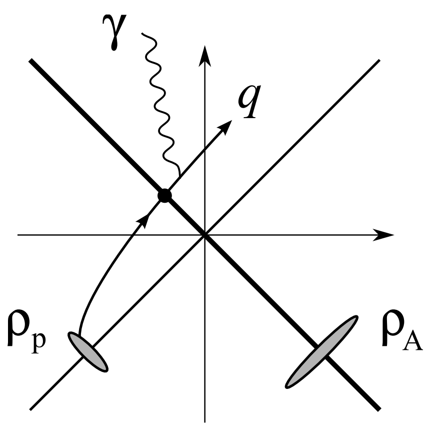


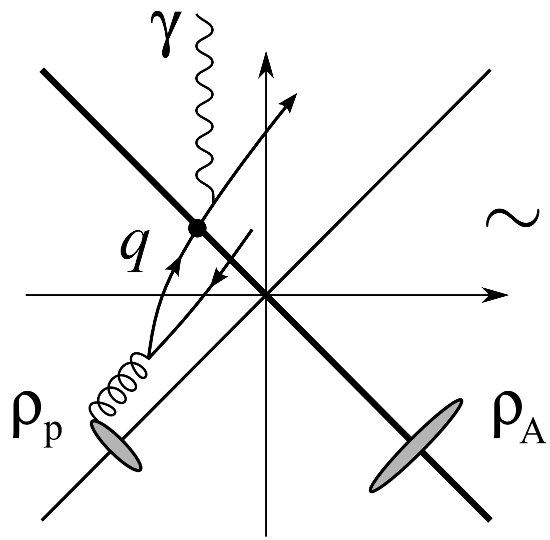
+ crossed diagram
(photon emitted first)



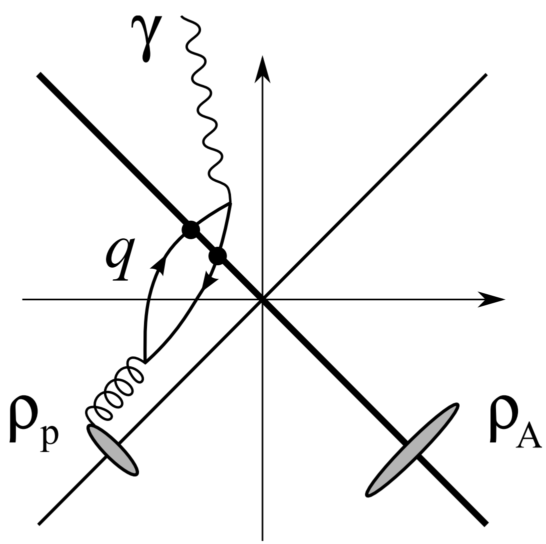
Gelis-Jalilian-Marian (2002)

Higher-order Processes





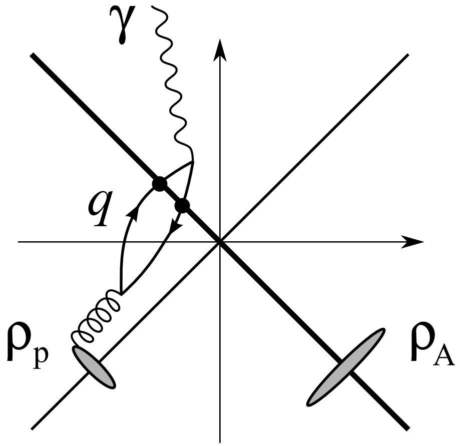
$$\sim \alpha_e \delta n_q^{(\mathrm{p})} \langle U U^\dagger \rangle$$



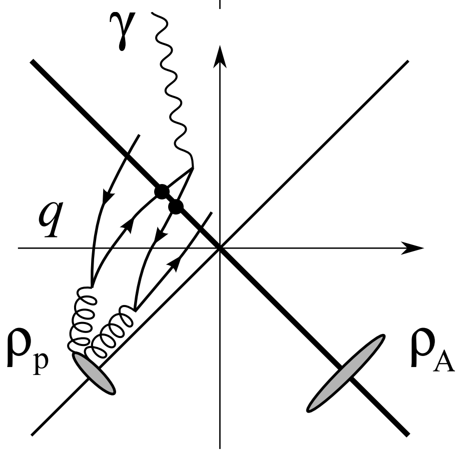
$$\sim \alpha_e \langle (g\rho_{\mathrm{p}})^2 \rangle \langle U U^\dagger U U^\dagger \rangle$$

Higher-order Processes

Should be more and more important as approaching AA



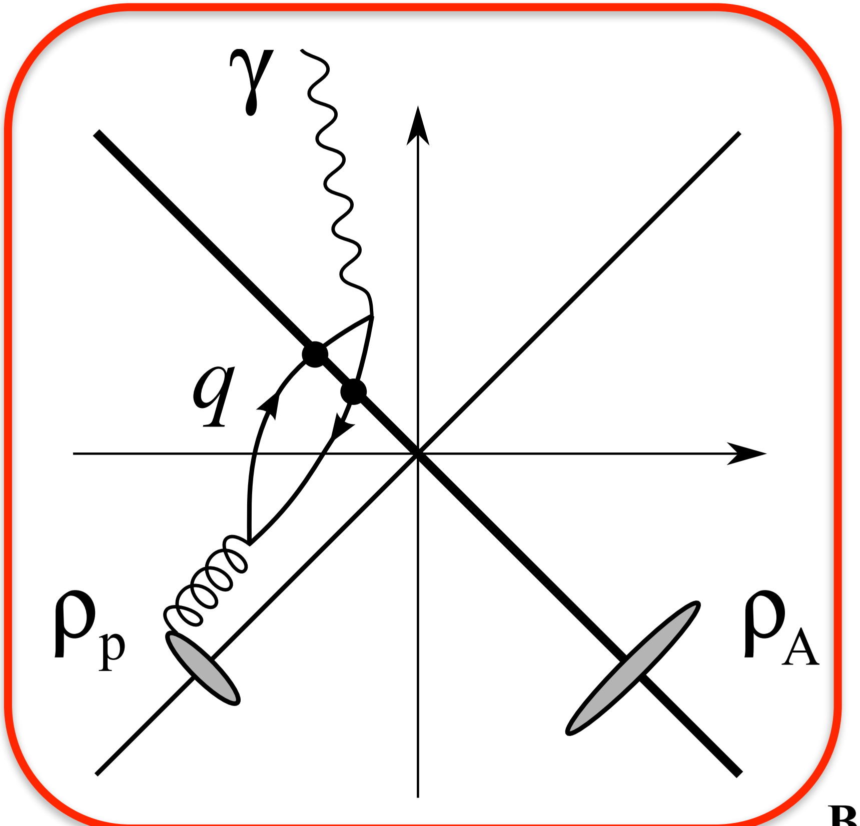
$$\sim \alpha_e \langle (g\rho_{\mathrm{p}})^2 \rangle \langle U U^\dagger U U^\dagger \rangle$$



$$\sim \alpha_e \langle (g\rho_{\mathrm{p}})^4 \rangle \langle U U^\dagger U U^\dagger \rangle$$

Interested Regime

$$(g\rho_{\rm p})^2 < g\rho_{\rm p} \sim n_q^{(\rm p)}$$

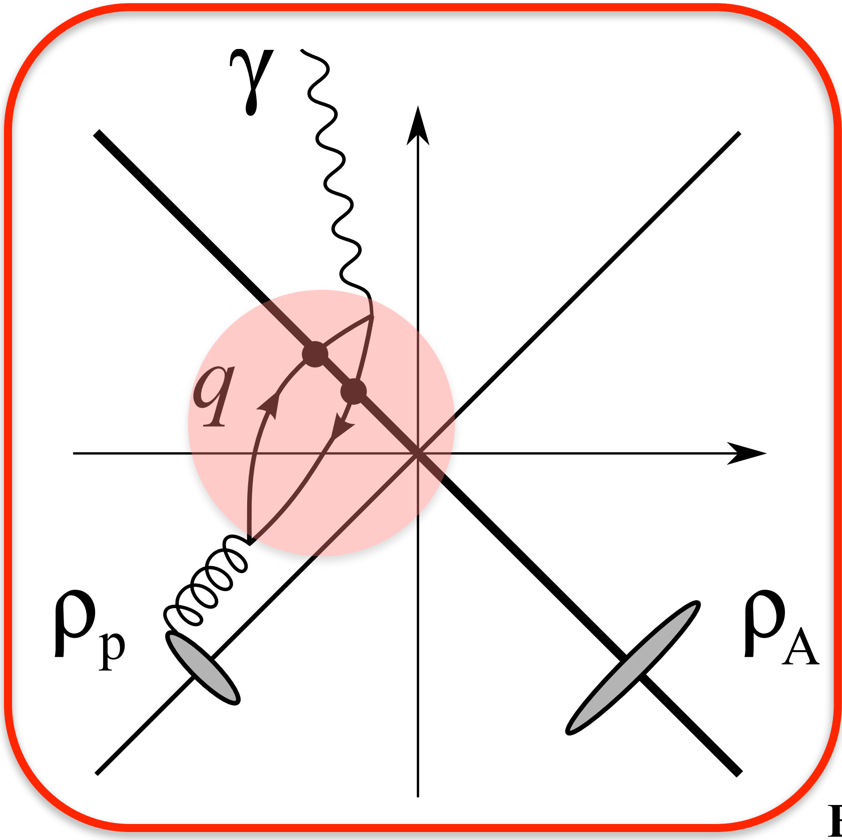


Benic-Fukushima (2015)

Interested Regime

$$(g\rho_{\rm p})^2 < g\rho_{\rm p} \sim n_q^{(\rm p)}$$

Lowest-order vanishes due to Furry's theorem

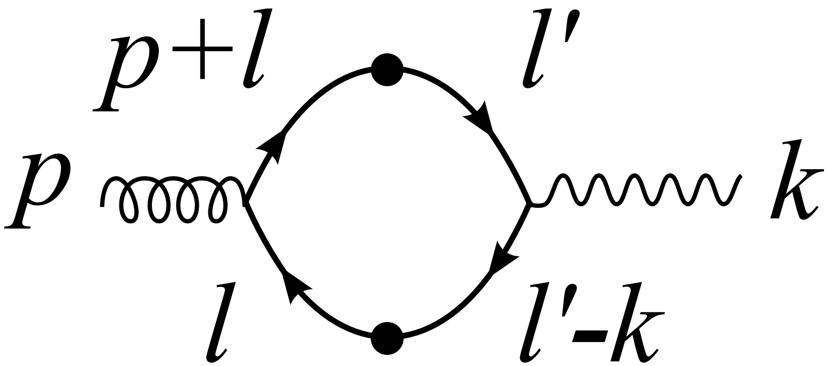


Anomaly sensitive?

Odderon needed?

Benic-Fukushima (2015)

Amplitude (preliminary)



Same approximation as GJ
~ massless with $\boldsymbol{p} = 0$

$$\langle \boldsymbol{k}, \lambda | \mathrm{pA} \rangle = 2\sqrt{\alpha_e}\sqrt{\alpha_s} \int \frac{d^2\boldsymbol{p}_\perp d^2(\boldsymbol{l}_\perp - \boldsymbol{l}'_\perp)}{(2\pi)^4} \frac{\mathcal{T}(\boldsymbol{k}_\perp, \boldsymbol{p}_\perp, \boldsymbol{l}_\perp - \boldsymbol{l}'_\perp)}{\boldsymbol{p}_\perp^2} \int \frac{d^2(\boldsymbol{l}_\perp + \boldsymbol{l}'_\perp)}{(2\pi)^2} \int_0^1 dx \, \frac{x\boldsymbol{k}_\perp - \boldsymbol{l}'_\perp}{(x\boldsymbol{k}_\perp - \boldsymbol{l}'_\perp)^2 + \boldsymbol{l}_\perp^2 - \boldsymbol{l}'^2_\perp}$$

Color matrix

"Calculable" part requiring some gauge inv. regularization

$$\mathcal{T}(\boldsymbol{k}_\perp, \boldsymbol{p}_\perp, \boldsymbol{l}_\perp - \boldsymbol{l}'_\perp) \equiv \mathrm{tr}\big[U(-\boldsymbol{p}_\perp - \boldsymbol{l}_\perp + \boldsymbol{l}'_\perp)\rho_\mathrm{p}(\boldsymbol{p}_\perp)U^\dagger(-\boldsymbol{l}_\perp + \boldsymbol{l}'_\perp - \boldsymbol{k}_\perp)\big]$$

Color Average (MV Model)

$$\langle \mathcal{T}(\boldsymbol{k}_\perp, \boldsymbol{p}_\perp, \boldsymbol{\Delta}_\perp)\mathcal{T}(\boldsymbol{k}_\perp, \boldsymbol{p}'_\perp, \boldsymbol{\Delta}'_\perp)\rangle$$
$$= \langle \rho^a_{\mathrm{p}}(\boldsymbol{p}_\perp)\rho^b_{\mathrm{p}}(\boldsymbol{p}'_\perp)\rangle \langle \mathrm{tr}[U(-\boldsymbol{p}_\perp - \boldsymbol{\Delta}_\perp)t^a U^\dagger(-\boldsymbol{\Delta}_\perp - \boldsymbol{k}_\perp)]\mathrm{tr}[U(-\boldsymbol{p}'_\perp - \boldsymbol{\Delta}'_\perp)t^b U^\dagger(-\boldsymbol{\Delta}'_\perp - \boldsymbol{k}_\perp)\rangle$$

Average on p : trivial

$$\langle \rho^a_{\mathrm{p}}(\boldsymbol{p}_\perp)\rho^{b*}_{\mathrm{p}}(\boldsymbol{p}'_\perp)\rangle = \delta^{ab}\, g^2\mu^2_{\mathrm{p}}(2\pi)^2\delta^{(2)}(\boldsymbol{p}_\perp - \boldsymbol{p}'_\perp)$$

Average on A : MV model (singlet extracted)

Fukushima-Hidaka (2007)

$$\langle \mathrm{tr}[U(\boldsymbol{x}_1)t^a U^\dagger(\boldsymbol{x}_2)]\mathrm{tr}[U(\boldsymbol{x}_3)t^b U^\dagger(\boldsymbol{x}_4)]\rangle$$
$$= \frac{\delta^{ab}}{2N_c}\cdot \frac{B_2(\boldsymbol{x}_1-\boldsymbol{x}_4)+B_2(\boldsymbol{x}_2-\boldsymbol{x}_3)-B_2(\boldsymbol{x}_1-\boldsymbol{x}_3)-B_2(\boldsymbol{x}_2-\boldsymbol{x}_4)}{B_2(\boldsymbol{x}_1-\boldsymbol{x}_4)+B_2(\boldsymbol{x}_2-\boldsymbol{x}_3)-B_2(\boldsymbol{x}_1-\boldsymbol{x}_2)-B_2(\boldsymbol{x}_3-\boldsymbol{x}_4)}$$
$$\times\left(e^{-B_2(\boldsymbol{x}_1-\boldsymbol{x}_4)-B_2(\boldsymbol{x}_2-\boldsymbol{x}_3)} - e^{-B_2(\boldsymbol{x}_1-\boldsymbol{x}_2)-B_2(\boldsymbol{x}_3-\boldsymbol{x}_4)}\right).$$

Numerical calculations to be performed…

Technical Remark

$$\left\langle U(\boldsymbol{x}_{1\perp})_{\beta_1\alpha_1} U(\boldsymbol{x}_{2\perp})_{\beta_2\alpha_2} \cdots U(\boldsymbol{x}_{n\perp})_{\beta_n\alpha_n} \right\rangle$$

$$U(\boldsymbol{x}_\perp) = \mathcal{P}\exp\left[-\mathrm{i}g^2 \int_{-\infty}^{+\infty} \mathrm{d}x^- \mathrm{d}^2\boldsymbol{z}_\perp\, G_0(\boldsymbol{x}_\perp - \boldsymbol{z}_\perp)\, \rho_a(x^-, \boldsymbol{z}_\perp)\, t^a\right]$$

$$\omega(\rho) = \exp\left[-\int_{-\infty}^{+\infty} \mathrm{d}x^- \mathrm{d}\boldsymbol{x}_\perp\, \frac{\rho_a^2(x^-, \boldsymbol{x}_\perp)}{2\mu^2(x^-)}\right]$$

$\rho_a(x^-, \boldsymbol{x}_\perp) \;\to\; \delta(x^-)\rho_a^{(t)}(\boldsymbol{x}_\perp)$ Very delicate limit

Fukushima (2007)

Summary

- **Early-time dynamics**

  Glasma (longitudinal chromo-*E/B*) + U(1) *B*

- **Particle production**

  Quark pair production on P- and CP-odd *E/B* leading to the CME current

  Photons in pA~AA

  Loop diagrams become more important

- **One loop diagram evaluated**

  Structure looks quite similar to Bremsstrahlung

  *** Introduction of U(1) *B***

  *** Color average in a different way?**