John F. Fuini III

Far-from-equilibrium dynamics of a strongly coupled non-abelian plasma with non-zero charge density or external magnetic field

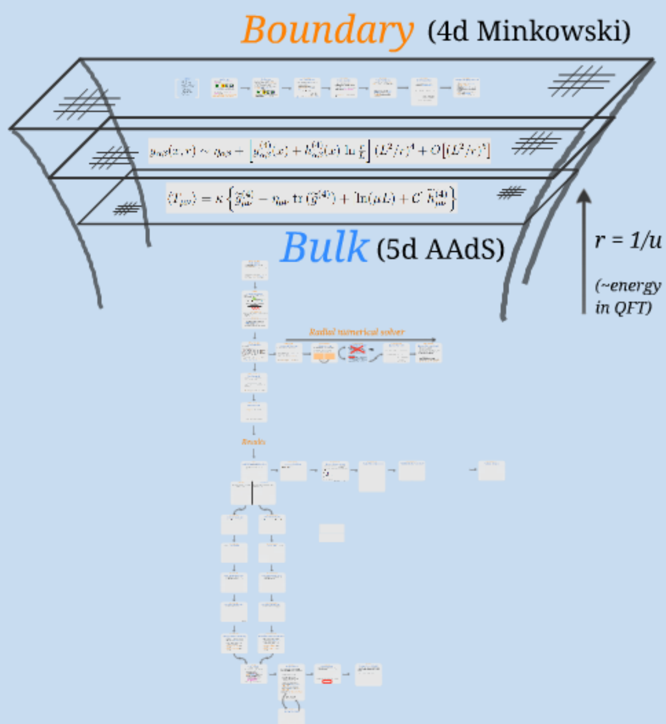
with Laurence Yaffe

JHEP (2015), arXiv: 1503.07148

This is the PDF version of this talk.

To properly view the talk as it was presented, using the website prezi, please use the following url:

http://prezi.com/h0j70g5cb4n1/?utm_campaign=share&utm_medium=copy&rc=ex0share



Conclusions

- developed far-from-equilibrium numerical evolution to include charge density and magnetic fields (including log difficulties).
- explored the surprising amount of linearity exhibited by the evolution.
- found that both a substantially large chemical potential or magnetic field has a very small effect on equilibration times.
- developed a simple model to understand the anisotropy seen in the boundary theory, given our initial anisotropy profiles.

This is the PDF version of this talk.

To properly view the talk as it was presented, using the website prezi, please use the following url:

http://prezi.com/klj70g3cb4nl/?utm_campaign=share&utm_medium=copy&rc=ex0share

John F. Fuini III

Far-from-equilibrium dynamics of a strongly coupled non-abelian plasma with non-zero charge density or external magnetic field

with Laurence Yaffe

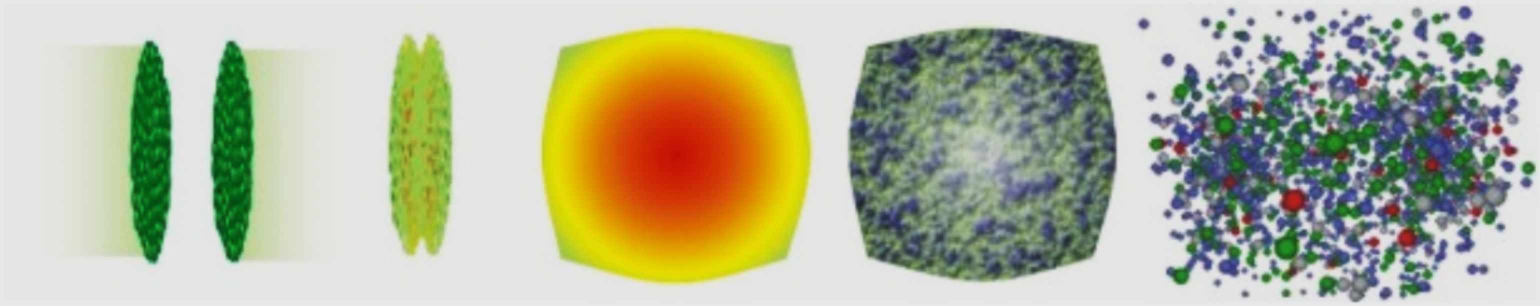
JHEP (2015), arXiv: 1503.07148

Outline

- Motivation
- Proxy Field Theory
  - $N = 4$ SYM + EM
- Holographic Implementation
  - Solving Maxwell-Einstein
- Results
  - Neutral Plasma
  - Charged Plasma
  - Magnetized Plasma
- Discussion
  - Simple Model

Motivation

- Want to understand strongly coupled and far-from-equilibrium QCD plasma.



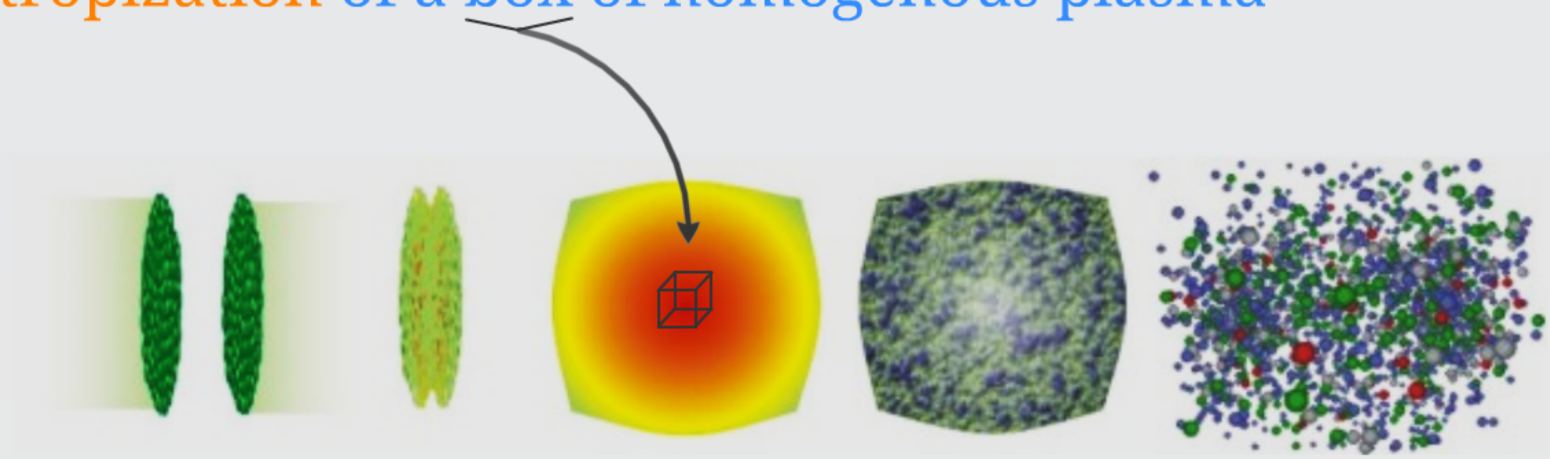
- Gauge/gravity duality (Holography) examines certain strongly coupled gauge theories by considering gravity.

- Numerical techniques allow us to study far-from-equilibrium dynamics.

The problem of a strongly coupled, far-from-equilibrium plasma turns into a calculation of numerical General Relativity.

Motivation

- Heavy Ion Collisions have a small but finite chemical potential and associated baryon density (charge density)
- Large but transient EM fields may play a significant role in Heavy Ion Collisions, despite small coupling
- Our work here is to extend previous work of analyzing the isotropization of a box of homogenous plasma



- Specifically, we want to analyze the effect that either a charge density OR a constant magnetic field has on isotropization/ equilibration timescales.

Proxy Field Theory

Our Field Theory

Simplest Gauge/Gravity example is $\mathcal{N}=4$ SYM $SU(N_c)$

qualitatively good toy theory for QCD

- on 4d minkowski space
- translational inv., x-y rotation inv.
- couple conserved current of $U(1)$ subgroup of $SU(4)_R$ global symmetry to a background Abelian gauge field.

$$S = S_{\mathrm{SYM}} + \int d^4x\, j^\alpha(x) A^{\mathrm{ext}}_\alpha(x)$$

$$A^{\mathrm{ext}}_\alpha(x) \equiv \mu\, \delta^0_\alpha + \tfrac{1}{2}\mathcal{B}\,(x^1\delta^2_\alpha - x^2\delta^1_\alpha)$$

interested in either baryon density (chem pot) OR magnetic field

Proxy Field Theory

Trace Anomaly

$$A_\alpha^{\rm ext}(x) \equiv \mu\,\delta_\alpha^0 + \tfrac{1}{2}\mathcal{B}\,(x^1\delta_\alpha^2 - x^2\delta_\alpha^1)$$

- chemical potential μ (conjugate to charge density in equilibrium) adds a scale, but does not effect the tracelessness of $T^{\alpha\beta}$

- In contrast, the magnetic field effects the microscopic dynamics - generates a non-zero trace anomaly:

$$T^\alpha{}_\alpha = -\tfrac{1}{4}\,\kappa\,\left(F_{\mu\nu}^{\rm ext}\right)^2 = -\tfrac{1}{2}\,\kappa\,\mathcal{B}^2$$

$$\kappa \equiv (N_{\rm c}^2 - 1)/(2\pi^2)$$

- implies logarithmic dependence on renormalization point

Enlarged Theory

To interpret this, we should consider the enlarged theory $\mathcal{N}=4$ SYM $SU(N_c)$ + EM, with total Action

$$S_{\text{SYM+EM}} = S_{\text{SYM, min. coupled}} + S_{\text{EM}}$$

$$S_{\text{EM}} \equiv -\int d^4x \, \frac{1}{4e^2} \, F_{\mu\nu}^2$$

- where the electromagnetic coupling $1/e^2$ is arbitrarily weak ➔ EM fields are classical background fields
- SYM fields charged under this $U(1)$ cause a running coupling (four fermions and three complex scalars)

Scale Dependence

$$\mu \frac{d}{d\mu} e^{-2} \equiv \beta_{1/e^2}(e^{-2}) = -b_0 + O(e^2) \qquad b_0 \equiv \kappa \Big[\tfrac{1}{6} \sum_{\alpha} (q_{\mathrm{f}}^{\alpha})^2 + \tfrac{1}{12} \sum_{a} (q_{\mathrm{s}}^{a})^2 \Big]$$

While the total stress energy of the theory is well-defined, the partitioning is ambiguous

$$T^{\alpha\beta}_{\mathrm{tot}} \equiv T^{\alpha\beta}_{\mathrm{EM}}(\mu) + \Delta T^{\alpha\beta}_{\mathrm{SYM}}(\mu) \qquad T^{\alpha\beta}_{\mathrm{EM}}(\mu) \equiv \frac{1}{e^2(\mu)} \left[F^{\alpha\nu} F^{\beta}{}_{\nu} - \tfrac{1}{4} \eta^{\alpha\beta} F^{\mu\nu} F_{\mu\nu} \right]$$

$$\Delta T^{\alpha\beta}_{\mathrm{SYM}}(\mu) \equiv T^{\alpha\beta}_{\mathrm{SYM,\, min.\ coupled}}(\mu)$$

scale dependence between each piece must cancel...

$$\mu \frac{d}{d\mu} \Delta T^{\alpha\beta}_{\mathrm{SYM}}(\mu) = -\mu \frac{d}{d\mu} T^{\alpha\beta}_{\mathrm{EM}}(\mu) = b_0 \left[F^{\alpha\nu} F^{\beta}{}_{\nu} - \tfrac{1}{4} \eta^{\alpha\beta} F^{\mu\nu} F_{\mu\nu} \right]$$

specialized to zero-temperature, ground state energy density:

$$\varepsilon(\mu) \equiv \Delta T^{00}_{\mathrm{SYM}}(\mu)/\kappa$$

$$T^{\alpha\beta}_{\text{tot}} \equiv T^{\alpha\beta}_{\text{EM}}(\mu) + \Delta T^{\alpha\beta}_{\text{SYM}}(\mu) \qquad T^{\alpha\beta}_{\text{EM}}(\mu) \equiv \frac{1}{e^2(\mu)}\left[F^{\alpha\nu}F^{\beta}{}_{\nu} - \tfrac{1}{4}\eta^{\alpha\beta}F^{\mu\nu}F_{\mu\nu}\right]$$

$$\Delta T^{\alpha\beta}_{\text{SYM}}(\mu) \equiv T^{\alpha\beta}_{\text{SYM, min. coupled}}(\mu)$$

scale dependence between each piece must cancel...

$$\mu\frac{d}{d\mu}\,\Delta T^{\alpha\beta}_{\text{SYM}}(\mu) = -\mu\frac{d}{d\mu}\,T^{\alpha\beta}_{\text{EM}}(\mu) = b_0\left[F^{\alpha\nu}F^{\beta}{}_{\nu} - \tfrac{1}{4}\eta^{\alpha\beta}F^{\mu\nu}F_{\mu\nu}\right]$$

$$\varepsilon(\mu) \equiv \Delta T^{00}_{\text{SYM}}(\mu)/\kappa$$

specialized to zero-temperature, ground state energy density:

$$\varepsilon(\mu) = c_0\,\mathcal{B}^2 - \tfrac{1}{4}\mathcal{B}^2\ln(|\mathcal{B}|/\mu^2)$$

We've found two choices of scale useful

$$\varepsilon_L \equiv \varepsilon(1/L)\,, \qquad \varepsilon_{\mathcal{B}} \equiv \varepsilon(|\mathcal{B}|^{1/2})$$

Proxy Field Theory

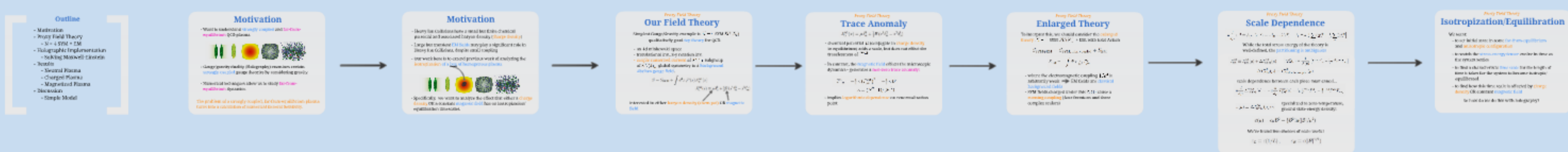
Isotropization/Equilibration

We want:

- to set initial state in some far-from-equilibrium and anisotropic configuration
- to watch the stress-energy tensor evolve in time as the system settles
- to find a characteristic time scale for the length of time it takes for the system to become isotropic/ equilibrated
- to find how this time scale is affected by charge density OR constant magnetic field

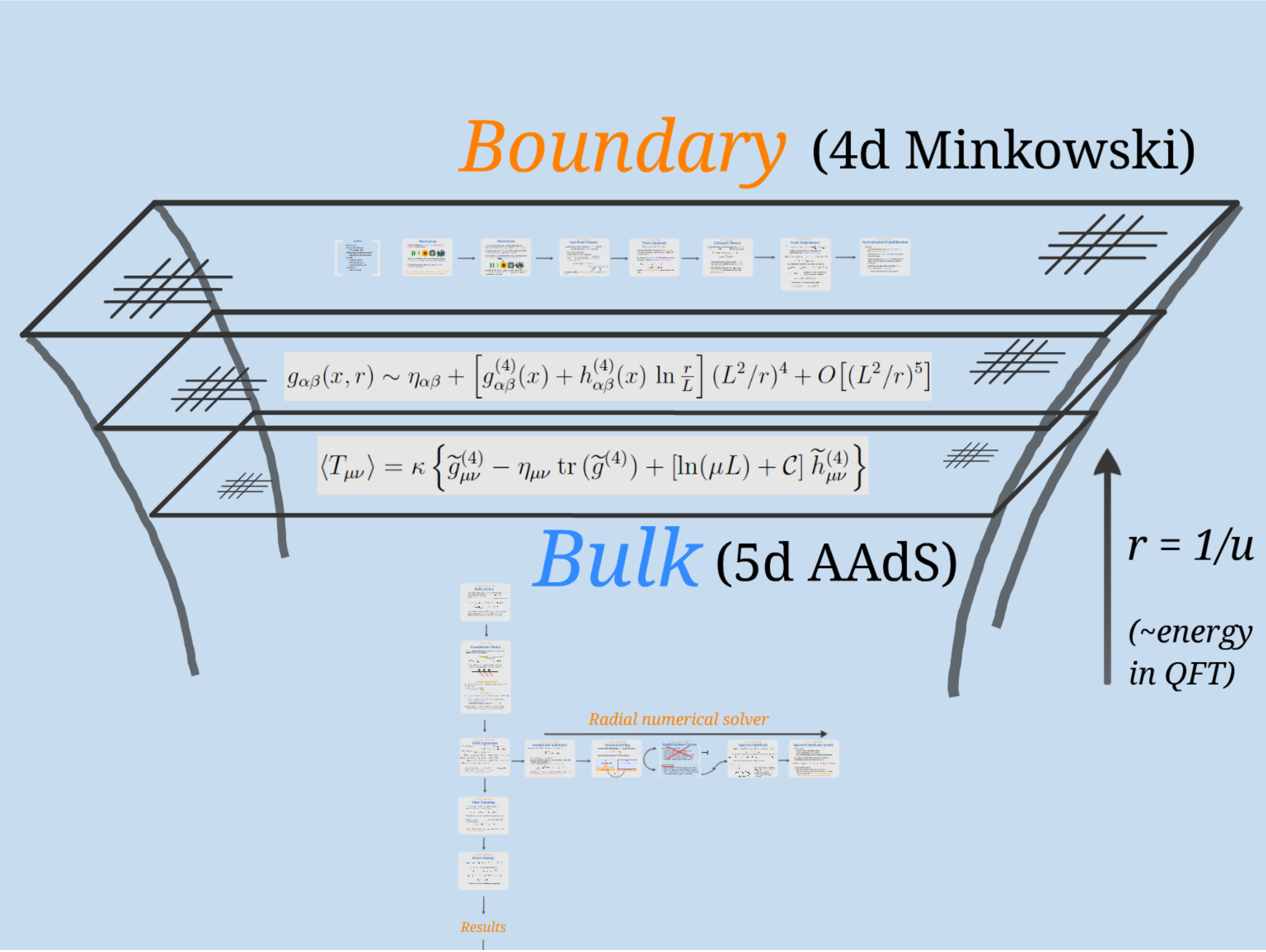
So how do we do this with holography?

Boundary (4d N



$$(x,r) \sim \eta_{\alpha\beta} + \left[g^{(4)}_{\alpha\beta}(x) + h^{(4)}_{\alpha\beta}(x)\ \ln\frac{r}{L}\right](L^2/r)^4 + O[(L^2/r)^5$$

$$\langle T_{\mu\nu}\rangle = \kappa\left\{\widetilde{g}^{(4)}_{\mu\nu} - \eta_{\mu\nu}\,\mathrm{tr}\,(\widetilde{g}^{(4)}) + [\ln(\mu L) + \mathcal{C}]\,\widetilde{h}^{(4)}_{\mu\nu}\right\}$$


Boundary (4d Minkowski)
$g_{\alpha\beta}(x,r) \sim \eta_{\alpha\beta} + \left[g^{(4)}_{\alpha\beta}(x) + h^{(4)}_{\alpha\beta}(x)\,\ln\frac{r}{L}\right](L^2/r)^4 + O\left[(L^2/r)^5\right]$
$\langle T_{\mu\nu}\rangle = \kappa\left\{\widetilde{g}^{(4)}_{\mu\nu} - \eta_{\mu\nu}\,\mathrm{tr}\,(\widetilde{g}^{(4)}) + [\ln(\mu L) + \mathcal{C}]\,\widetilde{h}^{(4)}_{\mu\nu}\right\}$
Bulk (5d AAdS)
r = 1/u
(~energy in QFT)
Radial numerical solver
Results

Holographic Implementation

Bulk Action

Consistent truncation (top-down construction) for our boundary theory, $S = S_{\rm SYM} + \int d^4x\, j^\alpha(x) A^{\rm ext}_\alpha(x)$ gives Einstein + Maxwell:

(Myers et al. arXiv:hep-th/9902170)

$$S_5 \equiv \frac{1}{16\pi G_5} \int d^5x \sqrt{-G} \left(R - 2\Lambda - L^2\, F_{MN} F^{MN}\right)$$

$$G_5 \equiv \tfrac{\pi}{2} L^3 / N_{\rm c}^2 \qquad\qquad \Lambda \equiv -6/L^2$$

- 5d Chern-Simons term could be present, but not relevant if only turning on charge density OR magnetic field
- familiar story by now for this initial value problem

Coordinate Choice

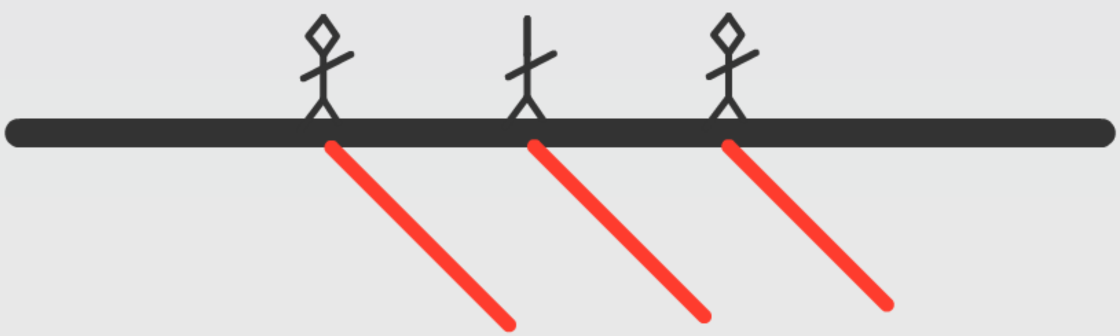
Following *Chesler, Yaffe arXiv:1309.1439*, we chose generalized Eddington Finklestein coordinates:

$$\{v, x, y, z, r\}$$

Boundary coordinates

$$ds^2 = \frac{r^2}{L^2} g_{\mu\nu}(x,r) dx^\mu dx^\nu - 2\omega_\mu(x) dx^\mu dr$$

- coordinates remain regular across future horizon
- infalling radial null geodesics (affine parameter, r)



Ansatz/Symmetries

Since we are interested in homogenous isotropization, we demand the following:

- trans. invariance in x, y, z
- rot. invariance in x-y plane

Bulk Metric

$$ds^2 = -A(v,r)dv^2 + 2drdv + \Sigma(v,r)^2 \left(e^{B(v,r)}(dx^2 + dy^2) + e^{-2B(v,r)}dz^2\right)$$

- at the boundary, v coincides with boundary time t.
- anisotropy isolated in B
- radial diffeomorphism freedom $r \to r + \lambda(v)$

Demanding boundary metric minkowski space fixes leading terms of near boundary expansion of our metric functions

Holographic Implementation

Field Equations

Maxwell's EQs give: $F_{xy}(x,r) \equiv \mathcal{B} = const$

$$F_{rv}(x,r) \equiv \mathcal{E}(v,r) = \rho\, L\, \Sigma^{-3}(v,r)$$

Einstein EQs give:

1) $$\Sigma'' + \tfrac{1}{2}(B')^2\,\Sigma = 0\,,$$

4) $$A'' - 6(\Sigma'/\Sigma^2)\, d_+\Sigma + \tfrac{3}{2}\, B' d_+ B = +\tfrac{5}{3}\mathcal{B}^2 L^2\, e^{-2B}\,\Sigma^{-4} + \tfrac{7}{3}\,\mathcal{E}^2 L^2 - 2/L^2$$

3) $$(d_+B)' + \tfrac{3}{2}(\Sigma'/\Sigma)\, d_+B + \tfrac{3}{2}B'\,(d_+\Sigma)/\Sigma = -\tfrac{2}{3}\mathcal{B}^2 L^2\, e^{-2B}\,\Sigma^{-4}\,,$$

2) $$(d_+\Sigma)'/\Sigma + 2(\Sigma'/\Sigma^2)\, d_+\Sigma = -\tfrac{1}{3}\mathcal{B}^2 L^2\, e^{-2B}\,\Sigma^{-4} - \tfrac{1}{3}\mathcal{E}^2 L^2 + 2/L^2$$

5) $$d_+(d_+\Sigma) - A'\,(d_+\Sigma) + \tfrac{1}{2}\Sigma\,(d_+B)^2 = 0\,,$$

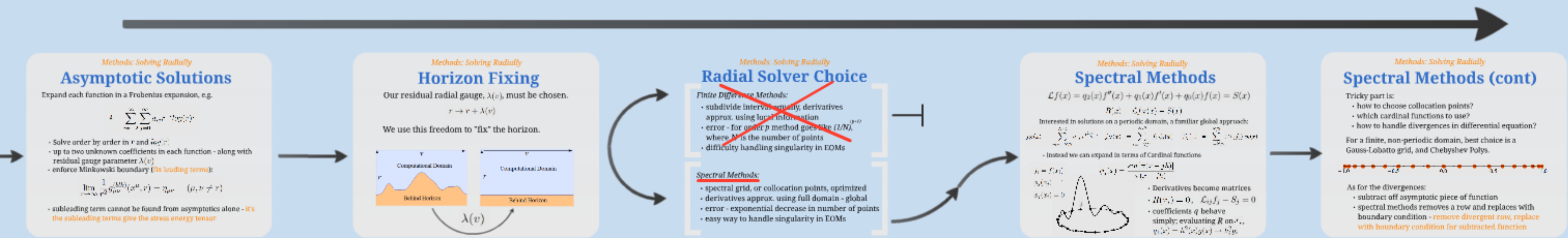
$$d_+ \equiv \partial_v + A(v,r)\,\partial_r$$

At some initial time, v_0, Given: $\mathcal{E}$, λ, $B(v_0,r)$ and (ρ or $\mathcal{B}$)

System reduces to nested set of radial ODEs

B encodes essential propogating d.o.f's.

Radial numerical solver



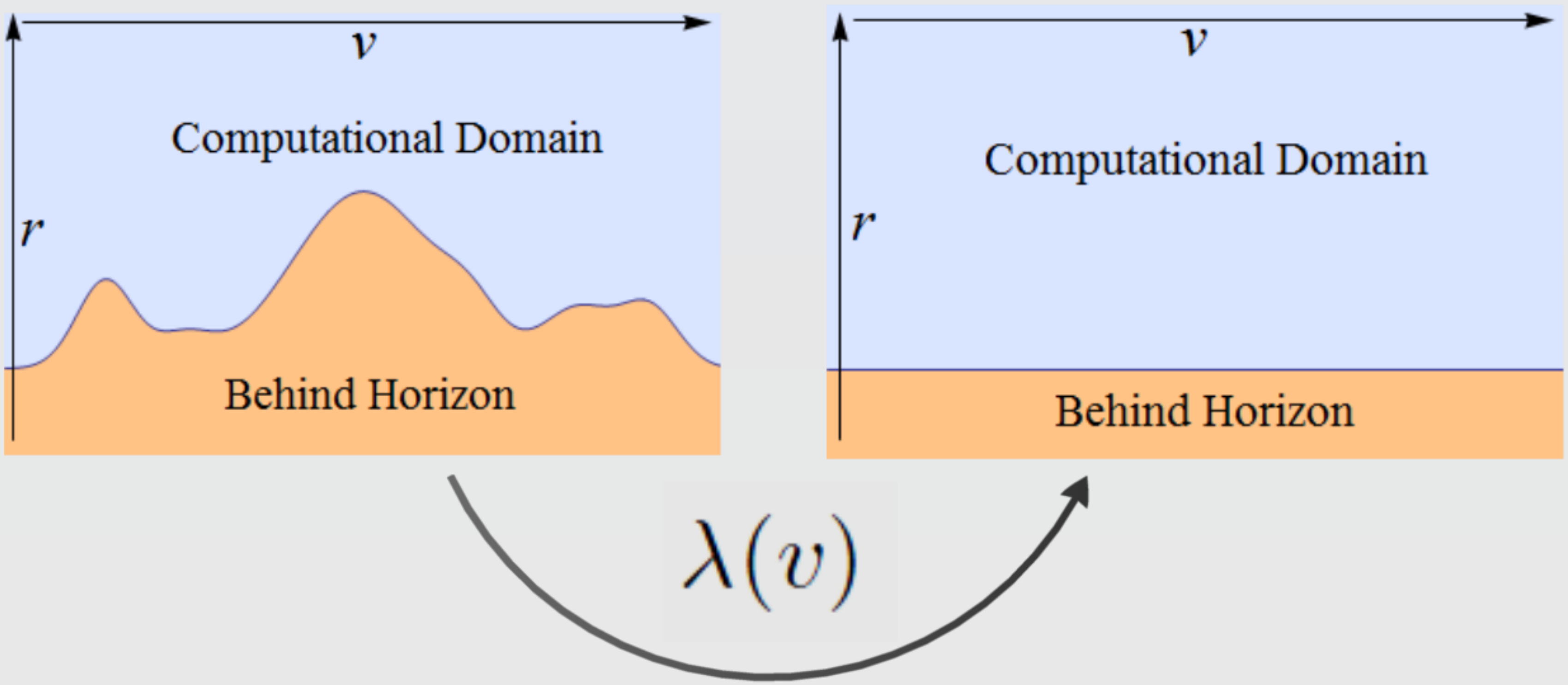
Methods: Solving Radially

Horizon Fixing

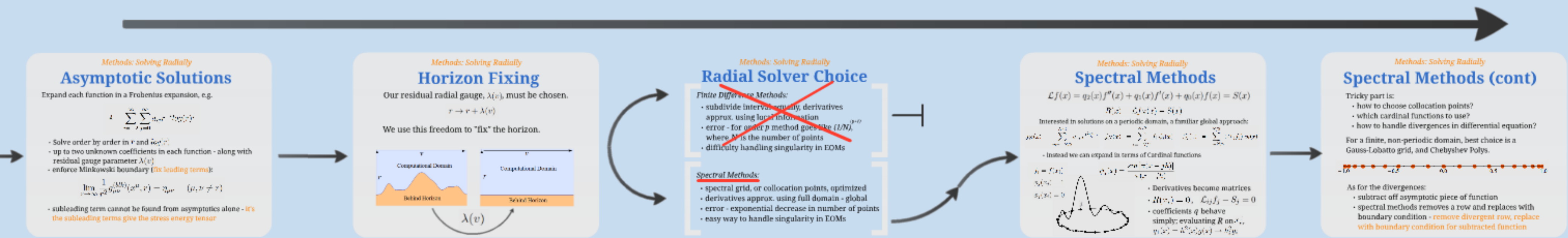
Our residual radial gauge, $\lambda(v)$, must be chosen.

$$r \to r + \lambda(v)$$

We use this freedom to "fix" the horizon.



Radial numerical solver



Holographic Implementation

Time Stepping

Once we have $d_+ B$ and B' on a given timeslice v_0,
(as well as our other metric functions)

$$\partial_v B(v_0, r) = d_+ B(v_0, r) - A(v_0, r)\, \partial_r B(v_0, r)$$

The stationarity condition on the horizon gives us $\partial_v \lambda$

Integrate forward, $v_0 \to v_0 + \Delta v$, with a Runge-Kutta 4th order method to find:

$$B(v_0 + \Delta v), \quad \lambda(v_0 + \Delta v)$$

- Restart the entire radial process, now on the new timeslice
- watch system equilibrate

Holographic Implementation

Stress Energy

$$g_{\alpha\beta}(x,r) \sim \eta_{\alpha\beta} + \left[g^{(4)}_{\alpha\beta}(x) + h^{(4)}_{\alpha\beta}(x)\ \ln\frac{r}{L}\right](L^2/r)^4 + O\left[(L^2/r)^5\right]$$

Holographic RG prescription finds us...

$$\langle T_{\mu\nu}\rangle = \kappa\left\{\widetilde{g}^{(4)}_{\mu\nu} - \eta_{\mu\nu}\,\mathrm{tr}\,(\widetilde{g}^{(4)}) + \left[\ln(\mu L) + \mathcal{C}\right]\widetilde{h}^{(4)}_{\mu\nu}\right\}$$

$$\widetilde{h}^{(4)}_{\mu\nu} \equiv h^{(4)}_{\mu\nu} + \tfrac{1}{4}\,\eta_{\mu\nu}\,h^{(4)}_{00} \qquad \widetilde{g}^{(4)}_{\mu\nu} \equiv g^{(4)}_{\mu\nu} + \tfrac{1}{4}\,\eta_{\mu\nu}\left(g^{(4)}_{00} + \tfrac{1}{4}\,h^{(4)}_{00}\right)$$

$$T^{\alpha}{}_{\alpha} = -\tfrac{1}{2}\,\kappa\,\mathcal{B}^2$$

Checks out with our field theory perspective

Results

Equilibrium Configurations

Neutral Plasma

$$(\rho = 0, \mathcal{B} = 0)$$

Equilibrates to Schwarzschild Black Brane:

$$ds^2 = -U(\tilde{r})\,dt^2 + \frac{d\tilde{r}^2}{U(r)} + \frac{\tilde{r}^2}{L^2}\,(dx^i)^2$$

$$U(\tilde{r}) \equiv \frac{\tilde{r}^2}{L^2} - \frac{m\,L^2}{\tilde{r}^2}$$

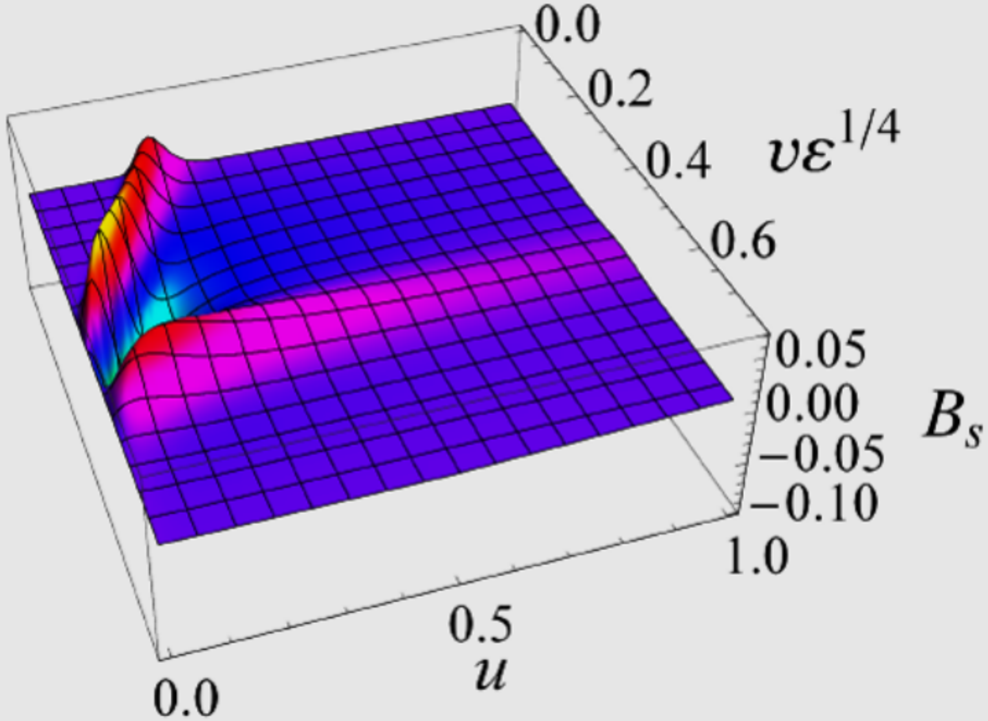
$$\varepsilon = \frac{3}{4} m L^{-4} = \frac{3}{4}(\pi T)^4$$

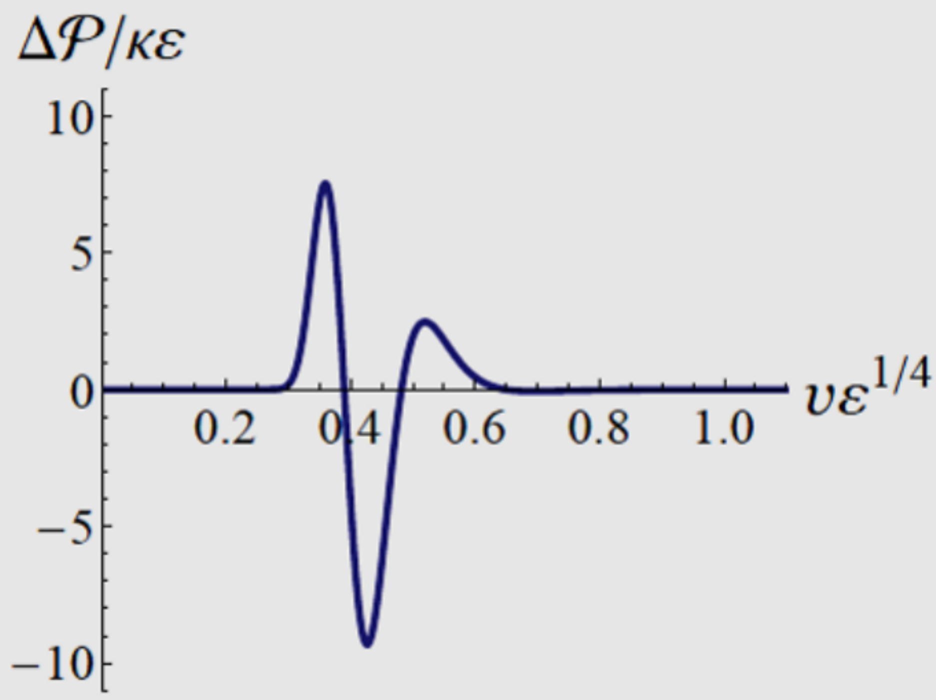
Results: Neutral Plasma

General Evolution

$$B(v_0, r) = \mathcal{A}\, e^{-\frac{1}{2}(r-r_0)^2/\sigma^2}$$

$$\Delta\mathcal{P} \equiv \frac{1}{2}(T^{11} + T^{22}) - T^{33}$$



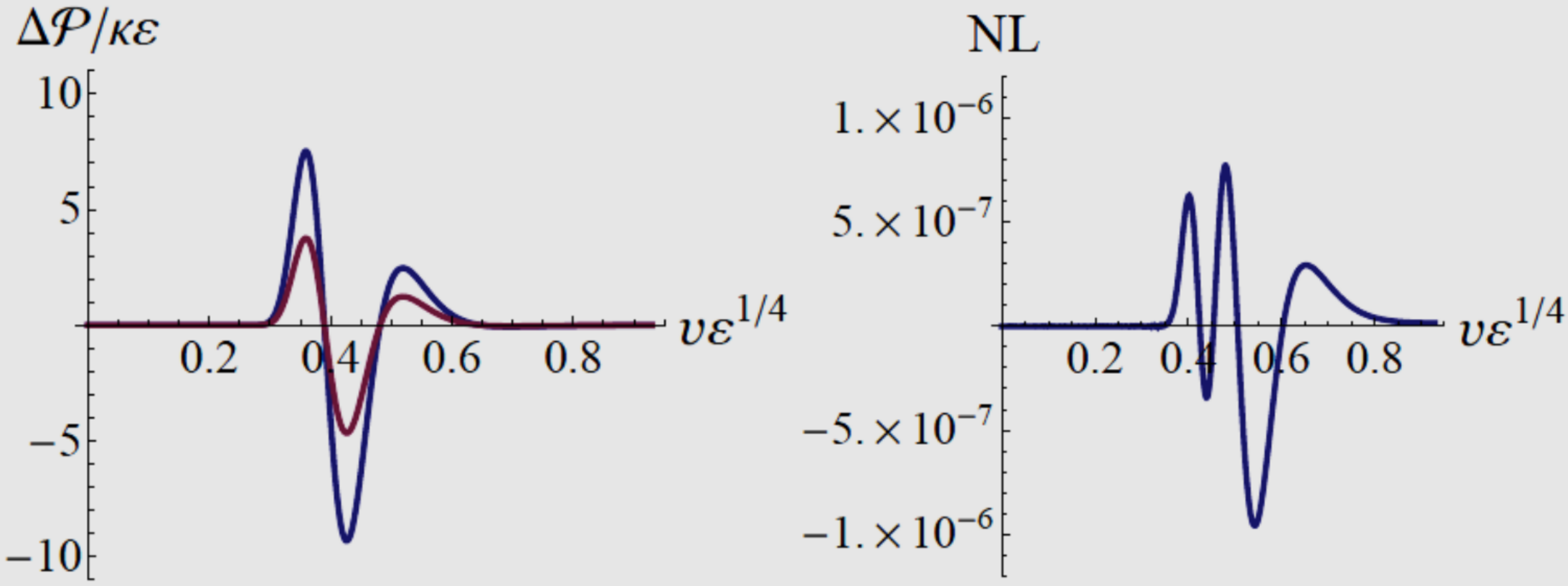


Sensitivity to Gaussian

$$B(v_0, r) = \mathcal{A}\, e^{-\frac{1}{2}(r-r_0)^2/\sigma^2}$$

- Amplitude and r_0 interesting

Amplitude test. Double initial Gaussian's Amp, check results.

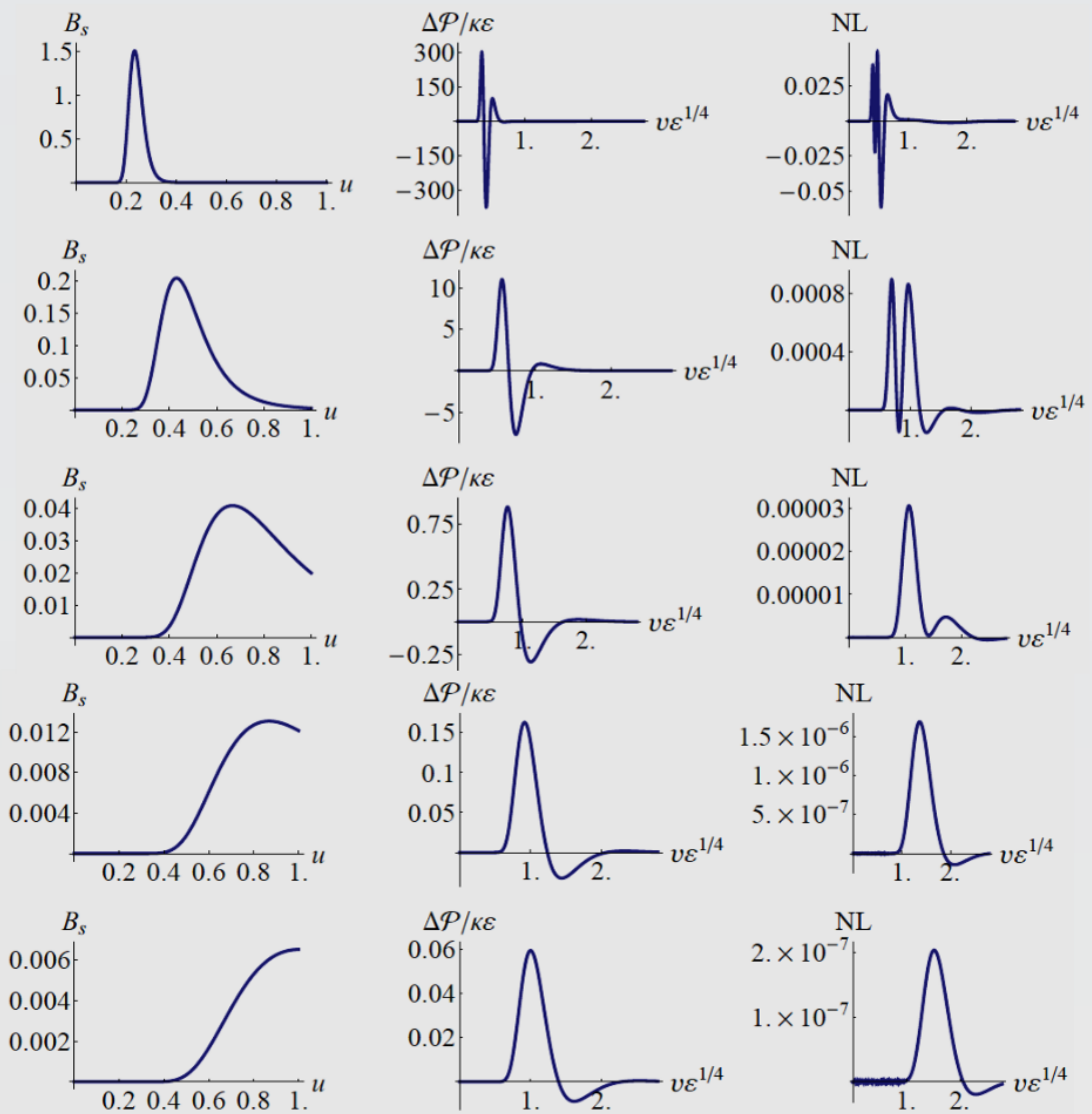


Response is quite linear

van der Schee et al. arXiv: 1202.0981, 1304.5172

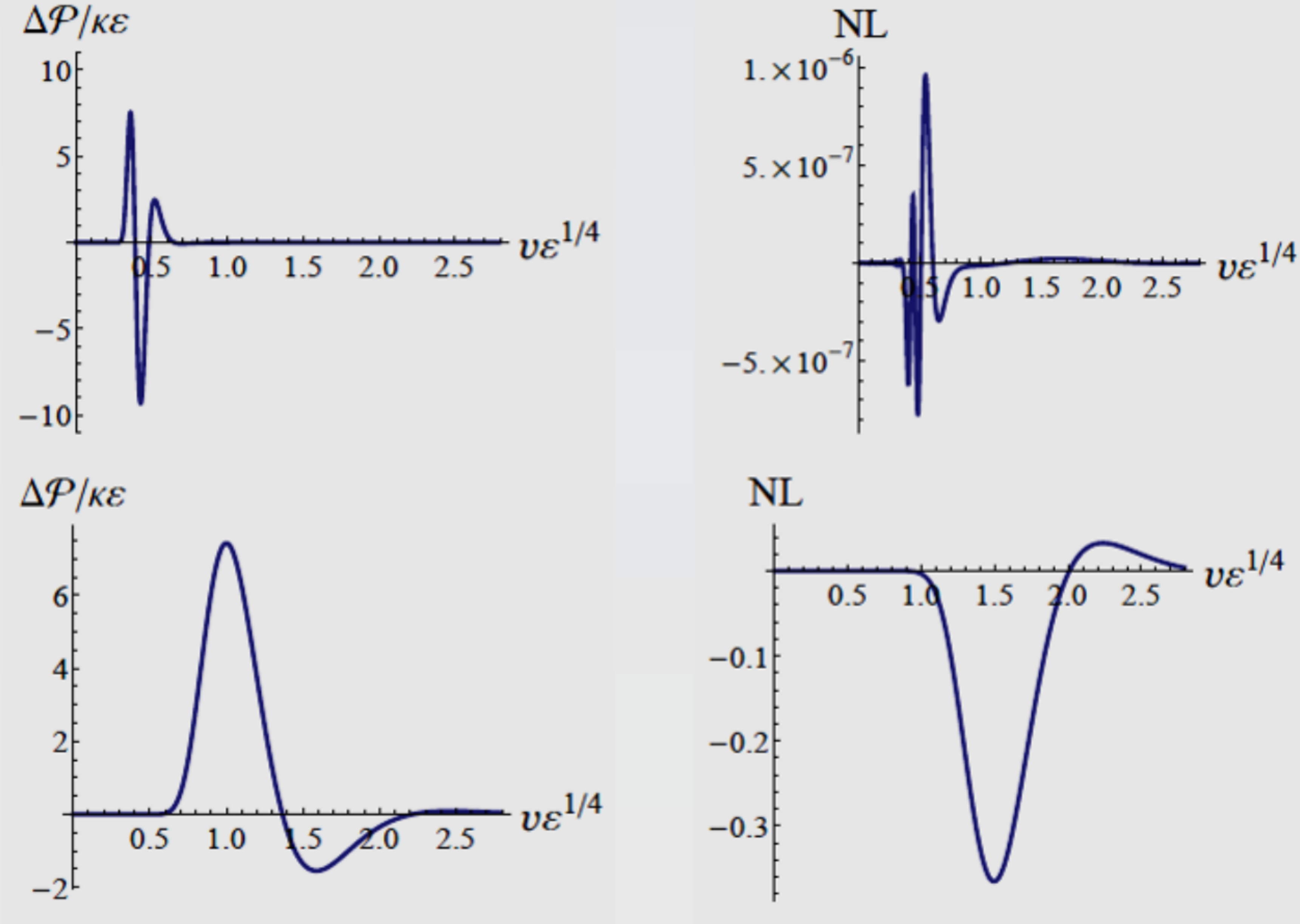
Results: Neutral Plasma

Depth & Non-linearities



Results: Neutral Plasma

Matched Bndry Anisotropy



Results

Equilibrium Configurations

Neutral Plasma

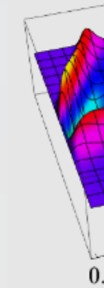
$$(\rho = 0, \mathcal{B} = 0)$$

Equilibrates to Schwarzschild Black Brane:

$$ds^2 = -U(\tilde{r})\, dt^2 + \frac{d\tilde{r}^2}{U(r)} + \frac{\tilde{r}^2}{L^2}\,(dx^i)^2$$

$$U(\tilde{r}) \equiv \frac{\tilde{r}^2}{L^2} - \frac{m\,L^2}{\tilde{r}^2}$$

$$\varepsilon = \tfrac{3}{4} m L^{-4} = \tfrac{3}{4}(\pi T)^4$$



Charged Plasma

$$(\rho \neq 0, \mathcal{B} = 0)$$

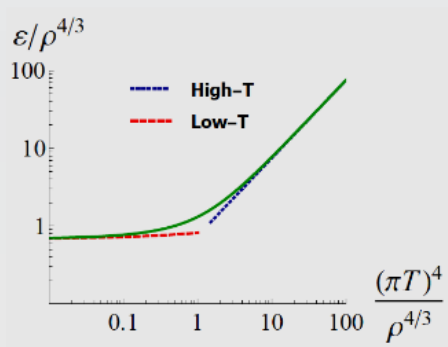
Equilibrates to Reissner-Nordstrom Black Brane

Myers et al. hep-th/9902170

$$ds^2 = -U(\tilde{r})\, dt^2 + \frac{d\tilde{r}^2}{U(r)} + \frac{\tilde{r}^2}{L^2}\,(dx^i)^2$$

$$U(\tilde{r}) \equiv \frac{\tilde{r}^2}{L^2} - m\,\frac{L^2}{\tilde{r}^2} + \tfrac{1}{3}\,(\rho L^3)^2\,\frac{L^4}{\tilde{r}^4}$$

$$(\rho_{\max} L^3)^4 = \tfrac{4}{3}\, m^3 \qquad \mu = \tfrac{1}{2}\,\rho\,(L^2/\tilde{r}_{\rm h})^2$$

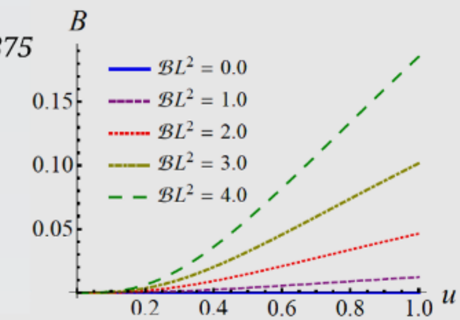


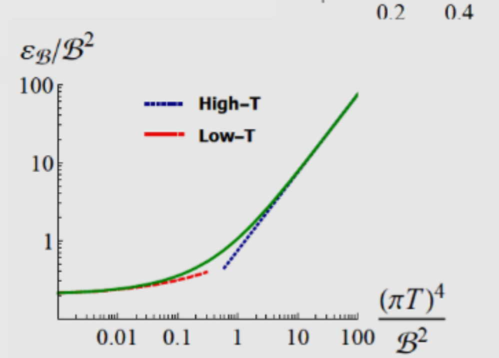
Magnetized Plasma

$$(\rho = 0, \mathcal{B} \neq 0)$$

Equilibrates to Magnetic Black Brane (Numerical Solution)

D'Hoker, Kraus 0908.3875





Charged Plasma

$$(\rho \neq 0, \mathcal{B} = 0)$$

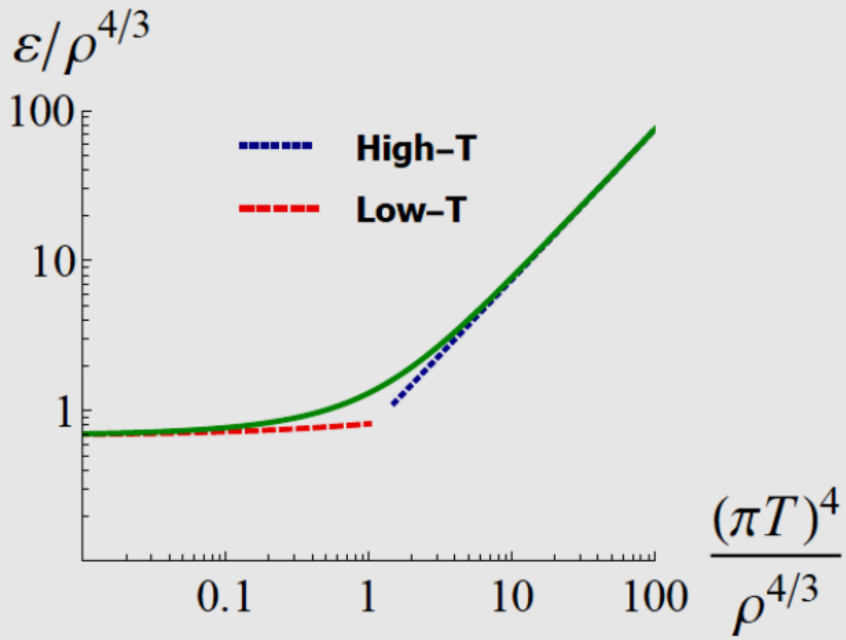
Equilibrates to Reissner-Nordstrom Black Brane

Myers et al. hep-th/9902170

$$ds^2 = -U(\tilde{r})\,dt^2 + \frac{d\tilde{r}^2}{U(r)} + \frac{\tilde{r}^2}{L^2}\,(dx^i)^2$$

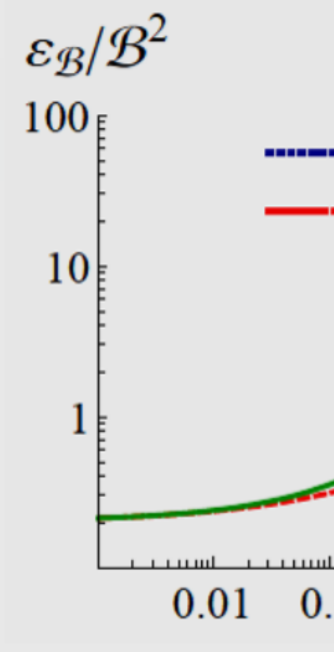
$$U(\tilde{r}) \equiv \frac{\tilde{r}^2}{L^2} - m\,\frac{L^2}{\tilde{r}^2} + \tfrac{1}{3}\,(\rho L^3)^2\,\frac{L^4}{\tilde{r}^4}$$

$$(\rho_{\max} L^3)^4 = \tfrac{4}{3}\,m^3 \qquad \mu = \frac{1}{2}\,\rho\,(L^2/\tilde{r}_{\rm h})^2$$



Equilibrates to M

D'Hoker, Kraus 09

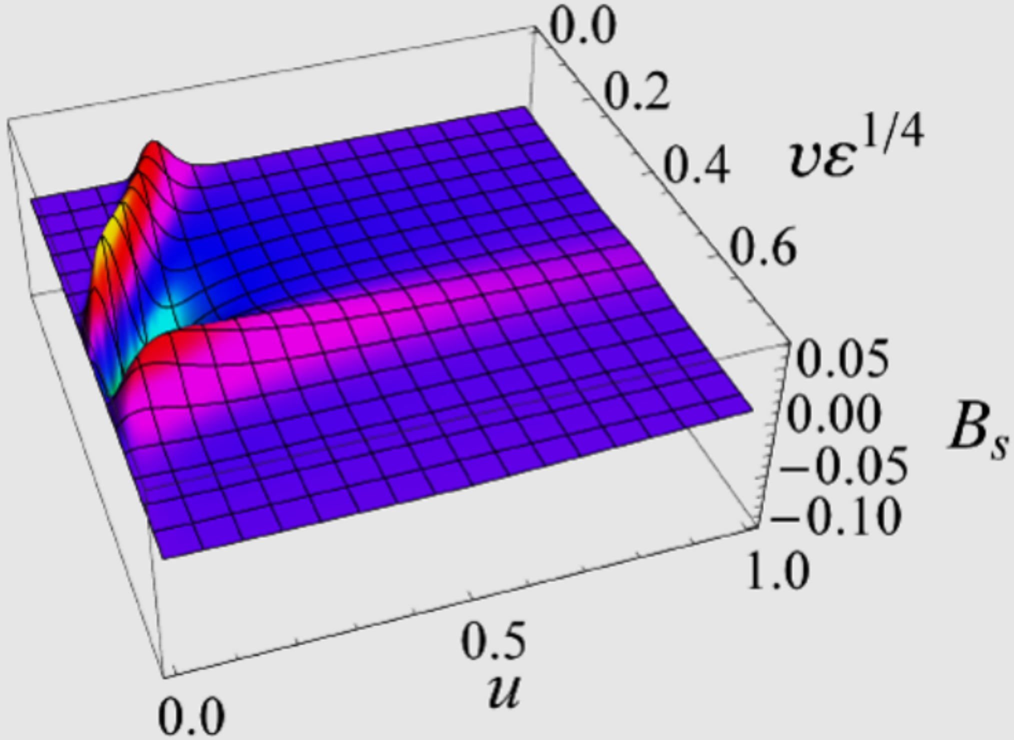


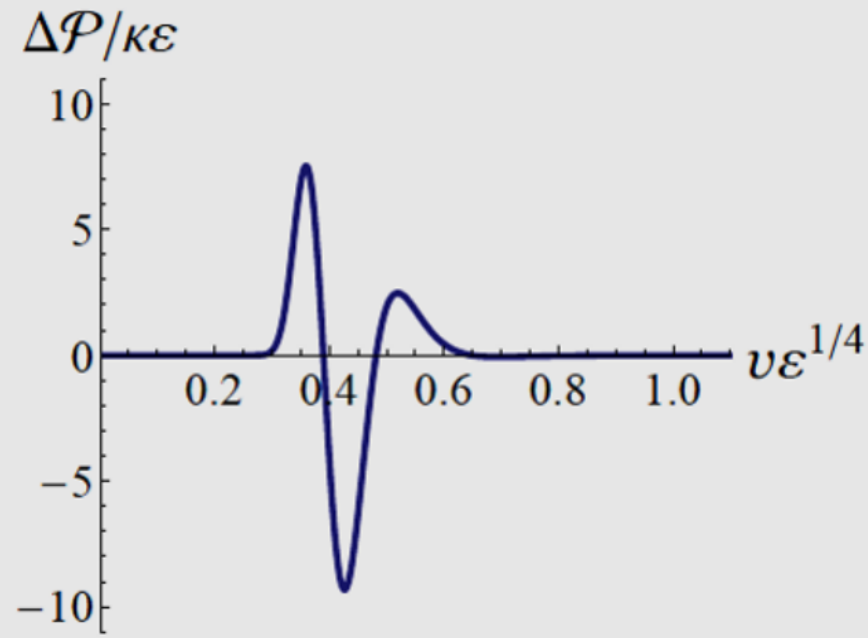
Results: Charged Plasma

Generic Evolution

$$B(v_0, r) = \mathcal{A}\, e^{-\frac{1}{2}(r-r_0)^2/\sigma^2}$$

$$\Delta\mathcal{P} \equiv \tfrac{1}{2}(T^{11} + T^{22}) - T^{33}$$





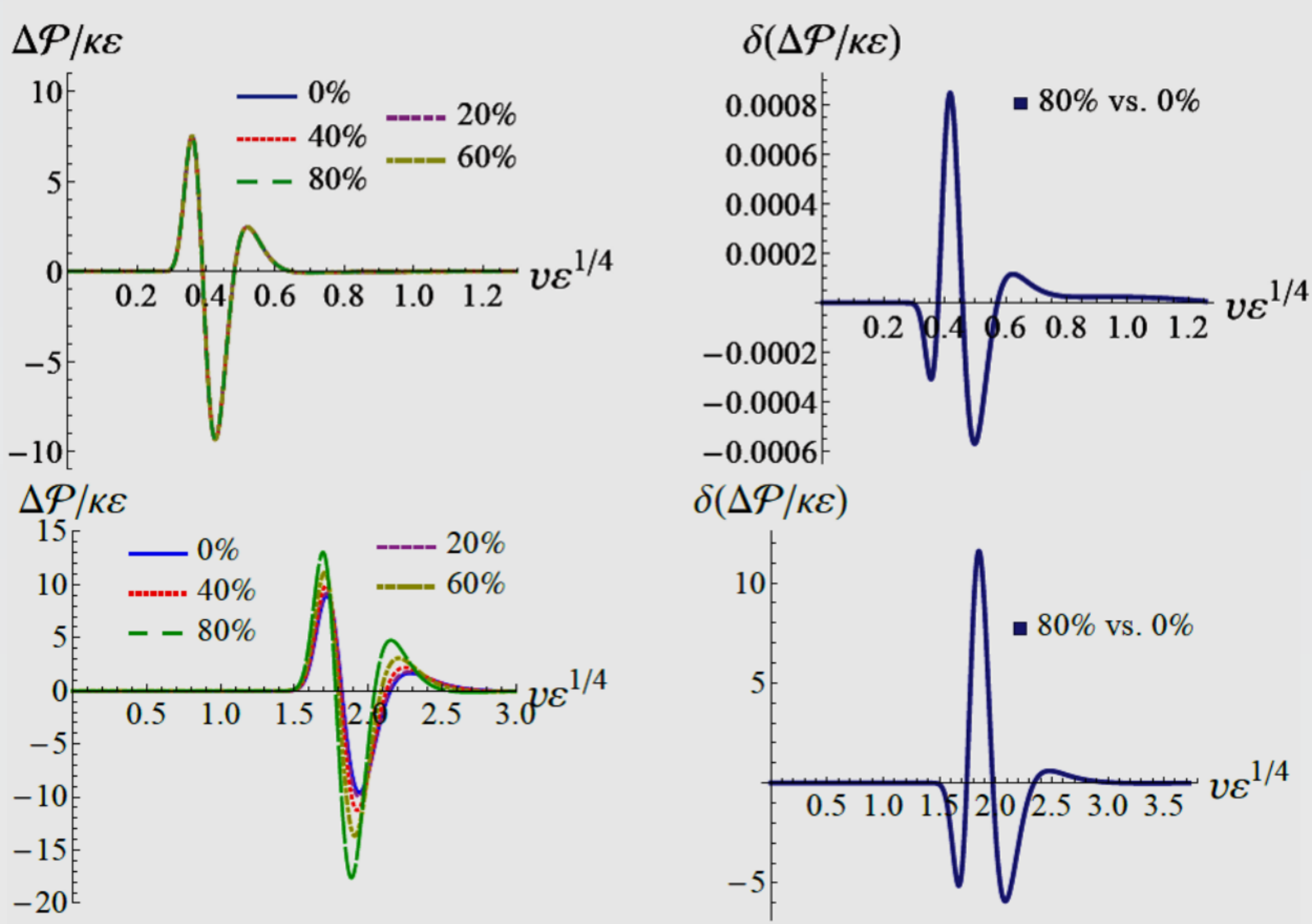
Results: Charged Plasma

ε held fixed

Anisotropy

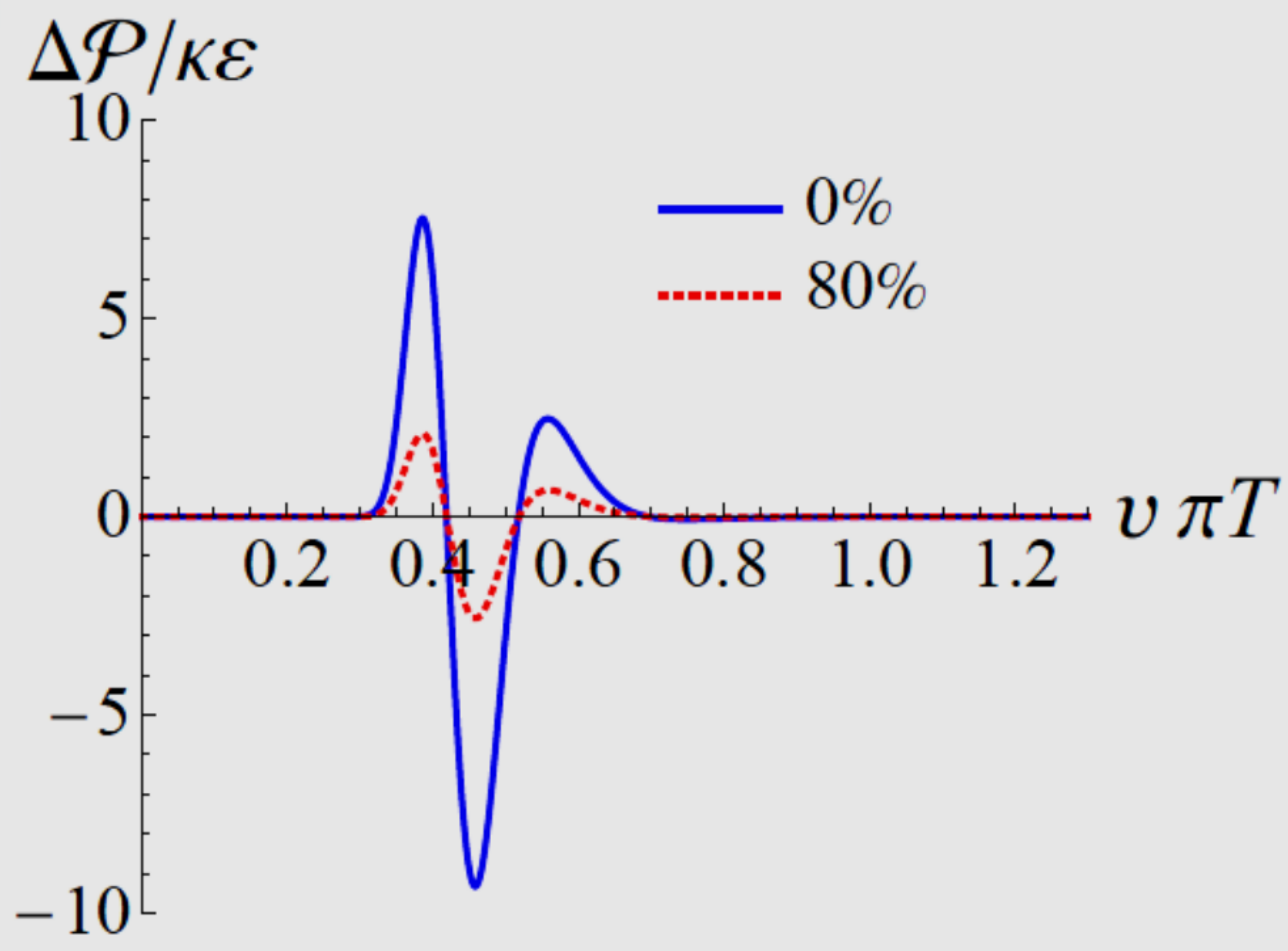
$$(\rho_{\max} L^3)^4 = \frac{4}{3}\, m^3$$

$$\mu/T = 0,\ 0.34,\ 0.73,\ 1.26,\ 2.21$$



Constant Temperature

- if we instead keep T fixed (as opposed to ε), and express time in units of T



Results: Charged Plasma

Quasi-normal Modes

From asymptotically late time of $\Delta\mathcal{P}/\kappa\varepsilon$

$$\sim e^{\lambda_I t} cos[\lambda_R t + \phi]$$

Charged ($\rho \neq 0$, $\mathcal{B} = 0$)				
$\rho/\rho_{\max}$	$\mathrm{Re}\,\lambda/\varepsilon^{1/4}$	$\mathrm{Im}\,\lambda/\varepsilon^{1/4}$	$\lambda/\pi T$	Linearized $\lambda/\varepsilon^{1/4}$
0.0	3.35208 ± 0.00004	-2.95144 ± 0.00013	$3.11946 - 2.74663\,i$	$3.35207 - 2.95150\,i$
0.1	3.34564 ± 0.00016	-2.95468 ± 0.00019	$3.11948 - 2.75763\,i$	$3.34568 - 2.95460\,i$
0.2	3.32624 ± 0.00020	-2.96429 ± 0.00019	$3.13222 - 2.79139\,i$	$3.32630 - 2.96444\,i$
0.3	3.29319 ± 0.00028	-2.98266 ± 0.00036	$3.14987 - 2.85285\,i$	$3.29327 - 2.98287\,i$
0.4	3.24572 ± 0.00007	-3.01376 ± 0.00008	$3.17857 - 2.95141\,i$	$3.24574 - 3.01377\,i$
0.5	3.18366 ± 0.00016	-3.06498 ± 0.00007	$3.22529 - 3.10506\,i$	$3.18370 - 3.06491\,i$
0.6	3.11311 ± 0.00002	-3.15177 ± 0.00002	$3.31032 - 3.35142\,i$	$3.11311 - 3.15176\,i$
0.7	3.07021 ± 0.00006	-3.29402 ± 0.00006	$3.50617 - 3.76176\,i$	$3.07022 - 3.29399\,i$
0.8	3.11863 ± 0.00265	-3.41376 ± 0.00295	$3.99199 - 4.36977\,i$	$3.11848 - 3.42004\,i$

(Janizewski, Kaminski)

Charged Plasma Summary

- charge sensitivity more pronounced with pulses originating closer to horizon
- non-linearity sizes comparable to what was seen for neutral plasma
- low sensitivity to charge density when ε held fixed
- low senitivity to charge density when T held fixed

Results

Equilibrium Configurations

Neutral Plasma

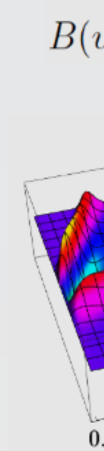
$(\rho = 0, \mathcal{B} = 0)$

Equilibrates to Schwarzschild Black Brane:

$$ds^2 = -U(\tilde{r})\,dt^2 + \frac{d\tilde{r}^2}{U(r)} + \frac{\tilde{r}^2}{L^2}\,(dx^i)^2$$

$$U(\tilde{r}) \equiv \frac{\tilde{r}^2}{L^2} - \frac{m\,L^2}{\tilde{r}^2}$$

$$\varepsilon = \tfrac{3}{4} m L^{-4} = \tfrac{3}{4}(\pi T)^4$$



Charged Plasma

$(\rho \neq 0, \mathcal{B} = 0)$

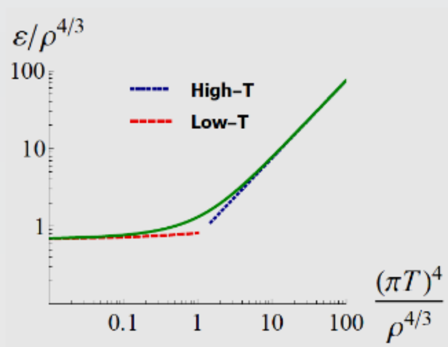
Equilibrates to Reissner-Nordstrom Black Brane

Myers et al. hep-th/9902170

$$ds^2 = -U(\tilde{r})\,dt^2 + \frac{d\tilde{r}^2}{U(r)} + \frac{\tilde{r}^2}{L^2}\,(dx^i)^2$$

$$U(\tilde{r}) \equiv \frac{\tilde{r}^2}{L^2} - m\,\frac{L^2}{\tilde{r}^2} + \tfrac{1}{3}\,(\rho L^3)^2\,\frac{L^4}{\tilde{r}^4}$$

$$(\rho_{\max} L^3)^4 = \tfrac{4}{3}\,m^3 \qquad \mu = \tfrac{1}{2}\,\rho\,(L^2/\tilde{r}_{\rm h})^2$$

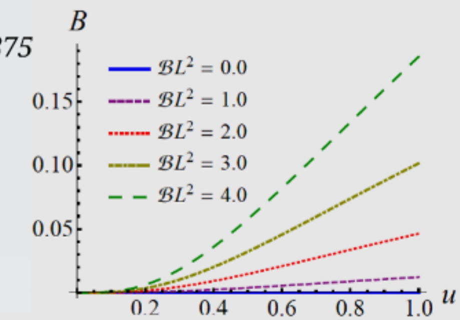


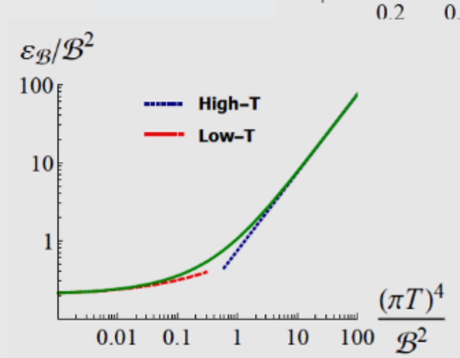
Magnetized Plasma

$(\rho = 0, \mathcal{B} \neq 0)$

Equilibrates to Magnetic Black Brane (Numerical Solution)

D'Hoker, Kraus 0908.3875





ack Brane

$dx^i)^2$

$_2\ \frac{L^4}{\tilde{r}^4}$

$L^2/\tilde{r}_{\rm h})^2$

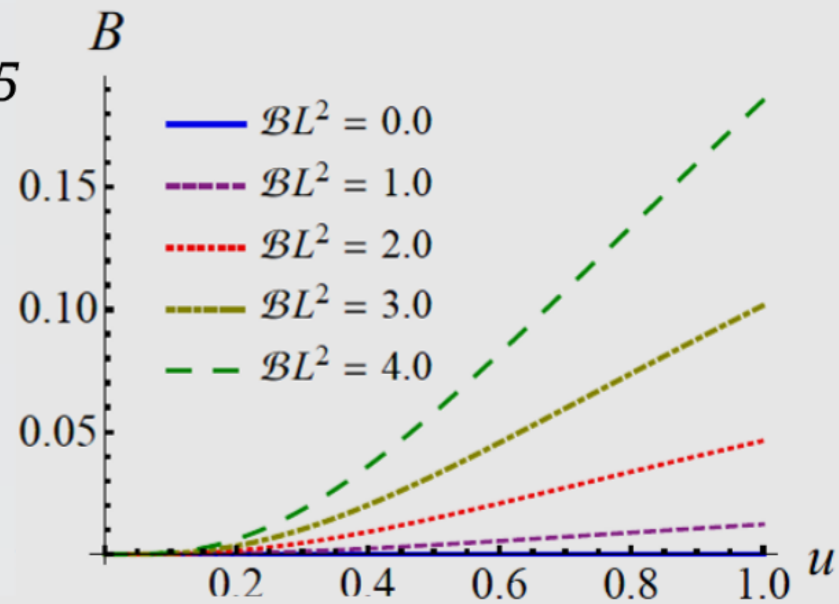
$)^4$

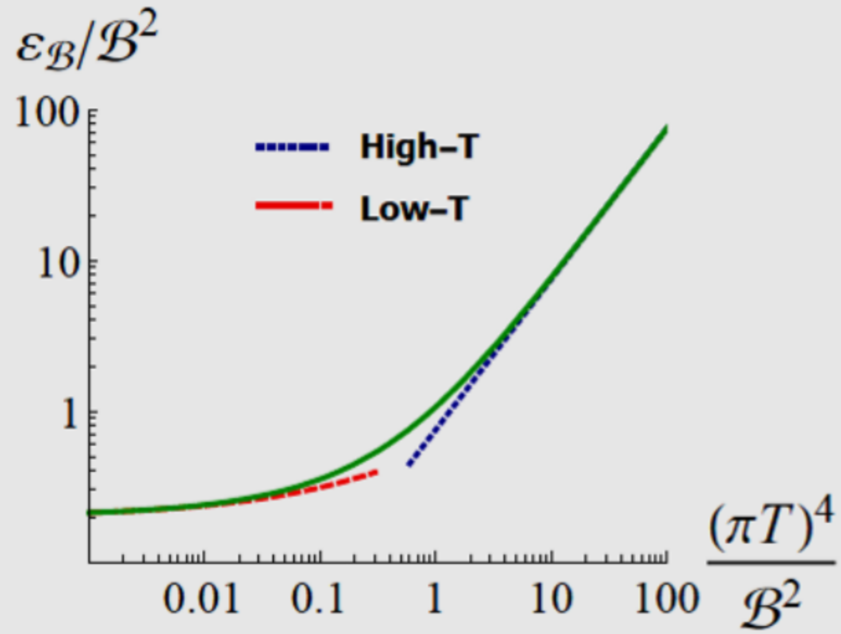
Magnetized Plasma

$(\rho = 0, \mathcal{B} \neq 0)$

Equilibrates to Magnetic Black Brane (Numerical Solution)

D'Hoker, Kraus 0908.3875



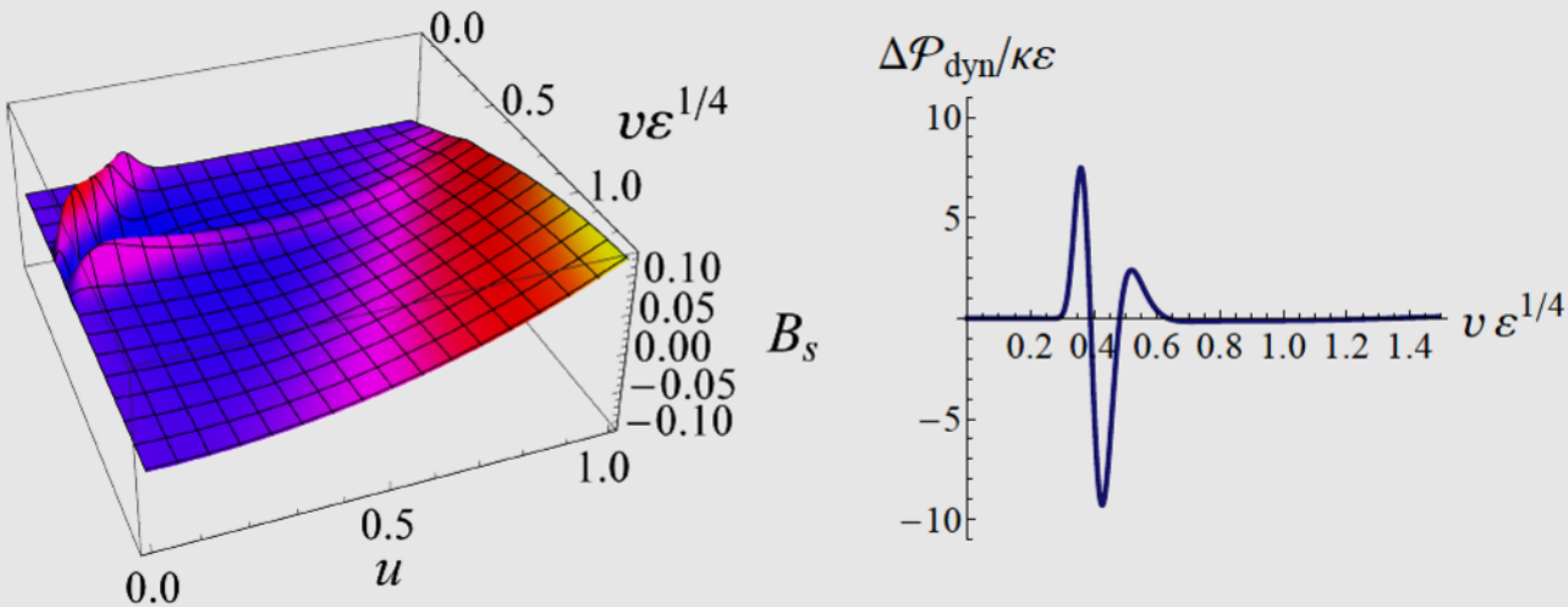


Results: Magnetized Plasma

Generic Evolution

$$B(v_0, r) = \mathcal{A}\, e^{-\frac{1}{2}(r-r_0)^2/\sigma^2} + \frac{1}{3}\mathcal{B}^2\, r^{-4} \ln r$$

$$\Delta\mathcal{P}_{\rm dyn} \equiv \frac{1}{2}(T^{11} + T^{22}) - T^{33} + \frac{1}{4}\kappa\, \mathcal{B}^2$$

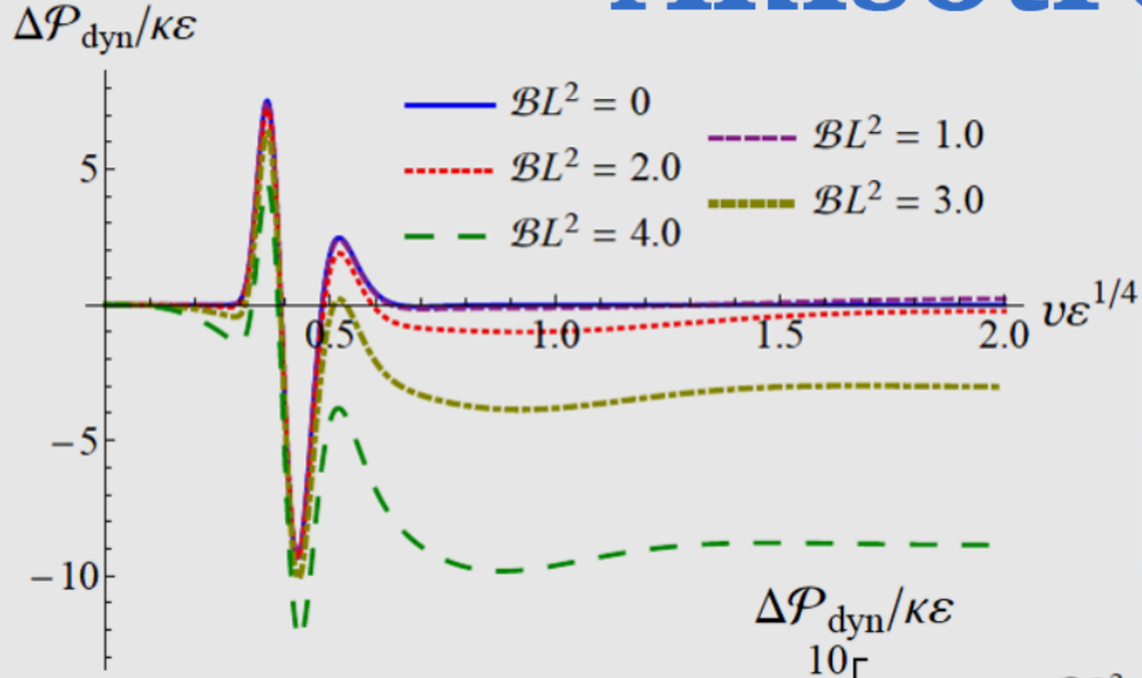


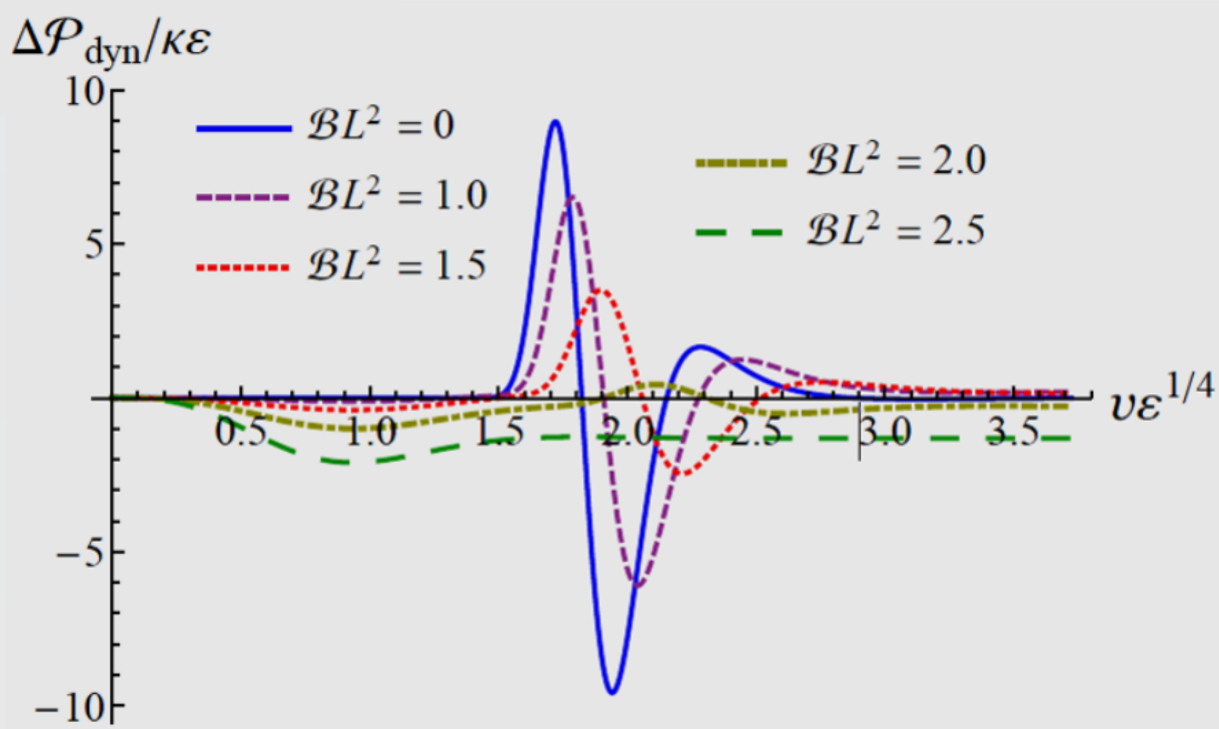
Results: Magnetized Plasma

Anisotropy

ε_L held fixed

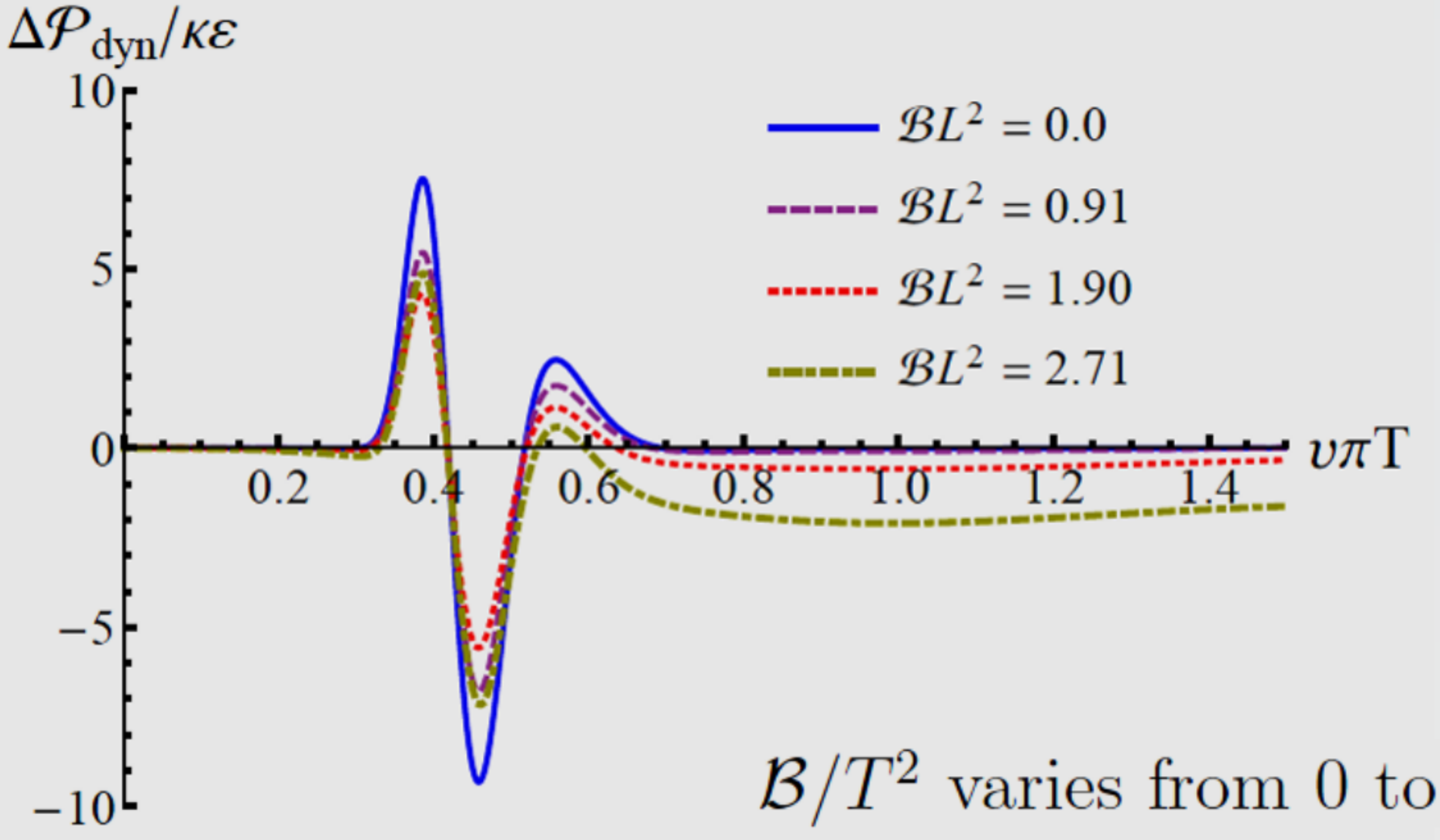
$$\mathcal{B}/T^2 = 0,\ 13.0,\ 30.2,\ 30.5,\ 26.3$$





Constant Temperature

- if we instead keep T fixed (as opposed to ε_L), and express time in units of T



$\mathcal{B}/T^2$ varies from 0 to 26.7

Results: Magnetized Plasma

Quasi-normal Modes

From asymptotically late time of $\Delta\mathcal{P}_{\text{dyn}}/\kappa\varepsilon$

$$\sim e^{\lambda_I t} cos[\lambda_R t + \phi]$$

Magnetic ($\mathcal{B} \neq 0$, $\rho = 0$)						
$\mathcal{B}/T^2$	$\varepsilon_\mathcal{B}/T^4$	$\overline{\mathcal{P}}/\kappa T^4$	$\Delta\mathcal{P}/\kappa T^4$	$\text{Re}\,\lambda/\varepsilon_\mathcal{B}^{1/4}$	$\text{Im}\,\lambda/\varepsilon_\mathcal{B}^{1/4}$	$\lambda/(\pi T)$
0	73.06	24.35	0	3.3521 ± 0.0001	-2.9514 ± 0.0001	$3.1195 - 2.7466\,i$
0.990	72.98	24.16	-1.13	3.357 ± 0.001	-2.93 ± 0.06	$3.124 - 2.73\,i$
5.344	80.74	22.15	-10.60	3.372 ± 0.002	-2.92 ± 0.06	$3.217 - 2.79\,i$
12.953	125.85	13.98	-15.76	3.264 ± 0.007	-2.78 ± 0.01	$3.480 - 2.96\,i$
17.821	170.16	3.79	-9.36	3.161 ± 0.002	-2.69 ± 0.04	$3.634 - 3.09\,i$
22.836	226.69	-11.35	5.00	3.061 ± 0.008	-2.60 ± 0.03	$3.780 - 3.21\,i$
30.161	328.21	-42.21	39.97	2.94 ± 0.01	-2.49 ± 0.03	$3.98 - 3.38\,i$

Magnetized Plasma Summary

- magnetic sensitivity more pronounced with pulses originating closer to horizon
- non-linearity sizes comparable to what was seen for neutral plasma
- low sensitivity to magnetic field when ε_L held fixed
- low senitivity to magnetic field when T held fixed

Results: Charged Plasma

Charged Plasma Summary

- charge sensitivity more pronounced with pulses originating closer to horizon
- non-linearity sizes comparable to what was seen for neutral plasma
- low sensitivity to charge density when $\mathcal{E}$ held fixed
- low senitivity to charge density when T held fixed

Results: Magnetized Plasma

Magnetized Plasma Summary

- magnetic sensitivity more pronounced with pulses originating closer to horizon
- non-linearity sizes comparable to what was seen for neutral plasma
- low sensitivity to magnetic field when ε_L held fixed
- low senitivity to magnetic field when T held fixed

Discussion

Conclusions

What have we we learned?

To a good (often extremely good) level of accuracy:

- the pressure anisotropy is a linear functional of the initial anisotropy profile
- the timecourse is insensitive to charge

Consider
case, t_{pea}
initial pro

Discussion

Conclusions

What have we we learned?

To a good (often extremely good) level of accuracy:

- the pressure anisotropy is a linear functional of the initial anisotropy profile
- the timecourse is insensitive to charge density or magnetic field when the initial pulse profile is held fixed and either:
  - time in units of energy density, and energy density held fixed
  - time in units of temperature and temperature held fixed

How do we synthesize this information?

Discussion

Simple Model

Consider the time of peak anisotropy in the charged case, $t_{\rm peak}$, as a function of equilibrium parameters and initial profile parameters

$$t_{\rm peak}/L = f(\varepsilon L^4, TL, r_0/L, \sigma/L, \mathcal{A})$$

where we've exchanged charge fraction, $\rho/\rho_{\rm max}$, for T

- small non-linearity size: indep of $\mathcal{A}$
- consider small width pulses, or for non-negligible width combine r_0 and σ

$$t_{\rm peak}/L \approx g(\varepsilon L^4, TL, r_{\rm eff}/L) \qquad r_{\rm eff} = r_0 + n\sigma$$

- our results show small dependence on

$$t_{\mathrm{peak}}/L \approx g(\varepsilon L^4, TL, r_{\mathrm{eff}}/L) \qquad r_{\mathrm{eff}} = r_0 + n\sigma$$

- our results show small dependence on charge density at fixed energy density - i.e. indep of temp
- also results show small dependence on charge density at fixed temp - i.e. indep of energy density

Main dependence on effective dept of initial pulse

$$t_{\mathrm{peak}}/L \approx h(r_{\mathrm{eff}}/L)$$

functional form limited by scaling relations, we find:

$$t_{\mathrm{peak}} \approx C L^2/r_{\mathrm{eff}}$$

Discussion

Scaling Relations

$$x \to \tilde{x} = \alpha^{-1} x \qquad r \to \tilde{r} = \alpha\gamma^{-2} r$$

$$\widetilde{B}(\widetilde{x}, \widetilde{r}) \equiv B(x(\widetilde{x}), r(\widetilde{r}))\,,$$
$$\widetilde{\Sigma}(\widetilde{x}, \widetilde{r}) \equiv (\alpha/\gamma)\,\Sigma(x(\widetilde{x}), r(\widetilde{r}))\,,$$
$$\widetilde{A}(\widetilde{x}, \widetilde{r}) \equiv (\alpha/\gamma)^2 A(x(\widetilde{x}), r(\widetilde{r}))\,,$$

$$\widetilde{L} \equiv \gamma^{-1} L \qquad \widetilde{\mathcal{B}} \equiv \alpha^2 \mathcal{B} \qquad \widetilde{\rho} \equiv \alpha^3 \rho$$

$$\alpha = \gamma \qquad \alpha = 1, \gamma > 0$$

scaling by dimension

AdS radius scaling

$$t_{\text{peak}}/L \approx g(\varepsilon L^4, TL, r_{\text{eff}}/L) \qquad r_{\text{eff}} = r_0 + n\sigma$$

- our results show small dependence on charge density at fixed energy density - i.e. indep of temp
- also results show small dependence on charge density at fixed temp - i.e. indep of energy density

Main dependence on effective dept of initial pulse

$$t_{\text{peak}}/L \approx h(r_{\text{eff}}/L)$$

functional form limited by scaling relations, we find:

$$t_{\text{peak}} \approx CL^2/r_{\text{eff}}$$

Discussion

Simple Model

Same argument goes for our Magnetic results, with the replacements

$$t_{\rm peak}/L = f(\varepsilon_L L^4, \mathcal{B}L^2, r_0/L, \sigma/L, \mathcal{A})$$

$$t_{\rm peak}/L \approx g(\varepsilon_L L^4, \mathcal{B}L^2, r_{\rm eff}/L)$$

$$t_{\rm peak} \approx C L^2/r_{\rm eff}$$

For charged *and* magnetized plasmas our model suggests:

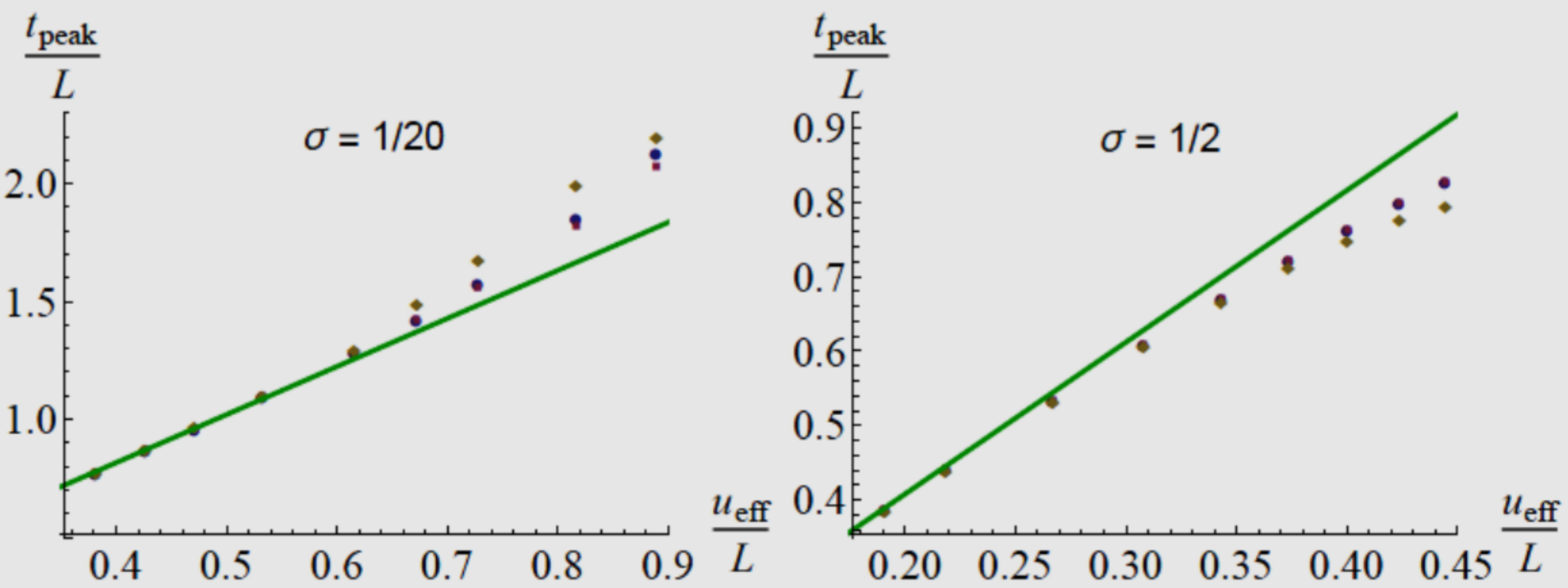
$$t_{\rm peak} \approx C L^2/r_{\rm eff}$$

Comparison to Model

We test against a sharper width pulse, and wider pulse, in uncharged, charged, and magnetized plasmas, at varying depths.

$$r_{\text{eff}} = r_0 + n\sigma$$

We find good agreement using n = 2.5, and $C \approx 2.04$



Conclusions

- developed far-from-equilibrium numerical evolution to include charge density and magnetic fields (including log difficulties).
- explored the surprising amount of linearity exhibited by the evolution.
- found that both a substantially large chemical potential or magnetic field has a very small effect on equilibration times.
- developed a simple model to understand the anisotropy seen in the boundary theory, given our initial anisotropy profiles.