Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Alexander von Humboldt
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Inhomogeneous thermalization and strongly coupled thermal flow via holography

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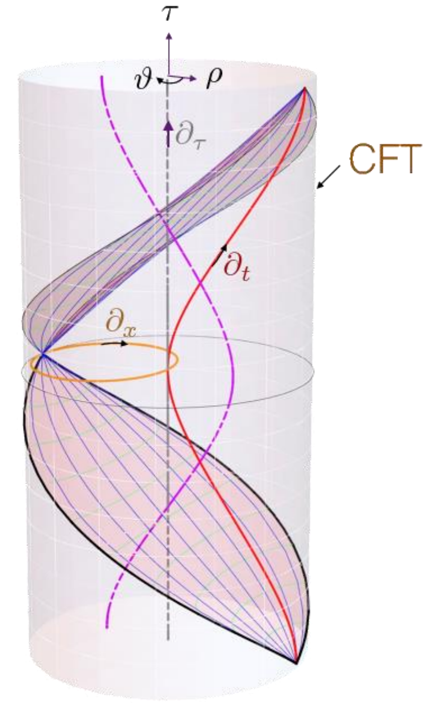
The holographic path



Out-of-equilibrium strongly coupled field theory

- Pros of gauge/gravity approach:
  - Provides real-time analysis.
  - Allows for finite temperature setups.
  - New angle can lead to fresh new insights.
- Cons of gauge/gravity approach:
  - Caveats and limitations ($\mathcal{N}$=4 SYM)
  - Time dependence → Solve PDEs

Applications

- Quark-gluon plasma thermalization *[Chesler, Yaffe, Heller, Romatschke, Mateos, vd Schee, Bantilan]*
- Turbulence in Gravity *[Lehner, Green, Yang, Zimmerman, Chesler, Adams, Liu]*
- Driven superconductors *[Rangamani, Rozali, Wong]*
- Quantum quenches *[Balasubramanian, Buchel, Myers, van Niekerk, Das]*
- Revivals *[Mas, Lopez, Serantes, da Silva]*

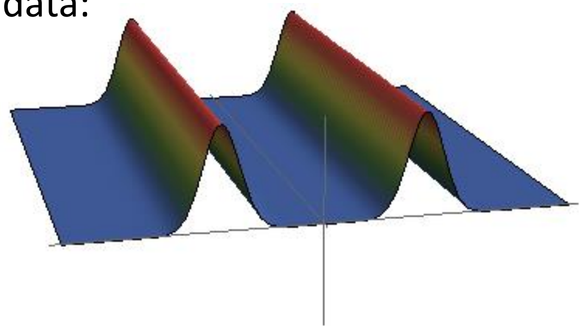
Outline

1

Collisions modeled as shockwaves

- Extraction of S-E tensor at boundary.
- Comparison with experiment.
- Main result: Thermalization time.

Initial data:



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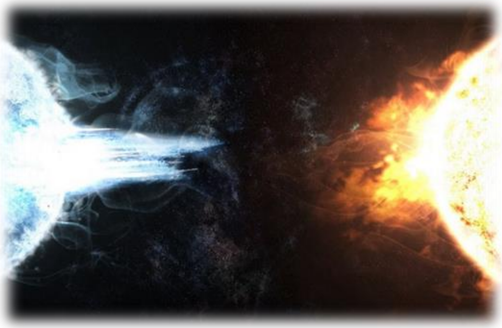
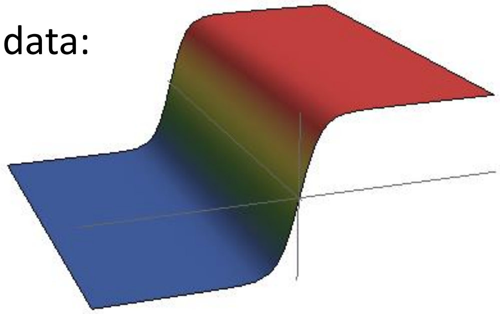


Thermal flow modeled as dynamical horizon

- Extraction of steady state regime.
- Comparison with CFT and hydro.
- New result: Information flow.

Initial data:



Hydrodynamics approach

Conservation Law:

$$\partial_\mu T^{\mu\nu} = 0$$

- Hydro: Effective field theory of the slow evolution of expectation values of conserved currents close to equilibrium.
- Assumption: Some notion of local quasi-equilibrium.

Gradient expansion:

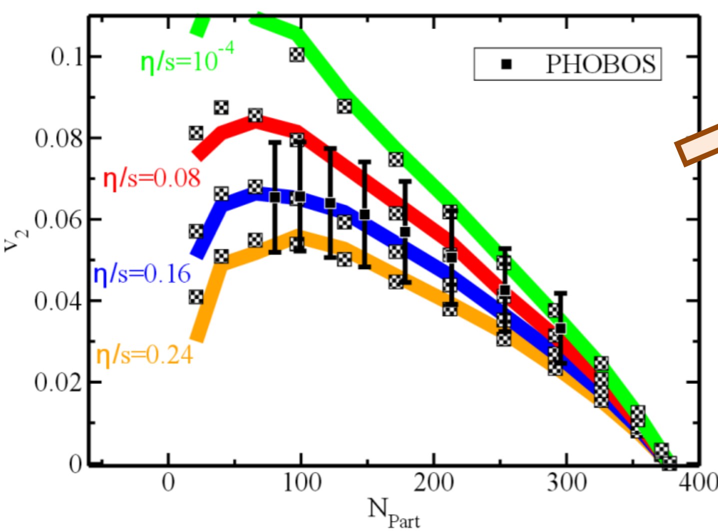
$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + p\, g^{\mu\nu} + \Pi^{\mu\nu}$$

energy density — *pressure* — *4-velocity* — *metric*

$\Pi^{\mu\nu}$: *1st and higher order gradient terms*

Include viscosity term:

$$\Pi^{\mu\nu} = -2\eta g^{\mu\nu}$$



In HICs, the kind of physics that dominates is viscous fluid dynamics.

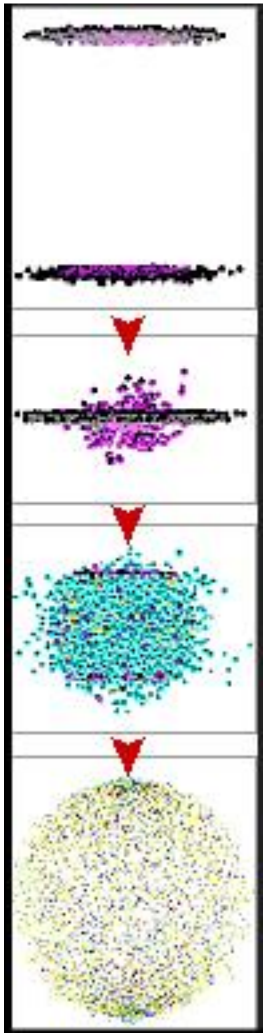
Connection to holography: $\frac{\eta}{s} = \frac{1}{4\pi} \sim 0.08$

- ➢ It should work when gradients are small.
- ➢ However, it seems to describe well HIC data!

[Luzum, Romatschke '09]

Evolution of a Heavy Ion Collision

(Picture not to scale)



Out of equilibrium

- Non-hydro degrees of freedom are dominant (at eq, they're damped).
- Gravity dual is useful description.
- Explicit and simple model: Shock wave collisions.

← - - - - - - - - - - $\Delta p \sim 0.7$

Hydrodynamization

- Transition to the hydrodynamics approximation.

Thermalization

- Isotropy of the diagonal components of $T^{\mu\nu}$ in local rest frame.
- Not necessary for the applicability of viscous hydro.

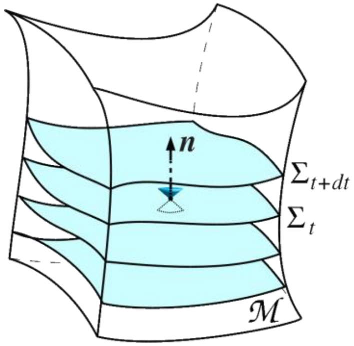
Hadronization

- The hadrons are back.
- Kinetic theory is applicable.

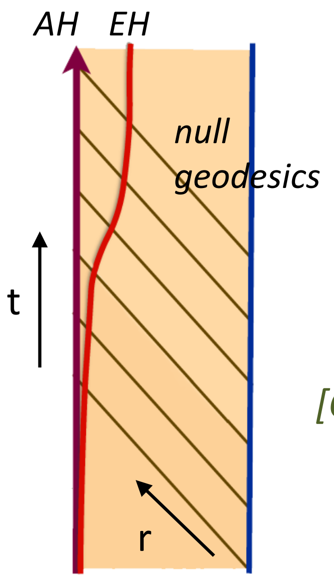
Gravitational formulation

Cauchy Formulation

- ADM decomposition based on Hamiltonian formulation of GR.
- Generalized harmonic evolution scheme.
- Foliation of spacelike hypersurfaces.



[Pretorius ,05]
[Pretorius, Gubser, Bantilan '12]



Characteristic Formulation

- Ingoing Eddington-Finkelstein coordinates.
- Foliation of null hypersurfaces.
- IR cutoff → require fixed-*r* Apparent Horizon condition.

[Chesler, Yaffe '08]

Gravitational formulation

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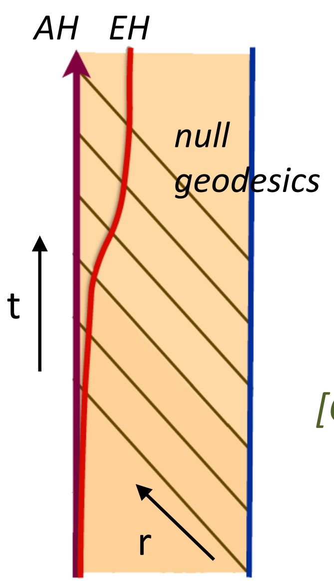

Characteristic Formulation

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[Chesler, Yaffe '08]

$$\partial_{rt} g = \mathcal{F}[S,g] \quad \Rightarrow \quad \partial_t \mathcal{G} = \mathcal{F}, \quad \partial_r g = \mathcal{G}$$

with conditions $\mathcal{C}_{ri}[S] = 0, \quad \mathcal{C}_{ti}[S] = 0$

Bianchi identities imply they are boundary constraints: $\nabla_r \mathcal{C}_{ri} = 0$

Gauge freedom:
coord. reparametrization

$r \to r + \xi(x^i) \quad \Rightarrow$ AH fixed

Outcome overview

- Plasma thermalizes very quickly: Hydro applicable within $\tau < 1/T$. *[Chesler, Yaffe '11]*
- Linearizing far-from-eq. State around final state:
  Surprisingly accurate description. *[Heller, Mateos, van der Schee, Trancanelli '12]*
- Fully dynamical simulation of a HIC:
  Holography + Hydro + cascade *[Pratt, Romatschke, van der Schee '13]*
- Simulation of BH collisions in AdS *[Bantilan, Romatschke '14]*
- Successful simulation of 4+1 evolution for off-center collisions *[Chesler, Yaffe '15]*

Ansatz equations

Null holographic coord.

Required initial data

$$ds^2 = 2\,dt\,dr - A\,dt^2 + 2F_i\,dt\,dx^i + \Sigma^2 \tilde{g}_{ij}\,dx^i dx^j$$

Determinant of spatial part

Boundary EOM:

$$\partial_t a_4 = -\frac{4}{3}\partial_y f_4$$

$$\partial_t f_4 = -\frac{1}{4}\partial_y a_4 - 2\partial_y b_4$$

Introducing ripples

$$ds^2 = 2\,dt\,dr - A\,dt^2 + 2dt\,(F\,dy + G\,dx_1) +$$
$$\Sigma^2 \left[e^{C-2B}\cosh D\,dy^2 + e^{B-C}\cosh D\,dx_1^2 + 2e^{-B/2}\sinh D\,dy\,dx_1 + e^B dx_2^2 \right]$$

Simplification from 3+1 to 2+1 by:

$$h(t,r,y,x_1) \to h_0(t,r,y) + \epsilon\, e^{ikx_1}\,\delta h(t,r,y)$$

Alternative: Use polar coordinates.
Assuming boost invariance and breaking rotational symmetry,

$$x_1,\, x_2 \to \rho,\, \phi$$
$$t = \tau\cosh\eta$$
$$y = \tau\sinh\eta$$

$$h(\tau,r,\rho,\phi) \to h_0(\tau,r,\rho) + \epsilon\, e^{im\phi}\,\delta h(\tau,r,\rho)$$

Initial Condition

Gaussian shocks

- Two separated shocks with finite thickness and energy density, moving toward each other:

$$ds^2 = \frac{L^2}{z^2}\left[-dt^2 + dz^2 + dy^2 + d\vec{x}^2 + \Phi\left(t, y, \vec{x}, z\right)\left(dy - dt\right)^2\right]$$

Exact analytic solution, in Fefferman-Graham coordinates.

$\Phi(t, y, \vec{x}, z = 0)$ is the bdry. energy density, arbitrary if

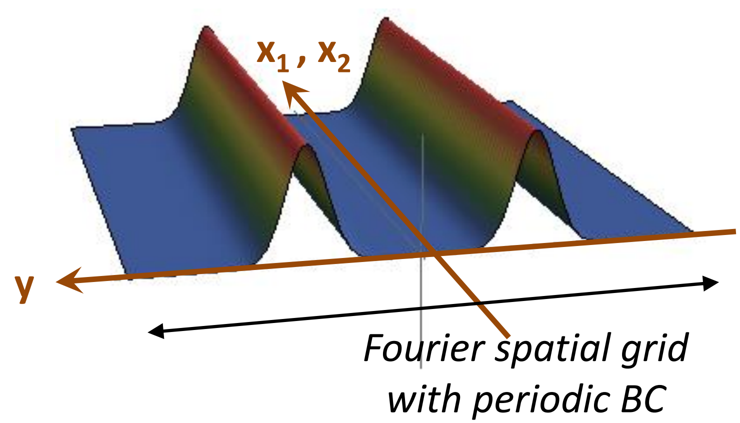
$$\left(\partial_z^2 - \frac{3}{z}\partial_z + \nabla_{\vec{x}}^2\right)\Phi = 0$$

choose profile:

$$\Phi(t, y) = \frac{\mu^3}{2\pi w^2} e^{-(t \pm y)^2/2w^2}$$

- Transform from FG to EF coord. → Infalling geodesic congruence → Read initial data:

$$B(0, r, y),\ a_4(0, y),\ f_4(0, y)$$



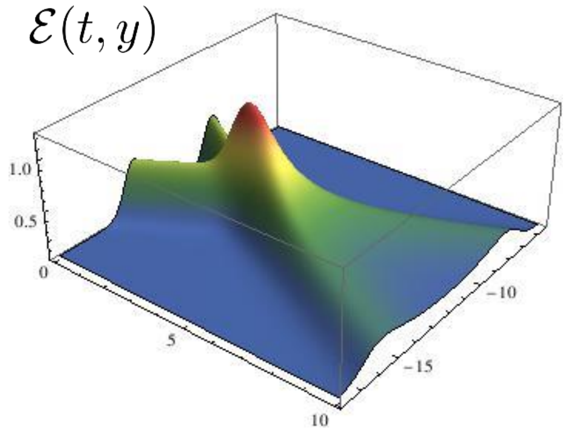
- Generalize for perturbations. Freedom of choice, simple example: $\delta\mathcal{E} = \epsilon e^{ik\, x_1}\mathcal{E}$

- Need to add a (small) regulator energy density.

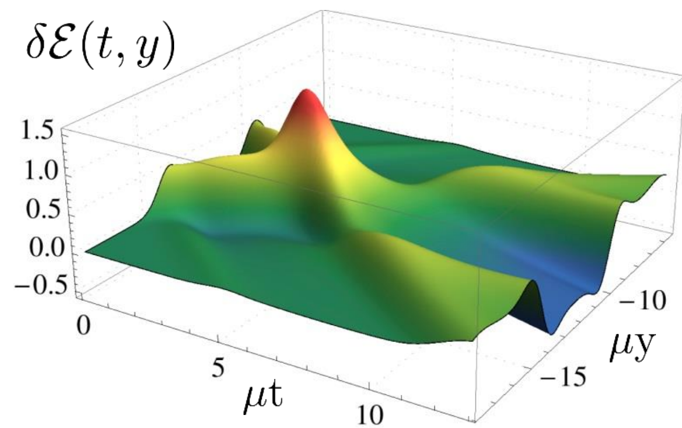
Stress-Energy Tensor results

- The inhomogeneities are evolved on top of the dynamical background.

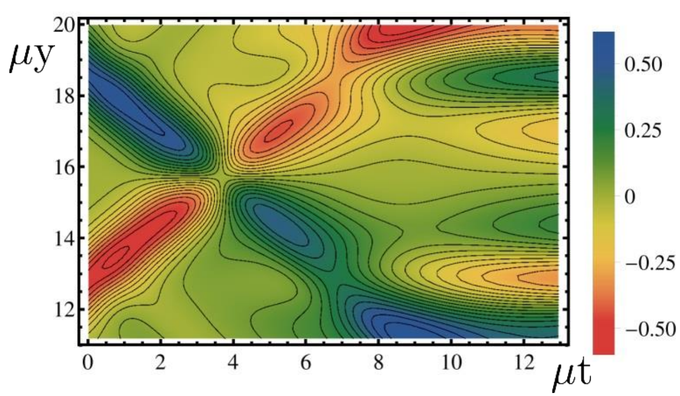
Energy density:



$+ \, \epsilon \, e^{ikx}$

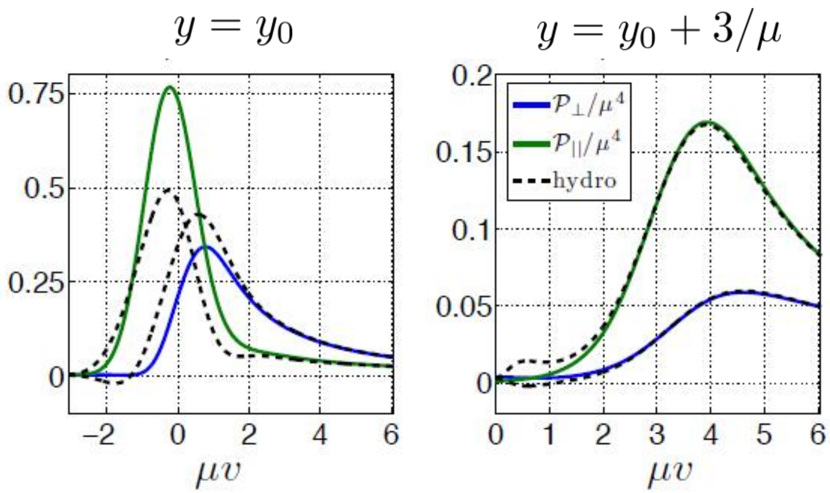


Longitudinal energy flux:



[Fernández '15]

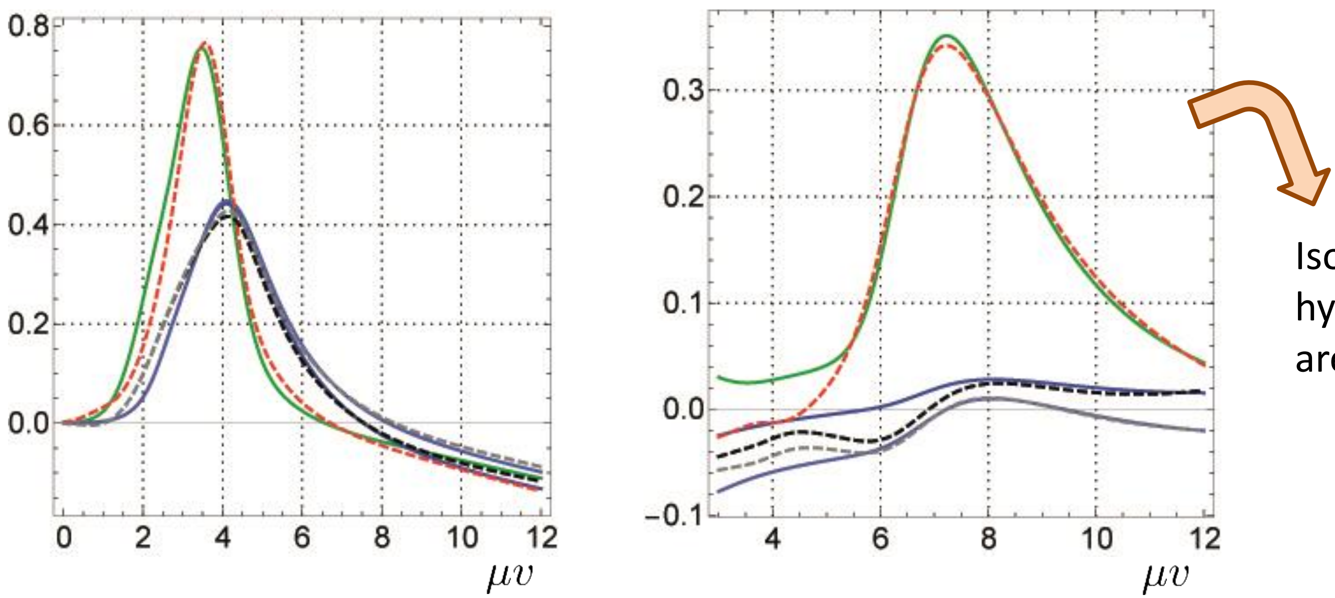
Comparing with hydrodynamics



- Compare with the pressures that would follow from the viscous hydrodynamic const. relations.

Mid-rapidity

- Dramatic rise in the pressures.
- Very anisotropic and out-of-eq.
- Hydro holds almost from the beginning.



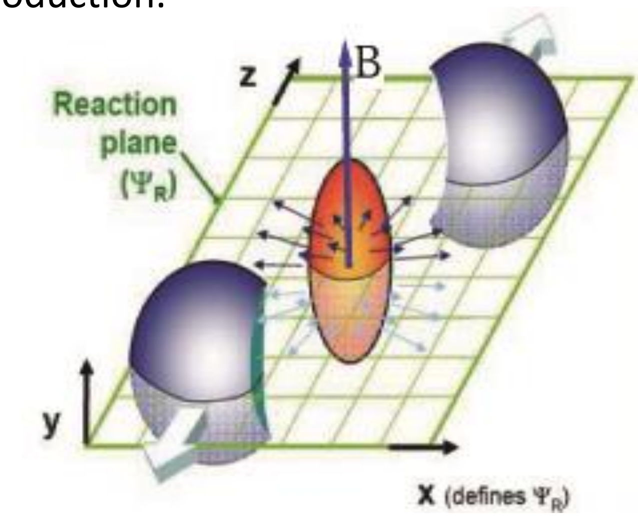
Isotropization time and hydrodynamization time are completely different.

[Fernández '15]

Outlook: QGP chiral anomaly

In a Heavy Ion Collision:

- Strong magnetic fields are produced by the charged "spectators" ($B \sim 5 - 15 m_\pi^2/e$)
- Anomalous electric currents → Observable effects in hadron production.



In relativistic chiral plasma, the anomaly is present:

$$\partial_\mu J^\mu = a_1 F \wedge F + a_3 \,\mathrm{Tr}\, G \wedge G$$

→ Anomalous electric current: $\vec{J} = \sigma_B \vec{B}$

Holomodel ingredients

- Massive A_μ → Emulate dynamical gluons for $\partial_\mu J^\mu \neq 0$.
- Scalar field θ → Recover gauge invariance.
- CS term → Non-dynamical part of anomaly.

[Rebhan, Schmitt, Stricker '09]
[Gürsoy, Kharzeev, Rajagopal '14]

❖ Stückelberg-Chern-Simons theory:

$$S = \int d^5x \sqrt{-g} \left[R + \Lambda - \frac{1}{4e} F^2 - \frac{m^2}{2} (A_\mu - \partial_\mu \theta)^2 + \frac{\kappa}{3} \epsilon^{\mu\alpha\beta\gamma\delta} (A_\mu - \partial_\mu \theta) F_{\alpha\beta} F_{\gamma\delta} \right]$$

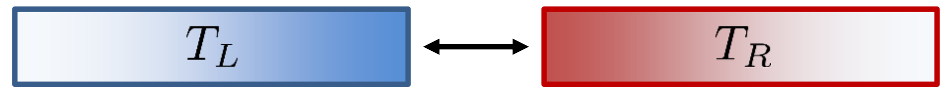
...evolution of axial charge?

Universal regime of thermal transport

[Bernard, Doyon '12]

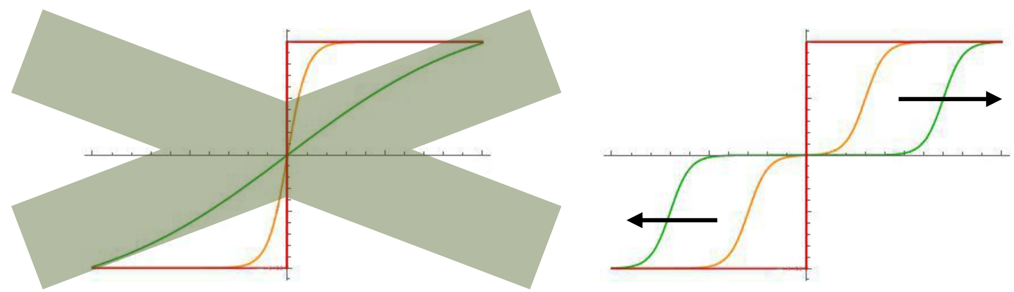
Thermal quench in 1+1

Two exact copies initially at equilibrium, independently thermalized.



Conservation eqs & tracelessness:

$$\partial_x \langle T^{xx} \rangle = -\partial_t \langle T^{tx} \rangle = 0$$
$$\langle T^{xx} \rangle = \langle T^{tt} \rangle$$



$$\langle T^{tx} \rangle = F(x-t) - F(x+t)$$
$$\langle T^{tt} \rangle = F(x-t) + F(x+t)$$

shock waves emanating from interface, converge to non-equilibrium *Steady State*.

Long time limit

$$\langle T^{tx} \rangle = cg(T_L - T_R)$$

↔

Energy flow

$$\langle J_E \rangle \neq 0$$

(Lorentz-boosted thermal distribution)

Thermal transport in d>1

[Bhaseen, Doyon, Lucas, Schalm '13]

Holographic dual

- In 1+1, holographic dual is unique: Boosted BTZ black hole.
- In d+1, assume ctant. homogeneous heat flow as well:

$$\langle T^{\mu\nu}\rangle = a_d\, T^{d+1}\,(\eta^{\mu\nu} + (d+1)u^\mu u^\nu)$$

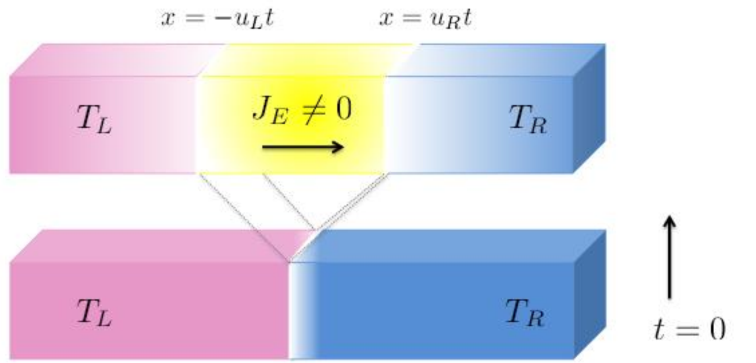
➡ also unique non-singular solution: Boosted black brane.

Generalization to any d:

- Effective dimension reduction to 1+1.
- *Dissipation*: Energy can be exchanged among the various constituents.
- Linear response regime:

$$|T_L - T_R| << T_L + T_R$$

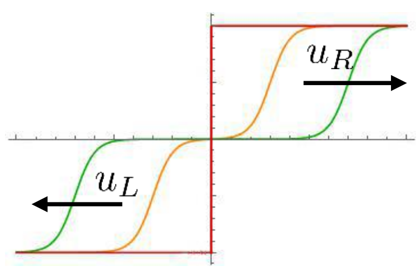
→ Hydro eqs. explicitly solvable.



$\langle \vec{J}_E \rangle \neq 0$ even if systems asymptotically far apart.

❖ Heat transport dominated by diffusion instead of flow?

➡ $\left\{ \begin{array}{l} \text{width} \propto \sqrt{t} \\ \text{distance} \propto t \end{array} \right.$

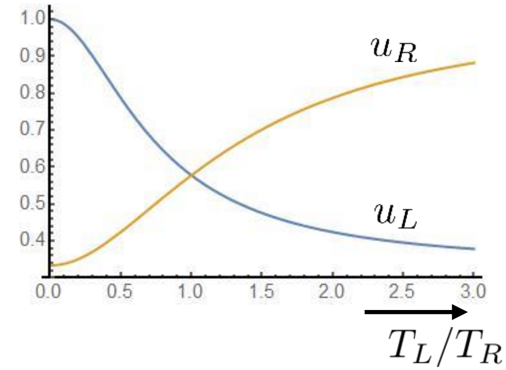


Hydrodynamics approach

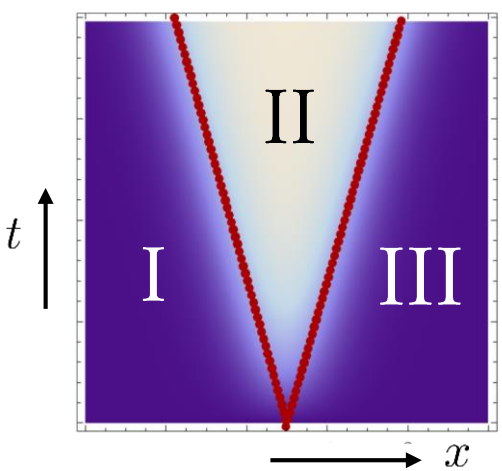
- Generic d prediction:

Final steady state characterized by

$$\langle T^{tx} \rangle = a_d \frac{T_L^{d+1} - T_R^{d+1}}{u_L + u_R} \quad \text{where} \quad a_d \sim \frac{L^d}{G_N} \quad \text{and}$$



- Shockwave velocities: $u_L u_R = \frac{1}{d} = c_s^2$



<u>Hydrodynamical evolution of 3 regions</u>

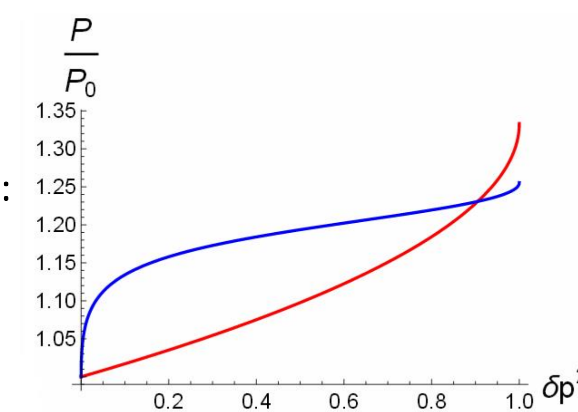
Match solutions → asymptotics of the central region?

$$T^{\mu\nu} = P(d\, u^\mu u^\nu + \eta^{\mu\nu}) + \pi^{\mu\nu} + \mathcal{O}(\partial^2) \quad \text{with} \quad p = \frac{\epsilon}{d-1}$$

looking for sols. of the form:

$$u^\mu = (\cosh[\theta(t,x)], \sinh[\theta(t,x)], \vec{0})$$

Two posible Steady State configurations:
Thermodynamic branch, second branch



[Bhaseen, Doyon, Lucas, Schalm '13]
[Chang, Karch, Yarom '13]

Late time prediction
is matched

Holographic approach

[Amado, Yarom '15]

Energy transport: Lorentz-boosted thermal distribution

→ Gravity dual: Boosted black brane

$$ds^2 = \frac{L^2}{z^2}\left[\frac{dz^2}{f(z)} - f(z)\left(\cosh\theta\, dt - \sinh\theta\, dx\right)^2 + \left(\cosh\theta\, dx - \sinh\theta\, dt\right)^2 + dx_\perp^2\right]$$

Check via holography

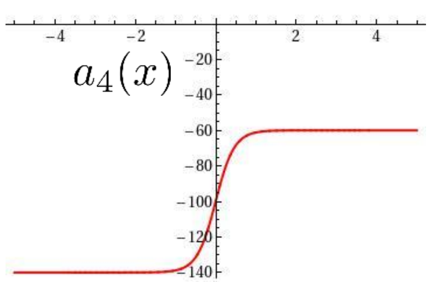
Numerical details

- Full out-of-equilibrium evolution of Einstein's equations.
  - Ansatz for Characteristic Evolution (Eddington-Finkelstein coord.):

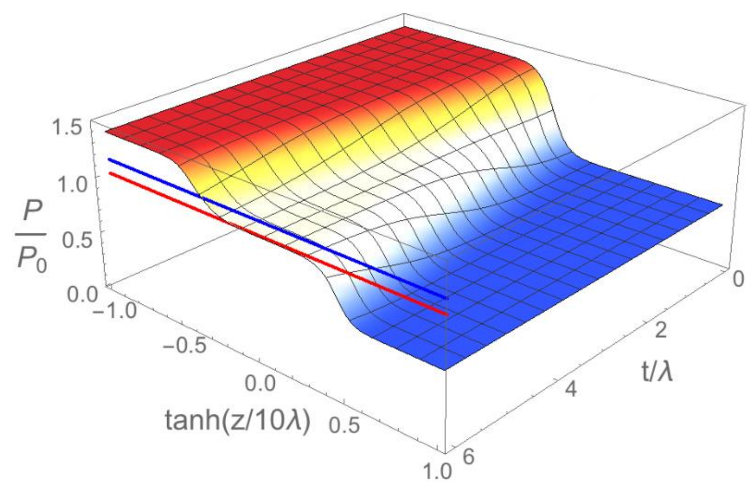
$$ds^2 = 2dt\left(dr - A\, dt - F\, dx\right) + \Sigma^2\left(e^{B}\, dx_\perp^2 + e^{-B}\, dx^2\right)$$

- Initial data:

$B = 0,$
$f_4 = 0,$
and



evolution

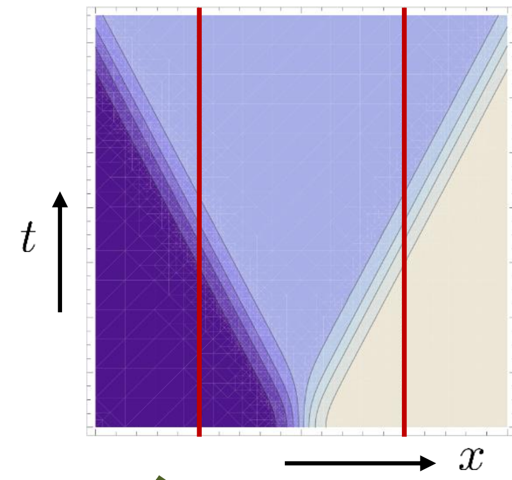


Information flow

How does information get exchanged between the systems which are isolated at t=0?

- Entanglement entropy
  → Holographic measure: $$S_E = \frac{\text{Area}(\gamma_A)}{4G_N^{d+2}}$$

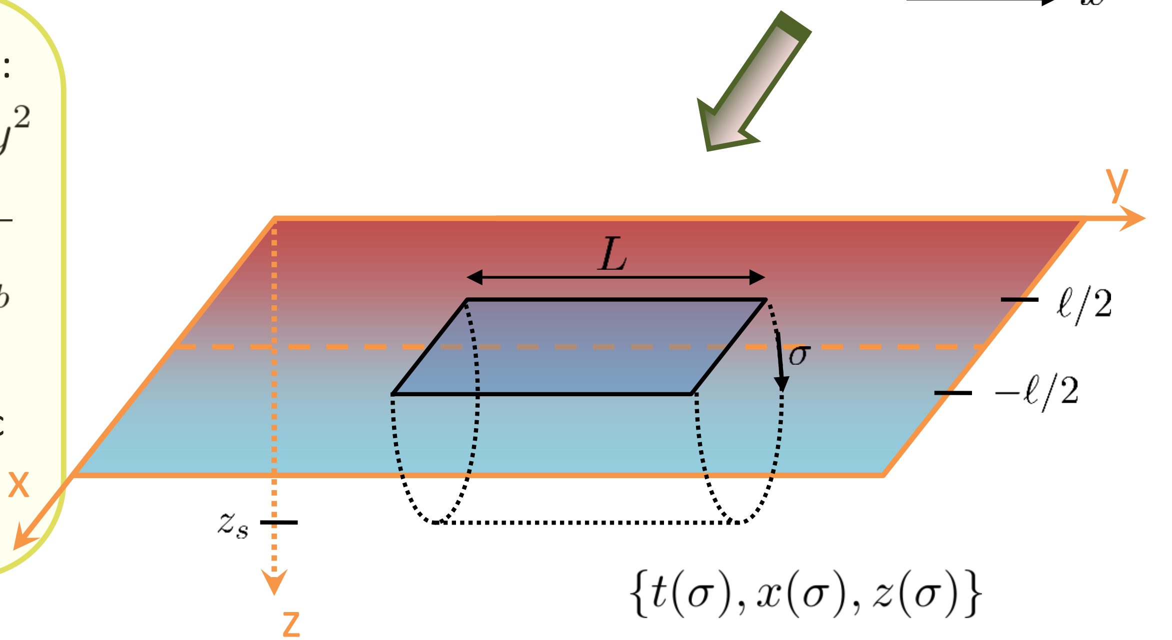
- Regularization: Substract result at T=0: $$S_{EE}^{div} \sim \frac{1}{z_{uv}}$$



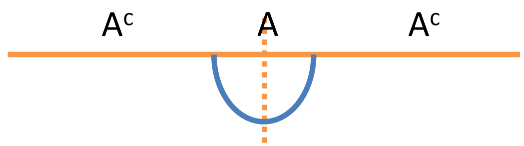
> Spatial induced metric on surface:
$$ds^2 = h_{\sigma\sigma} d\sigma^2 + g_{yy} dy^2$$

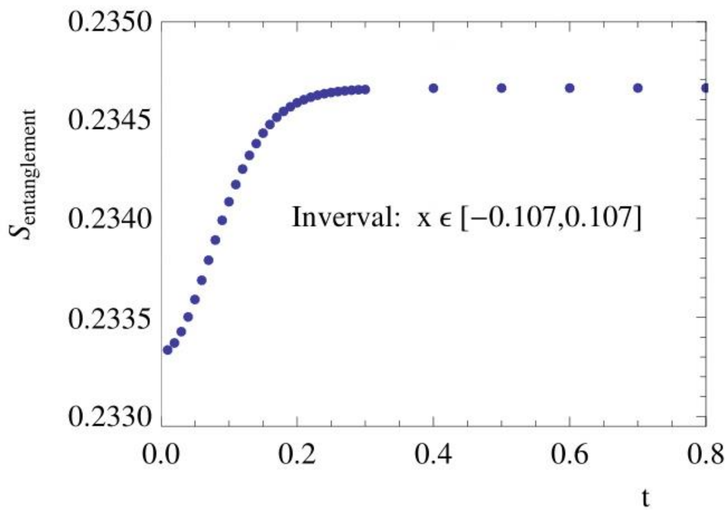
> Action:
$$\mathcal{I} = L \int ds \sqrt{\frac{\partial x^a}{\partial \sigma}\frac{\partial x^b}{\partial \sigma}\tilde{g}_{ab}}$$

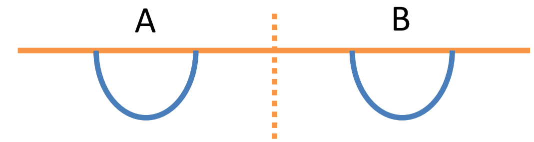
> Geodesic eqs. for effective metric
$$\tilde{g}_{ab} = g_{yy} g_{ab}$$

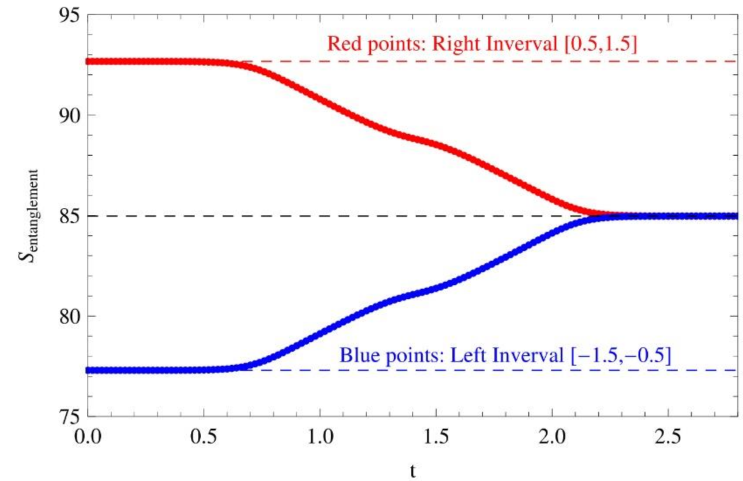


Entanglement Entropies







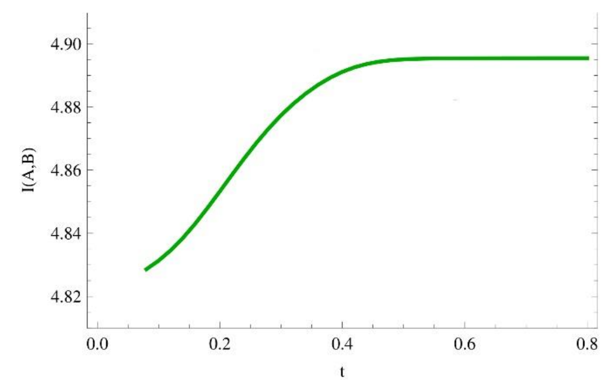


Mutual Information

What do we learn about A by looking at B?

$$I(A, B) = S(A) + S(B) - S(A + B)$$

- Shockwaves transport information.
- M.I. grows as shocks pass through the region.



[Erdmenger, Fernández, Flory, Megías, Straub '15]

Outlook

- QGP anomalies
  a. *Time evolution of the axial charge*
  b. *Axial asymmetry in hadronic distribution?*

- Azimuthal perturbations with radial flow
  a. *Comparison with elliptic flow*
  b. *Connection to hydrodynamics*

- Thermal flow analysis
  a. *Generalize: 2+1 → 3+1, what changes?*
  b. *Asymmetry – Intervals are not reached simultaneously*

Thank you!