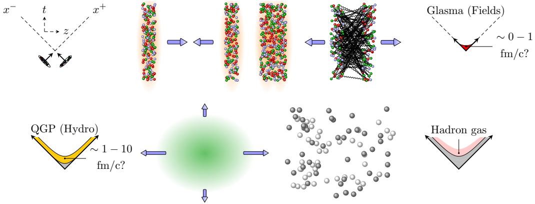
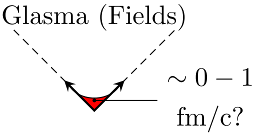


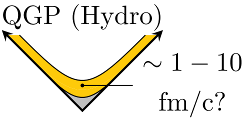
On the use of the classical approximation for the early stages of HIC

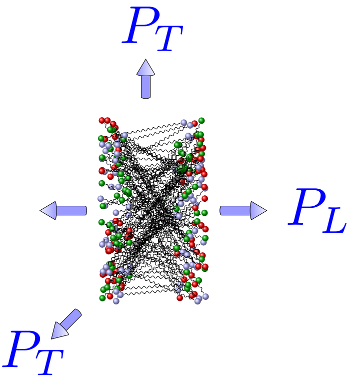
INT, Seattle, 24/08/15

Thomas EPELBAUM

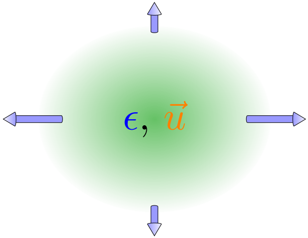
McGill


Glasma (Fields)
$\sim 0 - 1$
fm/c?

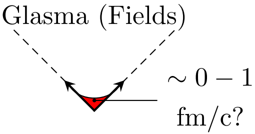

QGP (Hydro)
$\sim 1 - 10$
fm/c?

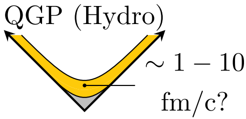

P_T
P_L
P_T

?


$\epsilon, \vec{u}$

How to match the pre-equilibrium stage with hydrodynamics?





$$T^{\mu\nu} = T^{\mu\nu}_{\text{ideal}} + \Pi^{\mu\nu} \qquad u_\mu T^{\mu\nu} = \epsilon u^\nu$$

$$T^{\mu\nu}_{\text{ideal}} = \epsilon u^\mu u^\nu - p\Delta^{\mu\nu}$$

~~$\Pi^{\mu\nu}$~~

THE COLOR GLASS CONDENSATE [MCLERRAN, VENUGOPALAN (1993)]

$$\mathcal{L} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + J_\mu \mathcal{A}^\mu \qquad \text{(no quarks)}$$

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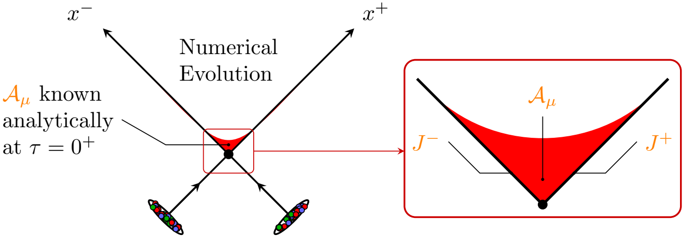
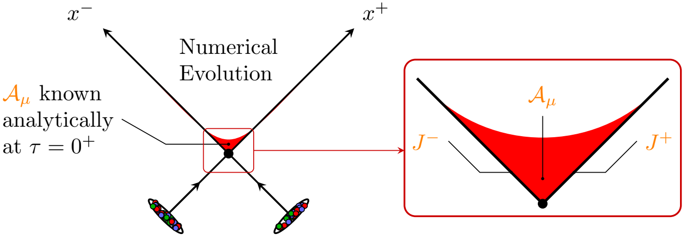


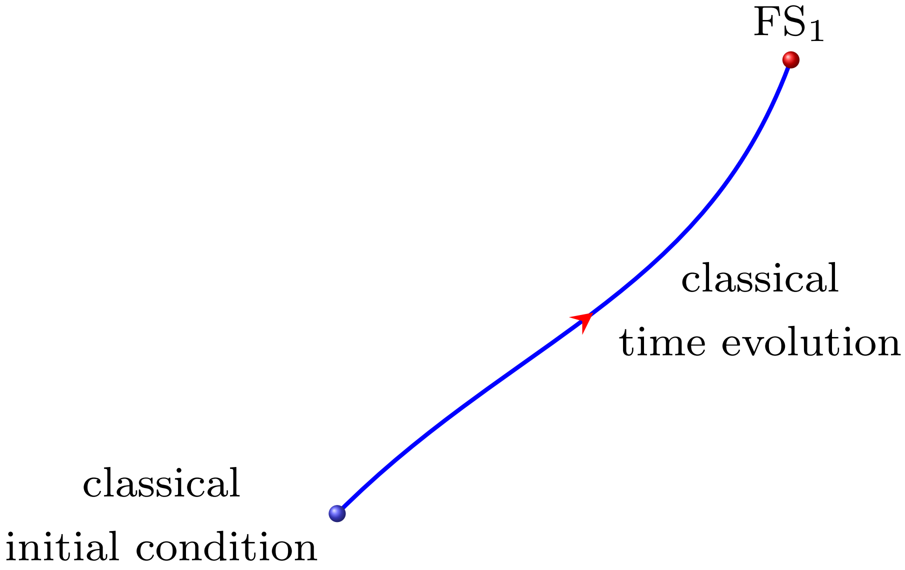
The Color Glass Condensate [McLerran, Venugopalan (1993)]

$$\mathcal{L} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + J_\mu \mathcal{A}^\mu \qquad \text{(no quarks)}$$



LO: $[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}] = J^\nu \sim \frac{Q_s^3}{g}$ (Color sources on the light cone) $\qquad \epsilon = \frac{1}{2}\left(\vec{\mathcal{E}}^2 + \vec{\mathcal{B}}^2\right) \sim \frac{Q_s^4}{g^2}$ (**Classical** color fields)

[Krasnitz, Venugopalan (1998)]


FS_1
classical
time evolution
classical
initial condition

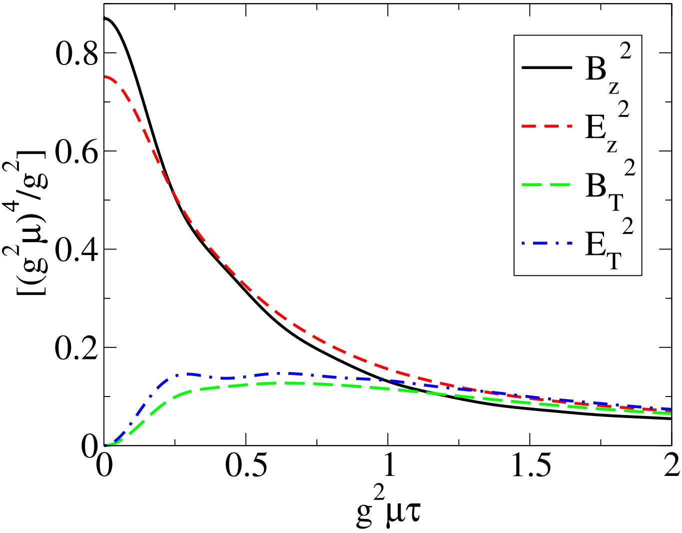
The Color Glass Condensate at its LO

$$\epsilon = \mathcal{E}_\perp^2 + \mathcal{B}_\perp^2 + \mathcal{E}_L^2 + \mathcal{B}_L^2$$

$$P_T = \mathcal{E}_L^2 + \mathcal{B}_L^2$$

$$P_L = \mathcal{E}_\perp^2 + \mathcal{B}_\perp^2 - \mathcal{E}_L^2 - \mathcal{B}_L^2$$

The Color Glass Condensate at its LO



[Lappi, McLerran (2006)]

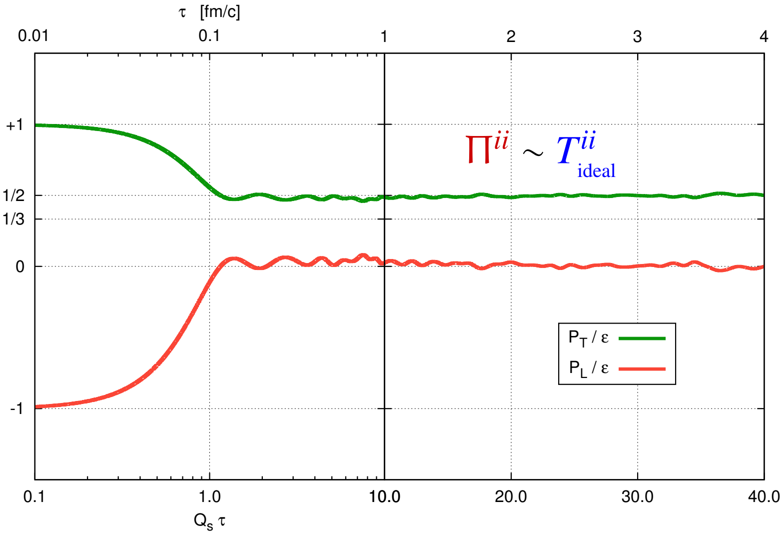
$$\epsilon = \underbrace{\mathcal{E}_\perp^2}_{0} + \underbrace{\mathcal{B}_\perp^2}_{0} + \mathcal{E}_L^2 + \mathcal{B}_L^2$$

$$P_T = \mathcal{E}_L^2 + \mathcal{B}_L^2$$

$$P_L = \underbrace{\mathcal{E}_\perp^2}_{0} + \underbrace{\mathcal{B}_\perp^2}_{0} - \mathcal{E}_L^2 - \mathcal{B}_L^2$$

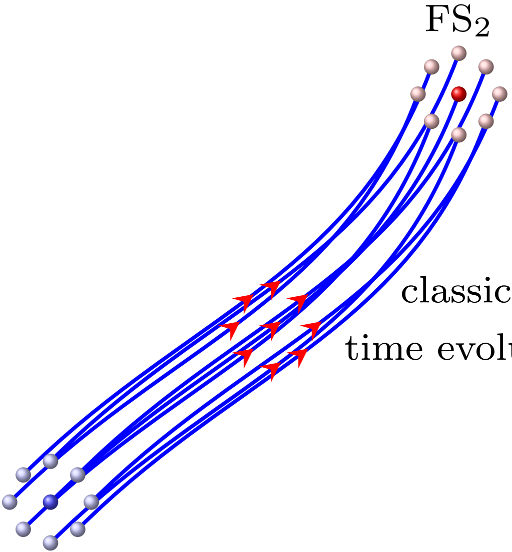
Initial $T^{\mu\nu}$ is $(\epsilon, \epsilon, \epsilon, -\epsilon)$!

THE COLOR GLASS CONDENSATE AT ITS LO
Strong anisotropy at early time



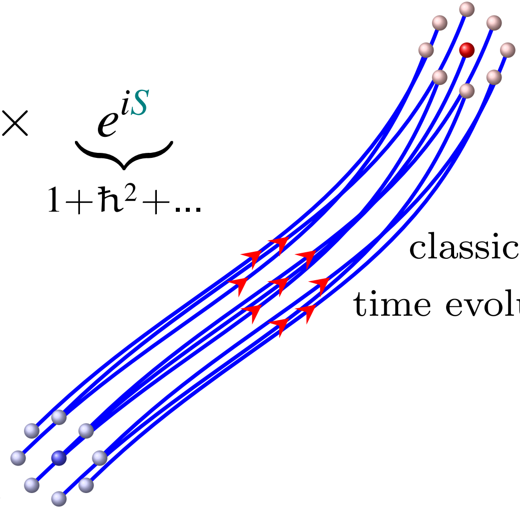
[LAPPI, MCLERRAN (2006), FUKUSHIMA, GELIS (2012)...]

quantum
initial condition

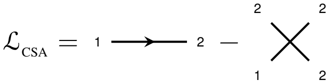


classical
time evolution

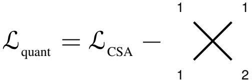
$$\mathcal{Z} \sim \underbrace{\rho}_{1+\hbar+\ldots} \times \underbrace{e^{iS}}_{1+\hbar^2+\ldots}$$

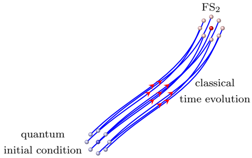


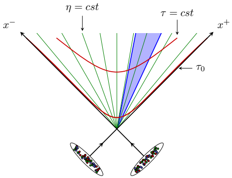
The classical Lagrangean reads



Differs from the full Lagrangean







Initial condition

$$A_0{}^{\mu a}(\tau_0, \boldsymbol{x}_\perp, \eta) = \mathcal{A}_0{}^{\mu a}(\tau_0, \boldsymbol{x}_\perp) + \int_{\boldsymbol{k}} c_{\boldsymbol{k}}\; a_{\boldsymbol{k}}^{\mu a}(\tau_0, \boldsymbol{x}_\perp, \eta)$$

Time evolution ($I = x, y, \eta$) for each configuration

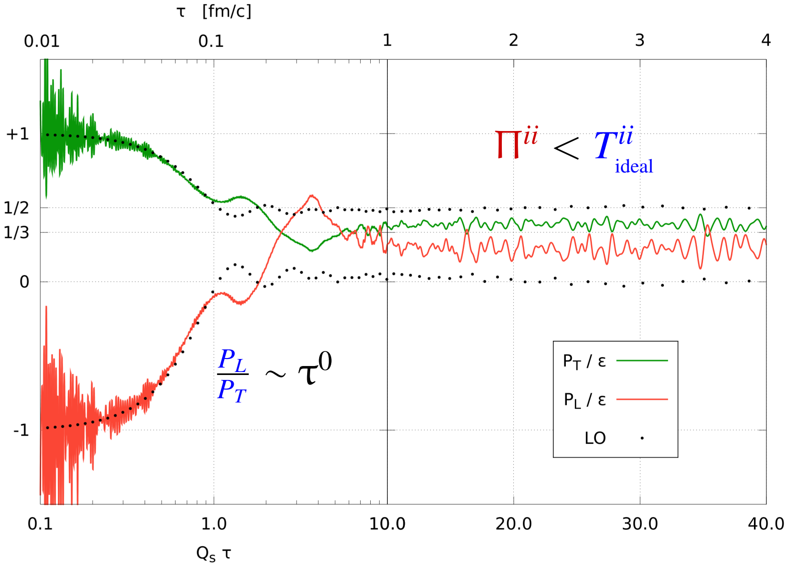
$$D_\mu F^{\mu I} = 0 \qquad \Rightarrow \qquad \epsilon, P_T, P_L$$

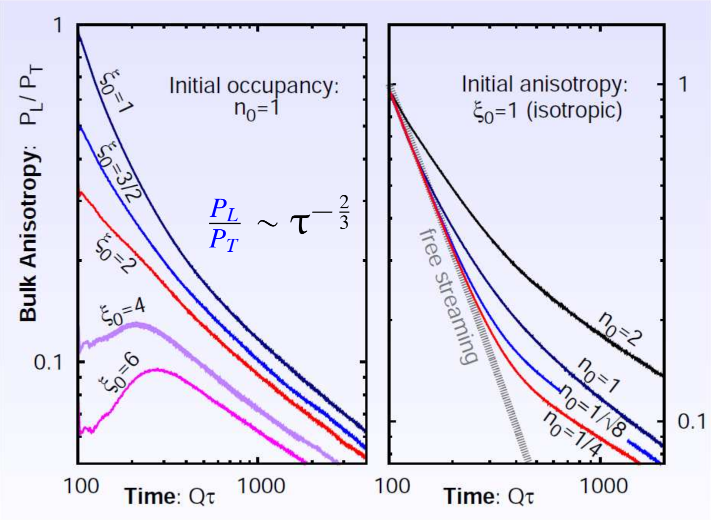
Cross checks: Gauss's law and Bjorken's law

$$D_\mu E^\mu = 0 \qquad\qquad \tau\partial_\tau \epsilon = -\epsilon - P_L$$

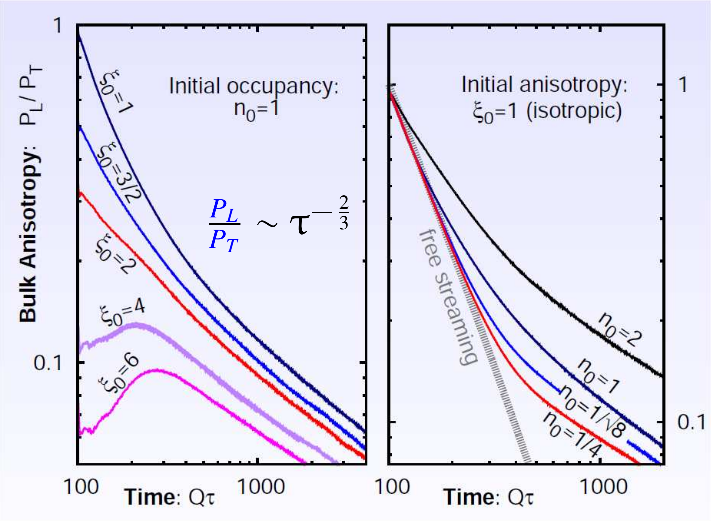
NUMERICAL RESULTS [TE,GELIS (2013)]

$\alpha_s = 2\,10^{-2}$ $(g = 0.5)$





$$f_0 = \frac{n_0}{2g^2}\theta\left(Q - \sqrt{p_\perp^2 + (\xi p_z)^2}\right)$$



$$\frac{P_L}{P_T} \sim \tau^{-\frac{2}{3}}$$

Why is it so different??

BBSV Scenario

- Start at $Q\tau \sim 100$ with $\mathcal{A} = 0$ and $a \sim \frac{1}{g} \to g$ scales out
- at $Q\tau \gtrsim 300$: $f(p_\perp, p_z) = (Q_s\tau)^\alpha f_0((Q_s\tau)^\beta p_\perp, (Q_s\tau)^\gamma p_z)$
- $\alpha, \beta, \gamma = (-\frac{2}{3}, 0, \frac{1}{3})$ "universals". deduced from

$$\epsilon = \text{cst} \times \tau^{-1} \qquad n = \text{cst} \times \tau^{-1} \qquad \partial_\tau f = \hat{q}\partial_z^2 f$$

BBSV Scenario

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$$\epsilon = \text{cst} \times \tau^{-1} \qquad n = \text{cst} \times \tau^{-1} \qquad \partial_\tau f = \hat{q}\partial_z^2 f$$

TG
- LO+NLO fully
- $Q_s\tau_{\text{init}} \ll 1$
- $g \lesssim 0.5$

upside: close to real situation

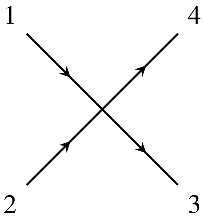
downside: Λ effects ?
$Q_s\tau_{\text{init}} \gg 1$ not accessible

BBSV
- not clear LO, not NLO
- $Q_s\tau_{\text{init}} \gg 1$
- $g \lesssim 10^{-6}$

upside: Almost no Λ effects

downside: Phenomenological relevance?
Fixed point IC dependent?
How to flow?

CSA non renormalizable: How to really see the Λ effects?[1]



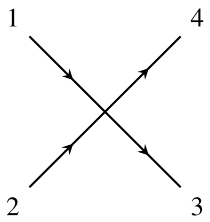
Prerequisites

$1 \ll f \ll g^{-2}$ $(Qt \gg 1)$

f isotropic: $f(\boldsymbol{p}) \to f(|\boldsymbol{p}|)$

[1] Gauge case: [ABRAAO, KURKELA, LU, MOORE (2014)]

CSA non renormalizable: How to really see the Λ effects?[1]



Prerequisites

$1 \ll f \ll g^{-2}$ $(Qt \gg 1)$

f isotropic: $f(\boldsymbol{p}) \to f(|\boldsymbol{p}|)$

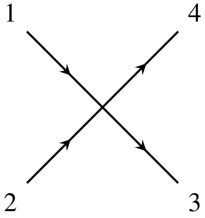
$$\partial_t f_1 = \frac{(2\pi)^4 g^4}{4E_1} \int_{2,3,4} \delta^4(P_1 + P_2 - P_3 - P_4) \underbrace{[(1+f_1)(1+f_2) f_3 f_4 - f_1 f_2 (1+f_3)(1+f_4)]}_{F[f]}$$

with

$$\int_k = \int \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\pi)^3 2E_k} \qquad E_k = \sqrt{|\boldsymbol{k}|^2 + m^2}$$

[1] Gauge case: [ABRAAO, KURKELA, LU, MOORE (2014)]

CSA non renormalizable: How to really see the Λ effects?[1]



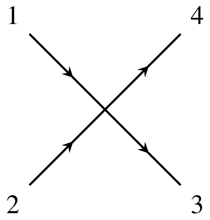
Prerequisites
$1 \ll f \ll g^{-2}$ $(Qt \gg 1)$
f isotropic: $f(\boldsymbol{p}) \to f(|\boldsymbol{p}|)$

Quantum theory Q: keep everything

$$F_{\mathrm{Q}}[f] = (1+f_1)(1+f_2)f_3f_4 - f_1f_2(1+f_3)(1+f_4)$$

[1] Gauge case: [Abraao, Kurkela, Lu, Moore (2014)]

CSA non renormalizable: How to really see the Λ effects?[1]



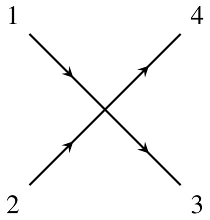
Prerequisites
$1 \ll f \ll g^{-2}$ $(Qt \gg 1)$
f isotropic: $f(\boldsymbol{p}) \to f(|\boldsymbol{p}|)$

Classical approximation $\mathcal{C}^0 \to f \gg 1$, keep the dominant term in $F_{\mathcal{Q}}[f]$

$$F_{\mathcal{C}^0}[f] = (f_1 + f_2) f_3 f_4 - f_1 f_2 (f_3 + f_4)$$

[1] Gauge case: [ABRAAO, KURKELA, LU, MOORE (2014)]

CSA non renormalizable: How to really see the Λ effects?[1]



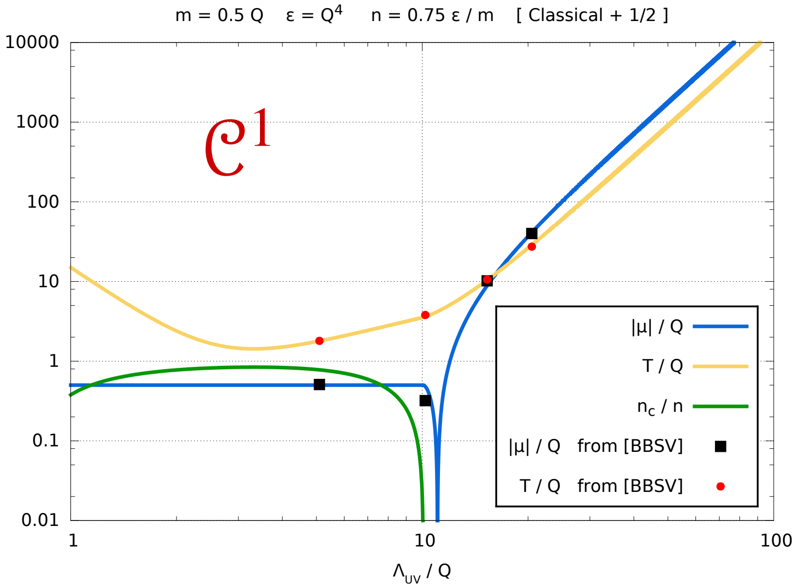
Prerequisites
$1 \ll f \ll g^{-2}$ $(Qt \gg 1)$
f isotropic: $f(\boldsymbol{p}) \to f(|\boldsymbol{p}|)$

Classical-Statistical approximation $\mathcal{C}^1 \to \mathcal{C}^0$ and then $f \to f + \frac{1}{2}$
[MUELLER, SON (2002)]

$$F_{\mathcal{C}^1}[f] = F_{\mathcal{Q}}[f] + \frac{1}{4}(f_3 + f_4 - f_1 - f_2)$$

[1] Gauge case: [ABRAAO, KURKELA, LU, MOORE (2014)]

KINETIC TREATMENT: NUMERICAL RESULTS FOR ISOTROPIC f
[TE, GELIS, TANJI, WU (2014)]



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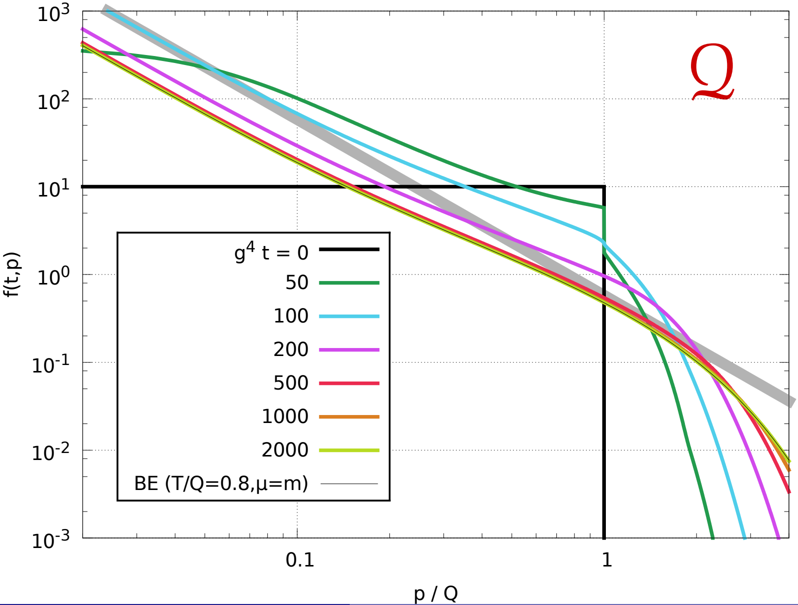

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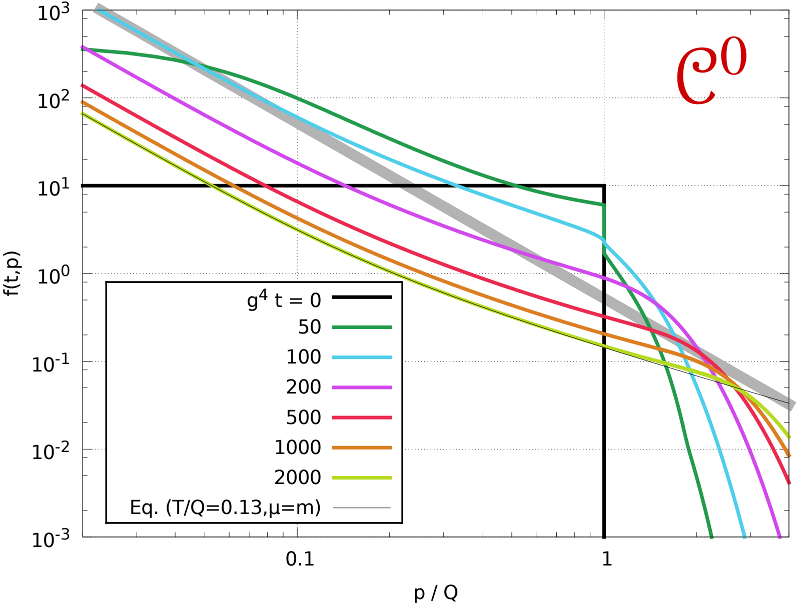

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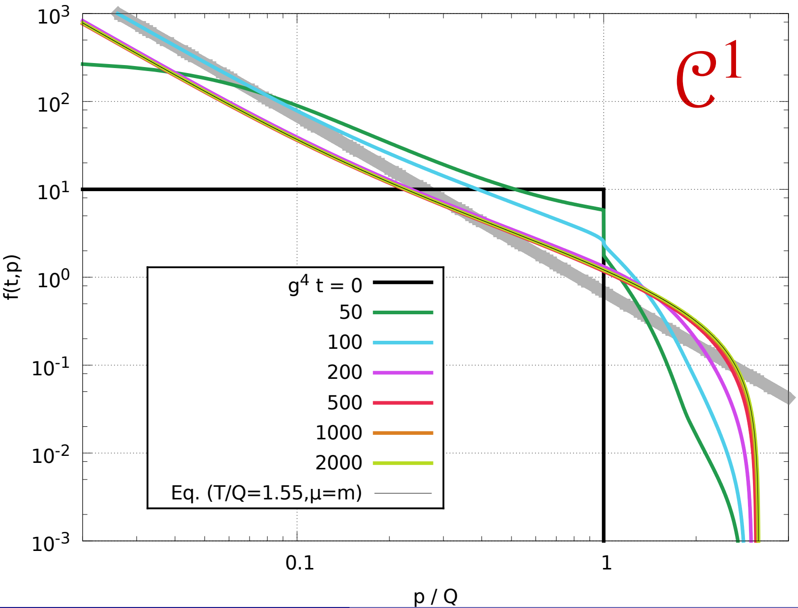

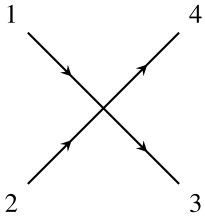

Prerequisites

$1 \ll f \ll g^{-2}$ ($Qt \gg 1$)

f anisotropic: $f(\boldsymbol{p}) \to f(|\boldsymbol{p}_\perp|, p_z)$

Boltzmann equation for $2 \leftrightarrow 2$ elastic scattering

$$\partial_t f_1 = \frac{(2\pi)^4 g^4}{4E_1} \int_{2,3,4} \delta^4(P_1 + P_2 - P_3 - P_4) F[f]$$

Now suppose f very anisotropic initially

$$f_p \sim \delta(p_z) f_0(p_\perp)$$

What can happen?

Now suppose f very anisotropic initially

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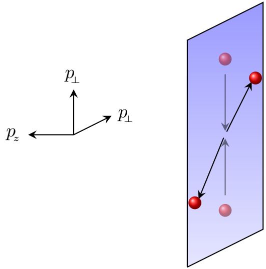
What can happen?

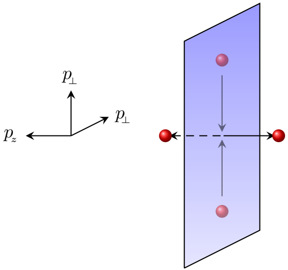


In plane collisions → no isotropization

Now suppose f very anisotropic initially

$$f_p \sim \delta(p_z) f_0(p_\perp)$$

What can happen?



Out of plane collisions $\rightarrow$ isotropization

Now suppose f very anisotropic initially

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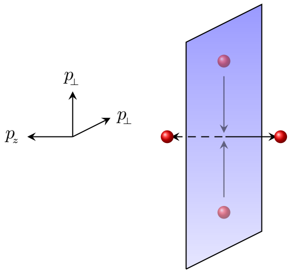
What can happen?



Out of plane collisions $\rightarrow$ isotropization

Can these large angle collisions happen?

Remember

$$F_{\mathcal{C}^0}[f] = (f_1 + f_2) f_3 f_4 - f_1 f_2 (f_3 + f_4)$$
$$F_{\mathcal{Q}}[f] = F_{\mathcal{C}^0}[f] + f_3 f_4 - f_1 f_2$$

Now take initially $f_{p_\perp, p_z} = 2\pi\delta(p_z) f_0(p_\perp)$

Let us inspect the Boltzmann equation in both cases

Remember

$$F_{\mathcal{C}^0}[f] = (f_1 + f_2)f_3 f_4 - f_1 f_2(f_3 + f_4)$$
$$F_{\mathcal{Q}}[f] = F_{\mathcal{C}^0}[f] + f_3 f_4 - f_1 f_2$$

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Let us inspect the Boltzmann equation in both cases

$\mathcal{C}^0$

$$\partial_t f_1 = \delta(p_{1z}) \frac{(2\pi)^4 g^4}{4E_1} \int_{2_\perp, 3_\perp, 4_\perp} \delta^3(P_{1\perp} + P_{2\perp} - P_{3\perp} - P_{4\perp}) F_{\mathcal{C}^0}[f_0]$$

Remember

$$F_{\mathcal{C}^0}[f] = (f_1+f_2)f_3f_4 - f_1f_2(f_3+f_4)$$
$$F_{\mathcal{Q}}[f] = F_{\mathcal{C}^0}[f] + f_3f_4 - f_1f_2$$

Now take initially $f_{p_\perp,p_z} = 2\pi\delta(p_z)f_0(p_\perp)$

Let us inspect the Boltzmann equation in both cases

$\mathcal{Q}$

$$\partial_t f_1 = \delta(p_{1z})\frac{(2\pi)^4g^4}{4E_1}\int_{2_\perp,3_\perp,4_\perp}\delta^3(P_{1\perp}+P_{2\perp}-P_{3\perp}-P_{4\perp})F_{\mathcal{C}^0}[f_0]$$
$$+\frac{(2\pi)^4g^4}{4E_1}\int_{2,3_\perp,4_\perp}\delta^3(P_{1\perp}+P_{2\perp}-P_{3\perp}-P_{4\perp})\delta(p_{1z}+p_{2z})f_3f_4$$
$$-\delta(p_{1z})\frac{(2\pi)^4g^4}{4E_1}\int_{2_\perp,3,4}\delta^3(P_{1\perp}+P_{2\perp}-P_{3\perp}-P_{4\perp})\delta(p_{3z}+p_{4z})f_1f_2$$

Remember

$$F_{\mathcal{C}^0}[f] = (f_1 + f_2) f_3 f_4 - f_1 f_2 (f_3 + f_4)$$
$$F_{\mathcal{Q}}[f] = F_{\mathcal{C}^0}[f] + f_3 f_4 - f_1 f_2$$

Now take initially $f_{p_\perp, p_z} = 2\pi\delta(p_z) f_0(p_\perp)$

Let us inspect the Boltzmann equation in both cases

$\mathcal{Q}$

$$\partial_t f_1 = \delta(p_{1z}) \frac{(2\pi)^4 g^4}{4E_1} \int_{2_\perp, 3_\perp, 4_\perp} \delta^3(P_{1\perp} + P_{2\perp} - P_{3\perp} - P_{4\perp}) F_{\mathcal{C}^0}[f_0]$$
$$+ \frac{(2\pi)^4 g^4}{4E_1} \int_{2, 3_\perp, 4_\perp} \delta^3(P_{1\perp} + P_{2\perp} - P_{3\perp} - P_{4\perp}) \delta(p_{1z} + p_{2z}) f_3 f_4$$
$$- \delta(p_{1z}) \frac{(2\pi)^4 g^4}{4E_1} \int_{2_\perp, 3, 4} \delta^3(P_{1\perp} + P_{2\perp} - P_{3\perp} - P_{4\perp}) \delta(p_{3z} + p_{4z}) f_1 f_2$$

Remember

$$F_{\mathcal{C}^0}[f] = (f_1 + f_2)f_3 f_4 - f_1 f_2 (f_3 + f_4)$$
$$F_{\mathcal{Q}}[f] = F_{\mathcal{C}^0}[f] + f_3 f_4 - f_1 f_2$$

Now take initially $f_{p_\perp, p_z} = 2\pi\delta(p_z) f_0(p_\perp)$

Let us inspect the Boltzmann equation in both cases

$\mathcal{C}^0$ artificially supresses large angle collisions.

Remember

$$F_{\mathcal{C}^0}[f] = (f_1 + f_2)f_3 f_4 - f_1 f_2 (f_3 + f_4)$$
$$F_{\mathcal{Q}}[f] = F_{\mathcal{C}^0}[f] + f_3 f_4 - f_1 f_2$$

Now take initially $f_{p_\perp, p_z} = 2\pi\delta(p_z) f_0(p_\perp)$

Let us inspect the Boltzmann equation in both cases

$\mathcal{C}^0$ artificially supresses large angle collisions.

$\mathcal{C}^0$ artificially "traps" anistropic distributions.

Remember

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$$F_{\mathcal{Q}}[f] = F_{\mathcal{C}^0}[f] + f_3 f_4 - f_1 f_2$$

Now take initially $f_{p_\perp, p_z} = 2\pi\delta(p_z) f_0(p_\perp)$

Let us inspect the Boltzmann equation in both cases

$\mathcal{C}^0$ artificially supresses large angle collisions.

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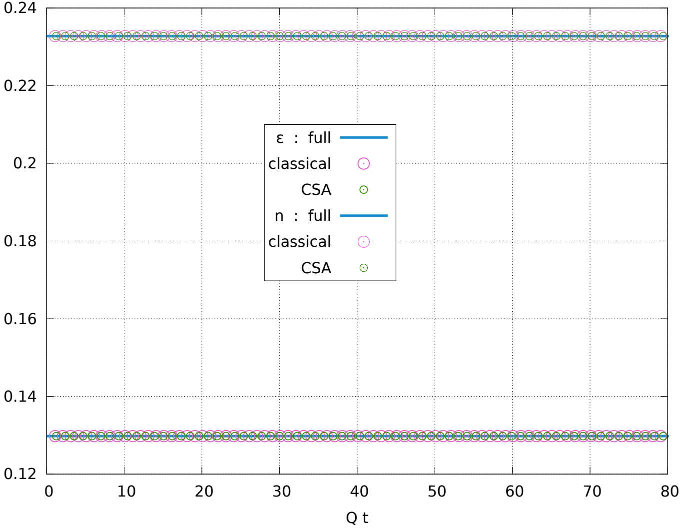
Could it be the reason why $\mathcal{C}^0$ so slow to isotropize?

ANISOTROPIC f: ILLUSTRATION OF THE PROBLEM WITH THE $\mathcal{C}^0$ SCHEME

[TE, GELIS, JEON, MOORE, WU (2015)]



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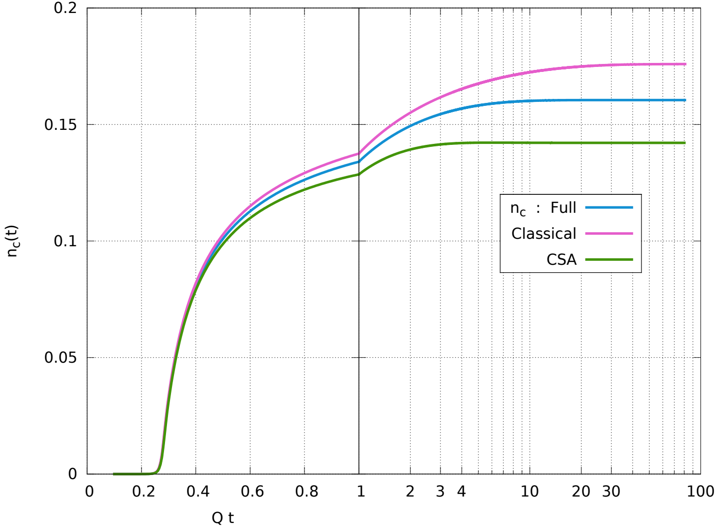

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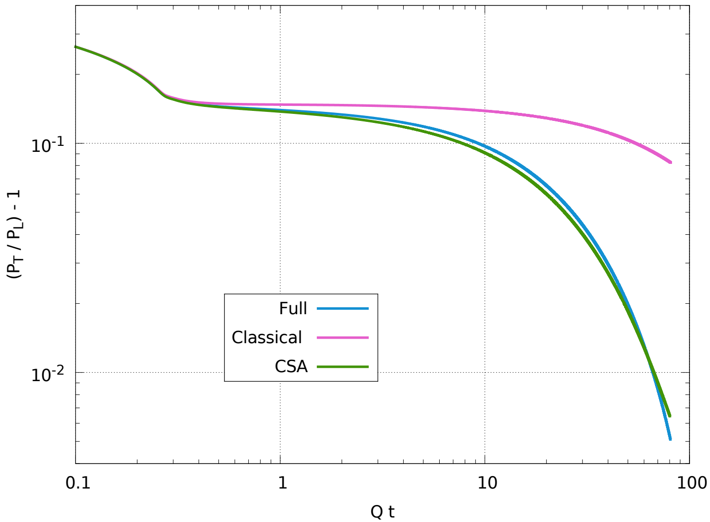
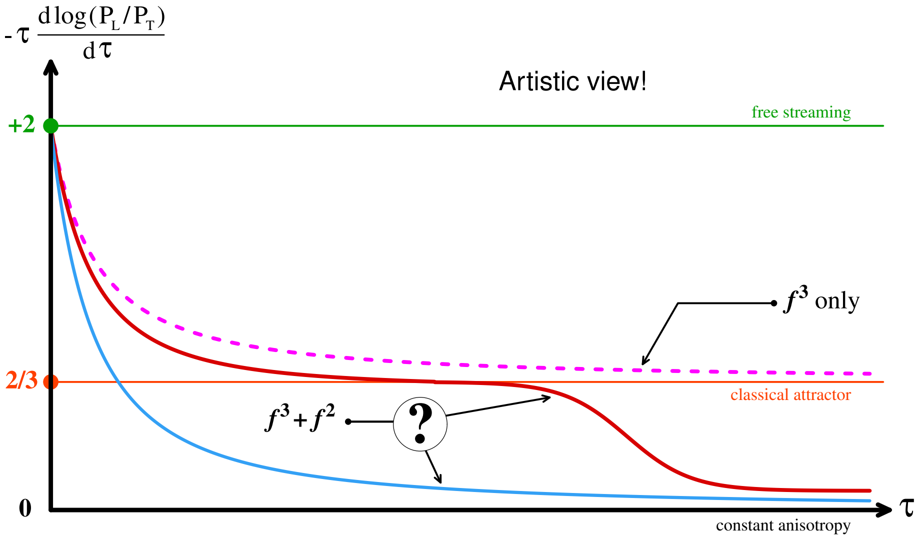


Expanding case, what to expect?



Boltzmann equation, expanding case, presence of a condensate

$$\left[\partial_\tau - \frac{p_{1z}^2}{\tau E_1}\partial_{E_1} - \frac{p_{1z}}{\tau}\partial_{p_{1z}}\right] f_1 = C_{\rm nc}[f_1] + C_{\rm c}^{1c\leftrightarrow 34}[f_1] + C_{\rm c}^{12\leftrightarrow c4}[f_1]$$

$$\frac{1}{\tau}\partial_\tau\left(\tau n_c\right) = C_{\rm c}^{c2\leftrightarrow 34}[f_1]$$

Conservation laws

$$\partial_\tau(\tau N) = 0\,. \qquad \partial_\tau(\tau\epsilon) = -P_L\,. \qquad \tau\partial_\tau\rho_z = -2\rho_z$$

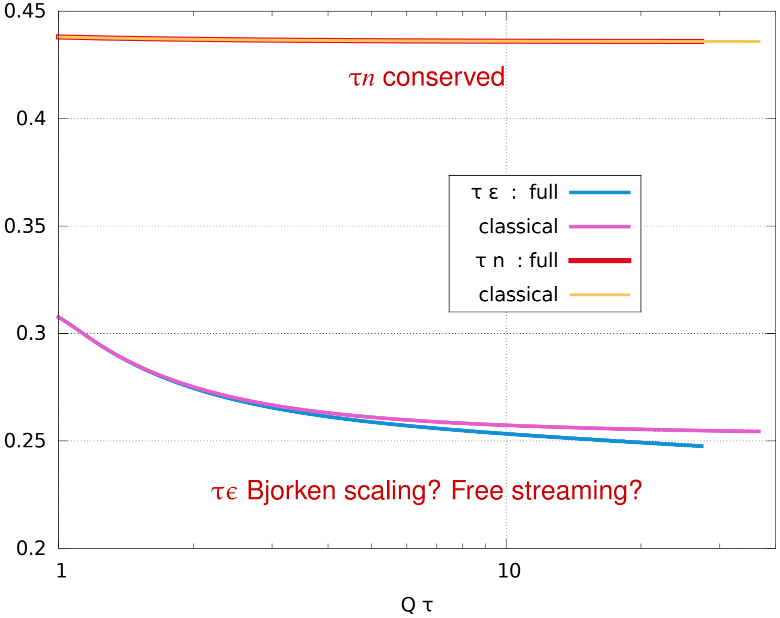
Collision kernel properties

$$\int E_p\,{\rm d}E_p{\rm d}p_z\,C[f_p] = 0\,. \qquad \int E_p^2\,{\rm d}E_p{\rm d}p_z\,C[f_p] = 0\,. \qquad \int E_p\,p_z^2\,{\rm d}E_p{\rm d}p_z\,C[f_p] = 0$$

Initial condition

$$f_p = \underbrace{f_0}_{\gg 1}\exp\left[-\frac{\alpha E_p^2 + \beta p_z^2}{Q^2}\right]\,. \qquad g^4 = 50$$

NUMERICAL RESULTS [TE, GELIS, JEON, MOORE, WU (2015)]



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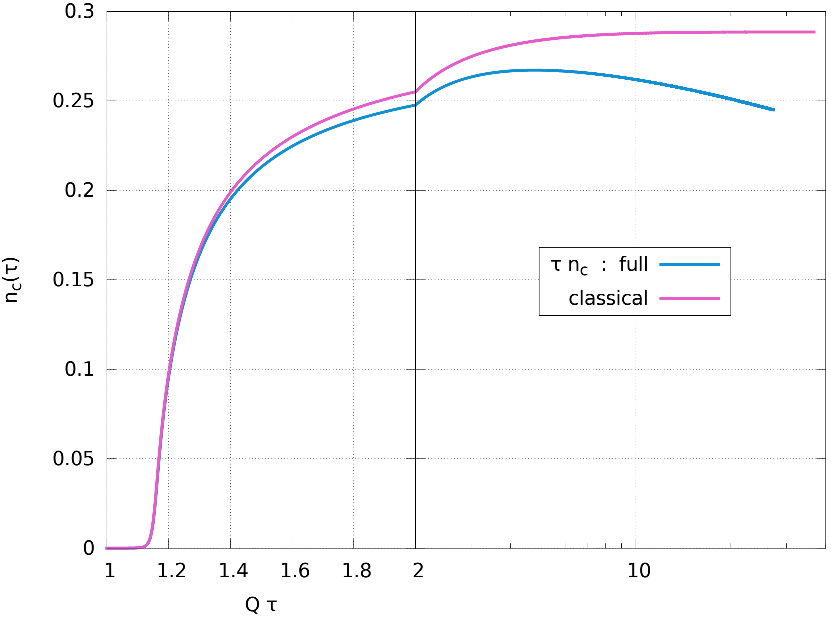
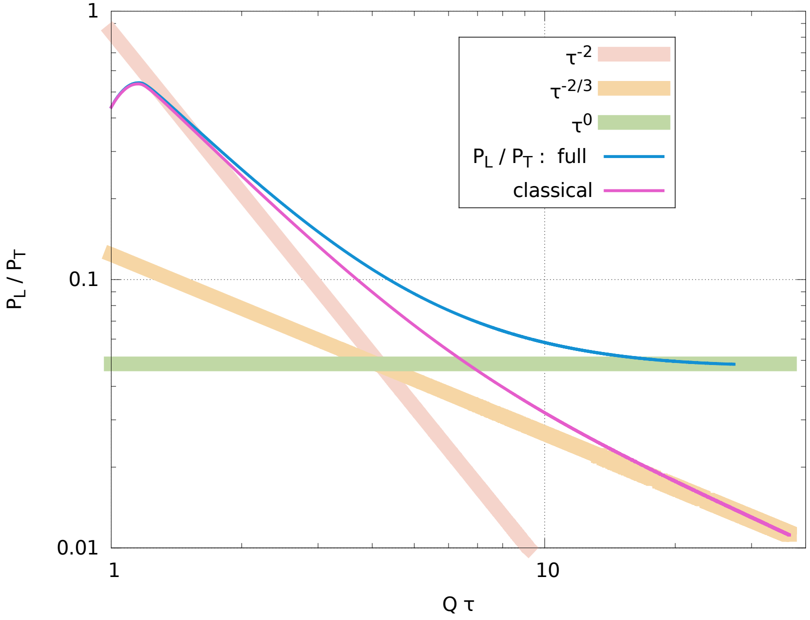


Numerical results [TE, Gelis, Jeon, Moore, Wu (2015)]



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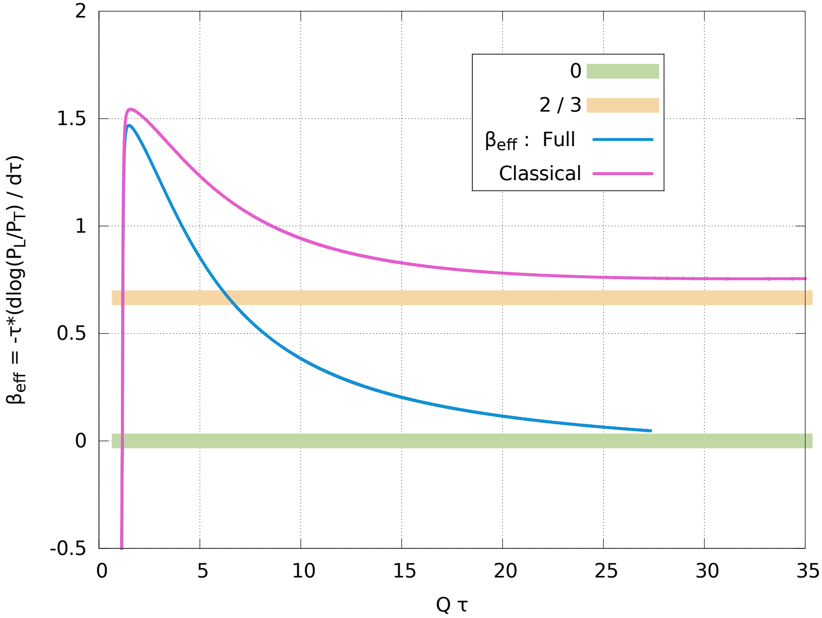

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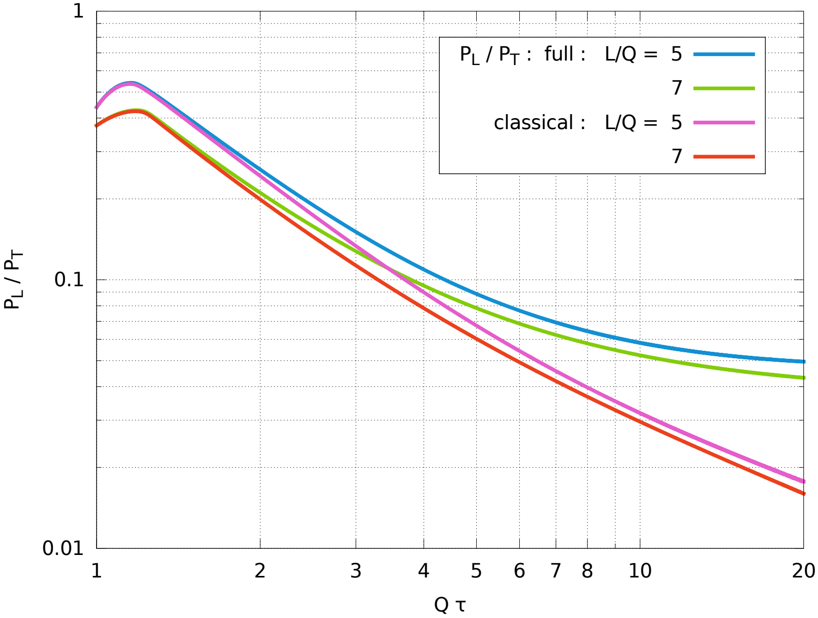

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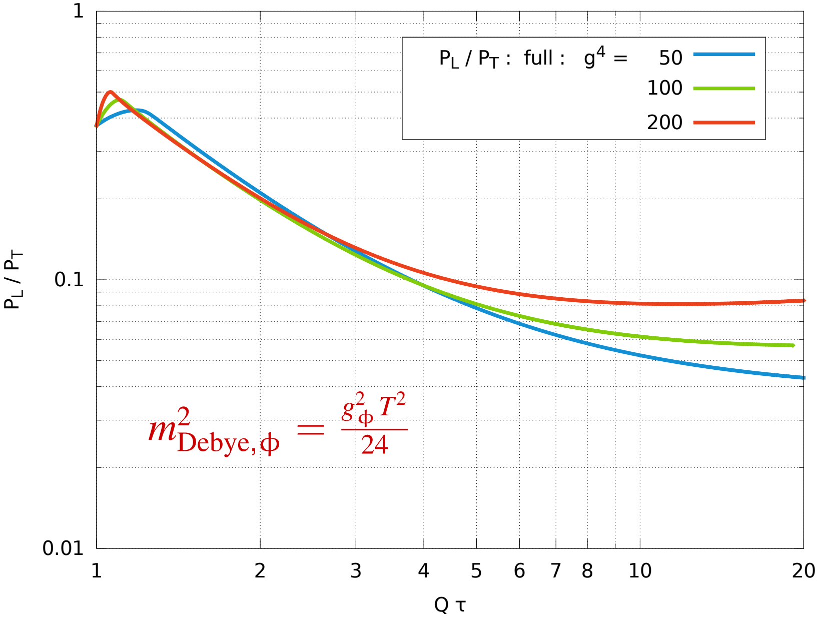
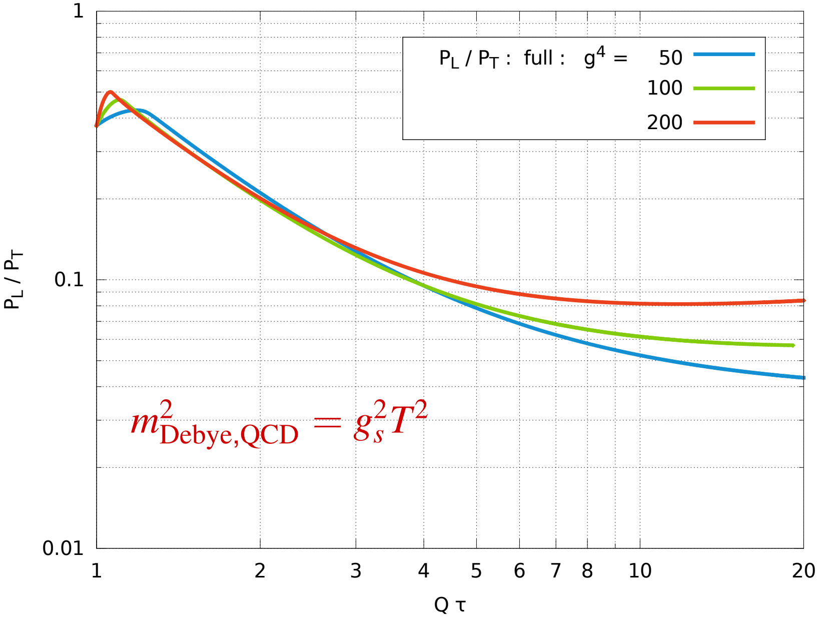
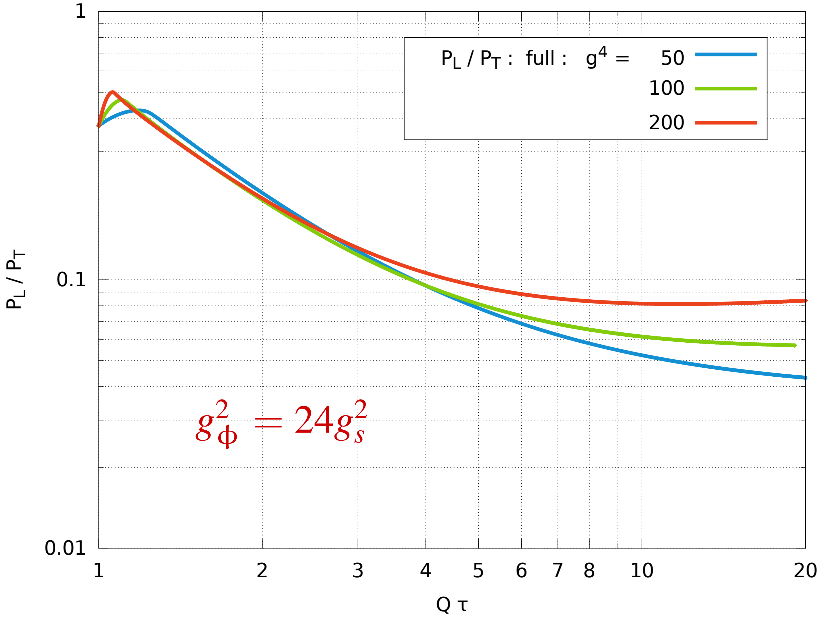




$$m^2_{\mathrm{Debye,QCD}} = g_s^2 T^2$$

NUMERICAL RESULTS [TE, GELIS, JEON, MOORE, WU (2015)]



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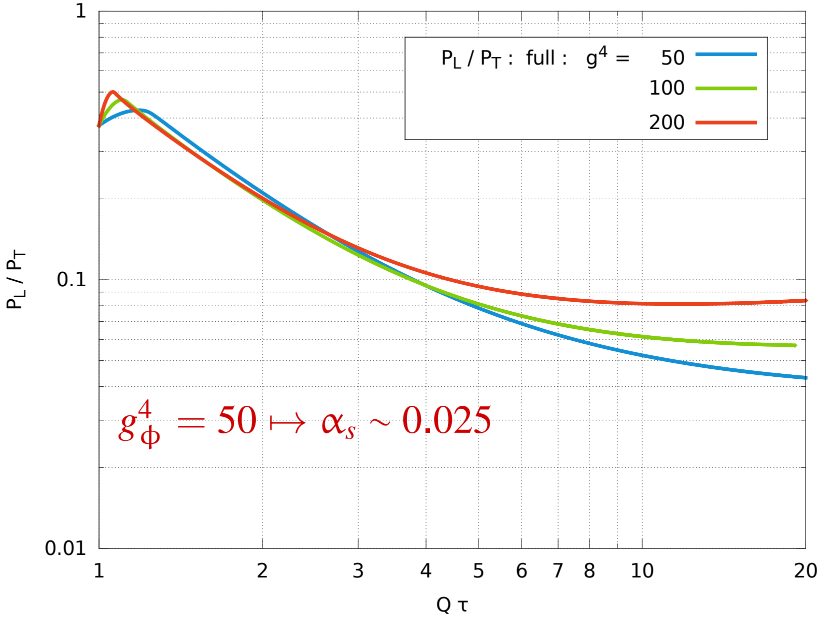

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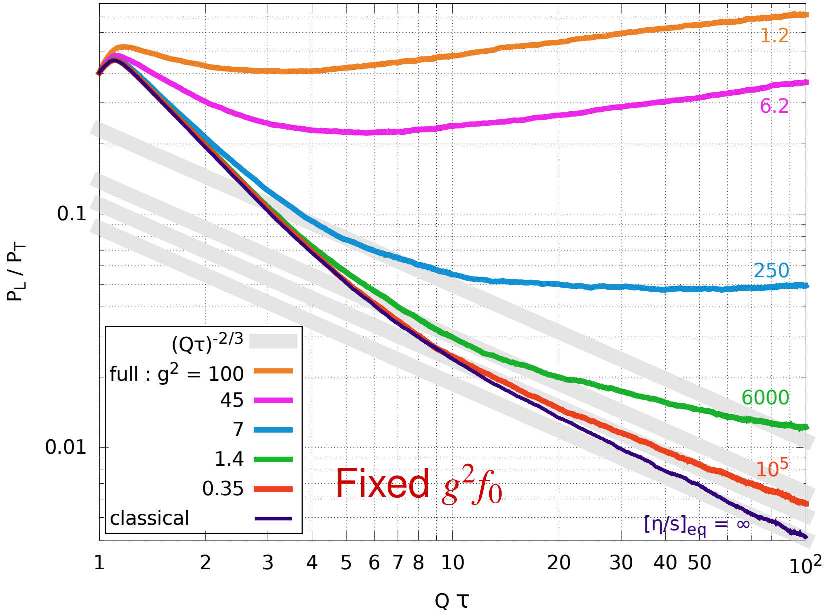

Why no $\mathcal{C}^1$ results?

$$p_z \sim \frac{\nu}{\tau}$$

Constant p_z cutoff means ν increases!

$\mathcal{C}^1$ non-renormalizable $\rightarrow$ no limit when $\nu \mapsto \infty$

Recall isotropic case..

ν-fixed calculations for $\mathcal{C}^1$ would be interesting...

ARE THE POSTULATES OF HYDRODYNAMICS SATISFIED DURING THE EARLY STAGES OF A HEAVY-ION COLLISION?

Conclusion

- Yang-Mills: Evidences for an early hydrodynamical onset
- Hydrodynamization already happens at weak coupling
- CSA non-renormalizable $\Rightarrow$ isotropic Boltzmann OK
- Anisotropic Boltzmann $\Rightarrow$ Classical attractors ruled out?

ARE THE POSTULATES OF HYDRODYNAMICS SATISFIED DURING THE EARLY STAGES OF A HEAVY-ION COLLISION?

Perspectives

- Boltzmann treatment for the $\mathcal{C}^1$ expanding case?
- Boltzmann treatment for the gauge expanding case?
- Renormalization in the YM case?
- Going beyond the CSA $\Rightarrow$ Quantum evolution?

Thank you!

FS$_2$
$\frac{\eta}{s}$ close to $\frac{1}{4\pi}$
quantum
initial condition
classical
time evolution
P_L / P_T
τ^{-2}
$\tau^{-2/3}$
τ^{0}
P_L / P_T : full
classical
Q τ
Full
Classical
Q t
τ [fm/c]
P_T / ϵ
P_L / ϵ
LO
$Q_s \tau$