Equilibration rates in a strongly coupled nonconformal quark-gluon plasma

Alex Buchel

(Perimeter Institute & University of Western Ontario)

Based on arXiv:

1503.07114 with M.Heller and R.Myers; 1505.05012 with A.Day; also on earlier work on quantum quenches with L.Lehner and R.Myers

$\Longrightarrow$ A lot of research in gauge/string correspondence with applications to QGP centers on answering the question:

Why QGP thermalization is fast?

$\Longrightarrow$ A lot of research in gauge/string correspondence with applications to QGP centers on answering the question:

Why QGP thermalization is fast?

$\Longrightarrow$ The difficulty in answering this question is that it implies understanding the evolution of the strong coupled gauge theories, where lattice techniques are not very useful

$\Longrightarrow$ A lot of research in gauge/string correspondence with applications to QGP centers on answering the question:

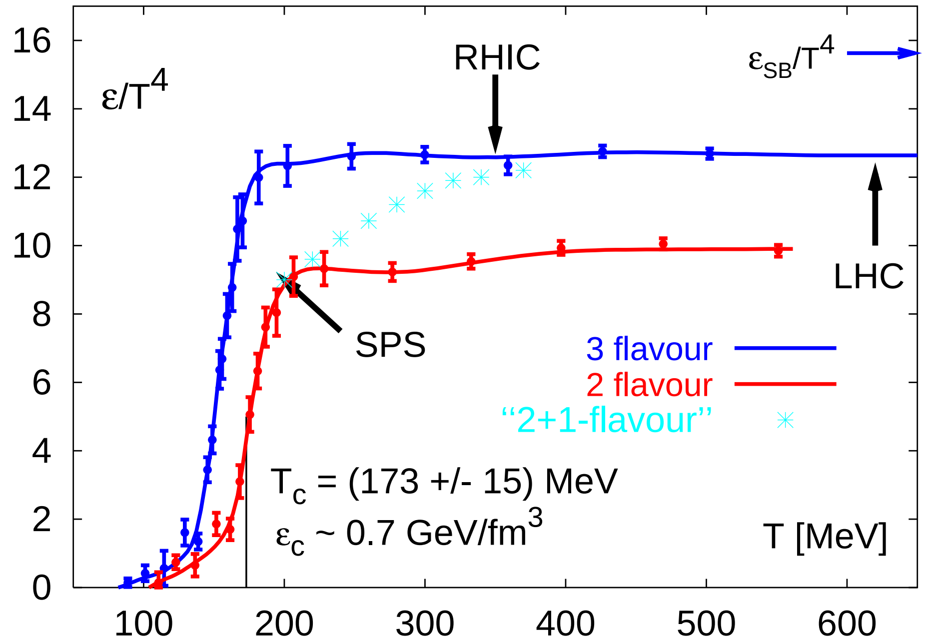
Why QGP thermalization is fast?

$\Longrightarrow$ The difficulty in answering this question is that it implies understanding the evolution of the strong coupled gauge theories, where lattice techniques are not very useful

$\Longrightarrow$ So, we like to refer to AdS/CFT correspondence, and use large-N SYM as a proxy for a real QGP

The motivations:

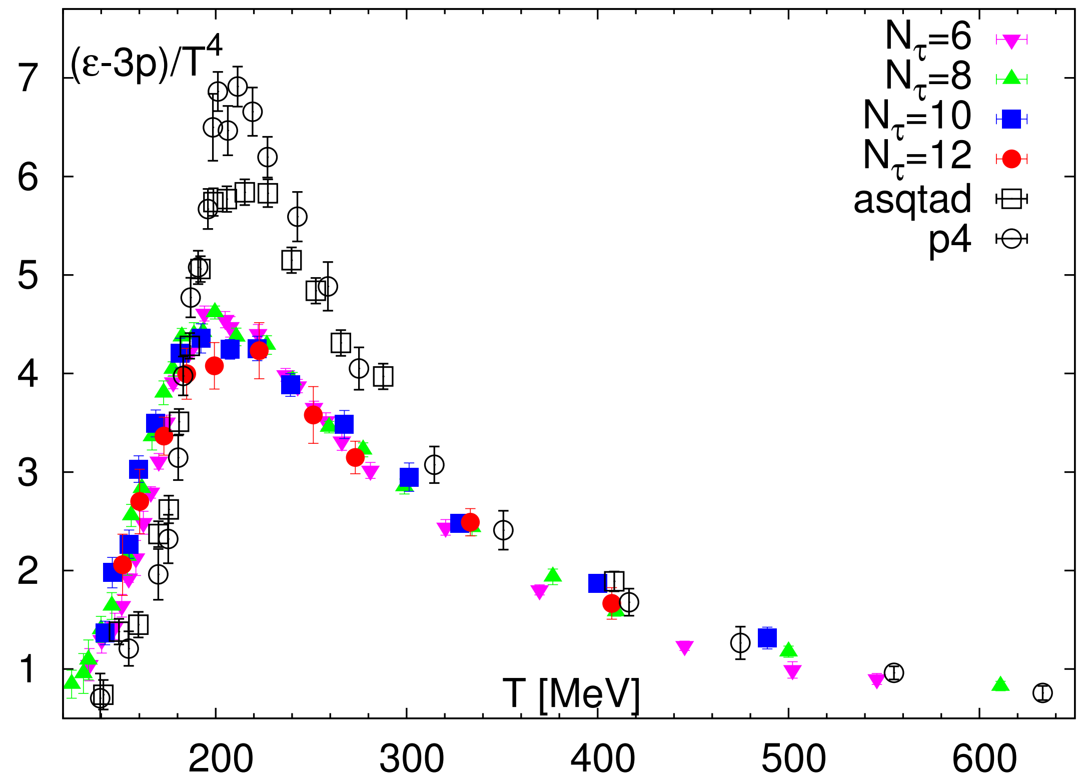
- QCD thermodynamics from lattice; (Karsch, Laermann, hep-lat/0305025). The plateau is $\sim 80\%$ of the SB result — close to 3/4 in SYM thermodynamics



- The small shear viscosity ratio (Policastro,Son,Starinets, hep-th/0104066)

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

⟹ From A.Bazarov et.al (HotQCD Collaboration), arXiv:1407.6387:



⟹ The violation of the conformality,

$$\frac{\epsilon - 3p}{\epsilon} \sim 50\%$$

at the maximum

So, should we use nonconformal models of gauge/gravity correspondence to model equilibration of QGP?

So, should we use nonconformal models of gauge/gravity correspondence to model equilibration of QGP?

$\Longrightarrow$ I am going to use top-down holographic model to address this question

Outline of the talk:

- A toy model for holographic equilibration
  - where the relaxation time(s) is(are) encoded?
- $\mathcal{N} = 2^*$ gauge theory/supergravity holography
  - Gauge theory perspective
  - Holographic Pilch-Warner RG flow
  - Matrix model and localization results
- Spectra of quasinormal modes
  - Relaxation rates for homogeneous/isotropic perturbations: $\{\mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4, T_{\mu\nu}, \cdots\}$
  - Relaxation rates for generic transverse/traceless perturbations of $T_{\mu\nu}$
- Conclusion — beyond $\mathcal{N} = 2^*$ holography
  - relaxation in other top-down models
  - relaxation with chemical potential in $\mathcal{N} = 4$ SYM
  - relaxation in bottom-up holographic models

- Consider $\mathcal{N}=4$ large-N $SU(N)$ SYM theory at strong coupling in thermal state:

$$\epsilon = \frac{3}{8}\pi^2 N^2 T^4 \,, \qquad p = \frac{1}{8}\pi^2 N^2 T^4 \,, \qquad s = \frac{1}{2}\pi^2 N^2 T^3$$

- The holographic dual to this state is a Schwarzschild black hole in Poincare-slice AdS_5. It has the thermodynamic properties (Hawking temperature, Bekenstein-Hawking entropy,...) as above

- SYM has gauge invariant fermion bi-linear operators of dimension $\Delta = 3$: $\mathcal{O}_3$, and gauge invariant scalar bi-linears of dimension $\Delta = 2$: $\mathcal{O}_2$. In thermal equilibrium,

$$\langle \mathcal{O}_3 \rangle_{T\neq 0} = 0 \,, \qquad \langle \mathcal{O}_2 \rangle_{T\neq 0} = 0$$

- We can 'prepare' a non-equilibrium states of the $\mathcal{N}=4$ plasma (thus inducing a non-trivial time-dependence of $\mathcal{O}_\Delta$) by quenching the coupling constants of these relevant operators:

$$H_{SYM} \ \to H_{SYM} + \lambda_\Delta \mathcal{O}_\Delta$$

$$\lambda_\Delta = \lambda_\Delta(t) \,, \qquad \lambda_\Delta(-\infty) = 0$$

- Specifically, we assume

$$\lambda_\Delta(t) = \lambda_\Delta^0 \left(\frac{1}{2} + \frac{1}{2}\tanh\frac{t}{\mathcal{T}}\right), \qquad \mathcal{T} = \frac{\alpha}{T_i},$$

where:

- ▪ T_i is the temperature of the thermal state at $t \to -\infty$,
- ▪ λ_Δ^0 is the amplitude of the quench, taken to be small compare to the initial temperature,

$$\frac{|\lambda_\Delta|}{T_i^{4-\Delta}} \ll 1$$

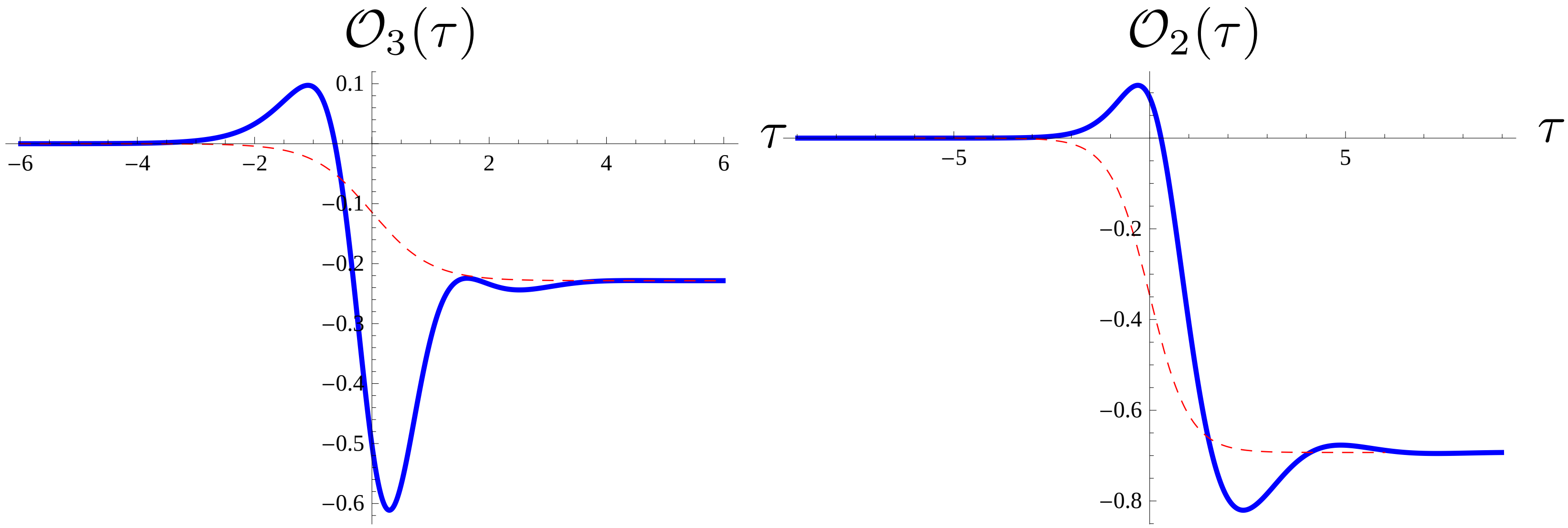
- ▪ α is the rate of quench, measure in units of inverse temperature.
- ▪ Note that α can be arbitrarily small/large, corresponding to abrupt/adiabatic quenches

- $\alpha \to 0$ limit (infinitely sharp – step-function — quench) can be thought as preparing a system in an excited state at $t = 0$ and allowing it to relax

$\Longrightarrow$ I am not going to explain how to set up about quench holographically, and rather move to to discuss the results

$\Longrightarrow$ Our primary observable is the expectation value of the quenching operator:

$$\mathcal{O}_\Delta = \mathcal{O}_\Delta(t)$$

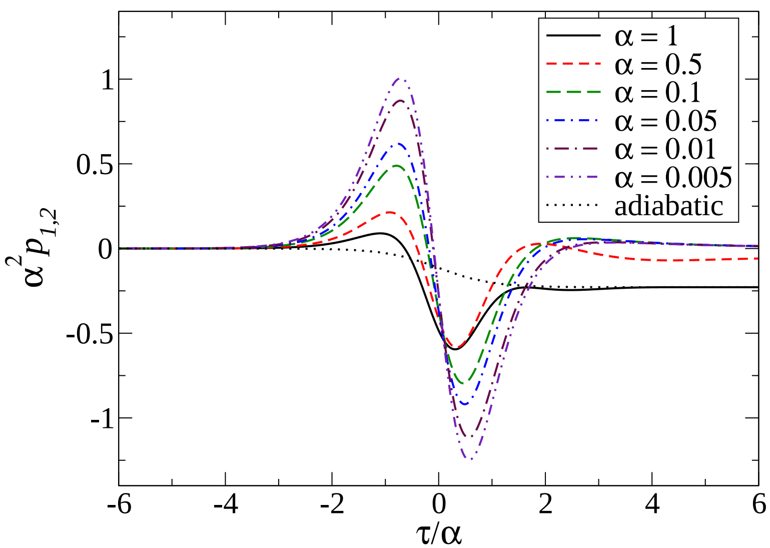


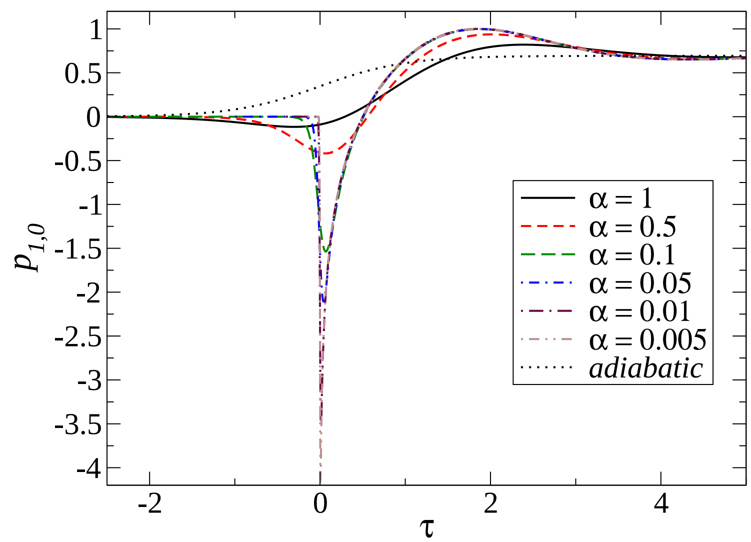
- Evolution of the normalizable component $\mathcal{O}_3$ (left panel) and $\mathcal{O}_2$ (right panel) during the quenches with $\alpha = 1$. The dashed red lines represent the adiabatic response.

- As $\tau \to +\infty$ the expectation values approach their equilibrium values in a damped-oscillatory manner (More on this later).

The response of $\mathcal{O}_\Delta$ depends on Δ:

- for fast quenches, α is small,





$\Uparrow$ $\Uparrow$

$\alpha^2\mathcal{O}_3(t)$ $\mathcal{O}_2(t)$

$\Longrightarrow$ The response is quite different!

$\Longrightarrow$ How do we characterize equilibration time?

- Introduce

$$\delta_{neq}(\tau) \equiv \left| \frac{\mathcal{O}_\Delta(\tau) - [\mathcal{O}_\Delta(\tau)]_{adiabatic}}{[\mathcal{O}_\Delta(\tau)]_{adiabatic}} \right| ,$$

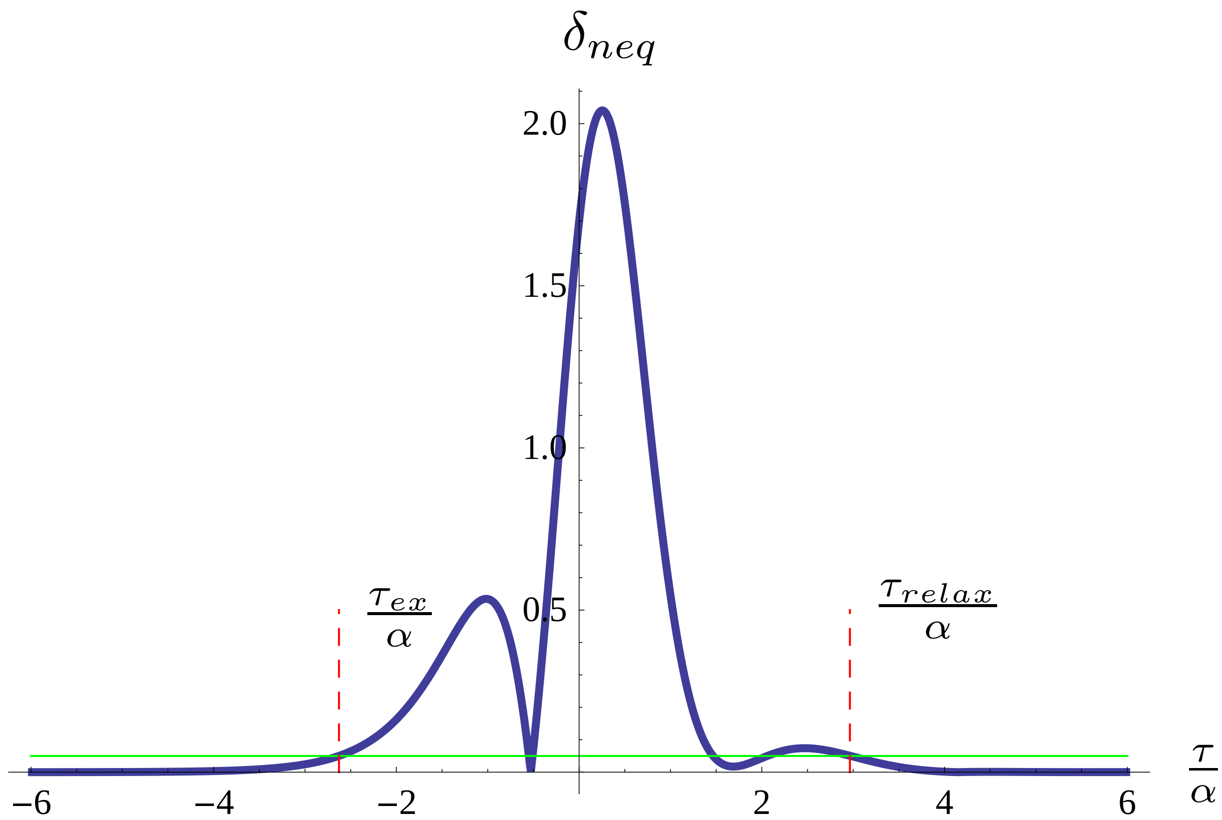
where $[\mathcal{O}_\Delta(\tau)]_{adiabatic}$ is the adiabatic response that can be computed analytically.

- Note,

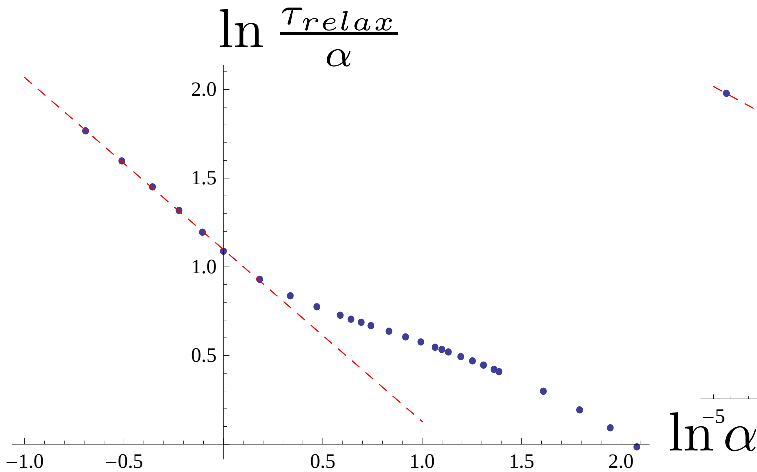
$$\lim_{\tau\to\pm\infty} \delta_{neq}(\tau) \to 0$$

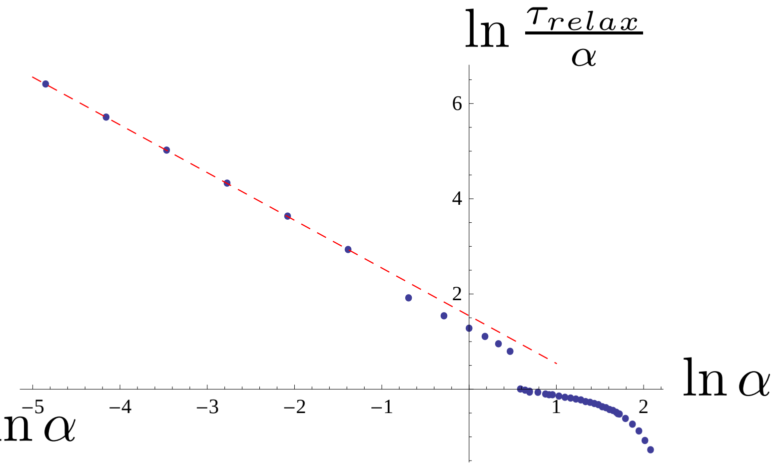
as at early/late times the system is in equilibrium.

$\Longrightarrow$ In practice,



Extraction of the excitation/equilibration rates for $\alpha = 1$ quench. The horizontal green line is the threshold for excitation/equilibration which we define to be 5% away from local equilibrium as determined by δ_{neq}. The dashed red lines indicate the earliest and latest times of crossing this threshold, which we denote as τ_{ex} (for excitation time) and τ_{relax} (for equilibration time), respectively.





$\Uparrow$ $\Uparrow$

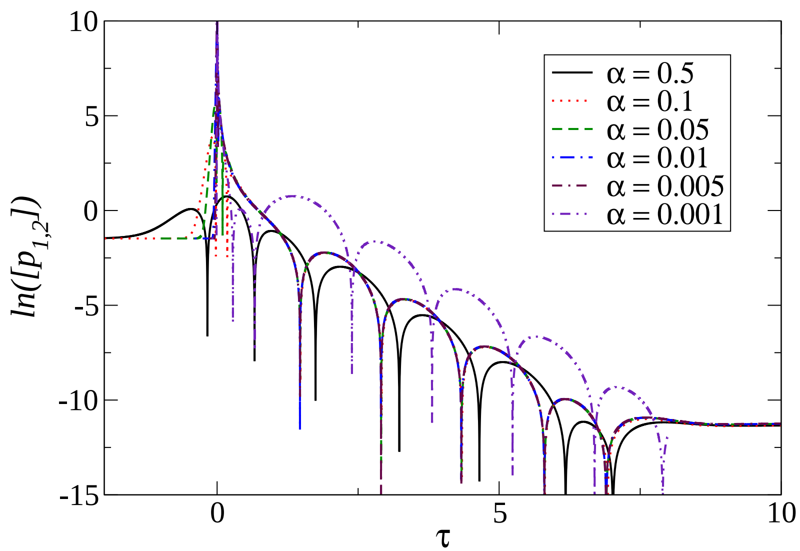
$\mathcal{O}_3(t)$ $\mathcal{O}_2(t)$

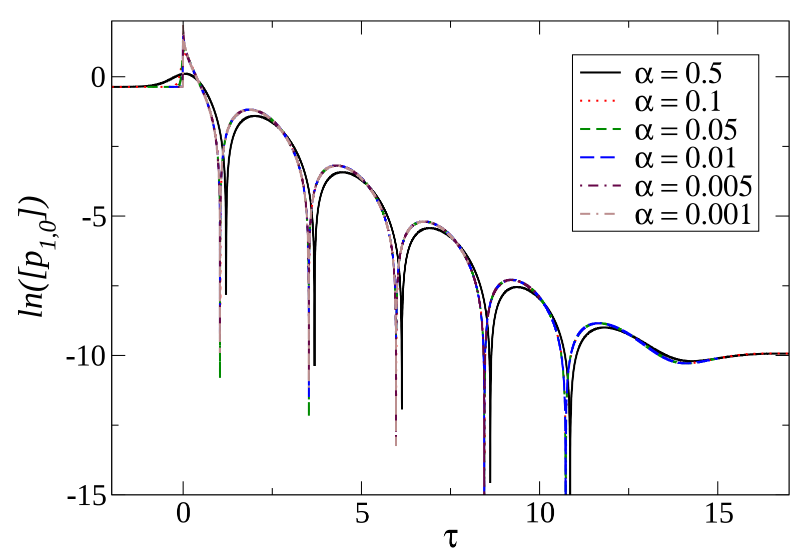
$\Longrightarrow$ Going to small α ($\ln \alpha \to -\infty$) corresponds to preparing the state with an abrupt quench of a dim-Δ operator. The dashed scaling line translates into a **universal relaxation time**:

$$t_{relax} \sim \frac{1}{T}$$

independent of α!

$\Longrightarrow$ We can do more:





Behavior of the response coefficients versus time for representative fast quenches. As is evident in the picture, the same quasinormal mode governs the dynamics very quickly after the quench:

$$\Delta = 3: \qquad \frac{\omega}{2\pi T}\bigg|_{fit} \simeq (1.095 - i\,0.87)\,, \qquad \frac{\omega}{2\pi T}\bigg|_{BH} \simeq (1.099 - i\,0.879)$$

$$\Delta = 2: \qquad \frac{\omega}{2\pi T}\bigg|_{fit} \simeq (0.64 - i\,0.4)\,, \qquad \frac{\omega}{2\pi T}\bigg|_{BH} \simeq (0.644 - i\,0.411)$$

$\Longrightarrow$ Moral of the story:

Lowest quasinormal modes of the black hole in the gravitational dual control the relaxation in strongly coupled gauge theory plasma

- Such feature was also observed in various other holographic examples
- It probably should not be a surprise that the relaxation rate is $\frac{1}{T}$, as after preparing the state in a high-temperature plasma after an abrupt quench temperature is **the only scale**

$\Longrightarrow$ This also motivates to look at non-conformal examples of gauge/gravity correspondence.

$\mathcal{N} = 2^*$ gauge theory (a QFT story)

$\Longrightarrow$ Start with $\mathcal{N} = 4$ $SU(N)$ SYM. In $\mathcal{N} = 1$ 4d susy language, it is a gauge theory of a vector multiplet V, an adjoint chiral superfield Φ (related by $\mathcal{N} = 2$ susy to V) and an adjoint pair $\{Q, \tilde{Q}\}$ of chiral multiplets, forming an $\mathcal{N} = 2$ hypermultiplet. The theory has a superpotential:

$$W = \frac{2\sqrt{2}}{g_{YM}^2} \mathrm{Tr} \left(\left[Q, \tilde{Q} \right] \Phi \right)$$

We can break susy down to $\mathcal{N} = 2$, by giving a mass for $\mathcal{N} = 2$ hypermultiplet:

$$W = \frac{2\sqrt{2}}{g_{YM}^2} \mathrm{Tr} \left(\left[Q, \tilde{Q} \right] \Phi \right) + \frac{m}{g_{YM}^2} \left(\mathrm{Tr} Q^2 + \mathrm{Tr} \tilde{Q}^2 \right)$$

This theory is known as $\mathcal{N} = 2^*$ gauge theory

When $m \neq 0$, the mass deformation lifts the $\{Q, \tilde{Q}\}$ hypermultiplet moduli directions; we are left with the $(N-1)$ complex dimensional Coulomb branch, parametrized by

$$\Phi = \mathrm{diag}\left(a_1, a_2, \cdots, a_N\right) \,, \qquad \sum_i a_i = 0$$

We will study $\mathcal{N} = 2^*$ gauge theory at a particular point on the Coulomb branch moduli space:

$$a_i \in [-a_0, a_0] \,, \qquad a_0^2 = \frac{m^2 g_{YM}^2 N}{\pi}$$

with the (continuous in the large N-limit) linear number density

$$\rho(a) = \frac{2}{m^2 g_{YM}^2}\sqrt{a_0^2 - a^2} \,, \qquad \int_{-a_0}^{a_0} da \; \rho(a) = N$$

Reason: we understand the dual supergravity solution only at this point on the moduli space.

When $m \neq 0$, the mass deformation lifts the $\{Q, \tilde{Q}\}$ hypermultiplet moduli directions; we are left with the $(N-1)$ complex dimensional Coulomb branch, parametrized by

$$\Phi = \text{diag}\,(a_1, a_2, \cdots, a_N)\ , \qquad \sum_i a_i = 0$$

We will study $\mathcal{N} = 2^*$ gauge theory at a particular point on the Coulomb branch moduli space:

$$a_i \in [-a_0, a_0]\,, \qquad a_0^2 = \frac{m^2 g_{YM}^2 N}{\pi}$$

with the (continuous in the large N-limit) linear number density

$$\rho(a) = \frac{2}{m^2 g_{YM}^2}\sqrt{a_0^2 - a^2}\,, \qquad \int_{-a_0}^{a_0} da\ \rho(a) = N$$

~~**Reason:** we understand the dual supergravity solution only at this point on the moduli space.~~

Reason: This moduli space point is a large-N saddle point obtained from localization (in 2 transparencies)

$\mathcal{N} = 2^*$ gauge theory (a supergravity story — a.k.a Pilch-Warner flow)

Consider 5d gauged supergravity, dual to $\mathcal{N} = 2^*$ gauge theory. The effective five-dimensional action is

$$S = \frac{1}{4\pi G_5} \int_{\mathcal{M}_5} d\xi^5 \sqrt{-g} \left(\tfrac{1}{4} R - (\partial\alpha)^2 - (\partial\chi)^2 - \mathcal{P} \right) ,$$

where the potential $\mathcal{P}$ is

$$\mathcal{P} = \frac{1}{16} \left[\left(\frac{\partial W}{\partial \alpha} \right)^2 + \left(\frac{\partial W}{\partial \chi} \right)^2 \right] - \frac{1}{3} W^2 ,$$

with the superpotential

$$W = -\frac{1}{\rho^2} - \frac{1}{2} \rho^4 \cosh(2\chi) , \qquad \alpha \equiv \sqrt{3} \ln \rho$$

$\Longrightarrow$ The 2 supergravity scalars $\{\alpha, \chi\}$ are holographic dual to dim-2 and dim-3 operators which are nothing but (correspondingly) the bosonic and the fermionic mass terms of the $\mathcal{N} = 4 \to \mathcal{N} = 2$ SYM mass deformation.

PW geometry ansatz:

$$ds_5^2 = e^{2A}\left(-dt^2 + d\vec{x}^{\,2}\right) + dr^2$$

solving the Killing spinor equations, we find a susy flow:

$$\frac{dA}{dr} = -\frac{1}{3}W\,, \qquad \frac{d\alpha}{dr} = \frac{1}{4}\frac{\partial W}{\partial \alpha}\,, \qquad \frac{d\chi}{dr} = \frac{1}{4}\frac{\partial W}{\partial \chi}$$

Solutions to above are characterized by a single parameter k:

$$e^A = \frac{k\rho^2}{\sinh(2\chi)}\,, \qquad \rho^6 = \cosh(2\chi) + \sinh^2(2\chi)\,\ln\frac{\sinh(\chi)}{\cosh(\chi)}$$

In was found (Polchinski,Peet,AB) that

$$k = 2m$$

$\Longrightarrow$ Precision test on $\mathcal{N}=2^*$ holography from Pestun's localization

- Supersymmetrically compactify $\mathcal{N}=2^*$ gauge theory on S^4
- Moduli of the theory are conformally coupled scalars, so they will all be lifted via coupling to S^4 curvature
- The exact partition function of the compactified theory is known due to Pestun's localization (reduces to a matrix model):

$$Z_{\mathcal{N}=2^*} = \int d^{N-1}\hat{a} \prod_{i<j} \frac{(\hat{a}_i-\hat{a}_j)^2 H^2(\hat{a}_i-\hat{a}_j)}{H(\hat{a}_i-\hat{a}_j-mR)H(\hat{a}_i-\hat{a}_j+mR)}$$

$$\times\ e^{-\frac{8\pi^2 N}{\lambda}\sum_j \hat{a}_j^2}\ \left|\mathcal{Z}_{\rm inst}\right|^2$$

where

$$H(x) \equiv \prod_{n=1}^{\infty}\left(1+\frac{x^2}{n^2}\right)^n\ e^{-\frac{x^2}{n}}$$

- In the large-N limit the partition function is dominated by the saddle point, that can be computed analytically (AB, J.G.Russo and K.Zarembo, 1301.1597)

$\Longrightarrow$ One recovers:

- the moduli space point picked out by supergravity (as a matrix model saddle point)
- the susy Wilson loops agree both in matrix model and in holographic dual
- the matrix model free energy agree with the holographic free energy (Bobev et.al, arXiv:1311.1508)

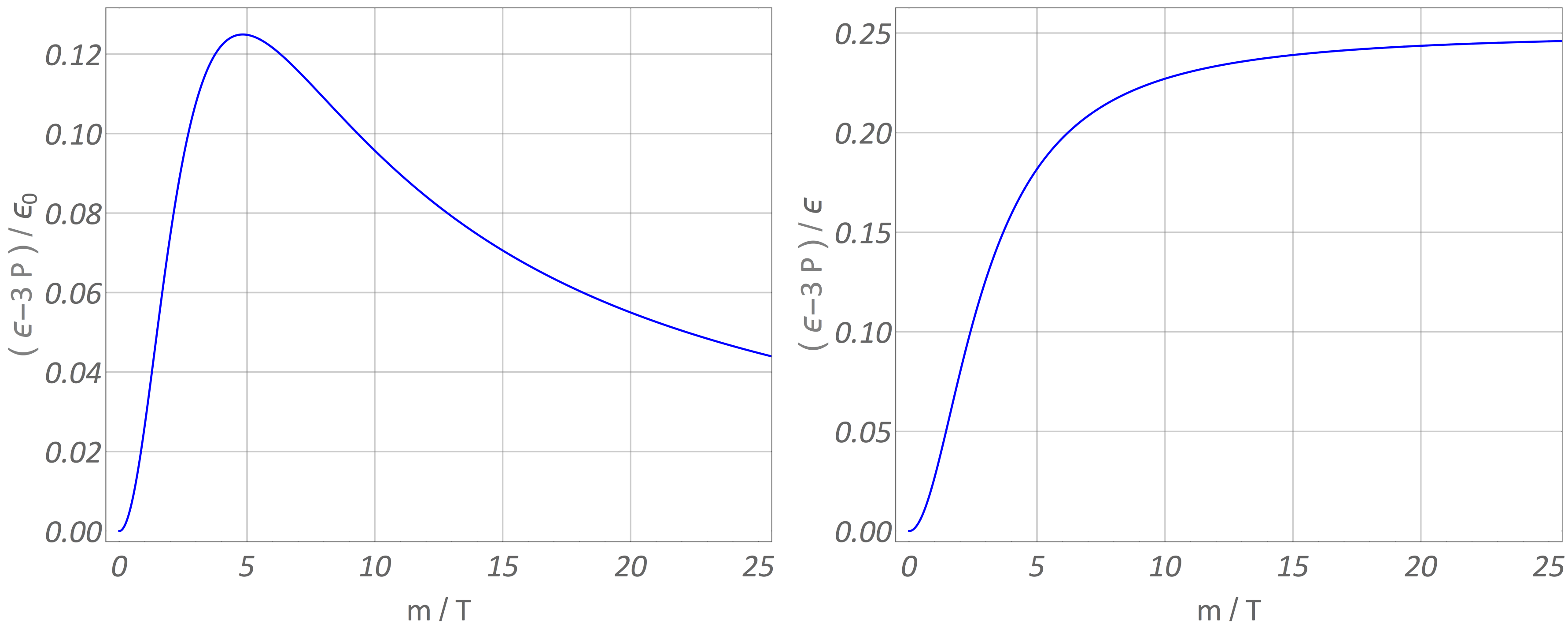
$\Longrightarrow$ All these checked are in addition to earlier agreement with the metric on the moduli space computed either in supergravity or from QFT using Seiberg-Witten techniques

$\Longrightarrow$ What do we do:

- Take PW gravitational dual to $\mathcal{N} = 2^*$ gauge theory and construct black hole solutions. The thermodynamics of the black hole has a nontrivial dependence of 2 scales: T and m. The m dependence is quite profound:

$$s(T) \propto \begin{cases} N^2\ T^3\,, & \frac{m}{T} \ll 1 \\ N^2\ \frac{T^4}{m}\,, & \frac{m}{T} \gg 1 \end{cases}$$

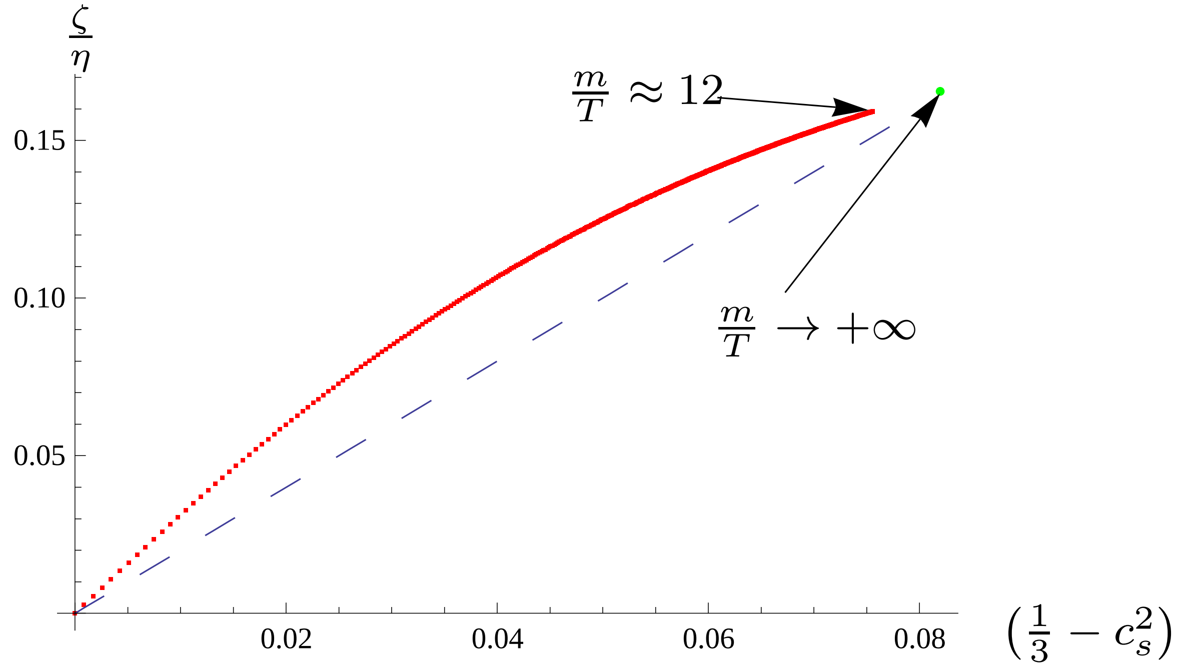
- Using the standard techniques we compute quasinormal modes of the BH corresponding to
  - the stress-energy tensor $T_{\mu\nu}$
  - operators $\{\mathcal{O}_2, \mathcal{O}_3\}$ inducing the RG flow
  - 'passive' operators $\{\mathcal{O}_2, \mathcal{O}_3\}$
  - also study the momentum dependence on the quasinormal frequencies to get an idea of relaxation of spatially inhomogeneous excitations



$\Longrightarrow$ (L) Trace of the energy-momentum tensor normalized to the energy density of $\mathcal{N}=4$ SYM ($\epsilon_0 = \frac{3}{8}\pi^2 N_c^2 T^4$ with N_c denoting the number of colors) as a function of m/T. The results indicate that, thermodynamically, the effects of the conformal symmetry breaking are the strongest at $m/T \approx 4.8$.

$\Longrightarrow$ (R) Trace anomaly in deep IR — approach to a CFT_5

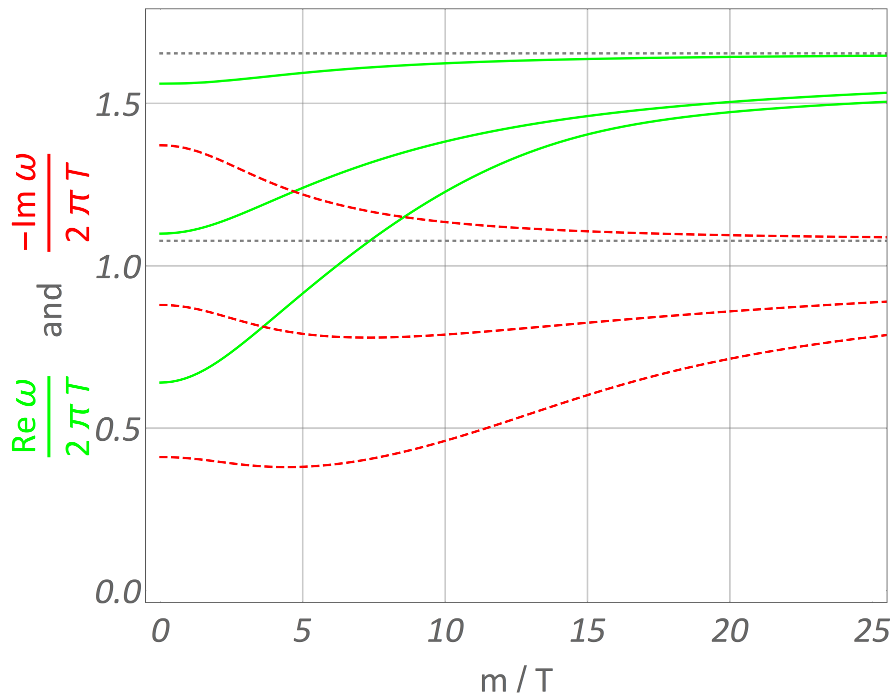
$\Longrightarrow$ There is an interesting story with the IR properties of the flow:



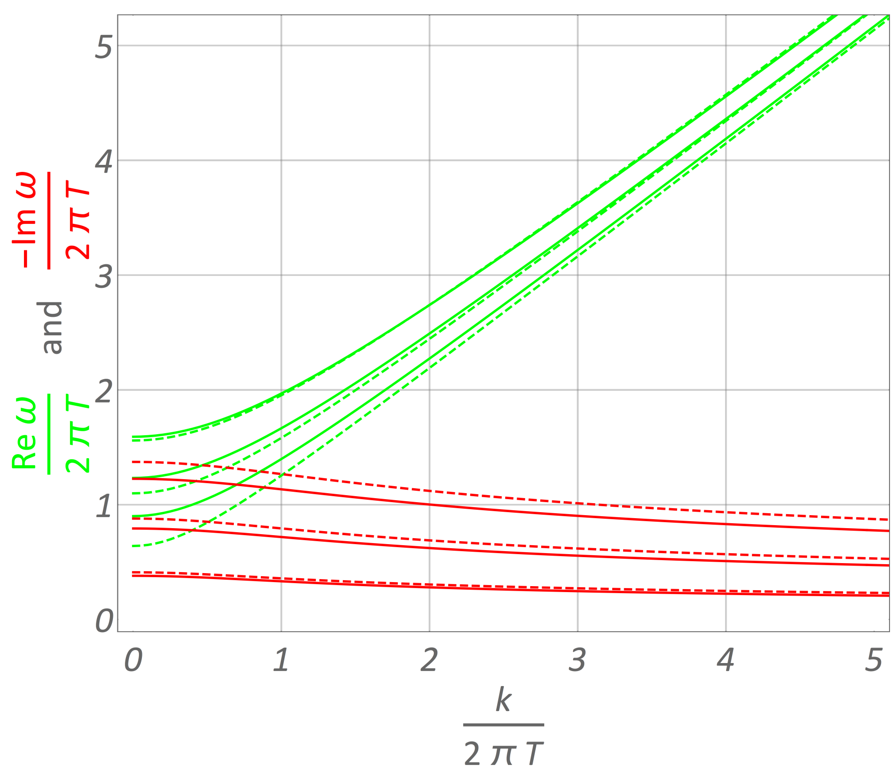
$\Longrightarrow$Ratio of viscosities $\frac{\zeta}{\eta}$ versus the speed of sound in $\mathcal{N} = 2^*$ gauge theory plasma (AB, arXiv:0708.3459). Dashed line is the bulk viscosity bound, $\frac{\zeta}{\eta} \geq 2\left(\frac{1}{3} - c_s^2\right)$. A single point represents extrapolation of the speed of sound and the viscosity ratio to $T \to +0$.

$\Longrightarrow$ Note, for a CFT_5, $\epsilon = 4p$, so $c^2_{s,CFT_5} - \frac{1}{4}$, and

$$\frac{1}{3} - c_s^2 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12} = 0.08333\cdots$$



Real (green continuous) and minus imaginary (red dashed) parts of the lowest quasinormal mode frequencies for operators of dimensions $\Delta = 2, 3$ and 4 (from bottom to top). The frequencies do not change significantly as a function of m/T, which leads to universal equilibration in $1/T$. One can also infer from this plot that all the frequencies asymptote at low temperatures to the quasinormal mode of a massless scalar field living in the (1+5)-dimensional AdS-Schwarzschild geometry (dotted curves).



- Momentum dependence of the real (green) and minus imaginary part (red) of the QNM frequency of operators with $\Delta = 2, 3$ and 4 (from bottom to top) for $m/T = 0$ ($\mathcal{N} = 4$ SYM, dashed) and $m/T = 4.8$ (continuous). Surprisingly, corresponding curves are very close to each other despite of the fact that $m/T = 4.8$ matches the locus of the maximal deviation from conformal invariance in thermodynamics of $\mathcal{N} = 2^*$.
- There is **very weak** dependence on k in $\mathrm{Im}\omega$

- So far I discussed a top-down holographic model with the maximum violation of nonconformality (over the full range of temperatures)

$$\Theta \equiv \frac{\epsilon - 3P}{\epsilon}\bigg|_{max} \sim\ 25\%$$

- One might argue that weak dependence of the relaxation rates on Θ are somehow results of smallness of Θ
- To address this concern we look at another model of non-conformal QGP in holography, namely, the **cascading gauge theory plasma** (aka KS gauge theory plasma)
- Some notable differences between PW and KS models:
  - in PW the scale invariance is broken explicitly (mass terms); in KS the scale invariance is broken by quantum effects (nonzero β-function)
  - unlike PW, KS gauge theory confines in the IR with the spontaneous chiral symmetry breaking
  - in KS,

$$\Theta \gtrsim 1 - 10$$

$\Longrightarrow$ First, we would like a *universal* framework to compare different non-conformal models.

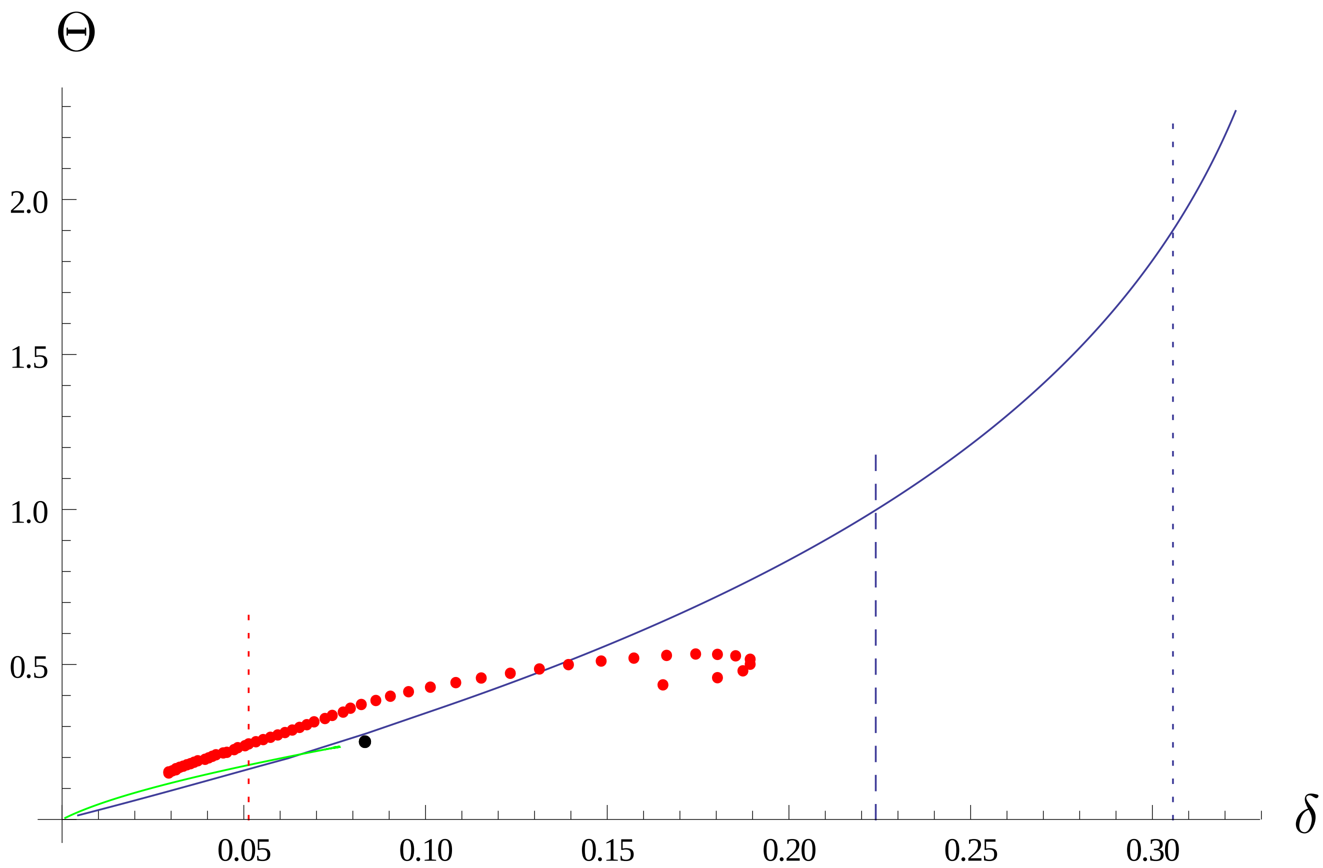
$\Longrightarrow$ We can do that by comparing the thermodynamics of the models in the Θ – vs. – δ plane, where

$$\Theta \equiv \frac{\epsilon - 3P}{\epsilon}\,, \qquad \delta \equiv \frac{1}{3} - c_s^2$$

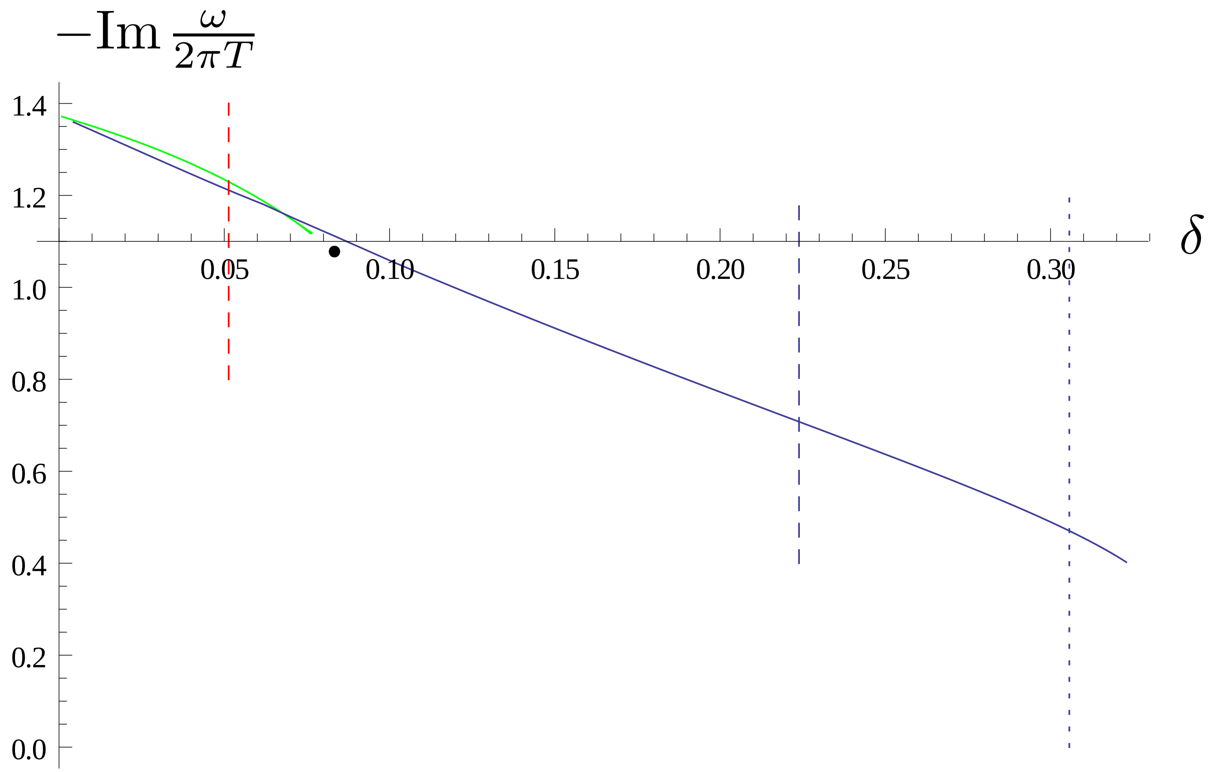
with c_s being the speed of sound waves in plasma,

$$c_s^2 = \frac{\partial P}{\partial \epsilon}$$

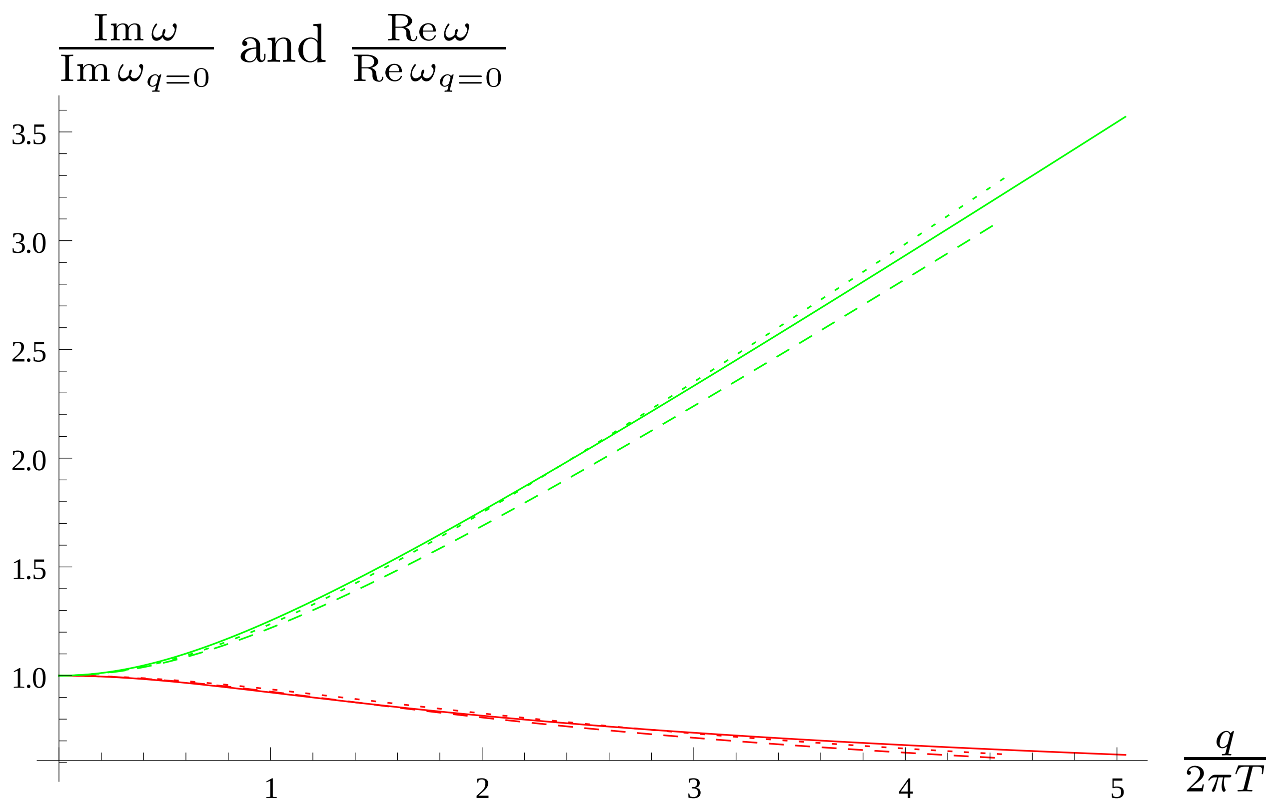
$\Longrightarrow$ The advantage of this framework is that we can also compare with lattice QCD



Parameterization of $\Theta = \frac{\epsilon - 3p}{\epsilon}$ with $\delta = \frac{1}{3} - c_s^2$ in strongly coupled gauge theory plasma for QCD (the red dots), $\mathcal{N} = 2^*$ (the solid green line), and cascading gauge theory (the solid blue line). The dashed red line represents the conformal violation parameter δ in QCD at $T = 0.3$GeV. The black dot is the $\frac{m}{T} \to \infty$ limit of $\mathcal{N} = 2^*$ thermodynamics. Vertical blue lines represent the phase transitions in cascading gauge theory plasma.



Minus imaginary part of the lowest quasinormal mode at zero spatial momentum of the transverse traceless fluctuations of the stress-energy tensor in $\mathcal{N}=2^*$ (the solid green line) and KS (the solid blue line) gauge theory plasma as a function of $\delta=\frac{1}{3}-c_s^2$. The black dot denotes the lowest quasinormal mode of dimension $\Delta=5$ operator of the effective five-dimensional CFT in the deep IR of $\mathcal{N}=2^*$ plasma. The dashed red line represents the conformal violation parameter δ in QCD at $T=0.3$GeV. Vertical blue lines represent the phase transitions in KS plasma.



Momentum dependence of the lowest quasinormal mode of the transverse traceless fluctuations of the stress-energy tensor in cascading gauge theory plasma at the ultraviolet fixed point (solid lines), the deconfinement phase transition (dashed lines), and the chiral symmetry breaking phase transition (dotted lines). The green/red lines represent the real/minus imaginary parts of the frequencies. The data is normalized to zero momentum values of the frequencies.

Conclusion:

- I argued that relaxation time in strongly coupled plasma is encoded in the spectrum of quasinormal modes of BH in the holographic dual
- One can have controlled examples of the top-down holography where it is possible to systematically study effects of non-conformality on the relaxation time
- We found that

$$\tau_{relax} \propto \frac{1}{T}$$

universally, even though are other microscopic scales in the plasma (masses, etc)

- The spatial relaxation is **ultralocal** — imaginary parts of the quasinormal modes are almost flat in momentum k
- Similar conclusions are reached in other models:
  - thermalization in $\mathcal{N} = 4$ plasma in the presence of charge densities/magnetic fields (J.Fuini, L.Yaffe, arXiv:1503.07148)
  - Bottom-up pheno models of holography (R.Janik et.al, arXiv:1503.07148; T.Ishii et.al, arXiv:1503.07766)