Universality classes far from equilibrium of scalar and gauge theories

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Talk based on:

J. Berges, KB, S. Schlichting, and R. Venugopalan,
arXiv: 1508.03073 ;
PRL 114, 061601 (2015)

A. Piñeiro Orioli, KB, and J. Berges,
PRD 92, 025041 (2015)

In collaboration with:

Jürgen Berges
Asier Piñeiro Orioli
Sören Schlichting
Raju Venugopalan

INT thermalization workshop / week 2

Aug 14, 2015

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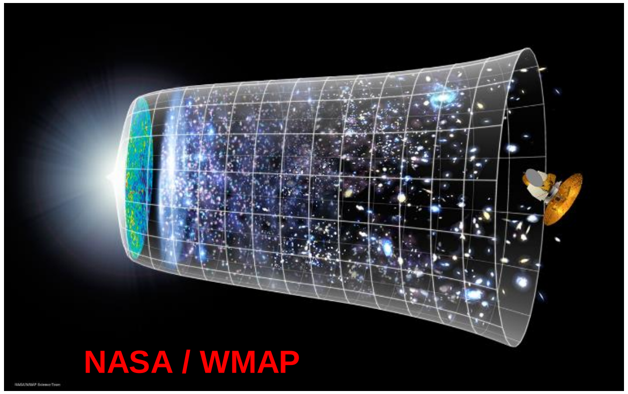
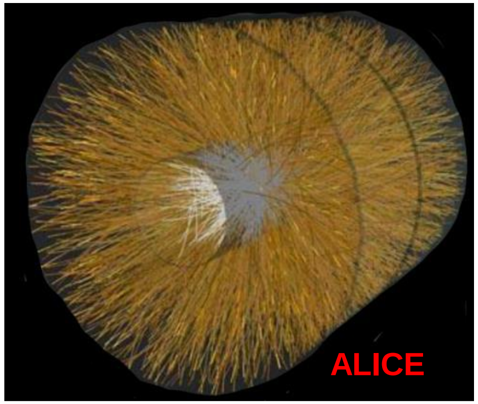
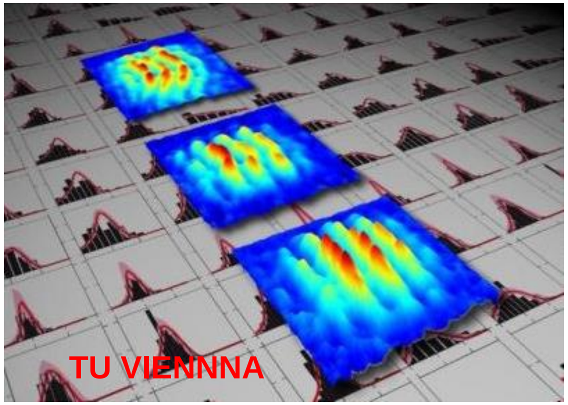
Introduction

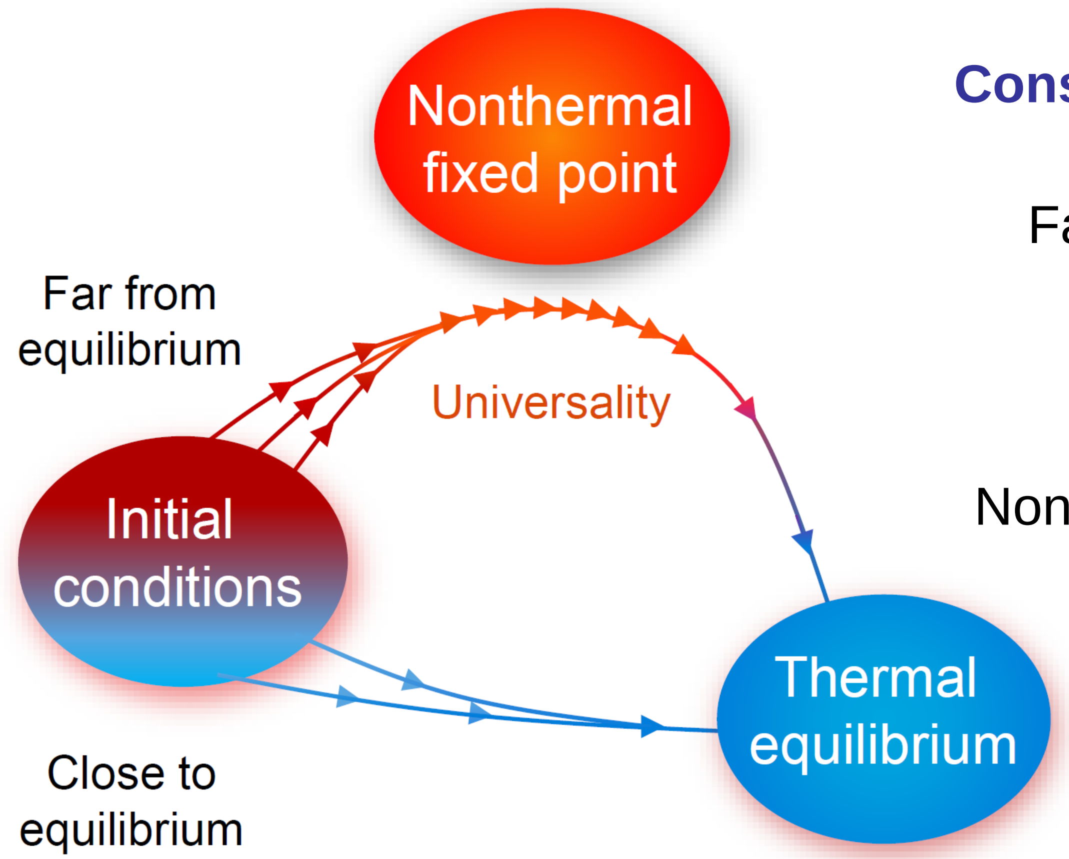
Real-time systems far from equilibrium

Inflationary cosmology	*Heavy-ion collisions at early stages*	*Ultracold atoms*
		
Relativistic O(N) scalars	Longitudinally expanding non-Abelian plasmas	Non-relativistic (Gross-Pitaevski) scalars

Very different field theories and energy scales!

For weak coupling limit: **Common properties?** **Universality classes?**

Introduction



Consider early time evolution:

Far-from-equilibrium initial conditions (IC)

↓

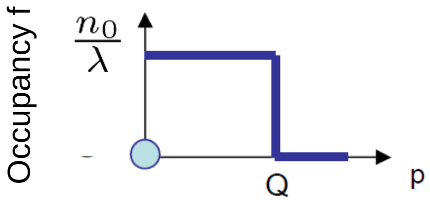
Nonthermal fixed point (NTFP)

- ✓ Memory loss
- ✓ Time scale independence
- ✓ Universal dynamics

Introduction

Typical initial conditions

Over-occupation IC



$$\langle\{\hat{\varphi}\,,\,\hat{\varphi}\}\rangle \sim Q^2 n_0/\lambda$$

or for distribution function f

$$\langle\hat{\varphi}\rangle = \phi = 0\;, \qquad f \sim n_0/\lambda$$

Weak couplings but highly correlated system

Weak coupling limit $\lambda \to 0$ while $\lambda f = const$

Fields follow *classical* evolution!

Observables averaged over (quantum) IC

Many examples:

Micha & Tkachev ; Smit & Tranberg; Nowak, Sexty & Gasenzer ; Berges, KB, Schlichting & Venugopalan; Kurkela & Moore; …

$$\varphi_a(\boldsymbol{x}, t_0) = \phi_a(t_0) + \int_{\boldsymbol{p}} \sqrt{f(\boldsymbol{p}, t_0) + \frac{1}{2}}\,\left(c_{a,\boldsymbol{p}}\;\xi(\boldsymbol{p}, (t_0))\;e^{i\boldsymbol{p}\boldsymbol{x}} + c.c.\right)$$

Gaussian distributed complex random numbers

(initialization example: O(N) symmetric scalar field theory)

Universality class: expanding gauge and scalar theories

Example:

See also:
- Talk by S. Schlichting
- Contribution by A. Kurkela

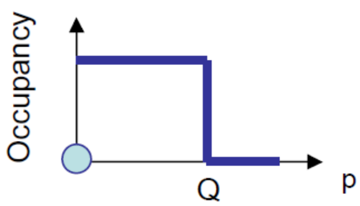
High energy (weak coupling) limit of heavy-ion collisions at early times

$$Q_s(s) \gg \Lambda_{\mathrm{QCD}} \qquad \alpha_s(Q_s) \ll 1$$

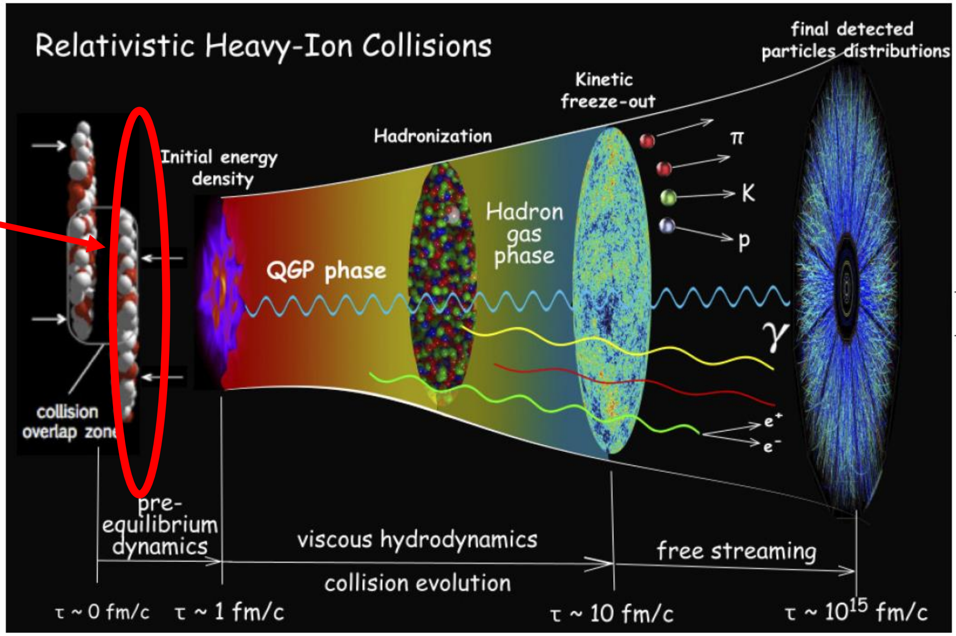
Far from equilibrium IC:

Large fluctuations (inspired by color-glass condensate)

$$\langle \{A\,,\,A\} \rangle \sim Q^2/g^2$$



$$f(p_T, p_z, \tau_0) = \frac{n_0}{\lambda}\Theta\left(Q - \sqrt{p_T^2 + (\xi_0 p_z)^2}\right)$$



Little Bang by P. Sorensen and C. Shen

Universality class: expanding gauge and scalar theories

Massless scalar field theory (O(N))

Non-Abelian gauge theory (SU(2))

In Bjorken coordinates: $\tau = \sqrt{t^2 - (x^3)^2}\,, \qquad \eta = \text{artanh}\left(\frac{x^3}{t}\right)$

Longitudinally expanding metric: $g_{\mu\nu}(\tau) = \text{diag}\,(1, -1, -1, -\tau^2)$

Distribution function: $f(p_T, p_z, \tau) \sim \tau \sqrt{\langle \varphi\varphi \rangle \langle \partial_\tau \varphi\, \partial_\tau \varphi \rangle}$

for gauge theory in Coulomb gauge → talk S. Schlichting

J. Berges, KB, S. Schlichting, and R. Venugopalan:

PRL 114, 061601 (2015) ; arXiv: 1508.03073 ;

PRD 89, 074011 (2014) ; PRD 89, 114007 (2014)

Universality class: expanding gauge and scalar theories

What we are after: Scaling regions and universality classes

Scaling region (close to a nonthermal fixed point)

- *Self-similar evolution* of distribution function f (→ slow dynamics, memory loss)

$$f(p_T, p_z, \tau) = \tau^{\alpha} f_S\left(\tau^{\beta} p_T, \tau^{\gamma} p_z\right)$$

with scaling behavior of typical scales $f \sim \tau^{\alpha}\ ,\ p_T \sim \tau^{-\beta}\ ,\ p_z \sim \tau^{-\gamma}$

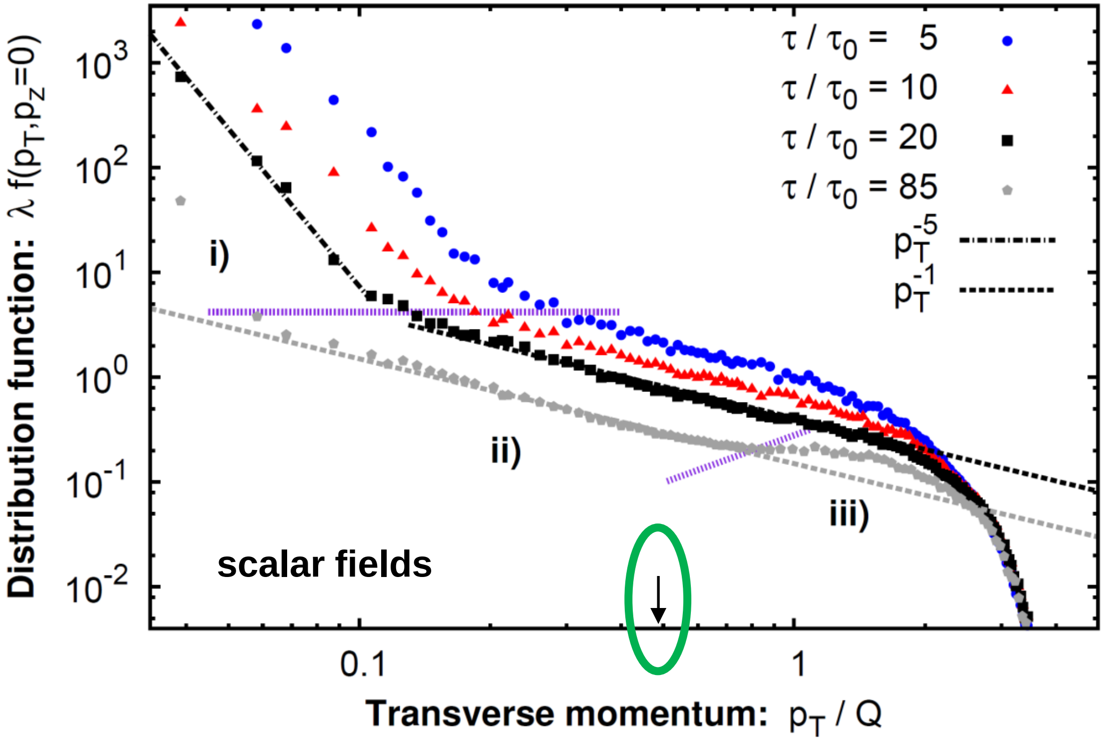
Classification: universality classes far from equilibrium

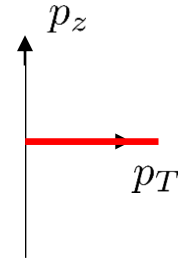
- Scaling regions, described by their exponents α, β, γ and the scaling function $f_S(x)$, may be classified in universality classes

Universality classes: Classification of theories in classes intriguing; comparison of theories within same class gives insight into dynamics; benchmarks for microscopic / kinetic descriptions (especially because of ‚memory loss')

Universality class: expanding gauge and scalar theories

Scalar nonthermal attractor: Different scaling regions i), ii) and iii)





Regions ii) and iii):

Local conservation of particle number and energy density in p_T:

$$\tau\, dn/dp_T\,, \quad \tau\, d\epsilon/dp_T$$

are time-independent

Effectively no flux in p_T!

Reminder: *Self-similar evolution*

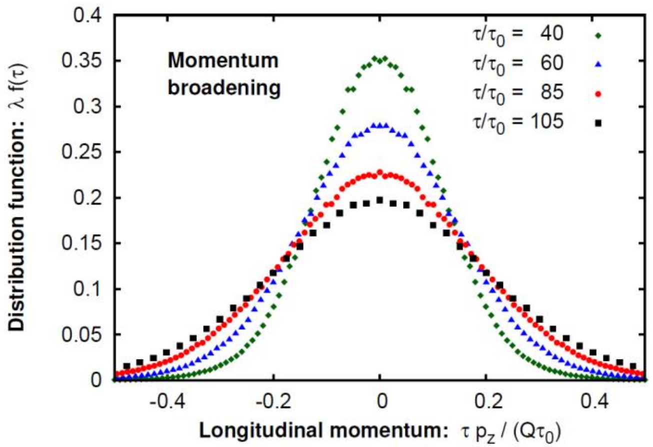
$$f(p_T, p_z, \tau) = \tau^{\alpha} f_S(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$

implies

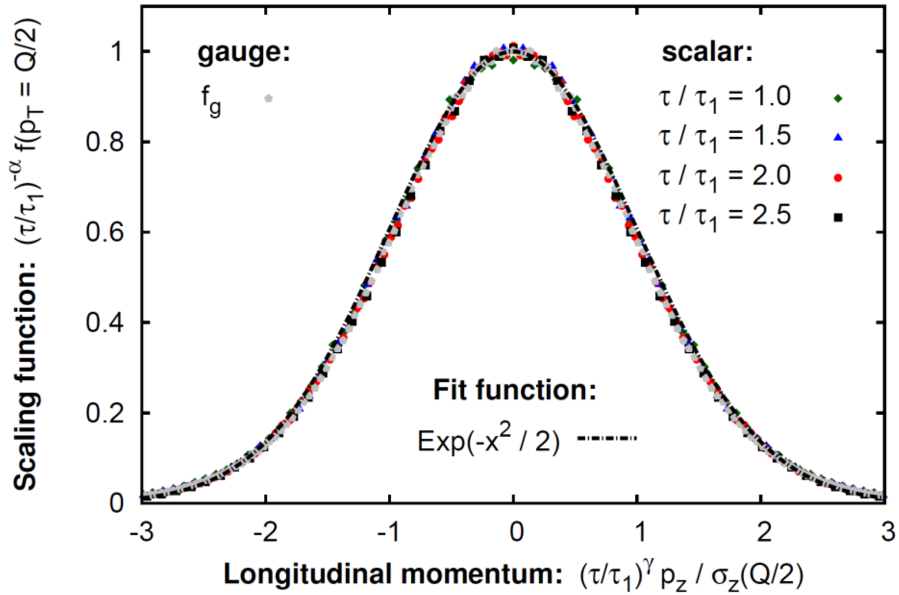
$$\begin{aligned} \alpha - \gamma &= -1 \\ \beta &= 0 \end{aligned}$$

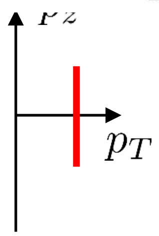
Universality class: expanding gauge and scalar theories

Longitudinal dynamics in scaling region ii)



Rescaling with





$$\begin{aligned} \alpha &= -2/3 \\ \gamma &= 1/3 \end{aligned}$$

leads to time-independent distribution!
Well described by Gaussian shape.

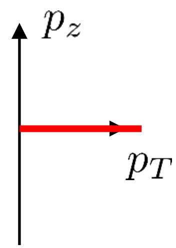
Reminder: *Self-similar evolution*

$$f(p_T, p_z, \tau) = \tau^{\alpha} f_S(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$

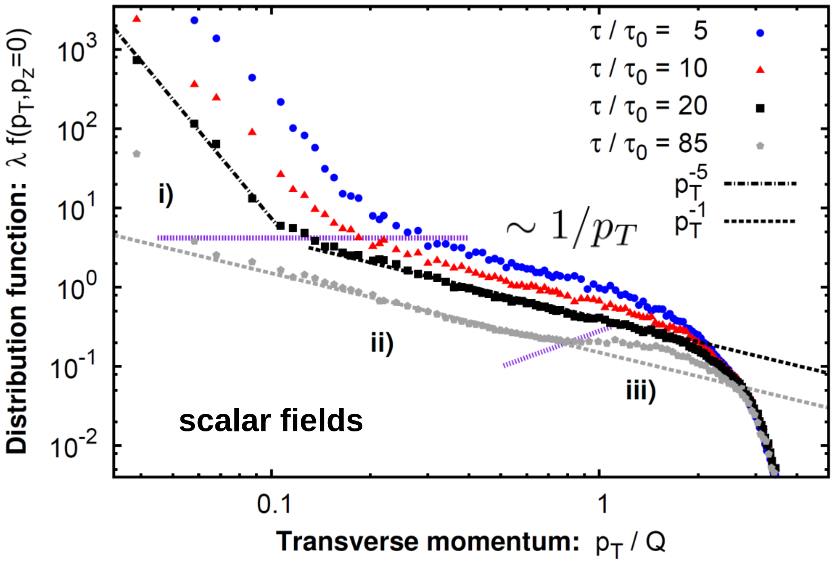
Same form as for gauge theory! (also exponents)

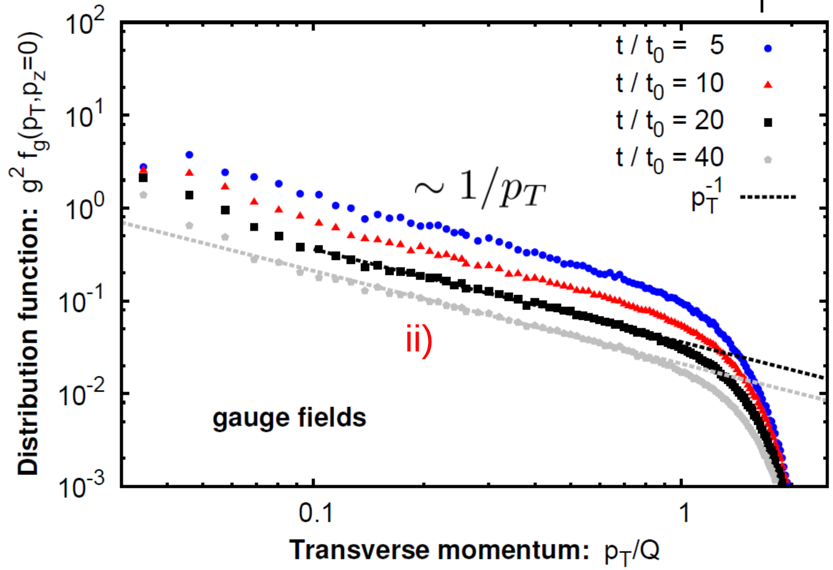
Common universality class!

Universality: expanding gauge and scalar theories



Where is common scaling region?





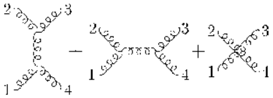
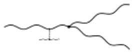
- Scaling range ii) given by $\lambda f \sim \frac{\tau^{-2/3}}{p_T} e^{-p_z^2/2\sigma_z^2}$ with $\sigma_z^2 = \frac{\int dp_z\ p_z^2 f}{\int dp_z f} \sim \tau^{-2/3}$
- Exponents and structure insensitive to initial conditions (memory loss)

J. Berges, KB, S. Schlichting, and R. Venugopalan: *PRD 89, 074011 + 114007 (2014) ; arXiv:1508.03073*

Universality class: expanding gauge and scalar theories

Puzzles

Gauge theory:

✓ Gauge theories for $p \gtrsim m_D$ and $f \gg 1/\alpha_S$ well described by effective kinetic theory (AMY) including  

Arnold, Moore & Yaffe, JHEP 0301, 030 (2003)

Baier, Mueller, Schiff & Son, PLB 502, 51 (2001)

Berges, KB, Schlichting & Venugopalan, PRD 89, 074011 (2014)

Kurkela & Zhou, 1506.06647

BUT: Thermalization scenarios suggested influence from IR (plasma instabilities, condensates, …) → **no influence from IR? Why?**

Bodeker (BD), (2005)

Kurkela, Moore (KM), (2011)

Blaizot, Gelis, Liao, McLerran, Venugopalan (BGLMV), (2012)

Universality class: expanding gauge and scalar theories

Puzzles

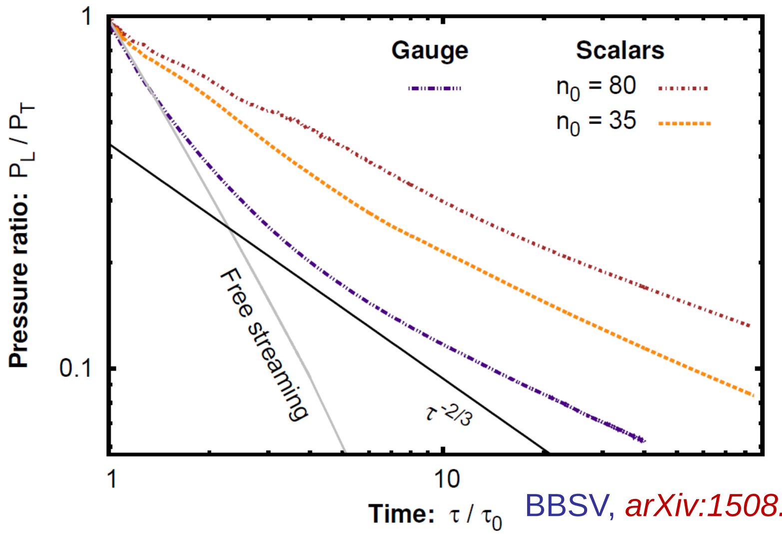
Pressure ratio:

Parametrically:

$$\frac{P_L}{P_T} \overset{\text{kinetic theory}}{\sim} \frac{\int d^3p\, p\, p_z^2/\omega f}{\int d^3p\, p\, p_T^2/\omega f} \overset{\text{late times}}{\sim} (Q\tau)^{-2/3}$$

Discrepancies because of IR?

In scalar theory nontrivial IR dynamics!



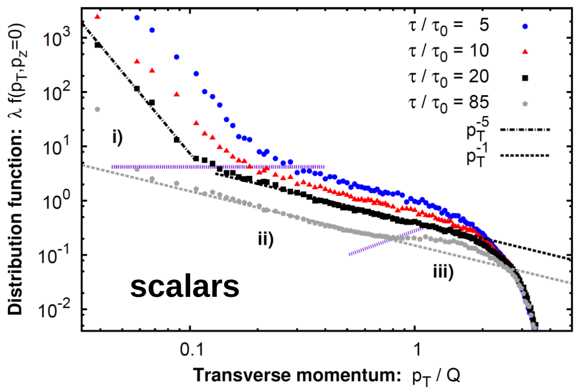
BBSV, *arXiv:1508.03073*

Scalar theory:

How can region ii) be microscopically understood?

How important is soft region for it?

How does Bose condensation emerge?

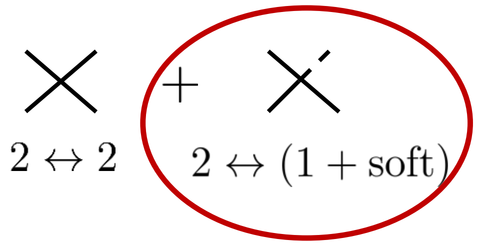


Scalars in the IR

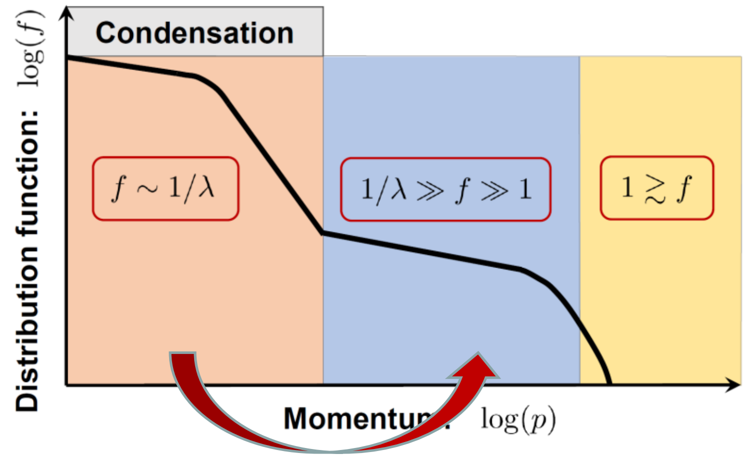
Kinetic approach: Vertex-resummed kinetic theory

Scalar theory:

Often one uses for hard momenta below mass with perturbative occupancies $1/\lambda \gg f$



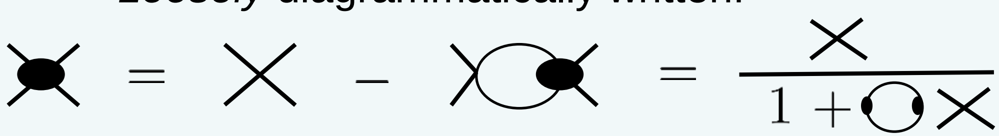
as kinetic theory



BUT: IR dynamics may influence hard momentum evolution. Consistent kinetic description, even for $1/\lambda \gg f$, should need proper IR treatment.

Possible solution: vertex-resummed kinetic theory based on 2PI 1/N expansion to NLO

Loosely diagrammatically written:

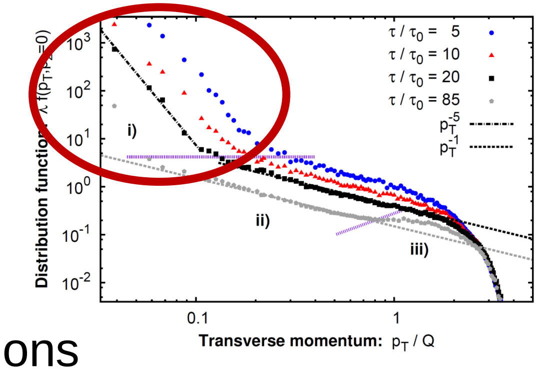


Rigorously:

Berges & Sexty, *PRD 83, 085004 (2011)*

Pinerio Orioli, KB & Berges, *PRD 92, 025041 (2015)*

Universality class of scalars in the infrared



Strategy: Dual description

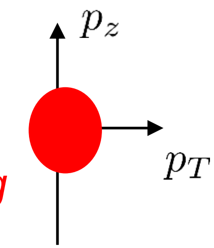
1) It is important to know dynamics in IR → classical simulations

2) It is important to understand dynamics in IR → vertex-resummed kinetic theory (VRKT)

Longitudinally **expanding** scalars: Berges, KB, Schlichting & Venugopalan, *arXiv:1508.03073*

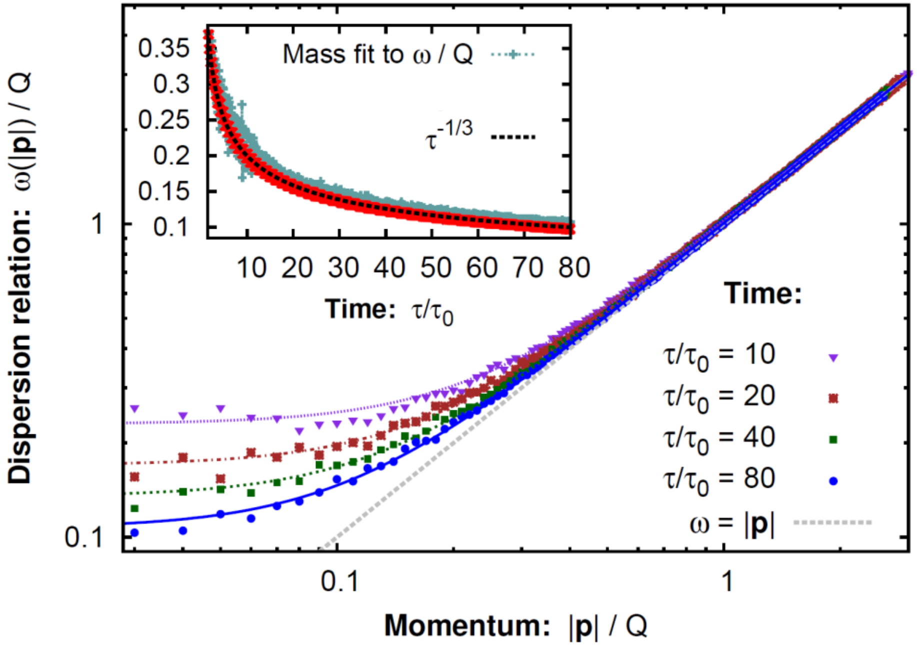
Nonexpanding scalars: (both relativistic and nonrelativistic) Piñerio Orioli, KB & Berges, *PRD 92, 025041 (2015)*

Universality class: scalars in the IR

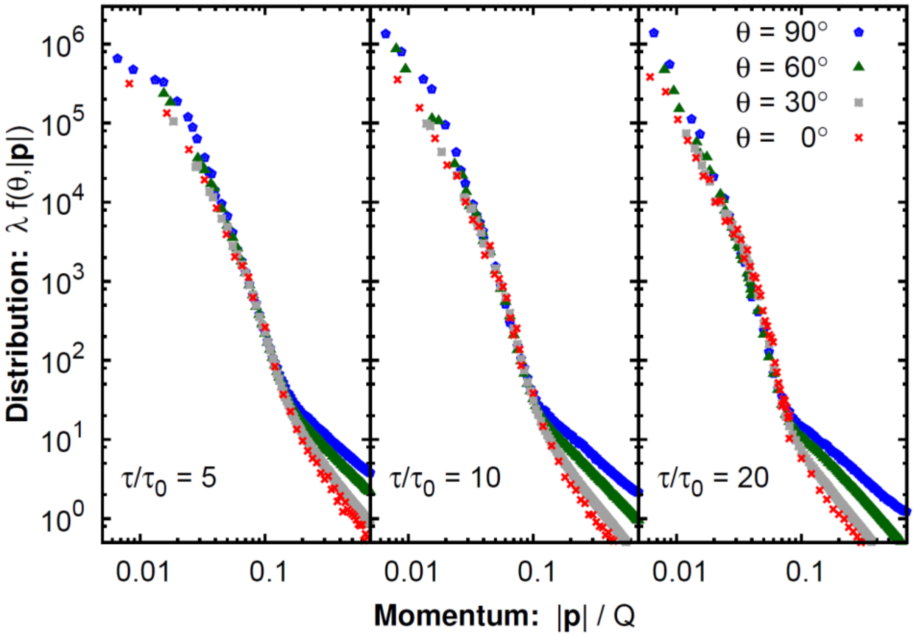


Scalar fields infrared scaling region: i)

Dynamically generated *mass*



(approx.) *isotropic* distribution



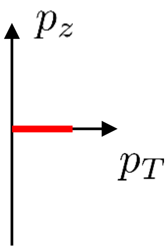
Dispersion relation fit: $\sqrt{m^2 + p^2}$

$m(\tau) \sim \tau^{-1/3}$

All momenta in i) below mass! $p \lesssim m$

Effectively nonrelativistic infrared region

Universality class: scalars in the IR



Expanding

Self-similar evolution

Reminder: *Self-similar evolution*

$$f(p_T, p_z, \tau) = \tau^\alpha f_S(\tau^\beta p_T, \tau^\gamma p_z)$$

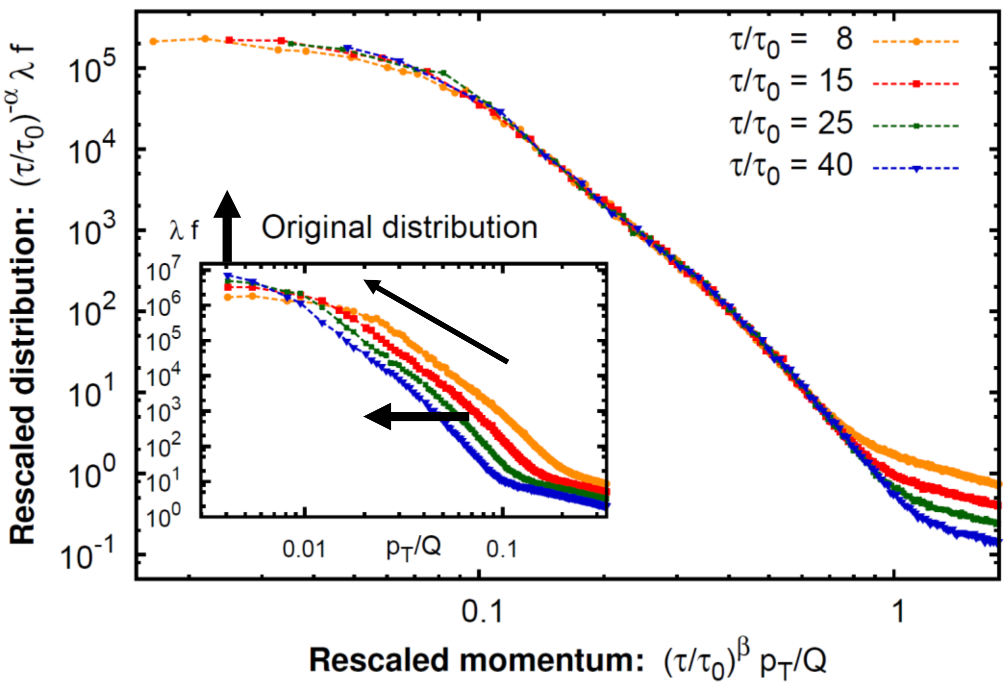
Exponents:

$$\alpha = 1$$
$$\beta = 2/3$$

From isotropic evolution $\gamma = \beta$

Particle number conservation

$$n \sim \int d^3p f \sim 1/\tau$$

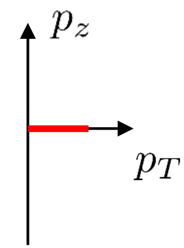


Physical picture: Inverse particle cascade to IR

Bose condensation far from equilibrium!

J. Berges & D. Sexty, *PRL 108 (2012), 161601*

Universality class: scalars in the IR

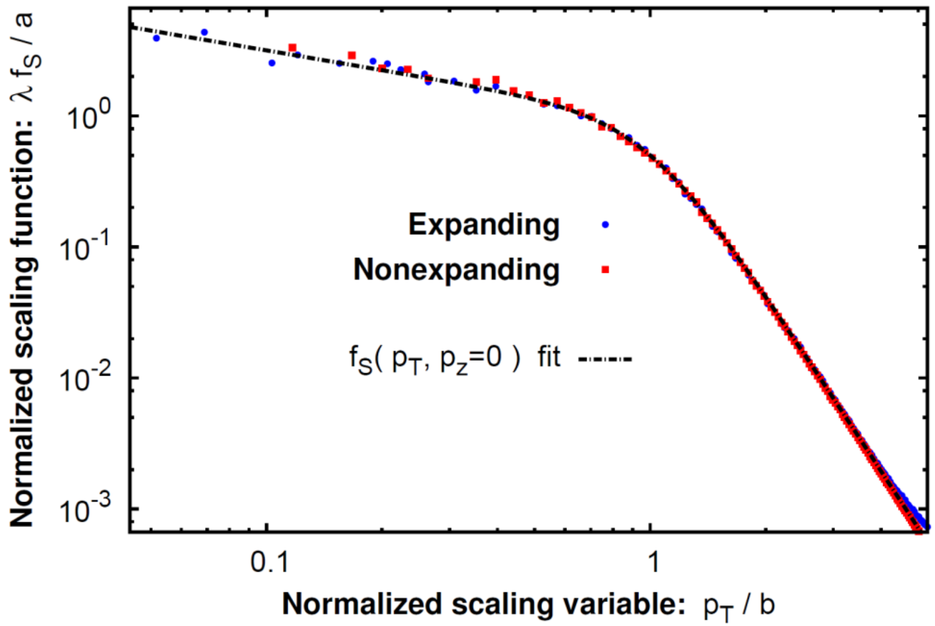


Universal scaling function

Expanding

$$\lambda f_S = \frac{a}{(|\mathbf{p}|/b)^{\kappa_<} + (|\mathbf{p}|/b)^{\kappa_>}}$$

with $\kappa_< \simeq 0.5\,,\ \kappa_> \simeq 4.5 - 5$



Same function for **nonexpanding**:

- *O(N) scalar theories* (N > 1)
- *Nonrelativistic scalars*

Nonexp.

Reminder: *Self-similar evolution*

$$f(p_T, p_z, \tau) = \tau^{\alpha} f_S(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$

Universality class: scalars in the IR

Universality class in IR of **nonexpanding** scalars *Nonexp.*

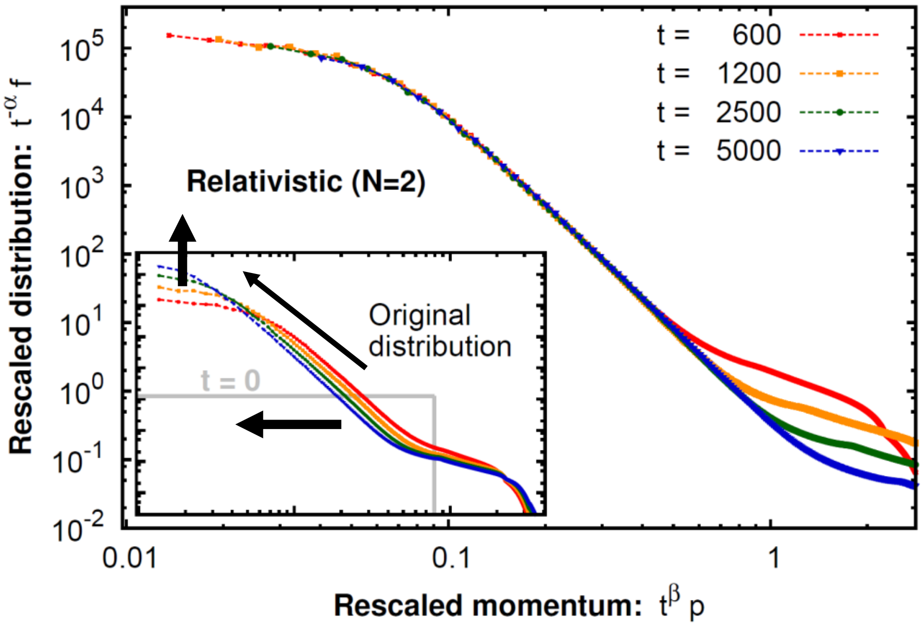
Slightly different exponents

$$\alpha = 3/2\,,\;\; \beta = 1/2$$

Same for relativistic and nonrelativistic!

Particle number conservation

$$n \sim \int d^3p f = const$$



Inverse particle cascade to IR, as in expanding case!

Bose condensation far from equilibrium? Yes!

Self-similar evolution

$$f(p,\tau) = \tau^{\alpha} f_S(\tau^{\beta} p)$$

Universality class: scalars in the IR

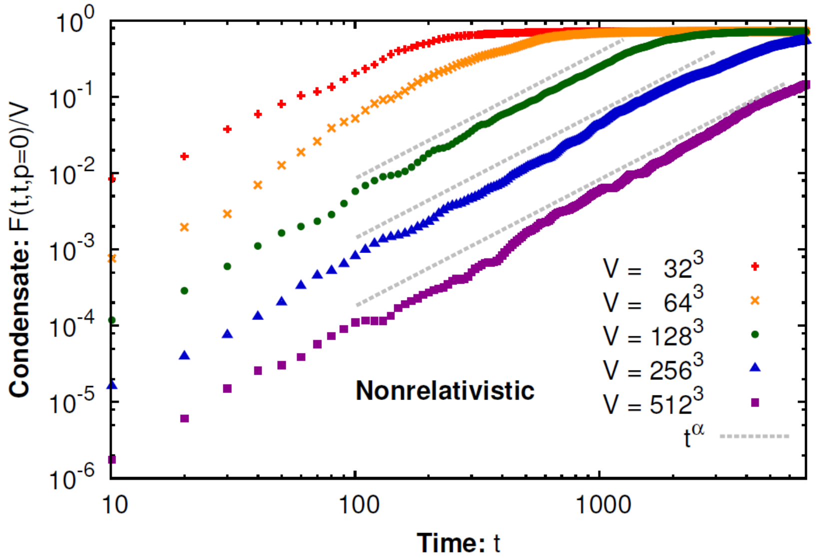
Self-similar evolution

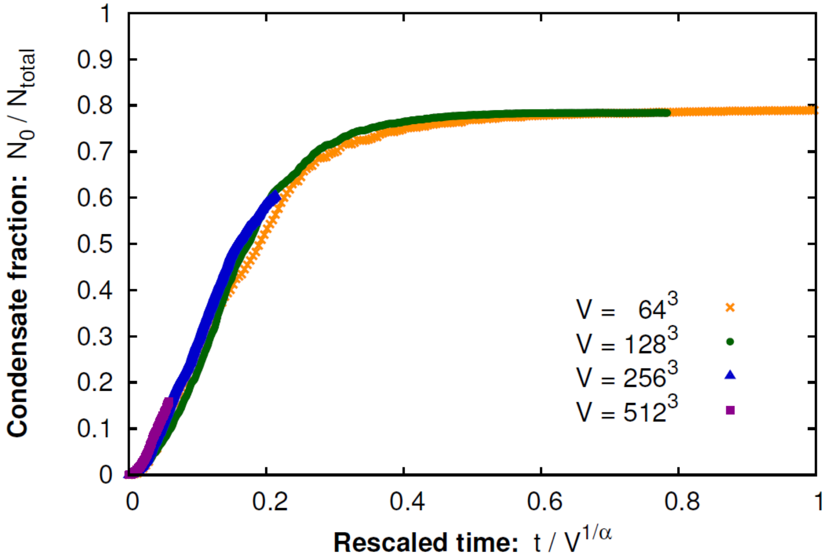
$$f(p,\tau) = \tau^{\alpha} f_S(\tau^{\beta} p)$$

Bose-Einstein condensation

Pinerio Orioli, KB & Berges,
PRD 92, 025041 (2015)

Nonexpanding





Correlation function: $F(p,\tau) = f(p,\tau) + \delta^{(3)}(\boldsymbol{p})\,\phi^2$

Condensation time: $\tau_c \sim V^{1/\alpha}$

Condensation: $F(p=0,\tau) \sim \tau^{\alpha}$

Stops when: $F(p=0,\tau_c) \sim V$

In contrast: perturbative kinetic theory provides finite τ_c Semikoz & Tkachev (1994)

Universality class: scalars in the IR

Vertex-resummed kinetic theory

Berges & Sexty,
PRD 83, 085004 (2011)

Pinerio Orioli, KB & Berges,
PRD 92, 025041 (2015)

(Nonexp.)

$$\frac{\partial f(t;\mathbf{p})}{\partial t} = C^{\mathrm{NLO}}[f](t;\mathbf{p})$$

$$C^{\mathrm{NLO}}(\mathbf{p}) = \int \mathrm{d}\Omega^{2\leftrightarrow 2}\; |M^{2\leftrightarrow 2}[f]|^2 \;\; [(1+f_p)(1+f_l)f_q f_r - f_p f_l (1+f_q)(1+f_r)] + \cdots$$

Usual phase-space integral

$$\int \mathrm{d}\Omega^{2\leftrightarrow 2}(p,l,q,r) = \int \frac{d^d l}{(2\pi)^d} \int \frac{d^d q}{(2\pi)^d} \int \frac{d^d r}{(2\pi)^d} \frac{(2\pi)^{d+1}}{16\; \omega_p \omega_l \omega_q \omega_r} \delta^{(d+1)}(p+l-q-r)$$

4-vectors!

Universality class: scalars in the IR

Vertex-resummed kinetic theory

Berges & Sexty, *PRD 83, 085004 (2011)*

Pinerio Orioli, KB & Berges, *PRD 92, 025041 (2015)*

(Nonexp.)

$$\frac{\partial f(t;\mathbf{p})}{\partial t} = C^{\mathrm{NLO}}[f](t;\mathbf{p})$$

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Usual phase-space integral

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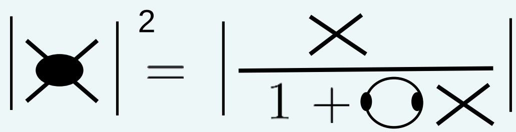
Resummed matrix element

$$|M^{2\leftrightarrow 2}[f]|^2 = \frac{1}{18N}\left(\lambda^2_{\mathrm{eff}}[f](p+l) + \lambda^2_{\mathrm{eff}}[f](p-q) + \lambda^2_{\mathrm{eff}}[f](p-r)\right)$$

$$\lambda^2_{\mathrm{eff}}[f](p) =$$



$$= \frac{\lambda^2}{|1+\lambda \Pi^R(p)|^2}$$

4-vectors!

with

$$\Pi^R[f](p) = \frac{1}{3}\int \frac{d^d q}{(2\pi)^d} \frac{f_q + 1/2}{2\ \omega_q} G^R(p+q)$$

Possible: generalization to off-shell processes ; $1 \leftrightarrow 3$, $0 \leftrightarrow 4$ scatterings ; also condensate

Universality class: scalars in the IR

Self-similarity analysis

- Use kinetic equation $\partial_\tau f(\mathbf{p},\tau) - \frac{p_z}{\tau}\partial_{p_z} f(\mathbf{p},\tau) = C^{\mathrm{NLO}}[f](\mathbf{p};\tau)$ for $f \gg 1$

- Plug in self-similarity ansatz $f(p,\tau) = \tau^\alpha f_S(\tau^\beta p)$

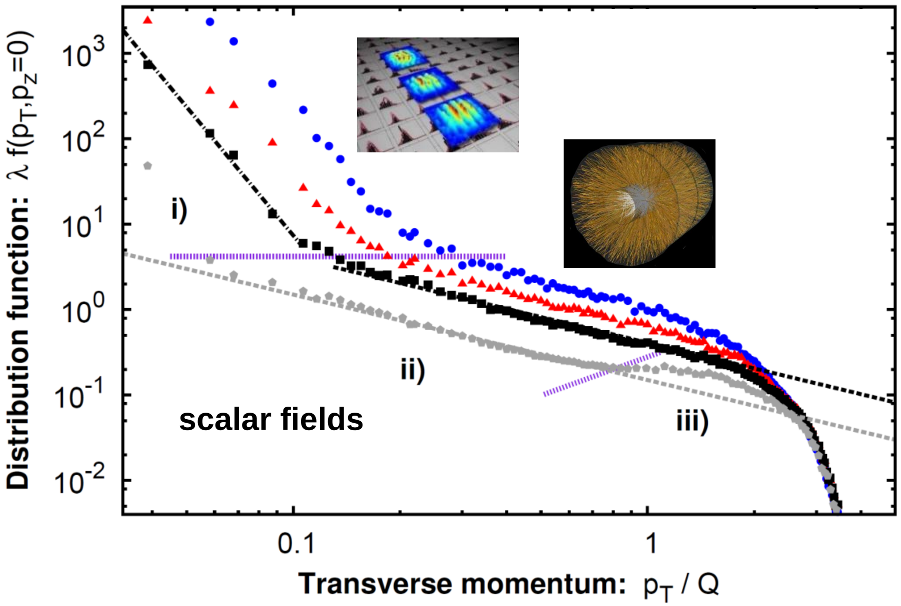
- Comparison of temporal exponents of both sides yields an equation for the exponents: $\alpha - 1 = \alpha - 2\beta + \sigma$ with mass evolution exponent $m(\tau) \sim \tau^{-\sigma}$

- Use particle number conservation: $n \sim \int d^3p f \sim 1/\tau$ ⇒ $\alpha - 3\beta = -1$

(similar for nonexpanding: $\alpha - 3\beta = 0$)

- Exponents $\alpha = 1,\ \beta = 2/3$ for expanding and $\alpha = 3/2,\ \beta = 1/2$ for nonexpanding scalars (both relat. & nonrel.) same as in classical simulations!

Nonthermal fixed point of expanding scalars



Reminder: *Self-similar evolution*

$$f(p_T, p_z, \tau) = \tau^{\alpha} f_S(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$

Berges, KB, Schlichting & Venugopalan,

arXiv:1508.03073

Fixed points	α	β	γ	λf_S
i)	1	$2/3$	$2/3$	$\left((p/b)^{-1/2} + (p/b)^{-5}\right)^{-1}$
ii)	$-2/3$	0	$1/3$	$p_T^{-1}\ e^{-p_z^2/2\sigma_z^2}$
iii)	$-1/2$	0	$1/2$	$\mathrm{sech}(p_z/\sigma_z)$

Conclusion:

- Nonthermal scaling regions may be classified in universality classes; remarkable features of overoccupied field theories; useful tools to test microscopic descriptions

- Different theories in same univ. class may provide insight into their dynamics.

  Example: Expanding scalar and gauge theories

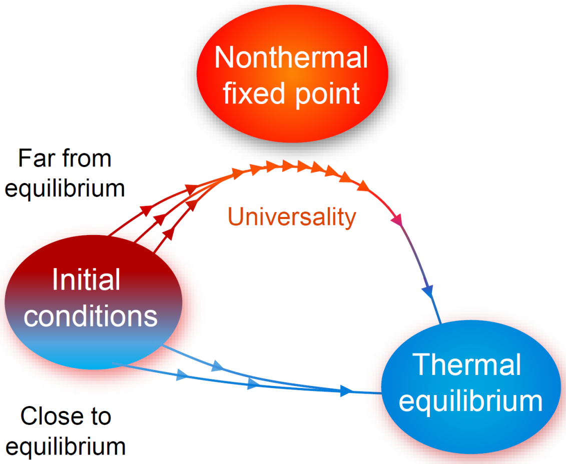
- Infrared region may alter dynamics at high momenta → description of IR needed

- In scalar theories IR universality class; key features of scalars in different geometry can be understood from a vertex-resummed kinetic theory (VRKT)

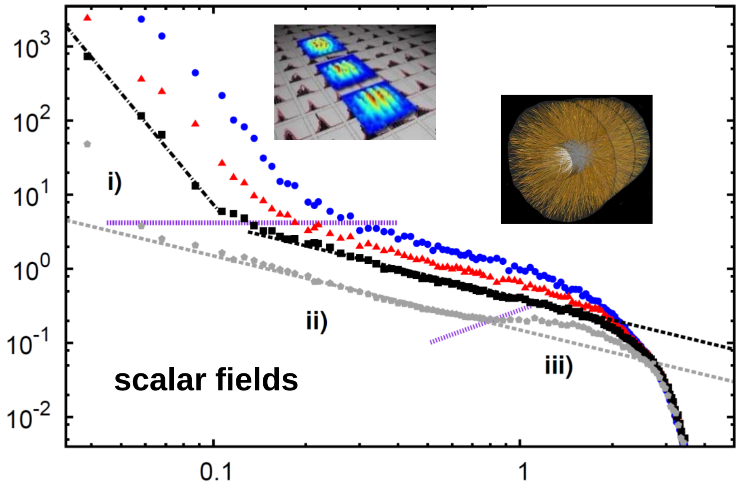
Outlook:

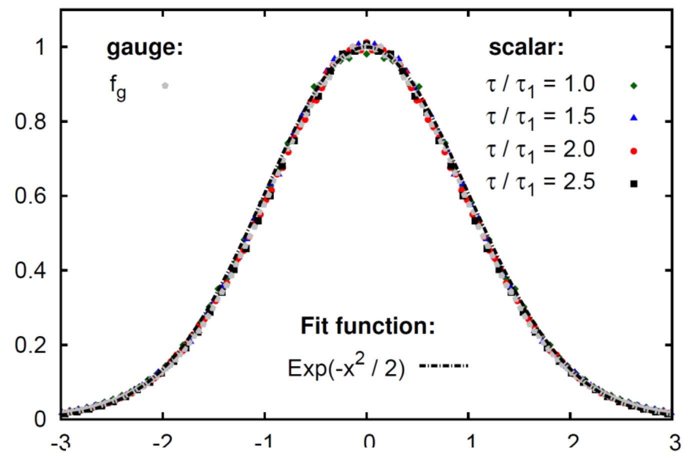
- IR region needs to be better understood in gauge theories; in scalars further IR studies also need (how much influence on hard p; is VRKT consistent with scaling fs)

- How comes that exp. scalars show same scaling region as gauge theory?



Thank you for your attention!

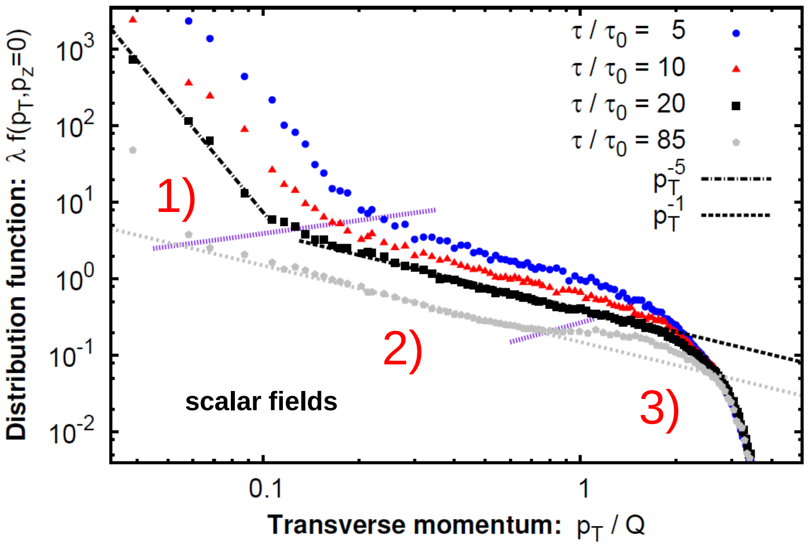




BACKUP SLIDES

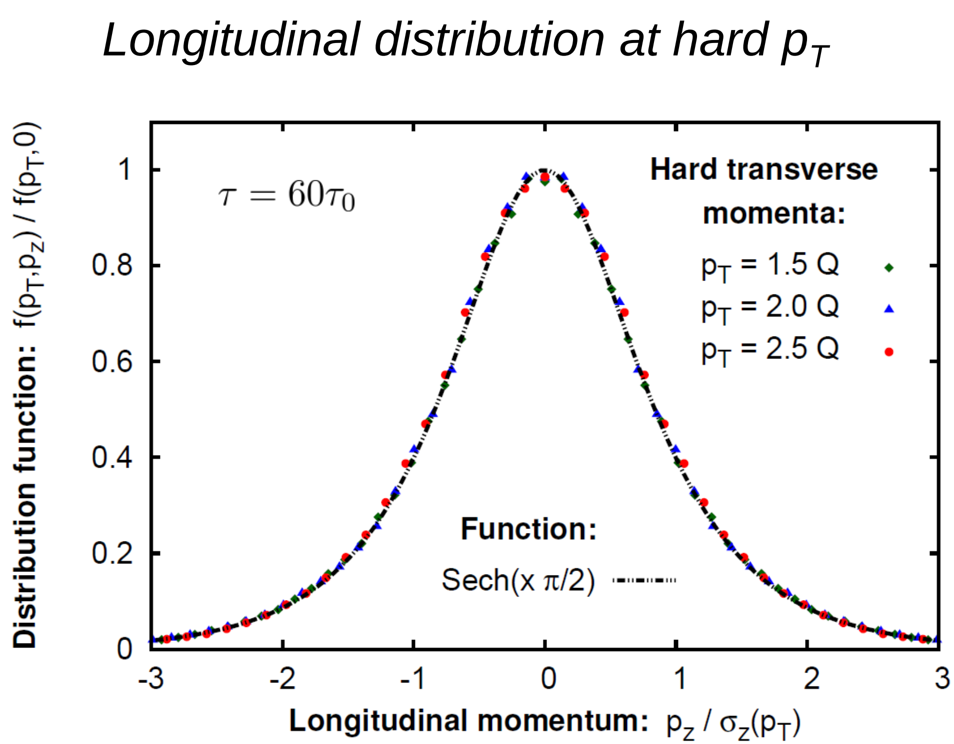
Longitudinally expanding systems

Scalars fields hard-momentum fixed-point: Inertial range 3)



At late times a non-thermal fixed-point emerges at *large momenta*.

Longitudinal distribution at hard p_T



It has a hyperbolic secant shape, which has a broader tail than the Gaussian function.

Sign for large angle scatterings? ✕ $(2 \leftrightarrow 2)$

Longitudinally expanding systems

Scalars fields hard-momentum fixed-point: Inertial range 3)

Self-similar evolution at large p_T

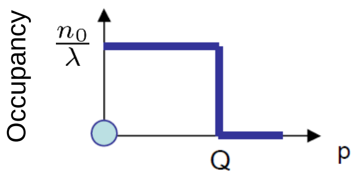
$$f(p_T, p_z, \tau) = \tau^{\alpha'} f'_S(p_T, \tau^{\gamma'} p_z)$$

Longitudinal hard scale

$$\Lambda_L^2(\tau) \approx \frac{\int d^2p_T \int dp_z\; p_z^2\, \omega(\boldsymbol{p})\; f(p_T, p_z, \tau)}{\int d^2p_T \int dp_z\; \omega(\boldsymbol{p})\; f(p_T, p_z, t)}$$

$$\sim \tau^{-2\gamma'}$$

Independent of initial conditions!



$$f(p_T, p_z, \tau_0) = \frac{n_0}{\lambda} \Theta\left(Q - \sqrt{p_T^2 + (\xi_0 p_z)^2}\right)$$

Logarithmic slope of hard scale