Poincaré AdS Black Hole Formation to Model Head-on Heavy Ion Collision

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Outline

- Motivation
- Setup
- Global AdS black holes
- Poincaré AdS black holes
- Summary

Motivation

Open Questions

Gravitational collapse in AdS:
how generic is it?

Hairy AdS black holes:
what is the dynamical endpoint of thermodynamically unstable black hole solutions?

Charged AdS black holes:
what are the dynamical properties of holographic gauge theories at finite density and temperature?

Black hole collisions in AdS:
what are the dynamical properties of holographic gauge theories far from equilibrium?

Within the context of $\mathcal{L} = \frac{1}{16\pi}\left[R - 2\Lambda - \frac{1}{2}g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi - V(\varphi) - \frac{f(\varphi)}{4}F_{\mu\nu}F^{\mu\nu}\right]$

Motivation

Quark gluon plasma

- a non-perturbative problem in QCD
- lattice QCD has no access to real-time dynamics
- experimental data are well described by hydro simulations
- but, hydro always needs initial conditions

Description in terms of classical gravity: AdS/CFT

RHIC → CFT → AdS

goal: *a black hole merger model of heavy ion collisions, where* M_1,M_2,γ_1,γ_2,b *are the only tunable parameters, the rest is determined by AdS dynamics*

Motivation

AdS/CFT correspondence

between an asymptotically anti-de Sitter (AdS) spacetime in (d+1)D and a conformal field theory (**CFT**) in (d)D

Proposed use

to find a gravity description of non-perturbative problems in **QCD**

Major obstacle is the current lack of a gravity dual for QCD

Possible approach: try to capture some features of QCD with a CFT toy model for which there *is* a known gravity dual

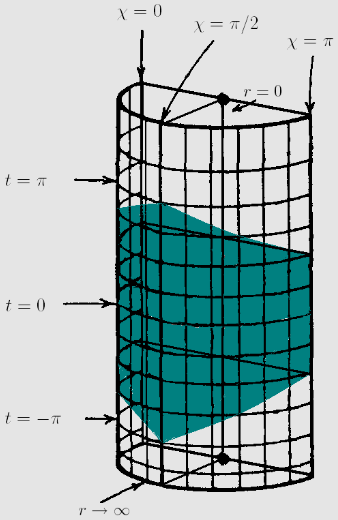
Motivation

- **Previous work relating RHIC to AdS simulations**

  Gravitational shock waves in AdS (single horizon with planar topology) [1] [2] [3] [4] [5] [6] [7] [8] [9] [10] [11]

  Black hole collisions in AdS (merger of horizons with spherical topology) [12] [13] [14] [15]

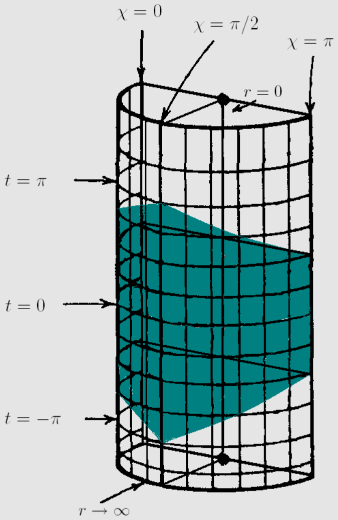
[1] *PRD* **78** (2008), S. Gubser, S. Pufu, A. Yarom
[2] *PRD* **82** (2010), P. Chesler, L. Yaffe
[3] *PRL* **106** (2011), P. Chesler, L. Yaffe
[4] *PRL* **108** (2012), M. Heller, R. Janik, P. Witaszczyk
[5] *PRD* **85** (2012), M. Heller, R. Janik, P. Witaszczyk
[6] *PRL* **111** (2013), W. v. d. Schee, P. Romatschke, S. Pratt
[7] *PRL* **111** (2013), J. Casalderrey-Solana, M. Heller, D. Mateos, W.v.d.S.
[8] *JHEP* **1507** (2015), D. Fernandez
[9] `hep-th/1501.04644` (2015), P. Chesler, L. Yaffe
[10] `hep-th/1506.02209` (2015), P. Chesler
[11] `hep-th/1507.08195` (2015), W. v. d. Schee, B. Schenke
[12] *JHEP* **04** (2007), J. Freiss, S. Gubser, G. Michalogiorgakis, S. Pufu
[13] *JHEP* **04** (2009), P. Figueras, V. Hubeny, M. Rangamani, S. Ross
[14] *PRD* **85** (2012), HB, F. Pretorius, S. Gubser
[15] *PRL* **114** (2014), HB, P. Romatschke



Poincaré coordinates	(t', z', x_1, x_2, x_3)
global coordinates	$(t, r, \chi, \theta, \phi)$

Figure: Shaded region depicting the Poincaré patch, defined by $W(t, r, \chi, \theta, \phi) \equiv (1 + r)/(\sqrt{L^2 + r^2}\cos(t/L) + r \sin\chi \cos\theta) > 0$.

Setup



asymptotically AdS$_5$ spacetime

- $g_{\mu\nu}$ is a solution of Einstein field equations with negative $\Lambda = -6/L^2$
- the bulk metric may deviate from the pure AdS metric $g_{\mu\nu}^{AdS}$
- $g_{\mu\nu}|_{\partial M}$ belongs to the same conformal class as $g_{\mu\nu}^{AdS}|_{\partial M}$
- the boundary of global AdS is identified with $\mathbb{R} \times S^3$
- the boundary of the Poincaré patch is identified with $\mathbb{R}^{3,1}$

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Global AdS$_5$: Scalar Field Variable $\bar{\varphi}$

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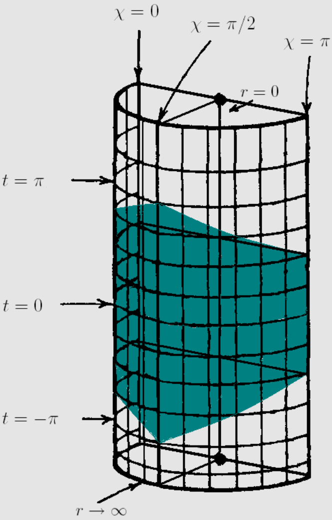
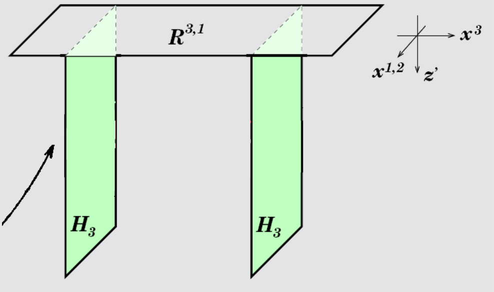
Setup





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A Poincaré Patch of AdS$_5$: $\bar{g}_{x_3x_3}$ on (z', x_3)-slice

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A Poincaré Patch of AdS$_5$: $\bar{g}_{x_3 x_3}$ on (x_2, x_3)-slice

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Setup

Evolution Equations:

$$\begin{aligned}
0 &= -\frac{1}{2}g^{\alpha\beta}g_{\mu\nu,\alpha\beta} - g^{\alpha\beta}{}_{,(\mu}g_{\nu)\alpha,\beta} \\
&\quad -H_{(\mu,\nu)} + H_\alpha\Gamma^\alpha{}_{\mu\nu} - \Gamma^\alpha{}_{\beta\mu}\Gamma^\beta{}_{\alpha\nu} \\
&\quad -\kappa\left(2n_{(\mu}C_{\nu)} - (1+P)g_{\mu\nu}n^\alpha C_\alpha\right) \\
&\quad -\frac{2}{3}\Lambda_5 g_{\mu\nu} - 8\pi\left(T_{\mu\nu} - \frac{1}{3}T^\alpha{}_\alpha g_{\mu\nu}\right) \\
&\quad \downarrow \\
0 &= \mathcal{L}_f|^n_{ijkl} \qquad \text{(15 such equations, one for each } \mu,\nu)
\end{aligned}$$

$$C^\mu \equiv H^\mu - \Box x^\mu \qquad \text{(seek solutions for which } C^\mu = 0)$$

Setup

Evolution Equations:

$$0 = \mathcal{L}_f|^n_{ijkl} \qquad \text{(15 such equations, one for each } \mu\nu\text{)}$$

Solve by a Newton-Gauss-Seidel iterative scheme:

- three-level scheme at time levels t^{n+1}, t^n, t^{n-1}
- knowns: f^n_{ijkl}, f^{n-1}_{ijkl}, unknowns: f^{n+1}_{ijkl}
- the f^n_{ijkl} are used as an initial guess for the f^{n+1}_{ijkl}
- one iteration step:

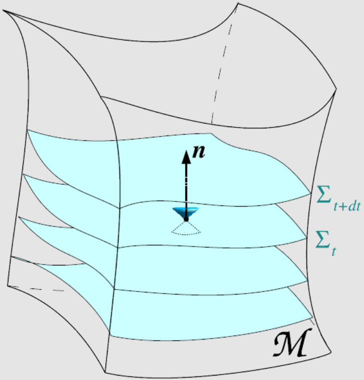
  $$\tilde{f}^{n+1}_{ijkl} \to \tilde{f}^{n+1}_{ijkl} - \frac{\mathcal{R}_f|^n_{ijkl}}{\mathcal{J}_f|^n_{ijkl}} \; (\mathcal{R}_f|^n_{ijkl} = \mathcal{L}_{\tilde{f}}|^n_{ijkl}, \, \mathcal{J}_f|^n_{ijkl} = \frac{\partial \mathcal{L}_f|^n_{ijkl}}{\partial f^{n+1}_{ijkl}})$$

  (letting $\tilde{f}^{n+1}_{ijkl}$ be an approximate solution)
- iterate until $\mathcal{R}_f|^n_{ijkl}$ becomes sufficiently small
  this is done at each time step

Setup

Initial Data: need $g_{\mu\nu}|_{t=0}$, $\partial_t g_{\mu\nu}|_{t=0}$ on the spatial slice $\Sigma_{t=0}$

- The spatial components $g_{ijkl}|_{t=0}$, $\partial_t g_{ijkl}|_{t=0}$ are constrained by the field equations on $\Sigma_{t=0}$
- Other components are set by the choice of coordinates



Adapted from `gr-qc/0703035`

Setup

Key Features:

- Direct discretization of 4+1 field equations
- Cauchy evolution scheme
- Spatially compactified coordinates
- Dynamical horizon tracking
- Adaptive mesh refinement
- Modified cartoon method

Single Deformed BH: Metric Variable $\bar{g}_{\chi\chi}$

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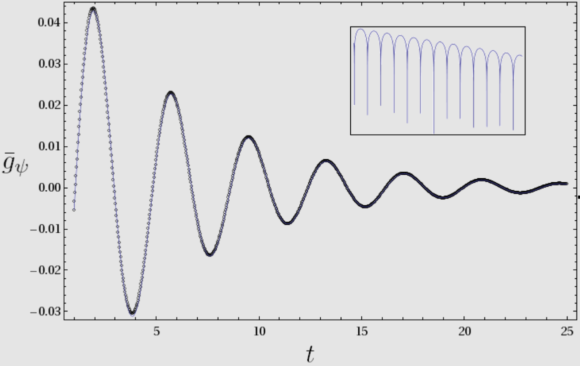
QNM Frequencies: Dependence on BH size



r_h/L=12.2	$1.640 - i(0.902)L/r_h$
r_h/L=11.3	$1.640 - i(0.895)L/r_h$
r_h/L=10.5	$1.641 - i(0.876)L/r_h$
r_h/L=9.0	$1.643 - i(0.864)L/r_h$
r_h/L=6.5	$1.650 - i(0.841)L/r_h$
r_h/L=5.0	$1.661 - i(0.837)L/r_h$
r_h/L=4.5	$1.666 - i(0.823)L/r_h$
r_h/L=4.0	$1.675 - i(0.823)L/r_h$
r_h/L=3.3	$1.692 - i(0.808)L/r_h$

Figure: Fundamental qnm frequencies taken from $\bar{g}_{\chi\chi}$ by projecting onto $\mathbb{S}(200)$ near $r \to \infty$ then fitting to damped sinusoids $\sim e^{-i\omega t}$.

Energy Density ϵ on $\mathbb{R}^{3,1}$

$$\epsilon_{\mathbb{R}^{3,1}} = W^{-4}\epsilon_{\mathbb{R}\times S^3}$$

$x_2 \uparrow$

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$\longrightarrow$
x_3

$$W = \sqrt{(t')^2 + (1 + x_1^2 + x_2^2 + x_3^2 - (t')^2)^2/4}$$

BH Collision: Scalar Field Variable $\bar{\varphi}$

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BH Collision: Scalar Field Variable $\bar{\varphi}$

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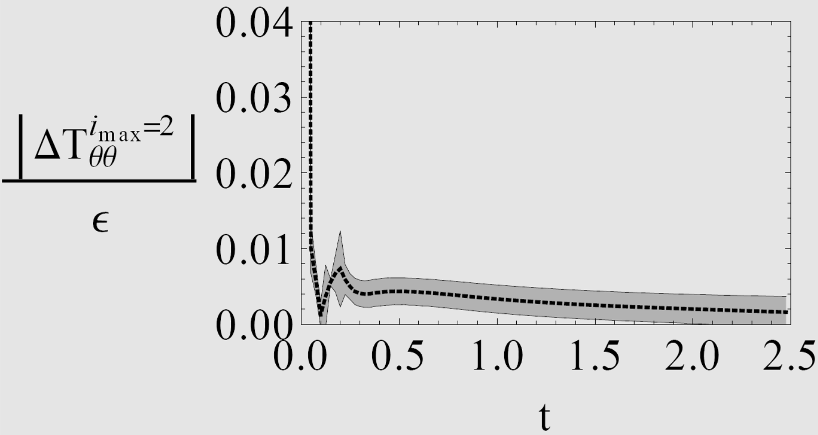
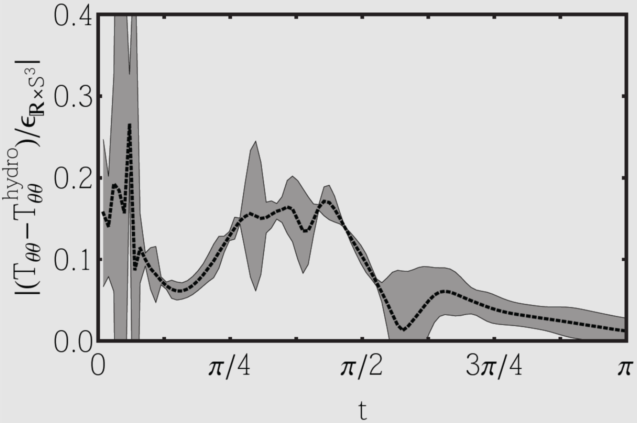
Comparison to Hydrodynamics on $\mathbb{R} \times S^3$: single deformed BH



Comparison to Hydrodynamics on $\mathbb{R} \times S^3$: BH collision



A Poincaré Patch of AdS_5

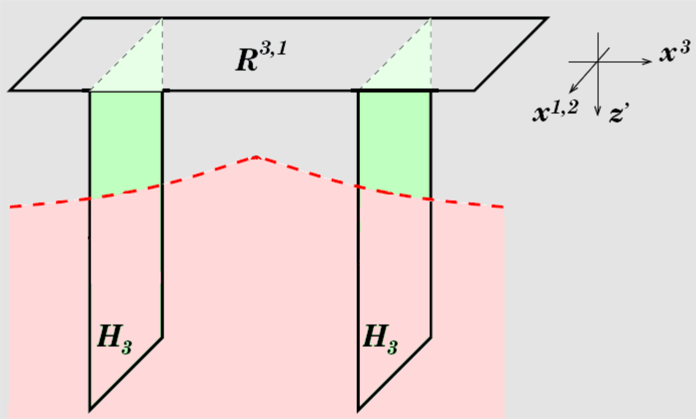
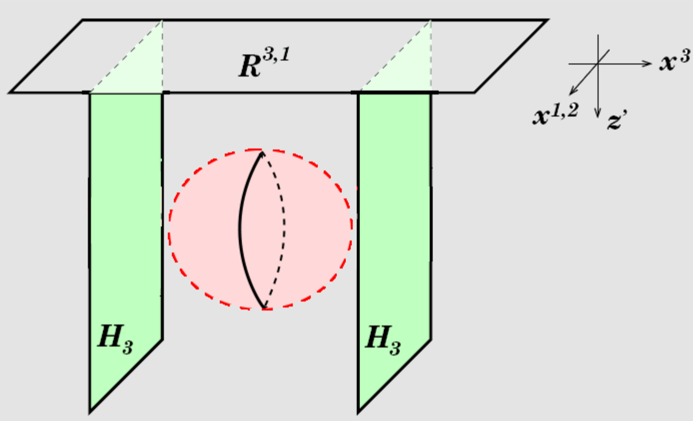


Figure: The Poincaré patch of AdS_5 drawn in coordinates adapted to the $\mathbb{R}^{3,1}$ boundary; constant-x_3 slices are copies of the hyperbolic plane H_3.

Adapted from `hep-th/0805.1551`

A Poincaré Patch of AdS_5



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Planar BH: $\bar{\varphi}$ on (z', x_3)-slice

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Planar BH: $\bar{g}_{x_3x_3}$ on (z', x_3)-slice

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Spherical BH: $\bar{\varphi}$ on (z', x_3)-slice

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Spherical BH: $\bar{\varphi}$ on (x_2, x_3)-slice

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BH collision: $\bar{\varphi}$ on (z', x_3)-slice

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BH collision: $\bar{\varphi}$ on (x_2, x_3)-slice

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Summary

What physics can we hope to extract from these simulations?

- dynamics of CFT far from equilibrium, relevant to head-on heavy ion collisions

What has been done?

- black hole collisions in global AdS
- control over imposed symmetries on the Poincaré patch

What remains to be done?

- stable merger of two black holes on the Poincaré patch
- $\langle T_{\mu\nu} \rangle_{CFT}$ calculations relevant to RHIC observables