Thermal fluctuations in heavy-ion collisions

Clint Young

Michigan State University

July 21, 2015

Fluctuation-dissipation and the importance of noise

Thermal noise in a viscous, evolving fluid

Momentum eccentricity in LHC 10-20%

Statistical fluctuations in static fluids

Fluids are effective descriptions of a material in terms of macroscopic quantites e, p, u^μ, ...

If a fluid is ideal, its energy-momentum tensor is described assuming local thermal equilibrium:

$$T^{\mu\nu} = -p(e)g^{\mu\nu} + (e+p)u^\mu u^\nu.$$

However, the underlying microscopic dynamics comes back to haunt us in the form of viscous corrections. At first order,

$$\Delta T^{\mu\nu} = \eta(\Delta^\mu u^\nu + \Delta^\nu u^\mu - \tfrac{2}{3}(\partial \cdot u)\Delta^{\mu\nu}) + \zeta(\partial \cdot u)\Delta^{\mu\nu}.$$

What about fluctuations in e, u^μ, n,...? Fluctuations in a static fluid are described statistically [from Landau and Lifshitz]:

$$\left\langle \delta u^i(\mathbf{x},t)\delta u^j(\mathbf{x}',t)\right\rangle = (T/(e+p))\delta^{ij}\delta^3(\mathbf{x}-\mathbf{x}'),$$

$$\left\langle \delta e(\mathbf{x},t)\delta e(\mathbf{x}',t)\right\rangle = (e+p)T\left(\frac{\partial e}{\partial p}\right)_{n/s}\delta^3(\mathbf{x}-\mathbf{x}'),$$

$$\left\langle \delta n(\mathbf{x},t)\delta n(\mathbf{x}',t)\right\rangle = T(\tfrac{\partial n}{\partial \mu})\delta^3(\mathbf{x}-\mathbf{x}'),...$$

Easy to see why: by equipartition of energy $\left\langle \frac{1}{2}\rho(\delta u^i)^2\Delta V\right\rangle = 3T/2$.

Already, fluctuations provide one way of measuring the speed of sound squared $\left(\frac{\partial p}{\partial e}\right)_{n/s}$.

Fluctuations at unequal times and the dynamics of noise

Autocorrelations $\langle \delta q(\mathbf{x}, t) \delta q(\mathbf{x}', t') \rangle$ when $t \neq t'$ are no longer ideal quantities but depend on dynamical coefficients η, ζ, σ...

The presence of dissipation (nonzero $\mathrm{Im}(\mathrm{G_R})$) and thermal expectation values of δq also requires the existence of noise terms in the conserved currents: for example, I^μ in $J^\mu = nu^\mu + \sigma T \Delta^\mu(\beta\mu) + I^\mu$.

The form of the noise can be determined in great generality: if $G_R(\omega, \mathbf{k}) = \frac{\omega}{A(\omega, \mathbf{k})}$, then

$$\langle \delta q \delta q(\omega, \mathbf{k}) \rangle = \frac{2T}{\omega} \mathrm{Im}\{G_R\} = iT \left(\frac{1}{A} - \frac{1}{A^*} \right),$$

and the noise in conserved currents is determined using the conservation law:

$$\frac{1}{3} k^2 \left\langle I^l I^l(\omega, \mathbf{k}) \right\rangle = -iT(A - A^*).$$

Getting more specific: relativistic hydrodynamics has superluminal modes at first order but not at second order. The homogeneous Israel-Stewart equations are

$$\partial_\mu \left(T^{\mu\nu}_{id.} + \Pi^{\mu\nu} \right) = 0,$$

$$T^{\mu\nu}_{id.} = -pg^{\mu\nu} + (e+p)u^\mu u^\nu,$$

$$\Delta^\mu_\alpha \Delta^\nu_\beta (u \cdot \partial)\Pi^{\mu\nu} = -\frac{1}{\tau_\Pi}(\Pi^{\mu\nu} - \eta(\Delta^{\langle\mu} u^{\nu\rangle}) - \zeta(\partial \cdot u)\Delta^{\mu\nu}),$$

which are closed by the equation of state $p(e)$ and a *choice of frame*: one commonly chooses the Landau frame $u_\mu \Pi^{\mu\nu} = 0$ so that $u^\mu =$ the flow of energy through the fluid.

Finding $G_R(\omega)$ for linear perturbations of these equations will determine the dynamics and statistics of thermal noise in an evolving fluid...

For $T^{\mu\nu} = T^{\mu\nu}_{id.} + \Pi^{\mu\nu} + \delta T^{\mu\nu}_{id.} + \delta\Pi'^{\mu\nu}_{id.}$, the linearized equations of motion for perturbations become

$$\partial_t \delta T^{t\nu}_{\text{id.}} = -\partial_i \delta T^{i\nu}_{\text{id.}} - \partial_\mu \delta\Pi'^{\mu\nu},$$

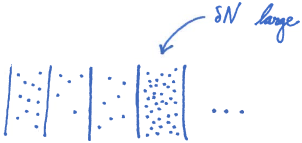
$$(u\cdot\partial)\delta\Pi'^{\mu\nu} = -\frac{1}{\tau_\pi}(\delta\Pi'^{\mu\nu} - \delta S^{\mu\nu} - \xi^{\mu\nu}) - \frac{4}{3}(\partial\cdot\delta u)\Pi'^{\mu\nu} - \frac{4}{3}(\partial\cdot u)\delta\Pi'^{\mu\nu} - (\delta u\cdot\partial)\Pi^{\mu\nu}$$
$$-\delta u^\mu((u\cdot\partial)u_\alpha)\Pi^{\alpha\nu} - u^\mu((\delta u\cdot\partial)u_\alpha)\Pi^{\alpha\nu} + (u\cdot\partial)\delta u_\alpha)\Pi^{\alpha\nu} + (u\cdot\partial)u_\alpha)\delta\Pi'^{\alpha\nu})$$
$$-\delta u^\nu((u\cdot\partial)u_\alpha)\Pi^{\alpha\mu} - u^\nu((\delta u\cdot\partial)u_\alpha)\Pi^{\alpha\mu} + (u\cdot\partial)\delta u_\alpha)\Pi^{\alpha\mu} + (u\cdot\partial)u_\alpha)\delta\Pi'^{\alpha\mu}),$$

$$\left\langle \xi^{\mu\nu}(x)\xi^{\alpha\beta}(x')\right\rangle = \left[2\eta T(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}) + 2T(\zeta - 2\eta/3)\Delta^{\mu\nu}\Delta^{\alpha\beta}\right]\delta^4(x-x').$$

The equations are discretized: $\xi(x) \to \xi_i = \frac{1}{\Delta V \Delta t}\int d^4x\xi(x)$.
$\langle\xi(x)\xi(x')\rangle \propto \delta^4(x-x') \to \left\langle \xi^i\xi^j\right\rangle \propto \frac{1}{\Delta V\Delta t}$:
hyperbolic equations with large gradients and sources.

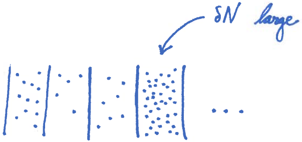
$\sqrt{\langle N^2\rangle - \langle N\rangle^2} \sim \sqrt{N}$:

high T



$\sqrt{\langle N^2\rangle - \langle N\rangle^2} \sim \sqrt{N}$:

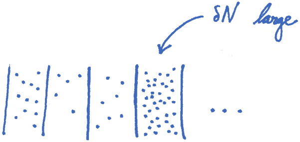
high T



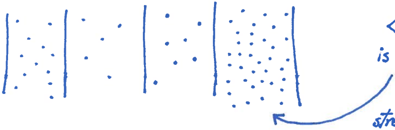
low T, after expanding, $t \to \infty$:

$\sqrt{\langle N^2\rangle - \langle N\rangle^2} \sim \sqrt{N}$:

high T



low T, after expanding $t \neq \infty$:



$\langle \delta n(\vec{x},t)\, \delta n(\vec{x}',t)\rangle$ is not yet $\propto \delta^3(\vec{x}-\vec{x}')$, the earlier δN is stretched over the sky.

low T, after expanding, $t \to \infty$:

Baryon fluctuations away from equilibrium

At first order, $J^\mu = nu^\mu + \sigma\Delta^\mu(\beta\mu) + I^\mu$, and the diffusion equation

$$\left[\frac{\partial}{\partial t} - D\nabla^2\right] n = -\partial_\mu I^\mu,$$

where $D = \sigma\frac{\partial n}{\partial\mu}$ and the fluid is at rest. For this,

$$\left\langle \delta n(\mathbf{x},t)\delta n(\mathbf{x}',t')\right\rangle = T(\tfrac{\partial n}{\partial\mu})(\tfrac{1}{4\pi Dt})^{3/2}\exp(-|\mathbf{x}-\mathbf{x}'|^2/4Dt) \text{ and}$$

$$\left\langle I^i(\mathbf{x},t)I^j(\mathbf{x}',t')\right\rangle = 2\sigma T\delta^{ij}\delta^3(\mathbf{x}-\mathbf{x}').$$

First order diffusion also suffers from superluminal modes.

Baryon fluctuations at second and third order

The Gurtin-Pipkin equation

$$\left[\frac{\partial}{\partial t} - D\nabla^2 + \tau_1\frac{\partial^2}{\partial t^2} + \tau_2^2\frac{\partial^3}{\partial t^3} - \tau_3' D\frac{\partial}{\partial t}\nabla^2\right] n = 0$$

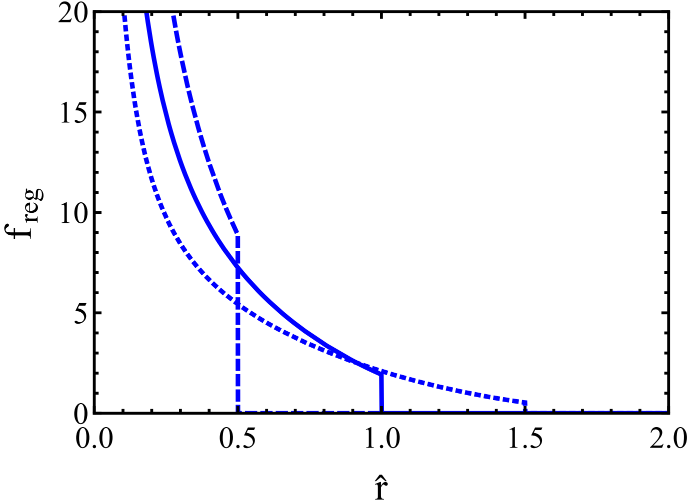
reduces to the Cattaneo equation when $\tau_2 = \tau_3' = 0$, and further to the usual diffusion equation when $\tau_1 = 0$.

A in G_R is now

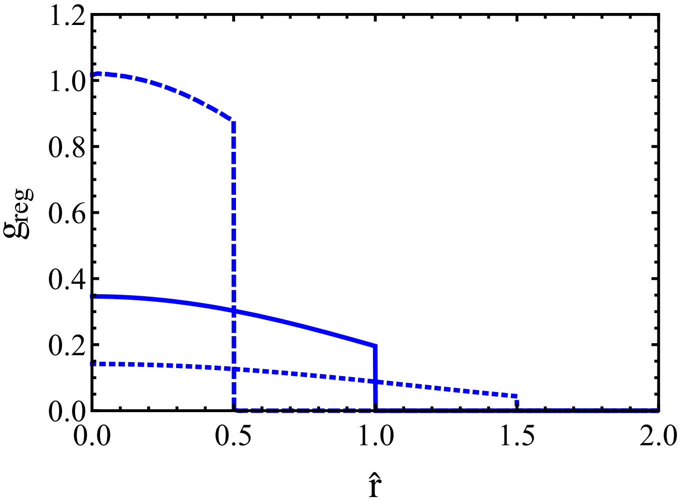
$$A = \frac{\omega - i\tau_1\omega^2 - \tau_2^2\omega^3 + iDk^2 + \tau_3' Dk^2\omega}{1 - i\tau_1\omega - \tau_2^2\omega^2 + \tau_3 Dk^2},$$

whose zeros and poles are determined by cubic equations.

$\langle \delta n \delta n(\hat{r}, \hat{t}) \rangle = \frac{\sigma T}{2\pi^2 v_0^2 \tau_2^4} [f_{\mathrm{reg}}(\hat{r}, \hat{t}) + \text{delta functions}]$:



$\left\langle I^i I^j(\hat{r},\hat{t})\right\rangle = \frac{3\sigma T}{8\pi^2 v_0^3 r_2^4}\delta^{ij}\left[g_{reg}(\hat{r},\hat{t}) + \text{delta functions}\right]$:



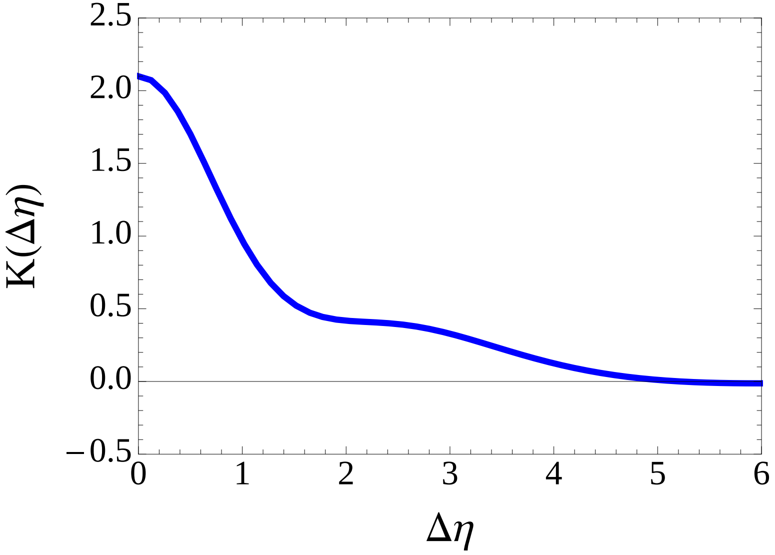
Thermal noise in heavy-ion collisions

Kapusta, Müller, Stephanov: Thermal noise exists in viscous relativistic fluids, represented by $S^{\mu\nu}_{\mathrm{heat}}$ and $S^{\mu\nu}_{\mathrm{visc.}}$ (in the Eckart frame) and $S^{\mu\nu}$ and I^{μ} in the (Landau frame).

$K(\Delta\eta) \propto \left\langle \frac{dN}{d\eta}(\eta+\Delta\eta)\frac{dN}{d\eta}(\eta) \right\rangle - \left\langle \frac{dN}{d\eta} \right\rangle^2$, the two-particle correlation as a function of rapidity gap, has a contribution from thermal noise.

Analytical calculations possible for the Bjorken expansion of an ultrarelativistic gas.

Thermal noise in ultrarelativistic gases

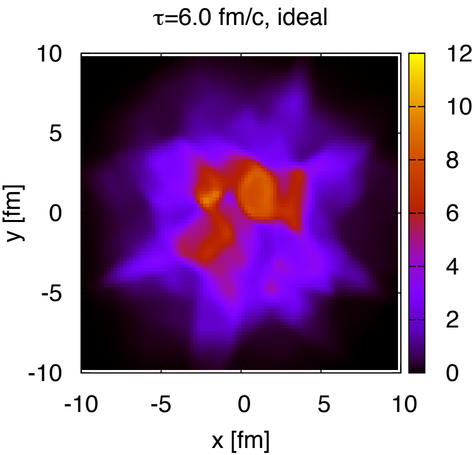


MUSIC for 3+1-dimensional viscous hydrodynamics

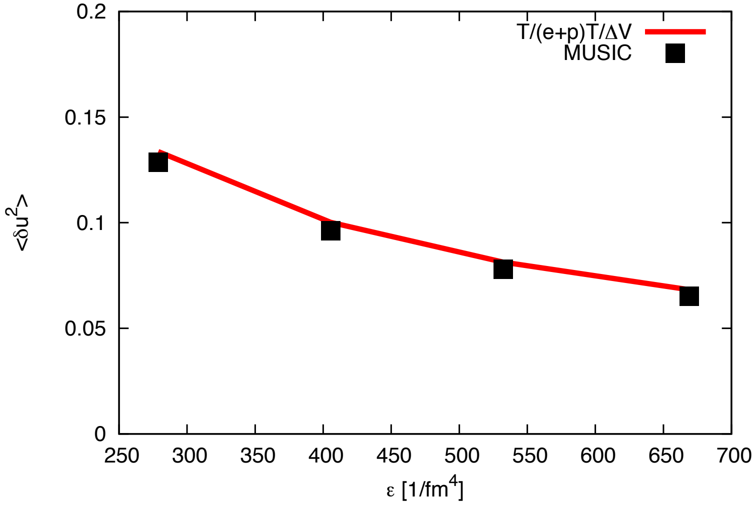
Noise breaks boost invariance
$\rightarrow$ (3+1)-dimensional simulation necessary for $K(\Delta\eta, \Delta\phi)$.

Fluctuations related to dissipation
$\rightarrow$ viscous hydrodynamics necessary.

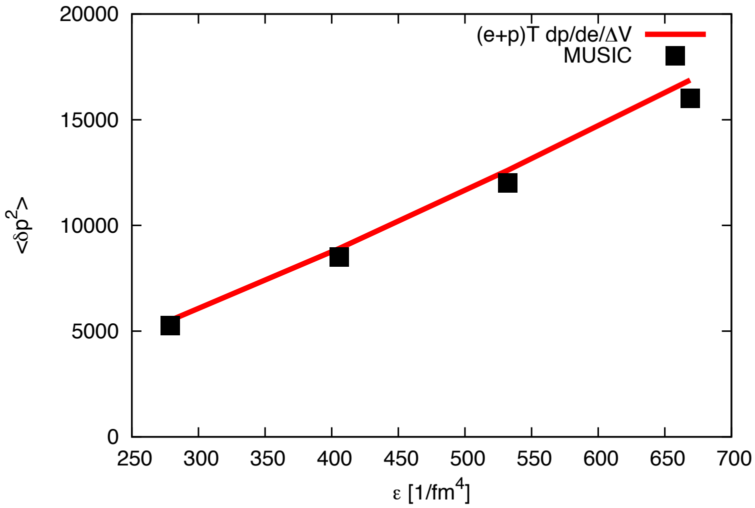
MUSIC (Schenke, Jeon, and Gale) solves the Israel-Stewart model of viscous hydrodynamics in (τ, x, y, η), uses lattice EOS, determines 3-dimensional freeze-out surface for hadron production.



Testing MUSIC with thermal noise

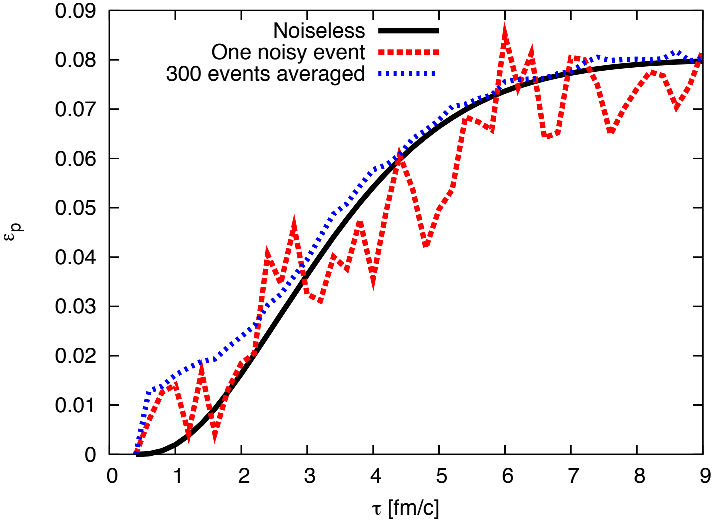


Testing MUSIC with thermal noise

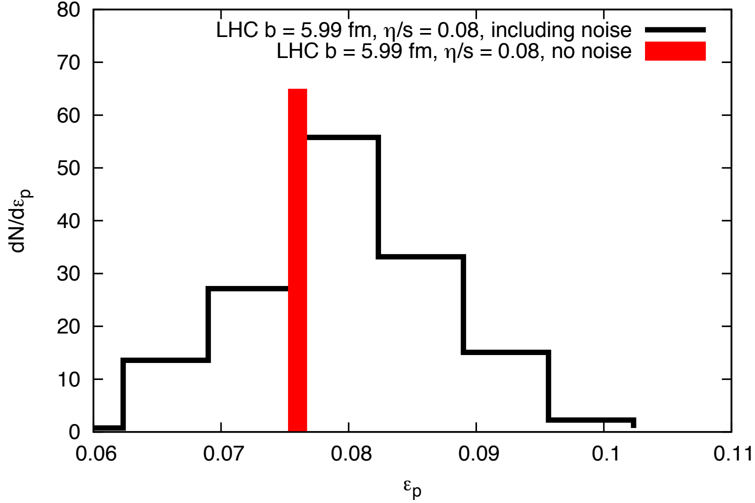


ϵ_p in LHC's 10-20% centrality class

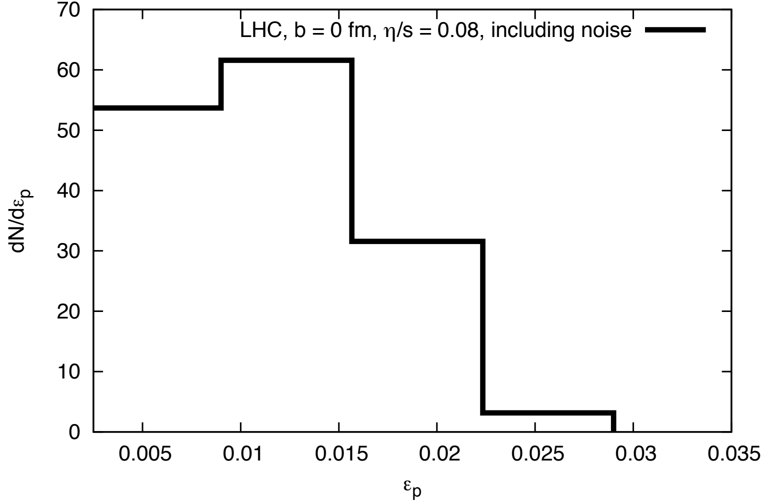
$\epsilon_p = \frac{\langle T^{xx} - T^{yy} \rangle}{\langle T^{xx} + T^{yy} \rangle}$ and $\frac{2\langle T^{xy} \rangle}{\langle T^{xx} + T^{yy} \rangle}$ added in quadrature:



The variance of ϵ_p



The variance of ϵ_p



Thermal fluctuations and freeze-out

Ultimately, thermal fluctuations in hydrodynamics propagate into the particle yields. For Cooper-Frye freeze-out,

$$
\begin{aligned}
\delta f \;=\; & \Big[\delta f_0 + \delta f_0 (1 \pm 2 f_0) \frac{W_{\alpha\beta} p^\alpha p^\beta}{2 s T^3} \\
& + f_0 (1 \pm f_0) \frac{\delta W'_{\alpha\beta} p^\alpha p^\beta}{2 s T^3} \\
& + f_0 (1 \pm f_0)) \frac{W_{\alpha\beta} p^\alpha p^\beta}{2 s T^3} \big(-2 \tfrac{\delta T}{T} - \tfrac{\delta e + \delta p}{e + p}\big)\Big] p \cdot d\Sigma
\end{aligned}
$$

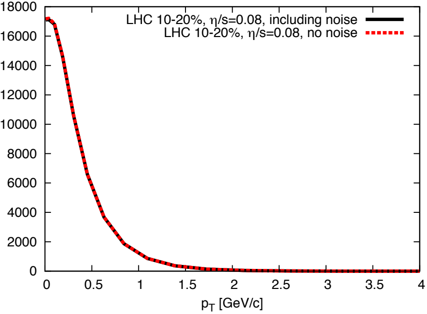
where

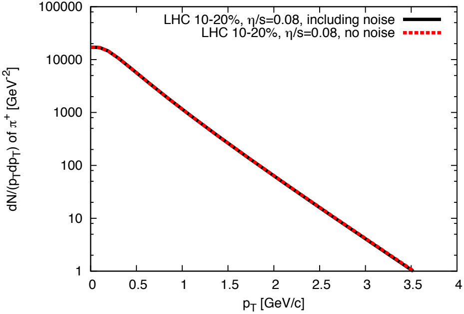
$$
\delta f_0 \;=\; -\frac{\exp(-p \cdot u / T)}{(1 \pm \exp(-p \cdot u / T))^2} (p \cdot \delta u / T - (p \cdot u) \delta T / T^2).
$$

The thermal fluctuations, which propagate from the earliest times and across rapidities, show up at the freeze-out surface.

One-particle observables

$\langle \Xi^{\mu\nu} \rangle = 0$: averaging 70 runs,





Summary

- The fluctuation-dissipation relation requires the existence of thermal noise in viscous fluids.
- Thermal hydrodynamical fluctuations are a *well-controlled* source of e-by-e fluctuations (no new parameters, just those of viscous hydrodynamics).
- Source for *small* e-by-e variations in v_2, possibly visible in ultra-central events.

References I

- L. D. Landau, E. M. Lifshitz, and L. P. Pitaevskii, *Statistical Physics Part 2, Landau and Lifshitz Course of Theoretical Physics Volume 9*, 1980 Pergamon Press plc.
- Joseph I. Kapusta and Charles Gale, *Finite Temperature Field Theory: Principles and Applications*, Cambridge University Press, 2006.
- B. Schenke, S. Jeon and C. Gale, Phys. Rev. C **82**, 014903 (2010) [arXiv:1004.1408 [hep-ph]].
- D. T. Son and D. Teaney, JHEP **0907**, 021 (2009) [arXiv:0901.2338 [hep-th]].
- J. I. Kapusta, B. Muller and M. Stephanov, Phys. Rev. C **85**, 054906 (2012) [arXiv:1112.6405 [nucl-th]].
- W. Rümelin, *SIAM Journal on Numerical Analysis* **19**, No. 3 (Jun., 1982), pp. 604-613

References II

B. Schenke, S. Jeon and C. Gale, Phys. Rev. Lett. **106**, 042301 (2011) [arXiv:1009.3244 [hep-ph]].