Chiral Anomaly Induced Phenomena in QGP: How to Test Them in AA and pA?

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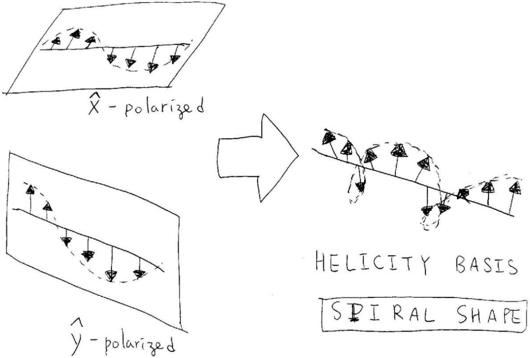
Correlations and Fluctuations in pA and AA Collisions
INT, Seattle, July, 2015

Summary of Chiral Anomaly Induced Transports in QGP

$B = 0$	$B \neq 0$
Chiral Shear Wave ($\mu \neq 0$) Chiral Vortical Effect ($\mu = 0$)	Chiral Magnetic Effect ($\mu \neq 0$) Chiral Magnetic Wave ($\mu = 0$)

Chiral Shear Wave (Sahoo-HUY)

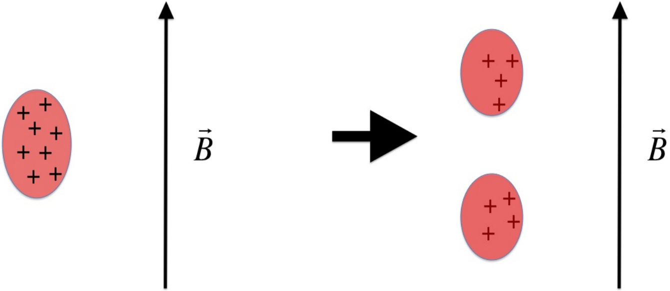


Shear velocity fluctuations δu^i decay as

$$\omega \approx -i\frac{\eta}{4p}k^2 \pm i\frac{\lambda_1}{16p}k^3 + \cdots$$

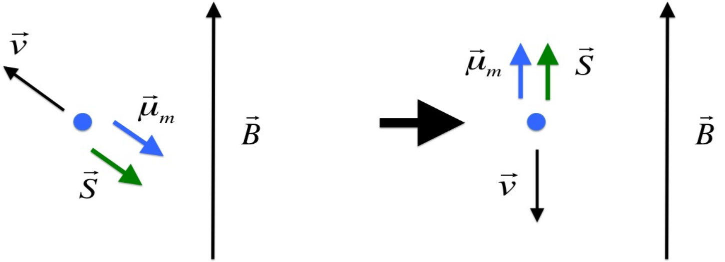
where $T^{\mu\nu}_{(2)} \sim \lambda_1 \Pi^{\mu\nu}_{\alpha\beta}\mathcal{D}^\alpha\omega^\beta$ (Kharzeev-HUY)

Chiral Magnetic Wave (CMW) on Electric Charges



A peculiar motion of electric charges under a magnetic field !!!

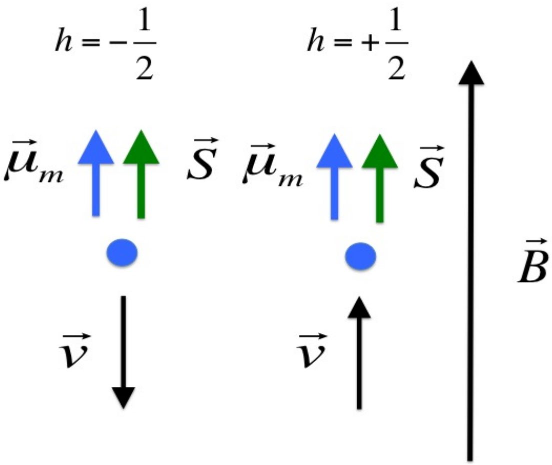
Quantum Picture of Fermionic Charge Carriers in a Magnetic Field (Kharzeev-Warringa)



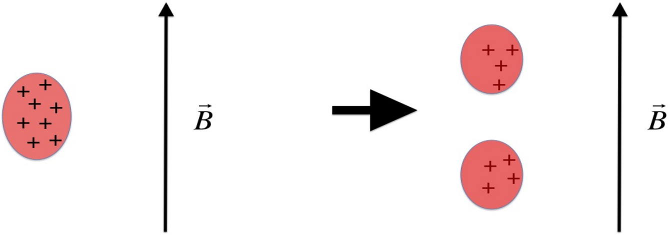
Helicity : $\vec{S} = h\frac{\vec{v}}{|\vec{v}|}$, $\quad h = \pm\frac{1}{2}$ **(Chirality)**

Wigner-Eckart Theorem : $\vec{\mu}_m \propto q\vec{S}$, $\quad q =$ **charge**

Spin Magnetic Moment Interaction : $H = -\vec{\mu}_m \cdot \vec{B}$



A motion along the direction of the magnetic field is induced. Some go up, others go down, depending on their helicity



An intricate interplay of

- **Quantum Spin**
- **Charge and Magnetic Moment**
- **Helicity (Chirality)**

Theoretical Description of CMW
(Kharzeev-HUY)

CMW arises from the interplay of

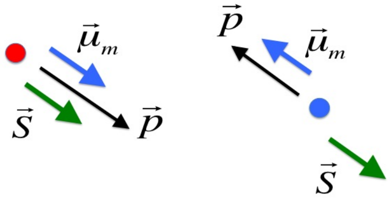
- **Chiral Magnetic Effect :** $\vec{J}_V = \frac{e\mu_A}{2\pi^2}\vec{B}$ (Fukushima-McLerran-Kharzeev-Warringa)
- **Chiral Separation Effect :** $\vec{J}_A = \frac{e\mu_V}{2\pi^2}\vec{B}$ (Son-Zhitnitsky)

The velocity is

$$v_\chi = \frac{eB}{2\pi^2}\frac{1}{\chi}\,, \quad \chi = \text{susceptibility}$$

Quasi-particle picture of CME

Quantized Weyl particles (p) and anti-particles ($\bar{p}$)

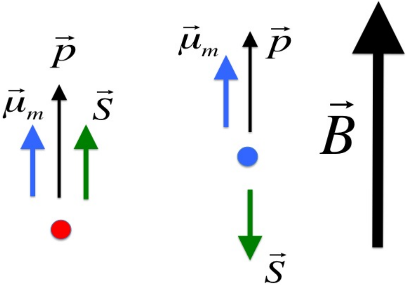


$$Q = +1 \qquad\qquad Q = -1$$

$$h = +\frac{1}{2} \qquad\qquad h = -\frac{1}{2}$$

$$\vec{S} = \pm\frac{1}{2}\frac{\vec{p}}{|\vec{p}|}, \qquad \vec{\mu}_M = \pm\frac{\vec{S}}{|\vec{p}|} = \frac{1}{2}\frac{\vec{p}}{|\vec{p}|^2}$$

Quasi-particle picture of CME



Energy shift in a magnetic field: $\Delta E = -\vec{\mu}_M \cdot \vec{B} = -\frac{1}{2}\frac{\vec{p}\cdot\vec{B}}{|\vec{p}|^2}$

It gives rise to a tendency to align the momentum along the magnetic field direction

Quantitative Understanding of CME

The energy shift $\Delta E = -\frac{1}{2}\frac{\vec{p}\cdot\vec{B}}{|\vec{p}|^2}$ will modify the equilibrium distribution of particles ($f_+^{\rm eq}$) and anti-particles ($f_-^{\rm eq}$)

from

$$f_\pm^{(0)} \equiv \left(\exp[\beta(|\vec{p}| \mp \mu)] + 1\right)^{-1}$$

to

$$\begin{aligned} f_\pm^{\rm eq} &= \left(\exp[\beta(|\vec{p}| - \frac{1}{2}\frac{\vec{p}\cdot\vec{B}}{|\vec{p}|^2} \mp \mu)] + 1\right)^{-1} \\ &\approx f_\pm^{(0)} + \beta f_\pm^{(0)}(1 - f_\pm^{(0)})\frac{\vec{p}\cdot\vec{B}}{2|\vec{p}|^2} + \mathcal{O}(B^2) \end{aligned}$$

The net current is

$$\begin{aligned}\vec{J} &= \int \frac{d^3\vec{p}}{(2\pi)^3}\ \dot{\vec{x}}\ (f_+^{\rm eq} - f_-^{\rm eq}) = \int \frac{d^3\vec{p}}{(2\pi)^3}\ \frac{\vec{p}}{|\vec{p}|}\ (f_+^{\rm eq} - f_-^{\rm eq}) \\ &= \frac{\beta}{2}\int \frac{d^3\vec{p}}{(2\pi)^3}\ \frac{\vec{p}}{|\vec{p}|}\frac{\vec{p}\cdot\vec{B}}{|\vec{p}|^2}\left(f_+^{(0)}(1-f_+^{(0)}) - f_-^{(0)}(1-f_-^{(0)})\right) \\ &= \frac{1}{3}\cdot\frac{1}{4\pi^2}\vec{B}\times\beta\int_0^\infty dp\ p\left(f_+^{(0)}(1-f_+^{(0)}) - f_-^{(0)}(1-f_-^{(0)})\right) \\ &= \frac{1}{3}\cdot\frac{\mu}{4\pi^2}\vec{B}\end{aligned}$$

where

$$\beta\int_0^\infty dp\ p\left(f_+^{(0)}(1-f_+^{(0)}) - f_-^{(0)}(1-f_-^{(0)})\right) = \mu$$

independent of temperature

This contribution from the energy shift explains only $\frac{1}{3}$ of the full result

Identifying the remaining $\frac{2}{3}$ contribution to the CME needs a complete picture of microscopic motions of fermions under a magnetic field

Motion of Weyl Particle in a Magnetic Field

Using the relativistic energy

$$\mathcal{E} = |\vec{p}| - \frac{1}{2}\frac{\vec{p}\cdot\vec{B}}{|\vec{p}|^2}$$

the equation of motion from the action gives

$$\sqrt{G}\;\dot{\vec{x}} \;=\; \frac{\partial\mathcal{E}}{\partial\vec{p}} + \vec{B}\left(\frac{\partial\mathcal{E}}{\partial\vec{p}}\cdot\vec{b}\right) = \frac{\vec{p}}{|\vec{p}|} + \frac{\vec{p}(\vec{p}\cdot\vec{B})}{|\vec{p}|^4} + \mathcal{O}(B^2)$$

where $\sqrt{G} = (1+\vec{B}\cdot\vec{b})$ is the modified phase space measure

The second term is the new velocity from triangle anomaly (Stephanov-Yin)

The current from this new velocity is

$$\begin{aligned}
\vec{J} &= \int \frac{d^3\vec{p}}{(2\pi)^3} \sqrt{G}\, \dot{\vec{x}} \left(f_+^{(0)} - f_-^{(0)}\right) \\
&= \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{\vec{p}(\vec{p}\cdot\vec{B})}{|\vec{p}|^4} \left(f_+^{(0)} - f_-^{(0)}\right) \\
&= \frac{2}{3}\cdot\frac{1}{4\pi^2}\vec{B} \times \int_0^\infty dp \left(f_+^{(0)} - f_-^{(0)}\right) \\
&= \frac{2}{3}\cdot\frac{\mu}{4\pi^2}\vec{B}
\end{aligned}$$

where

$$\int_0^\infty dp \left(f_+^{(0)} - f_-^{(0)}\right) = \mu$$

independent of temperature

Back to CMW: Linearize for small fluctuations:

$$\mu_{V/A} \approx n_{V/A}/\chi$$

(χ: charge susceptibility)

$$\vec{J}_V = \frac{e\vec{B}}{2\pi^2\chi} n_A \,, \quad \vec{J}_A = \frac{e\vec{B}}{2\pi^2\chi} n_V$$

Add and subtract the two equations, and define $J_R = J_V + J_A$ and $J_L = J_V - J_A$

$$\vec{J}_R = \frac{e\vec{B}}{2\pi^2\chi} n_R \,, \quad \vec{J}_L = -\frac{e\vec{B}}{2\pi^2\chi} n_L$$

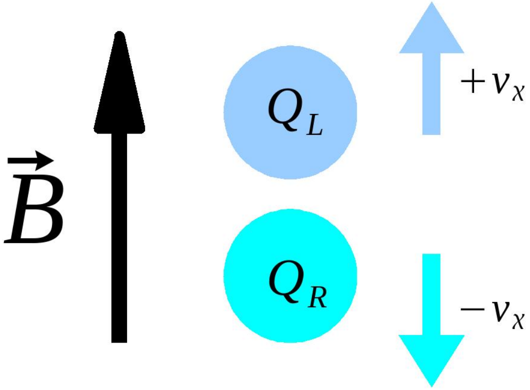
Conservation equation $\dot{n}_{R/L} + \vec{\nabla}\cdot\vec{J}_{R/L} = 0$ gives

$$(\partial_t + \vec{v}\cdot\vec{\nabla})n_R = 0 \,, \quad (\partial_t - \vec{v}\cdot\vec{\nabla})n_L = 0$$

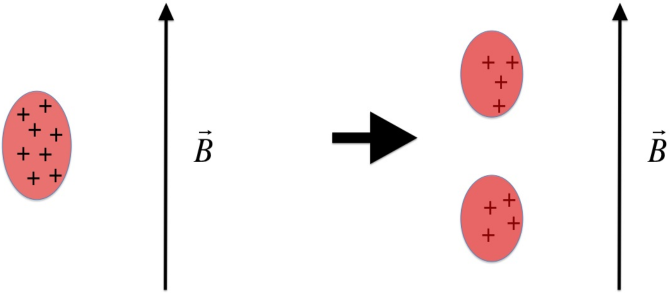
with velocity $\vec{v} = \frac{e\vec{B}}{2\pi^2}\frac{1}{\chi}$:

Two modes moving to opposite directions along the magnetic field!

Charge Splitting from CMW

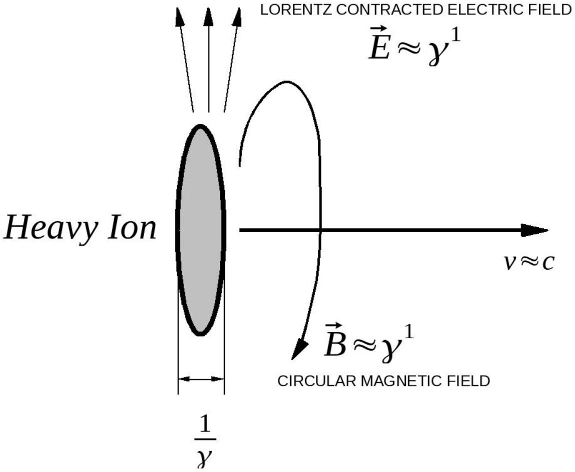


Charge Splitting from CMW

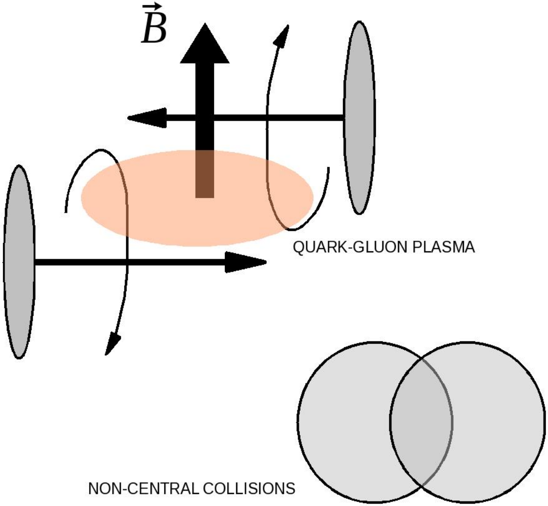


Initial vector (electric) charge is $n_V = n_R + n_L$

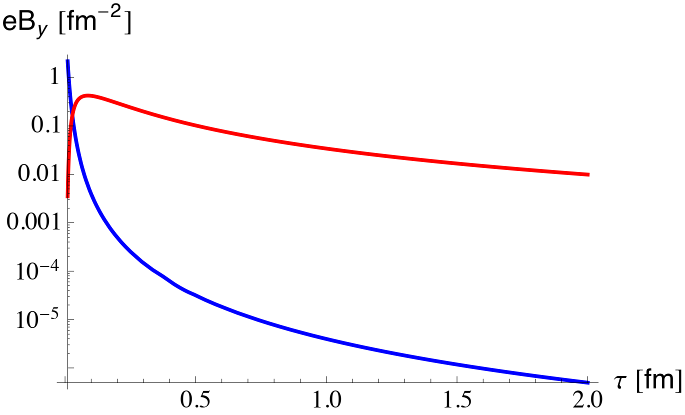
Magnetic field in heavy-ion collisions



In non-central collisions, two magnetic fields overlap along the same direction out of reaction plane



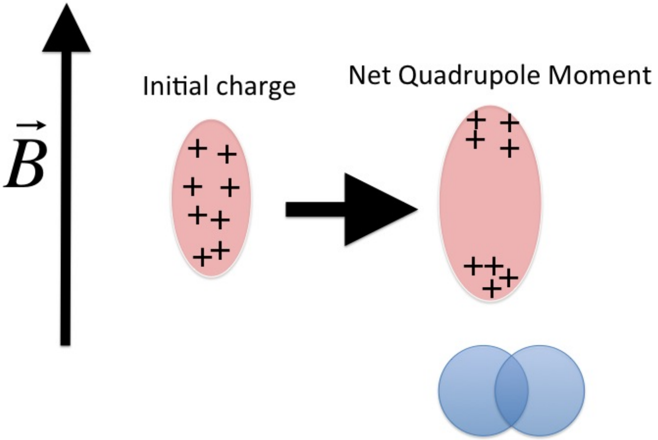
Magnetic Field in QGP Medium



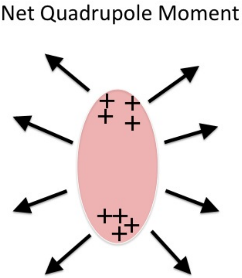
(Gursoy-Kharzeev-Ragagopal)

Faraday Effect with equilibrium QGP conductivity, which can be questioned

CMW in Heavy Ion Collisions
(Burnier-Kharzeev-Liao-HUY)

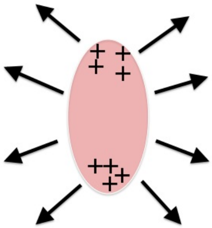


CMW in Heavy Ion Collisions



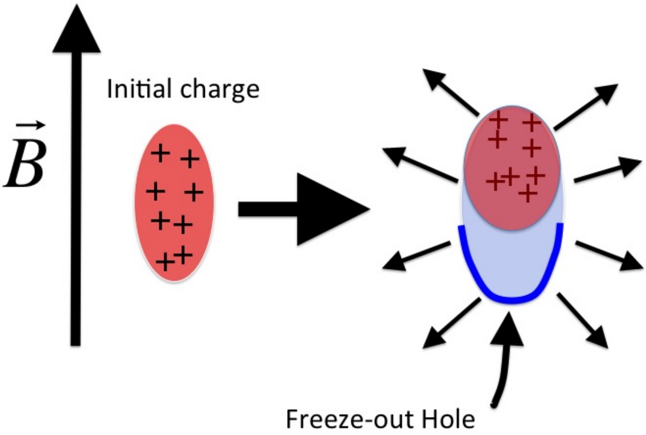
This leads to charge dependent elliptic flows of pions $v_2(\pi^+) < v_2(\pi^-)$ if the initial charge is positive $A_\pm > 0$

CMW in Heavy Ion Collisions

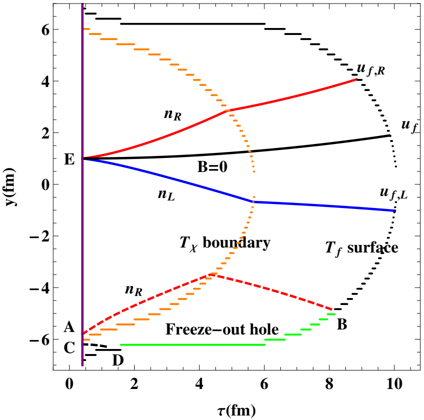


Prediction $\Delta v_2 \equiv v_2(\pi^-) - v_2(\pi^+) = r\ A_\pm\,, \quad A_\pm \equiv \frac{N_+ - N_-}{N_+ + N_-}$

Freeze-out Hole Effect (HUY-Yin):
A significant effect on the slope r



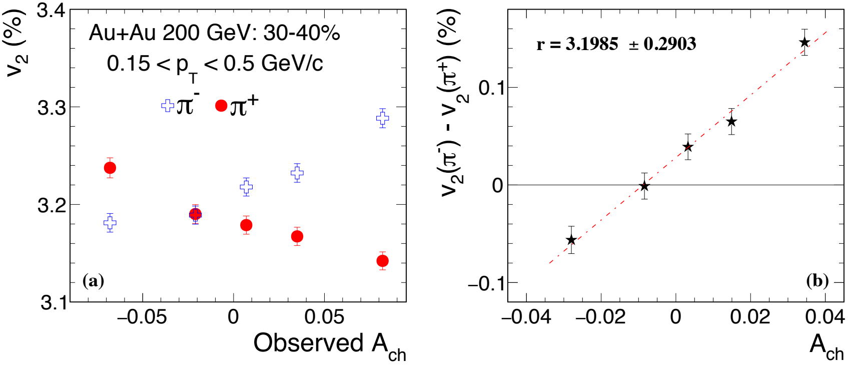
Freeze-out Hole Effect: A significant effect on the slope r



What it looks like in realistic simulations (HUY-Yin)

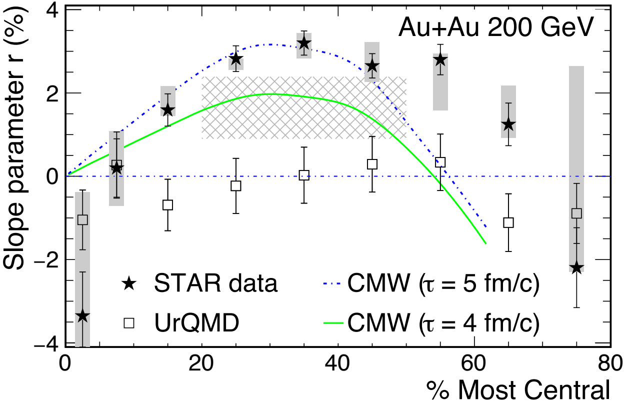
Confirmation of linear dependency at RHIC

(**Phys.Rev.Lett. 114 (2015) 25, 252302**)



$\Delta v_2 = r\ A_\pm + \Delta v_2^0$: **Note the intercept** Δv_2^0 **(Stephanov-Yee)**

Centrality Dependence of The Slope *r*



The colored curves are from the simulations
(Burnier-Kharzeev-Liao-HUY)

Other Sources for The Slope r?

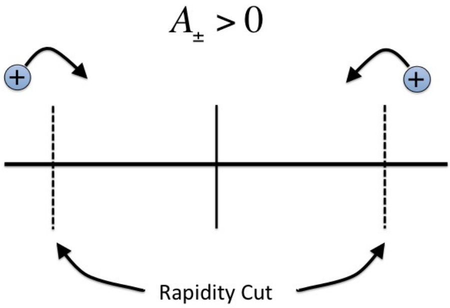
In the equation

$$\Delta v_2 = r\ A_\pm + \Delta v_2^0$$

both Δv_2 and $A_\pm$ are charge conjugation (C) odd, so that the slope r is C even.

This means that r can receive contributions from other sources unrelated to the magnetic field

Example: Bzdak-Bozek Scenario



Since v_2 drops sharply for a larger rapidity, the $v_2(\pi^+)$ will be relatively smaller than $v_2(\pi^-)$

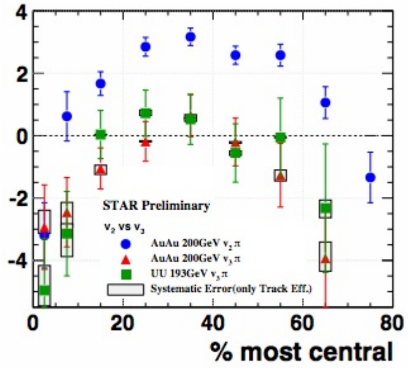
Can explain the 1/3 of the slope

Distinction in Δv_3?

The slope in $\Delta v_3 \equiv v_3(\pi^-) - v_3(\pi^+) = r_3 \ A_\pm$

CMW: $r_3 \approx 0$

Bzdak-Bozek: $r_3 \approx r \times \frac{v_3}{v_2} \approx r \times \frac{1}{3}$

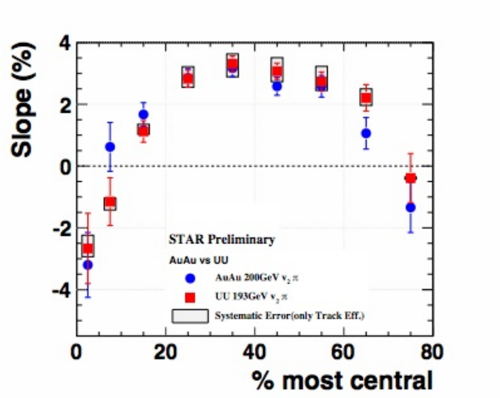

STAR Preliminary
v2 vs v3
AuAu 200GeV v2 π
AuAu 200GeV v3 π
UU 193GeV v3 π
Systematic Error(only Track Eff.)
4
2
0
-2
-4
0
20
40
60
80
% most central

Distinction in U-U Collisions?

What should be most interesting is near zero centrality

CMW: No magnetic field, so $r \approx 0$

Bzdak-Bozek: v_2 is finite even at zero centrality, so r should be non-zero



The slope *r* becomes negative near zero centrality:
no current theoretical understanding

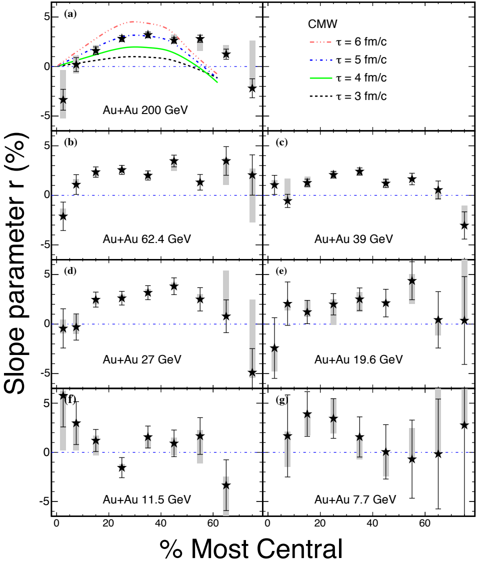
Other Background?: Mean-Field Potential

(C. M. Ko, *et al*)

Could explain $p - \bar{p}$ and $K^+ - K^-$ roughly, but its effect for $\pi^+ - \pi^-$ turns out to be too small

Beam energy Scan at RHIC

(Phys.Rev.Lett. 114 (2015) 25, 252302)



In Summary:

No overwhelming background has been identified. Experiments seem to be in line with CMW. Need more tests

Chiral Vortical Effect

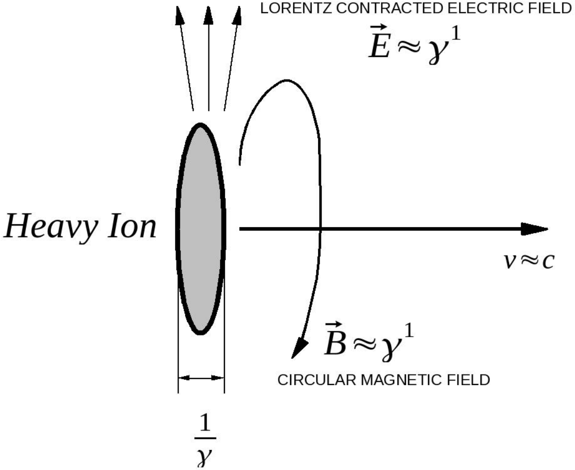
$$\vec{J}_V = \frac{e\mu_A}{2\pi^2}\,\vec{B} + \frac{\mu_A\mu_V}{\pi^2}\,\vec{\omega}$$

$\vec{\omega} = (1/2)\vec{\nabla} \times \vec{u}$ **is the fluid vorticity**

Non-zero v_1 can give a vorticity in off-central collisions, which can induce baryon charge transport by CVE:

Signal in $\Lambda^0 - \bar{\Lambda}^0$ baryon separation
(Kharzeev-Son)

pA Collisions ?



Magnetic field still exists with half a magnitude compared to AA

$\vec{B}$ orientation and magnitude are fluctuating, but since what has been measured in the CMW observable is $v_2\{2\}$ and $v_2\{4\}$, we should still see some effect if there is CMW effect of charge splitting along $\vec{B}$

However, the orientation of $\vec{B}$ is uncorrelated to the fluctuating v_2, so the net effect on the previous observable is expected to disappear

pA does not have vorticity, so CVE should disappear in pA. This should be also a good place for testing CVE

Thank you very much for listening