Thermal noise and Gubser flow

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Correlations and Fluctuations in p+A and A+A Collisions

with Hanna Grönqvist and Jean-Yves Ollitrault

Outline of this talk

- A brief introduction – thermal noise in heavy-ion collisions
- Theoretical formulation of thermal fluctuations in a thermal system
  - Thermal fluctuations in a fluid system and heavy-ion collisions.
- Thermal noise and hydrodynamics with analytical solutions :
  - 1+1D Bjorken hydro.(**Kapusta,Müller and Stephanov**)
  - 2+1D Gubser hydro.
- Applications to A+A, p+A and p+p .

Thermal fluctuations in heavy-ion collisions

- Thermal fluctuations – Physical quantities fluctuate around mean values
  - Thermal fluctuations are general for systems with $T > 0$.
  - Thermal fluctuations are related to dissipations – fluctuation-dissipation

$$\eta/s \sim O(1/4\pi)$$

  - Thermal fluctuations are more significant in small systems.

from Pb+Pb and Au+Au to p+Pb, d+Au, ^{3}He+Au, p+p

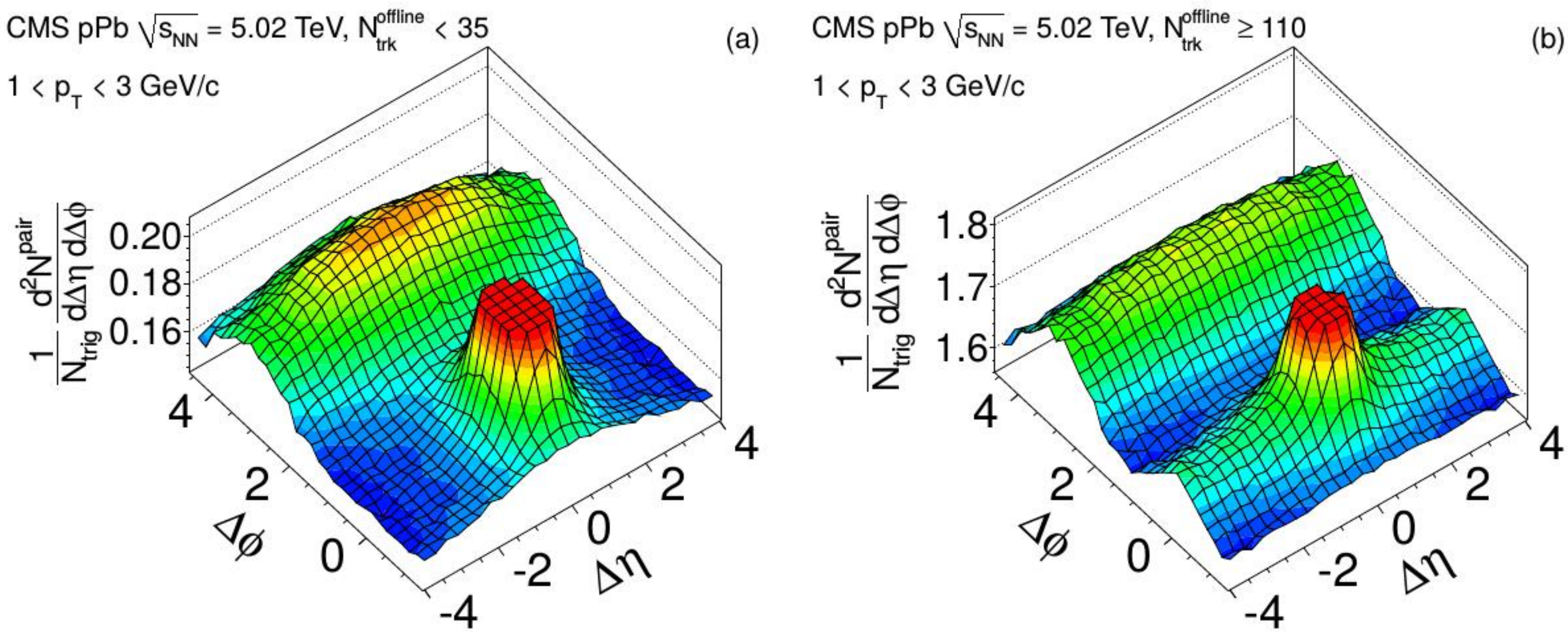
- Effective modeling heavy-ion collisions with hydro.,
  * Initial state fluctuations – hydro. event w.r.t. each initial state

$\Rightarrow$ Event-by-event hydro.

  * Thermal fluctuations – ensemble average for each initial state

Why thermal fluctuations might be important?

- Modeling medium collectivity in small colliding systems – hydro.



  * convergence of gradient expansion – Knudsen number
  * inclusion of thermal fluctuations

- A preliminary analysis of the effect of thermal fluctuations
  * How significant is the effect of thermal fluctuations in heavy-ion collisions?
  * In particular, effect of thermal fluctuations in small colliding systems?

Hydro. and thermal fluctuations in a fluid system

- Hydrodynamics – conservation of energy-momentum, etc.

$$T^{\mu\nu} = T^{\mu\nu}_{\text{ideal}} + \Pi^{\mu\nu} \qquad \leftrightarrow \qquad d_\mu T^{\mu\nu} = 0 \quad \text{(Euler and equ. of continuity)}$$

especially from hydro EoM, one recognizes that

$$d_\mu(su^\mu) = -\frac{1}{T}\nabla_{(\mu}u_{\nu)}\Pi^{\mu\nu} \quad \rightarrow \quad \frac{dS}{dt} = -\int d^3x \frac{1}{T}\nabla_{(\mu}u_{\nu)}\Pi^{\mu\nu}$$

Navier-Stokes hydro. (1st order):

$$\Pi^{\mu\nu} = -\eta\left[\frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}(d_\alpha u_\beta + d_\beta u_\alpha) - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\alpha\beta}d_\alpha u_\beta\right] = -\eta\sigma^{\mu\nu}$$

where

$$\Delta^{\mu\nu} = u^\mu u^\nu + g^{\mu\nu}$$

* We ignore bulk viscosity ζ in our work.

Hydro. and thermal fluctuations in a fluid system

- EoM of a set of physical quantities $\{x_a\}$ in a thermal system
  (**Landau and Lifshitz. J.Kapusta et al.**)

$$\dot{x}_a = \underbrace{-\sum_b \gamma_{ab} X_b}_{\text{drag}} + \underbrace{y_a}_{\text{fluc.}} \quad \Leftrightarrow \quad \dot{S} = -\sum_a \dot{x}_a X_a$$

$$\text{Maximization of S} \Rightarrow \quad \langle y_a(t_1) y_b(t_2) \rangle = (\gamma_{ab} + \gamma_{ba}) \delta(t_1 - t_2)$$

- Auto-correlations of thermal noise : fluctuation-dissipation
- For hydro., γ_{ab} is determined then by identifying

$$\dot{x} \to \Pi^{\mu\nu} \text{ and } X \to \frac{\Delta V}{T} \nabla_{(\mu} u_{\nu)}, \qquad \left(\frac{dS}{dt} = -\int d^3x \frac{1}{T} \nabla_{(\mu} u_{\nu)} \Pi^{\mu\nu} \right)$$

- Remarks:
  1. White noise – delta function $\delta() \sim \frac{1}{\Delta V \Delta t}$.
  2. Form of γ_{ab} corresponds to the detailed form of $\Pi^{\mu\nu}$, $\gamma_{ab} \sim$ dissipations.
  3. Noise y_a corresponds to thermodynamical quantity $\dot{x}_a$, also for $S^{\mu\nu}$ and $\Pi^{\mu\nu}$.

- Hydrodynamics with thermal noise (Navier-Stokes hydro):

(**Landau and Lifshitz, J. Kapusta et. al. and A. Kumar et al.**)

$$T^{\mu\nu} = T^{\mu\nu}_{\rm ideal} + \Pi^{\mu\nu} + S^{\mu\nu}$$

where thermal fluctuation tensor $S^{\mu\nu}$ are introduced w.r.t. $\Pi^{\mu\nu}$

$$\langle S^{\mu\nu}(x)\rangle = 0$$

$$\text{Navier-Stokes: } \langle S^{\mu\nu}(x_1) S^{\alpha\beta}(x_2)\rangle = 4T\eta\Delta^{\mu\nu\alpha\beta}\delta^{(4)}(x_1 - x_2)$$

where

$$\Delta^{\mu\nu\alpha\beta} = \frac{1}{2}\left[\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}\right] - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\alpha\beta}, \text{ and } \Delta^{\mu\nu\alpha\beta} d_\alpha u_\beta = \sigma^{\mu\nu}$$

* One-point functions of physical quantities are not affected.
* Thermal noise affects two-point correlations.

Solving hydro. with thermal noise

- Linearized hydro. EoM with thermal fluctuations around $d_\mu T_0^{\mu\nu} = 0$

$$\begin{aligned} T(x) =& T_0(x) + \delta T(x) \\ \varepsilon(x) =& \varepsilon_0(x) + \delta\varepsilon(x) \\ \mathcal{P}(x) =& \mathcal{P}_0(x) + \delta\mathcal{P}(x) \\ u^\mu(x) =& u_0^\mu(x) + \delta u^\mu(x) \end{aligned}$$

  therefore $d_\mu \delta T^{\mu\nu} = 0 \sim O(\delta)$,

$$\delta w D u_\alpha + w\delta u^\mu d_\mu u_\alpha + (Dw + w\partial\cdot u)\delta u_\alpha + \nabla_\alpha \delta\mathcal{P} + wD\delta u_\alpha + d_\mu(\delta\Pi^\mu_\alpha + S^\mu_\alpha) = 0$$
$$D\delta\varepsilon + \delta w \partial\cdot u + d_\mu(w\delta u^\mu) + w\delta u^\alpha D u_\alpha - u^\alpha d_\mu(\delta\Pi^\mu_\alpha + S^\mu_\alpha) = 0$$

  Note that $\delta\Pi^{\mu\nu}$ is induced by δT, etc. (**C. Young, J. Kapusta et al.**)

- Beyond linear order, thermal noise becomes large, e.g., phase transition.

Hydro. with symmetry simplification – Bjorken hydro.

- Bjorken hydro., 1+1D (Bjorken, 1982)
  - Bjorken boost – indep. of spatial rapidity ξ, so that in (τ, ξ) space-time,

$$ds^2 = -d\tau^2 + \tau^2 d\xi^2 \quad \rightarrow \quad u^\mu = (1, 0) \qquad \text{fluid at rest}$$

  - Only one equation left non-trivial – equation of continuity:

$$D\varepsilon + (\varepsilon + \mathcal{P})\nabla \cdot u + \nabla_{(\mu} u_{\nu)} \Pi^{\mu\nu} = 0$$

  - For Navier-Stokes (1st order) hydro., with conformal EoS $\varepsilon = 3\mathcal{P}$,

$$\varepsilon(\tau) = \varepsilon_0 \left(\frac{\tau_0}{\tau}\right)^{4/3} \left[1 - \frac{H_0}{2\tau\varepsilon_0^{1/4}} \left(\frac{\tau}{\tau_0}\right)^{1/3}\right]^4$$

where constant $H_0 \propto \eta/s$ is used to parameterize η.

Hydro. with symmetry simplification – Gubser hydro.

- Gubser hydro., 2+1D (**Gubser and Yarom, 2010**)
  - Bjorken boost – indep. of spatial rapidity ξ
  - Rotational symmetry w.r.t. to beam axis – for p+A and ultra-central A+A
- Change coordinates:
  - Via Weyl transformation, $g_{\mu\nu} \to \tilde{g}_{\mu\nu} = g_{\mu\nu}/\tau^2$

$$\mathbf{R}^{1,3} \Rightarrow dS_3 \times \mathbf{R} : \qquad d\tilde{s}^2 = \frac{1}{\tau^2}[-d\tau^2 + d\vec{x}_\perp^2] + d\xi^2 .$$

  - Reparameterize dS_3 by the mapping $(r, \tau) \leftrightarrow (\rho, \theta)$:

$$\sinh\rho = -\frac{1 - q^2\tau^2 + q^2 r^2}{2q\tau}$$

$$\tan\theta = \frac{2qr}{1 + q^2\tau^2 - q^2 r^2}$$

so that symmetry $SO(1,1) \times SO(3) \times \mathcal{Z}_2$ are now manifest

$$d\hat{s}^2 = -d\rho^2 + d\xi^2 + \cosh^2\rho \left(d\theta^2 + \sin^2\theta d\phi\right) .$$

ρ plays the role of 'time' in the 'hat' coordinate system.

- $d\hat{s}^2 = -d\rho^2 + d\xi^2 + \cosh^2\rho\left(d\theta^2 + \sin^2\theta d\phi\right)$ leads to allowed velocity profile

$$\hat{u}^\mu = (1, 0, 0, 0) \qquad \text{fluid at rest}$$

- Only one equation left non-trivial – equation of continuity:

$$D\hat{\varepsilon} + (\hat{\varepsilon} + \hat{\mathcal{P}})d_\mu \hat{u}^\mu + \nabla_{(\mu}\hat{u}_{\nu)}\hat{\Pi}^{\mu\nu} = 0$$

- For Navier-Stokes (1st order) hydro., with conformal EoS $\varepsilon = 3\mathcal{P}$,

$$\hat{\varepsilon}(\rho) = (\cosh\rho)^{-\frac{4}{3}}\left[\hat{T}_0 + \frac{1}{3}H_0 F_d(\rho)\right]^4$$

- To recover quantities in the original coordinates,

$$\begin{aligned}
\epsilon &= \tau^{-d}\hat{\epsilon} \\
u_\tau &= \tau\left(\frac{\partial\rho}{\partial\tau}\hat{u}_\rho + \frac{\partial\theta}{\partial\tau}\hat{u}_\theta\right) \\
u_\perp &= \tau\left(\frac{\partial\rho}{\partial\vec{x}_\perp}\hat{u}_\rho + \frac{\partial\theta}{\partial\vec{x}_\perp}\hat{u}_\perp\right) \\
&\ldots
\end{aligned}$$

- $\hat{T}_0 \to$ multiplicity, $q \to$ transverse size.

Bjorken hydro. and thermal noise

- Correlation of thermal noise in 1+1D Bjorken hydro,. (J. Kapusta et. al., 2012)

$$\langle S^{\mu\nu}(\tau_1,\xi_1)S^{\alpha\beta}(\tau_2,\xi_2)\rangle = \frac{8}{3\tau_1 A_\perp} T\eta\Delta^{\mu\nu}\Delta^{\alpha\beta}\delta(\tau_1-\tau_2)\delta(\xi_1-\xi_2)$$

1. $\delta(\vec{x}_{1\perp}-\vec{x}_{2\perp}) \to A_\perp$ characterizes transverse size of the system.
2. Tensor structure of $S^{\mu\nu}$ is factorized, due to the fact that $u^\mu=(1,0)$.

$$S^{\mu\nu} = w(\tau)f(\tau,\xi)\Delta^{\mu\nu},$$

   such that the unknown scalar and dimensionless function $f(\tau,\xi)$

$$\begin{aligned}\langle f(\tau_1,\xi_1)f(\tau_2,\xi_2)\rangle &= \frac{8T(\tau_1)\eta(\tau_1)}{3A_\perp w^2(\tau_1)\tau_1}\delta(\tau_1-\tau_2)\delta(\xi_1-\xi_2)\\ &= \frac{2\nu}{A_\perp w(\tau_1)\tau_1}\delta(\tau_1-\tau_2)\delta(\xi_1-\xi_2) \qquad \text{with } \nu=\frac{4}{3}\frac{\eta}{s}\end{aligned}$$

3. Magnitude of thermal noise is constrained by (in addition to η/s):

$$(A_\perp w(\tau)\tau) \sim A_\perp\left(\frac{dE}{\tau d^2x_\perp d\xi}\right)\tau \sim \frac{dE_\perp}{dy} \sim \text{multiplicity}$$

Multiplicity more crucial than system size.

Bjorken hydro. and thermal noise

- EoM of thermal fluctuations of modes (in the conjugate space of ξ):

$$d_\mu \delta T^{\mu\nu} = 0 \quad \rightarrow \quad \tilde{\mathcal{V}}'(\tau, k_\xi) = -\tilde{\Gamma}(\tau, k_\xi)\tilde{\mathcal{V}}(\tau, k_\xi) + \tilde{\mathcal{K}}(\tau, k_\xi),$$

Prime denotes $\tau\partial_\tau$, and

$$\tilde{\mathcal{V}}(\tau) = \begin{pmatrix} \tilde{n}(\tau, k_\xi) \\ \tilde{\alpha}(\tau, k_\xi) \end{pmatrix}, \qquad \tilde{n} = \frac{\delta\tilde{s}}{\tilde{s}} \quad \text{and} \quad \tilde{\alpha} = \delta u_\xi/\tau$$

and

$$\tilde{\mathcal{K}}(\tau) = \begin{pmatrix} -\tilde{f} \\ -ik_\xi \tilde{f} \end{pmatrix}, \qquad \tilde{f}(\tau, k_\xi) = \int d\xi e^{ik_\xi \xi} f(\tau, \xi)$$

and

$$\tilde{\Gamma}(\tau) = \begin{pmatrix} 0 & ik_\xi \\ ik_\xi c_s^2 & 1 - c_s^2 \end{pmatrix} + \frac{\nu}{T\tau} \begin{pmatrix} c_s^2 & -ik_\xi \\ -ik_\xi & 1 + c_s^2 + k_\xi^2 \end{pmatrix}$$

1. Coupled EoMs in 1+1D.
2. Thermal noises for u_ξ mode are larger for higher order (larger k_ξ) modes.
3. Langevin-type, but $\langle \tilde{\mathcal{K}}\tilde{\mathcal{K}} \rangle$ is NOT directly related to $\tilde{\Gamma}$ as in Langevin process.
4. Can be solved numerically, and analytically in some special limits, e.g., $k_\xi \rightarrow 0$.

Gubser hydro. and thermal noise

- Correlation of thermal noise in 2+1D Gubser hydro., $(X \to (\rho, \theta, \phi, \xi))$

$$\langle \hat{S}^{\mu\nu}(\rho_1, \theta_1, \phi_1, \xi_1)\hat{S}^{\alpha\beta}(\rho_2, \theta_2, \phi_2, \xi_2)\rangle = \frac{2\nu\hat{T}\hat{s}}{\cosh^2\rho_1 \sin\theta_1}\hat{\Delta}^{\mu\nu}\hat{\Delta}^{\alpha\beta}\delta(X_1 - X_2)$$

1. Tensor structure of $\hat{S}^{\mu\nu}$ is factorized, due to $\hat{u}^\mu = (1,0,0,0)$.

$$\hat{S}^{\mu\nu}(\rho, \theta, \phi, \xi) = \hat{w}(\rho)\hat{f}(\rho, \theta, \phi, \xi)\hat{\Delta}^{\mu\nu}\,,$$

   and again we have the correlation of scalar function

$$\langle \hat{f}(\rho_1, \theta_1, \phi_1, \xi_1)\hat{f}(\rho_2, \theta_2, \phi_2, \xi_2)\rangle = \frac{2\nu}{\hat{w}\cosh^2\rho_1 \sin\theta_1}\delta(X_1 - X_2)$$

2. For scalar function $\hat{f}(X)$, mode decomposition w.r.t. SO(3) symmetry leads to

$$\text{scalar modes: } \hat{f}(\rho, \theta, \phi, \xi) = \sum h(\rho)Y_{lm}(\theta, \phi)e^{ik_\xi \xi}$$

   and

$$\langle h(\rho_1)h(\rho_2)\rangle = \frac{2\nu}{\hat{w}\cosh^2\rho_1}\delta(\rho_1 - \rho_2)$$

3. Magnitude of thermal noise is constrained by

$$\hat{w} \sim \hat{T}_0 \sim \text{multiplicity}$$

Multiplicity more crucial than system size.

Gubser hydro. and thermal noise

- Decompose thermal fluctuations into modes – scalar and vector modes:

$$\delta \hat{T} = \hat{T} \sum \delta_l(\rho) Y_{lm}(\theta,\phi) e^{ik_\xi \xi}$$
$$\delta u_i = \sum \left[v_{ls}(\rho) \partial_i Y_{lm}(\theta,\phi) e^{ik_\xi \xi} + v_{lv}(\rho) \Phi_{i(lm)}(\theta,\phi) e^{ik_\xi \xi} \right]$$
$$\delta u_\xi = \sum v_{l\xi}(\rho) Y_{lm}(\theta,\phi) e^{ik_\xi \xi}$$

- EoM of each mode,

$$\tilde{\mathcal{V}}_l'(\rho) = -\tilde{\Gamma}(\rho, l, k_\xi) \tilde{\mathcal{V}}_l(\rho) + \tilde{\mathcal{K}}(\rho, k_\xi),$$

where prime denotes derivative w.r.t. ρ

$$\tilde{\mathcal{V}}_l(\rho) = \begin{pmatrix} \delta_l(\rho) \\ v_{ls}(\rho) \\ v_{l\xi}(\rho) \\ v_{lv}(\rho) \end{pmatrix}, \qquad \tilde{\Gamma} \text{ is a } 4\times 4 \text{ matrix}, \qquad \tilde{\mathcal{K}} = \begin{pmatrix} -\frac{2}{3}\tanh\rho\, h(\rho) \\ \frac{2\hat{T}}{3\hat{T}'}\tanh\rho\, h(\rho) \\ -\frac{ik_\xi \hat{T}}{\hat{T}+H_0\tanh\rho} h(\rho) \\ 0 \end{pmatrix}$$

1. Coupled EoMs in 3+1D.
2. Thermal noises for u_ξ mode are larger for higher order (larger k_ξ) modes.
3. Vector modes are decoupled, and NOT affected by thermal noise.
4. Can be solved numerically, and simplified in some special limits, e.g., $k_\xi \to 0$.

Analytical solution of the modes

- EoM of thermal noise, (for Bjorken take $\rho \to \ln(\tau/\tau_0)$.)

$$\tilde{\mathcal{V}}'(\rho) = -\tilde{\Gamma}(\rho, l, k_\xi)\tilde{\mathcal{V}}(\rho) + \tilde{\mathcal{K}}(\rho, k_\xi),$$

has formal solutions, $(K \to (k_\xi, l, m))$

$$\tilde{\mathcal{V}}(\rho, K) = \underbrace{\int_{\rho_0}^{\rho} d\rho' \tilde{\mathcal{G}}(\rho - \rho', K)\tilde{\mathcal{K}}(\rho', K)}_{\text{thermal fluct.}} + \underbrace{\tilde{\mathcal{G}}(\rho - \rho_0, K)\tilde{\mathcal{V}}(\rho_0, K)}_{\text{initial fluct.}},$$

with Green function (determined by hydro. evolution)

$$\tilde{\mathcal{G}}(\rho - \rho', K) = \exp\left[-\int_{\rho'}^{\rho} d\rho'' \tilde{\Gamma}(\rho'', K)\right]$$

- One-point function (linear response),

$$\langle \tilde{\mathcal{V}}(\rho, K)\rangle = \tilde{\mathcal{G}}(\rho - \rho_0, K)\langle \tilde{\mathcal{V}}(\rho_0, K)\rangle$$

NOT affected by thermal fluctuations.

- Two-point function,

$$\langle\tilde{\mathcal{V}}_i(\rho,K)\tilde{\mathcal{V}}_j(\rho,K')\rangle = \underbrace{\int_{\rho_o}^{\rho} d\rho' \left(\tilde{\mathcal{G}}(\rho-\rho',K)\Lambda_{th}(\rho')\tilde{\mathcal{G}}^T(\rho-\rho',K')\right)_{ij}\delta(K+K')}_{\text{thermal fluct.}}$$
$$+\underbrace{\left(\tilde{\mathcal{G}}(\rho-\rho_0,K)\Lambda_{ini}(\rho_0)\tilde{\mathcal{G}}^T(\rho-\rho_0,K')\right)_{ij}\delta(K+K')}_{\text{initial fluct.}}\,.$$

* Dirac delta is given by two-point correlations of thermal and initial fluct.,

$$\text{e.g. } \delta(K+K') \sim \delta(k_\xi + k'_\xi)\delta_{ll'}\delta_{m,-m'}(-1)^m \quad \leftrightarrow \quad \delta(\vec{x}_\perp - \vec{x}'_\perp)\delta(\xi-\xi')$$

* Λ_{ini} and Λ_{th} are matrices characterizing the strength of correlations.

$$\Lambda_{ini} \sim \frac{1}{N} \qquad (\text{assuming } \delta s/s \sim 1/\sqrt{N})$$
$$\Lambda_{th} \sim \frac{\eta}{s}$$

* 1+1D Bjorken hydro.: $k_\xi = 0$ mode and small η/s limit ($\tilde{n} \sim \delta s/s$)

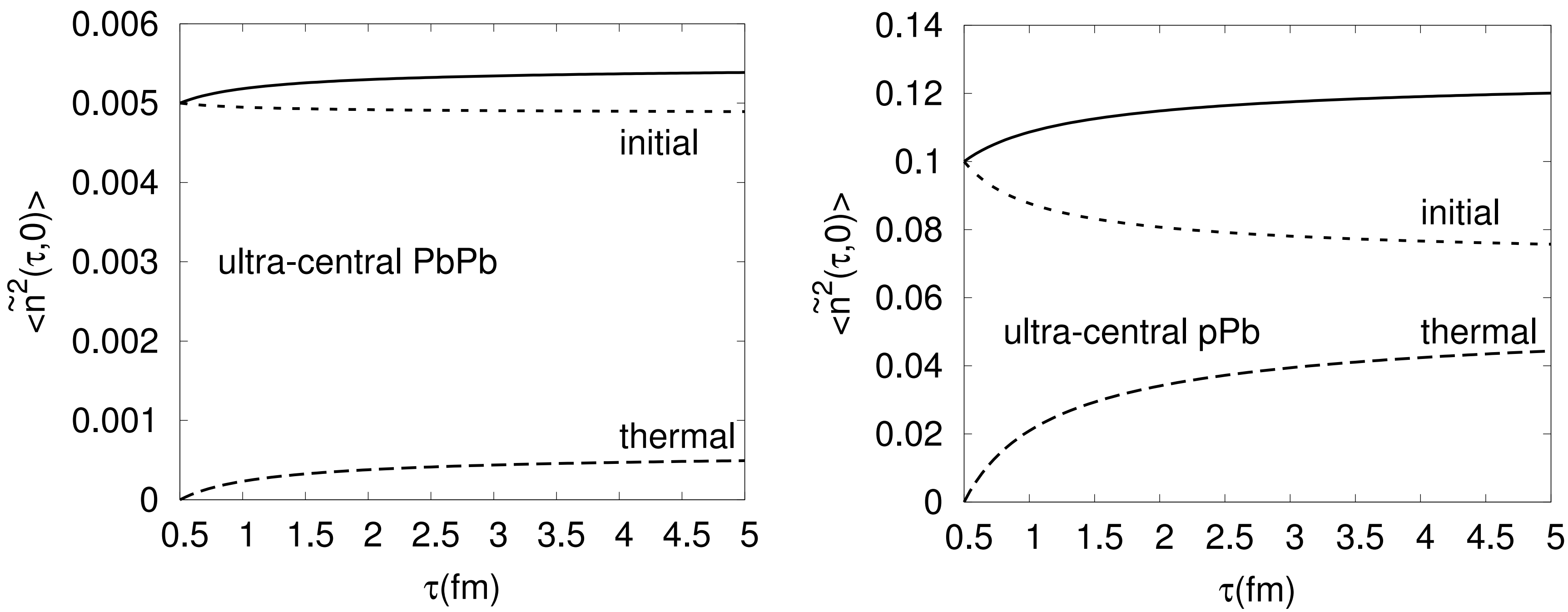
$$\langle \tilde{n}(\rho,0)^2 \rangle = \int_{\rho_0}^{\rho} d\rho' (\tilde{\mathcal{G}}\Lambda_{th}\tilde{\mathcal{G}}^T)_{11} + (\tilde{\mathcal{G}}\Lambda_{ini}\tilde{\mathcal{G}}^T)_{11}$$

$$= \frac{9\pi\nu}{2A_\perp \varepsilon_0 \tau_0^2}\left[1-\left(\frac{\tau_0}{\tau}\right)^{2/3}\right] + \frac{1}{N}\exp[-2D_1(\tau-\tau_0)c_s^2]$$

where

$$D_1(\tau-\tau_0) = \frac{3\nu}{2T_0\tau_0}\left[1-\left(\frac{\tau_0}{\tau}\right)^{2/3}\right] + O(\nu^2)$$

* Naïve estiamte for ultra-central PbPb and pPb:



Apply Gubser hydro. with thermal noise to heavy-ion collisions

- Ultra-central Pb-Pb, p-Pb and p-p.
- Approximates system evolution of first several fm's
  - conformal symmetry, e.g., $\varepsilon = 3\mathcal{P}$.
  - linearized hydro. EoM, treat noises as perturbations.
- $k_\xi = 0$ mode
  - Long rapidity range correlations, affected also by initial fluctuations.
  - Further simplification with v_ξ modes decoupled $\rightarrow$ 2 coupled equations.
- $\hat{T}_0$ and q determine the system.

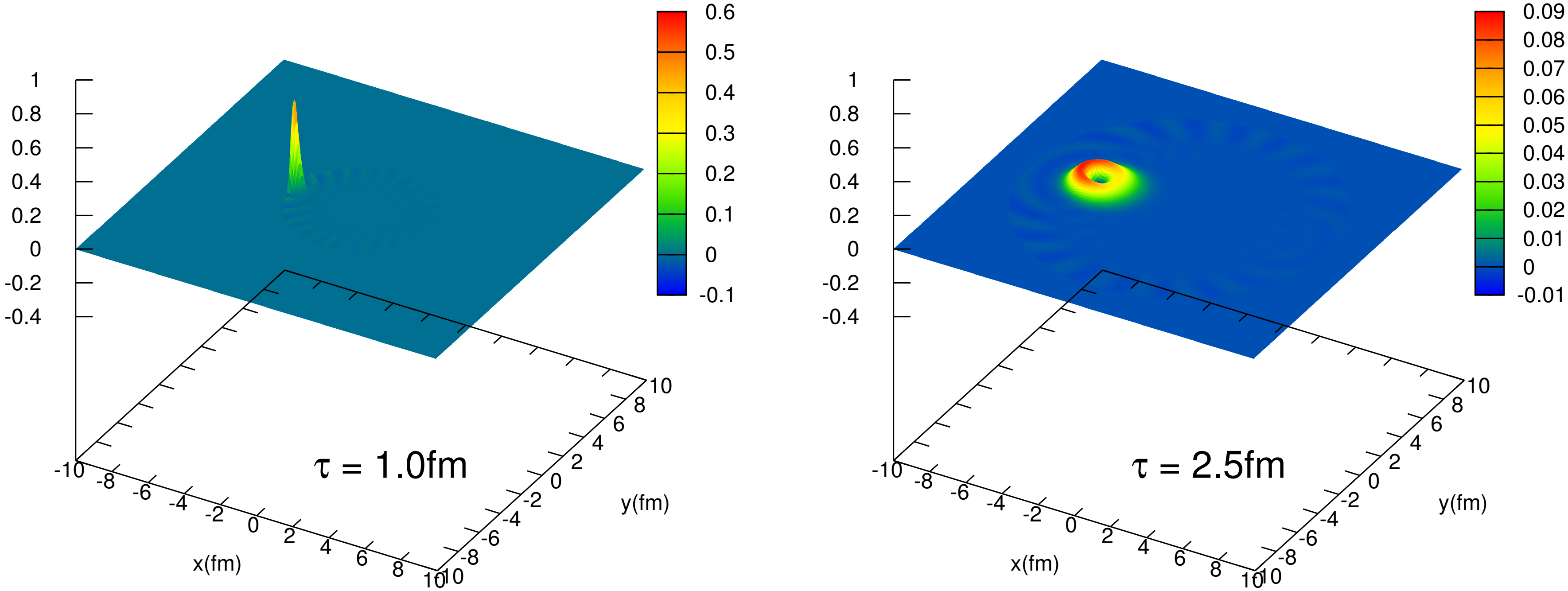
	PbPb	pPb	pp
$\hat{T}_0$	7.3	3.1	1.7
$q^{-1}(fm)^{-1}$	4.3	1.1	1.1

- Initial fluctuations: Gaussian, Dirac delta.

$$\text{magnitude} \sim 1/\sqrt{N}$$

Evolution of a Gaussian profile

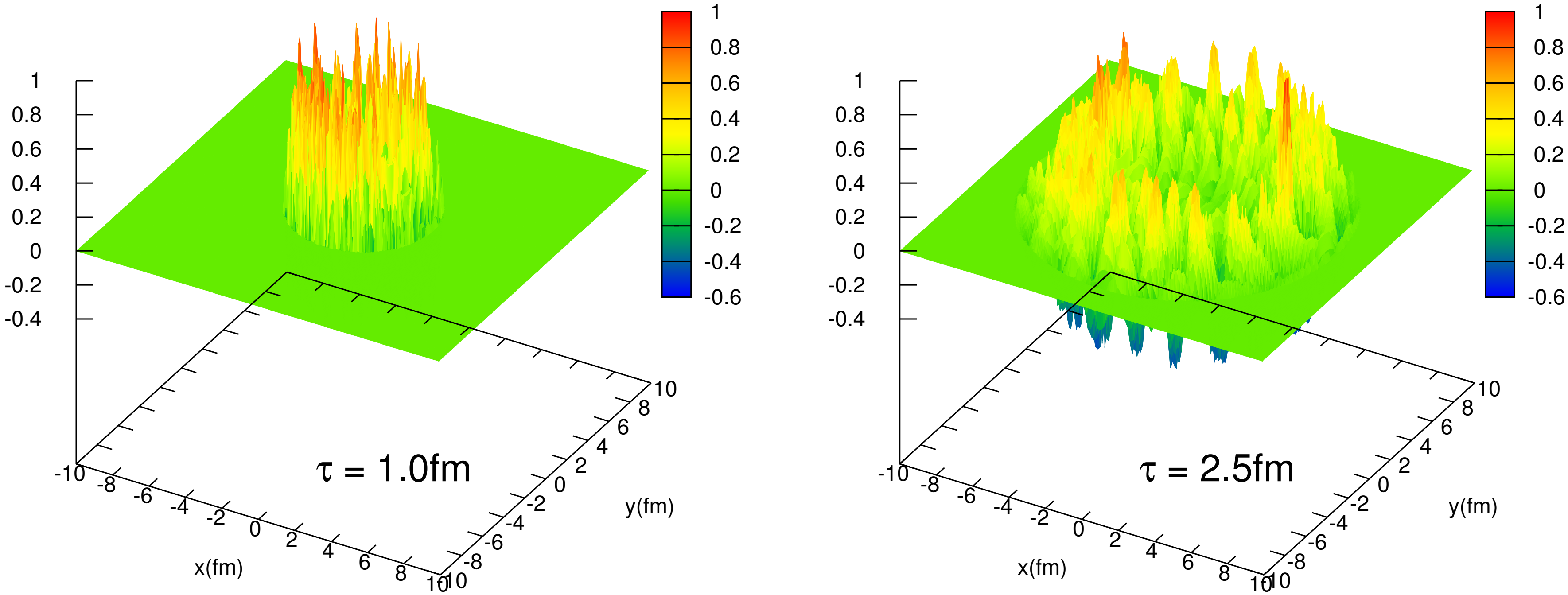
- $\delta\hat{T}_0 = \exp\left[-\frac{\theta^2+\theta_0^2-2\theta\theta_0\cos(\phi-\phi_0)}{2\sigma^2}\right]$ (**Staig and Shuryak**)
- $\delta T(\tau, \vec{x}_\perp)$ without thermal noise



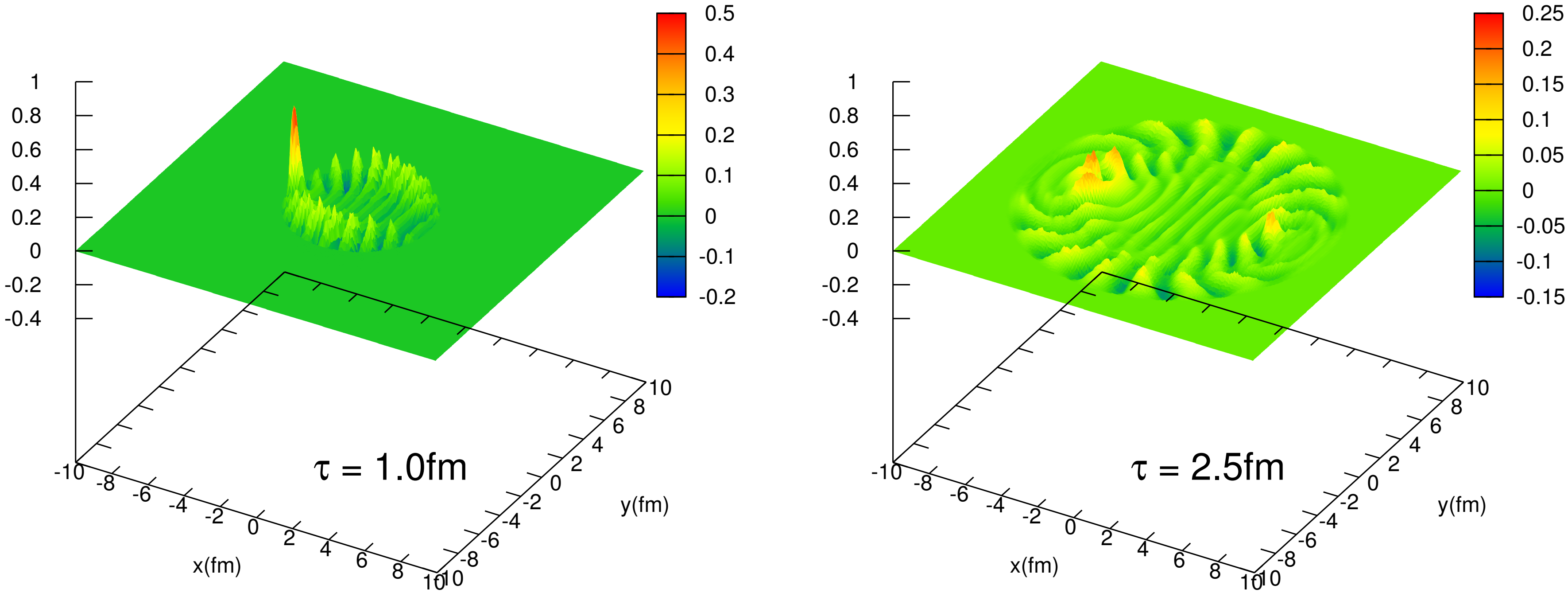
Evolution of a Gaussian profile

- $\delta\hat{T}_0 = \exp\left[-\frac{\theta^2+\theta_0^2-2\theta\theta_0\cos(\phi-\phi_0)}{2\sigma^2}\right]$ (**Staig and Shuryak**)

- $\delta T(\tau, \vec{x}_\perp)$ with thermal noise, one random event

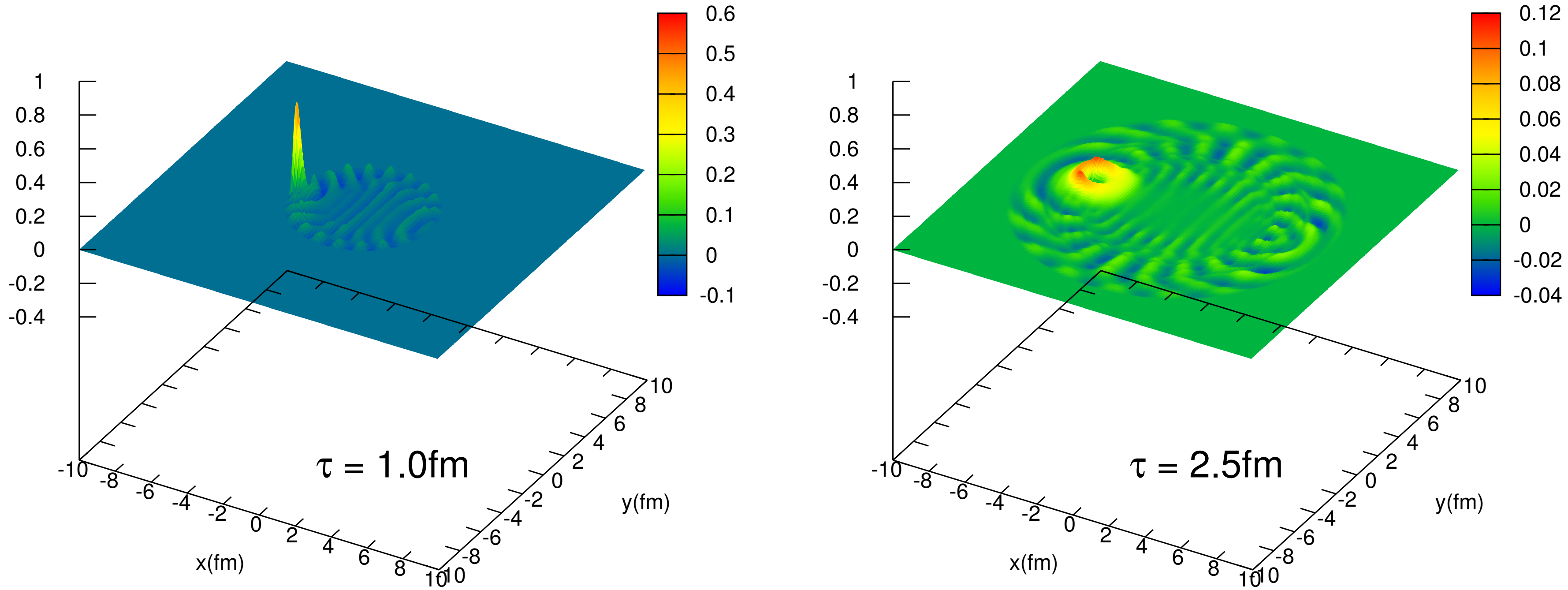


- $\delta\hat{T}_0 = \exp\left[-\frac{\theta^2+\theta_0^2-2\theta\theta_0\cos(\phi-\phi_0)}{2\sigma^2}\right]$ (**Staig and Shuryak**)
- $\delta T(\tau, \vec{x}_\perp)$ with thermal noise, average over 100 events



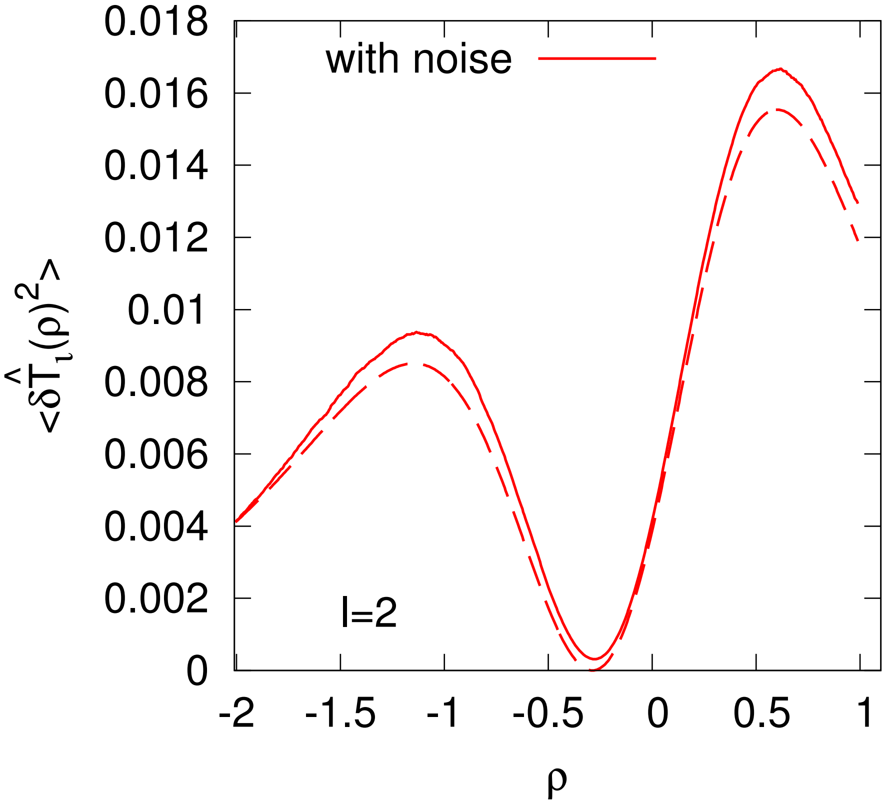
Evolution of a Gaussian profile

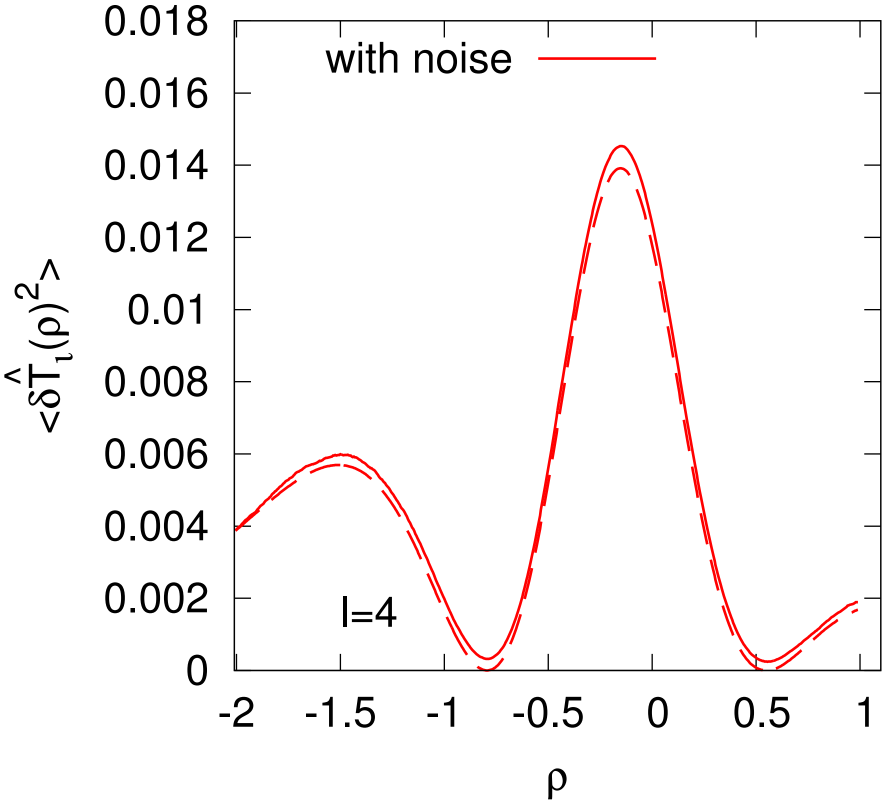
- $\delta\hat{T}_0 = \exp\left[-\frac{\theta^2+\theta_0^2-2\theta\theta_0\cos(\phi-\phi_0)}{2\sigma^2}\right]$ (**Staig and Shuryak**)
- $\delta T(\tau, \vec{x}_\perp)$ with thermal noise, average over 1000 events



Two-point correlations – PbPb

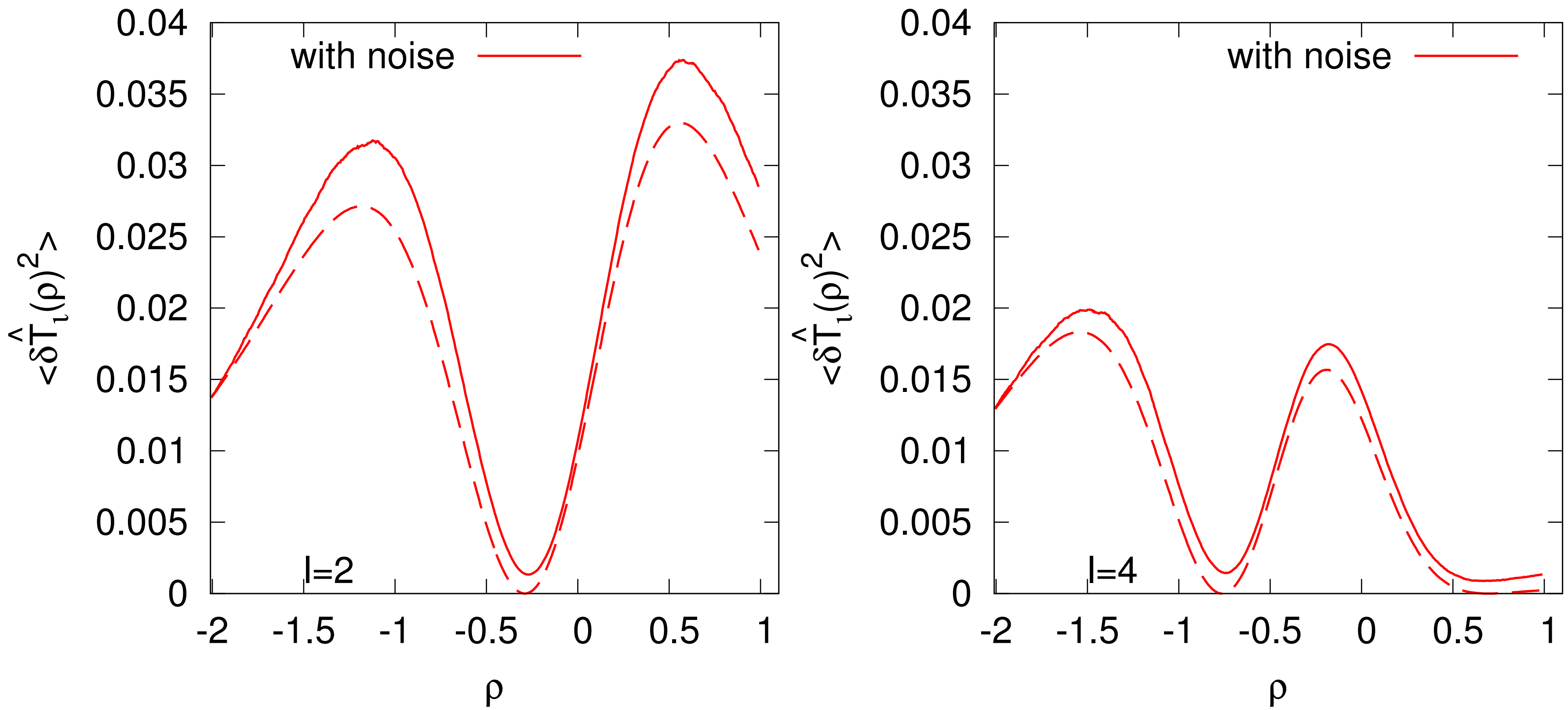
- Initial fluctuations: $\delta\hat{T}(\rho_0) \sim \delta(\theta - \theta_0)\delta(\phi - \phi_0)$, and $\theta_0 = 1.5$, $\phi_0 = \pi$.
- $\langle \delta\hat{T}_l(\rho)^2 \rangle$ with thermal noise (1000 events) and without thermal noise:





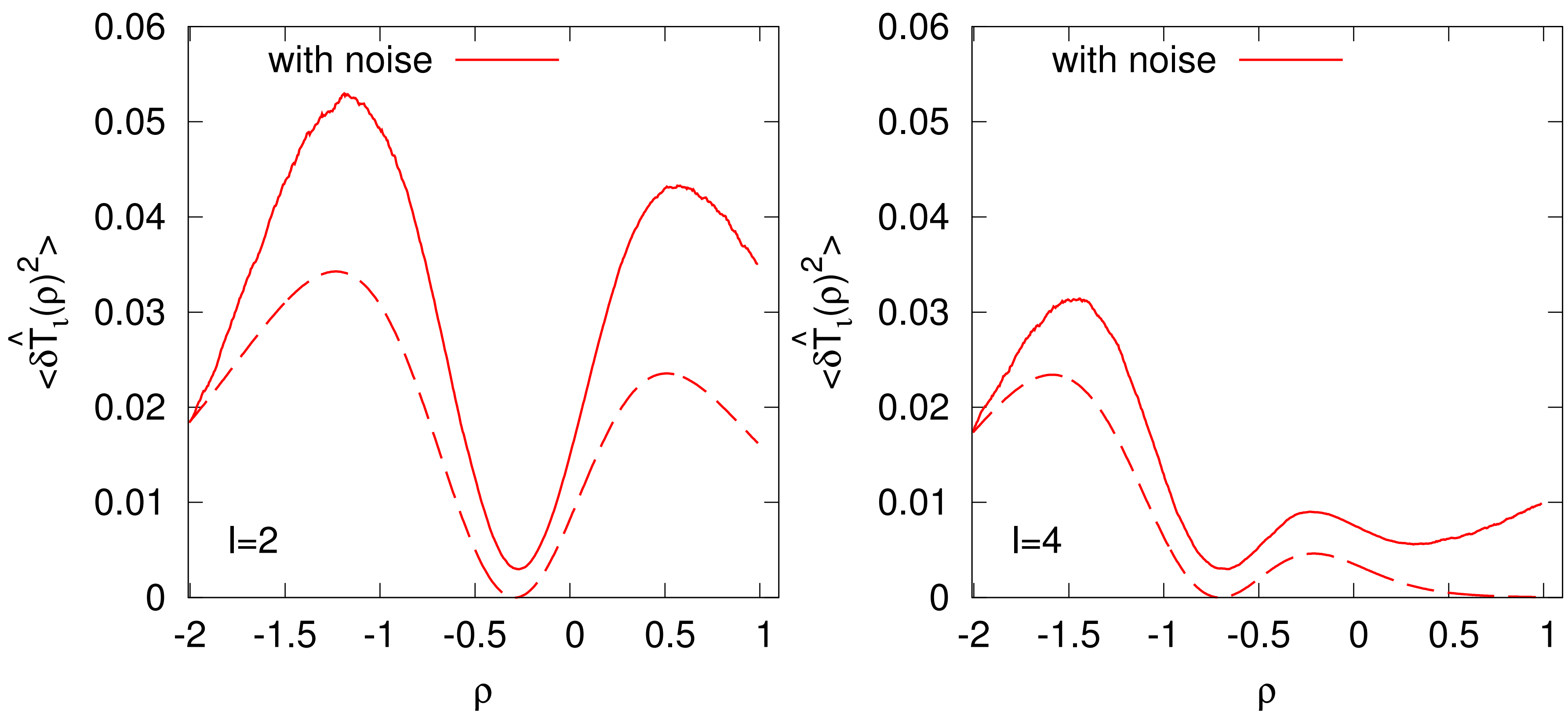
Two-point correlations – pPb

- Initial fluctuations: $\delta\hat{T}(\rho_0) \sim \delta(\theta - \theta_0)\delta(\phi - \phi_0)$, and $\theta_0 = 1.5$, $\phi_0 = \pi$.
- $\langle \delta\hat{T}_l(\rho)^2 \rangle$ with thermal noise (1000 events) and without thermal noise:



Two-point correlations – pp

- Initial fluctuations: $\delta\hat{T}(\rho_0) \sim \delta(\theta - \theta_0)\delta(\phi - \phi_0)$, and $\theta_0 = 1.5$, $\phi_0 = \pi$.
- $\langle \delta\hat{T}_l(\rho)^2 \rangle$ with thermal noise (1000 events) and without thermal noise:



Summary and outlook

- Formulate and solve 2+1D Gubser hydro. with thermal fluctuations.
- Effect of thermal noise in heavy-ion collisions:
  1. Absolute magnitude of thermal noise is mostly controlled by multiplicity.
  2. Relatively magnitude of initial fluctuations ($\sim 1/\sqrt{N}$).
  3. Thermal fluctuations are more significant in pp than pPb, but not in PbPb.
- Outlook
  * Two-point correlations in particle spectrum.
  * 2nd order hydro.
  * Colored noise.(**T. Hirano et al.**)