Cold atoms and hot quark-gluon plasma

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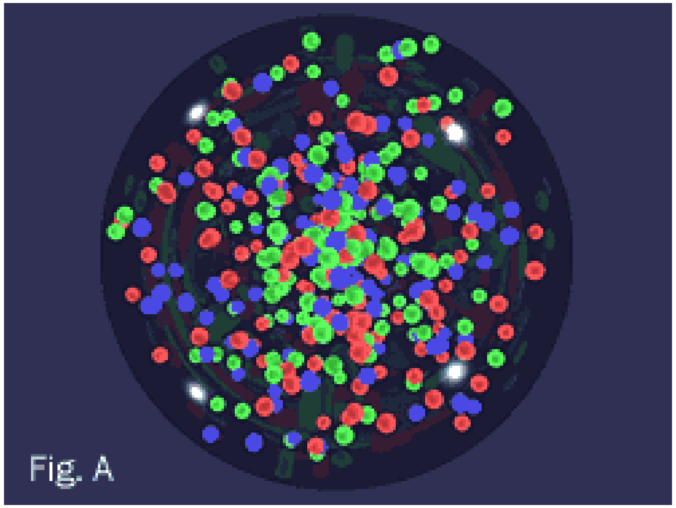
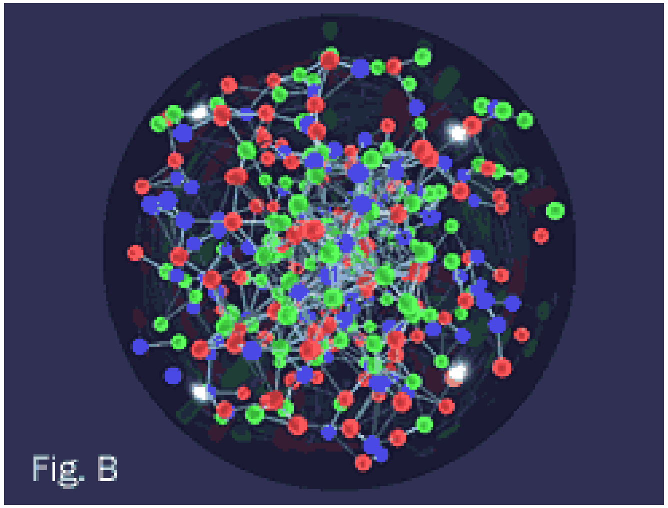
Our Paradigm

RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted -- raising many new questions

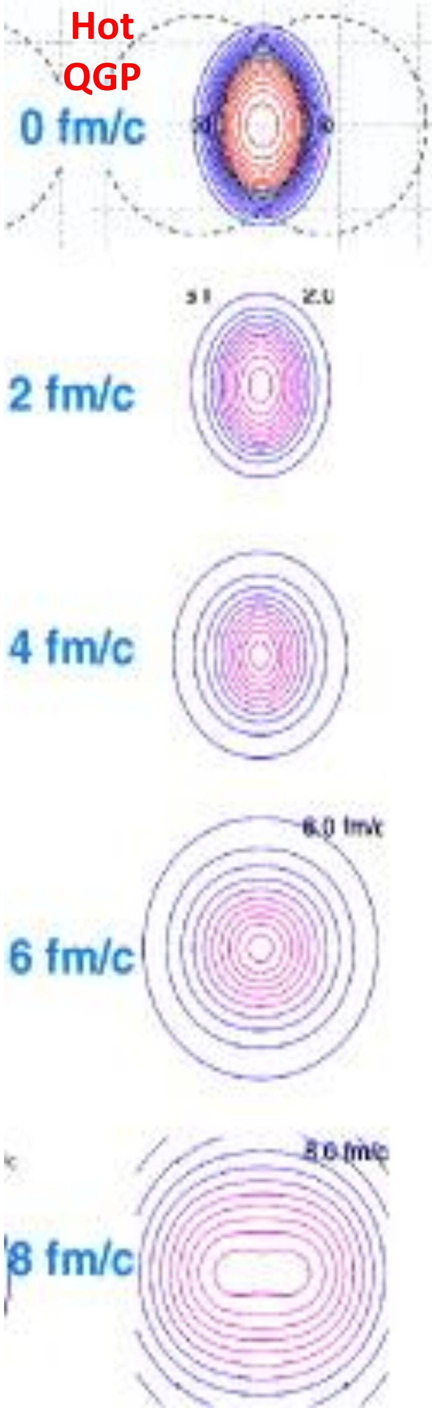


Infinite viscosity



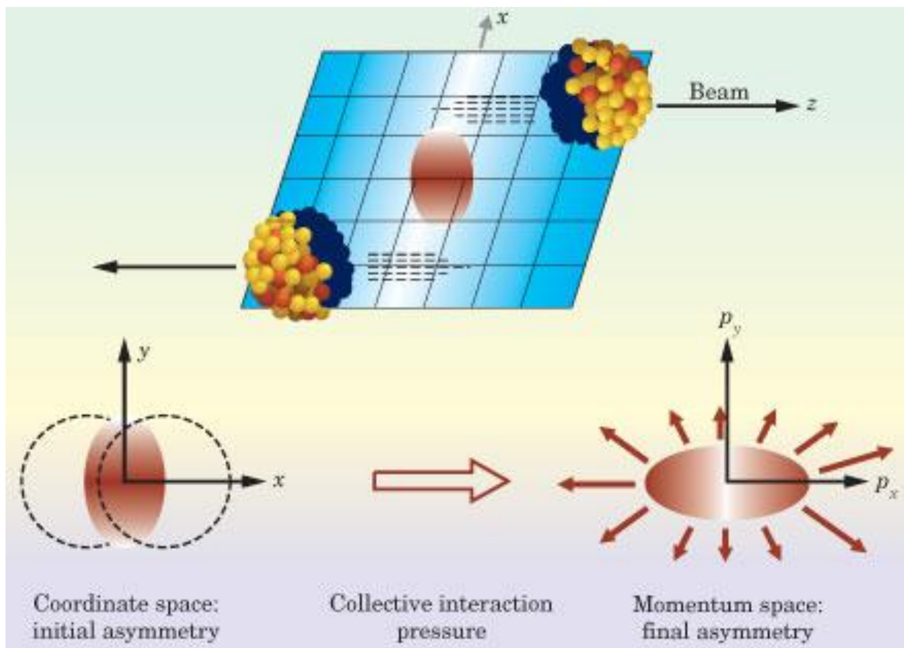
Very low viscosity

Main evidence: Anisotropy

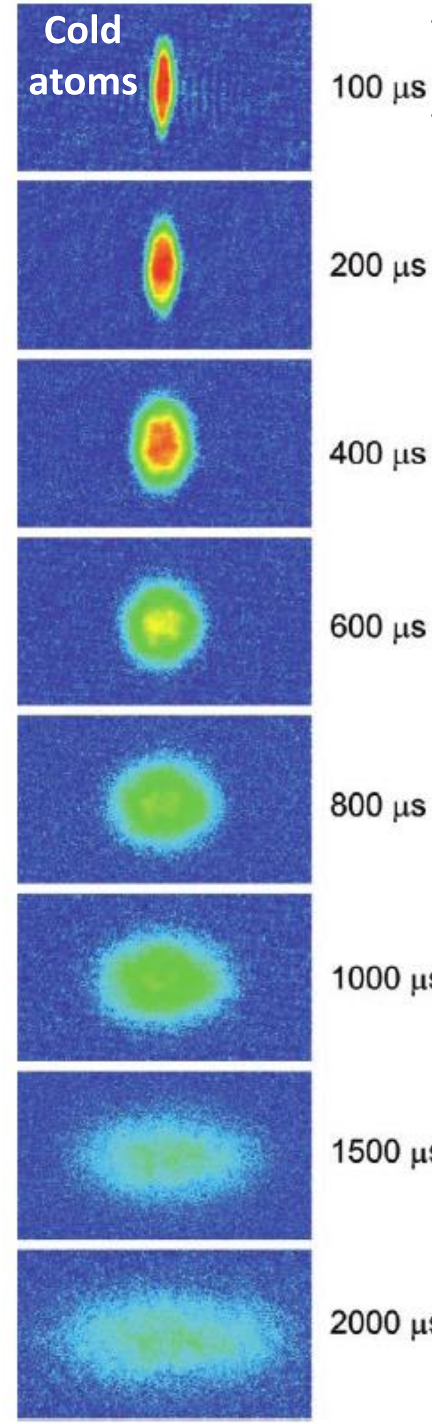


Kolb, Heinz, nucl-th/0305084; Lisa et al. New J. Phys. **13** (2011) 065006

$\vec{x}$-anisotropy $\Rightarrow$ $\vec{p}$-anisotropy

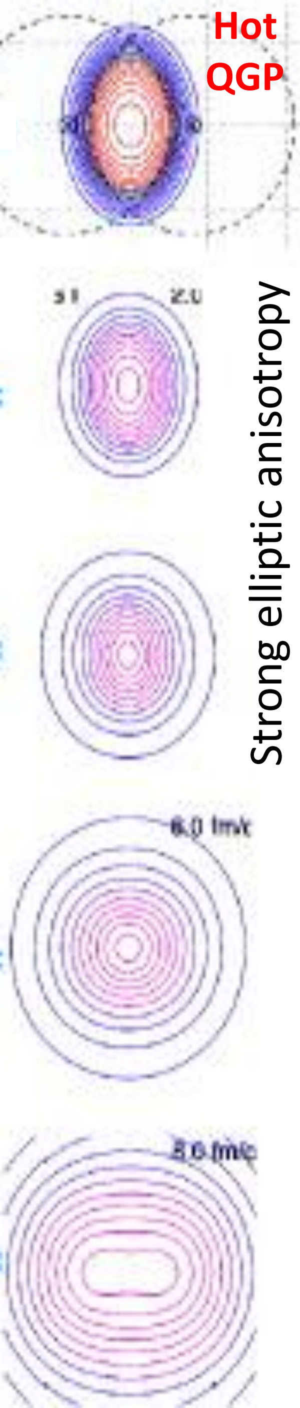


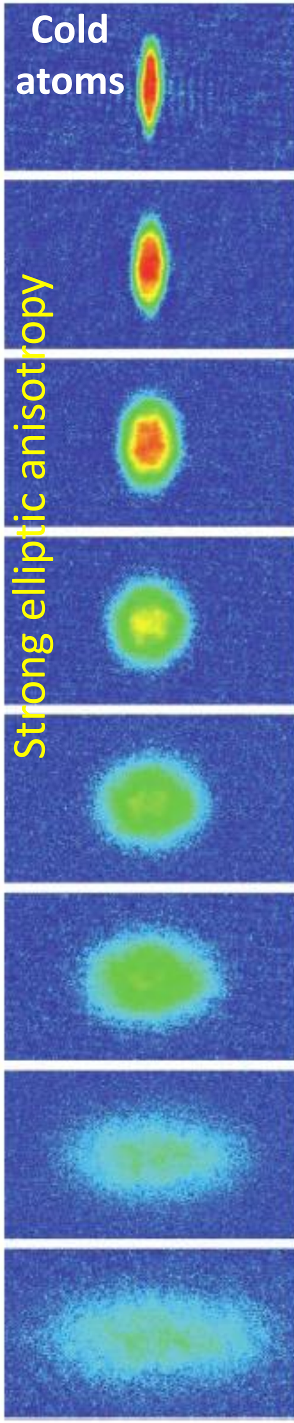
Strong elliptic anisotropy is measured



K. M. O'Hara *et al.*, Science 298, 2179 (2002)

Consistent with Hydrodynamics



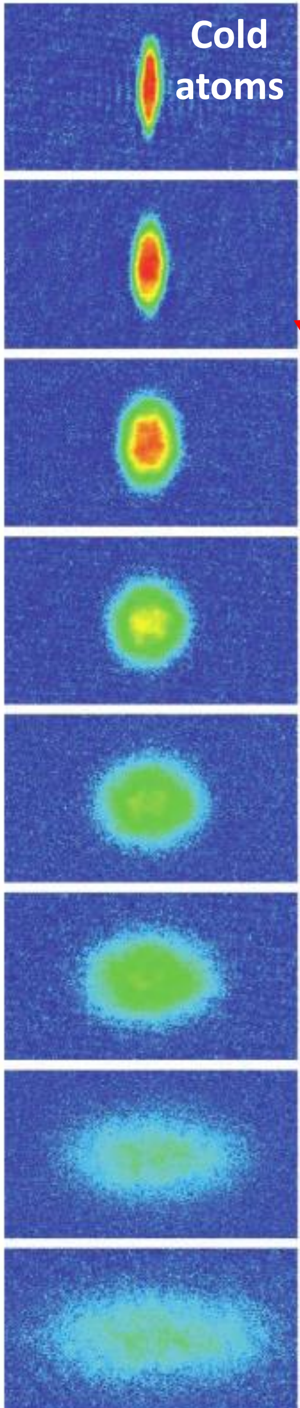


Some recent thinking…

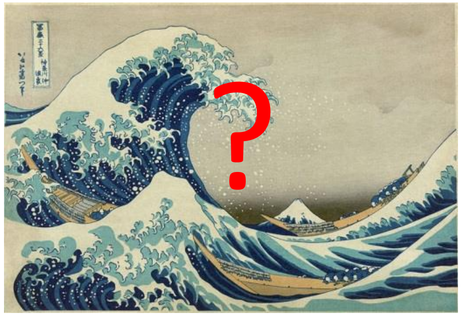
1. Opacity
2. Quantum effects

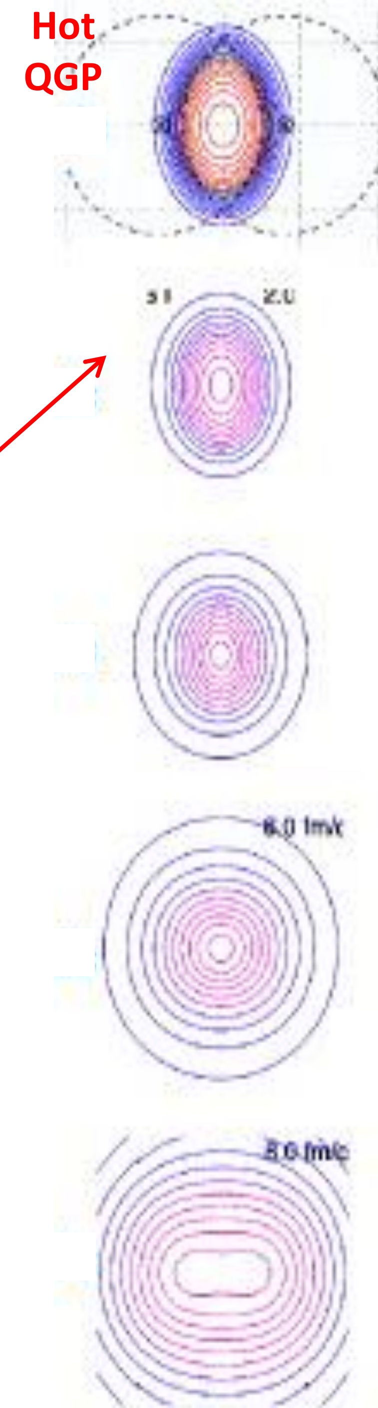
Motivated by small systems…

Opacity



Hydro!





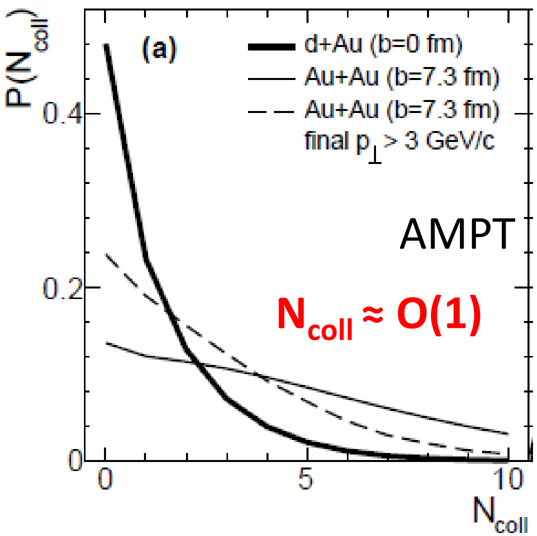
Hydro?

Mean free path?

$L_{mfp} = 1/\rho\sigma$, Prob. = $\exp(-L/L_{mfp})$

$a \approx 5\times10^{-5}$ cm
$\sigma_{int} \approx 10^{-8}$ cm^2
$\rho \approx 5\times10^{13}$ /cm^3
$L_{mfp} \approx 2\times10^{-6}$ cm
$L \approx 2\times10^{-3}$ cm
$L/L_{mfp} \approx 1000$

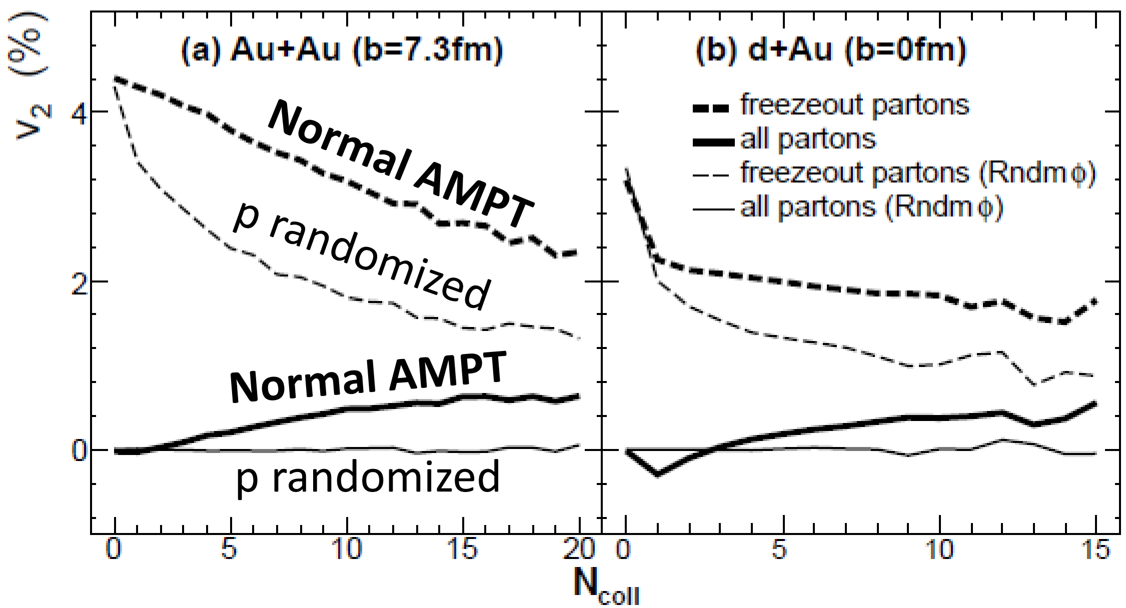
Very high opacity for the cold atom system



Low opacity in QGP modeled by AMPT

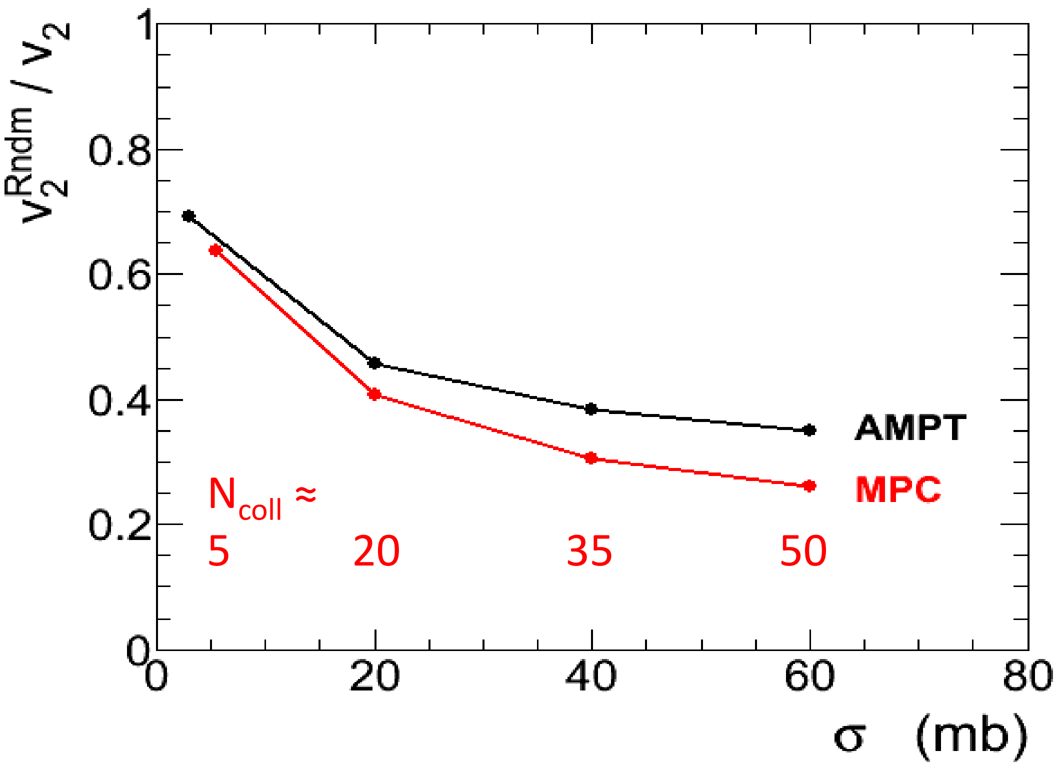
Majority anisotropy from escape

L. He, T. Edmonds, Z.-W. Lin, F. Liu, D. Molnar, FW, arXiv:1502.05572



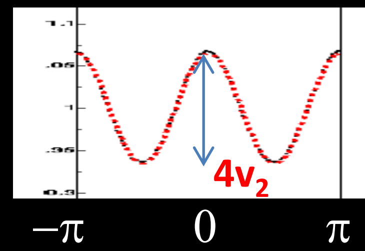
- Majority of anisotropy comes from the final-step “escape” mechanism.
- This small v_2 is due to dynamics, result of hydrodynamic pressure push. It is this flow that is most relevant. However it plays a minor role.
- May explain small system data and weak energy dependence.

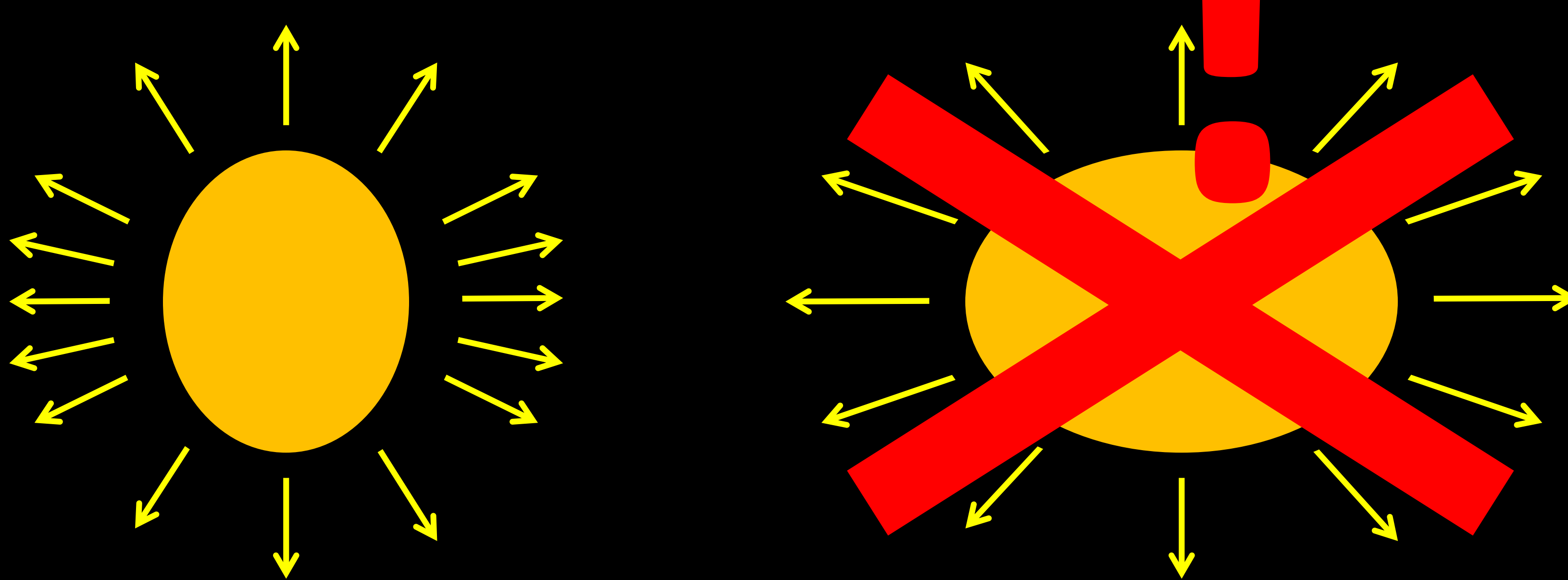
Relative escape contribution



- Escape contribution still sizeable even at x10 larger x-sections.

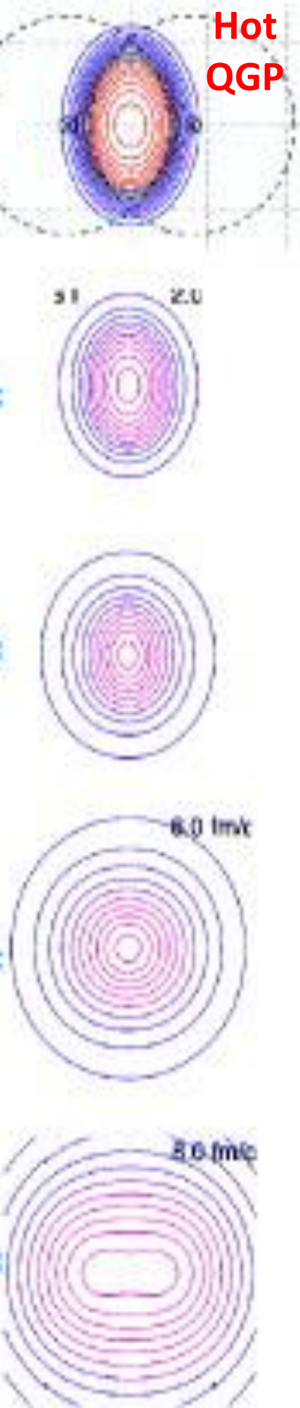
Anisotropy mechanism





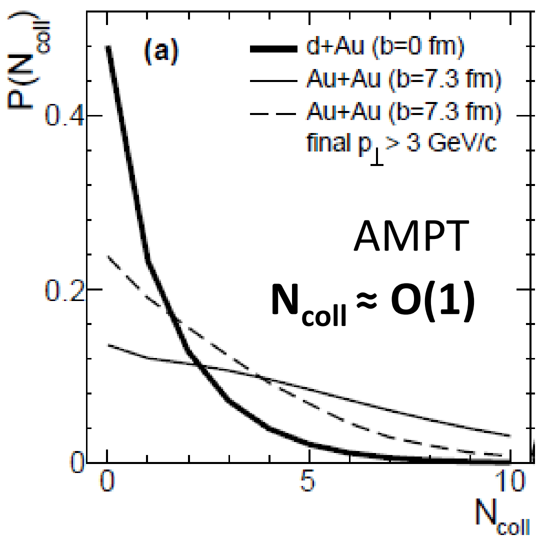
No expansion **Expansion, flow**

Emulate QGP with cold atoms



Mean free path?

$L_{mfp} = 1/\rho\sigma$, Prob. = $\exp(-L/L_{mfp})$



Low opacity in AMPT

$a \approx 5\times10^{-5}$ cm

$\sigma_{int} \approx 10^{-8}$ cm^2

$\rho \approx 5\times10^{13}$ /cm^3

$L_{mfp} \approx 2\times10^{-6}$ cm

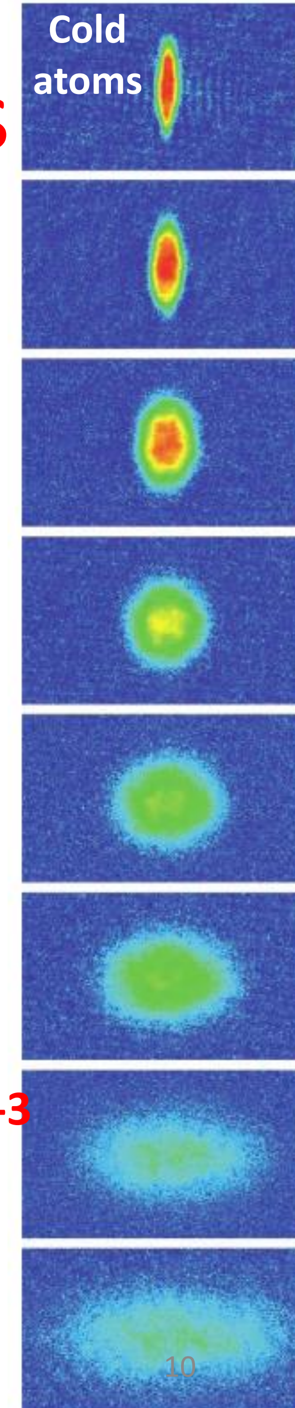
$L \approx 2\times10^{-3}$ cm

$L/L_{mfp} \approx$ ~~1000~~ 0-1

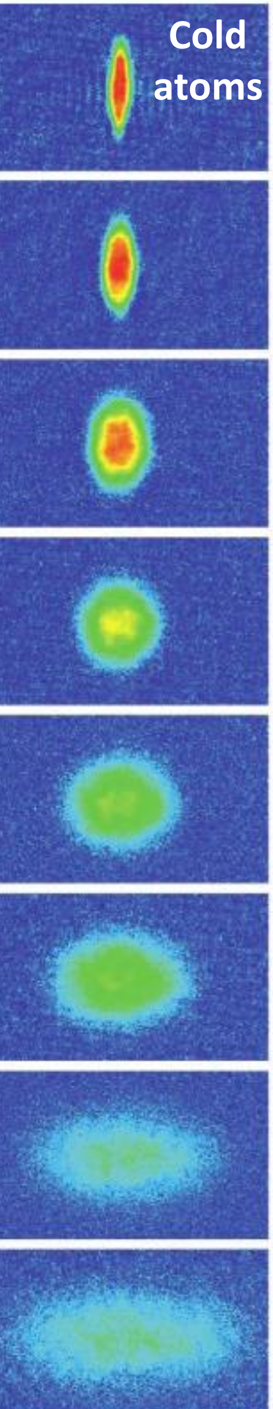
X10^{-3}

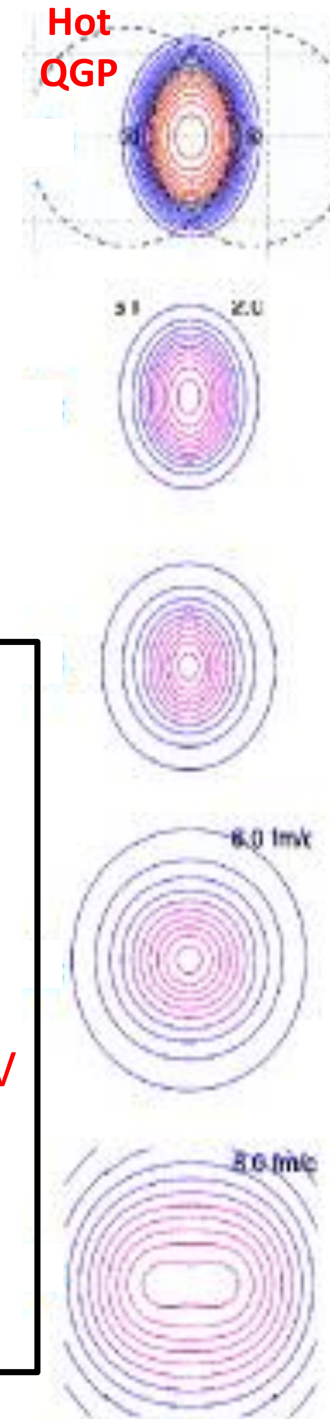
X10^{-3}

Very ~~high~~ low opacity for the cold atom system



Is QGP classical?





Li: M~6000 MeV
T~1μK~10^{-16}MeV
x~20μm, y~100μm
p~$(TM)^{1/2}$~10^{-6}MeV

E quantum~$1/(mr^2)$~10^{-20}MeV
p quanyum~1/r~10^{-8}MeV
Negligible!

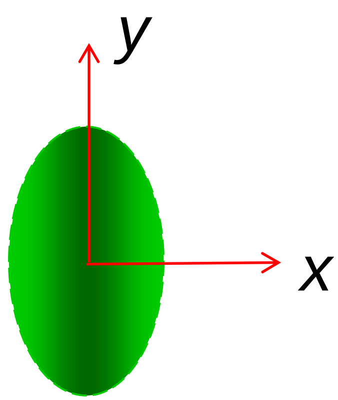
Cold atoms are **hot**,
"classical" w.r.t. trap size.

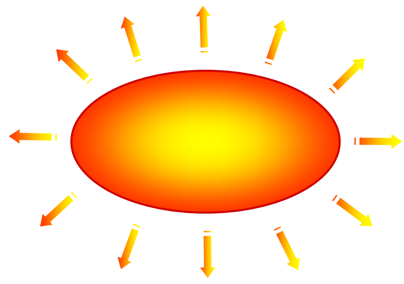
q,g: M~0 MeV
T~200 MeV
x~3fm, y~4fm
p~200MeV

E quanum~200 MeV
p quanum~1/r~200MeV
Comparable!

QGP is **cold**,
quantum mechanical.

QM uncertainty principle



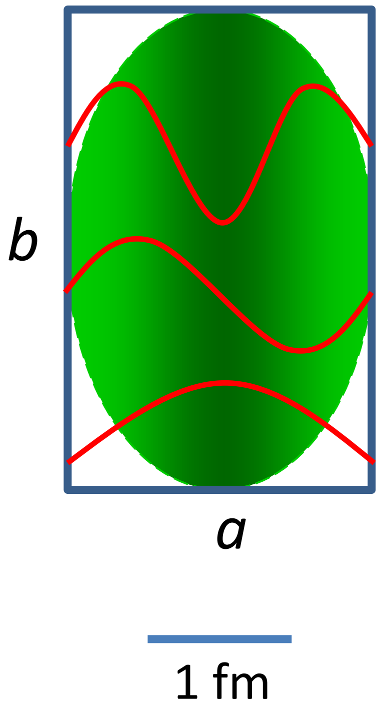


$$\Delta x \cdot \Delta p > \hbar / 2$$

$$p_x > p_y$$

$$\varepsilon = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle} \qquad v_2 = \langle \cos 2\varphi \rangle = \frac{\langle p_x^2 \rangle - \langle p_y^2 \rangle}{\langle p_x^2 \rangle + \langle p_y^2 \rangle}$$

Infinite square well



$$-\frac{\hbar^2}{2m}\nabla^2\psi = E\psi \quad \Rightarrow \quad \psi \propto \begin{cases} \cos\dfrac{n_{odd}\pi}{a}x \\ \sin\dfrac{n_{even}\pi}{a}x \end{cases}$$

Take even mode for example:

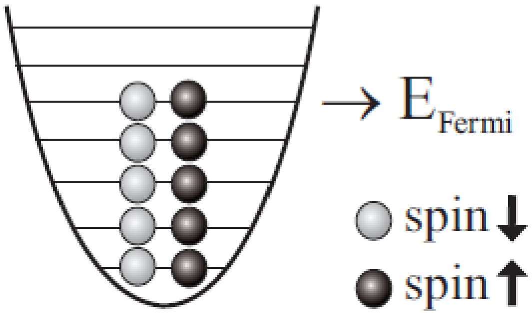
$$\langle p_x^2\rangle = \hbar^2 k^2 \ ; \quad \langle x^2\rangle = \frac{a^2}{4} - \frac{2}{k^2} \ ; \quad k = \frac{n_{odd}\pi}{a}$$

$$\sqrt{\langle p_x^2\rangle\cdot\langle x^2\rangle} = \hbar\sqrt{\frac{k^2a^2}{4} - 2} = \hbar\sqrt{\frac{\pi^2}{4}n_{odd}^2 - 2} > \hbar/2$$

$$v_2 = \frac{\langle p_x^2\rangle - \langle p_y^2\rangle}{\langle p_x^2\rangle + \langle p_y^2\rangle} = \frac{b^2 - a^2}{b^2 + a^2} = \varepsilon \quad \text{for all } n.$$

Single state anisotropy

Harmonic oscillator



$$\left(-\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}m\omega^2 x^2\right)\psi = E\psi \ ; \qquad E = \left(n + \frac{1}{2}\right)\hbar\omega$$

$$\left\langle \frac{p_x^2}{2m} \right\rangle = \left\langle \frac{1}{2}m\omega^2 x^2 \right\rangle = \frac{E}{2} = \frac{1}{2}\left(n + \frac{1}{2}\right)\hbar\omega$$

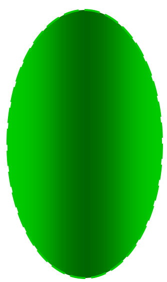
$$\sqrt{\left\langle p_x^2 \right\rangle \left\langle x^2 \right\rangle} = \left(n + \frac{1}{2}\right)\hbar$$

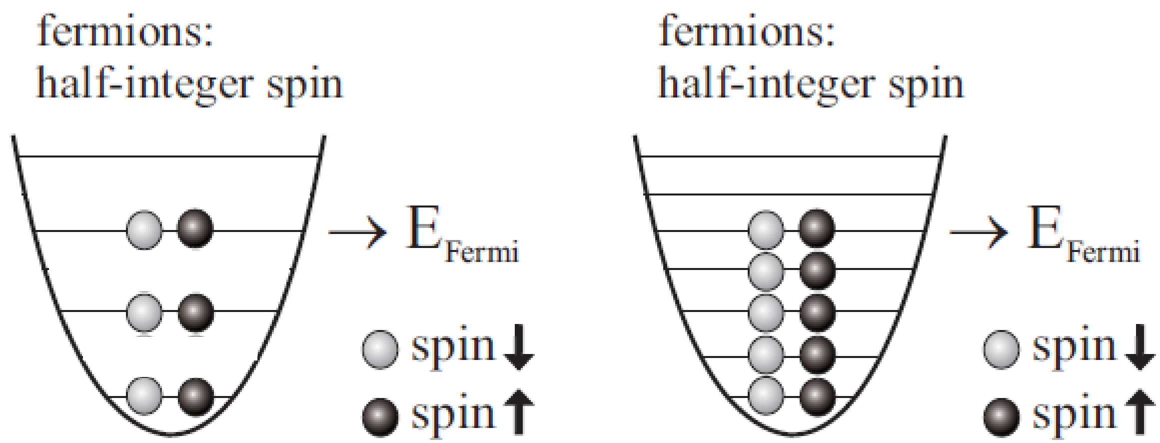
$$v_2 = \frac{\left\langle p_x^2 \right\rangle - \left\langle p_y^2 \right\rangle}{\left\langle p_x^2 \right\rangle + \left\langle p_y^2 \right\rangle} = \frac{\omega_x - \omega_y}{\omega_x + \omega_y}$$

$$\varepsilon = \frac{\left\langle y^2 \right\rangle - \left\langle x^2 \right\rangle}{\left\langle y^2 \right\rangle + \left\langle x^2 \right\rangle} = \frac{\omega_x - \omega_y}{\omega_x + \omega_y}$$

$v_2 = \varepsilon$ for each and all n

Thermal probability





x, y at same Fermi energy, so different number of filled energy levels.

At high temperature, classical limit, sum is approximated by integral:

$$\frac{dN}{d\mathbf{p}} = N\frac{\int d\mathbf{r}\, e^{-H_1(\mathbf{p},\mathbf{r})/T}}{\int d\mathbf{r}\, d\mathbf{p}\, e^{-H_1(\mathbf{p},\mathbf{r})/T}} = N\frac{e^{-K(\mathbf{p})/T}}{\int d\mathbf{p}\, e^{-K(\mathbf{p})/T}}$$

then it's independent of potential.

It's isotropic at all temperature because $K=(p_x^2+p_y^2)/2m$ is isotropic.

Thermal probability weight

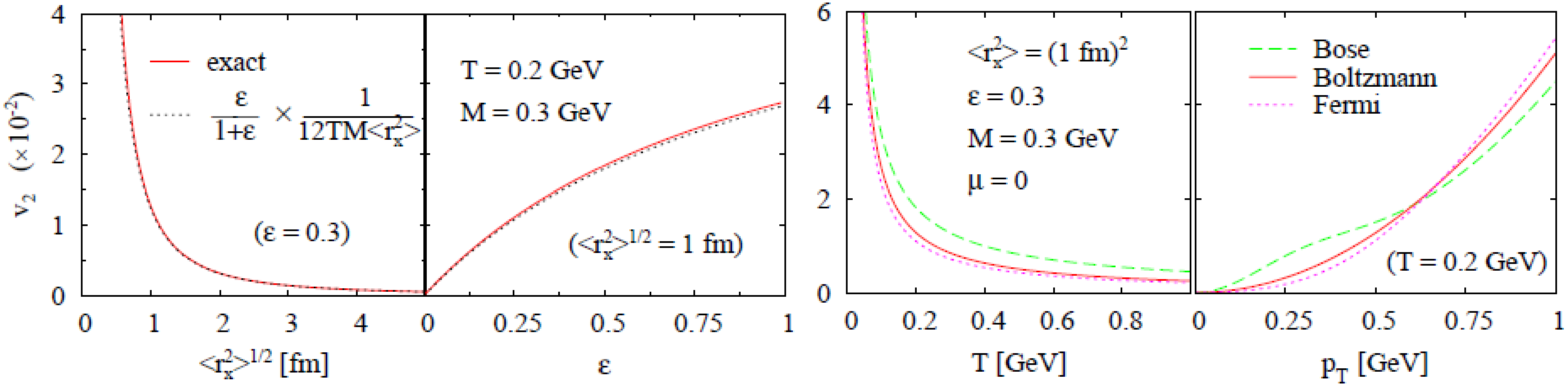
$$\rho(\mathbf{r}) \equiv \frac{dN}{d\mathbf{r}} = \frac{1}{Z}\sum_j |\psi_j(\mathbf{r})|^2\, e^{-E_j/T} \qquad f(\mathbf{p}) \equiv \frac{dN}{d\mathbf{p}} = \frac{1}{Z}\sum_j |\psi_j(\mathbf{p})|^2\, e^{-E_j/T}$$

$$Z \equiv \sum_j e^{-E_j/T}$$

$$\langle p_i^2\rangle = \frac{M\omega_i}{2}\coth\frac{\omega_i}{2T}\,, \quad \langle r_i^2\rangle = \frac{1}{2M\omega_i}\coth\frac{\omega_i}{2T}\,.$$

$$\bar{v}_2 \approx \frac{\hbar^2}{12 k_B T M \langle r_x^2\rangle}\cdot\frac{\varepsilon}{1+\varepsilon}$$

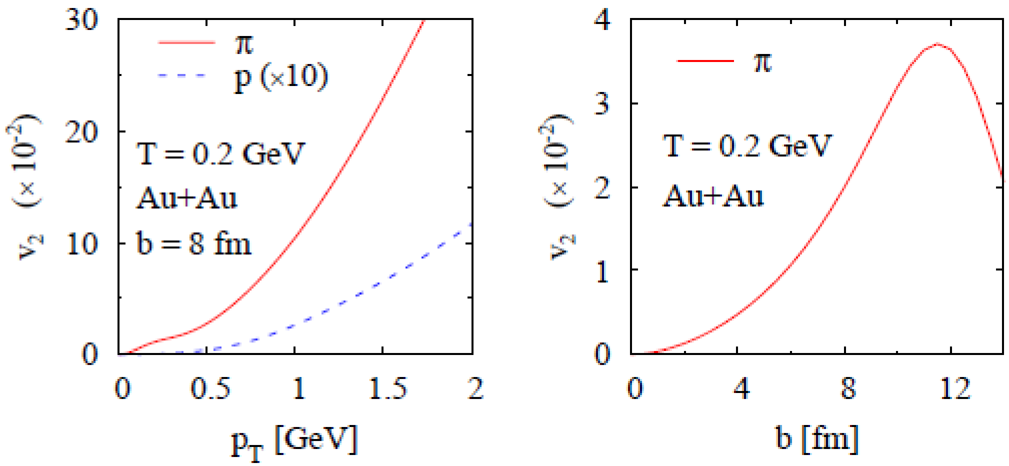
D. Molnar, FW, and C.H. Greene, arXiv:1404.4119



Quantum physics anisotropy

D. Molnar, FW, and C.H. Greene, arXiv:1404.4119

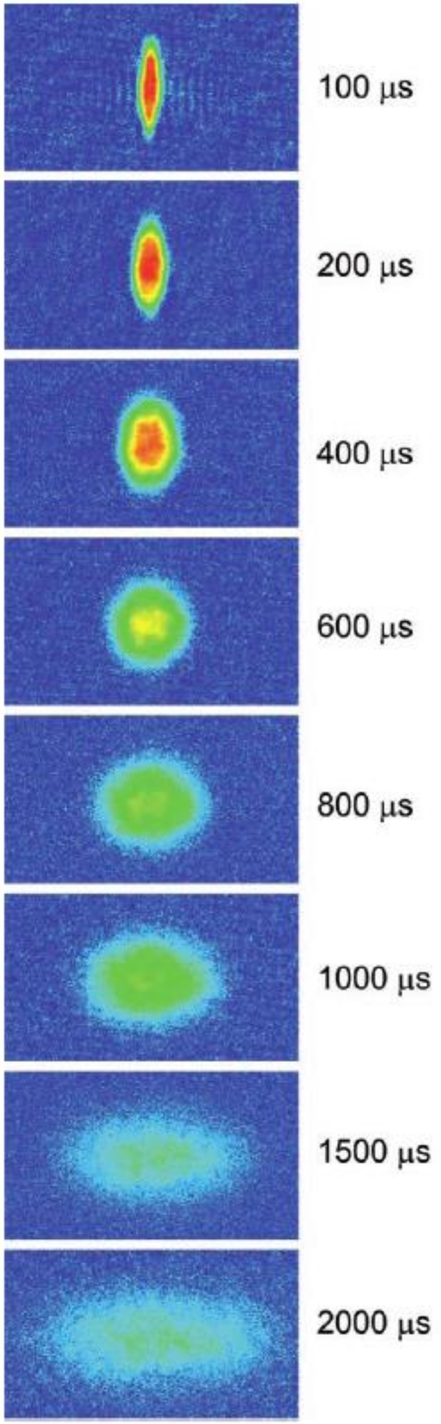
$$\bar{v}_2 \approx \frac{\hbar^2}{12 k_B T M \langle r_x^2 \rangle} \cdot \frac{\varepsilon}{1+\varepsilon}$$



$b = 8$ fm: $\langle r_x^2 \rangle^{1/2} = 1.5$ fm and $\langle r_y^2 \rangle^{1/2} = 2.2$ fm

$$\rho(\mathbf{r}) \propto \exp\left(-\sum_i \frac{r_i^2}{2\langle r_i^2 \rangle}\right), \quad f(\mathbf{p}) \propto \exp\left(-\sum_i \frac{p_i^2}{2\langle p_i^2 \rangle}\right)$$

Cold atoms



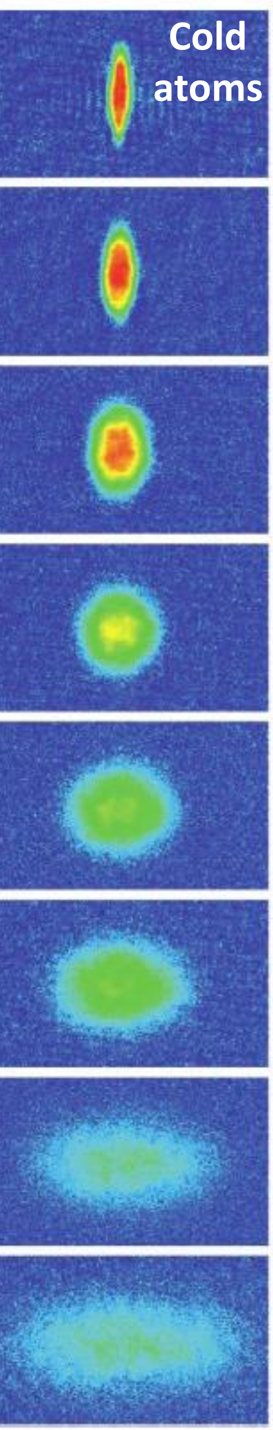
Strong elliptic anisotropy

K. M. O'Hara *et al.*, Science 298, 2179 (2002)

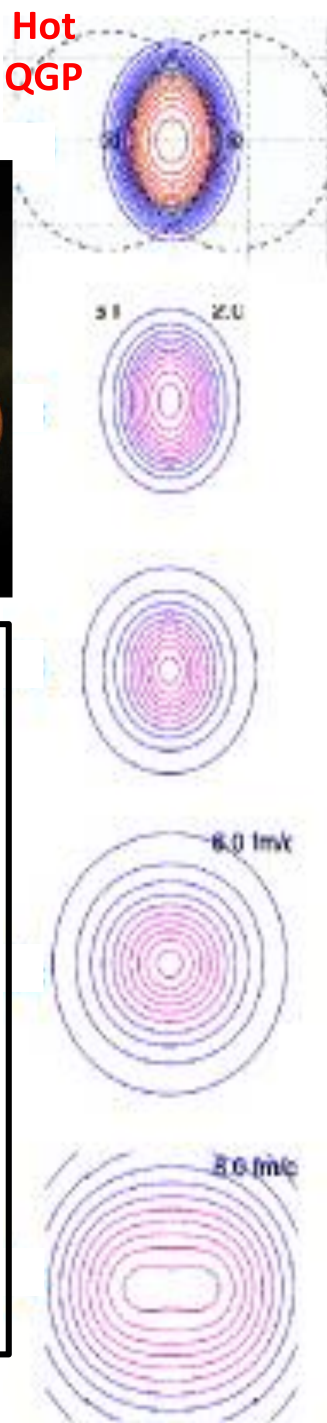
Lithium atoms M ~ 6000 MeV
Temperature T ~ 1 μK ~ 10^{-16} MeV
Trap size x ~ 20 μm, y ~ 100 μm

$$\bar{v}_2 \approx \frac{\hbar^2}{12 k_B T M \langle r_x^2 \rangle} \cdot \frac{\varepsilon}{1+\varepsilon} \approx 10^{-5}$$

The observed large v_2 is indeed due to strong interactions.



Is quantum v_2 real in QGP?



- It should be… but need experiment to verify (cold atom experiment)
- Cold atoms are "classical." Make it Quantum Mechanical.
- Would be neat to verify QM and uncertainty principle

Li: M~6000 MeV
T~1μK~10^{-16}MeV
x~20μm, y~100μm
p~$(TM)^{1/2}$~10^{-6}MeV

x10^{-4}

p quan~1/r~10^{-8}MeV
X10^{-2}
E quan~$1/(mr^2)$~10^{-20}MeV
Negligible!

Cold atoms are **hot**, **classical.**

q,g: M~0 MeV
T~200 MeV
x~3fm,y~4fm
p~200MeV

p quan~1/r~200MeV
E quan~200 MeV
Comparable!

QGP is **cold**, **quantum mechanical.**

Summary

- Close connections between hot QGP and cold atoms.
- Cold atoms are hydrodynamical; QGP may not be.

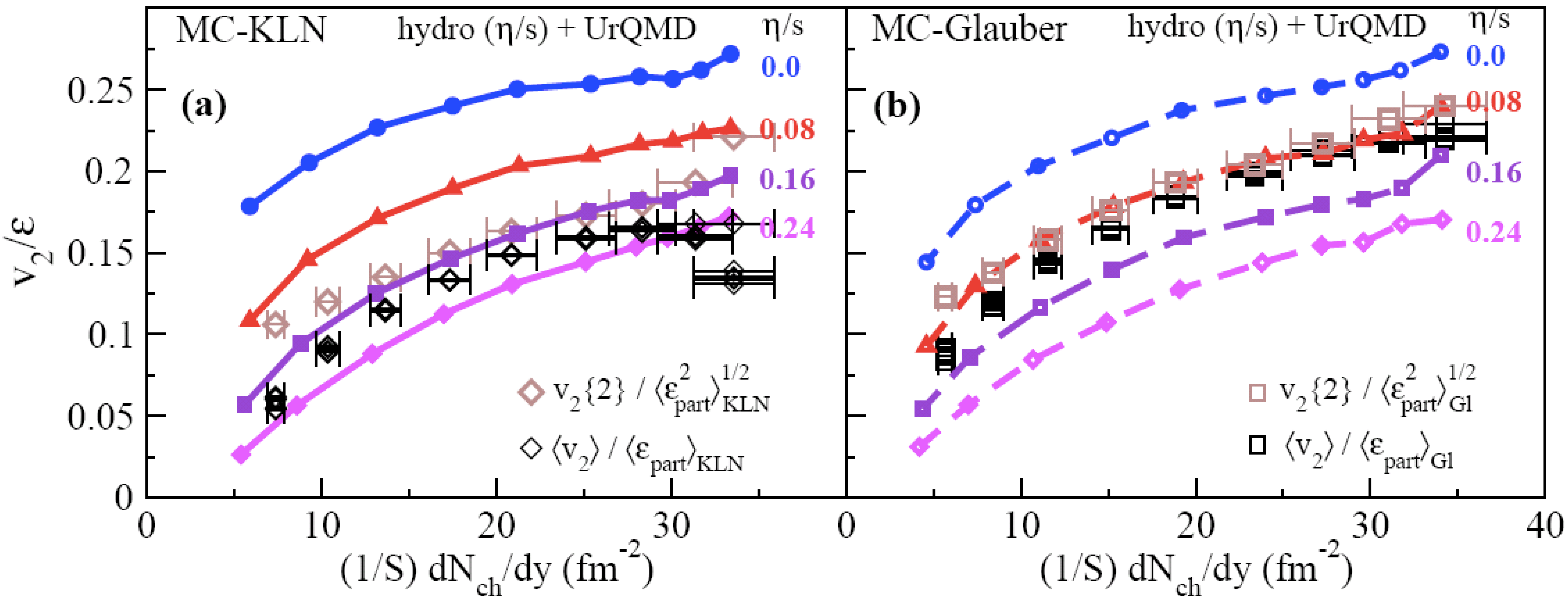
  Make it more dilute, or smaller, or less interacting to mimic QGP.

- QGP is quantum mechanical; cold atoms are "classical."

  Make it smaller, or colder to mimic QGP, and measure the uncertainty principle.

Comparison to Hydrodynamics

Strong elliptic anisotropy

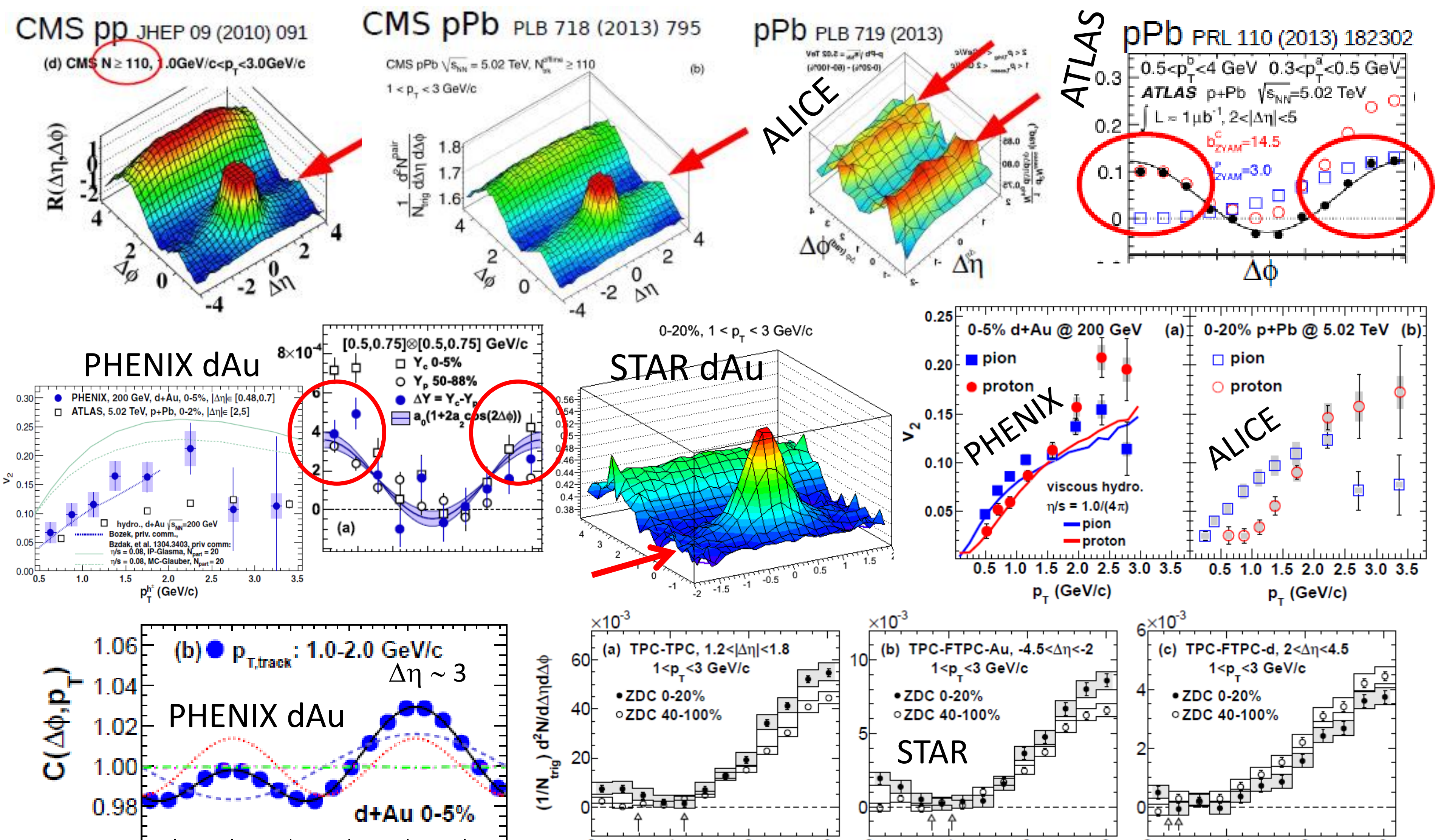


➔ ***Small value*** of specific viscosity over entropy ***η/s***

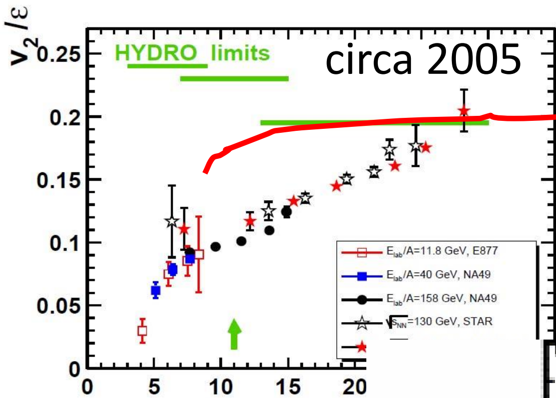
➔ Model uncertainty dominated by ***initial eccentricity ε***

Model: Song *et al. arXiv:1011.2783*

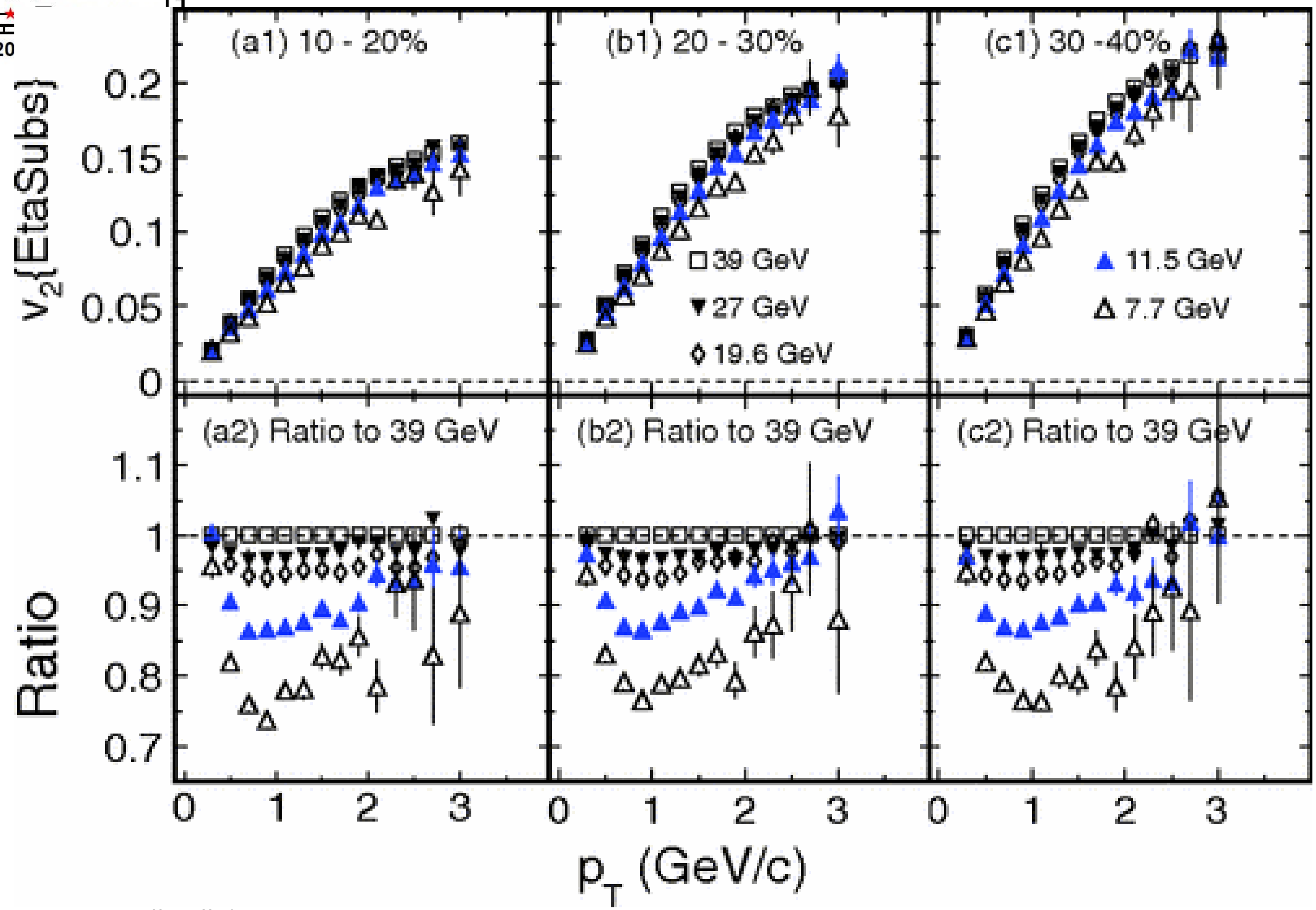
"flow" in small systems, and everywhere



Very little energy dependence

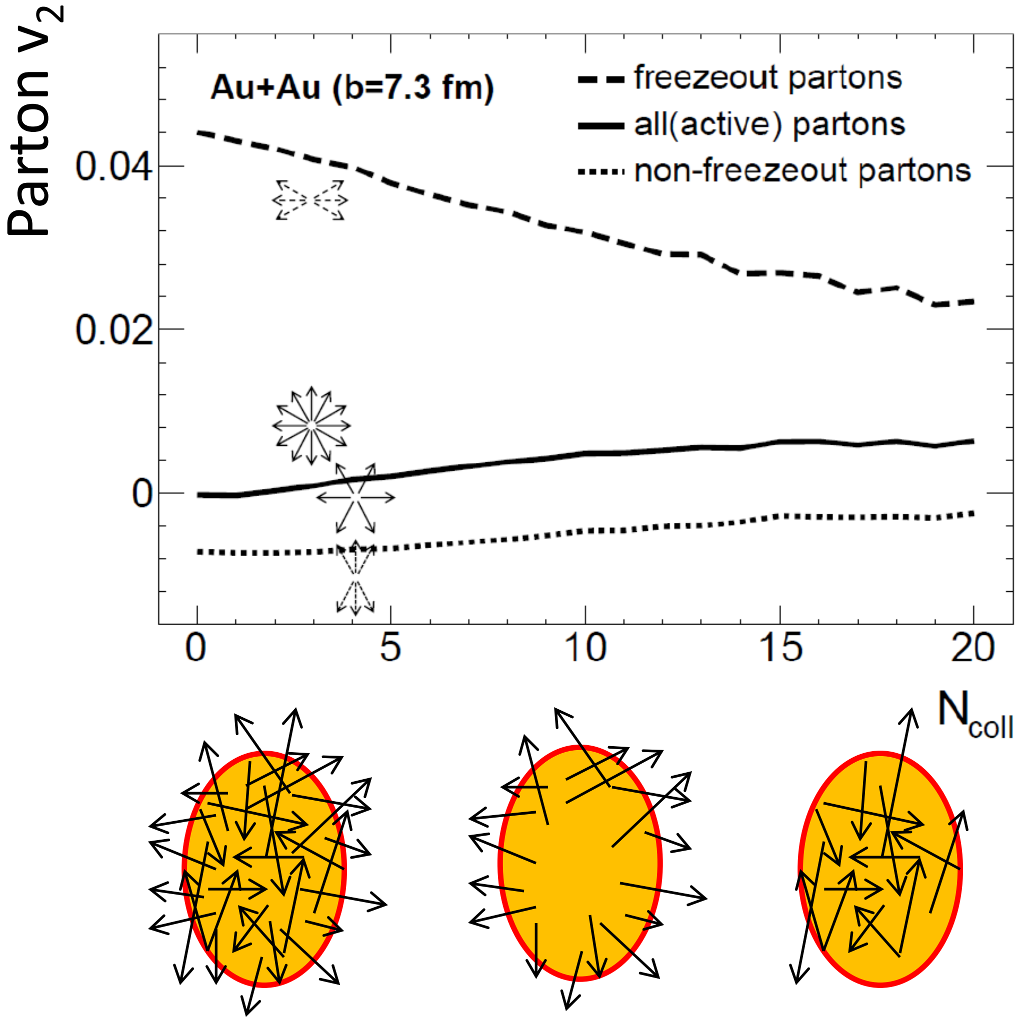


circa 2013



How is anisotropy developed in AMPT?

L. He, T. Edmonds, Z.-W. Lin, F. Liu, D. Molnar, FW, arXiv:1502.05572



- Partons freeze out with large positive v_2, even when they do not interact at all.
- This is due to larger escape probability along x than y.
- Remaining partons start off with negative v_2, and become ~isotropic (v_2~0) after one more collision.
- Process repeats itself.
- Similar for v_3.
- Similar for d+Au collisions.