Color fluctuation phenomena in h-A collisions at collider energies

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Workshop" Correlations and Fluctuations in p+A and A+A Collisions", INT July 2015

Outline

- Importance of coherence in high energy scattering
  example: positronium propagation through the medium

- Color fluctuations in hadron - new pattern of high energy hadron - nucleus scattering - going beyond single parton structure of nucleon.

- Evidence for x -dependent color fluctuations in nucleons

- Conditional nuclear pdfs

Several feature of NN interactions at the LHC relevant for pA and AA

two seem to be most important:

* Fluctuations of overall strength of NN interaction
* A factor of four difference of the transverse area scales for soft and hard NN interaction

Other fluctuations - gluon density in nucleon, nuclei, LT shadowing effects -- will mention briefly

Fluctuations of overall strength of high energy NN interaction

High energy projectile stays in a frozen configuration distances $l_{coh} = c\Delta t$

$$\Delta t \sim 1/\Delta E \sim \frac{2p_h}{m^2_{int} - m^2_h}$$

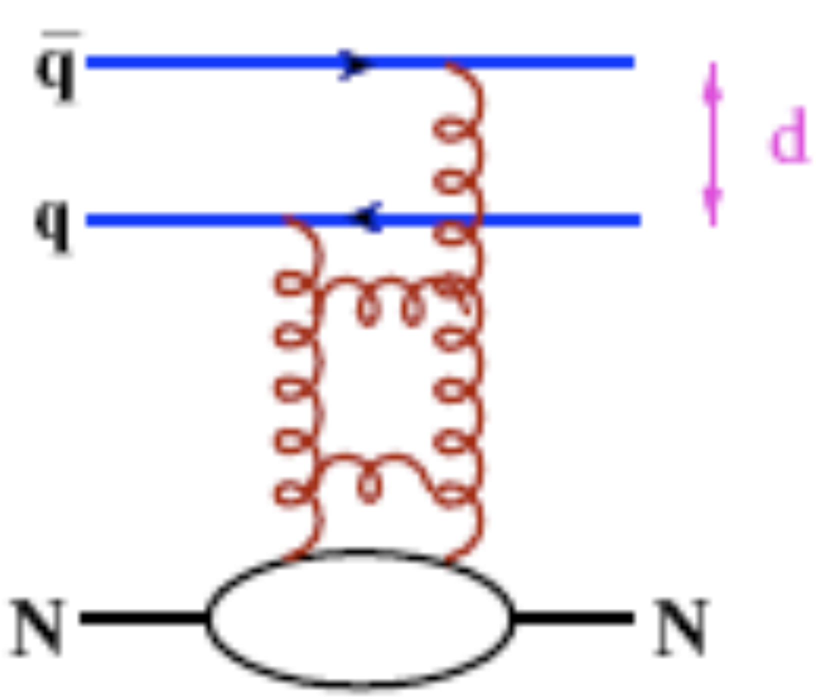
At LHC for $m^2_{int} - m^2_h \sim 1\mathrm{GeV}^2$ $l_{coh} \sim 10^7$ fm>> $2R_A$>> $2r_N$

coherence up to $m^2_{int} \sim 10^6\mathrm{GeV}^2$

Hence system of quarks and gluons passes through the nucleus interacting essentially with the same strength but changes from one event to another different strength

Strength of interaction of white small system is proportional to the area occupies by color.

QCD factorization theorem for the interaction of small size color singlet wave package of quarks and gluons.



For quark - antiquark dipole:

$$\sigma(d,x) = \frac{\pi^2}{3}\alpha_s(Q^2_{eff})d^2\left[xG_N(x,Q^2_{eff}) + \frac{2}{3}xS_N(x,Q^2_{eff})\right]$$
$$Q^2_{eff} = \lambda/d^2, \lambda = 4 \div 10$$

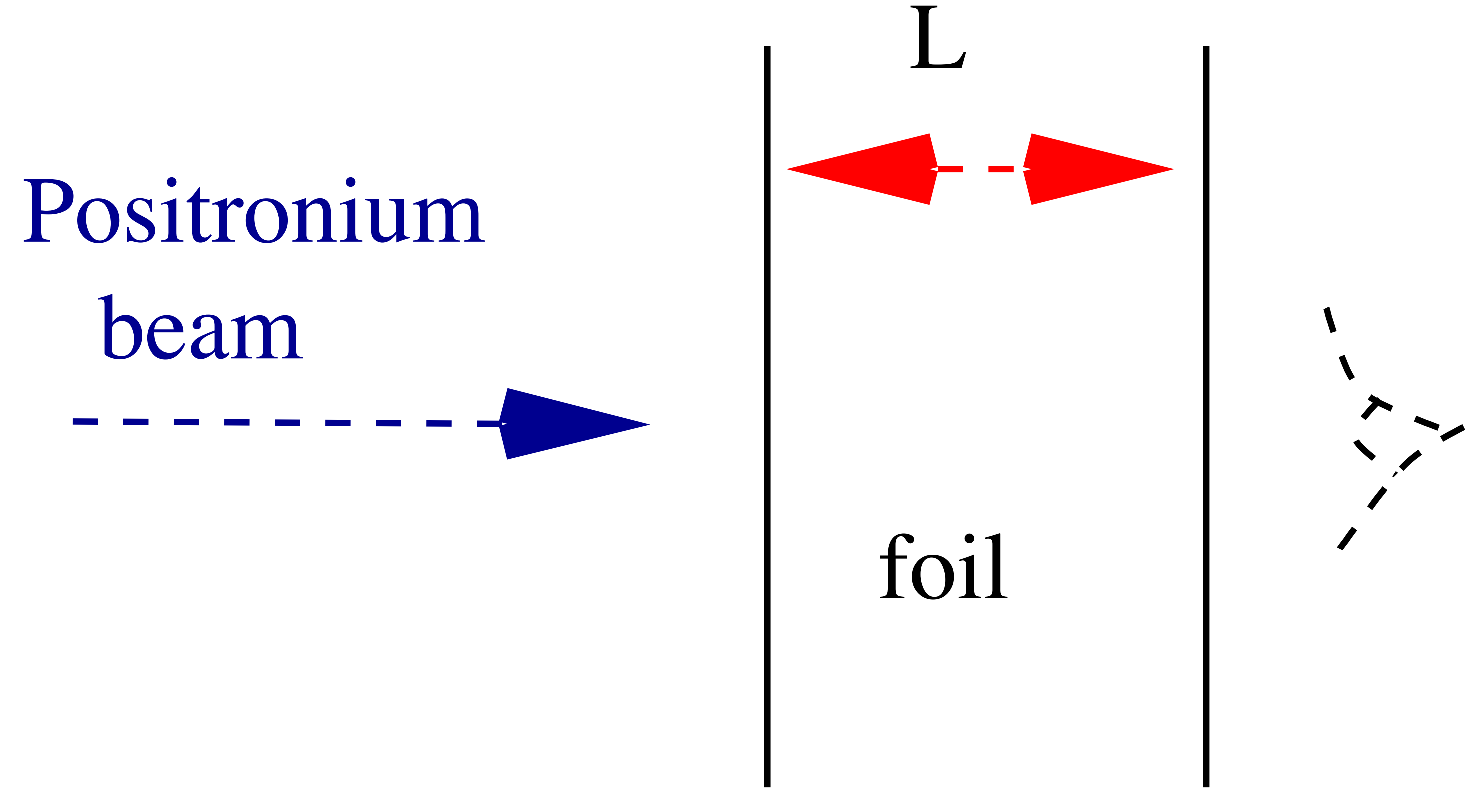
Baym, Blättel, Frankfurt, MS, 93;
Frankfurt,Miller, MS 93

compare: $\sigma(d,x) = cd^2$ in QED or two gluon exchange model of Low - Nussinov (1975)

Instructive example: propagation of a very fast positronium (bound state of electron and positron) through a foil

$$\frac{P_{pos}}{2m_e} \cdot \frac{1}{\Delta E(\sim few\ m_e\alpha^2)} \gg L(foil)$$

first qualitative discussion - Nemenov, 1981, quantitative treatment Frankfurt and MS 91)



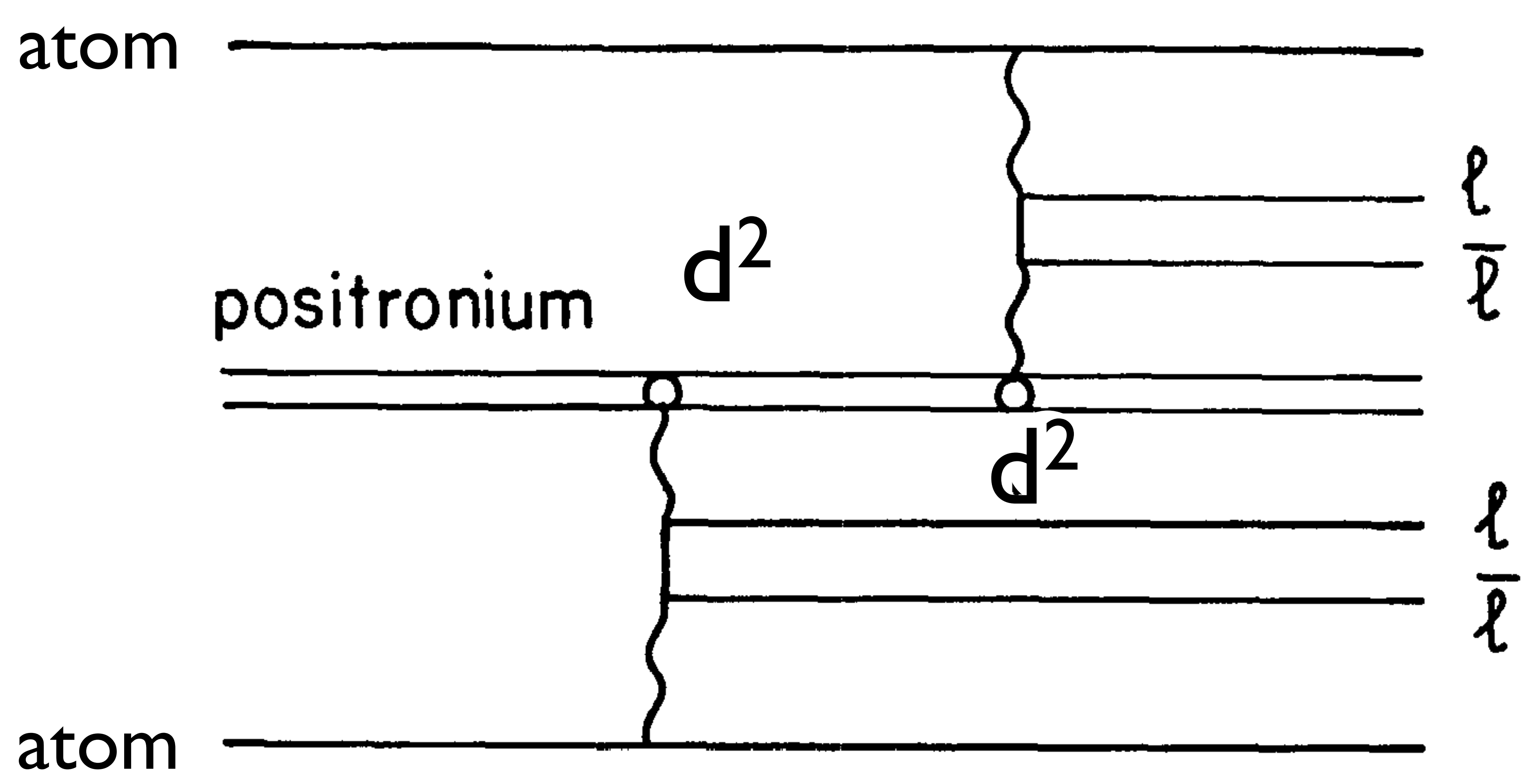
For the positronium at high energies transverse size is frozen during traversing through the foil - so interaction is of dipole-dipole type $\sigma(d) \propto d^2$ where $d = r_t^e - r_t^{e^+}$

Amplitude of i → f transition: $|M_{if}| = \left[\int d^3r \Psi_{pos} \Psi_f^* \exp(-\sigma(d)\rho L/2)\right]^2$

For large L: survival probability $\frac{16}{(<\sigma>\rho L)^2}$ absorption is not exponential !!!

Even larger probability to transform to electron - positron pair of the same momentum as positronium $\frac{2}{<\sigma>\rho L}$

Can we instead trigger on larger than average size configuration in positronium?

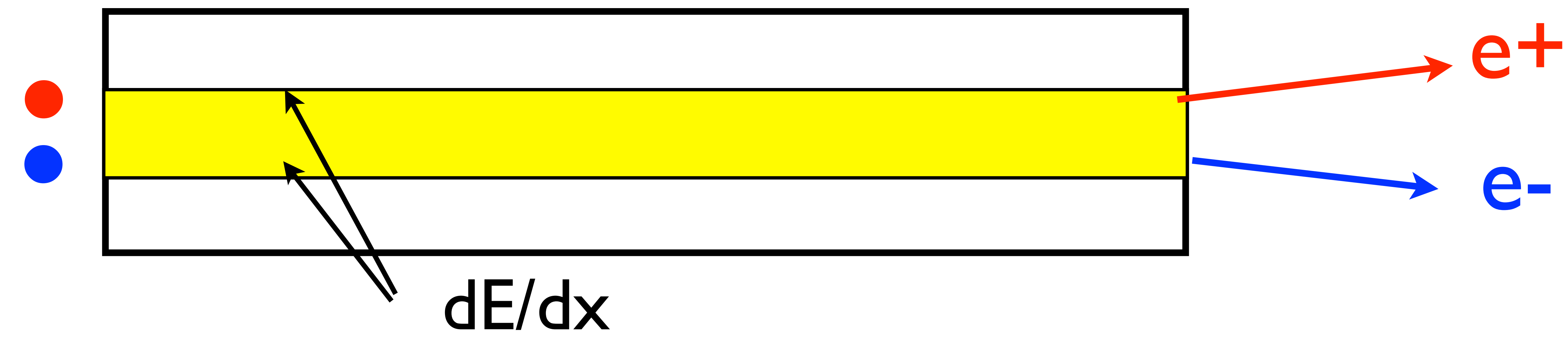


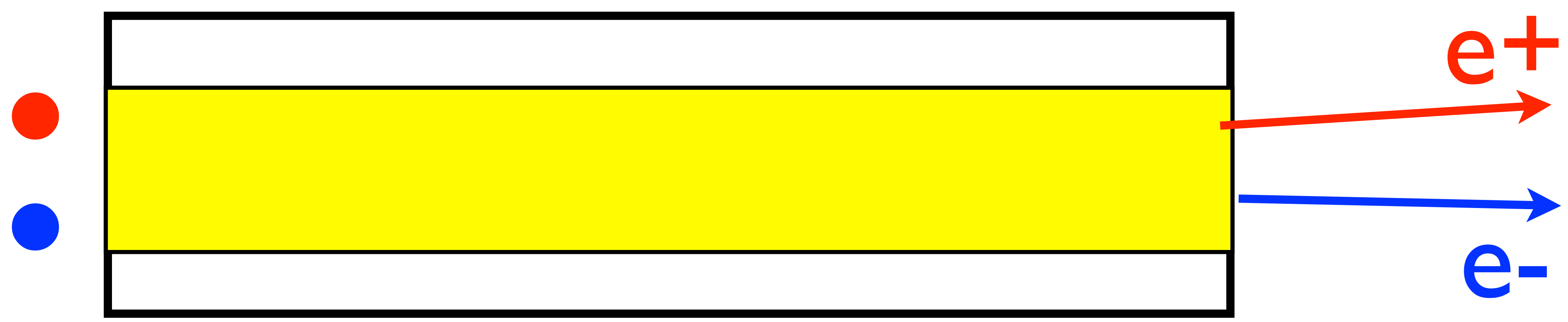
Consider production of one (two) lepton pairs with small momenta in the center of mass:
$\langle d^2 \rangle$ for these events is larger than in $\Psi^2_{pos}(d) = \int \Psi^2_{pos}(r)dz$ ➡ $\langle d^2_{2l\bar{l}} \rangle > \langle d^2_{l\bar{l}} \rangle > \langle d^2 \rangle$

Effects:

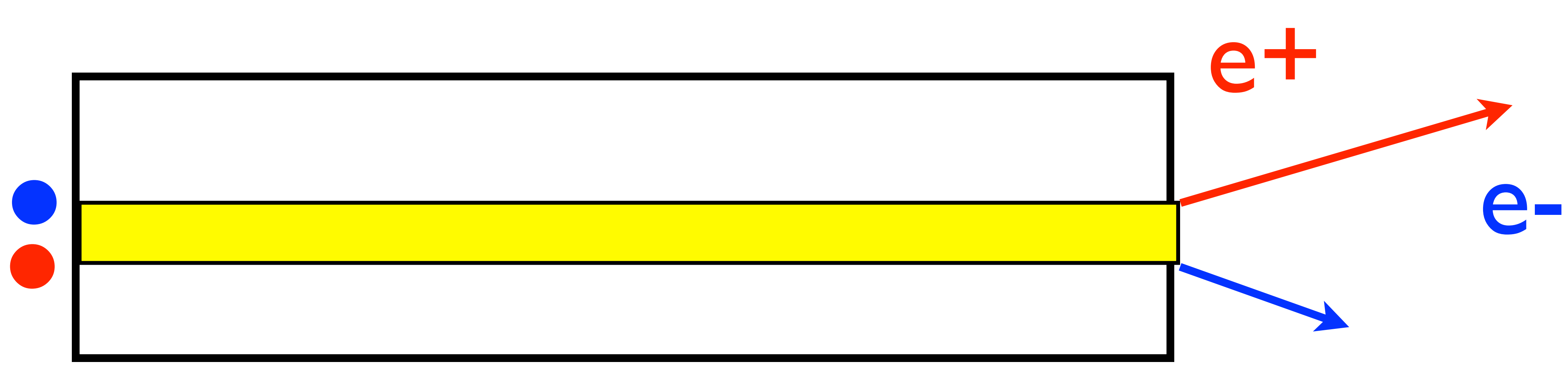
- *Positive correlation between production of one and two pairs*
- *Correlation between energy release along the positronium path and final momenta of e- e+ (next slide)*

Average configuration of incoming positronium





Post selection /Trigger on large d - large energy release along the path in the media -selects smaller than average transverse and longitudinal momenta in positronium - longitudinal momenta of electrons in the positronium fragmentation are softer (x-1/2 closer to 0)- looks as energy loss - but actually post selection.



Trigger on high p_t electron or electron with $x > 1/2$ (fraction of momentum of positronium carried by electron post selects events where excitations along the path were small.

Will discuss later similar effects for proton - nucleus interactions

$\Rightarrow$ The non exponential behavior is a manifestation of high energy coherence - slow down of space-time evolution

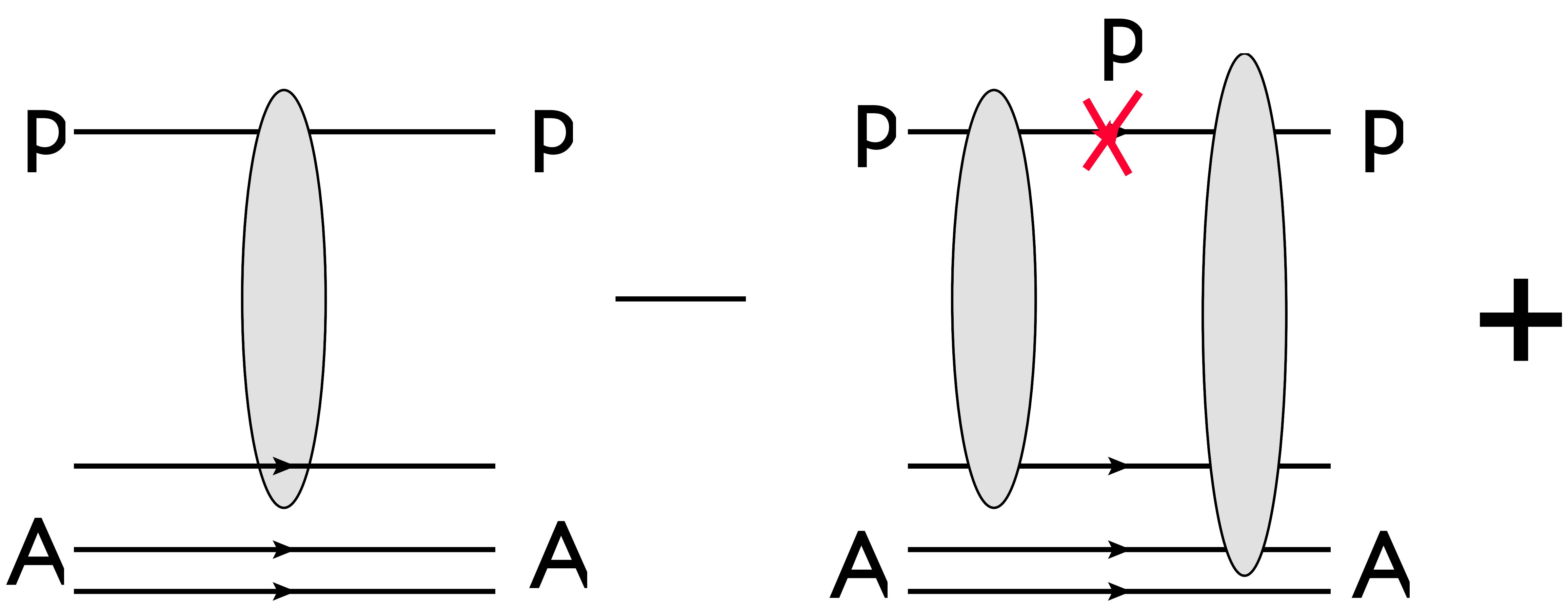
$\Rightarrow$ Various triggers allow to change proportion of small and large configurations in the data sample

$\Rightarrow$ Filtering enhances production of electron positron pairs with the same longitudinal momentum as the initial positronium but with transverse momenta >> than average momenta in positronium - an analog of inelastic ``hard" diffraction which I will discuss briefly

$\Rightarrow$ *Inelastic processes are sensitive to presence of large & small size configurations in projectile - longer the target (nucleus) --higher the sensitivity.*

Questions to the positronium example?

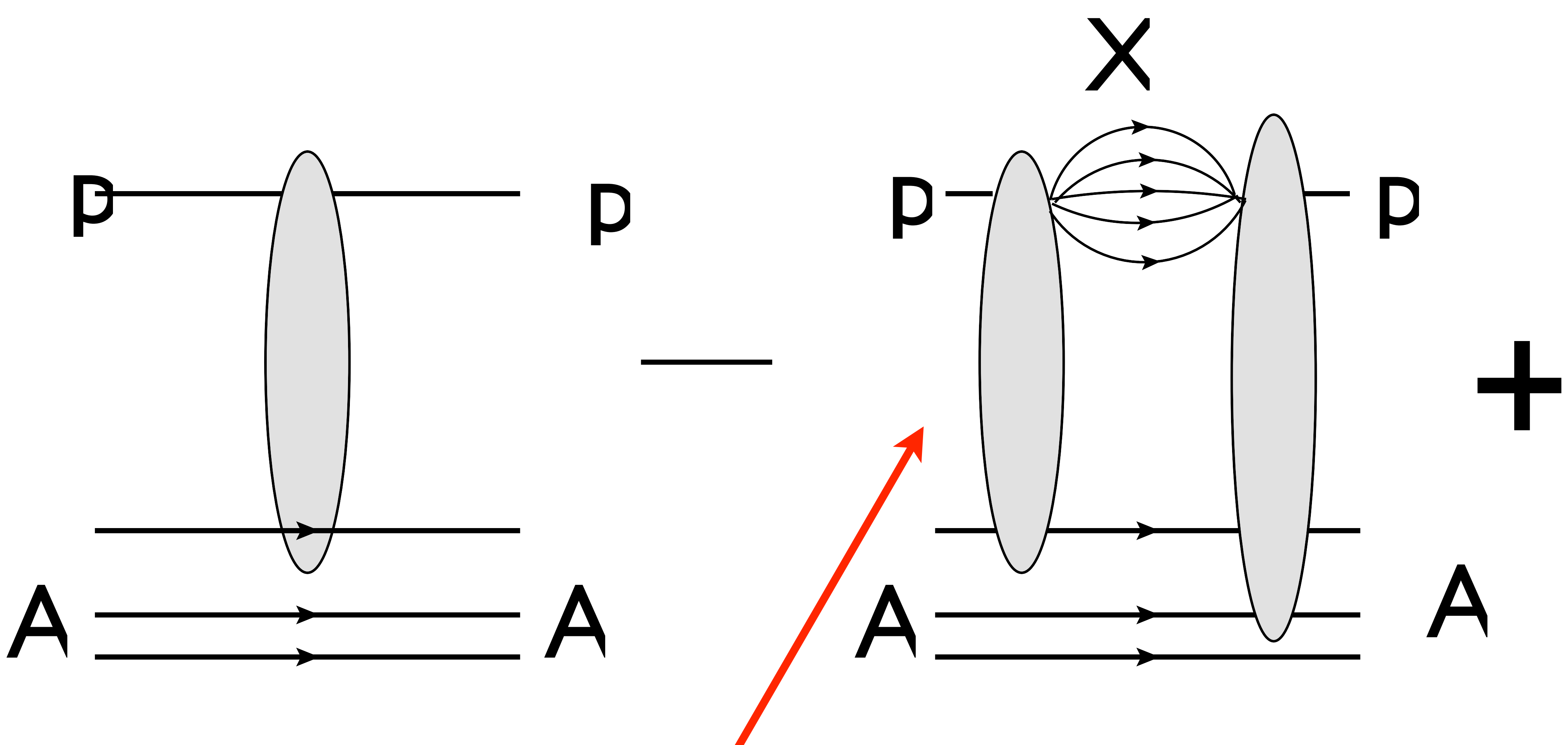
Formal account of large l_{coh} ➟ $p\,A$ scattering is described by different set of diagrams:



Glauber model

in rescattering diagrams proton propagates in intermediate state - zero at high energy - cancelation of planar diagrams (Mandelstam & Gribov)- no time for a proton to come back between interactions.

High energies = Gribov -Glauber model



X= set of frozen intermediate states the same as in pN diffraction

deviations from Glauber are small for E_{inc} < 10 GeV as inelastic diffraction is still small.

$$\sigma_2 \propto \int dt F_A^2(t) \frac{d\sigma(p+p \to p + X(p + \ inel\ diff))}{dt}$$

Convenient quantity - $P(\sigma)$ -probability that nucleon interacts with cross section σ with the target.

$$\int P(\sigma)d\sigma = 1, \quad \int \sigma P(\sigma)d\sigma = \sigma_{tot},$$

$$\left.\frac{\frac{d\sigma(pp\to X+p)}{dt}}{\frac{d\sigma(pp\to p+p)}{dt}}\right|_{t=0} = \frac{\int(\sigma-\sigma_{tot})^2 P(\sigma)d\sigma}{\sigma_{tot}^2} \equiv \omega_\sigma$$

variance

Pumplin &Miettinen

$$\int (\sigma - \sigma_{tot})^3 P(\sigma)d\sigma = 0,$$

Baym et al from pD diffraction

$$P(\sigma)_{|\sigma\to 0} \propto \sigma^{n_q-2}$$

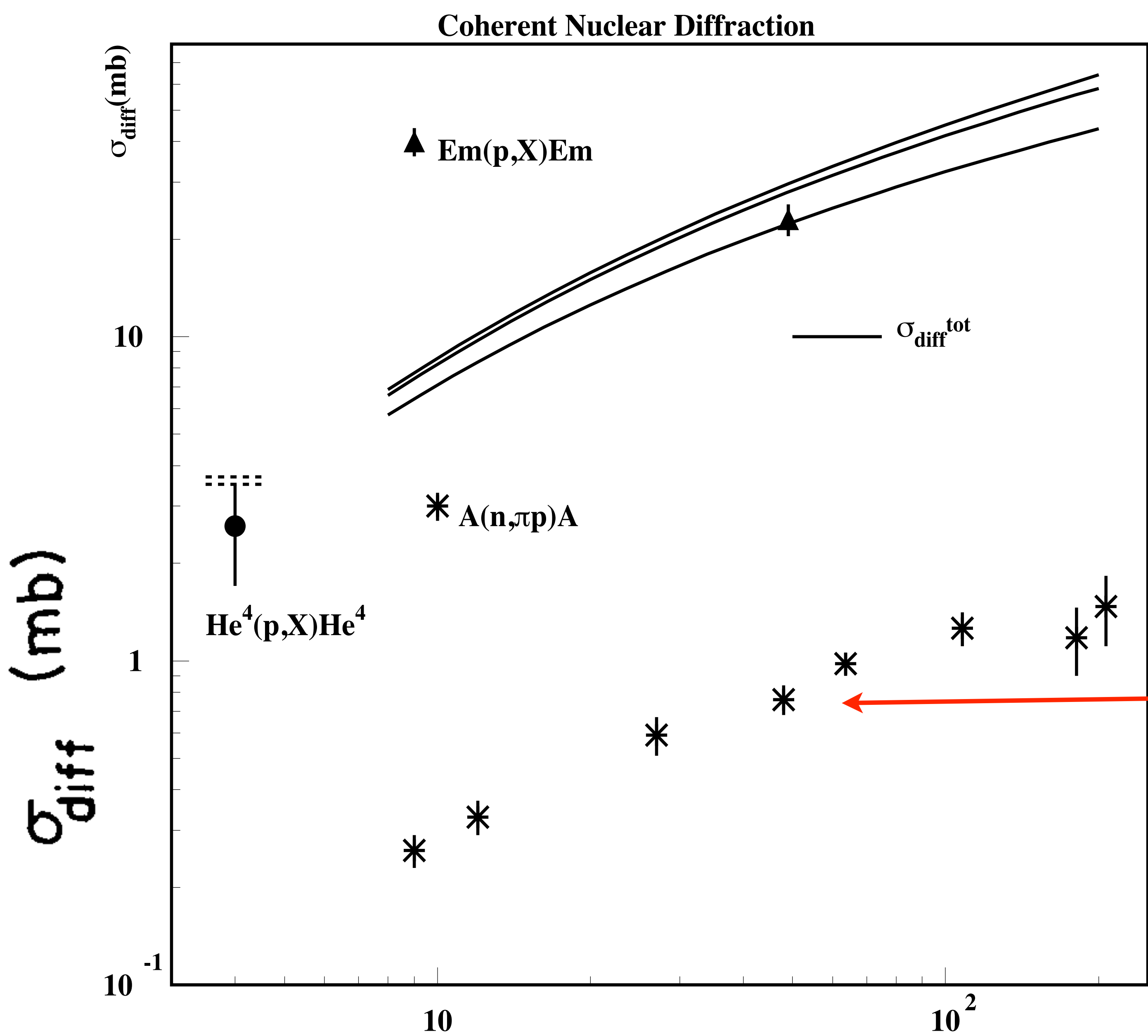
Baym et al 1993 - analog of QCD counting rules
probability for all constituents to be in a small transverse area

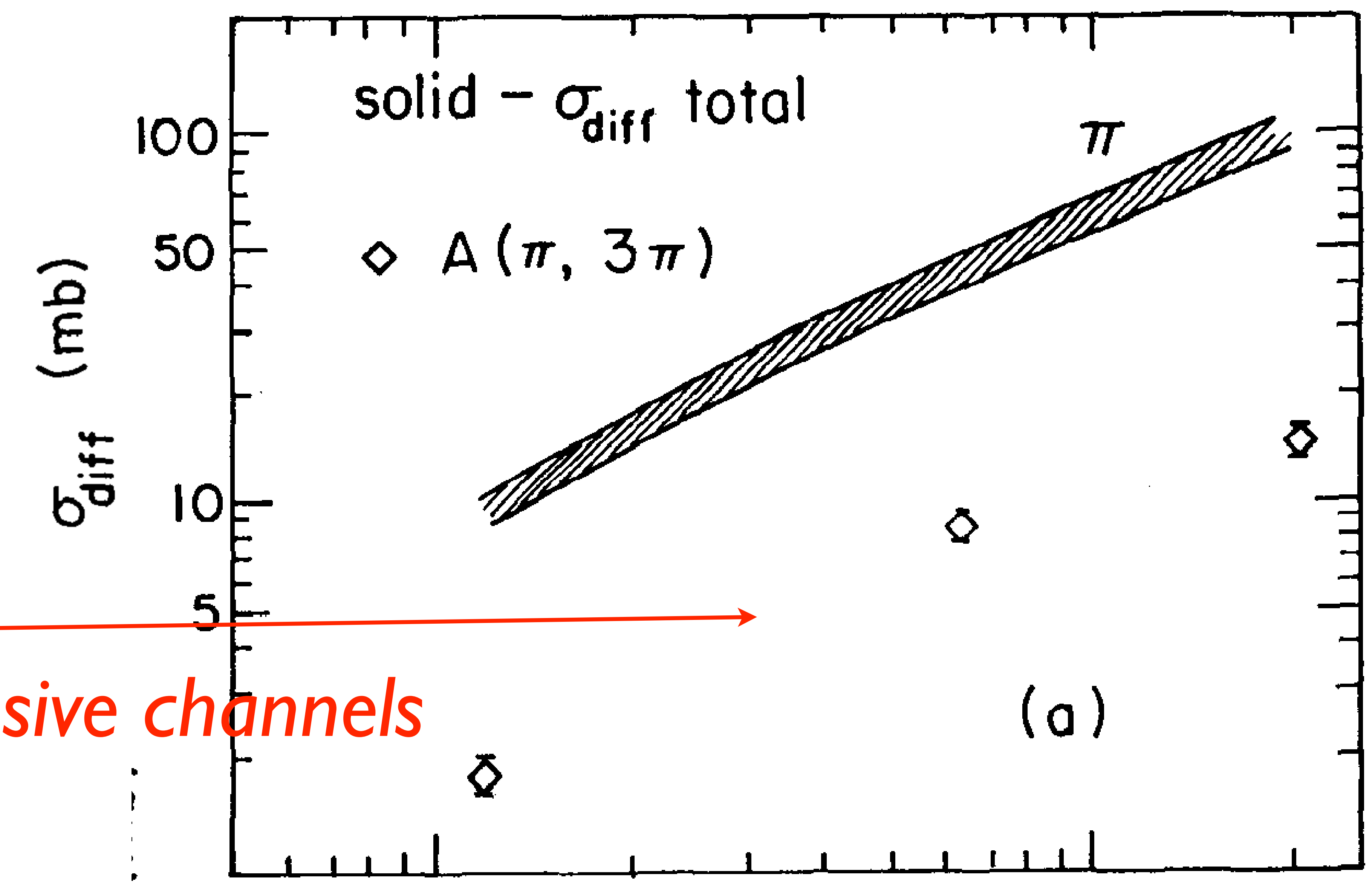
+ additional consideration that *for a many body system fluctuations near average value should be Gaussian*

$$P_{\bar{N}}(\sigma_{tot}) = r\,\frac{\sigma_{tot}}{\sigma_{tot}+\sigma_0}\,exp\{-\frac{(\sigma_{tot}/\sigma_0 - 1)^2}{\Omega^2}\}$$

Test: calculation of coherent diffraction off nuclei: $\pi A\to XA$, $p A\to XA$ through $P_h(\sigma)$

Test: Calculate inelastic diffraction off nuclei - no free parameters





exclusive channels

The inelastic small t coherent diffraction off nuclei provides one of the most stringent tests of the presence of the fluctuations of the strength of the interaction in NN interactions. The answer is expressed through $P(\sigma)$ - probability distribution for interaction with the strength σ. (Miller &FS 93)

$$\sigma_{diff}^{hA} = \int d^2b\left(\int d\sigma P_h(\sigma)|\langle h|F^2(\sigma,b)|h\rangle| - \left(\int d\sigma P(\sigma)|\langle h|F(\sigma,b)|h\rangle|\right)^2\right).$$

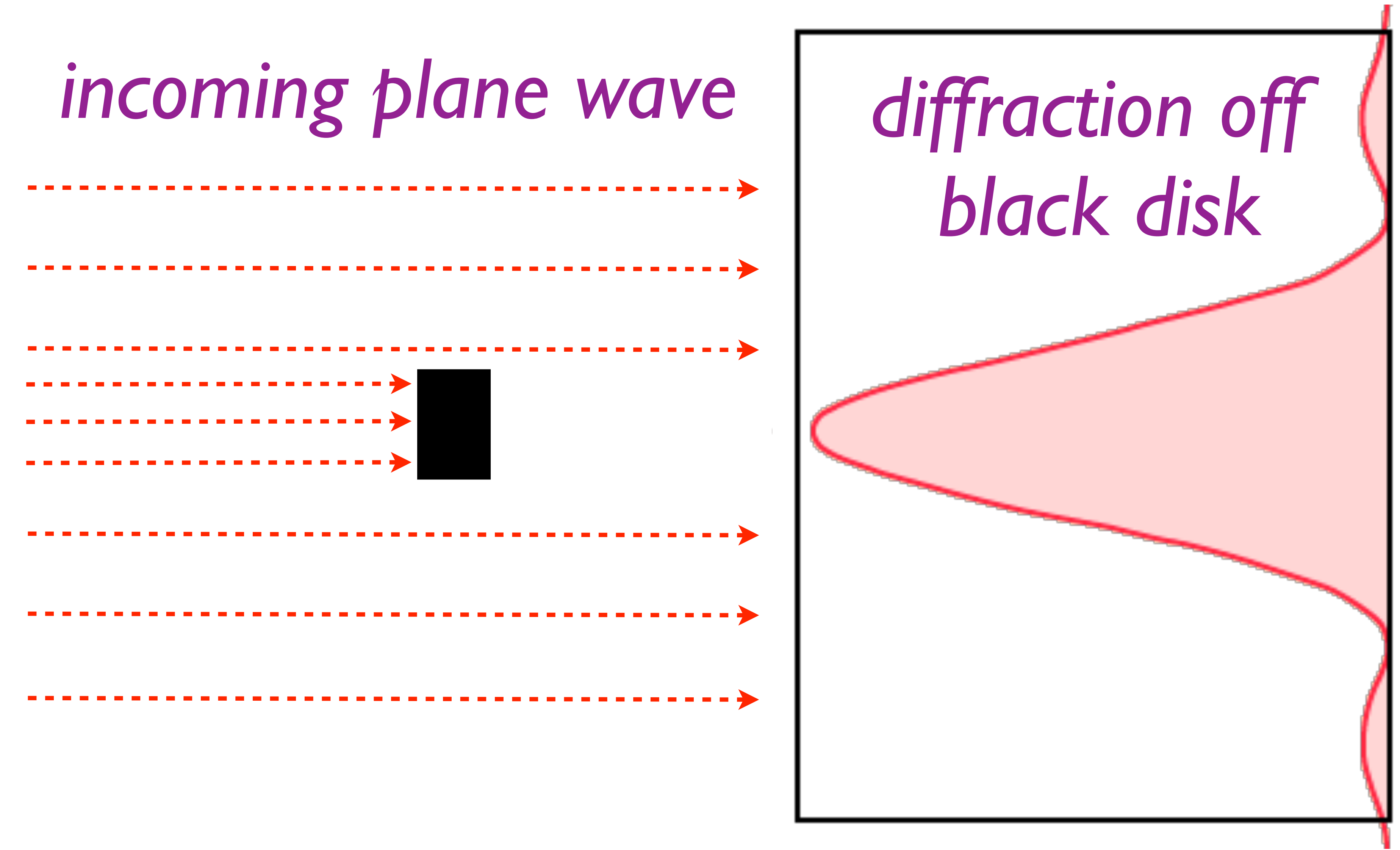
Here $F(\sigma,b) = 1 - e^{-\sigma T(b)/2}$, $T(b) = \int_{-\infty}^{\infty} \rho_A(b,z)dz$, and $\rho_A(b,z)$ is the nuclear density.

Why there is a connection between inelastic diffraction and variance of P(σ):

$$\left.\frac{\frac{d\sigma(pp\to X+p)}{dt}}{\frac{d\sigma(pp\to p+p)}{dt}}\right|_{t=0} = \frac{\int(\sigma-\sigma_{tot})^2 P(\sigma)d\sigma}{\sigma_{tot}^2} \equiv \omega_\sigma$$

Elastic diffraction is well known
in e.m. interactions and quantum mechanics:

projectile + Target → projectile + Target



Amazing high energy phenomenon of inelastic diffraction:

projectile + Target → inelastic state + Target

Target remains practically at rest, but projectile is excited!!!!

"Fluctuation of strength" explanation was put forward long time ago by Pomeranchuk & Feinberg, Good and Walker, Pumplin &Miettinen. Connected to color fluctuations in QCD in Baym, Bluttel, Frankfurt and MS in ~93.

If there were no fluctuations of strength of interaction - there will be no inelastic diffraction at t=0 (qualitative explanation)

$$|h\rangle = a_1 |1\rangle + a_2 |2\rangle$$

h

absorber with same absorption strength for "1" and "2"

h

$$|final\rangle = \lambda(a_1 |1\rangle + a_2 |2\rangle) = \lambda |h\rangle$$

only elastic scattering

$$|h\rangle = a_1 |1\rangle + a_2 |2\rangle$$

h

absorber with different absorption strength for "1" and "2"

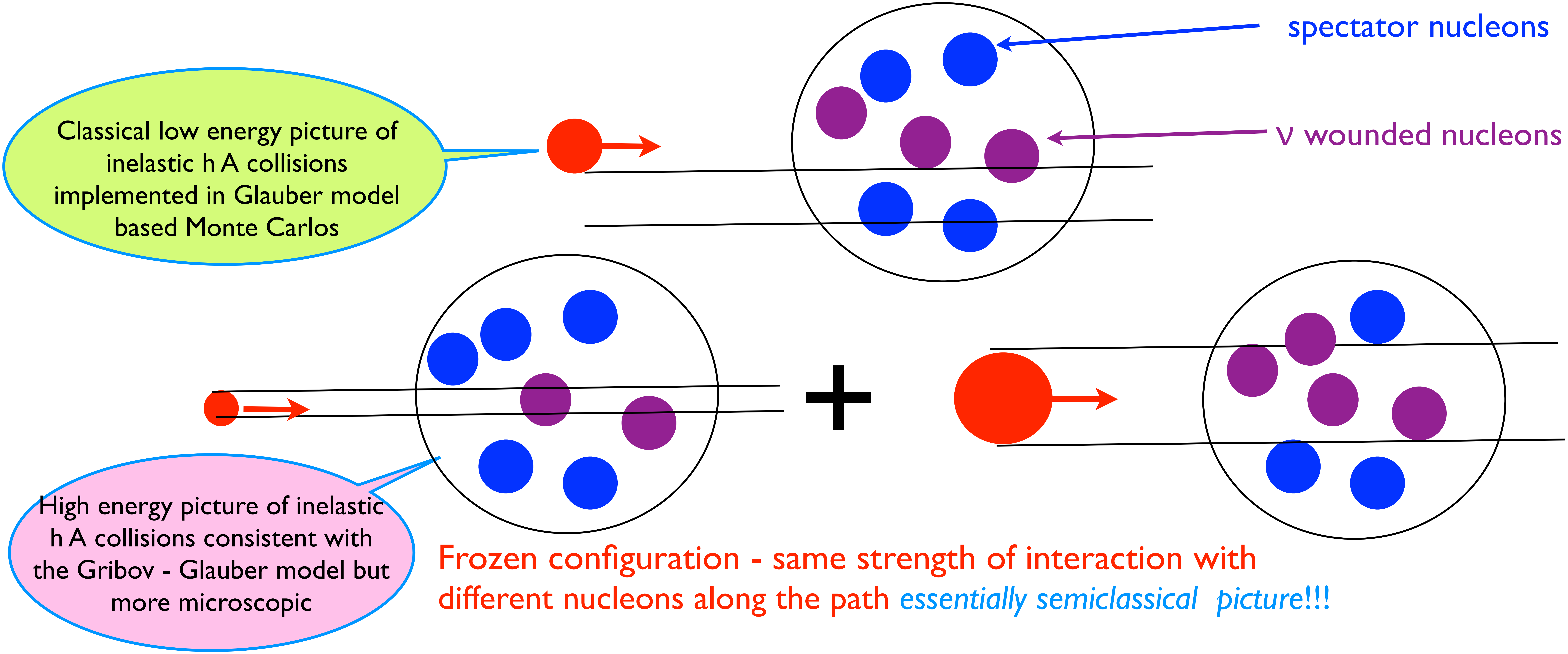
h+h'

$$|final\rangle = \lambda_1 a_1 |1\rangle + \lambda_2 a_2 |2\rangle)$$
$$= c_1 |h\rangle + c_2 |h'\rangle$$

elastic scattering

+inelastic diffraction

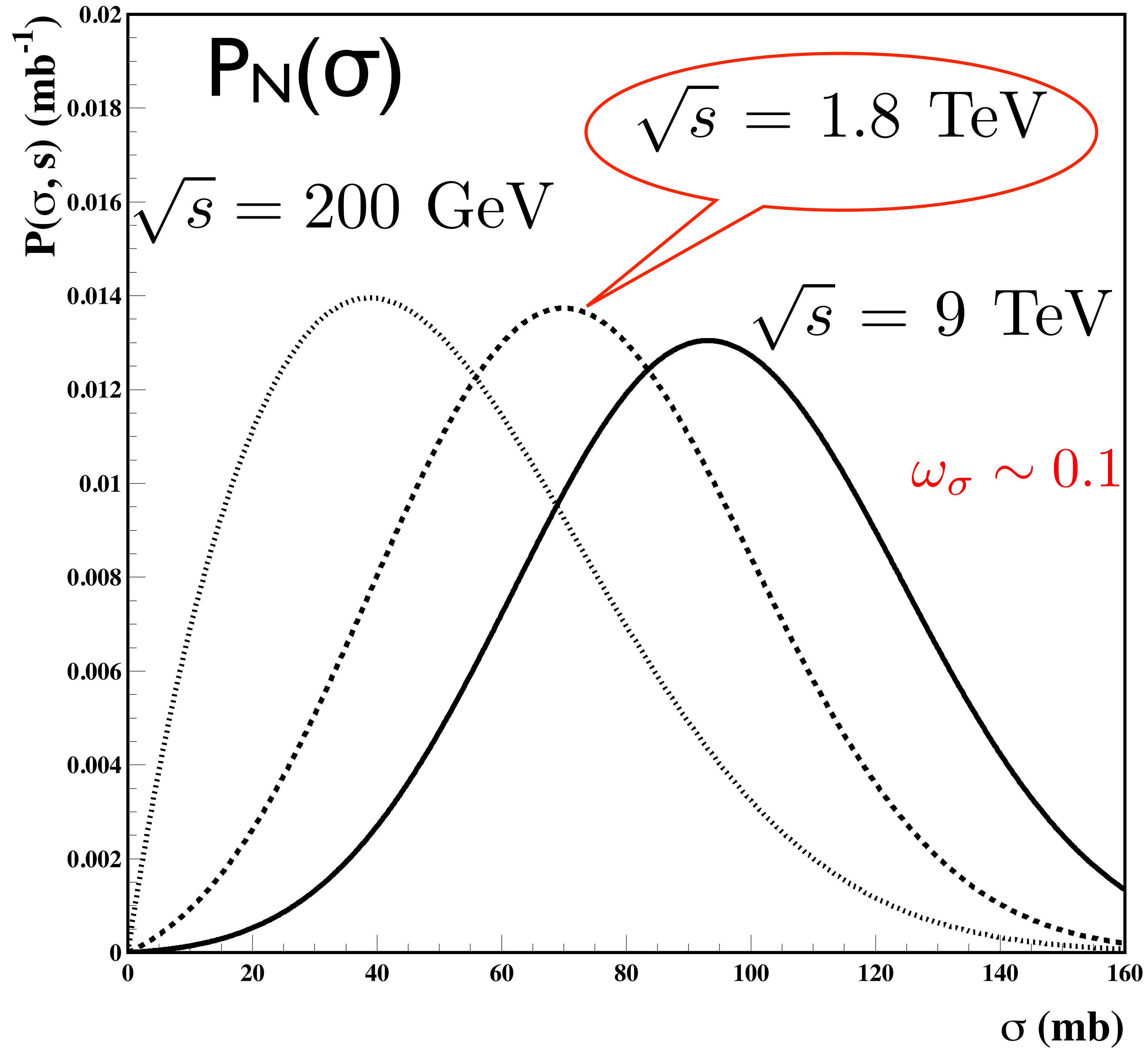
Constructive way to account for coherence of the high-energy dynamics is
Fluctuations of interaction cross section formalism.



Frozen configuration - same strength of interaction with different nucleons along the path *essentially semiclassical picture!!!*

$$\sigma_\nu = \int d\sigma P_h(\sigma) \cdot \frac{A!}{(A-\nu)!\,\nu!} \cdot \int d\boldsymbol{b} \left(\sigma T(b)/A\right)^\nu \left[1-\sigma T(b)/A\right]^{A-\nu}$$

simplified expression (optical limit)



Extrapolation of Guzey & MS before the LHC data

consistent with LHC data which are still not too accurate

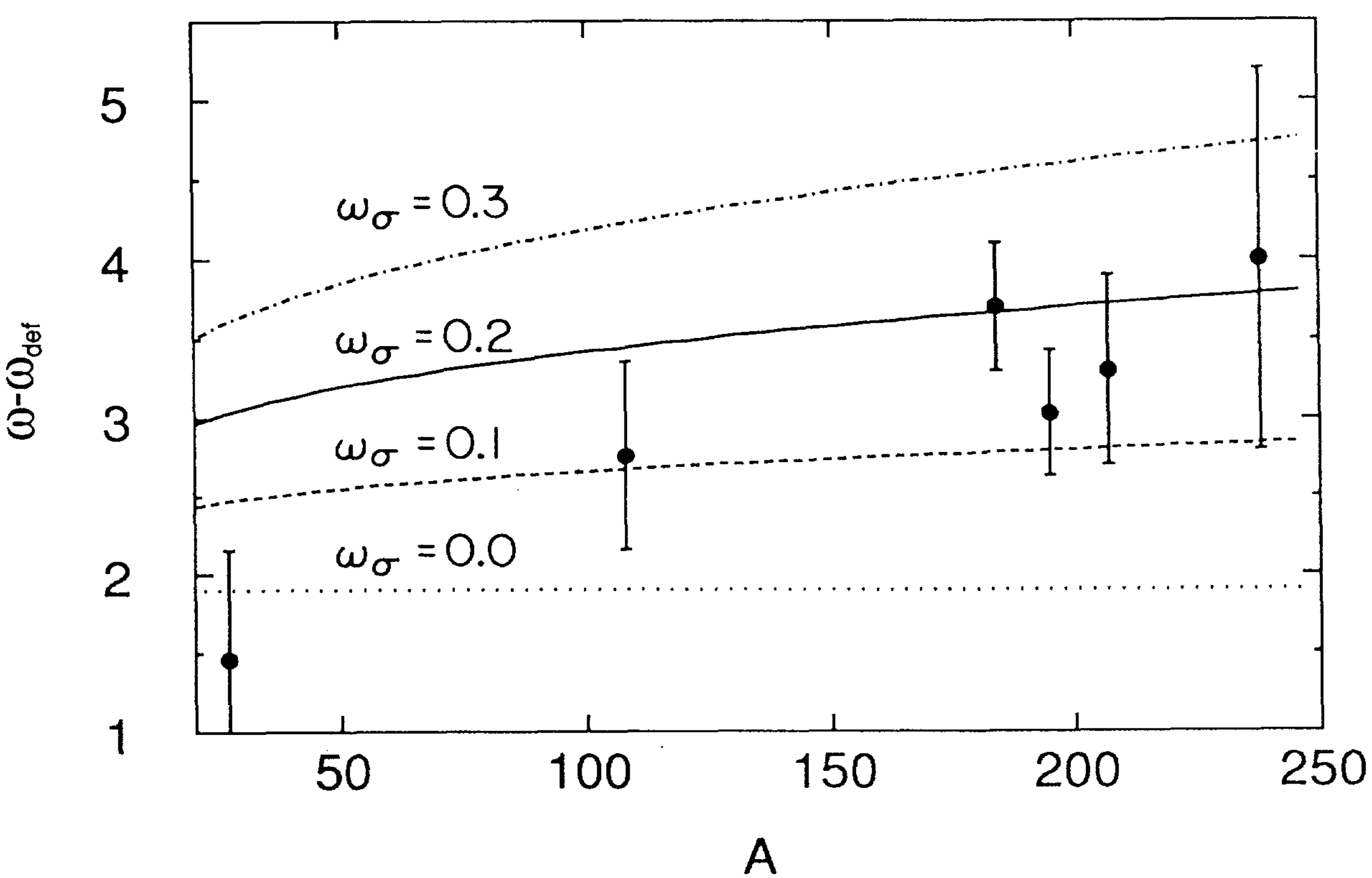
Qualitative expectation: CF increase fluctuations of a number of observables in pA and AB collisions.

First example: study of dispersion of E_T distribution in AB collisions as superposition of emission from binary collisions with variance ω_0:

$$\omega - \omega_{def} = \omega_0 + 2 - \alpha - \beta + (N_{pB} + N_{pA} - \alpha - \beta)\omega_\sigma$$

nucl. deform.

nucl. corr.: $\alpha \sim \beta \sim 0.3$



Dispersion of E_T distribution in central ^{32}S A collisions at SPS at E/A =200 GeV

H. Heiselberg, G. Baym, B. Blattel, L. L. Frankfurt, "' and M. Strikman PRL 1991

Large fluctuations in the number of wounded nucleons at fixed impact parameter

Simple illustration - two component model ≡ quasieikonal approximation:

$$\sigma_{1,2} = (1 \pm \sqrt{\omega_\sigma}) \cdot \sigma_{tot}$$

LHC $\sigma_1 = 70\, mb,\ \sigma_2 = 130\, mb$

number of wounded nucleons at small b differs by a factor of 2 !!!

Scattering at b=4 fm with probability ~ 1/2 generates the same number of wounded nucleons as an average collision at b=0. *Smearing of the centrality. More details in the Alvioli talk.*

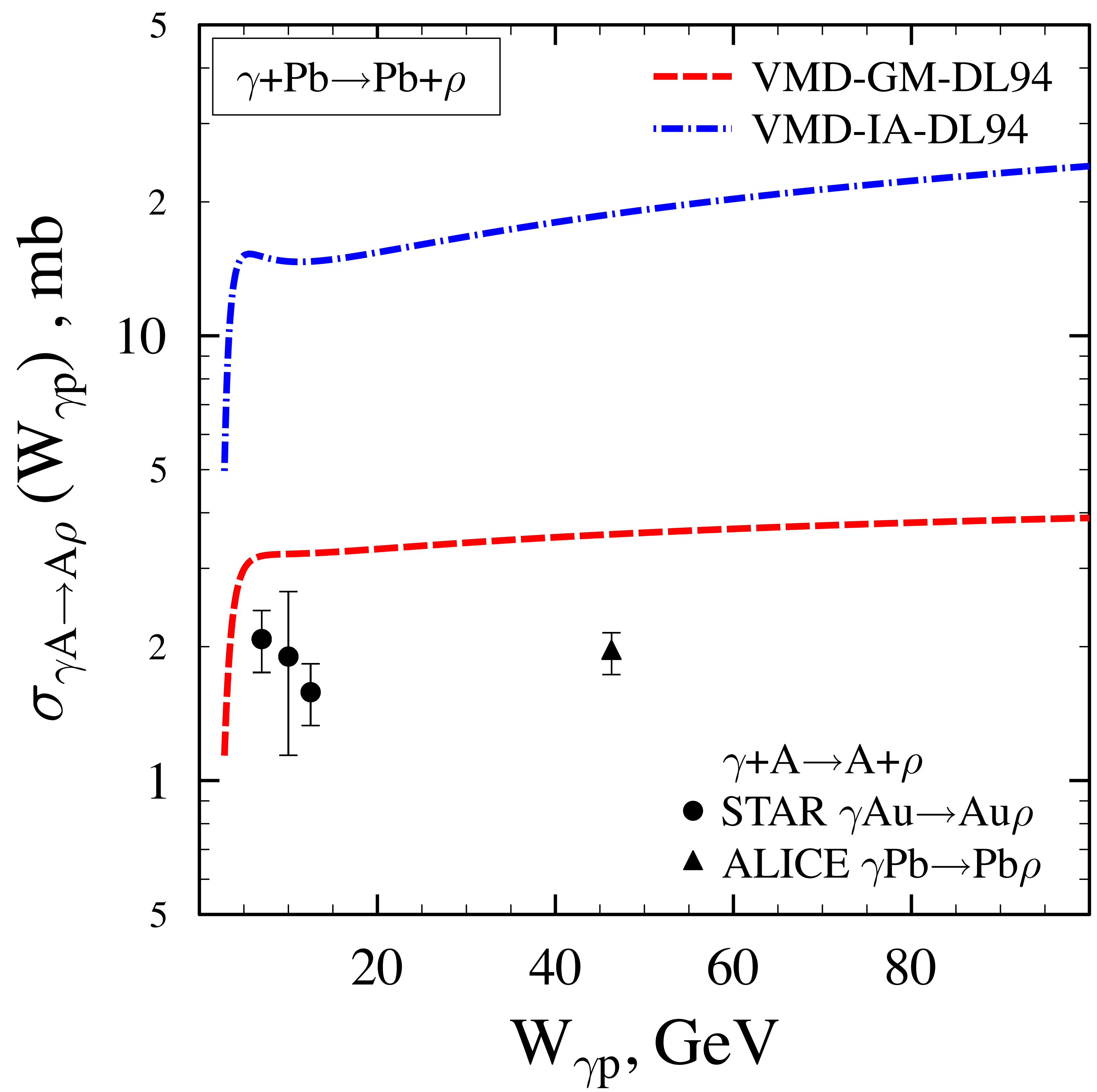
Fluctuations lead to broadening of the distribution over ν - number of participant nucleons as compared to Glauber model - reported by ATLAS and ALICE.

Large ν select configurations with larger than average σ

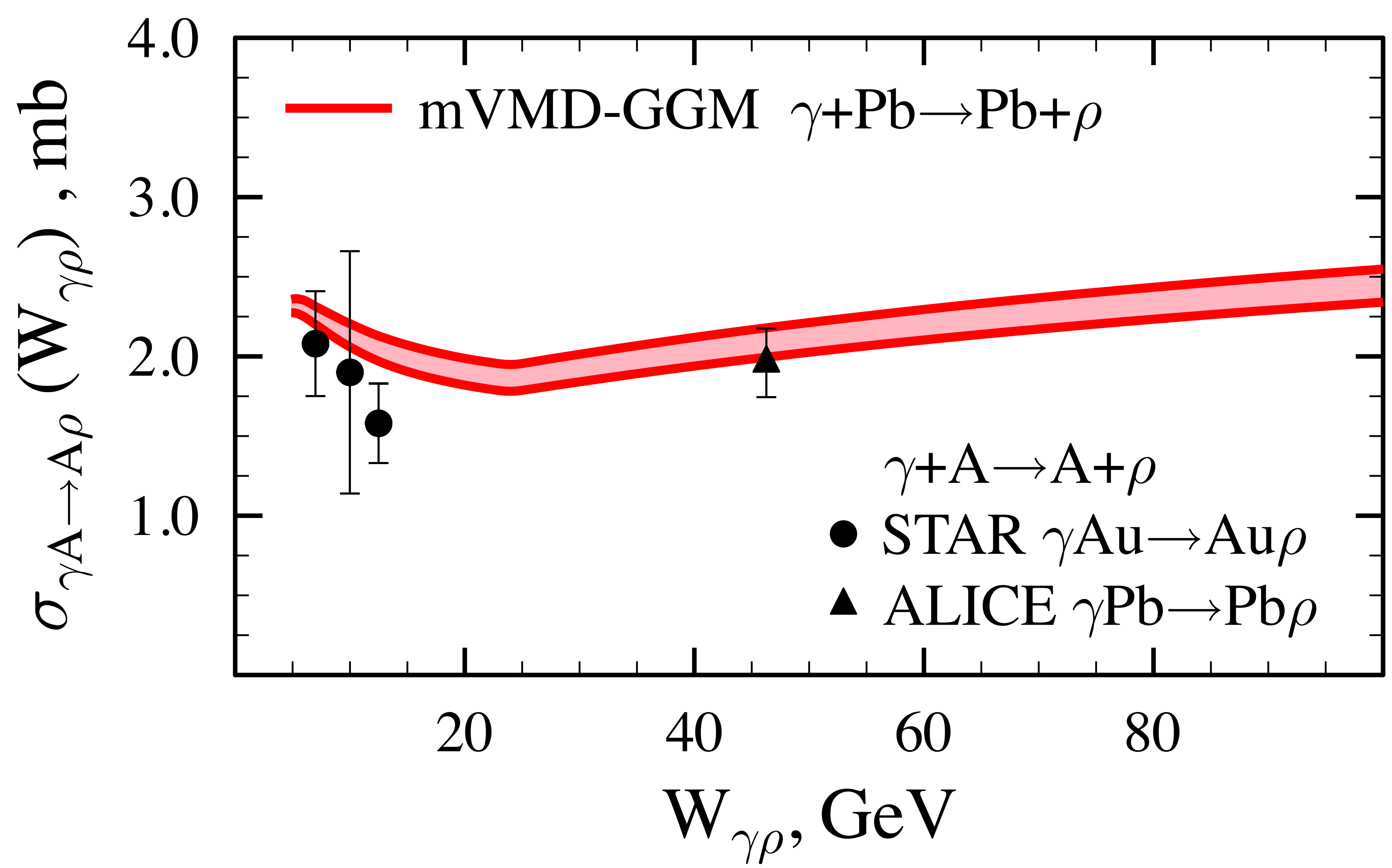
New experimental observation relevant for color fluctuation phenomenon: coherent photoproduction of ρ-meson in ultraperipheral heavy ion collisions at LHC (ALICE): $\gamma + A \to \rho + A$

Analysis of Guzey, Frankfurt, MS, Zhalov 2015 (1506.07150)

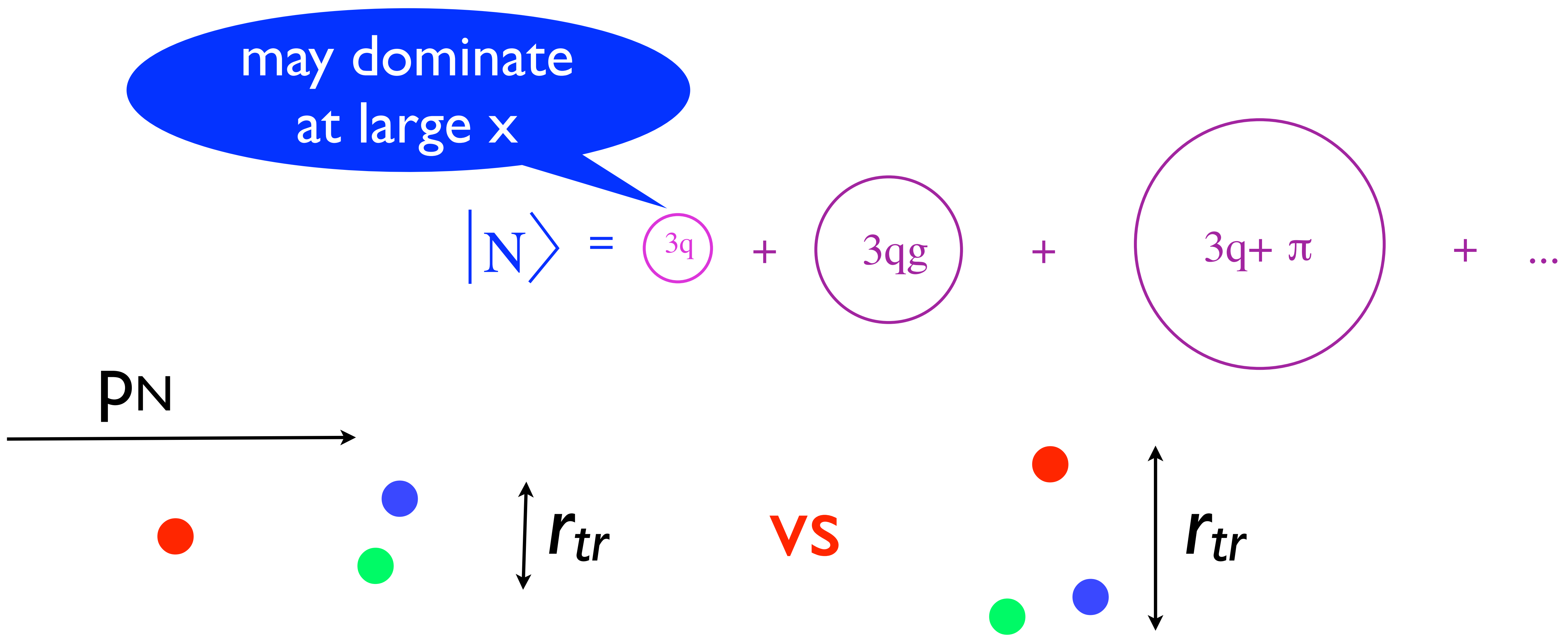
- Glauber model crossly overestimates the cross section

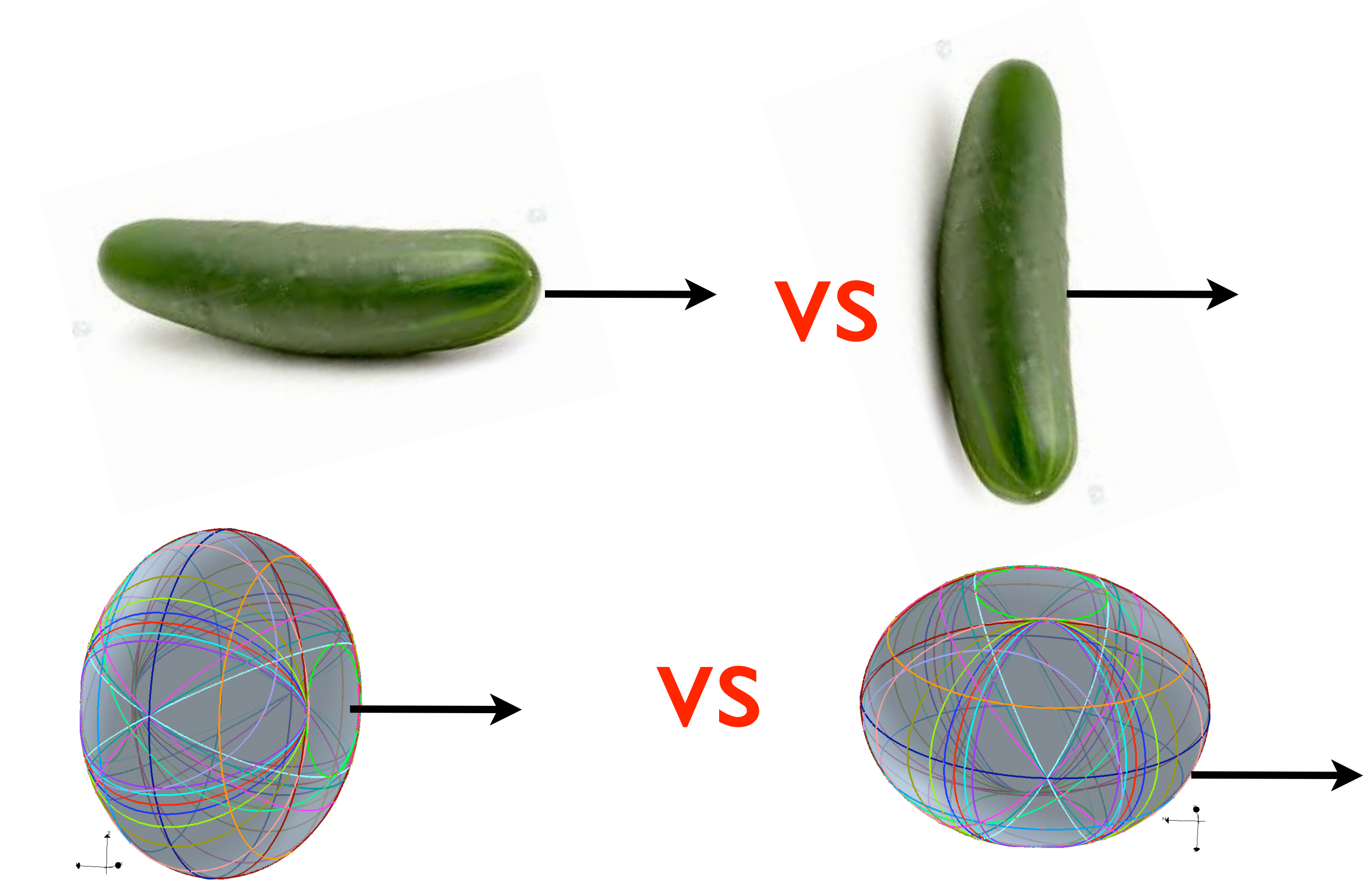


- Gribov - Glauber model with cross section fluctuations



There exist a number of dynamical mechanisms of the fluctuations of the strength of interaction of a fast nucleon/pion: fluctuations of the size, number of valence constituents, orientations





Localization of color certainly plays a role - so we refer to the fluctuations generically as color fluctuations.

Studing effects of CFs in pA aims at

(i) Mapping 3-dimensional global quark-gluon structure of the nucleon

(ii) Better understanding of the QCD dynamics of pA and AA collisions

Natural expectation is that there is a correlation between configuration of hard partons in the hadron and strength of interaction of the hadron:

π (ρ)-meson decay constants: f_π, f_ρ are determined by configuration with essentially no gluon field and of small transverse size

Operational success of quark counting rules → minimal Fock space configurations dominate at large x. Quarks in these configurations have to be close enough - otherwise generation of Weizsäcker -Williams gluons

IDEA

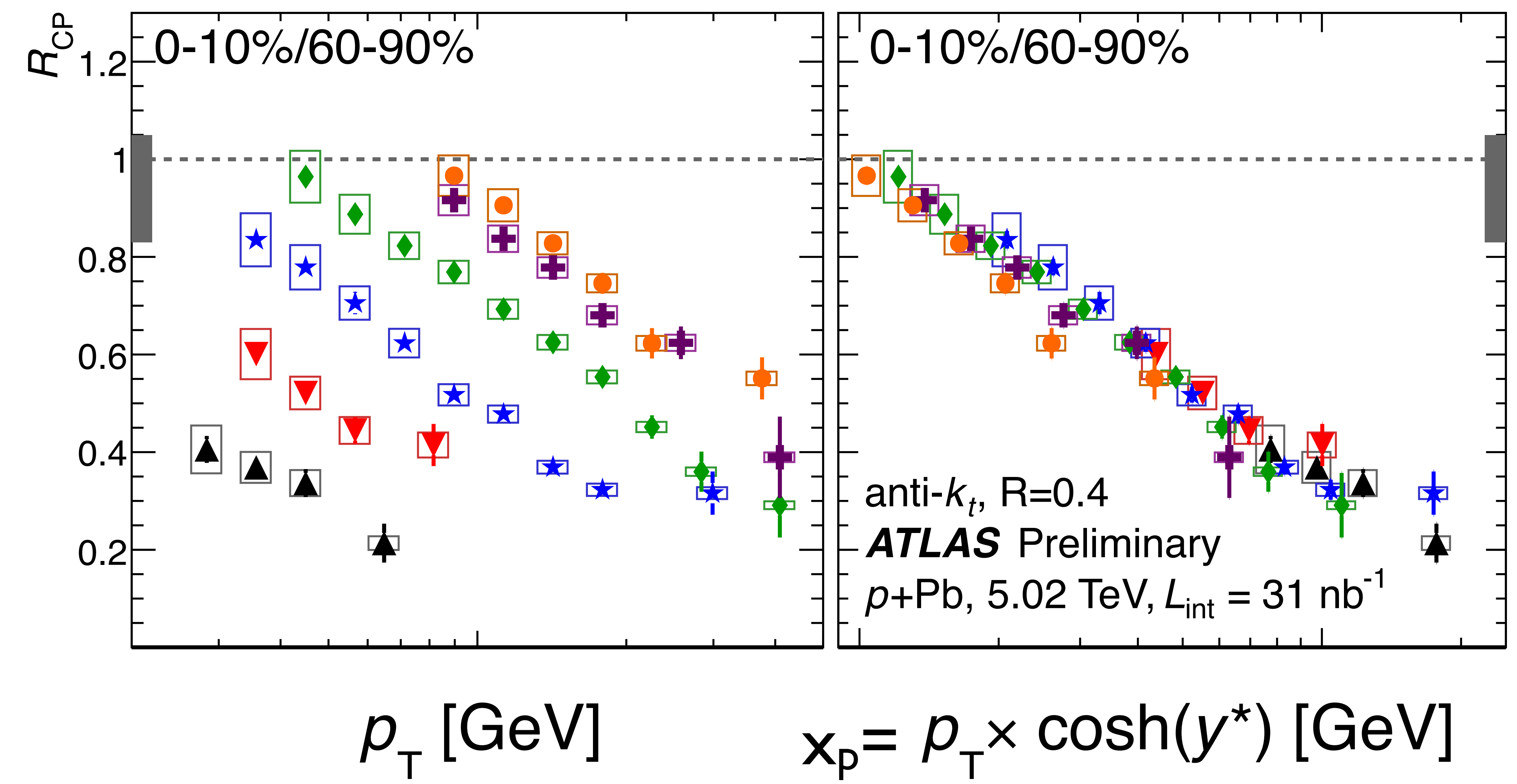
Use the hard trigger (dijet) to determine x of the parton in the proton (x_p) and low p_t hadron activity to measure overall strength of interaction σ_{eff} of configuration in the proton with given x FS83

Expectation: large x ($x \gtrsim 0.5$) correspond to much smaller σ → drop of # of wounded nucleons & overall hadron multiplicity for central collisions

Data - ATLAS & CMS on correlation of jet production and activity in forward rapidities

Key relevant observations:

- ✔ pQCD works fine for inclusive production of jets
- ✔ The jet rates for different centrality classes do not match geometric expectations. Discrepancy scales with x of the parton of the proton and maximal for large x_p

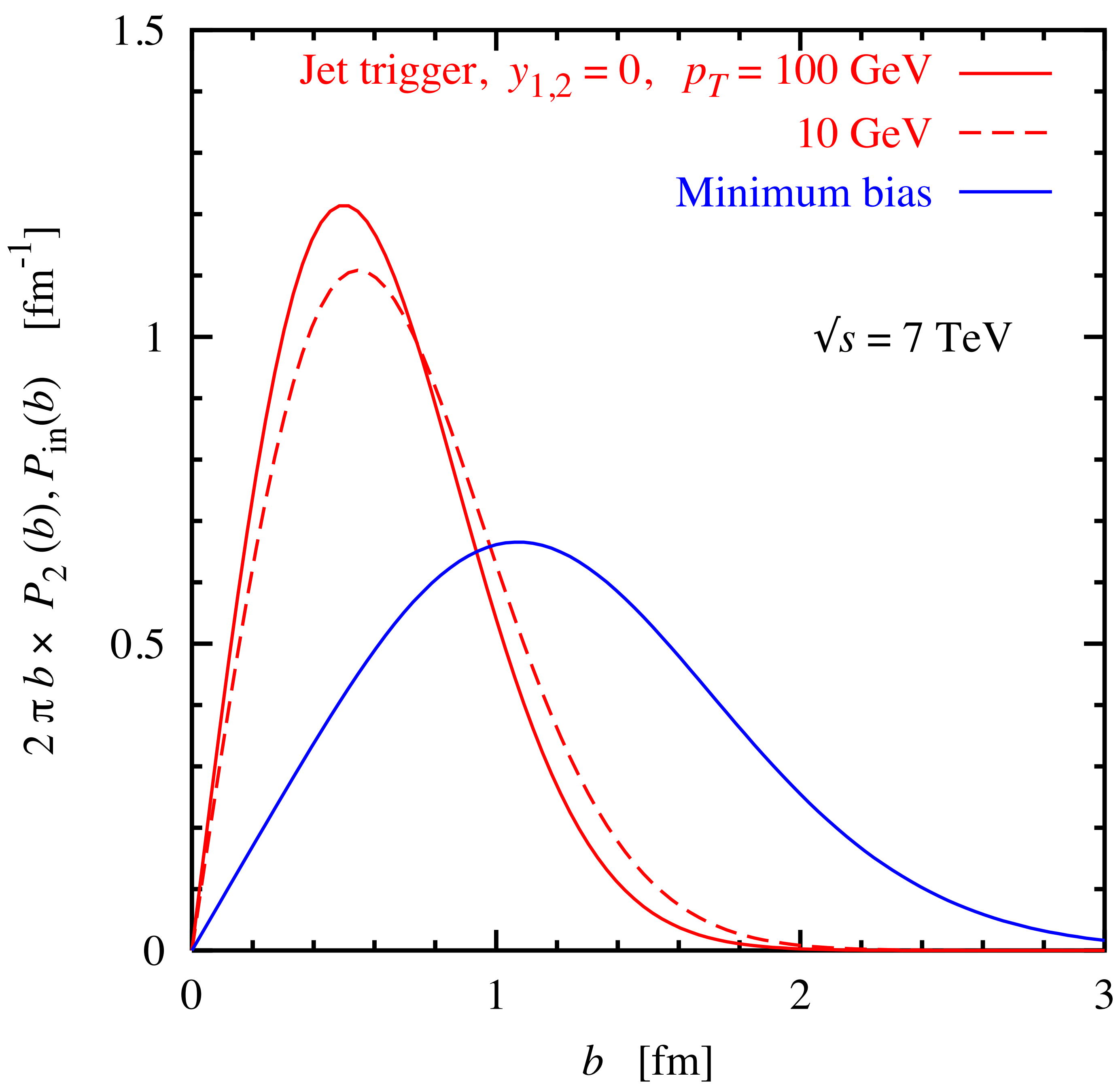


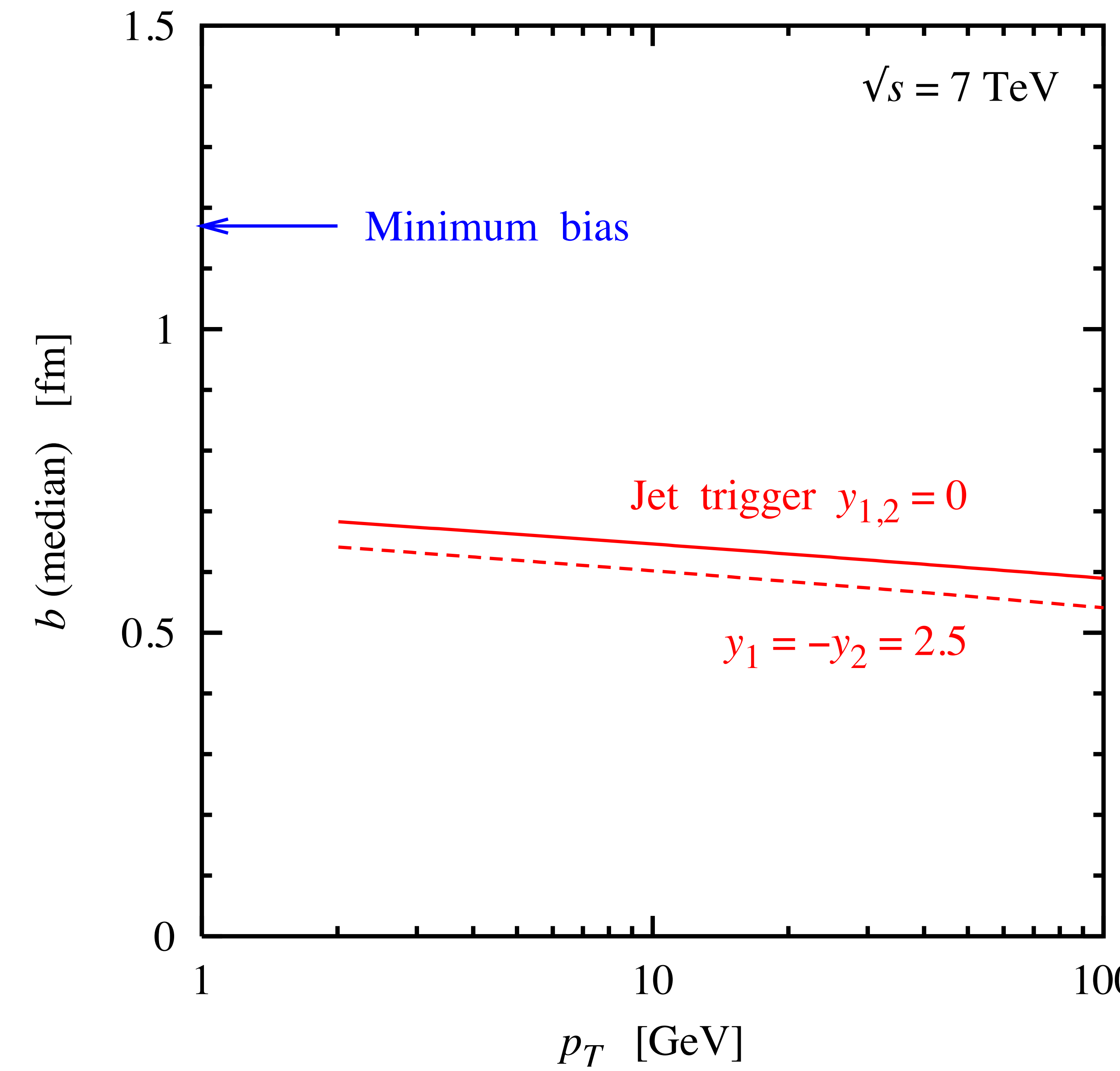
To calculate the expected CF effects accurately it is necessary to take into account grossly different geometry of minimum bias and hard NN collisions

LF, MS, Weiss 03 related hard dynamics in pp and DIS using generalized parton distribution extracted from analysis of exclusive hard processes

Two scale transverse dynamics of pp interactions at LHC -

Comparison of b -distributions for minimum bias and dijet collisions





Transverse area in which most of hard interactions occur in pp scattering is a factor of two smaller than that of minimum bias interactions

DISTRIBUTION OVER THE NUMBER OF COLLISIONS FOR PROCESSES WITH A HARD TRIGGER

M.Alvioli, L.Frankfurt, V.Guzey and M.Strikman, ``Revealing nucleon and nucleus flickering in pA collisions at the LHC,' Phys.Rev. C90 (2014) 3, 034914 arXiv 1402.2868

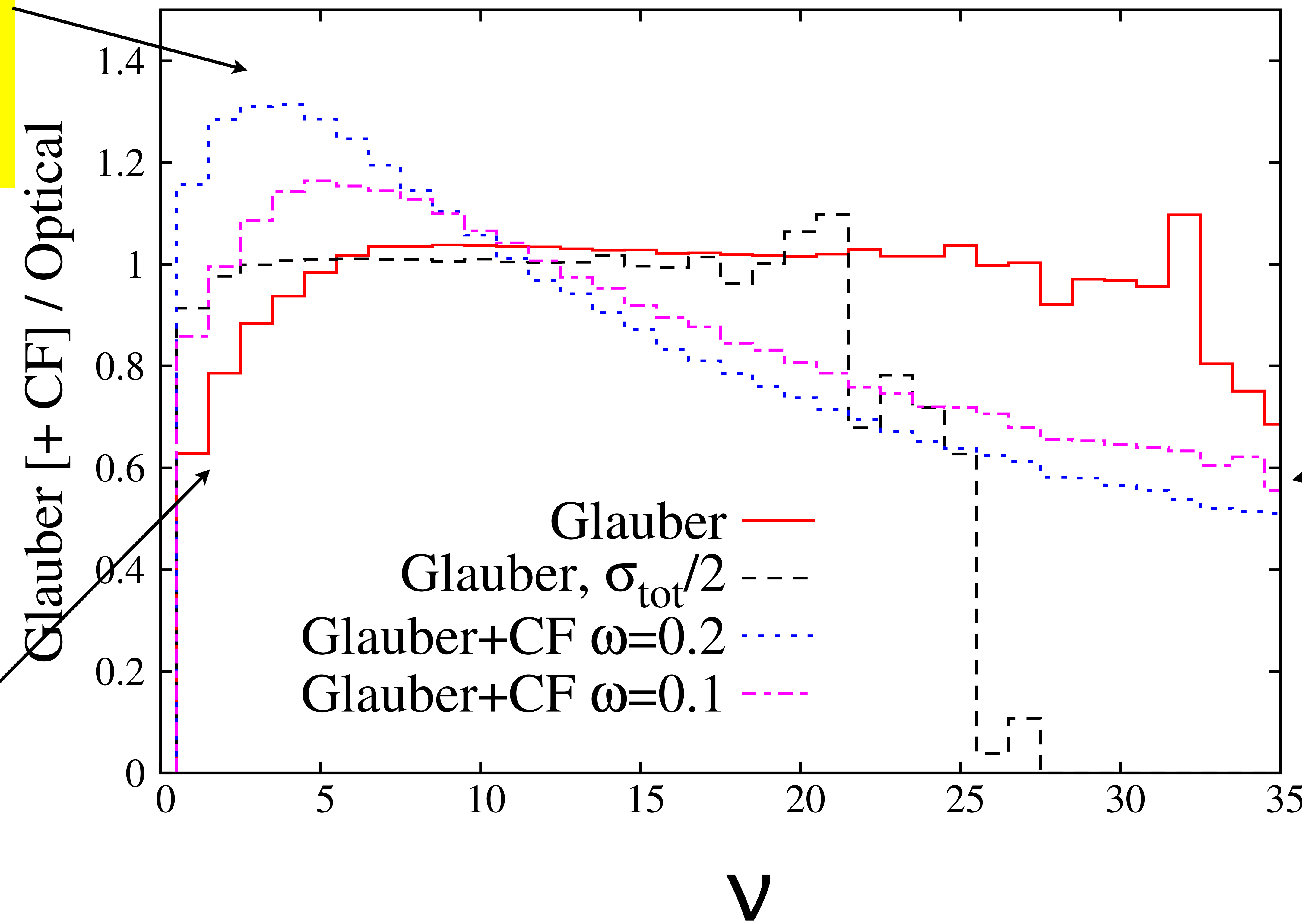
Consider multiplicity of hard events $Mult_{pA}(HT) = \sigma_{pA}(HT + X)/\sigma_{pA}(in)$ as a function of ν -- number of collisions

If the radius of strong interaction is small and hard interactions have the same distribution over impact parameters as soft interactions multiplicity of hard events:

$$R_{HT}(\nu) = \frac{Mult_{pA}(HT)}{Mult_{NN}(HT)\nu} = 1$$

Accuracy? Significant corrections due to presence of two transverse scale.

increase due to more central interactions of configurations with $\sigma < \sigma_{tot}$



drop due increased role of configurations with $\sigma > \sigma_{tot}$

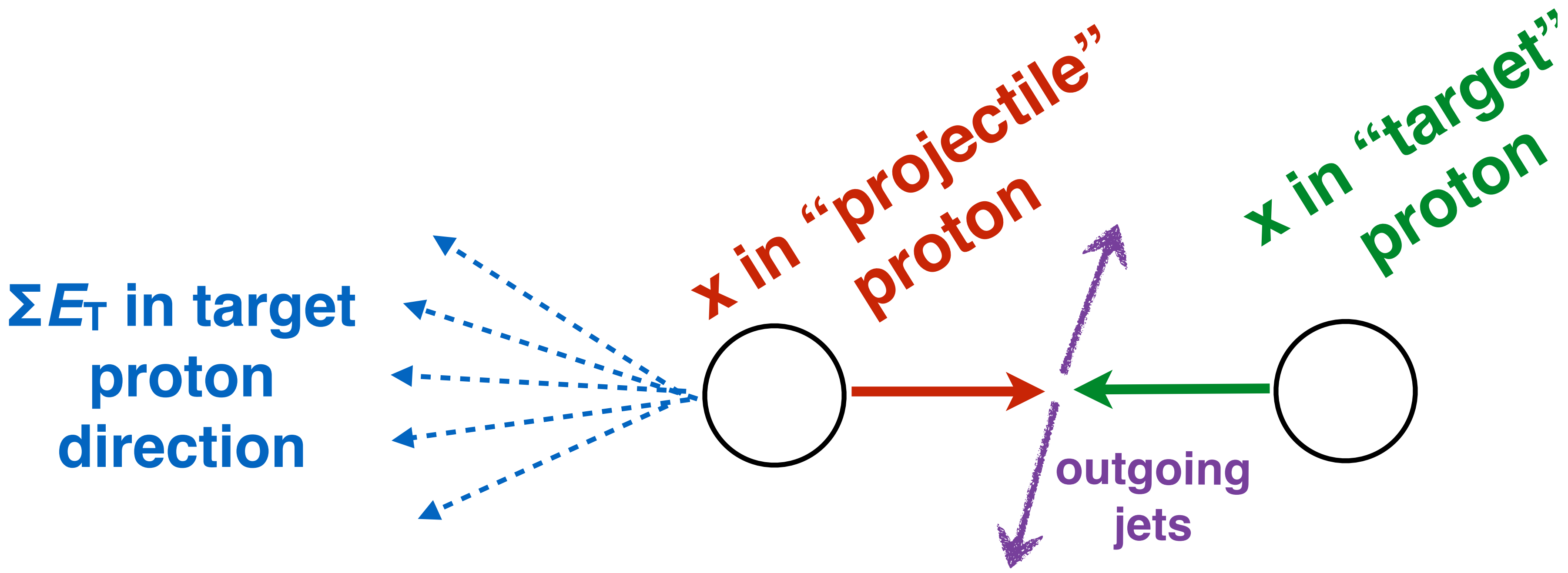
the cylinder in which interaction occur is larger but local density does not go up as fast in Glauber

drop due to more localized hard interactions

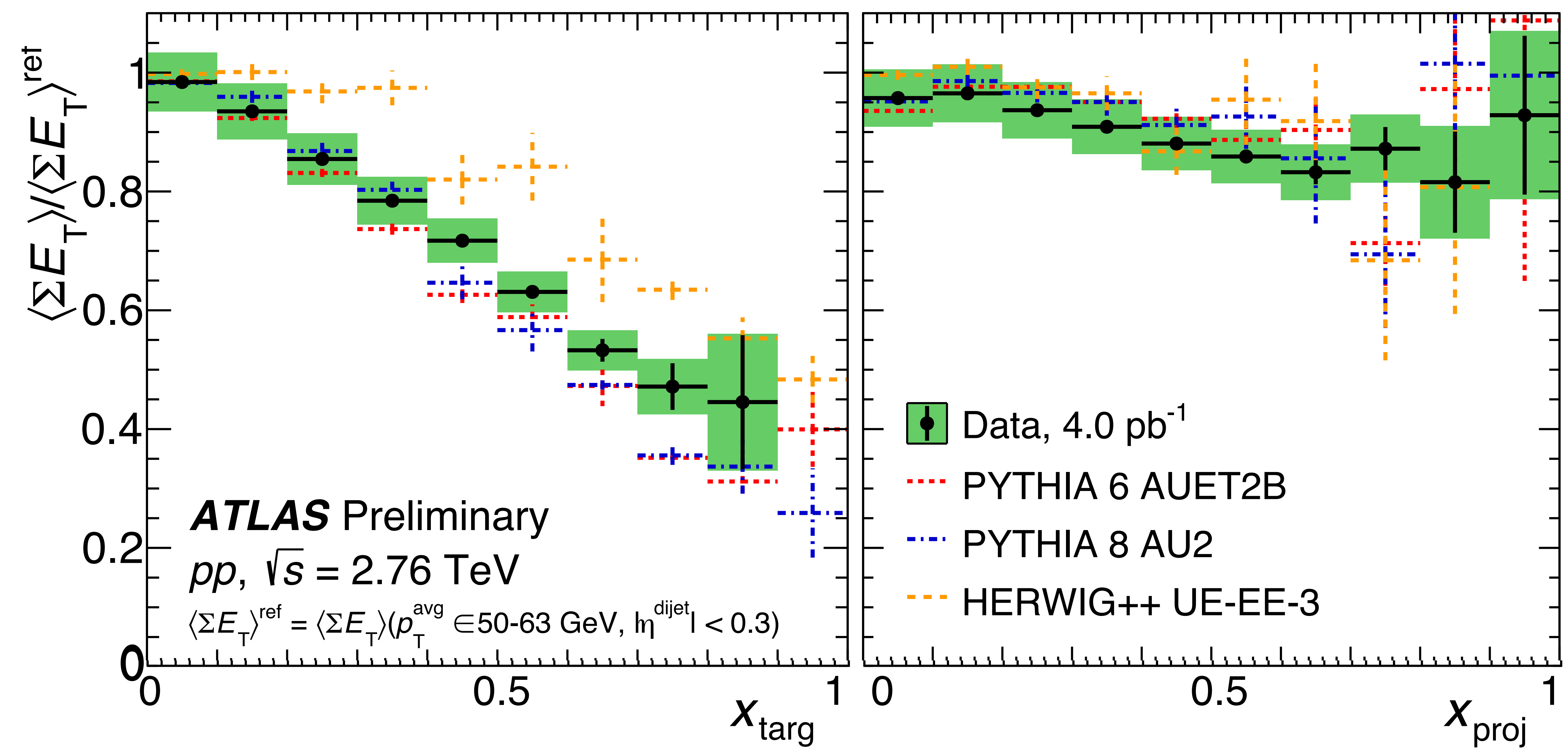
Deviation of $R_{HT}(\nu)$ from 1

In order to compare with the data we need to use a model for the distribution in E_T^{Pb} as a function of ν. We use the analysis of ATLAS . Note that E_T^{Pb} was measured at large negative rapidities which minimizes the effects of energy conservation (production of jets with large x_p)

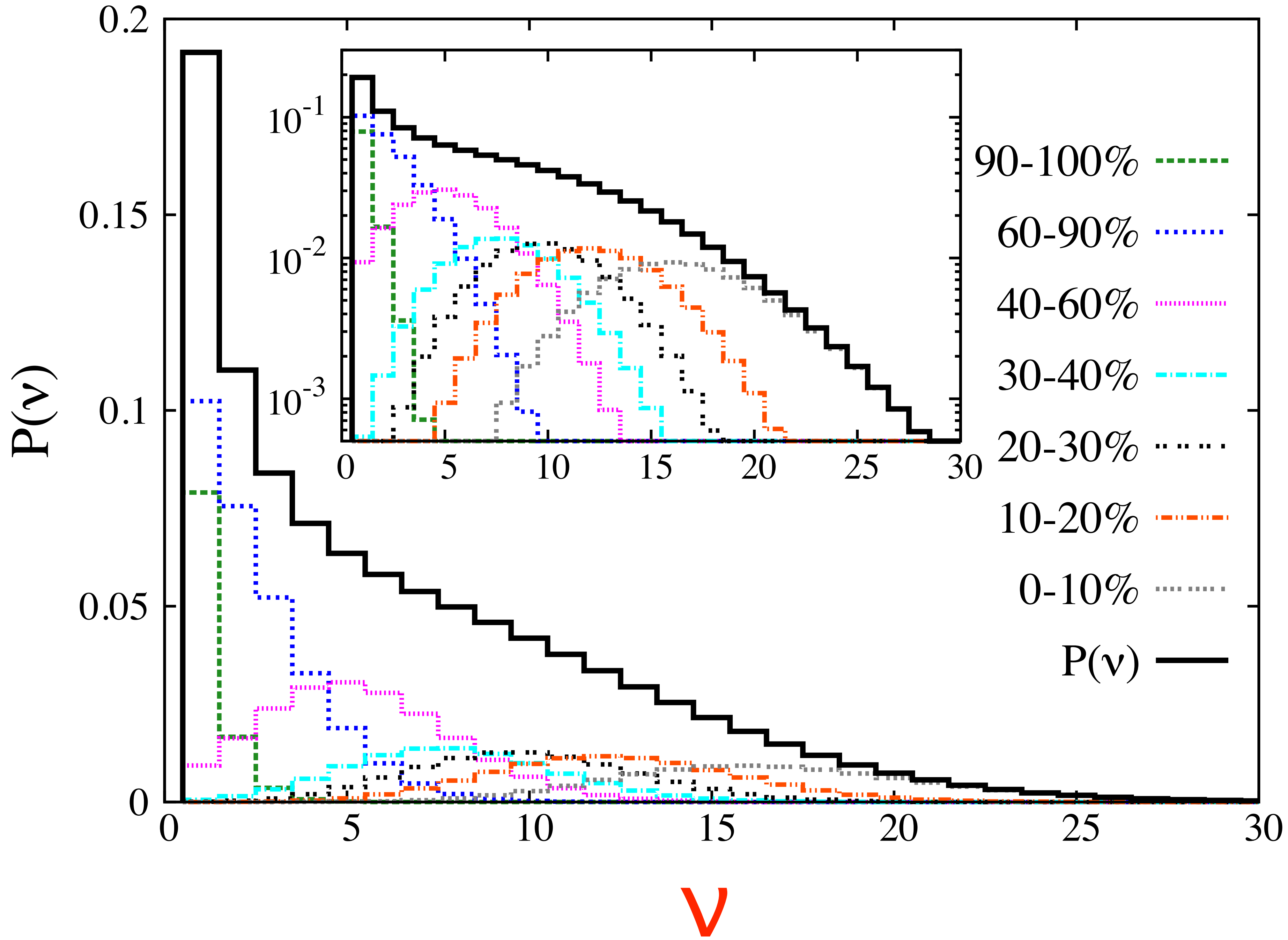
ATLAS-CONF-2015-019 analysis of pp data confirms this expectation



Measure ΣE_T at large pseudorapidity vs.
x in the **projectile** proton (moving away)
x in the **target** proton (moving towards)



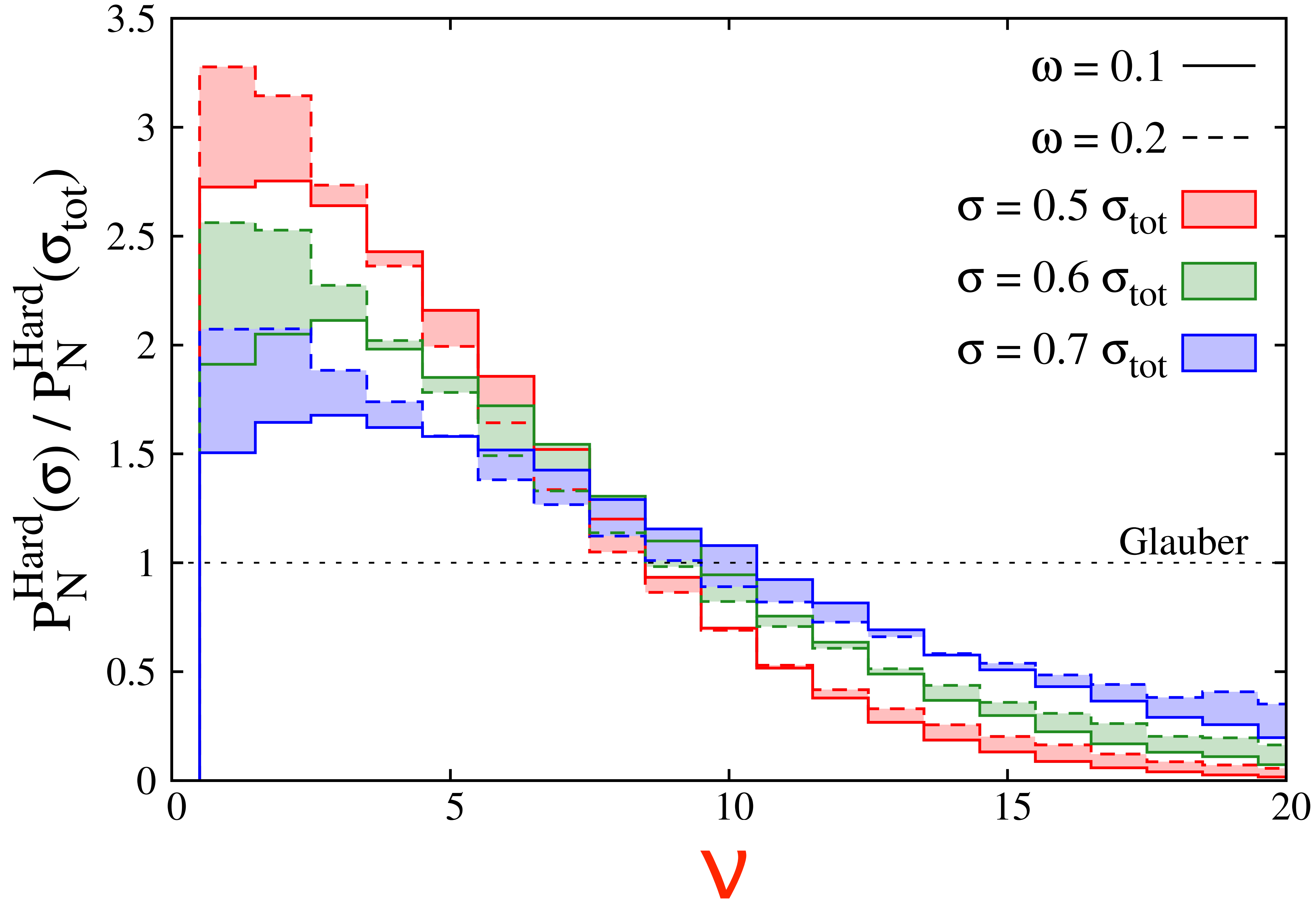
Dependence on x_{proj} and x_{targ}



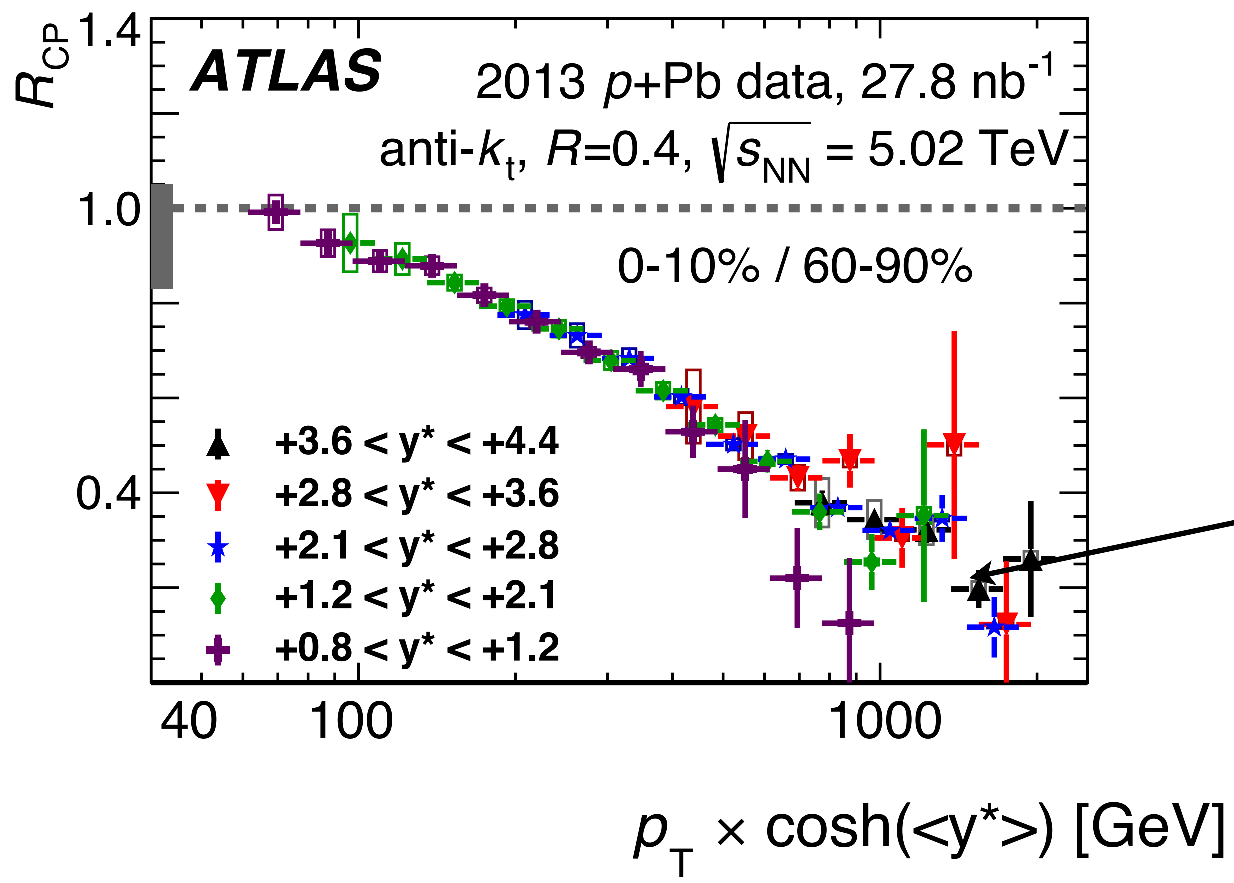
Alvioli, Cole, LF, Perepelitsa,MS, arXiv:1409.7381

Probability distributions in ν proton-nucleus collisions in all pA collisions and in those selected by different ΣE_T, or centrality, ranges. Note that ΣE_T, reasonably tracks ν's

Fluctuations for configurations with small σ maybe different than for average one so we considered both $\omega_\sigma(x\sim0.5) = 0.1$ & 0.2

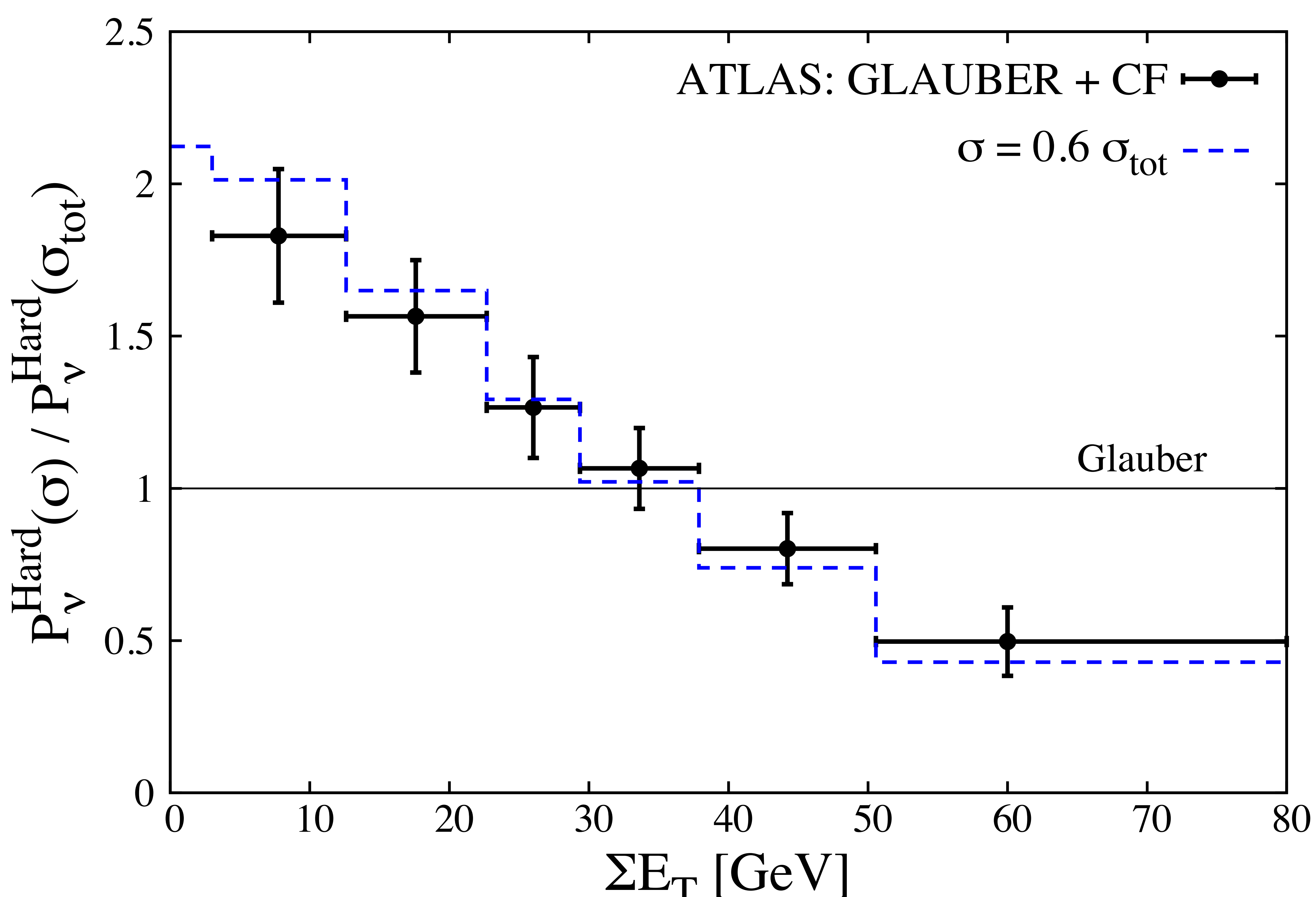


Sensitivity to ω_σ is small, so we use $\omega_\sigma = 0.1$ for following comparisons



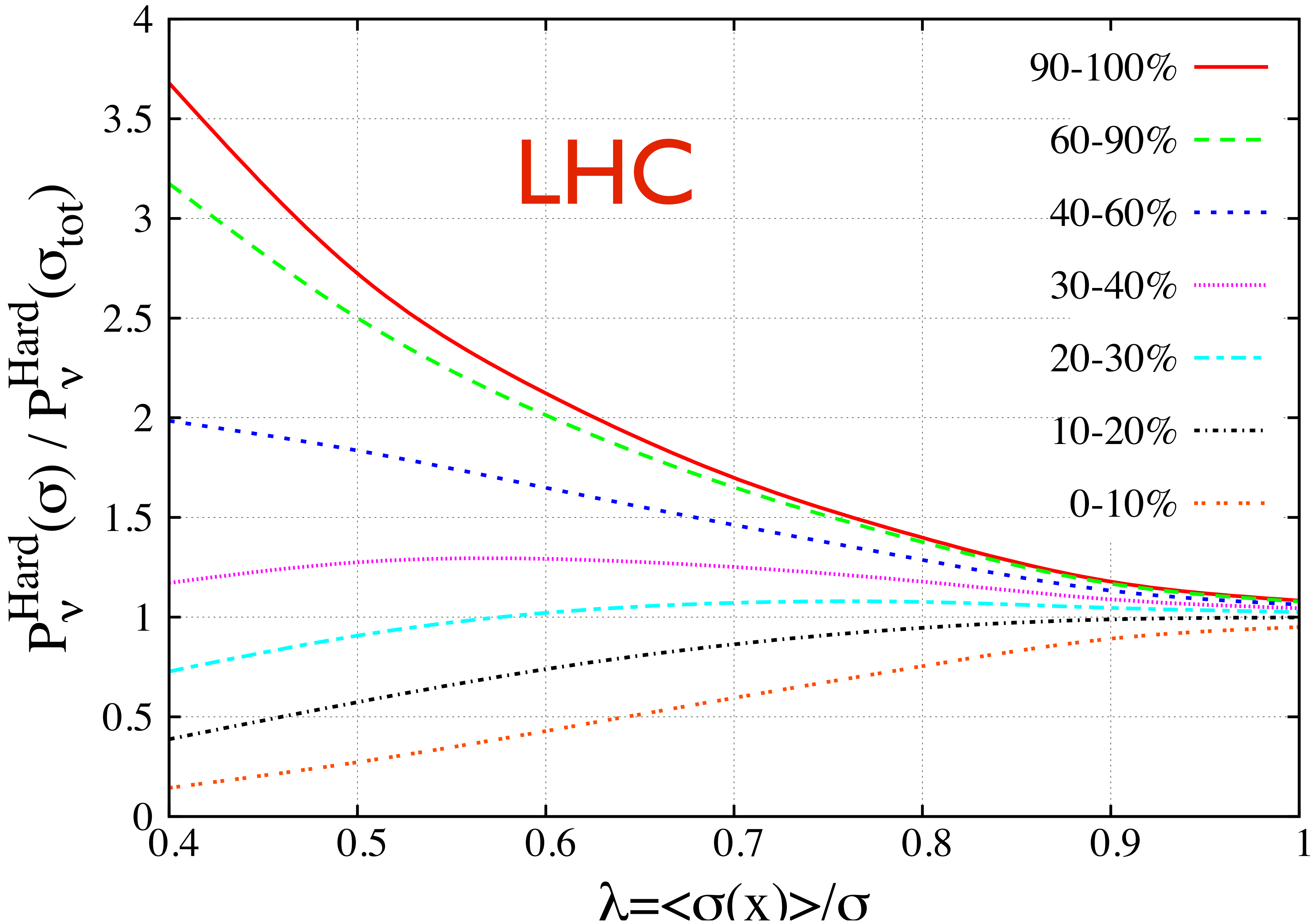
x_p =0.6

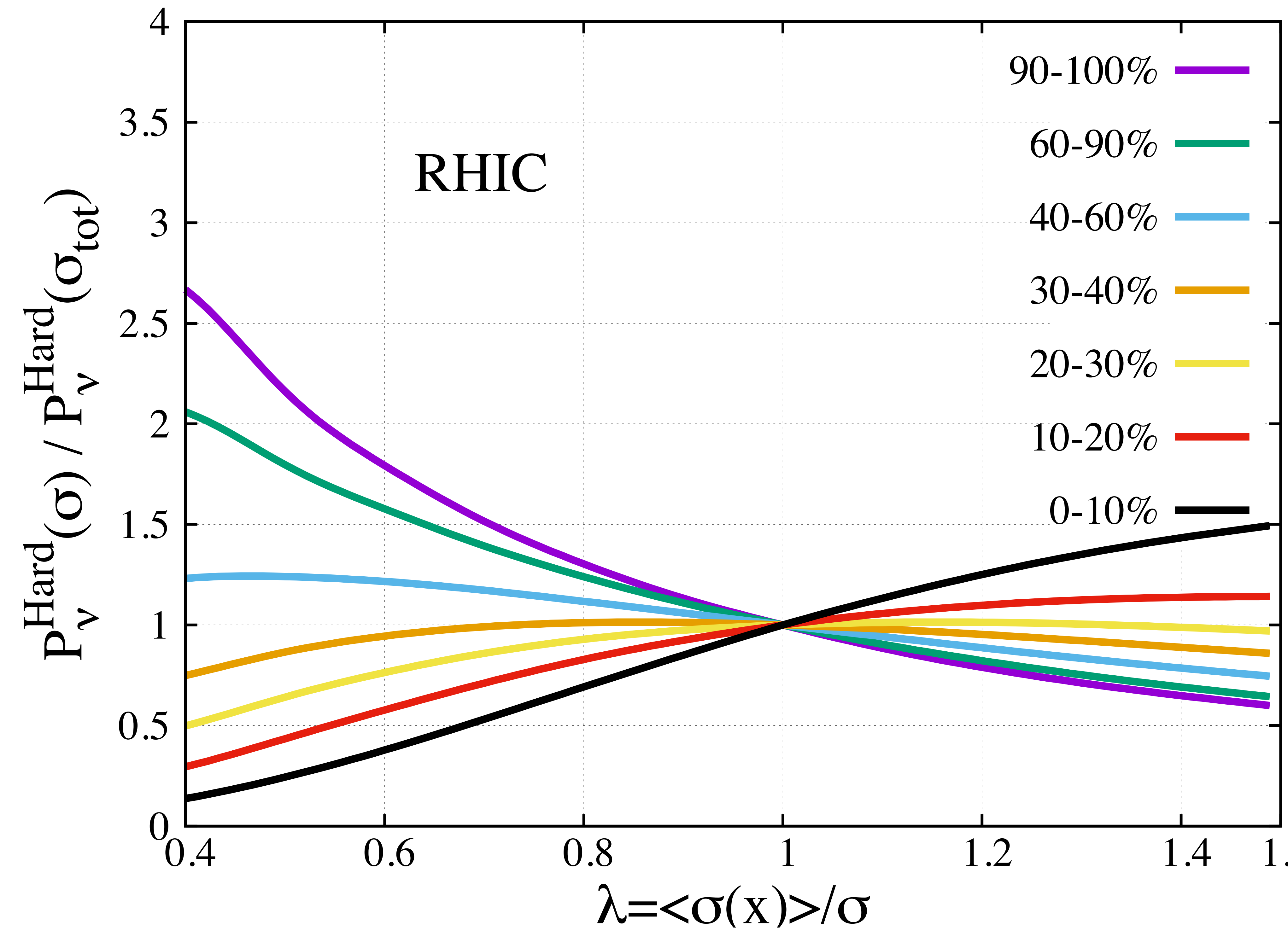
We focus on large x_p where effect is largest and hence corrections for transverse geometry are small (though we do include them)



R^{hard} for $x = E_{jet}/E_p = 0.6$ for centrality bins extracted from the ATLAS data using ν's of the CF model. Errors are combined statistical and systematic errors. The solid line is the Glauber model expectation.

R_{hard} for different centralities is calculated as a function of one x-dependent parameter $\lambda = \sigma(x)/\langle\sigma\rangle$





We can estimate $\sigma(x=0.6)/\sigma_{tot}$[fixed target]=1/4

from probability conservation relation: $$\int_0^{\sigma(s_1)} P(\sigma, s_1) d\sigma = \int_0^{\sigma(s_2)} P(\sigma, s_2) d\sigma$$

➠ $x \geq 0.5$ configurations have small transverse size ($\sim r_N/2$)

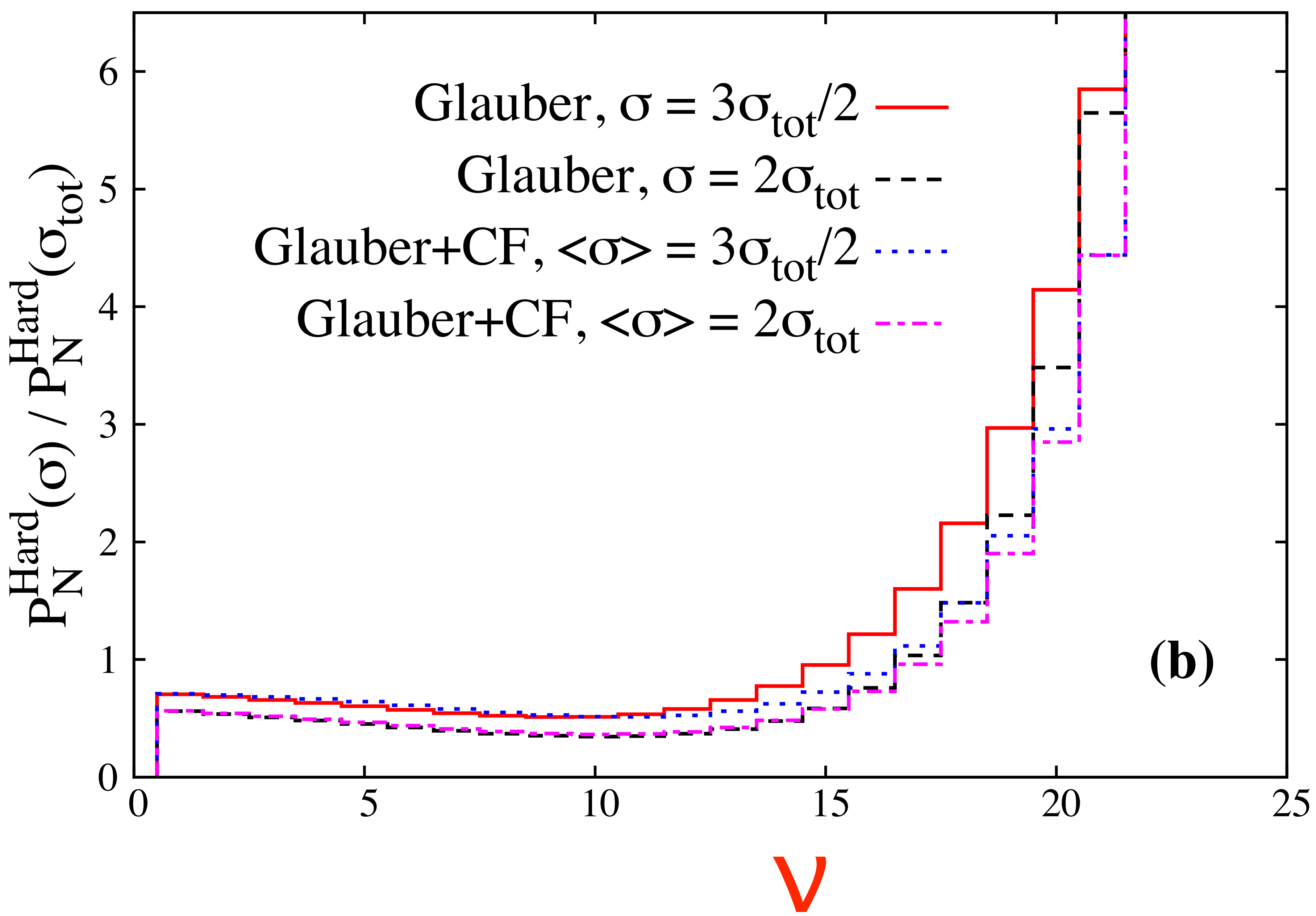
Small size configurations suppressed in bound nucleons (F83) ➠ explanation of the EMC effect

First rough estimates for smaller x:

$\sigma(x=0.2)/\langle\sigma\rangle=0.8$ gluon contribution sets in (smaller size than quarks for same x?)

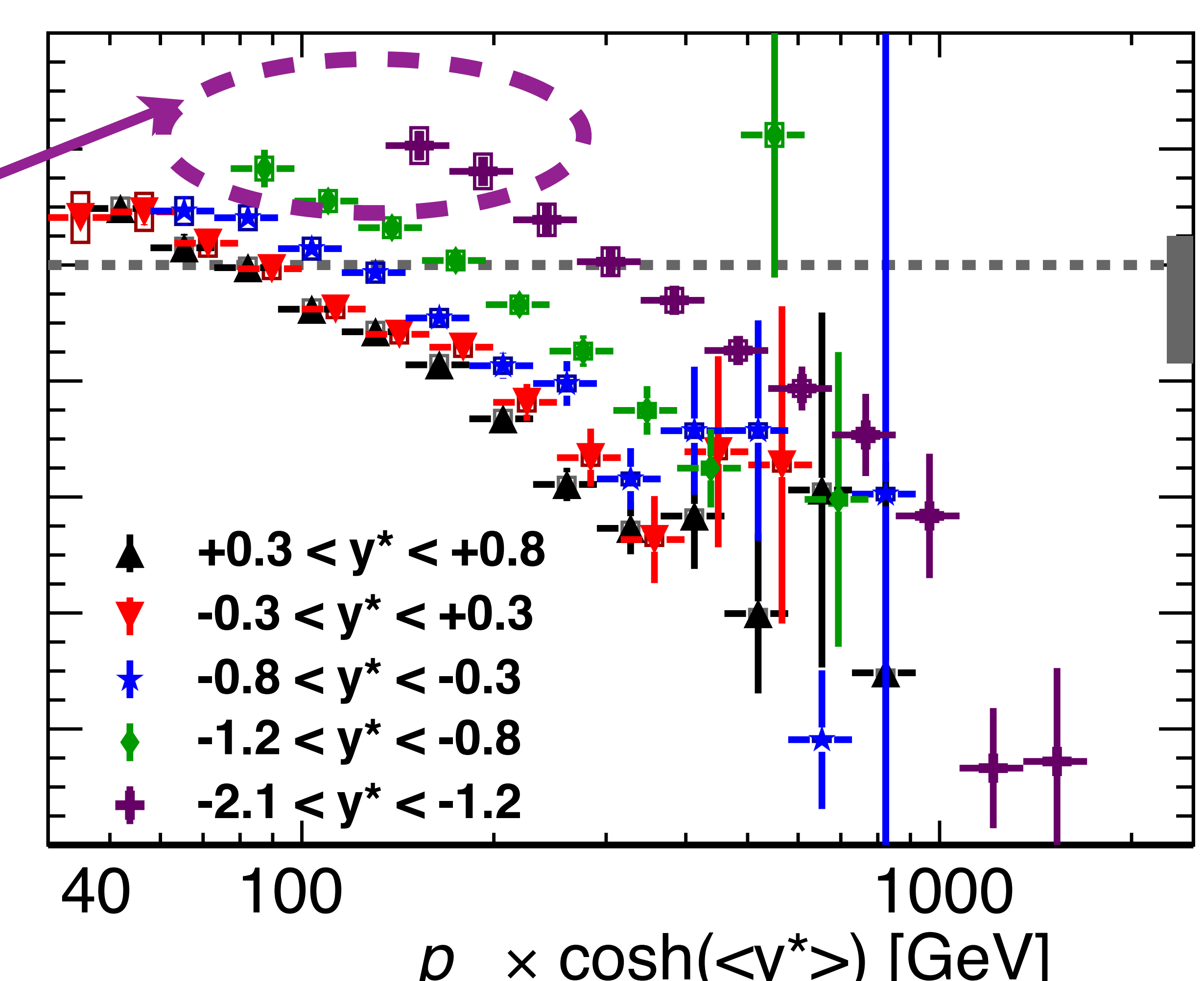
$\sigma(x=0.1)/\langle\sigma\rangle=1.0$

For σ > <σ> dependence on centrality is reversed



Ratio of the probabilities P_N of having ν wounded nucleons for scattering of the proton in configurations with different values of $\sigma(x)$ and P_N for $\sigma = \sigma_{tot}$ with CF (ω_σ=0.1) and without CF (marked as Glauber)

Transition to dominance of larger than average size - $x < 10^{-1}$?

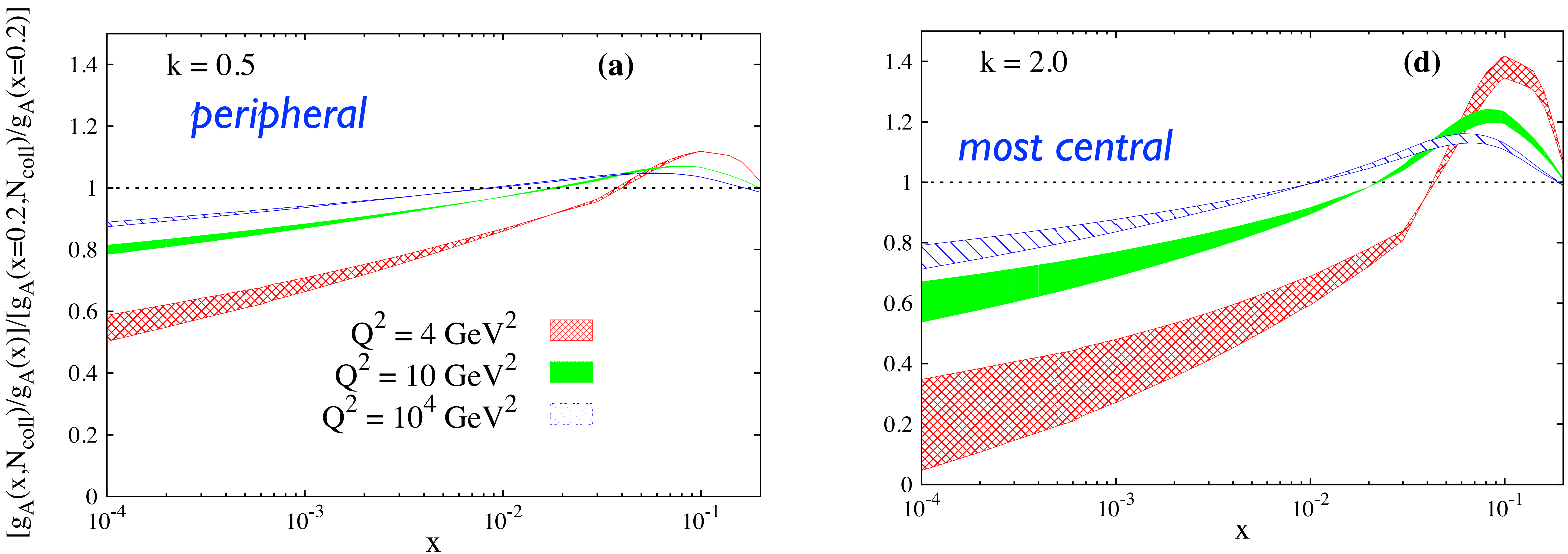


Outlook

✱ Observing effects of Large Hadronic Configurations - dijets at small x_p

✱ Study of the suppression / enhancement effects as a function of both x_p and x_A :

nuclear anti/shadowing for small x_A



EMC effect for $x_A \gtrsim 0.5$

EMC effect "peripheral collisions" ~ 0.5 inclusive EMC effect

EMC effect "central collisions"~ 1.5 inclusive EMC effect:

probes fluctuations of high density nuclear matter in the 10 fm tubes

Several effects (in addition to CF and nuclear pdf effects) which should be included in more detailed modeling of pA with jets:

- Fluctuations of small x gluon strength in nucleons: variance $\omega_g(x=10^{-3}) \sim 0.15$
- Strong dependence of the multiplicity on the impact parameter of the pp collision (Evidence from pp - supplementary slides)
- Influence of CF on impact parameters of the NN interactions in pA.
- Fluctuations of the gluon fields in nuclei - Swiss cheese

Experiment:

- Report data in the bins of x_p and x_A
- Study violation of the x_p scaling as a function of jet p_t
- quarks vs gluons for fixed x_p ; u-quarks vs d-quarks (W's)
- LHC vs RHIC for same x_p

Slides for discussion & supplementary slides

ΣE_T^{Pb} distribution: modeling by ATLAS

Transverse energy distributions in p+p collisions are typically well described by gamma distributions

$$\mathrm{gamma}(x;k,\theta) = \frac{1}{\Gamma(k)}\frac{1}{\theta}\left(\frac{x}{\theta}\right)^{k-1} e^{-x/\theta}$$

gamma distribution has convolution property:

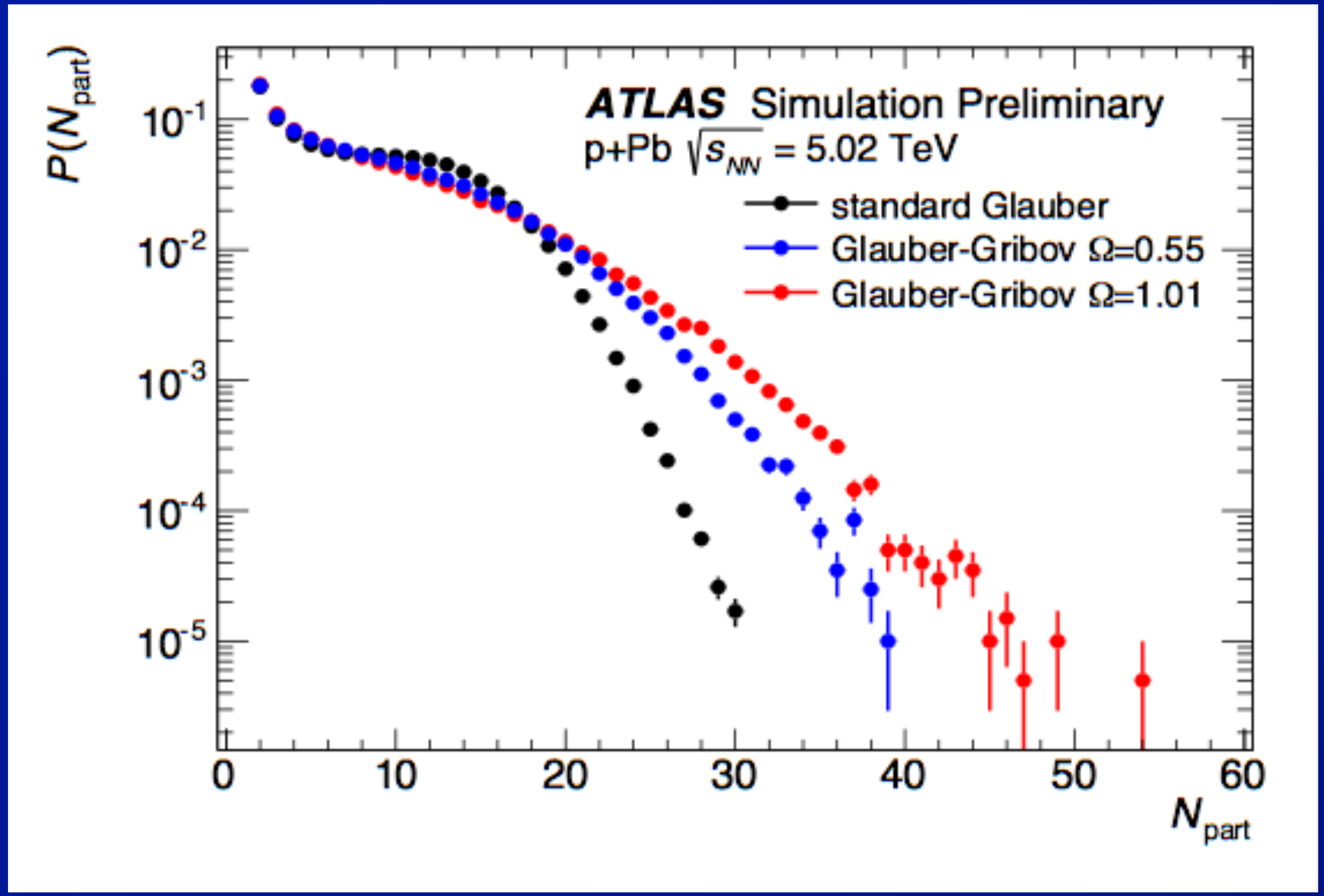
$$k\left(N_{\mathrm{part}}\right) = k_0 + k_1\left(N_{\mathrm{part}} - 2\right),$$

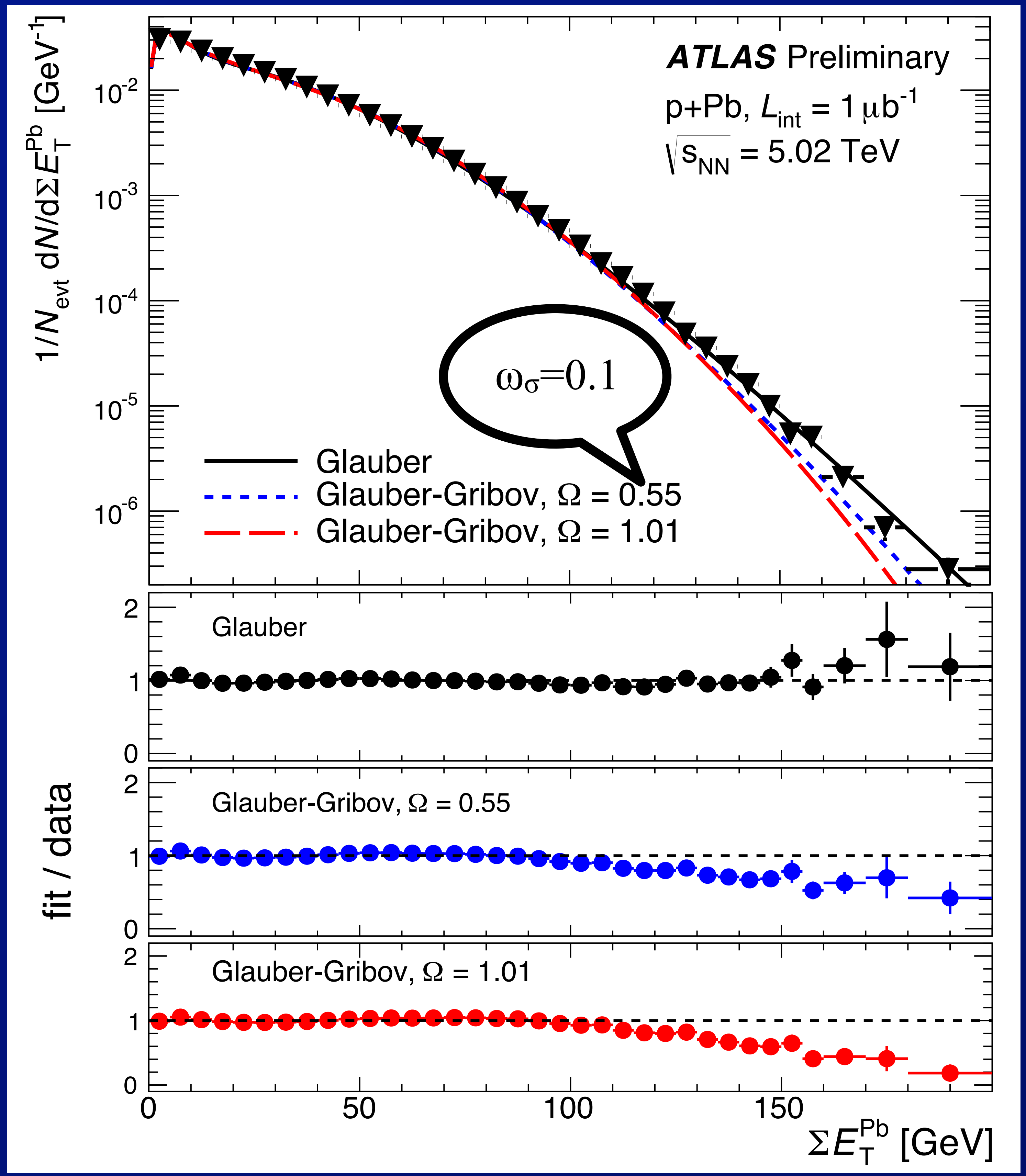
$$\theta\left(N_{\mathrm{part}}\right) = \theta_0 + \theta_1 \log\left(N_{\mathrm{part}} - 1\right).$$

N-fold conv. of gamma(x,k,θ)= $gamma(x,k,\theta) \equiv \frac{1}{\Gamma(Nk)}\frac{1}{\theta}\left(\frac{x}{\theta}\right)^{Nk-1} e^{-x/\theta}$

Note: for k = 1, gamma distribution is exponential, k < 1 is "super-exponential"

Glauber and Glauber-Gribov analysis





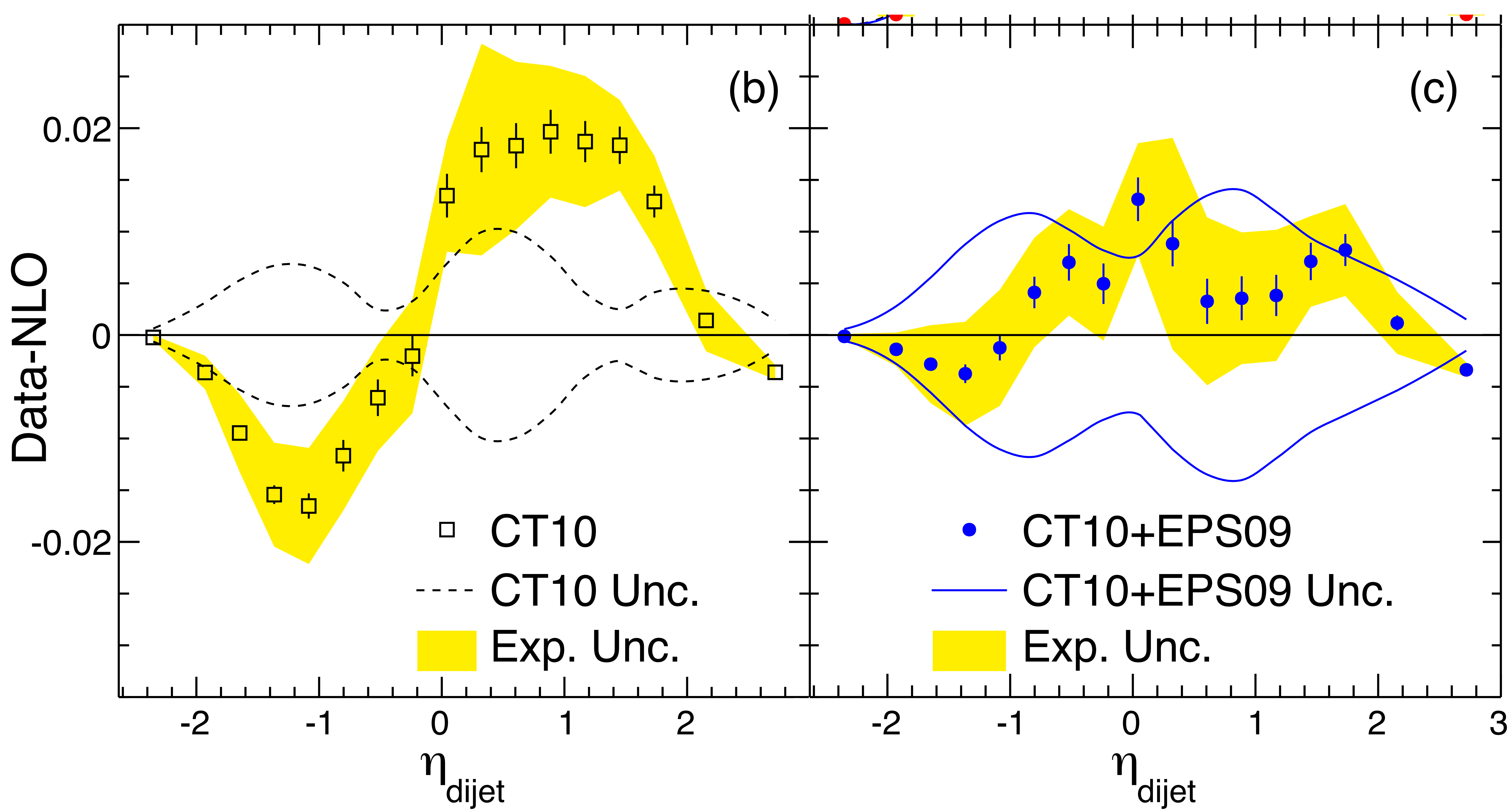
- **With Glauber-Gribov N_{part} distribution, the best fits become more WN-like**
  - e.g. for Ω = 0.55, k_1 = 0.9 (0.64 k_0), θ_1 = 0.07

From B.Cole

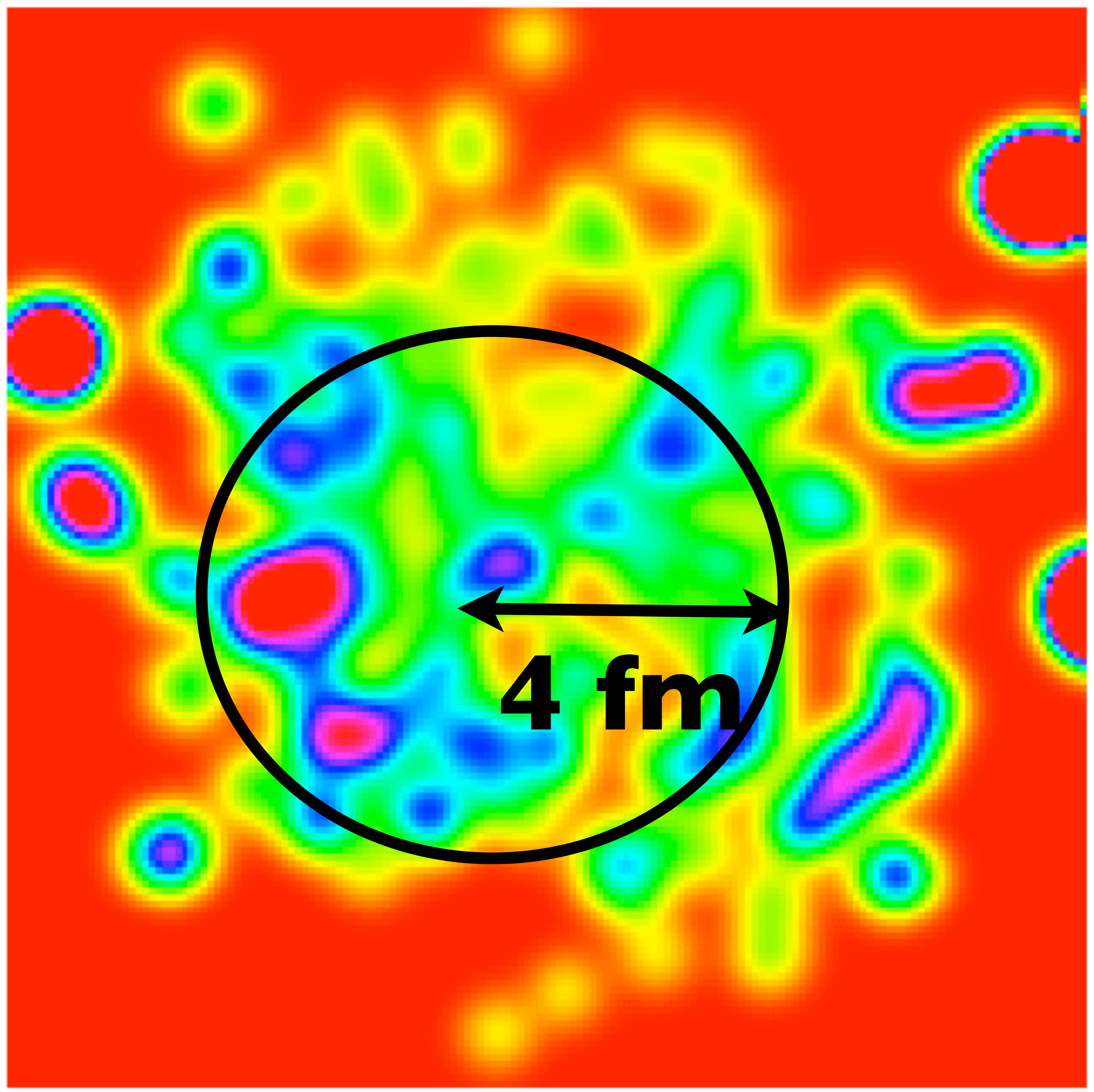
⇒ Glauber-Gribov smooths out the knee in the N_{part} distribution

Summary of some of the relevant experimental observations of CMS & ATLAS

❖ Inclusive jet production is consistent with pQCD expectations (CMS)



If two (three) nucleons are at a small relative impact parameter ($b < 0.6$ fm), the gluon shadowing strongly reduces the overall transverse. gluon density. However the thickness of the realistic nuclei is pretty low. So average number of overlapping nucleons is rather small (2.5 for $b \sim 0$) and hence fluctuations of the gluon transverse density are large



yellow < 1
1 green <2
2 <cyan < 3
3 <blue <4
4< magenta < 5
5< red

Fluctuations of transverse density of gluons in Pb on event by event basis (Alvioli and MS 09) for x outside the shadowing region

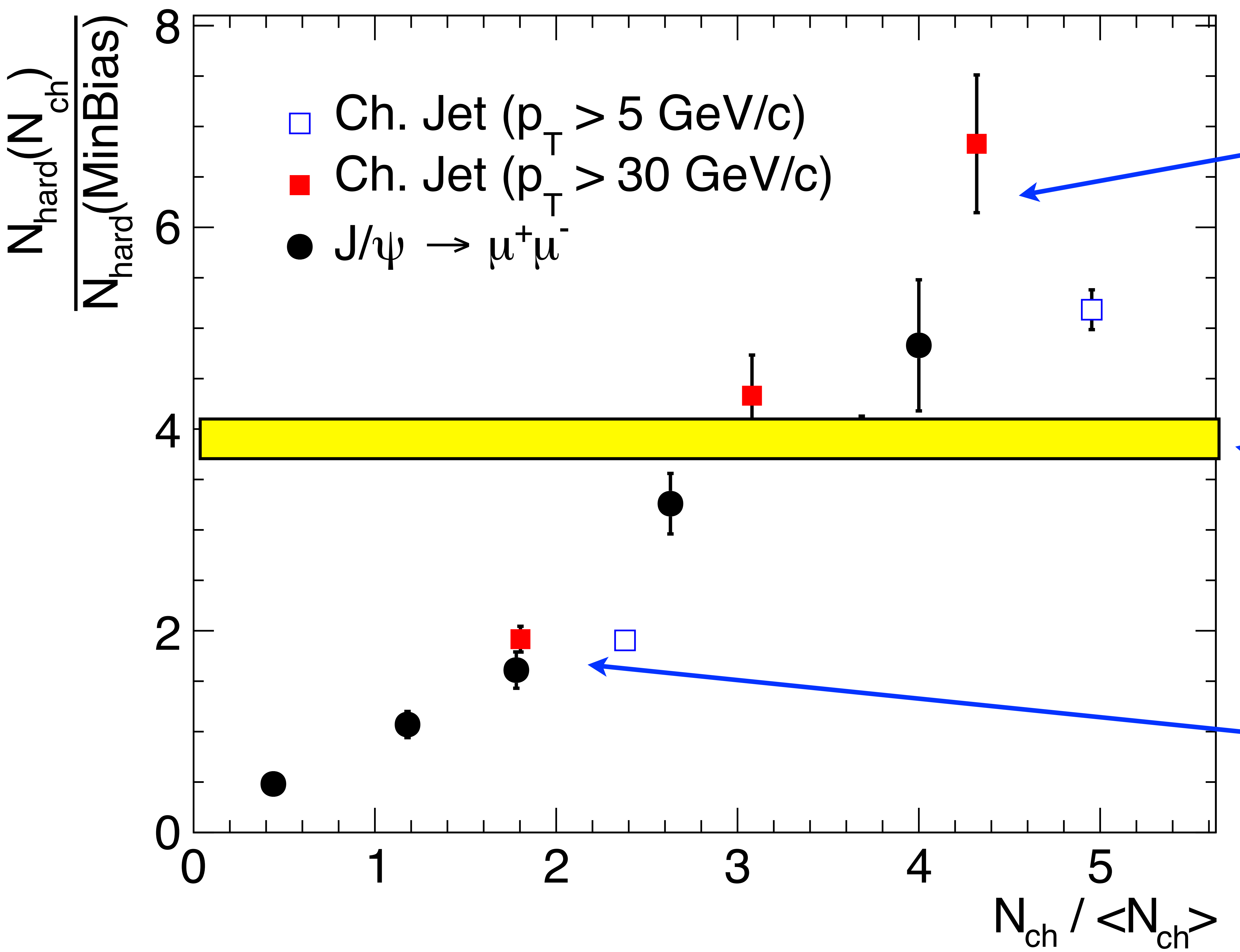
Heavy nuclei are not large enough to suppress fluctuations - A=200 nucleus for gluons with $x > 10^{-2}$ is like a *thin slice of Swiss cheese*.

Far from the $A \to \infty$ limit.

Leading twist shadowing observed at LHC does suppress some of fluctuations but new types of fluctuations

Universal relationship of soft and hard multiplicity

(Azarkin, Dremin , MS, 14)



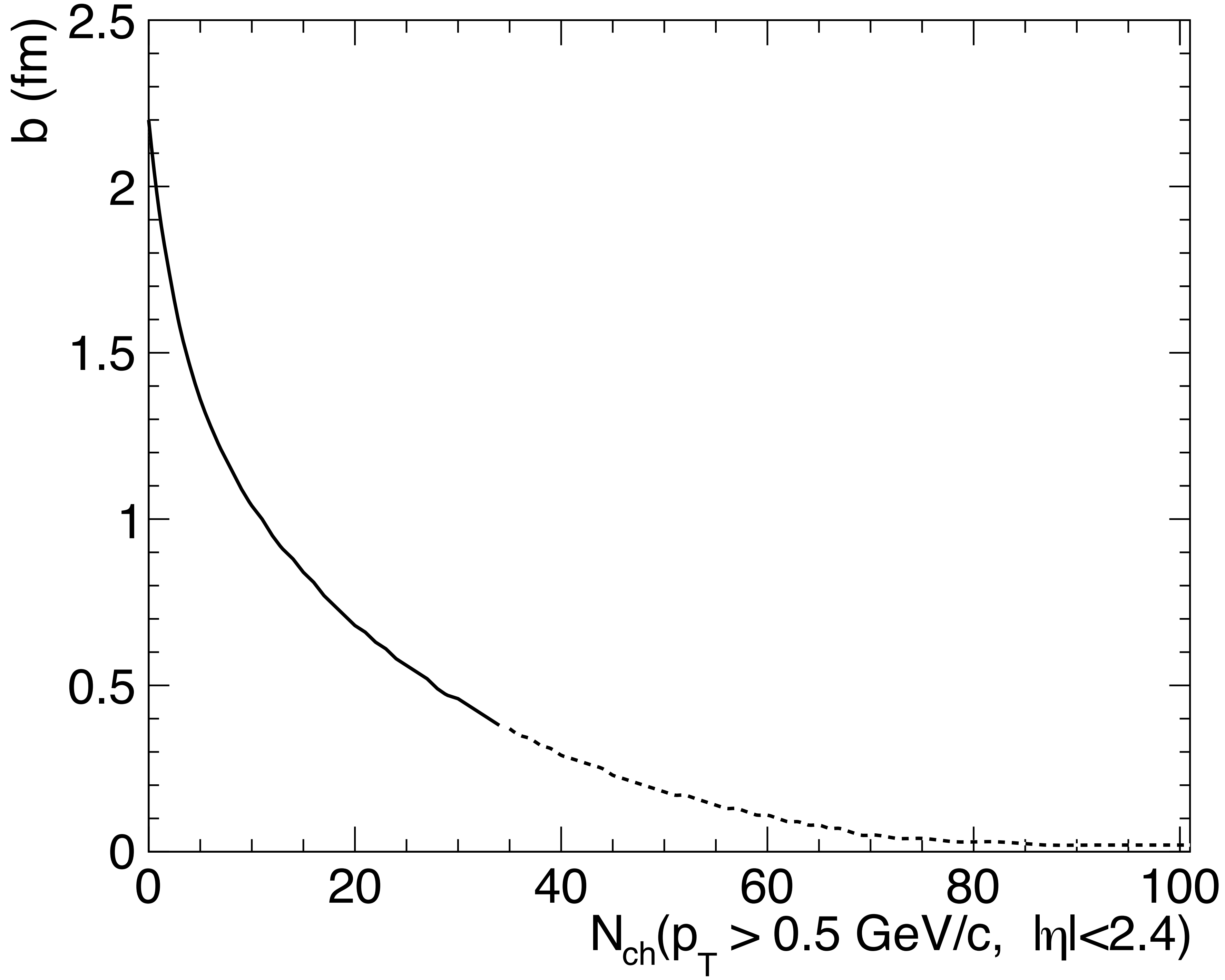
Superhigh multiplicities require special rare configurations in nucleons

max value from geometry

$$R = P_2(0)\sigma_{in}(pp) = \frac{m_g^2}{12\pi}\sigma_{in}(pp)$$

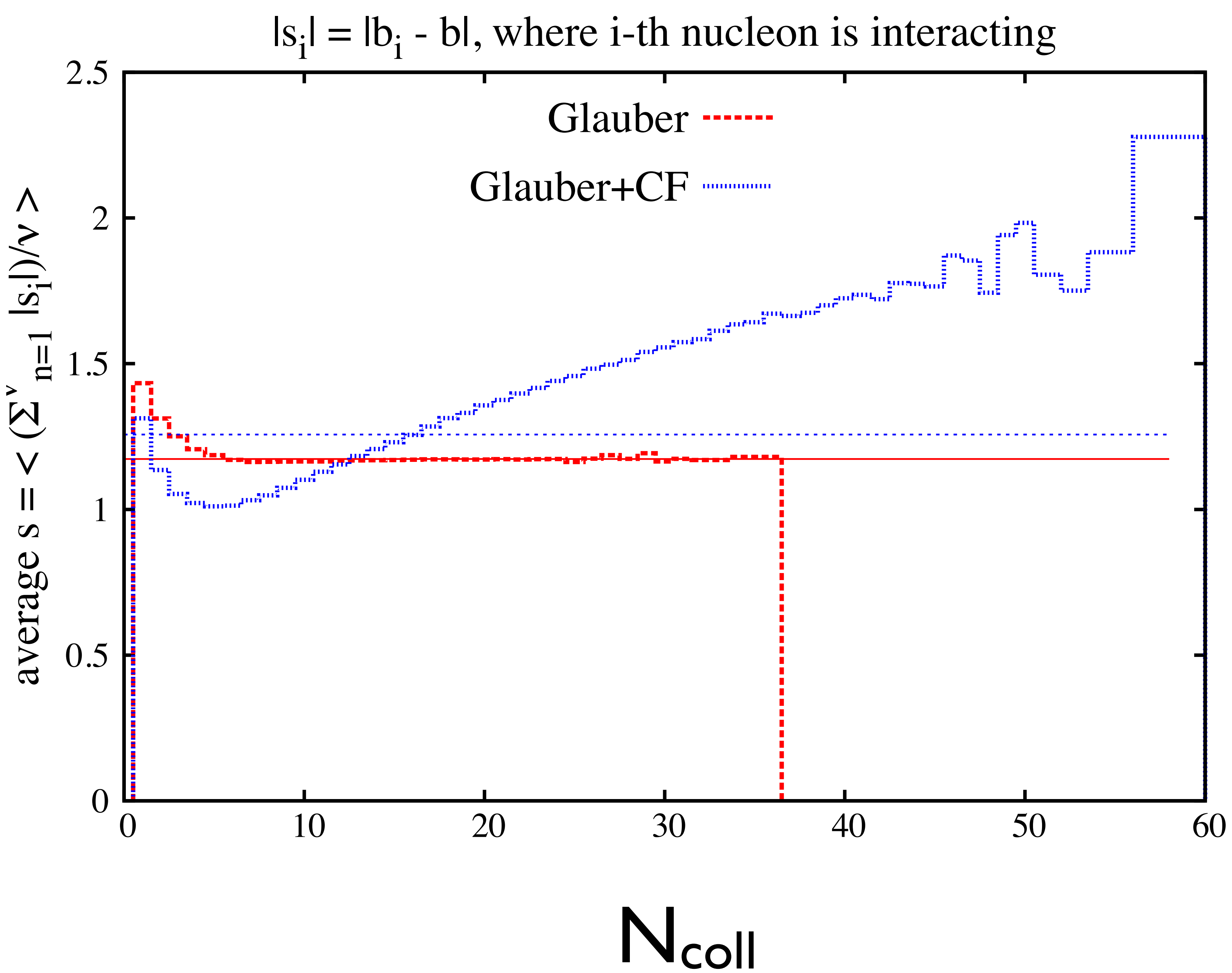
reproduced by $P_2(b)$

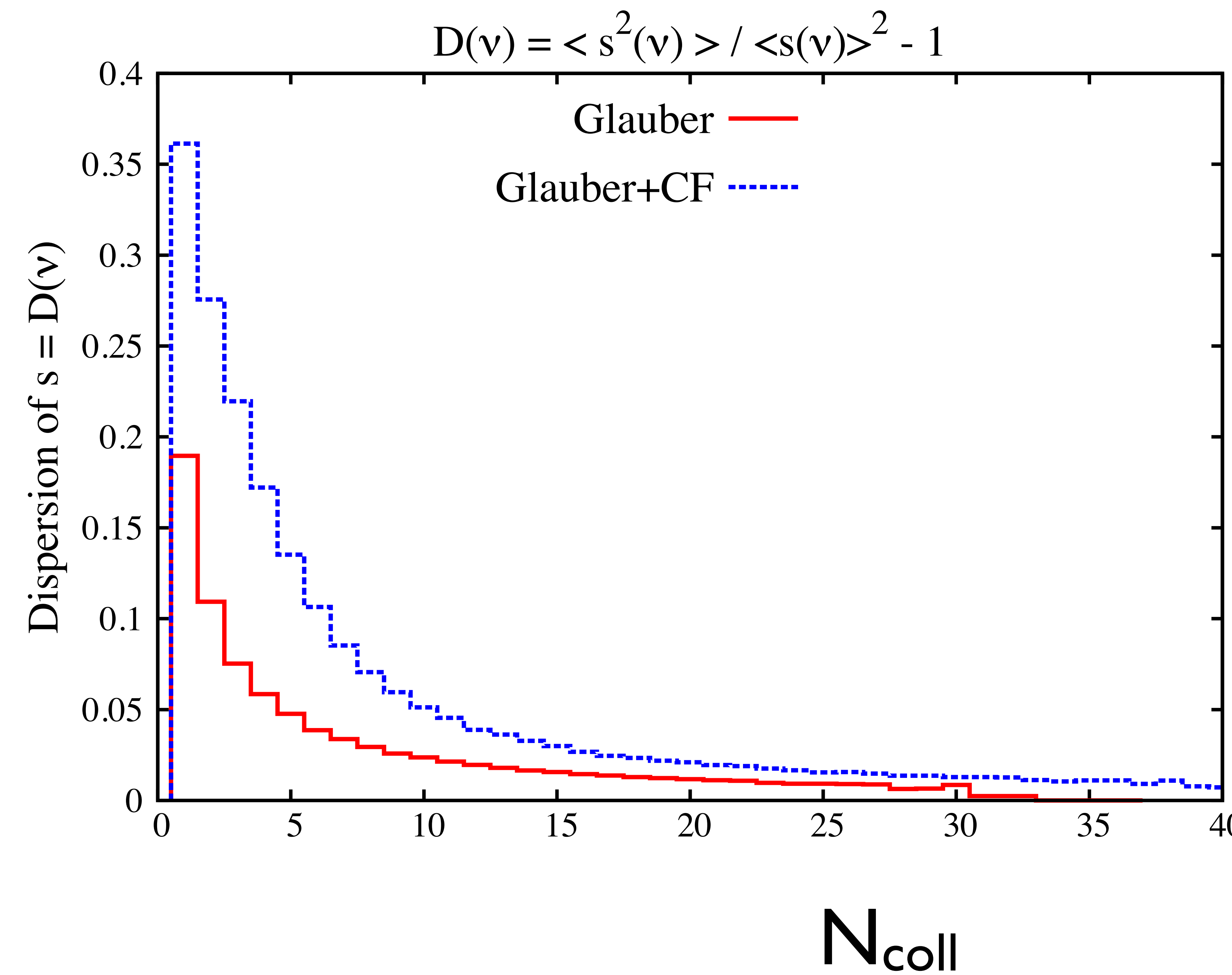
Universality of scaling of for hard processes scales with multiplicity: simple trigger - dijets(CMS) & direct J/ψ , D and B-mesons (Alice)



Correspondence between impact parameter and N_{ch}. N_{ch} is defined here as a number of charged particles with $|\eta| <$ 2.4 and $p_T >$ 0.5 GeV/c. Since events with $N_{ch} > 35$ are effectively central, the correspondence is not valid there.

- $b(N_{ch}/\langle N_{ch}\rangle \sim 2) \sim 0.7$ fm
- $b(N_{ch}/\langle N_{ch}\rangle \sim 3) \sim 0.5$ fm
- For $N_{ch}/\langle N_{ch}\rangle \gtrsim 4$ gluon fluctuations are important: jet multiplicity otherwise too high & probability of $N_{ch}/\langle N_{ch}\rangle > 4$ events is much smaller than given by $P_2(b)$.





Average impact parameter, s, for pN interactions as a function of number of wounded nucleons and its dispersion. Average s traces average σ.

Reminder -in the limit of small inelastic diffraction and neglecting radius of NN interaction as compared to internucleon distance, Gribov - Glauber model leads to

$$\sigma_{\rm in}^{\rm hA} = \int d\vec{b}\ \left[1-(1-x)^A\right] = \sum_{n=1}^{A} \frac{(-1)^{n+1}A!}{(A-n)!n!}\int d\vec{b}\, x^n$$

where $x = \sigma_{\rm in}^{\rm hN} T(\mathbf{b})/A$ $\int d\vec{b} T(b) = A$

Series can be rewritten as sum of positive terms corresponding to cross sections σ_n of exactly one, two ... inelastic interactions

Bertocchi, Treleani, 1976

$$\sigma_{\rm in}^{\rm hA} = \sum_{n=1}^{A} \sigma_n, \quad \sigma_n = \frac{A!}{(A-n)!\,n!}\int d\vec{b}\, x^n (1-x)^{A-n}$$

$$\langle N\rangle = \sum_{n=1}^{A} n\sigma_n \Big/ \sum_{n=1}^{A}\sigma_n = \frac{\sigma_{\rm in}^{\rm hN}}{\sigma_{\rm in}^{\rm hA}}\int {\rm d}^2b \sum_{n=1}^{A}\frac{A!}{(A-n)!(n-1)!}x^n(1-x)^{A-n}$$

$$= \frac{\sigma_{\rm in}^{\rm hN}}{\sigma_{\rm in}^{\rm hA}}\int {\rm d}^2b\, A\,T(\mathbf{b}) = \frac{A\sigma_{\rm in}^{\rm hN}}{\sigma_{\rm in}^{\rm hA}},$$

Simple geometric interpretation

Can use P(σ) to implement Gribov- Glauber dynamics of inelastic pA interactions. Baym et al 91-93

$$\sigma_{\rm in}^{\rm NA} = \int d\sigma_{in} P(\sigma_{in}) \int d\vec{b}\,\left[1-(1-x)^A\right]$$

$$\sigma_n = \int d\sigma_{in} P(\sigma_{in}) \frac{A!}{(A-n)!\,n!}\int d\vec{b}\, x^n (1-x)^{A-n}\,.$$

Probability of exactly n interactions is $P_n = \sigma_n/\sigma_{in}^{hA}$