Strong-Coupling Effects in a Plasma of Confining Gluons

Radoslaw Ryblewski

Institute of Nuclear Physics PAN

in collaboration with:
W. Florkowski, N. Su and K. Tywoniuk

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Motivation

- RHIC and LHC data suggest that QGP is a strongly-interacting dissipative fluid

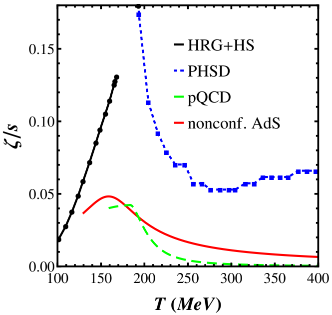
  $\Rightarrow$ relativistic dissipative fluid dynamics (equation of state?, transport coefficients?)

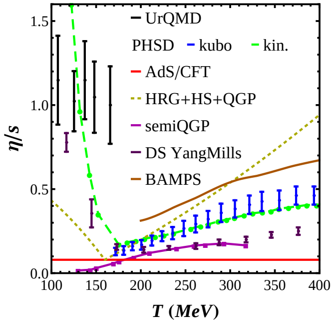
- equation of state (at high T)
  - lattice QCD
    (M. Cheng et al., Phys. Rev. D77, 014511 (2008);
    S. Borsanyi, G. Endrodi, Z. Fodor, A. Jakovac, S. D. Katz, S. Krieg, C. Ratti, and K. K. Szabo, JHEP 11, 077 (2010);
    S. Borsanyi, G. Endrodi, Z. Fodor, S. D. Katz, and K. K. Szabo, JHEP 07, 056 (2012))
  - re-summed perturbation theory
    (J.-P. Blaizot, E. Iancu, Phys. Rept. 359, 355 (2002), ;
    J. O. Andersen, M. Strickland, Annals Phys. 317, 281 (2005);
    G. D. Moore and O. Saremi, JHEP 09, 015 (2008)
    J. Hong and D. Teaney, Phys. Rev. C82, 044908 (2010);
    N. Su, Commun. Theor. Phys. 57, 409 (2012); Int. J. Mod. Phys. A30, 1530025 (2015);
    . . .)
    - large $T(\gtrsim 4T_C)$, asymptotic freedom $\Rightarrow$ pQCD
    - intermediate $T(\gtrsim 3T_C)$, poor convergence $\Rightarrow$ electric scale $\sim gT$ contributions resummation
    - low $T(\lesssim 3T_C)$, perturbative expansion breaks down (confinement effects at magnetic scale $\sim g^2 T$) $\Rightarrow$ non-perturbative approaches?
- transport coefficients

$$\begin{aligned}
\dot{\Pi} + \frac{\Pi}{\tau_\Pi} &= -\beta_\Pi \theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} \\
\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} &= 2\beta_\pi\sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \delta_{\pi\pi}\pi^{\mu\nu}\theta - \tau_{\pi\pi}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}
\end{aligned}$$

$\tau_\Pi\beta_\Pi = \zeta \rightarrow$ bulk viscosity, $\tau_\pi\beta_\pi = \eta \rightarrow$ shear viscosity

- transport coefficients (cont'd)
  - pQCD (P. B. Arnold, G. D. Moore, L. G. Yaffe, JHEP 11, 001 (2000); P. B. Arnold, G. D. Moore, L. G. Yaffe, JHEP 05, 051 (2003); G. D. Moore and O. Saremi, JHEP 09, 015 (2008); P. B. Arnold, C. Dogan, and G. D. Moore, Phys. Rev. D74, 085021 (2006);)
  - renormalization techniques (M. Haas, L. Fister, and J. M. Pawlowski, Phys. Rev. D90, 091501 (2014); N. Christiansen, M. Haas, J. M. Pawlowski, and N. Strodthoff, (2014);)
  - low energy theorems (D. Kharzeev and K. Tuchin, JHEP 09, 093 (2008); F. Karsch, D. Kharzeev, and K. Tuchin, Phys. Lett. B663, 217 (2008);)
  - lQCD (H. B. Meyer, Phys. Rev. Lett. 100, 162001 (2008); H. B. Meyer, Phys. Rev. D76, 101701 (2007);)
  - $N = 4$ supersymmetric plasma with broken conformal symmetry (G. Policastro, D. T. Son, and A. O. Starinets, Phys. Rev. Lett. 87, 081601 (2001); P. Benincasa, A. Buchel, and A. O. Starinets, Nucl. Phys. B733, 160 (2006); A. Buchel, Phys. Rev. D72, 106002 (2005); S. I. Finazzo, R. Rougemont, H. Marrochio, J. Noronha, JHEP 02, 051 (2015)))
  - . . .





see talk: J. Noronha-Hostler, *Extracting η/s in the presence of bulk viscosity in heavy ion collisions*

- Gribov quantization of Yang-Mills (YM) theory - fixing the infrared (IR) residual gauge transformations remaining after Faddeev-Popov procedure

$$\Downarrow$$

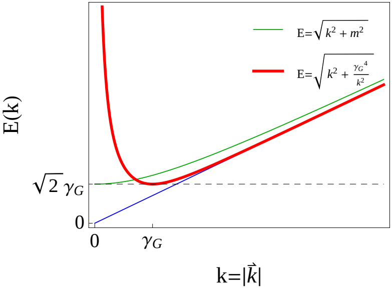
a new scale γ_G that leads to an IR improved dispersion relation for gluons (Coulomb gauge)
(V. Gribov, Nucl. Phys. B 139, 1 (1978);
D. Zwanziger, Nucl. Phys. B 323, 513 (1989);)

$$E(\mathbf{k}) = \mathbf{k} \quad \rightarrow \quad E(\mathbf{k}) = \sqrt{\mathbf{k}^2 + \frac{\gamma_G^4}{\mathbf{k}^2}}$$

- reduction of the physical phase space due to the large energy cost of the excitation of soft gluons

$$\Downarrow$$

essential feature of the confinement
(V. Gribov, Nucl. Phys. B139, 1 (1978);
Richard P. Feynman, Nucl. Phys. B188, 479 (1981);
D. Zwanziger, Nucl. Phys. B485, 185 (1997);)



Covariant setup

local rest frame

$$E(\mathbf{k}) = \sqrt{\mathbf{k}^2 + \frac{\gamma_G^4}{\mathbf{k}^2}} \quad \rightarrow$$

(explicitly breaks Lorentz invariance)

comoving frame

$$E(k \cdot u) = \sqrt{(k \cdot u)^2 + \frac{\gamma_G^4}{(k \cdot u)^2}}$$

$$k^\mu = (k_0 = |\mathbf{k}|, \mathbf{k}) \qquad k_0 \neq E(\mathbf{k})$$

(D. Zwanziger, Phys. Rev. Lett. 94, 182301 (2005);)

(W. Florkowski, R.R., N. Su, K. Tywoniuk, arXiv:1504.03176;)

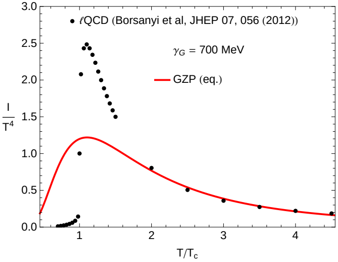
$$\varepsilon = g_0 \int \frac{d^3k}{(2\pi)^3} E(\mathbf{k}) \, f(\mathbf{k}) \quad \rightarrow \quad \varepsilon = \int dK \, E(k \cdot u) \, f(x,k)$$

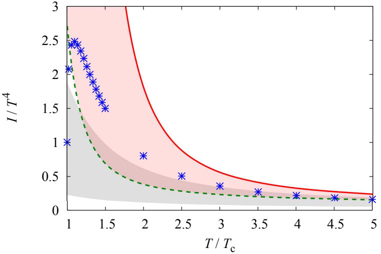
$$P = \frac{g_0}{3} \int \frac{d^3k}{(2\pi)^3} |\mathbf{k}| \frac{\partial E(\mathbf{k})}{\partial |\mathbf{k}|} \, f(\mathbf{k}) \quad \rightarrow \quad P = \frac{1}{3} \int dK \, \frac{(k \cdot u)^2}{E(k \cdot u)} \left(1 - \frac{\gamma_G^4}{(k \cdot u)^4} \right) f(x,k)$$

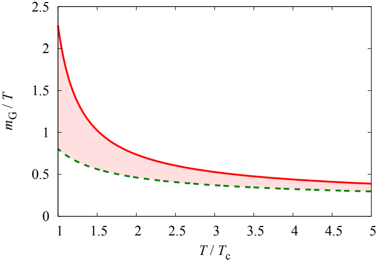
$$\int dK \, (\ldots) = g_0 \int \frac{d^3k}{(2\pi)^3 k^0} (k \cdot u) \, (\ldots)$$

$$g_0 = 2(N_c^2 - 1) \qquad (SU(N_c))$$

(W. Florkowski, R.R., N. Su, K. Tywoniuk, arXiv:1504.03176;
S. Borsanyi et al, J. High Energy Phys. 07 (2012) 056;)







- in vacuum $\gamma_G = \text{const.}$
  (D. Zwanziger, Phys. Rev. Lett. 94, 182301 (2005);)
- in high-temperature limit $\gamma_G \propto g^2 T$
  (D. Zwanziger, Phys. Rev. D 76, 125014 (2007);)
- $\gamma_G(T)$ may be derived numerically from the gap equation with running coupling from lQCD
  (K. Fukushima, N. Su, Phys. Rev. D 88, 076008 (2013);
  J. O. Andersen, M. Strickland, N. Su, Phys. Rev. Lett. 104, 122003 (2010);)
- $T \approx (2-4)\, T_c \quad \Rightarrow \quad \gamma_G \approx \text{const.}$

(0+1)D symmetry implementation

- consider the case of a transversely homogeneous boost-invariant system
- assume the Bjorken flow of matter in longitudinal direction (boost-invariance)
  (J. D. Bjorken, Phys. Rev. D 27, 140 (1983);)

$$u^\mu = (t/\tau, 0, 0, z/\tau)$$

- introduce convenient boost-invariant variables
  (A. Bialas, W. Czyz, A. Dyrek, W. Florkowski, Nucl. Phys. B 296, 611 (1988);)

$$v = k^0 t - k_\parallel z$$
$$w = k_\parallel t - k^0 z$$

- EOM follow from the conservation of $T^{\mu\nu}$

$$T^{\mu\nu} = \int dK\, k^\mu k^\nu f(x,k)$$

- $f = f(\tau, w, k_\perp)$
  $f(\tau, w, k_\perp) = f(\tau, -w, k_\perp)$
- within the assumed symmetries the $T^{\mu\nu}$ has the spherically anisotropic form
  (W. Florkowski, R. R., Phys. Rev. C 85, 044902 (2012); M. Martinez, R. R., M. Strickland, Phys. Rev. C 85, 064913 (2012);)

$$T^{\mu\nu} = (\varepsilon + P_\perp)u^\mu u^\nu - P_\perp g^{\mu\nu} + (P_\parallel - P_\perp)z^\mu z^\nu$$
$$z^\mu = (z/\tau, 0, 0, t/\tau)$$

$$\partial_\mu T^{\mu\nu} = 0 \qquad \Rightarrow \qquad \frac{\mathrm{d}\varepsilon}{\mathrm{d}\tau} + \frac{\varepsilon + P_\parallel}{\tau} = 0$$

Non-equilibrium dynamics

(A. Muronga, Phys. Rev. C69, 034903 (2004);
R. Baier, P. Romatschke, U. A. Wiedemann, Phys. Rev. C73, 064903 (2006);)

$$\frac{d\varepsilon}{d\tau} + \frac{\varepsilon + P_{\parallel}}{\tau} = 0 \quad \Leftrightarrow \quad \frac{d\varepsilon}{d\tau} + \frac{1}{\tau}\left(\varepsilon + P_{\rm GZ} + \Pi - \pi\right) = 0$$

$$\pi = \frac{4}{3}\frac{\eta_{\rm eff}}{\tau}$$

$$\Pi = -\frac{\zeta_{\rm eff}}{\tau}$$

(first-order dissipative fluid dynamics)

$$\pi = \frac{2}{3}\left(P_{\parallel} - P_{\perp}\right)$$

$$\Pi = P - P_{\rm GZ} = \frac{1}{3}\left(P_{\parallel} + 2P_{\perp}\right) - P_{\rm GZ}$$

what is the form of $\zeta_{\rm eff}$ and $\eta_{\rm eff}$ for GZ plasma?

Kinetic equation in the relaxation-time approximation

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}\tau} + \frac{\varepsilon + P_{\parallel}}{\tau} = \int dK\, E(\tau, w, k_{\perp})\, \frac{\partial f(\tau, w, k_{\perp})}{\partial \tau}$$

- kinetic equation in RTA
  P. L. Bhatnagar, E. P. Gross, M. Krook, Phys. Rev. 94, 511 (1954);
  G. Baym, Phys. Lett. B 138, 18 (1984);
  G. Baym, Nucl. Phys. A 418, 525C (1984);)

$$\frac{\partial f(\tau, w, k_{\perp})}{\partial \tau} = \frac{f_{\mathrm{GZ}}(\tau, w, k_{\perp}) - f(\tau, w, k_{\perp})}{\tau_{\mathrm{rel}}(\tau)}$$

- satisfied as long as Landau matching condition is satisfied $\varepsilon_{\mathrm{GZ}} = \varepsilon$

- formal solution
  W. Florkowski, R. R., M. Strickland, Nucl. Phys. A 916, 249 (2013);
  W. Florkowski, R. R., M. Strickland, Phys. Rev. C 88, 024903 (2013);
  W. Florkowski, E. Maksymiuk, R. R., M. Strickland, Phys. Rev. C 89, 054908 (2014);)

$$f(\tau, w, k_{\perp}) = f_0(w, k_{\perp}) D(\tau, \tau_0) + \int_{\tau_0}^{\tau} \frac{\mathrm{d}\tau'}{\tau_{\mathrm{rel}}(\tau')} D(\tau, \tau') f_{\mathrm{GZ}}(\tau', w, k_{\perp})$$

$$\varepsilon = \int \mathrm{d}K\, E(\tau, w, k_{\perp})\, f(\tau, w, k_{\perp})$$

$$P_{\parallel} = \int \mathrm{d}K \frac{w^2}{\tau^2 E(\tau, w, k_{\perp})} \left[1 - \frac{\gamma_{\mathrm{G}}^4}{(w^2/\tau^2 + k_{\perp}^2)^2} \right] f$$

$$P_{\perp} = \int \mathrm{d}K \frac{k_{\perp}^2}{2\, E(\tau, w, k_{\perp})} \left[1 - \frac{\gamma_{\mathrm{G}}^4}{(w^2/\tau^2 + k_{\perp}^2)^2} \right] f$$

$$D(\tau_2, \tau_1) = \exp\left[- \int_{\tau_1}^{\tau_2} \frac{\mathrm{d}\tau}{\tau_{\mathrm{rel}}(\tau)} \right]$$

see talk: M. Strickland, *Exact solution to the Boltzmann equation subject to Gubser flow*

- bulk viscous pressure within (0+1)D kinetic theory

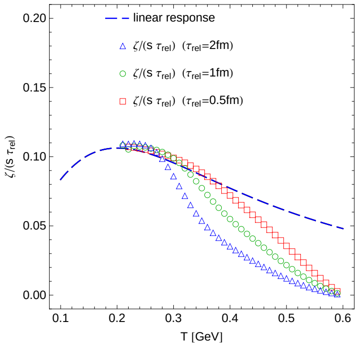
  $\Pi = P - P_{\rm GZ} = \frac{1}{3}(P_{\parallel} + 2P_{\perp}) - P_{\rm GZ}$

- bulk viscous pressure within (0+1)D 1st order viscous hydrodynamics

  $\Pi = -\frac{\zeta_{\rm eff}}{\tau}$

- close to equilibrium $f \approx f_{\rm GZ} + \delta f + \cdots$

$$\zeta(T,\gamma_{\rm G}) = \frac{g_0 \gamma_{\rm G}^5}{3\pi^2} \frac{\tau_{\rm rel}}{T} \int_0^\infty {\rm d}y \left[c_s^2 - \frac{1}{3}\frac{y^4-1}{y^4+1} \right] f_{\rm GZ}(1+f_{\rm GZ})$$



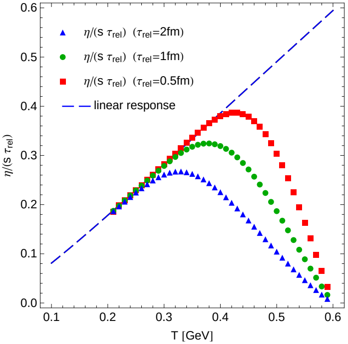
Shear viscosity

- shear viscous correction within (0+1)D kinetic theory

  $\pi = \frac{2}{3}\left(P_\parallel - P_\perp\right)$
- bulk viscous correction within (0+1)D viscous hydrodynamics

  $\pi = -\frac{4}{3}\frac{\eta_{\rm eff}}{\tau}$
- close to equilibrium $f \approx f_{\rm GZ} + \delta f + \cdots$

$$\eta(T,\gamma_{\rm G}) = \frac{1}{10}\frac{g_0\gamma_{\rm G}^5}{3\pi^2}\frac{\tau_{\rm rel}}{T}\int_0^\infty {\rm d}y\,\frac{\left(y^4-1\right)^2}{y^4+1} f_{\rm GZ}(1+f_{\rm GZ})$$



$$\frac{\zeta}{\eta} = \kappa \left(\frac{1}{3} - c_s^2 \right)^p$$

- $p = 2$, $\kappa = 15$

  photon gas coupled to hot matter
  (S. Weinberg, Astrophys. J. 168, 175 (1971);)

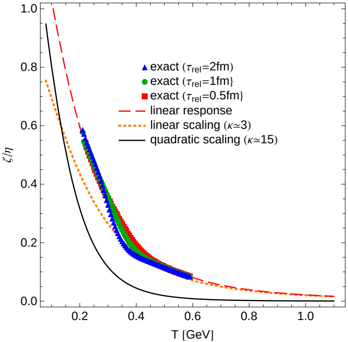
  scalar theory
  (A. Hosoya, M. Sakagami, M. Takao, Ann. Phys. (N.Y.) 154, 229 (1984);
  R. Horsley, W. Schoenmaker, Nucl. Phys. B280, 716 (1987);)

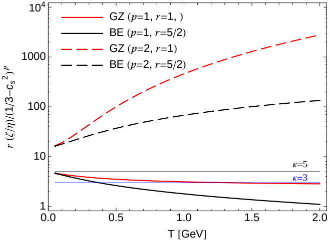
  weakly-coupled QCD (large-T limit)
  (P. B. Arnold, G. D. Moore, L. G. Yaffe, JHEP 0011 (2000) 001;
  P. B. Arnold, G. D. Moore, L. G. Yaffe, JHEP 0305 (2003) 051;
  P. B. Arnold, C. Dogan, G. D. Moore, Phys.Rev. D74 (2006) 085021;)

- $p = 1$, $\kappa = 2$, $\kappa = 4.558 - 4.935$

  strongly-coupled nearly-conformal gauge theory plasma using gauge theory–gravity duality (large-T limit)
  (P. Benincasa, A. Buchel, A. O. Starinets, Nucl. Phys. B 733, 160 (2006);
  A. Buchel, Phys. Rev. D 72, 106002 (2005);)

- $p = 1$, $\kappa = 3$
  Gribov plasma of confining gluons
  (W. Florkowski, R.R., N. Su, K. Tywoniuk arXiv:1504.03176;
  W. Florkowski, R.R., N. Su, K. Tywoniuk, forthcoming;)





Summary

- a dynamic and non-equilibrium description of a plasma consisting of confining gluons (obtained from the Gribov quantization of SU(3) YM theory) introduced for the first time
- the expressions for the shear and bulk viscosities of the Gribov-Zwanziger plasma were derived
- ζ/η T-scaling which is in line with the strong-coupling methods results was found

Outlook

- include the running $\gamma_{\rm G}(T)$
- use our formula in the hydrodynamic symulations

Thank you for your attention!

ACKNOWLEDGMENTS

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