

# Coarse-grained transport studies about local equilibrium and negative Cooper-Frye

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Hannah Petersen

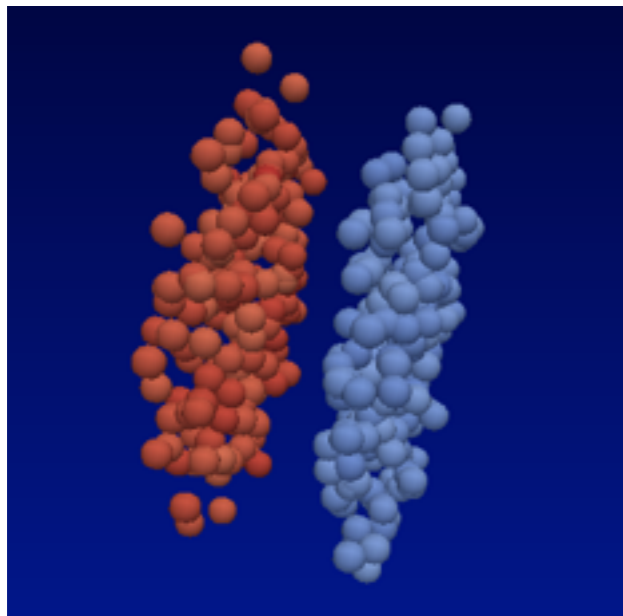
July 22, 2015, INT, Seattle

# Motivation

- **Hybrid** transport+hydrodynamics approaches are successfully applied for the description of the dynamics of heavy ion collisions
- There are **2 ad hoc** transitions
  - Initial assumption on local equilibration
    - Might not be fulfilled for lower beam energies, smaller systems, larger centralities
  - Final **Cooper-Frye** sampling/Particlization
    - Negative contributions: How large?
    - Are hydro and transport equivalent?
- **In this talk:** Investigation of these transitions in coarse-grained transport approach

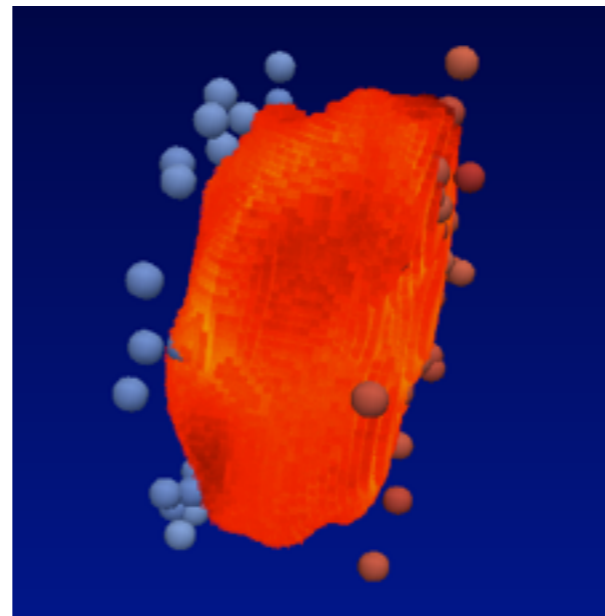
# Evolution of Heavy Ion Reactions

Nuclei at 99 %  
speed of light



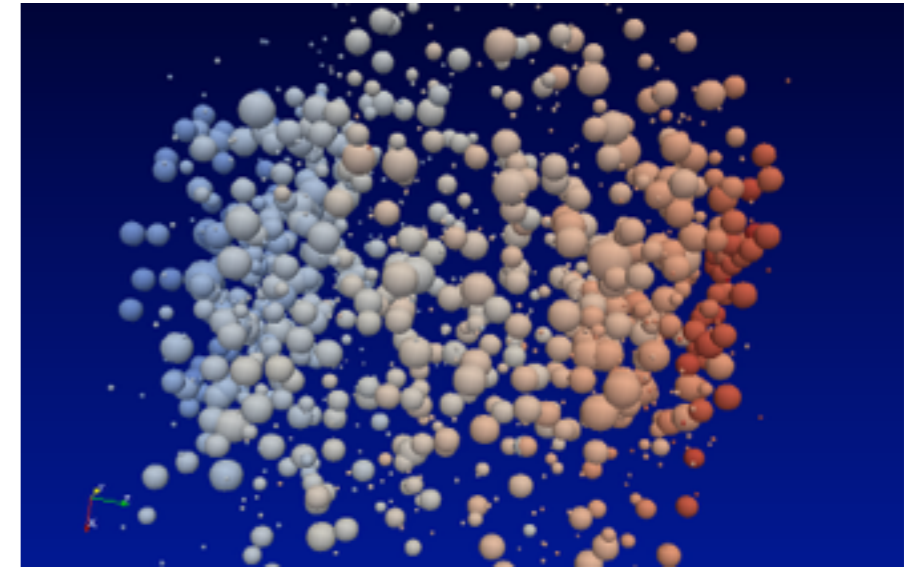
<1 fm/c

Quark gluon plasma



1-10 fm/c

Observable fragments  
in the detector



10-30 fm/c

- Initial and final state require non-equilibrium treatment
- Nearly ideal hydrodynamics provides framework for the hot and dense stage of the evolution including a phase transition

Hybrid models achieve realistic description

# Hybrid approaches

## Transport



Microscopic description of the whole phase-space distribution

Non-equilibrium evolution based on the Boltzmann equation

$$\left( p^\mu \partial_\mu \right) f = I_{coll}$$

Partonic or hadronic degrees of freedom

Cross-sections are calculable using different techniques

Phase transition?

## Hydrodynamics



Macroscopic description

Local equilibrium is assumed

$$\partial_\mu T^{\mu\nu} = 0 \quad \partial_\mu (n u^\mu) = 0$$

Propagation according to conservation laws

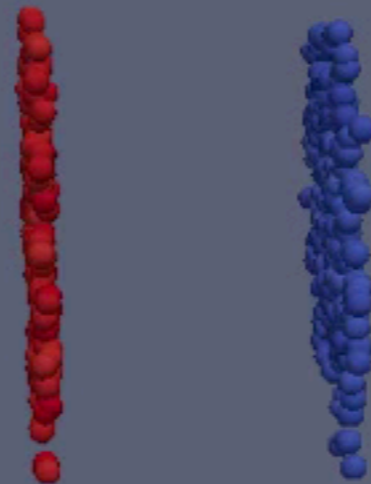
Equation of state is an explicit input

Boundary conditions: Breakdown of equilibrium assumptions?

- Combine the advantages of both approaches
- Successful description from initial to final state

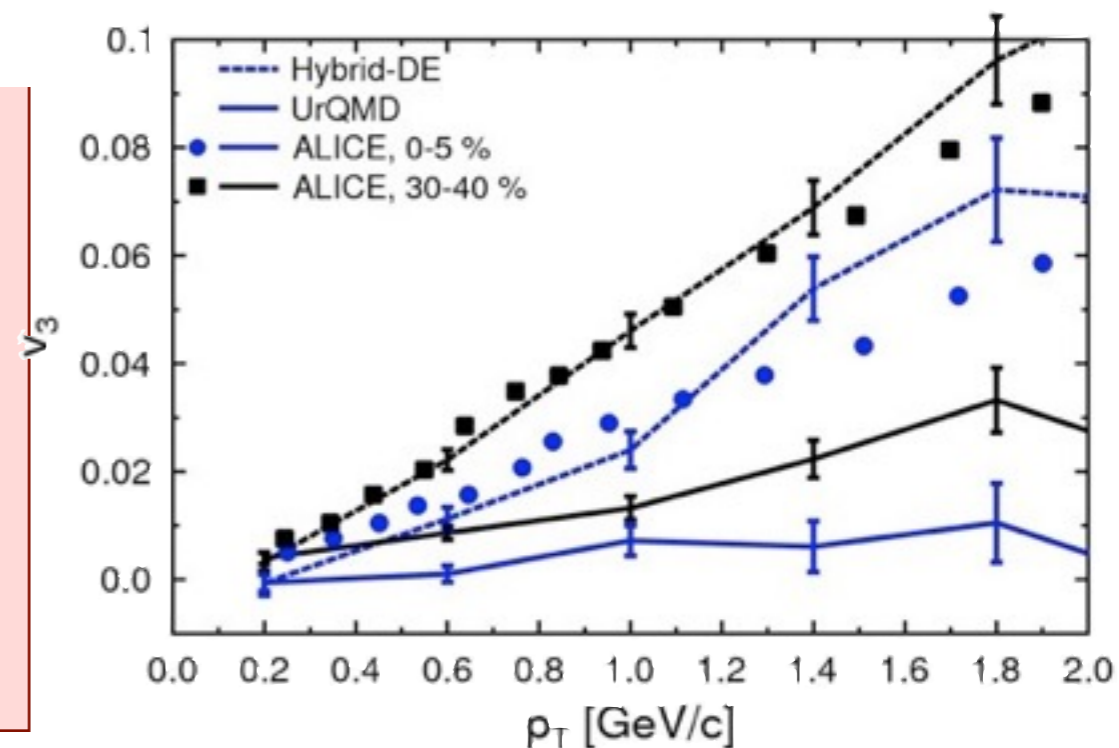
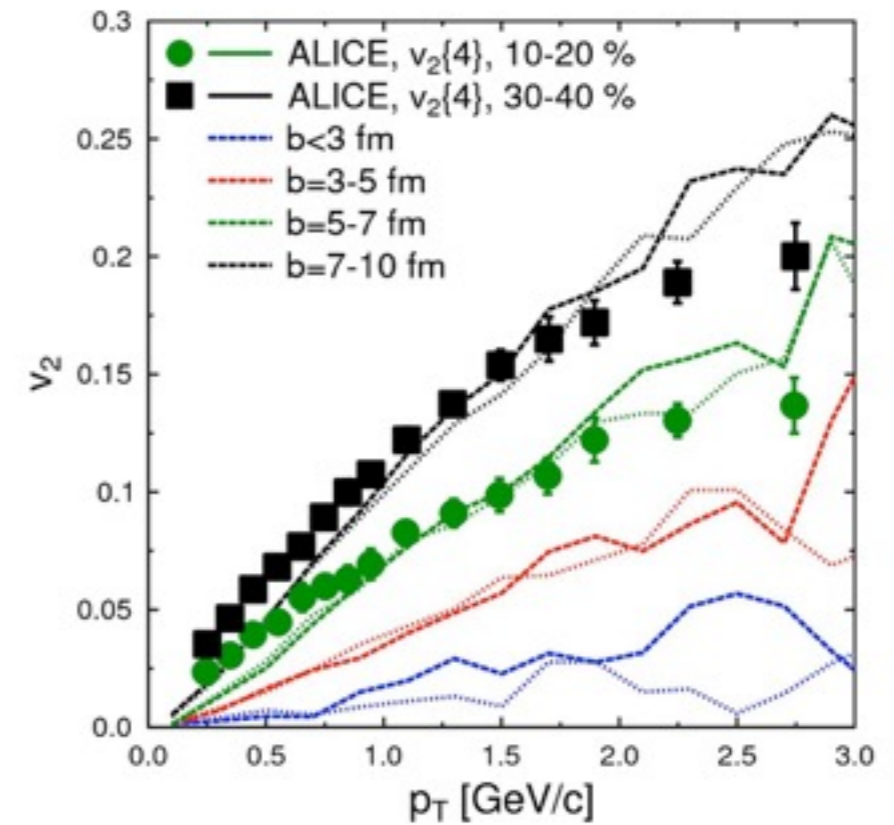
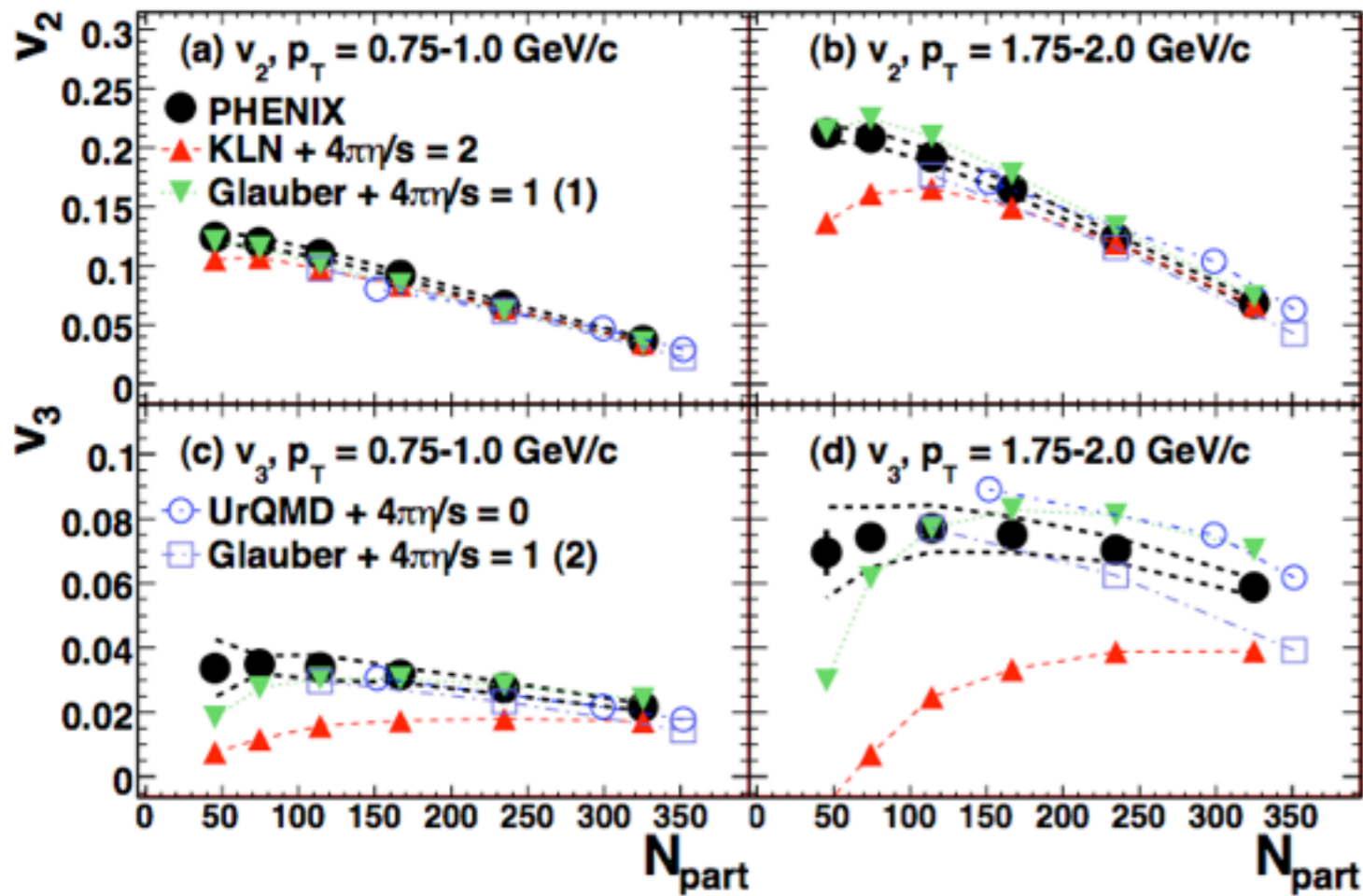
# One Event at RHIC Energies

Time:0.08



MADAI.us

# Comparison to Experimental Data



- Fluctuations from nucleon positions and energy deposition in binary collisions included
- Without tuning anything  $v_3$  at RHIC and flow at LHC in agreement with experimental data

# Initial State Transition

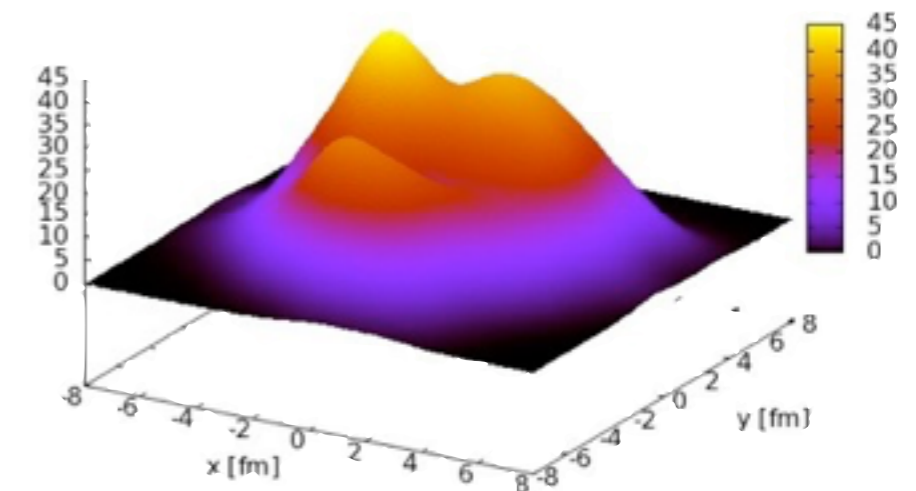
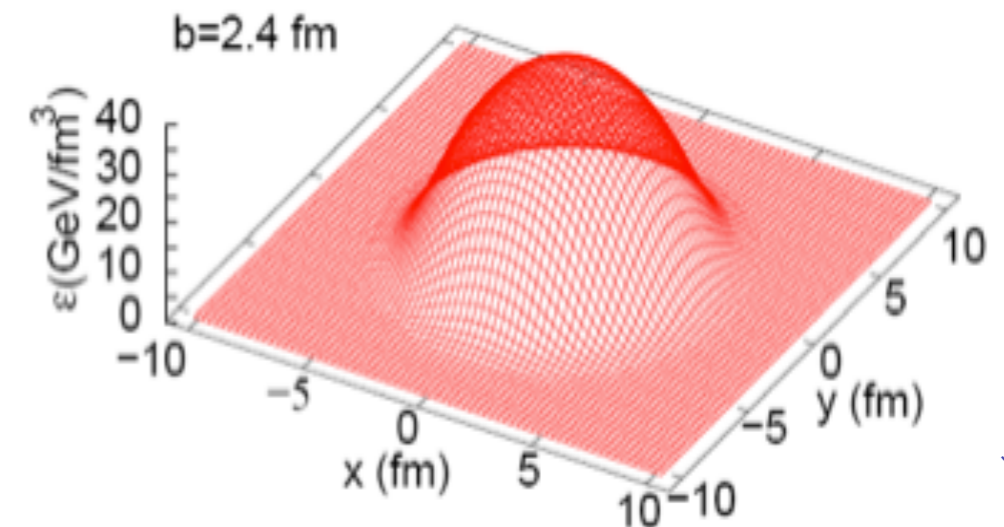
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based on work with D. Oliinychenko,  
in preparation

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# Fluctuating IC from Transport Approaches

- Density profiles are not smooth, but there are **local peaks** in transverse and longitudinal direction
  - **Impact parameter** fluctuations within one specific centrality class, multiplicity fluctuations and differences in initial geometry
  - Event plane rotation with respect to reaction plane in the laboratory
- All these effects are **averaged out** if assuming a smooth symmetric initial density profile



J.Steinheimer et al., PRC 77,034901,2008

Included in dynamical models of the initial state (e.g. a parton cascade, NEXUS/EPOS, UrQMD) or in Glauber or CGC Monte Carlo approaches

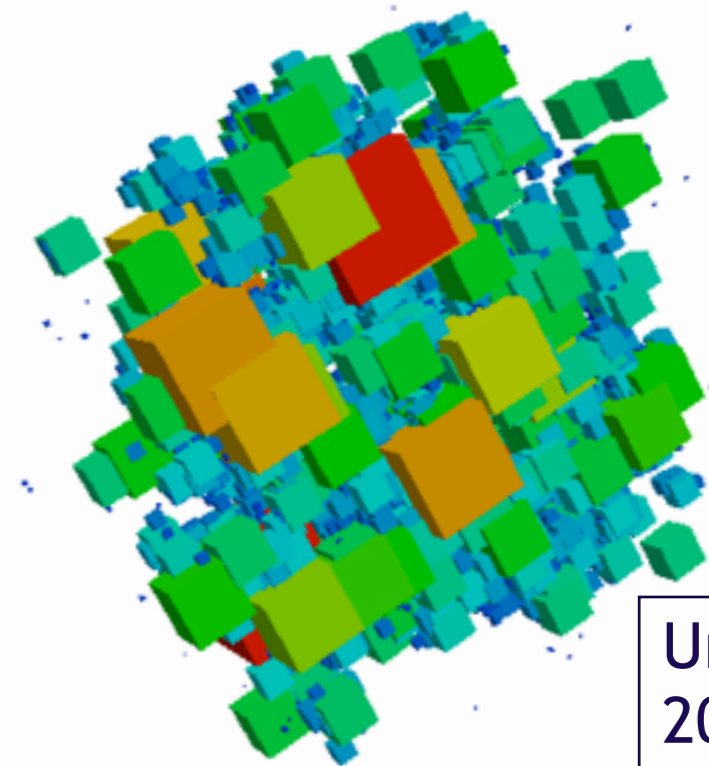


# Initial Conditions from Dynamical Approaches

- The initial  $T^{\mu\nu}$  for ideal hydrodynamics has to be given via:

$$\epsilon(x, y, z), p(x, y, z) \text{ and } n(x, y, z)$$

- **Energy deposition** model needs to describe final  $dE_T/dy$  in p-p and A-A correctly
- Granularity is influenced by
  - Shape of the incoming nuclei
  - Distribution of binary collisions
  - Interaction mechanism
  - Degree of thermalization
- Differences in **shape** and **fluctuations** need to be quantified
  - Challenge: How is local equilibrium reached so fast?



UrQMD @  
200 AGeV

# What is Usually Done?

- To calculate the energy-momentum tensor and four-current from particles a **smearing kernel** (Gaussian) is used:

$$T_{init}^{\mu\nu}(\mathbf{r}) = \sum_i \frac{p_i^\mu p_i^\nu}{p_i^0} K(\mathbf{r} - \mathbf{r}_i, \mathbf{p})$$

$$j_{init}^\mu(\mathbf{r}) = \sum_i \frac{p_i^\mu}{p_i^0} K(\mathbf{r} - \mathbf{r}_i, \mathbf{p})$$

- **Assuming** that the resulting tensor has the form for relativistic ideal fluid dynamics, the following equations are solved iteratively

$$\begin{cases} T^{00} = (\epsilon + p)\gamma^2 - p \\ T^{0i} = (\epsilon + p)\gamma^2 \mathbf{v} \\ j_B^0 = n\gamma \\ p = p_{EoS}(n, \epsilon) \end{cases}$$

- The other option: Solve the eigenvalue problem and decompose the tensor in the **Landau frame**

# Different Approaches

| Model                      | Initial condition            | Hydro               | Switching criterion  | Smearing kernel                                    | Getting $T_{ideal}^{\mu\nu}$ |
|----------------------------|------------------------------|---------------------|--|--|------------------------------|
| UrQMD hybrid [12]          | UrQMD cascade                | ideal 3+1D, SHASTA  | $t_{CM}[\text{fm}/c] = \max(2R\sqrt{\frac{E_{lab}}{2m_N}}, 1.0)$ | Gaussian z-contracted                              | $T^{\mu 0}, j^0$             |
| Skokov-Toneev hybrid [13]  | Quark-Gluon-String-Model     | ideal 3+1D, SHASTA  | $t_{CM}$ such that $S/Q_B = \text{const}$                        | not mentioned                                      | $T^{\mu 0}, j^0$             |
| EPOS [15]                  | Strings (Regge-Gribov model) | ideal 3+1D          | $\tau$   | Gaussian z-contracted                              | Landau frame                 |
| NeXSPheRIO hybrid [16, 17] | Strings (Regge-Gribov model) | ideal 3+1D, SPH     | $\tau = 1 \text{ fm}$ [18]                                       | Gaussian in $x, y, \tau\eta$                       | Landau frame                 |
| Gale et al [19]            | IP-glasma                    | viscous 3+1D, MUSIC | $\tau = 0.2 \text{ fm}/c$ ( $\sqrt{s_{NN}} = 2.76 \text{ TeV}$ ) | not mentioned                                      | Landau frame                 |
| Karpenko hybrid [20]       | UrQMD cascade                | viscous 3+1D        | $\tau_{geom}$  | Gaussian with $\sigma_{\perp}$ and $\sigma_{\eta}$ | $T^{\mu 0}, j^0$             |
| Pang et al hybrid [21]     | AMPT                         | ideal 3+1D, SHASTA  | $\tau$   | Gaussian with $\sigma_{\perp}$ and $\sigma_{\eta}$ | $T^{\mu 0}, j^0$             |

# Coarse-Grained UrQMD

1. Several thousands Au+Au collisions at  $E_{\text{lab}} = 5\text{-}160$  AGeV beam energy and different centralities
  2. Calculate  $T^{\mu\nu}$  on a space-time grid
  3. Transform to the Landau rest frame
- Investigate **locally** two measures of isotropization:

- Pressure anisotropy:

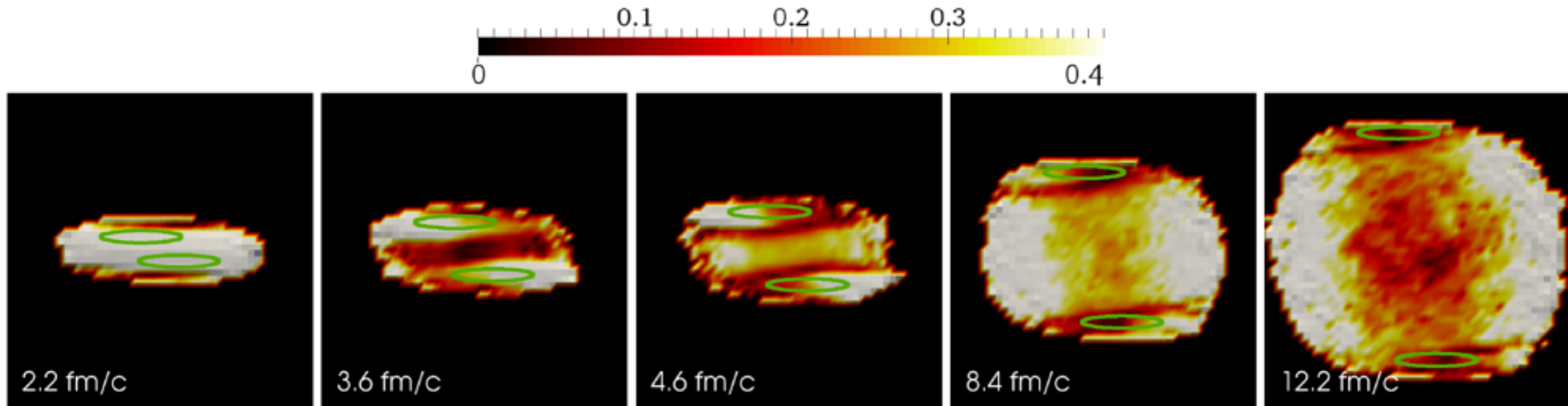
$$X \equiv \frac{|T_L^{11} - T_L^{22}| + |T_L^{22} - T_L^{33}| + |T_L^{33} - T_L^{11}|}{T_L^{11} + T_L^{22} + T_L^{33}} \ll 1$$

- Off-diagonality:

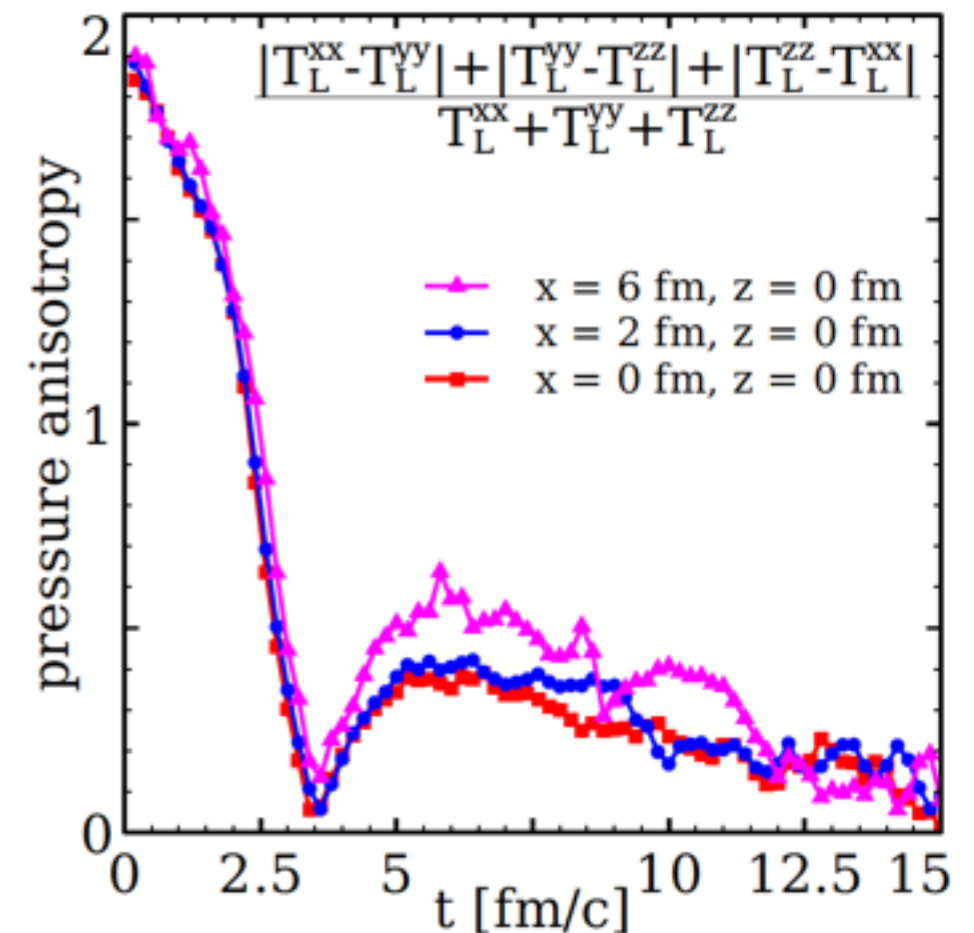
$$Y \equiv \frac{3(|T_L^{12}| + |T_L^{23}| + |T_L^{13}|)}{T_L^{11} + T_L^{22} + T_L^{33}} \ll 1$$

- $X, Y \leq 0.3 \rightarrow$  viscous hydrodynamics applicable

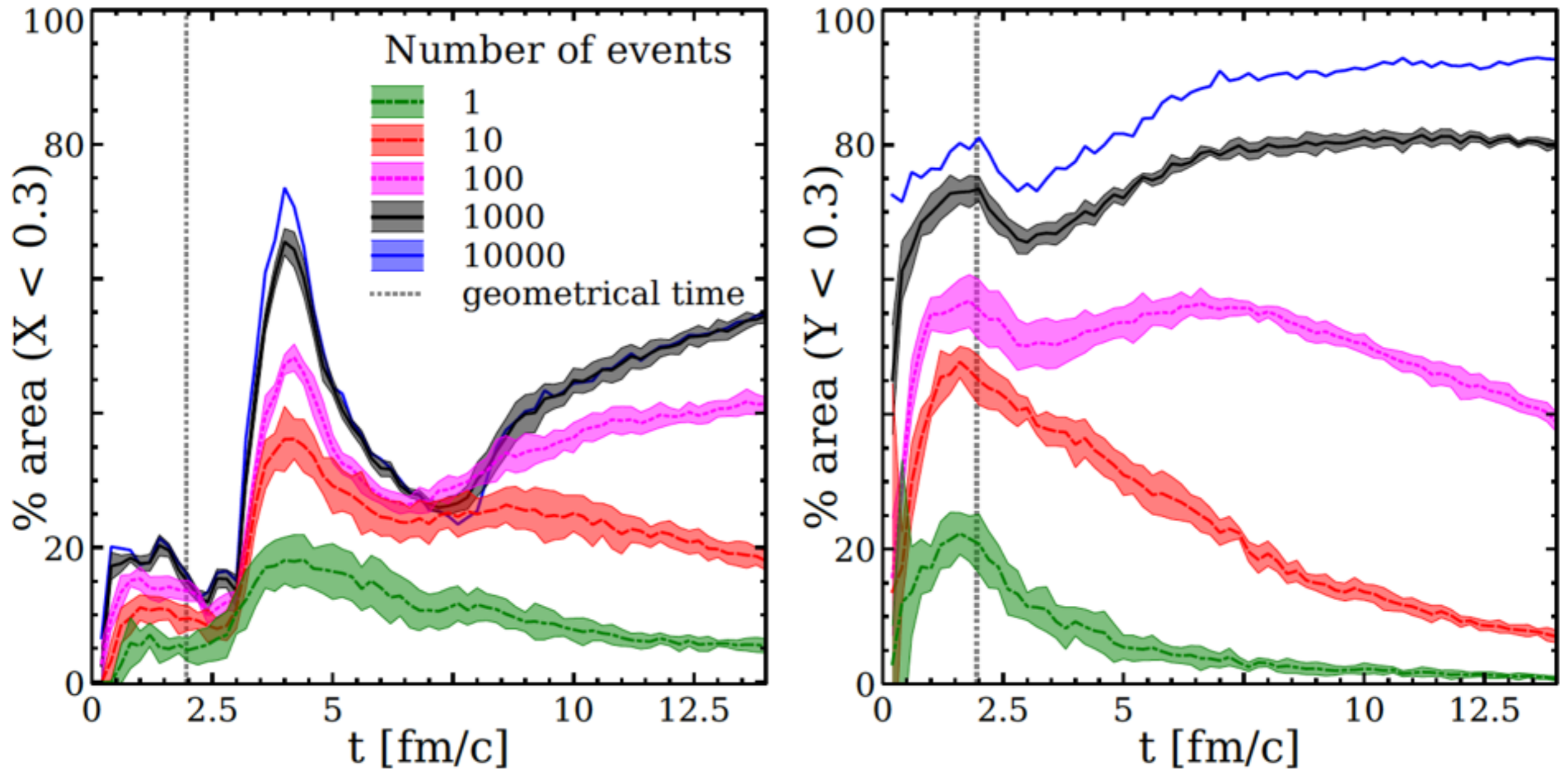
# Time Evolution



- $E_{\text{lab}} = 80A \text{ GeV}$ ,  $b=6 \text{ fm}$ , pressure anisotropy
- After initial collisions anisotropy develops minimum over a large region in space
- Later stages: Rise due to resonance decays



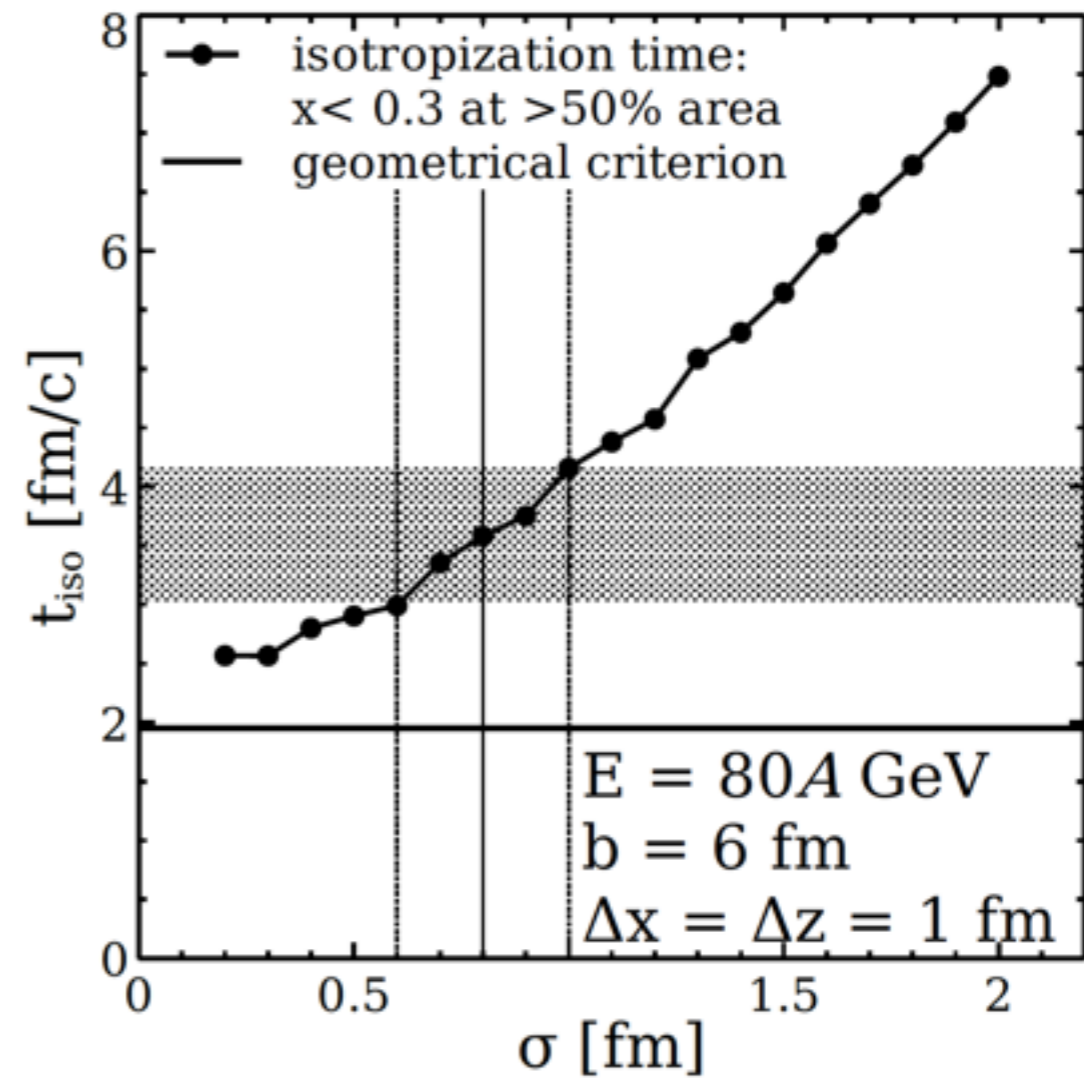
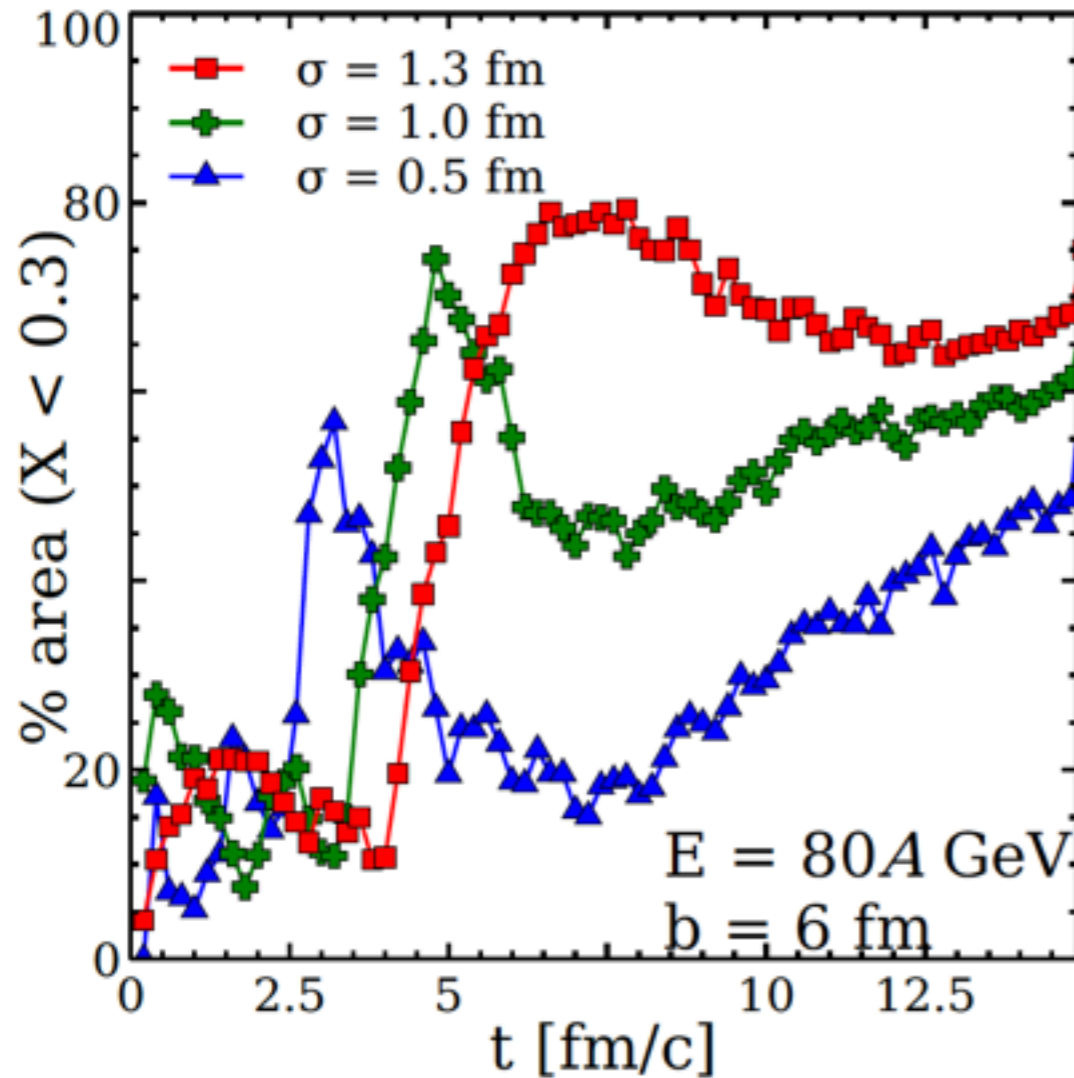
# Number of Events



- In single events only small amount of the area is isotropic
- Off-diagonality is small in more than 80 % of area

# Dependence on Gaussian Width

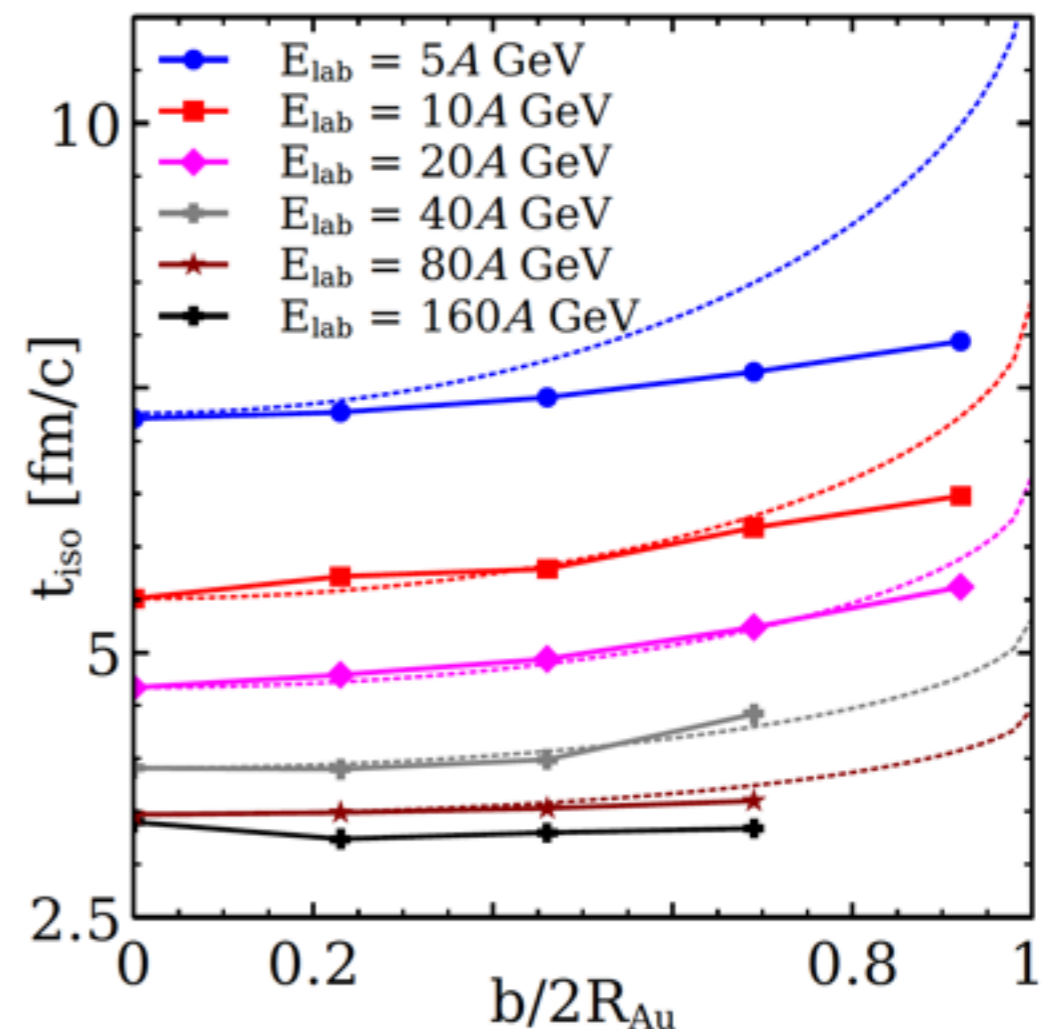
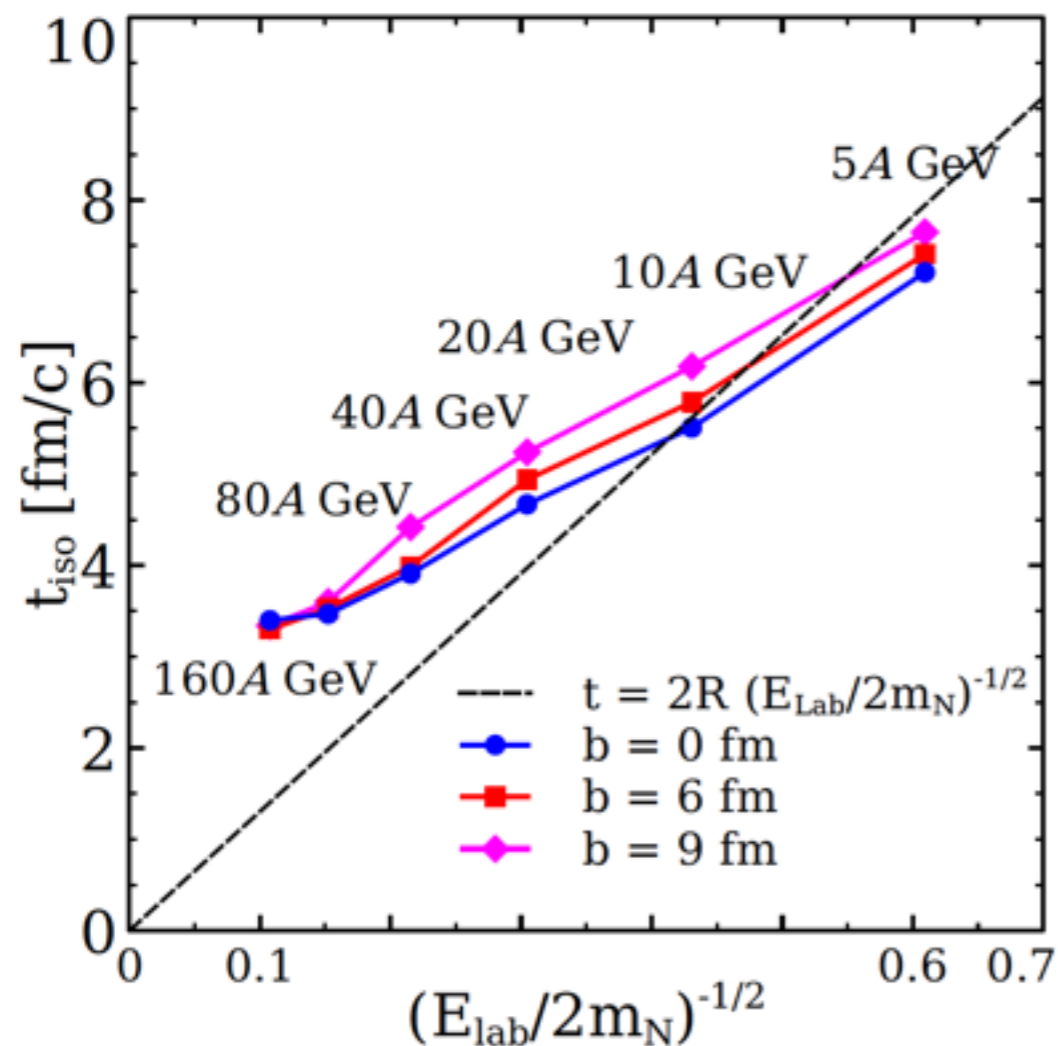
- Estimate systematic error associated with smearing parameter



- The qualitative pattern does not change

# Centrality Dependence

- Isotropization time deviates from geometrical overlap criterion for higher beam energies



- Centrality dependence is weaker than expected from geometry

$$t_0(b) = t_0(b=0) + \frac{R}{\gamma v} (\hat{1} - \sqrt{1 - (b/2R)^2})$$



# Cooper-Frye - Negative Contributions

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based on work with D. Oliinychenko and P. Huovinen,  
Phys.Rev. C91 (2015) 2, 024906

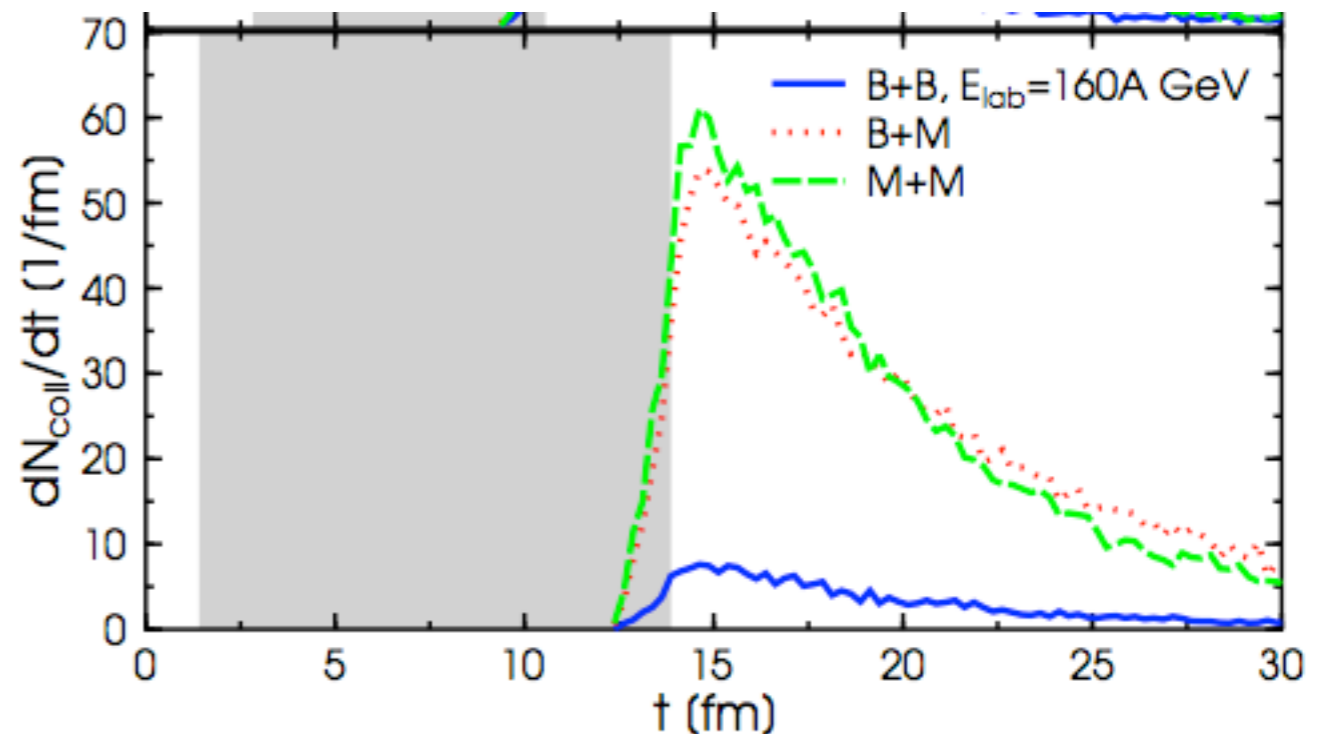
# Freeze-out Procedure

- **Deconfinement/Confinement** transition happens through equation of state in hydrodynamics
- **Transition** from hydro to transport when temperature/energy density is smaller than **critical value**
- Particle distributions are generated according to the **Cooper-Frye** formula

$$E \frac{dN}{d^3p} = \int_{\sigma} f(x, p) p^{\mu} d\sigma_{\mu}$$

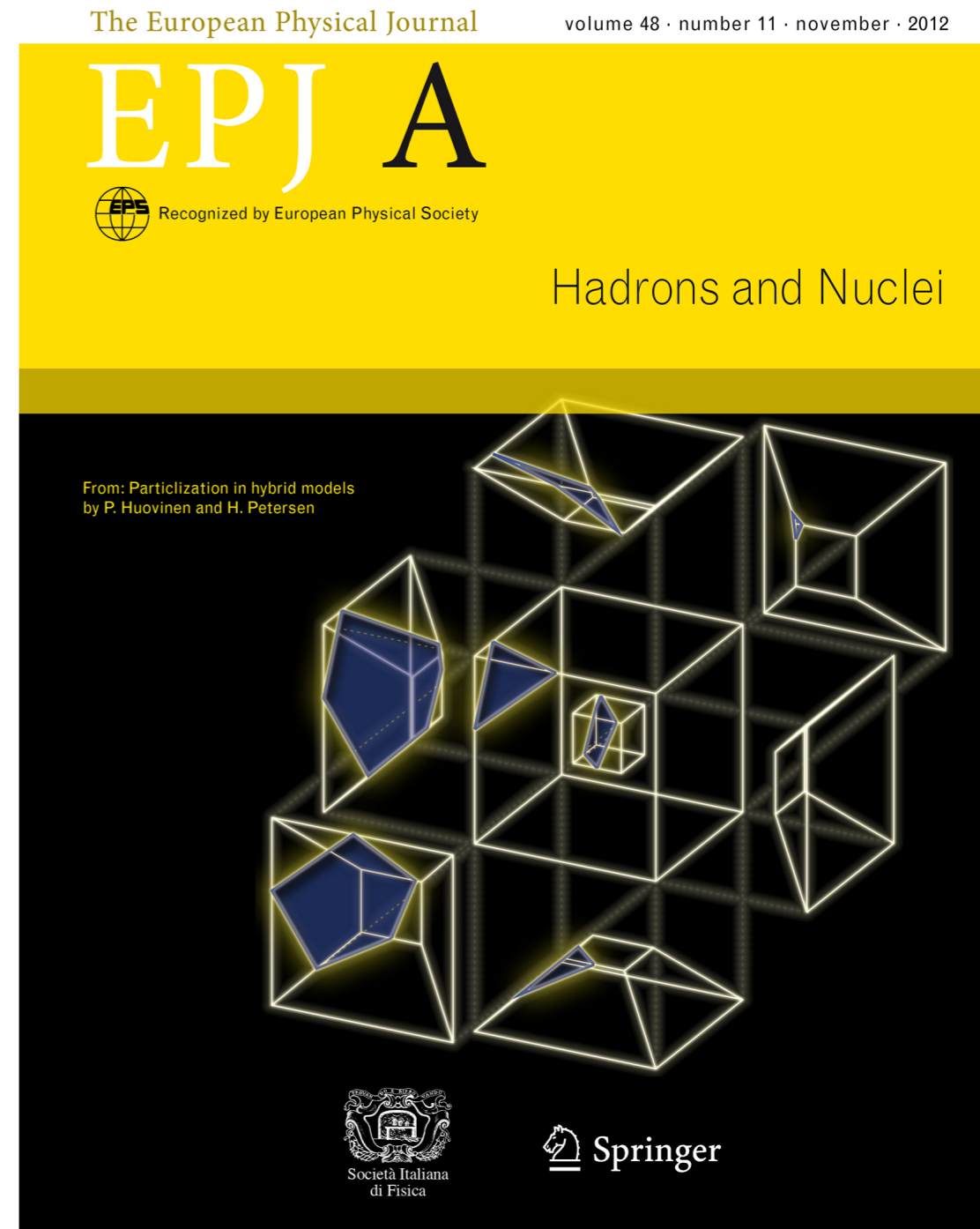
- Same EoS on both sides of the transition hypersurface
- Rescatterings and final decays calculated via **hadronic cascade** (UrQMD)

- Separation of **chemical** and **kinetic** freeze-out is taken into account
- **Large viscosity** in hadron gas stage!



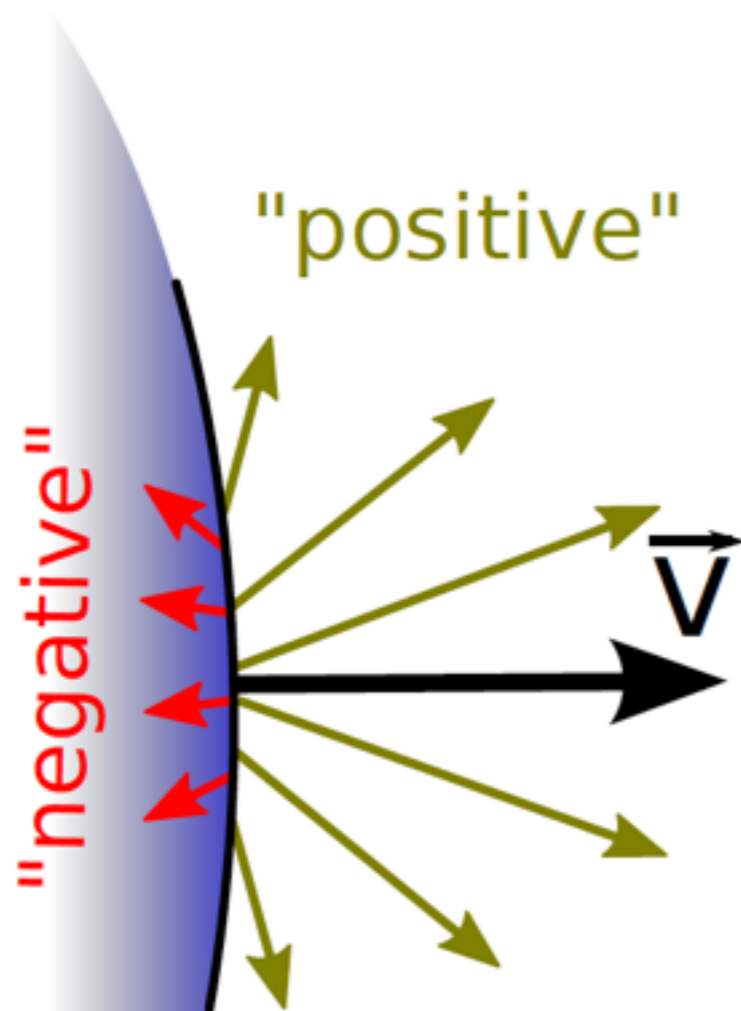
# Hypersurface Finding

- **Cornelius:** 3D hypersurface in 4 dimensions
- Constant energy density
- Avoiding holes and double-counting
- Applicable as a subroutine
  - Input: 16-tuples of spatio-temporal information
  - Output: Hypersurface vectors and interpolated thermodynamic quantities



P. Huovinen, H.P. arXiv: 1206.3371  
Fortran and C++ subroutines, cornelius,  
implementations of this algorithm in 3D and 4D, are  
available at <https://karman.physics.purdue.edu/OSCAR>

# Negative Contributions

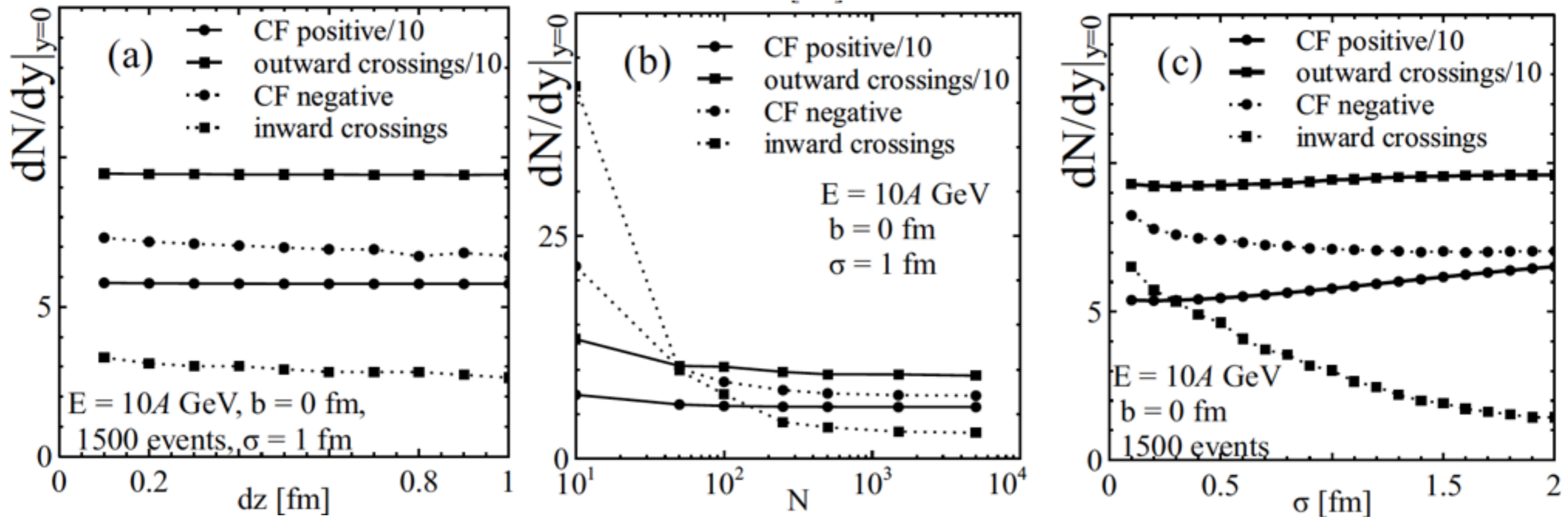


- Definition:
  - Particles outward:  $p^\mu d\sigma_\mu > 0$
  - Particles inward:  $p^\mu d\sigma_\mu < 0$
- Different options:
  - Account for feedback in hydro  
K. Bugaev, Phys Rev Lett. 2003; L. Czernai, Acta Phys. Hung., 2005
  - Account effectively by weights in transport  
S. Pratt, Phys.Rev. C89 (2014) 2, 024910
  - Neglect them and violate conservation laws
- **Systematic study of the size of negative contributions by comparison to actual transport**

$d\sigma_\mu$  - normal 4-vector  
 $u_\mu = (\gamma, \gamma \vec{v})$  - 4-velocity  
 $T$  - temperature  
 $\mu$  - chemical potential

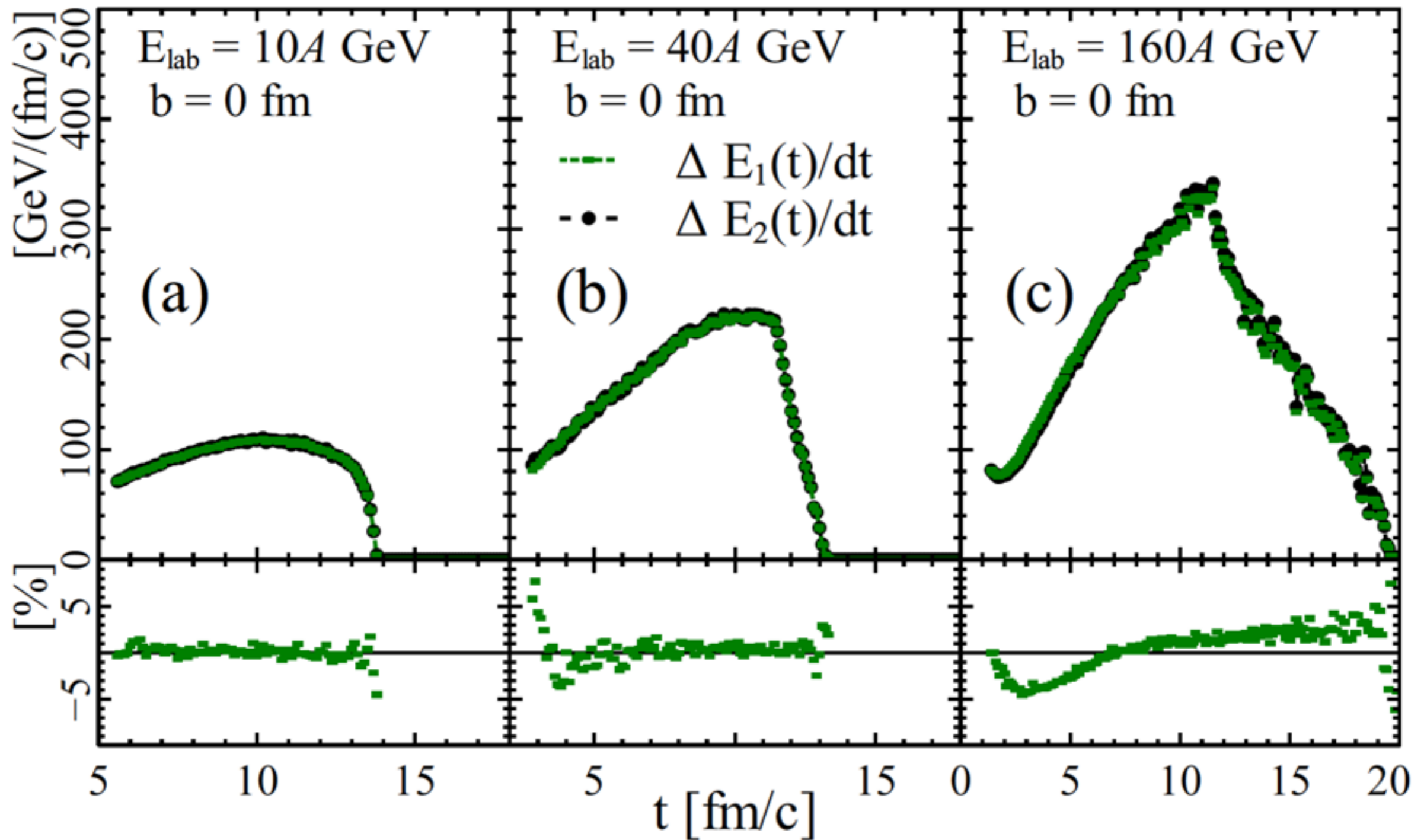
# Parameter Sensitivities

- Comparison of coarse-grained transport with Cooper-Frye calculation vs actual particles



- No significant dependence on **cell sizes**
- **Saturation** for large enough number of events
- Dependence on  $\sigma$  due to **smearing** of surface velocities

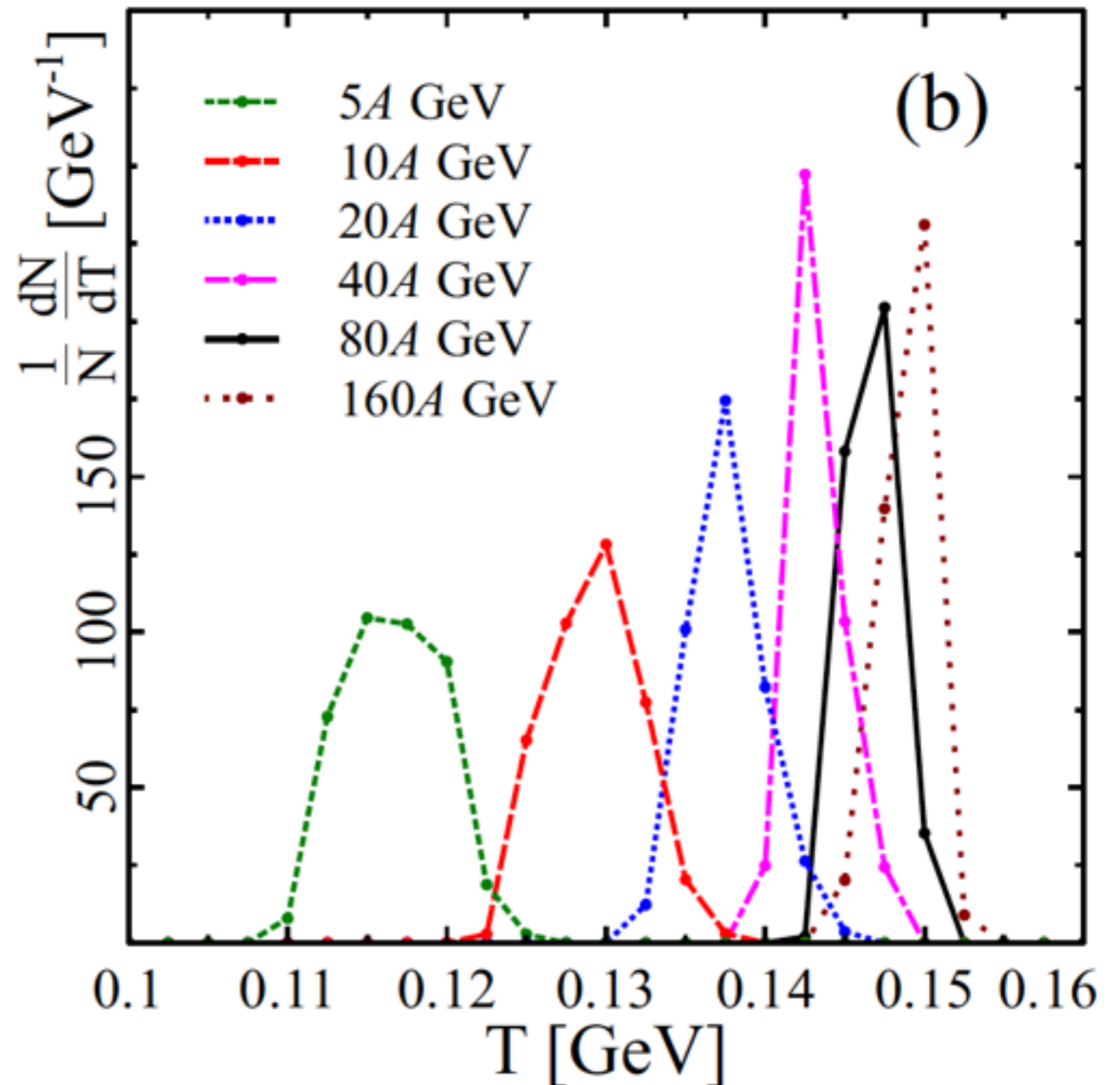
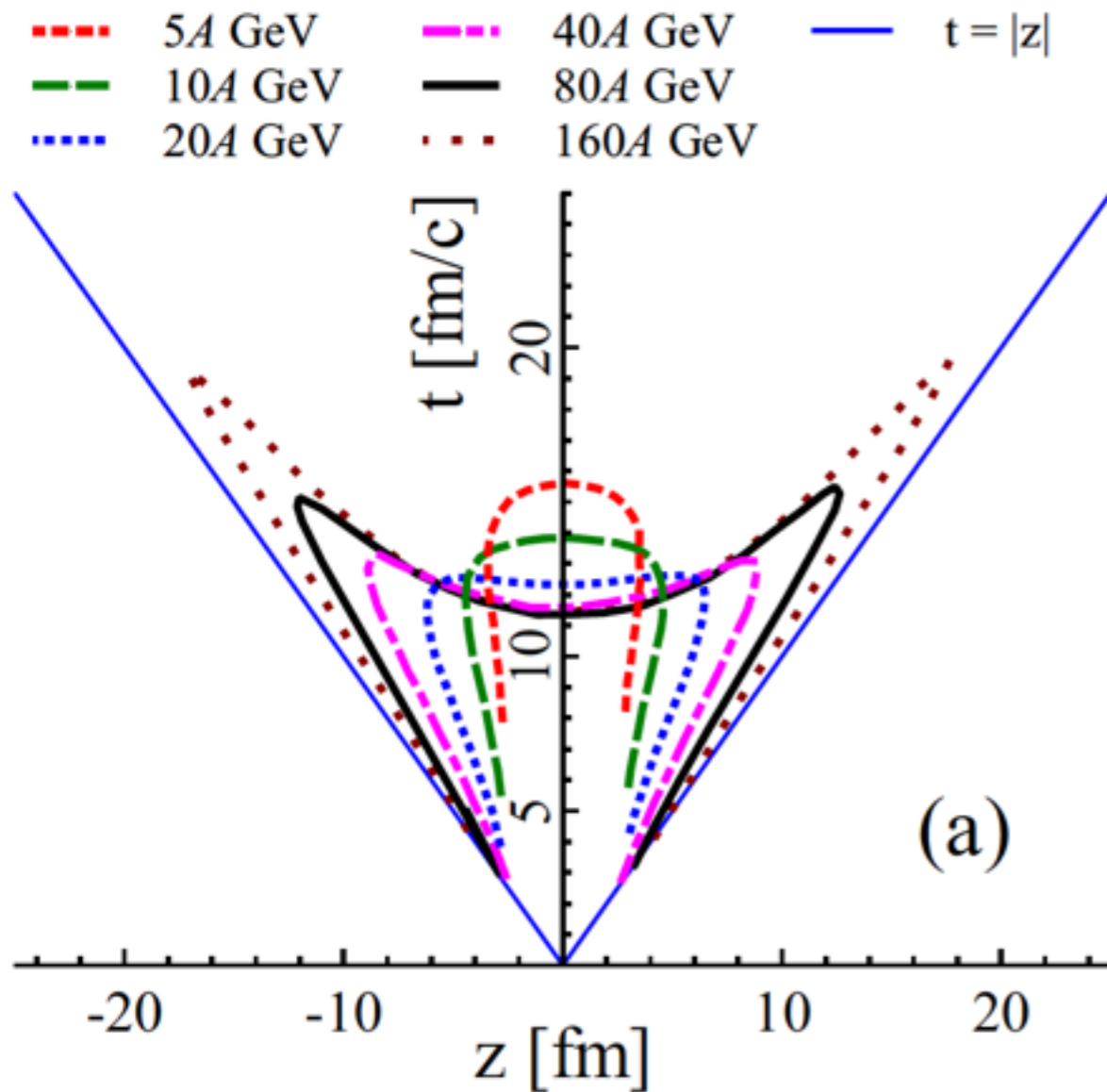
# Cross-Check Energy Conservation



- Energy is conserved at all times on hypersurface

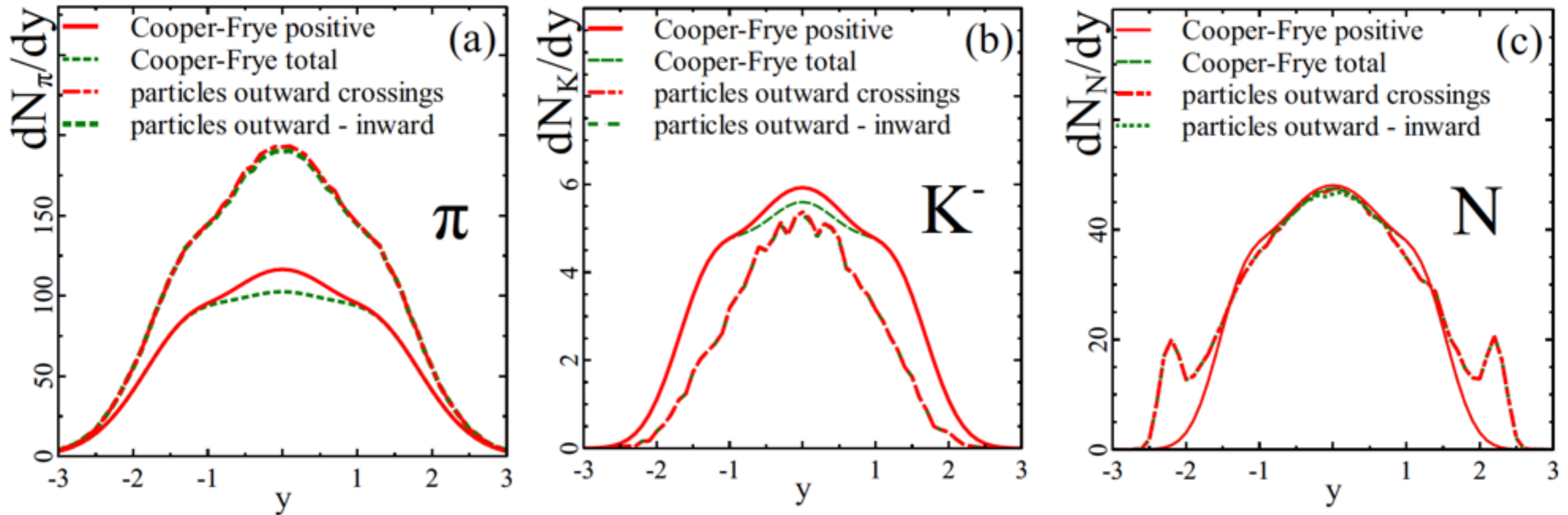
# Hypersurfaces

- Iso-energy density hypersurfaces ( $\epsilon_c = 0.3 \text{ GeV}/\text{fm}^3$ ) represent distributions in temperature



# Mass Dependence

- Rapidity spectra of pions, kaons and protons at  $E_{\text{lab}} = 40A$  GeV central Au+Au collisions

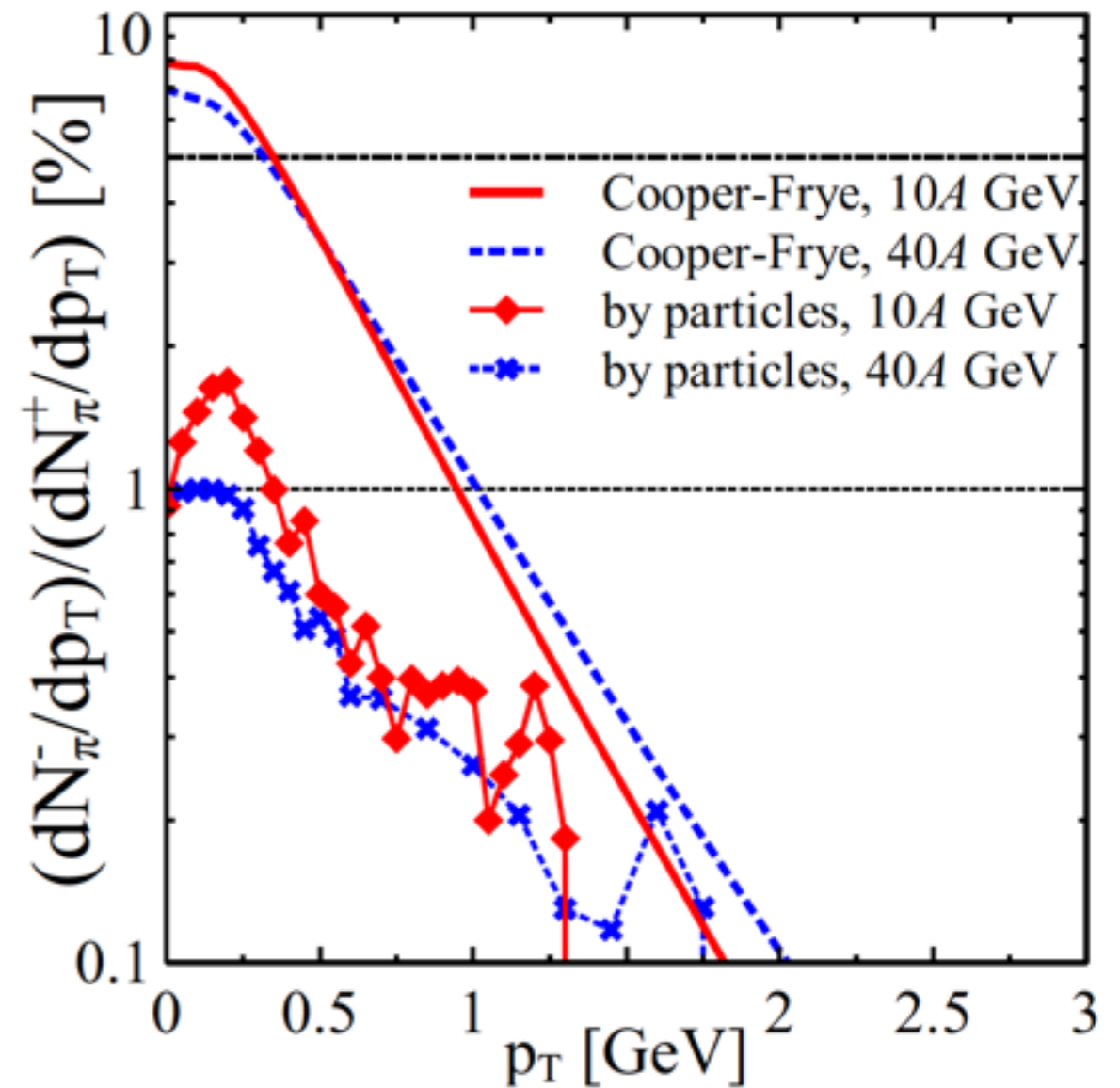
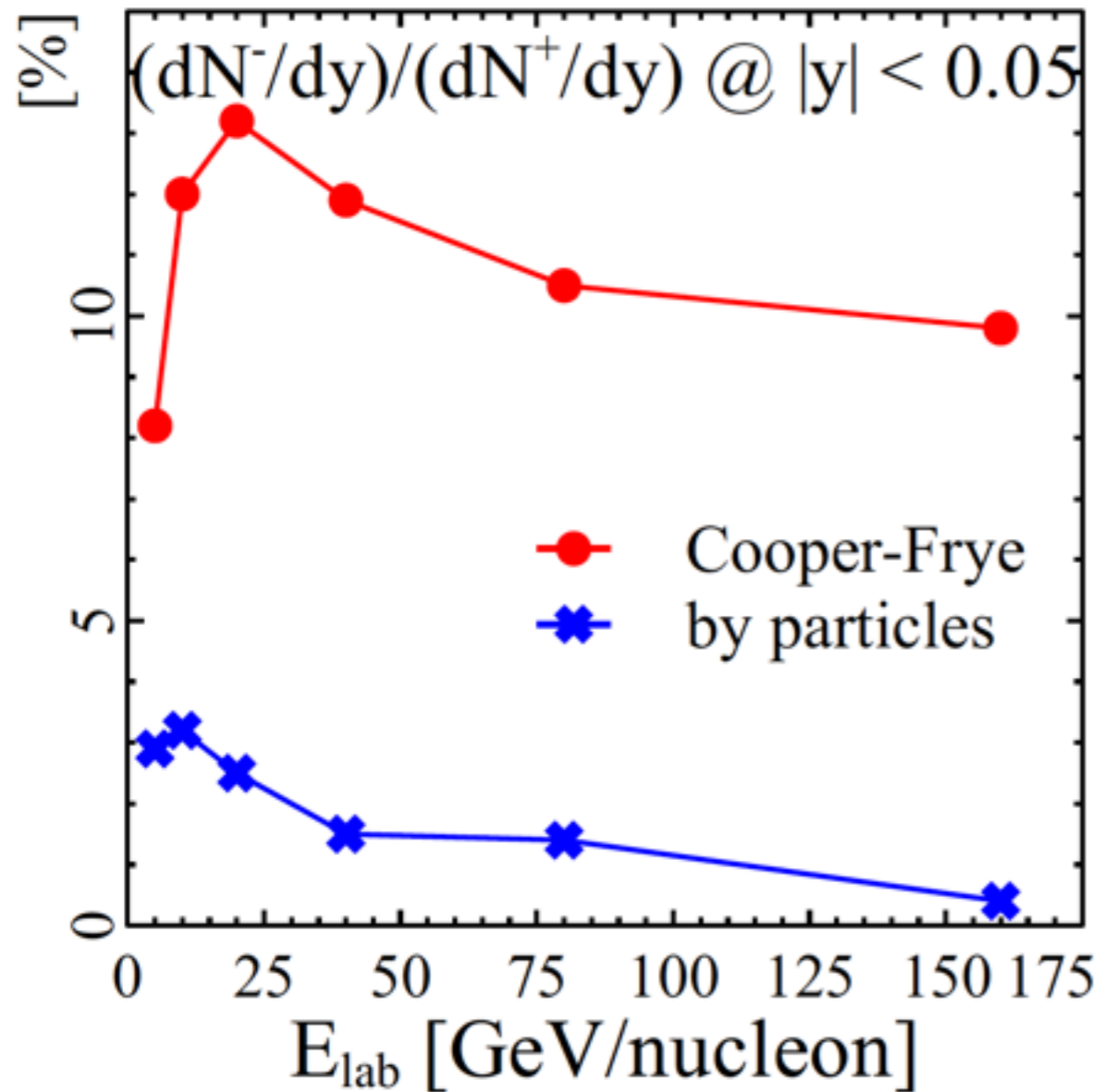


- Negative contributions are negligible for more massive particles
  - Concentrate on pions in the following



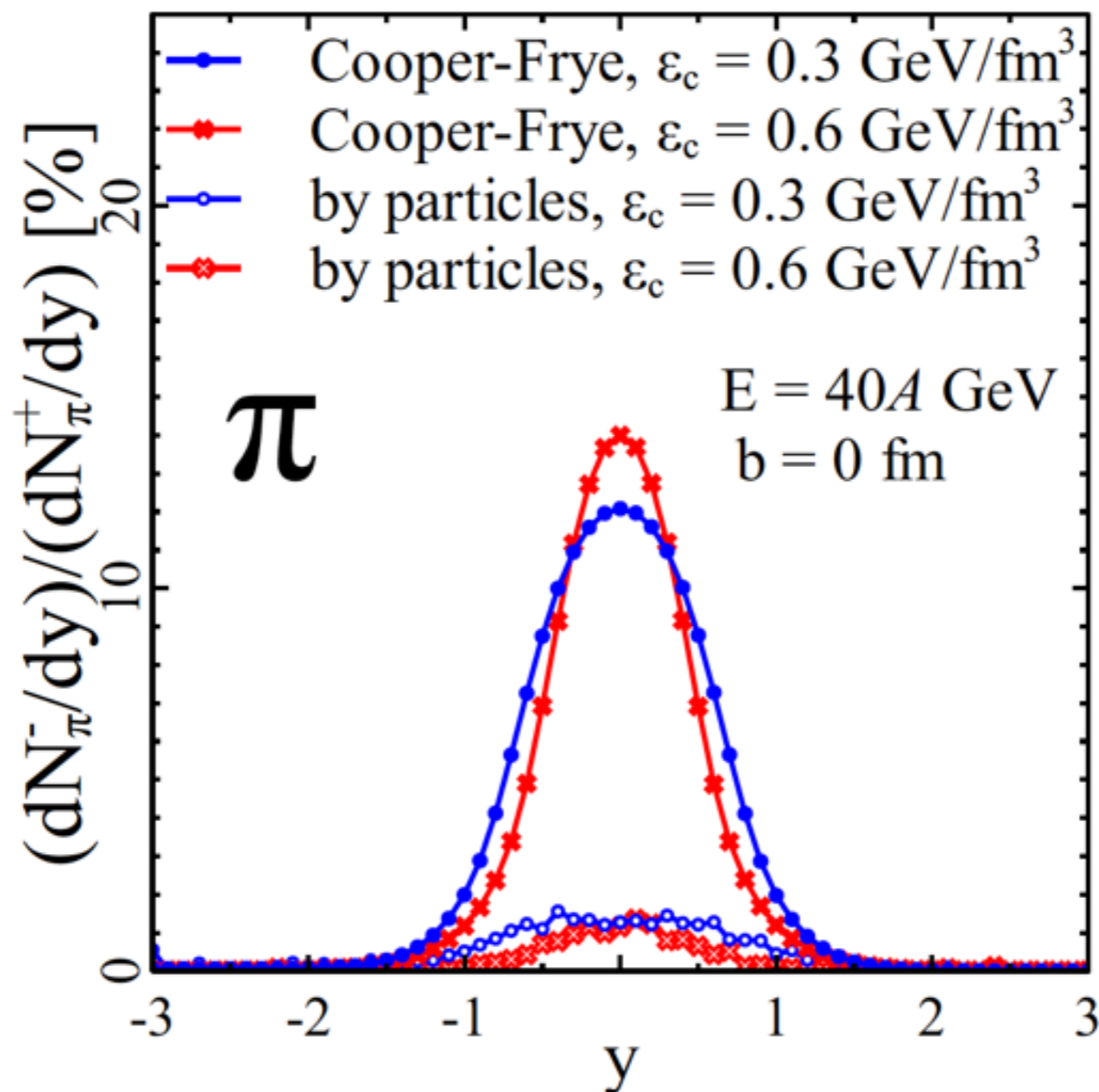
# Energy Dependence

- Maximum at  $E_{\text{lab}} \sim 25$  AGeV, decreasing at higher energies
- Actual particles are always less likely to fly inward



- Negative contributions are larger at small  $p_T$

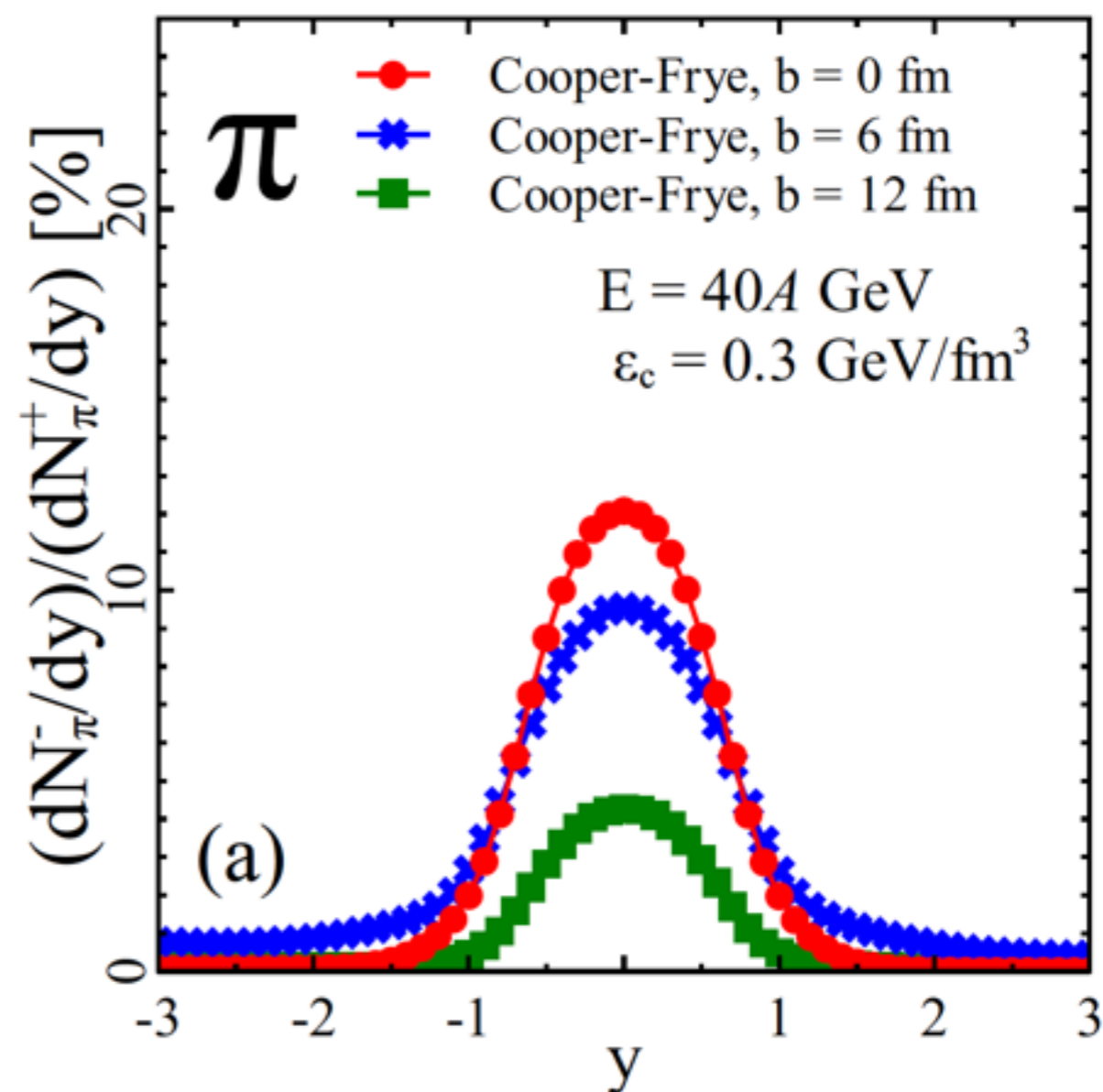
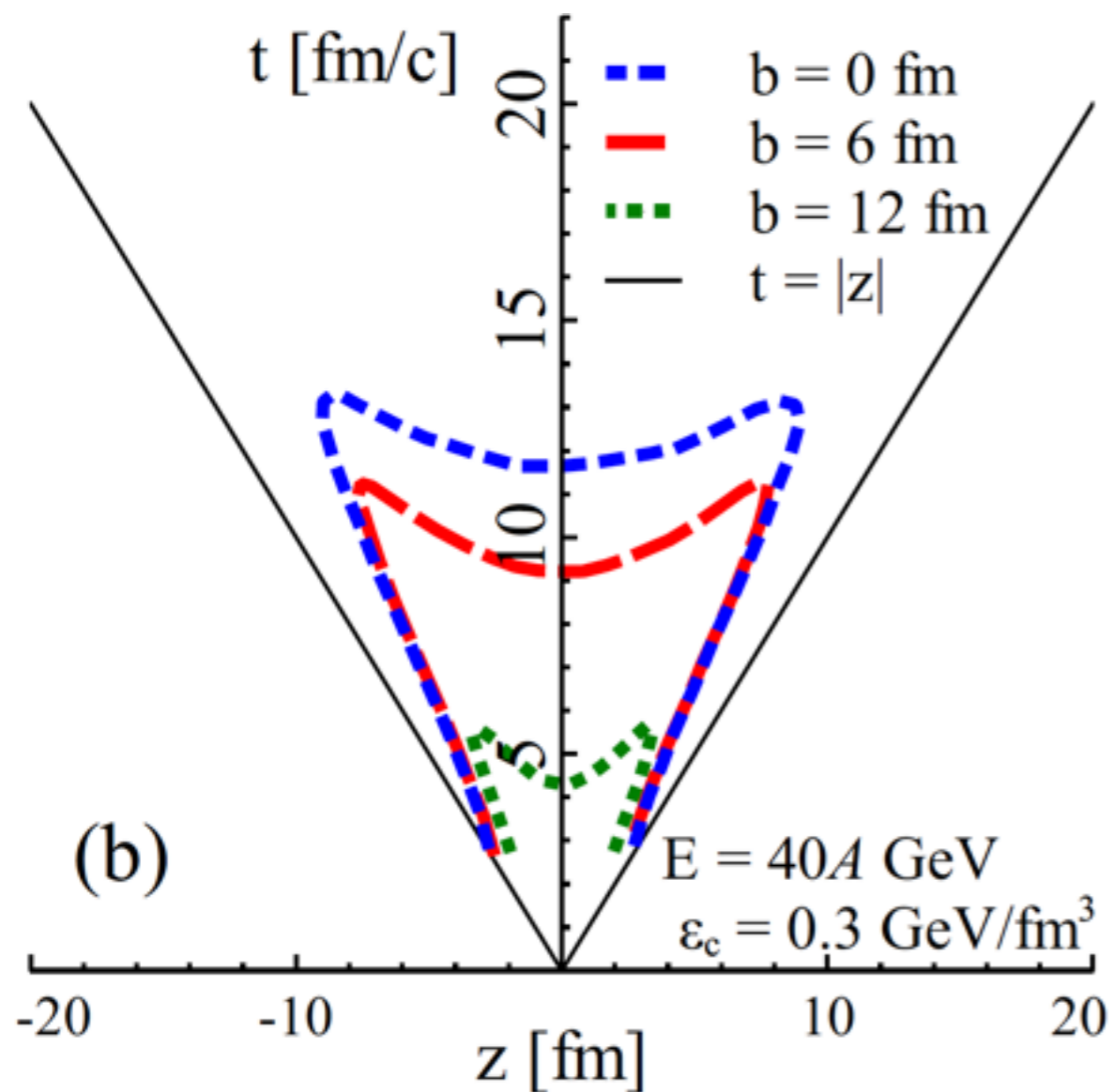
# Switching Criterion



- Shape of the negative contributions as a function of rapidity depends slightly on switching criterion

# Centrality Dependence

- Ratio of surface to volume emission varies with centrality
- Due to larger relative flow velocities the negative contributions are larger in more central events



# Summary

- **Hybrid approaches** based on relativistic hydrodynamics and hadron transport provide realistic dynamical description
- Two transitions have been studied systematically using **coarse-grained** UrQMD calculations
- Initial switching transition:
  - **Off-diagonality** is small for a large fraction of the system in the coarse-grained scenario, event by event larger effect
  - **Isotropy** (according to the weak criterion) is reached at times at maximum 3 fm/c larger than geometrical overlap criterion
- Cooper Frye negative contributions:
  - **Decrease** for higher **masses** and higher **beam energies**
  - Largest at **low transverse** momentum
  - Always **smaller for actual** particles in transport calculation
- Is dynamic switching between hydro and transport feasible?

# Backup

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# Influence of Statistics

- From  $N$  random thermal pions, the effect of finite particle statistics on the deviations of the energy-momentum tensor from equilibrium can be estimated

