Understanding non-linear hydrodynamic response in HI collisions via flow correlations

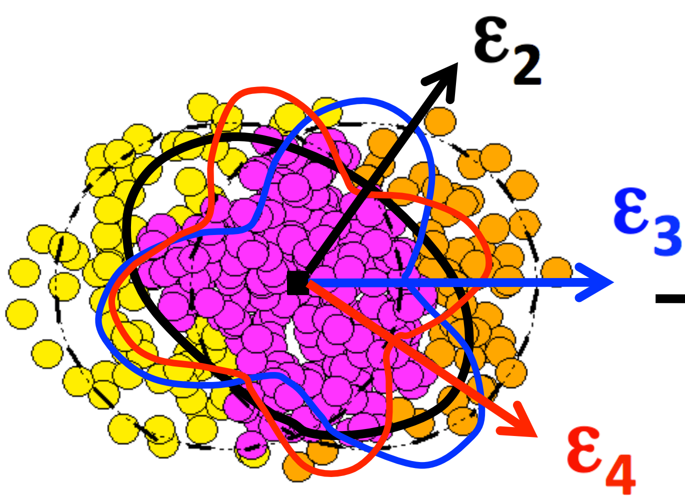
Soumya Mohapatra
Columbia University

COLUMBIA UNIVERSITY

COLUMBIA UNIVERSITY

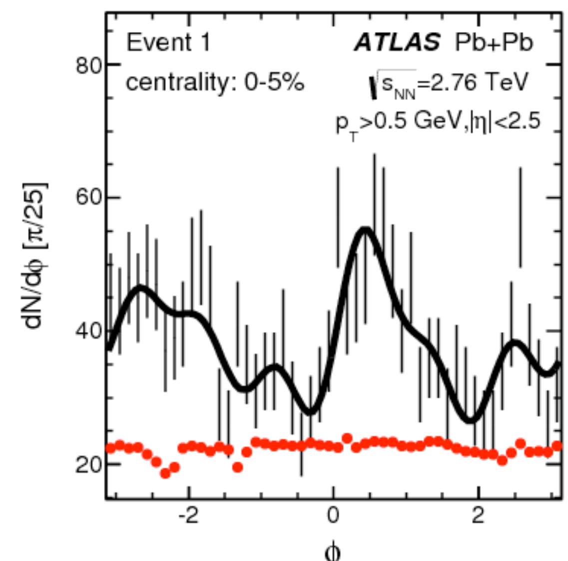
Introduction

- Initial spatial fluctuations of nucleons lead to higher moments of deformations in the fireball, each with its own orientation.
- The spatial anisotropy is transferred to momentum space by collective flow.



Collective Dynamics

$EOS,\ \eta / S$



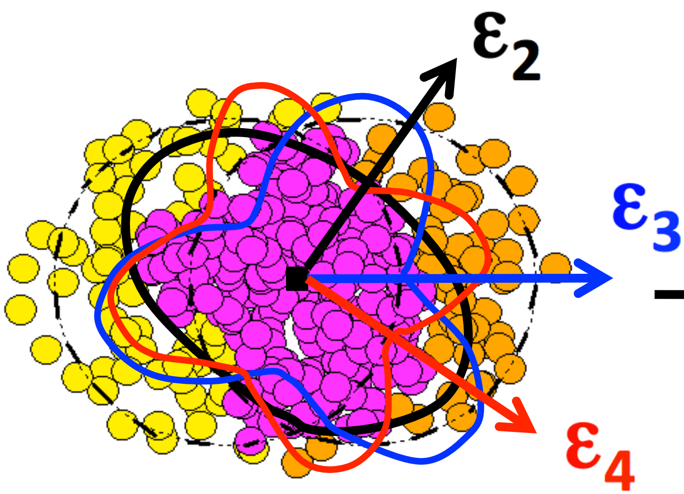
Initial Geometry

Final particle anisotropies

$\varepsilon_n \longrightarrow V_n$

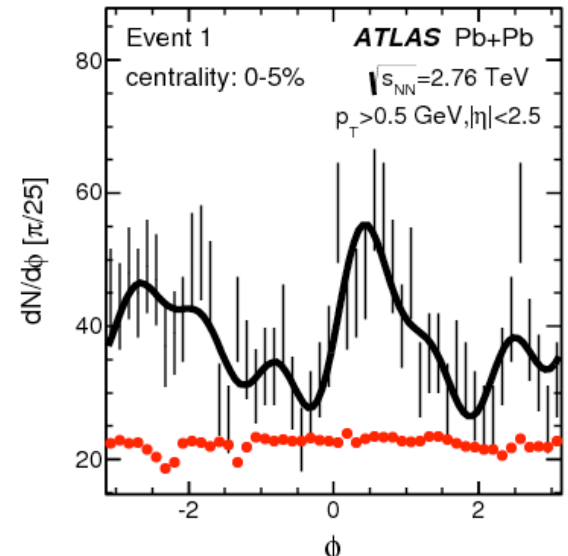
Introduction

- Initial spatial fluctuations of nucleons lead to higher moments of deformations in the fireball, each with its own orientation.
- The spatial anisotropy is transferred to momentum space by collective flow.



Collective Dynamics

$EOS,\ \eta / S$



Initial Geometry

Final particle anisotropies

$\varepsilon_n \rightarrow v_n$

$\varepsilon_{m \neq n} \rightarrow v_n$

Non-linear response

Origin of the flow correlations-I

Representation of flow vector: $\vec{v}_n \equiv (v_n \cos(n\Phi_n), v_n \cos(n\Phi_n)) \equiv v_n e^{in\Phi_n}$

Hydro response is linear for v_2 and v_3: $v_n \propto \varepsilon_n$ and $\Phi_n \approx \Phi_n^*$ i.e.

$$v_2 e^{i2\Phi_2} \propto \varepsilon_2 e^{i2\Phi_2^*}, \quad v_3 e^{i3\Phi_2} \propto \varepsilon_3 e^{i3\Phi_2^*}$$

PhysRevC.84.024911 (Qui & Heinz)
PhysRevC.87.054901 (Niemi et al.)

Non-linear terms possible for higher n

Eccentricities of initial geometry

$$v_4 e^{i4\Phi_4} = \alpha_4 \, \epsilon_4 e^{i4\Phi_4^*} + \alpha_{2,4} \left(\epsilon_2 e^{i2\Phi_2^*}\right)^2 + \ldots$$

Hydrodynamic response to eccentricities

PhysRevC.85.024908 Gardim et al.
j.nuclphysa.2013.02.025 Teaney & Yan

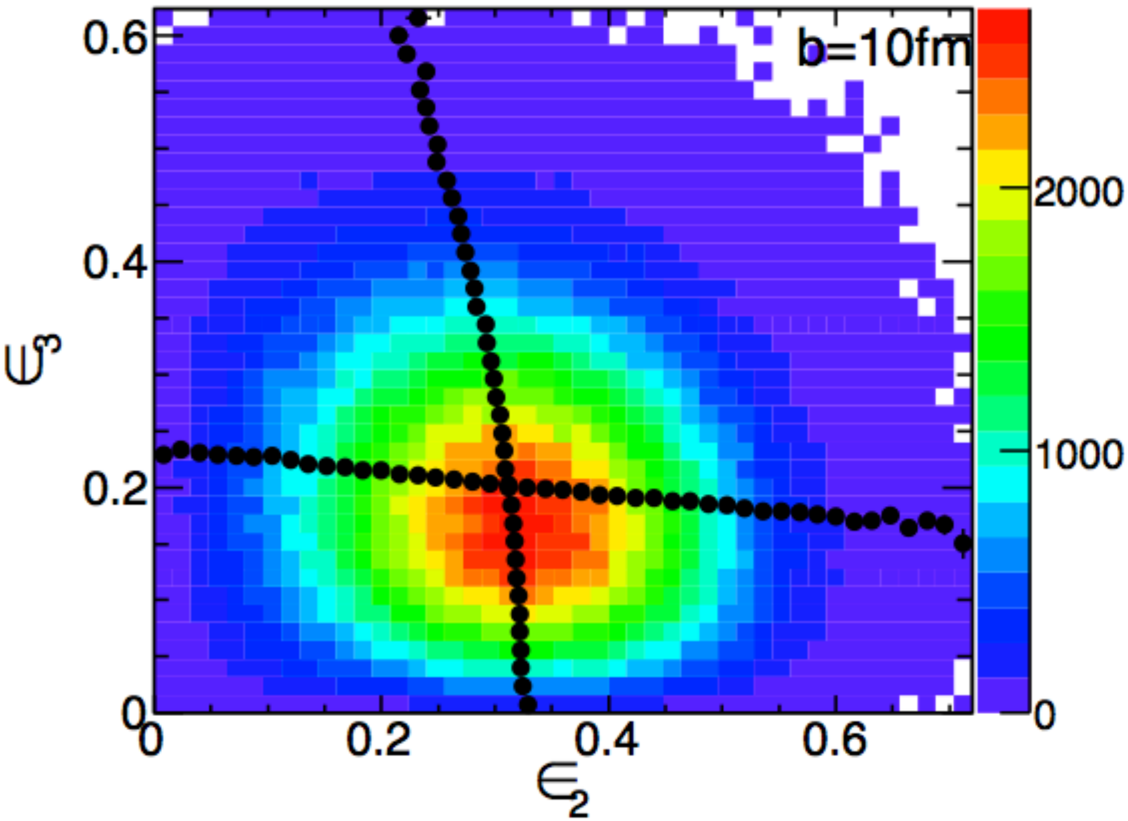
$$= \alpha_4 \, \epsilon_4 e^{i4\Phi_4^*} + \beta_{2,4} \, v_2^2 e^{i4\Phi_2} + \ldots,$$

Similarly correlations can occur between three harmonics of different orders:

$$v_5 e^{i5\Phi_5} = \alpha_5 \epsilon_5 e^{i5\Phi_5^*} + \alpha_{2,3,5} \epsilon_2 e^{i2\Phi_2^*} \epsilon_3 e^{i3\Phi_3^*} + \ldots.$$

$$= \alpha_5 \epsilon_5 e^{i5\Phi_5^*} + \beta_{2,3,5} v_2 v_3 e^{i(2\Phi_2 + 3\Phi_3)} + \ldots$$

Origin of the flow correlations-II



Pb+Pb , b_{imp}=10 fm
PhysRevC.90.024910
Huo, Jia & SM,

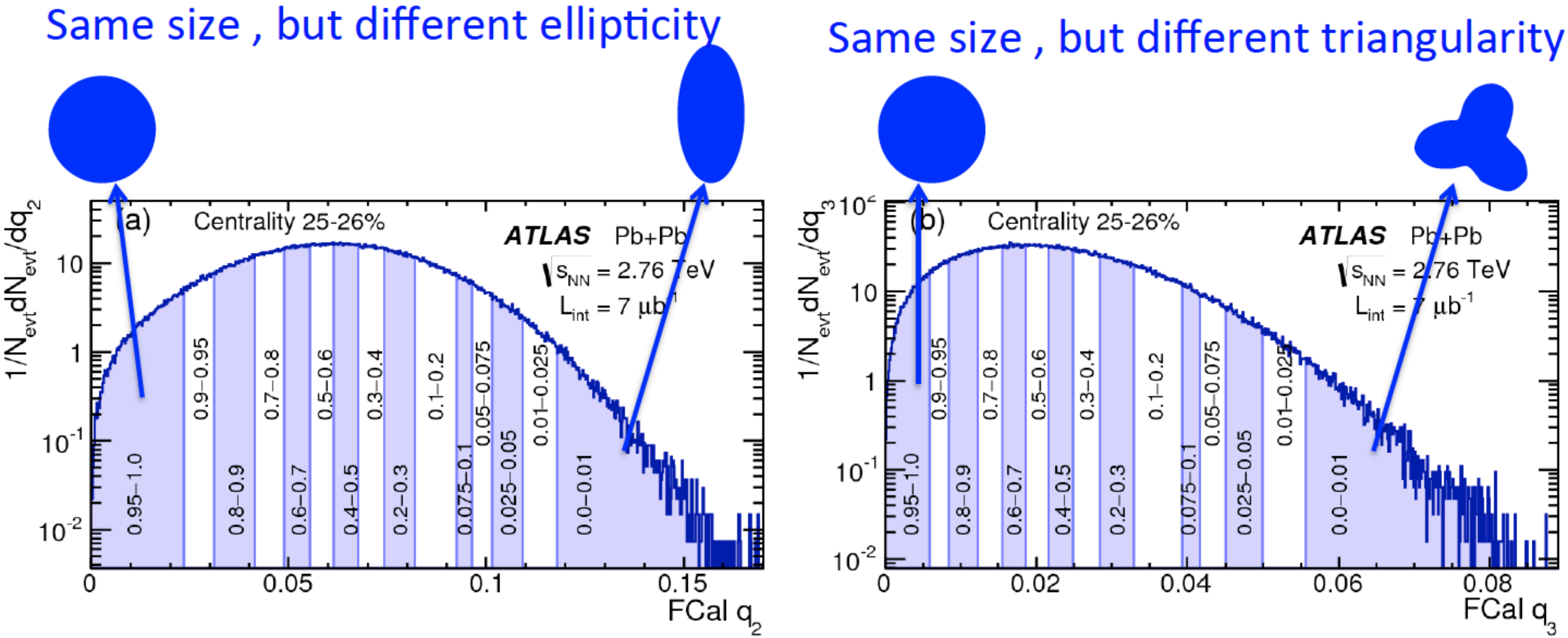
ε_2 and ε_3 are anti-correlated at fixed b_{imp} (centrality)

- Initial geometry effects.

$$v_2 e^{i2\Phi_2} \propto \epsilon_2 e^{i2\Phi_2^*}, \quad v_3 e^{i3\Phi_3} \propto \epsilon_3 e^{i3\Phi_3^*}$$

- Geometry Correlations = Flow correlations

Event-shape selection

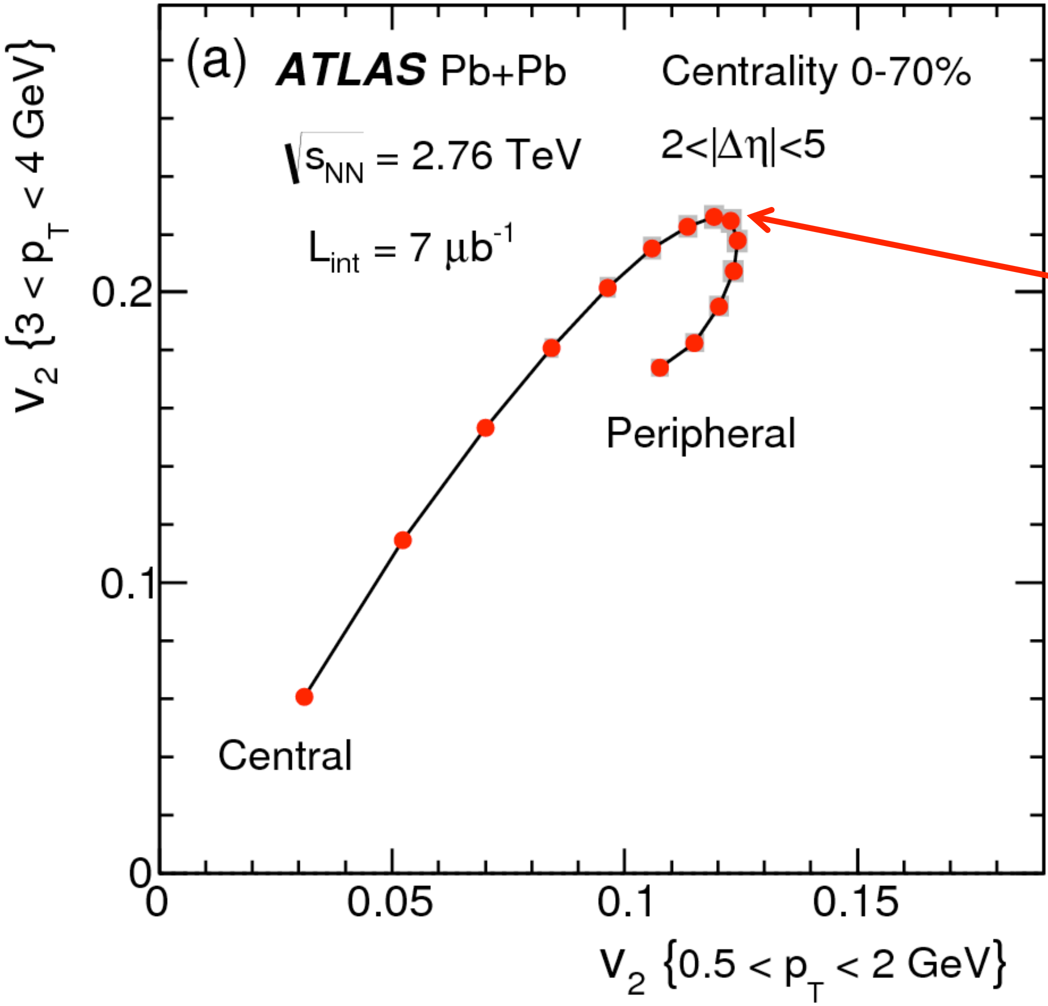


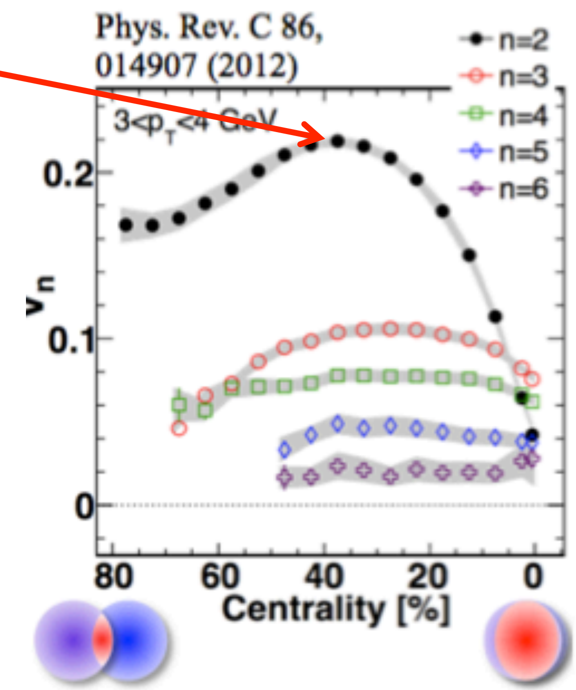
arXiv:1504.01289

- Select events within same centrality that have different geometries : different ellipticity or triangularity.
- Make geometry bins using integrated v_2 or v_3 measured in Forward detectors
- Measure correlations between flow harmonics ay mid-rapidity

v_2-v_2 correlations : Centrality bins only

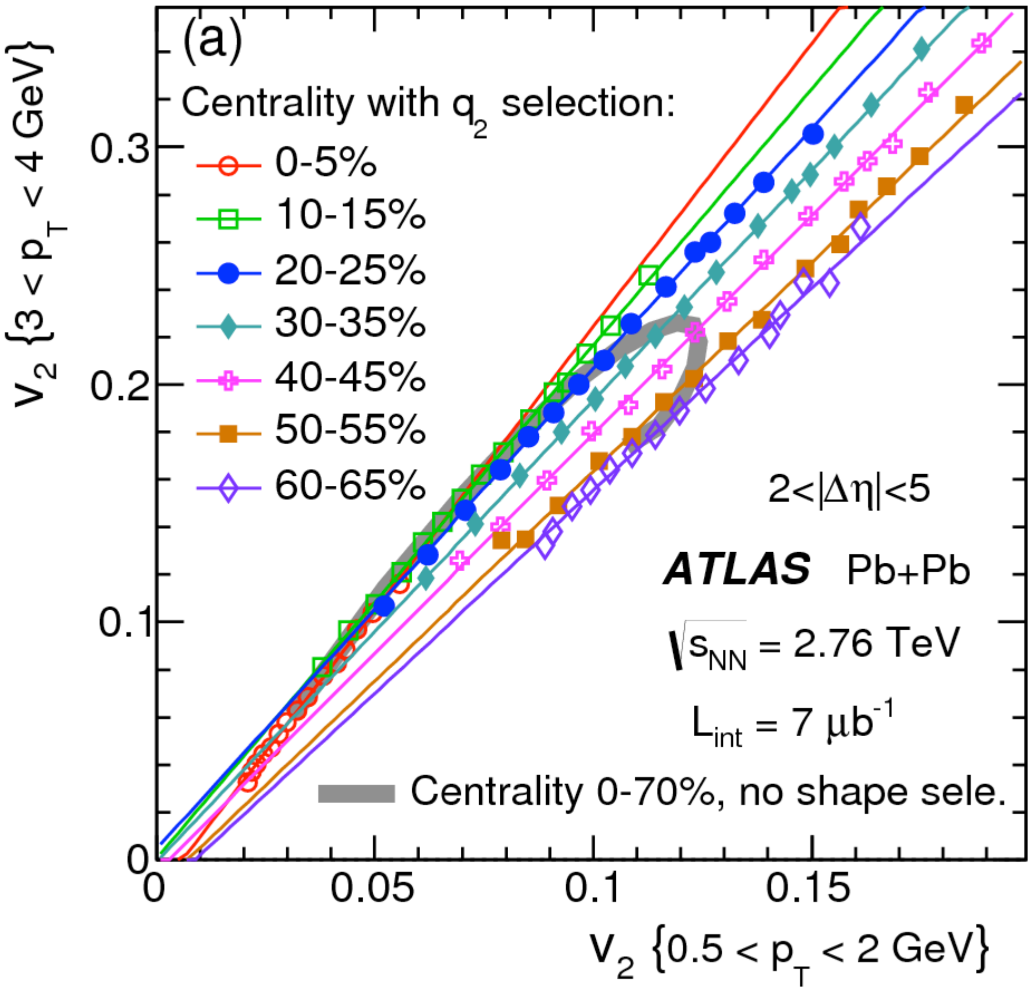
arXiv:1504.01289





- Plot shows low-p_T v_2 intermediate-p_T v_2 correlation as centrality varies
- See non-trivial dependence with centrality (boomerang-curve),
- Indicates that viscous correction larger in peripheral events

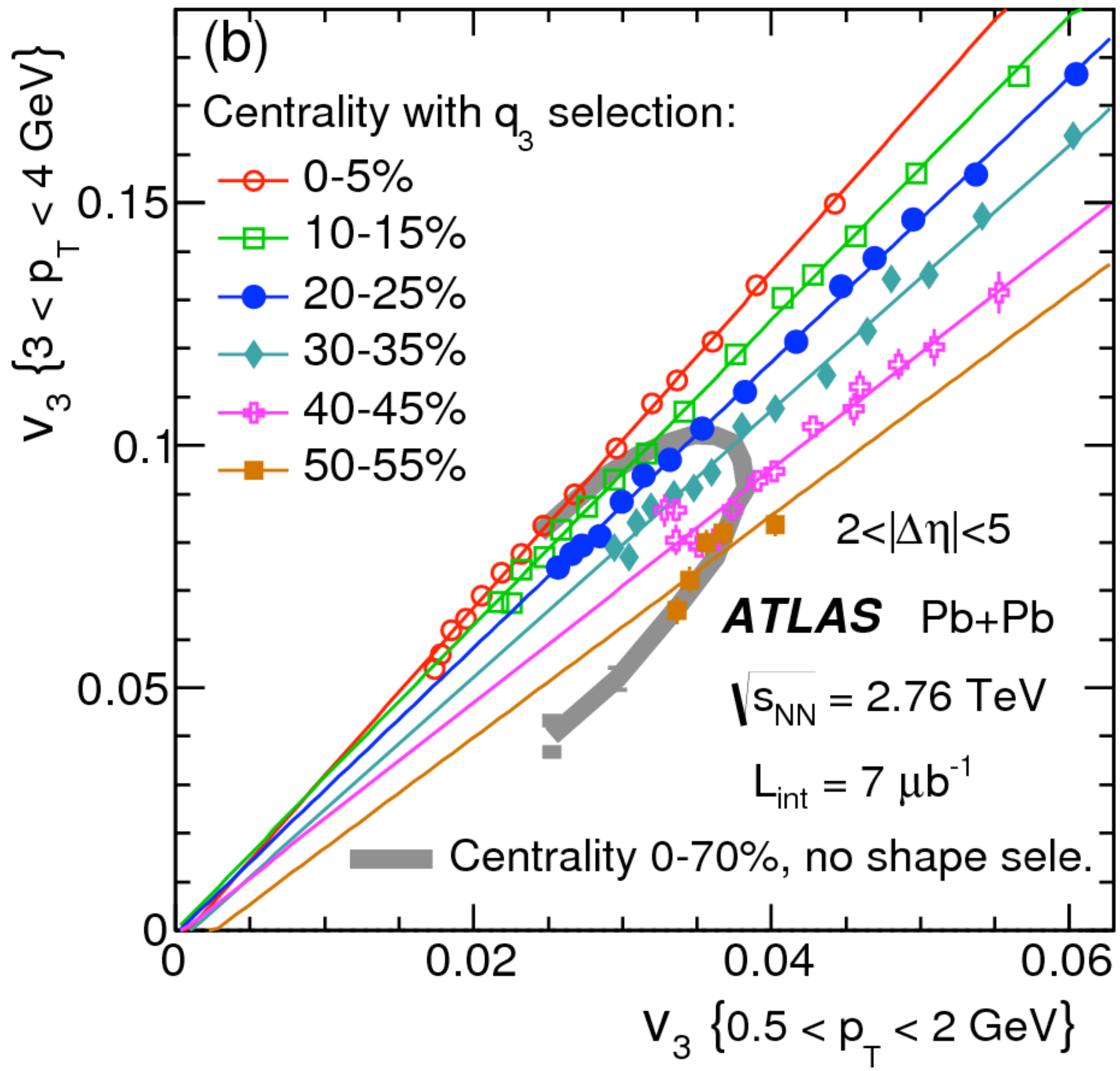
v_2-v_2 correlations : q_2-bins



arXiv:1504.01289

- Now for each centrality binning in event geometry (ellipticity) as well
- Saw non-trivial dependence with centrality (boomerang),
  - but within one centrality dependence is linear!
- Indicates that viscous correction mostly controlled by system size, not shape!

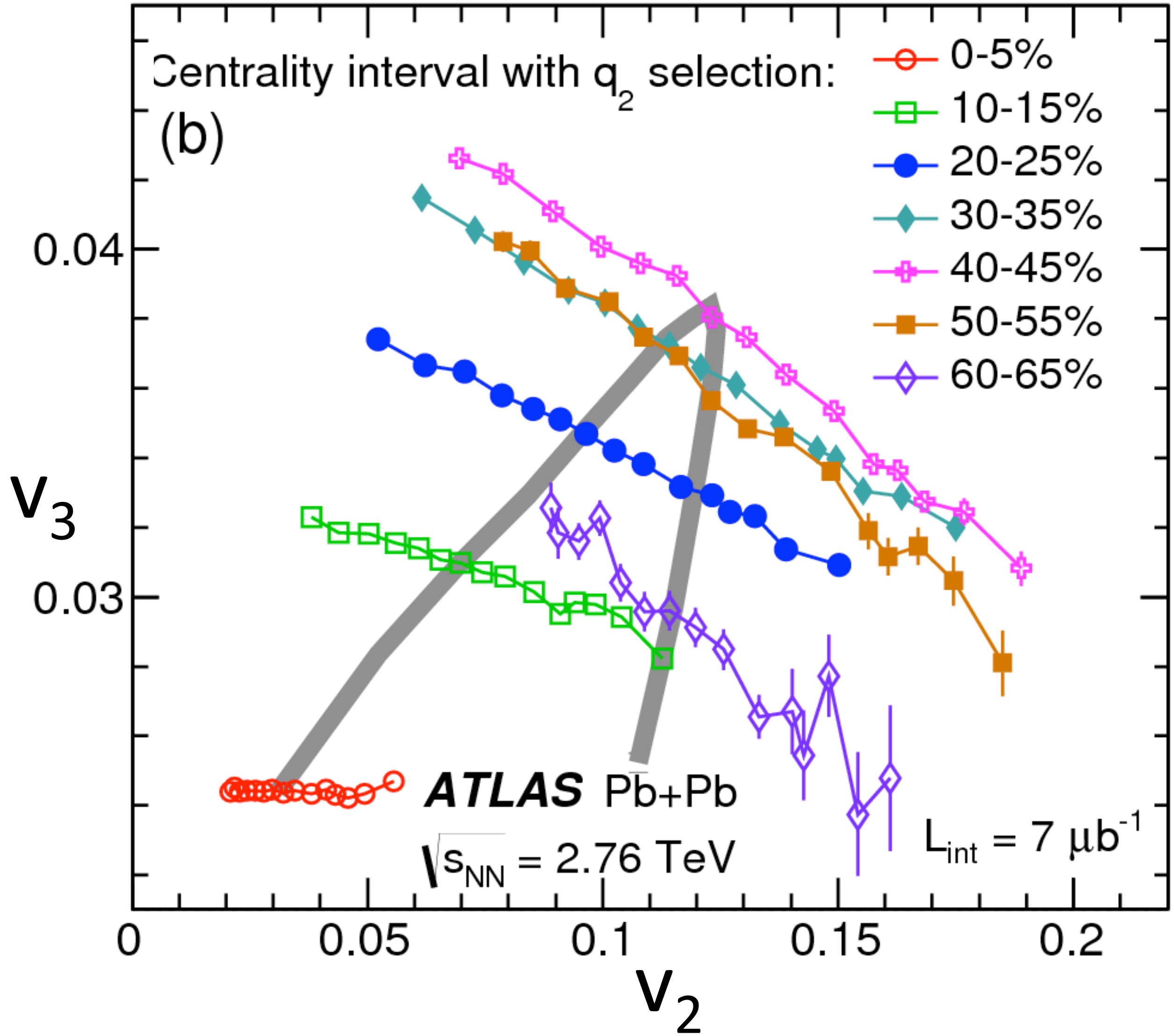
v_3-v_3 correlations : q_3-bins



arXiv:1504.01289

Same conclusions for v_3-v_3 correlations when binning in event triangularity

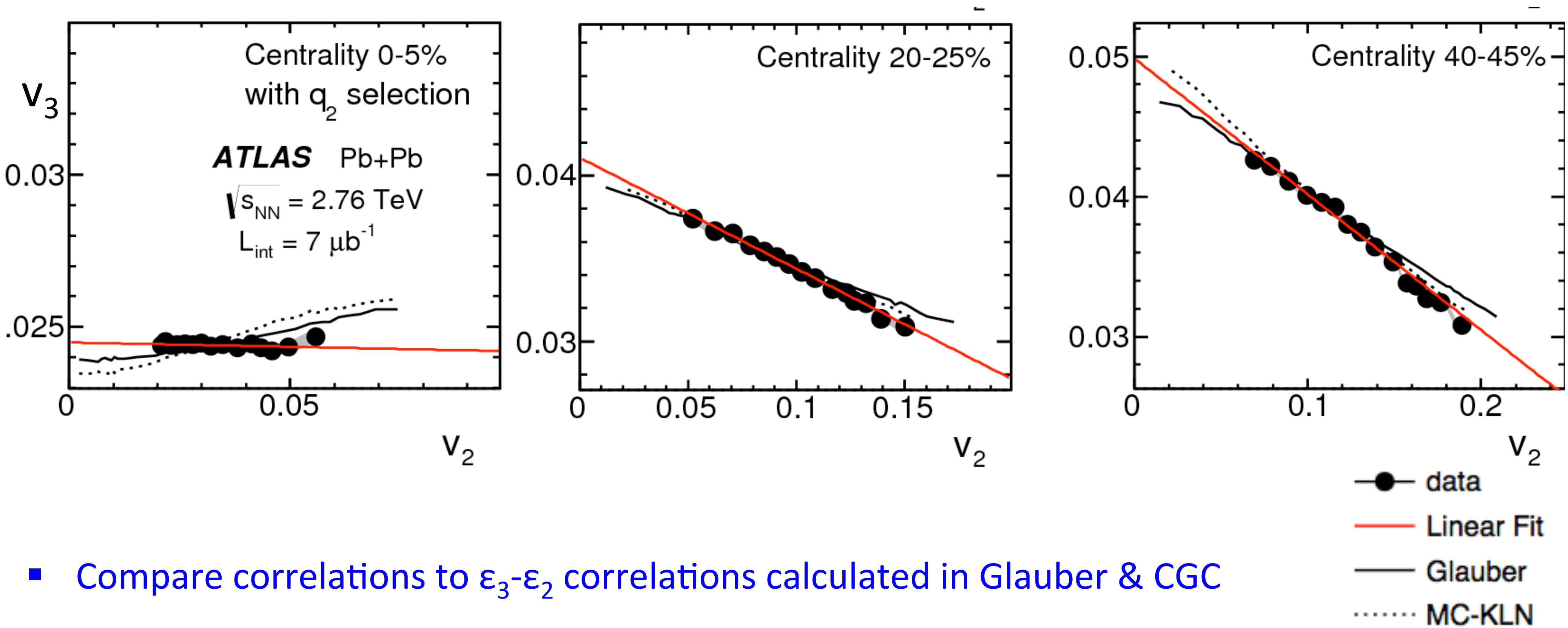
v_3-v_2 correlations : q_2-bins



arXiv:1504.01289

- See anti-correlation between v_2 and v_3 at fixed centrality!
- Initial geometry effect?

v_3-v_2 correlations : Glauber & CGC comparison



- Compare correlations to ε_3-ε_2 correlations calculated in Glauber & CGC models

$$(\varepsilon_3 - \varepsilon_2) \text{ correlation} \propto (v_3 - v_2) \text{ correlation}$$

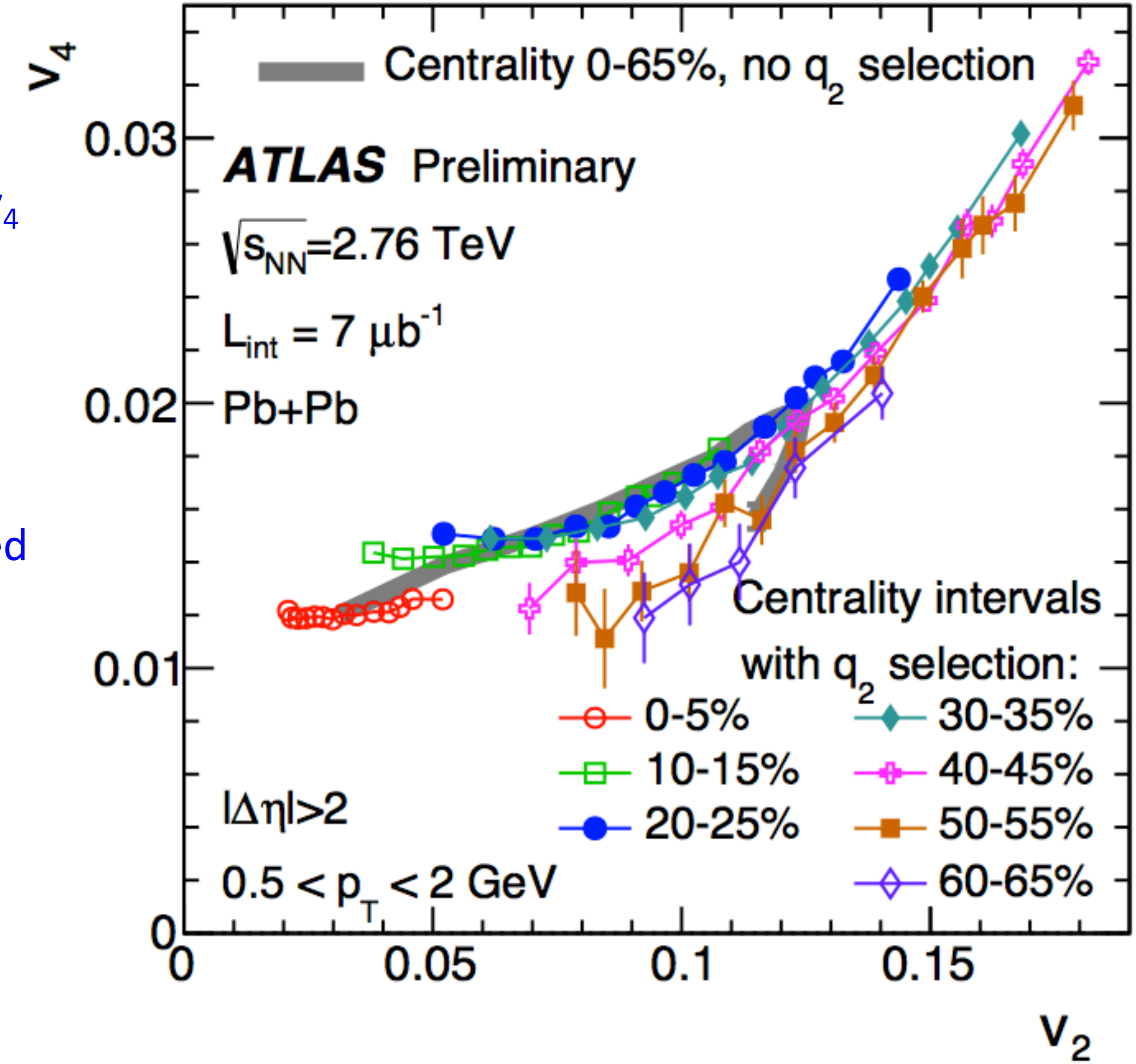
- See good agreement in most centralities but some deviation in (0-5)% central events
- Measurements can constrain initial geometry models
- Lines are linear fits $v_3 = kv_2 + v_3^0$

v_4-v_2 correlations : q_2-bins

- Clear non-linear correlations seen in v_4-v_2 case: upward bending of v_4 at large v_2.

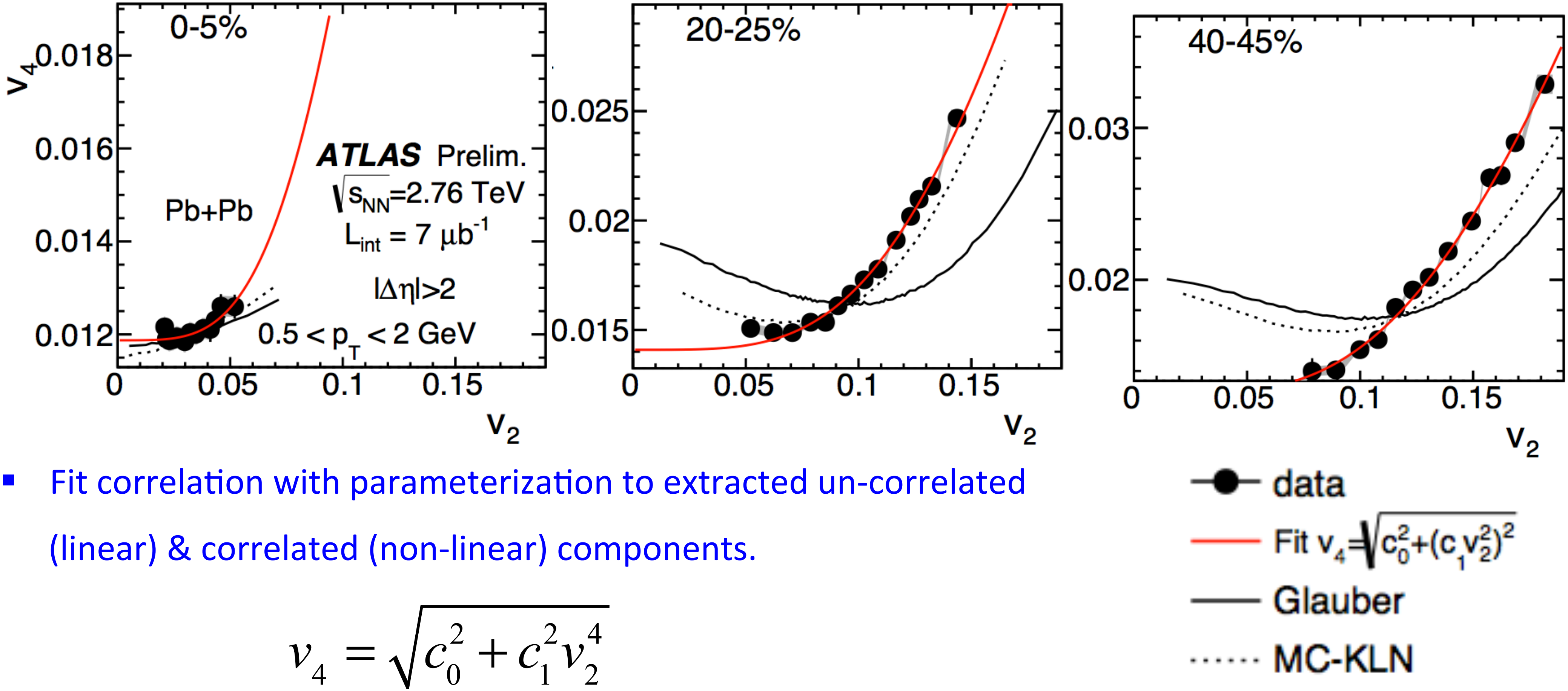
- Can parameterize v_4 into two components, one that is correlated to v_2 and one that is independent

$$v_4 e^{i4\Phi_4} = c_0 e^{i\Phi_4^*} + c_1 \left(v_2 e^{i2\Phi_2}\right)^2$$
$$\Rightarrow v_4 = \sqrt{c_0^2 + c_1^2 v_2^4}$$



- The c_0 component is driven by ε_4 while the c_1 component is driven by ε_2.

v_4-v_2 correlations : linear & non-linear components

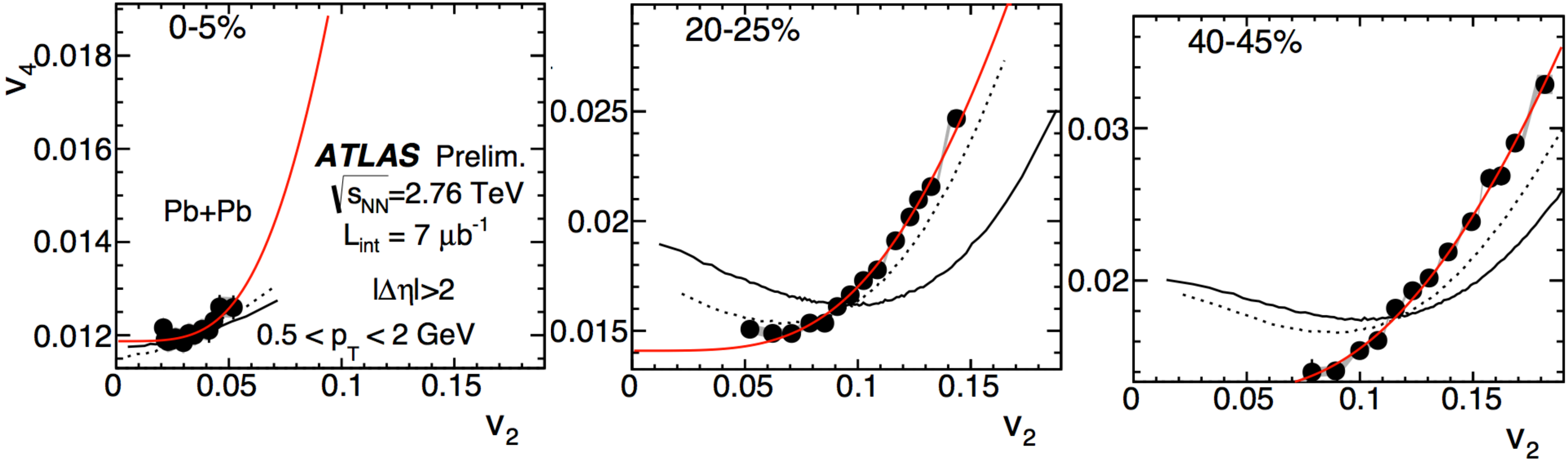


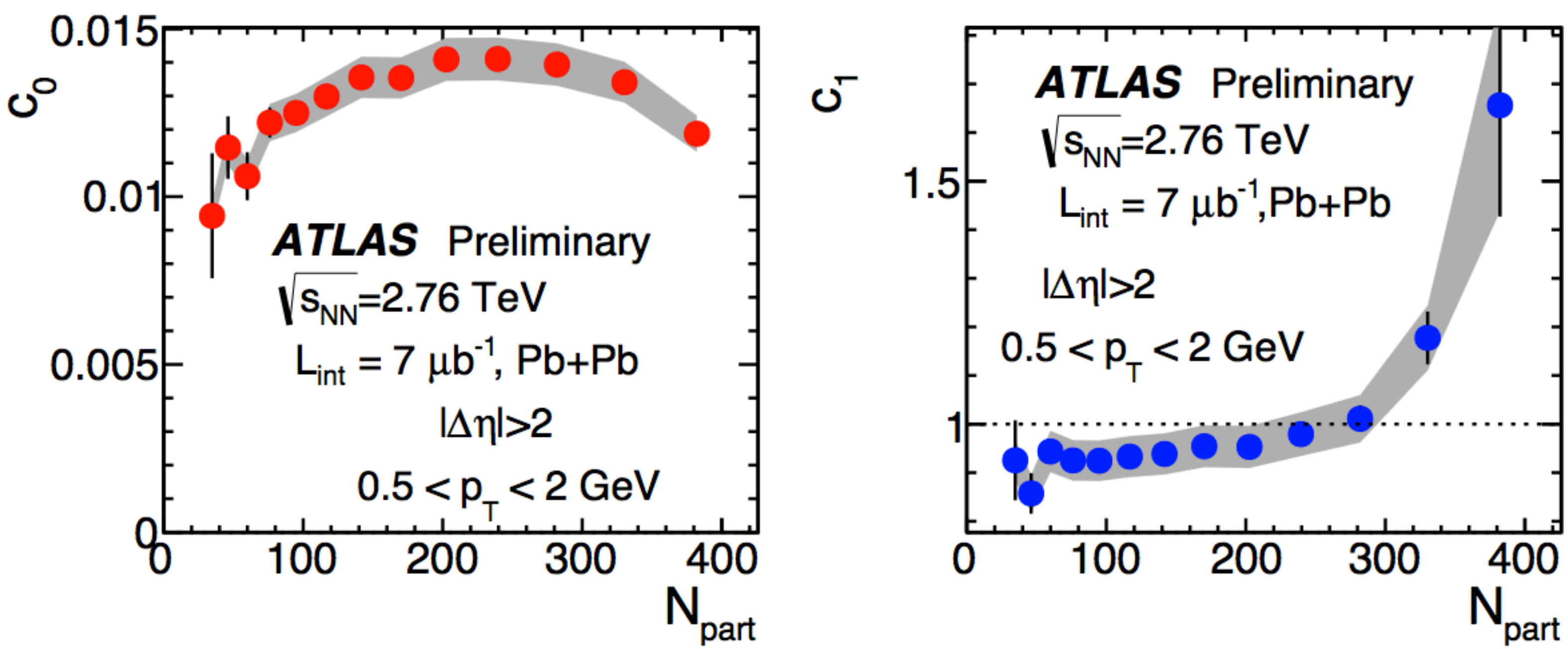
- Fit correlation with parameterization to extracted un-correlated (linear) & correlated (non-linear) components.

$$v_4 = \sqrt{c_0^2 + c_1^2 v_2^4}$$

- Also compare correlations to (rescaled) ε_4-ε_2 correlations calculated in Glauber & CGC models
  - Fits work quite well, but initial geometry models do not
  - Indicate that non-linear dynamical mixing produces these correlations

v_4-v_2 correlations : linear & non-linear components

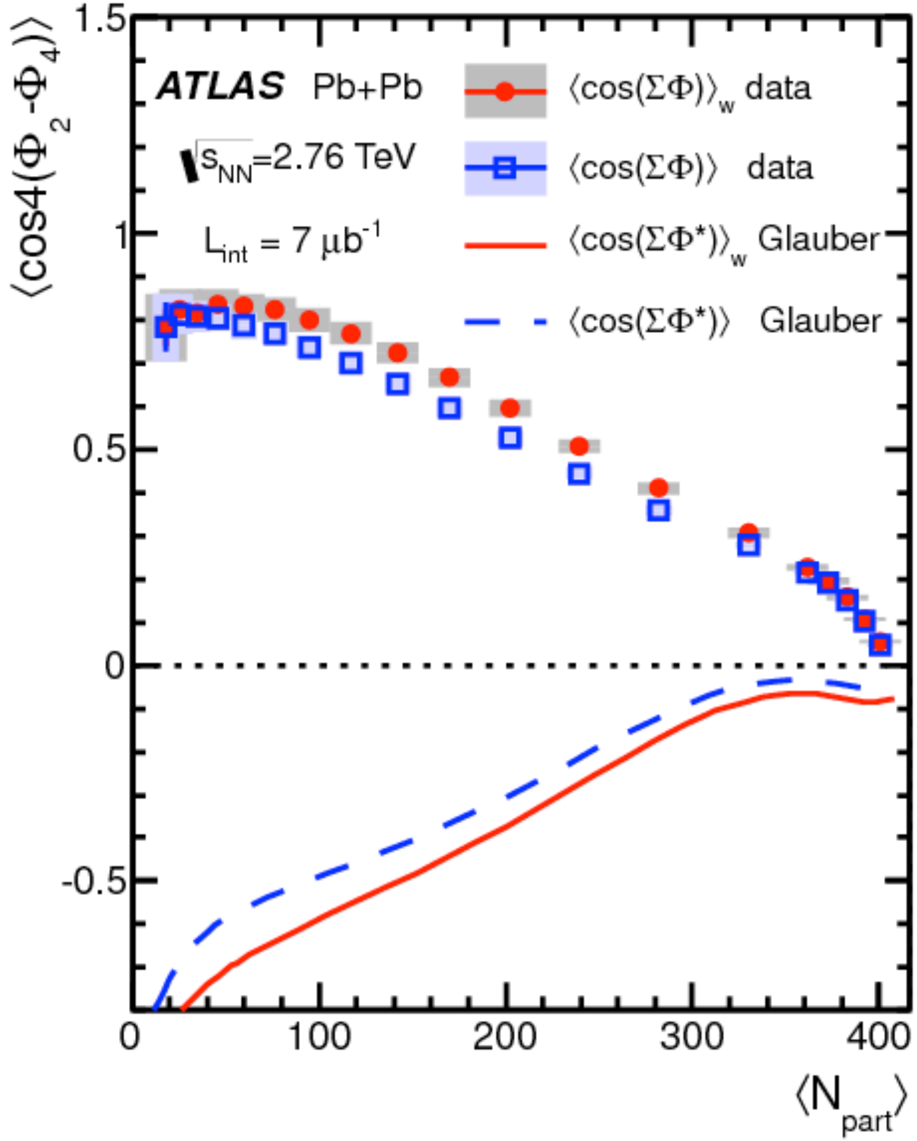




$$v_4 = \sqrt{c_0^2 + c_1^2 v_2^4}$$

Each N_{part} point corresponds to 5% centrality bin

Correlation between Φ_2 and Φ_4

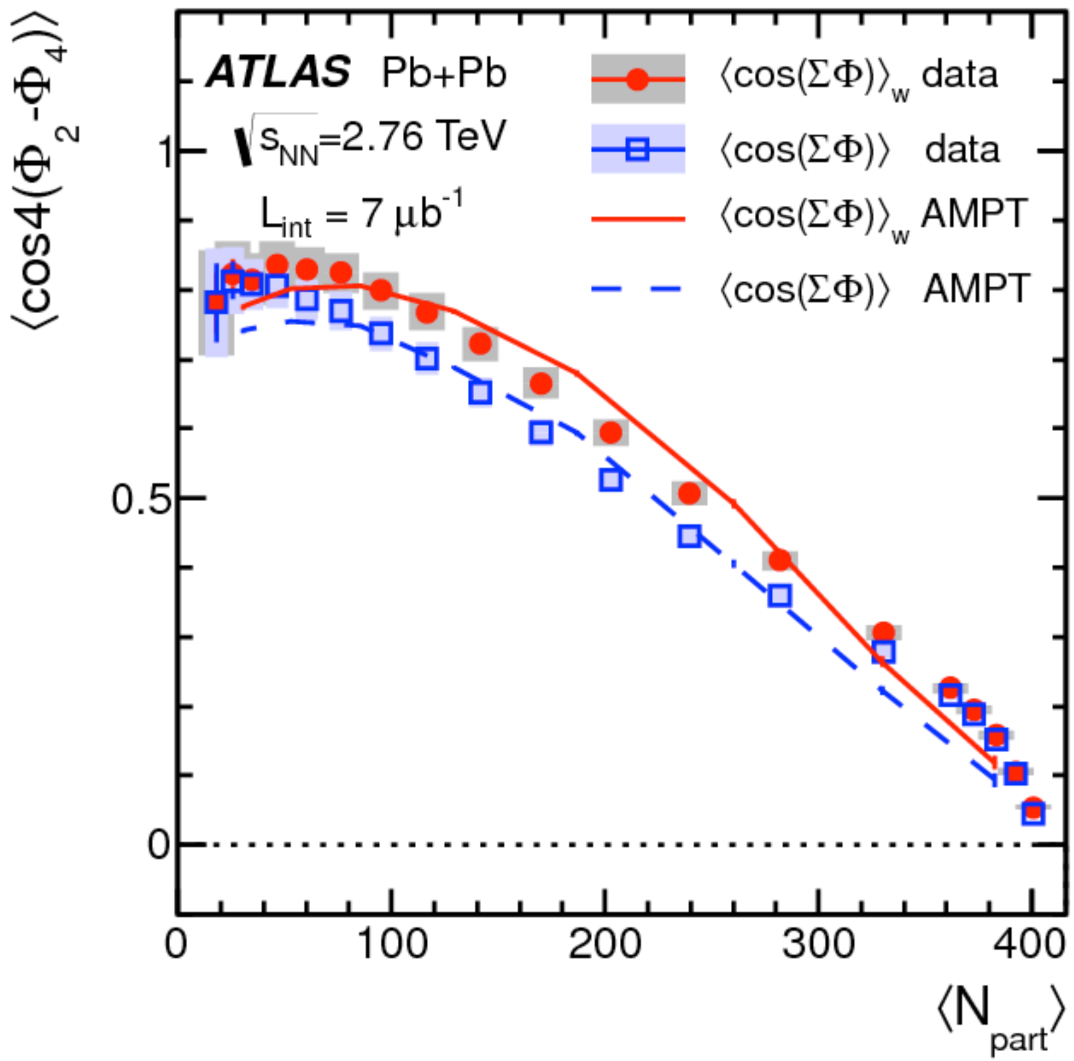


$$\left\langle \cos 4\left(\Phi_2 - \Phi_4\right)\right\rangle$$

PhysRevC.90.024905

- Very different from correlations in initial state (Glauber)
- What happens if we include final-state-interactions?

Correlation between Φ_2 and Φ_4

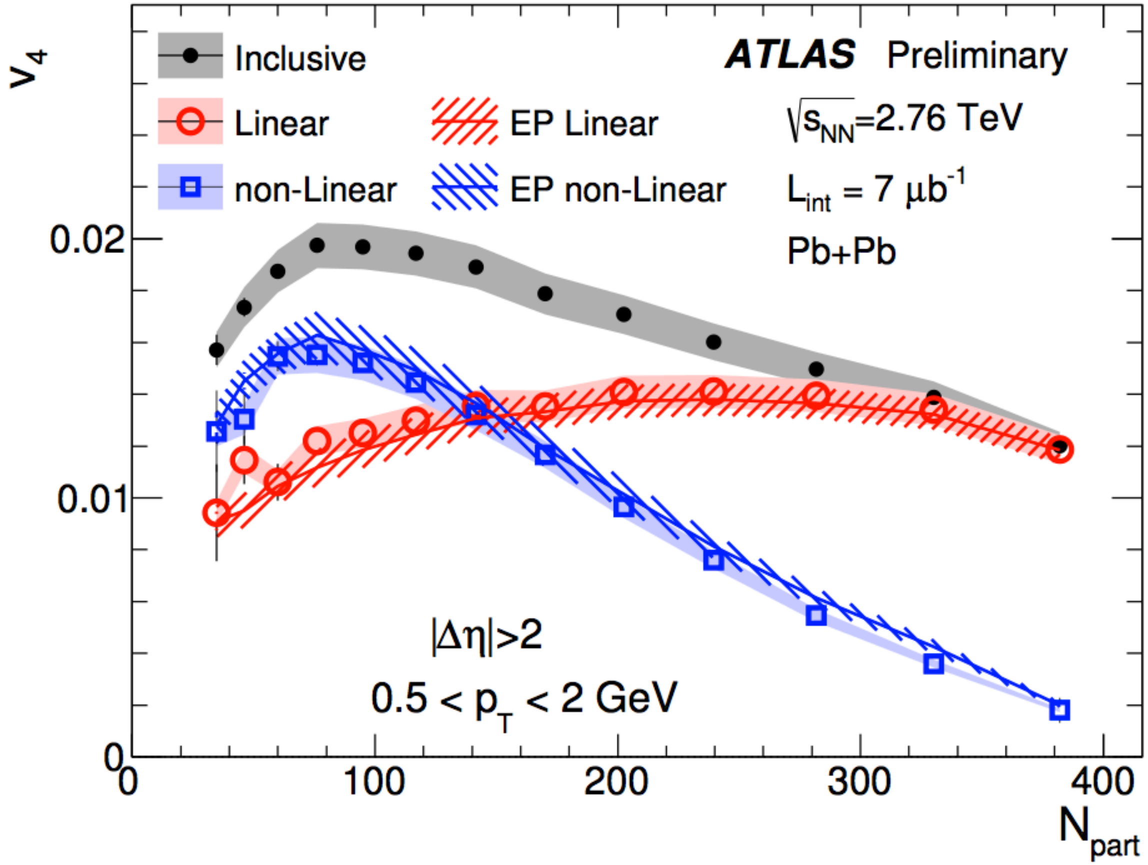


$$\left\langle \cos 4\left(\Phi_2 - \Phi_4\right)\right\rangle$$

PhysRevC.90.024905

- Correlations reproduced in AMPT model
  - AMPT results from PhysRevC.88.024909 (Bhalerao et. al.)
  - Model tuned to reproduce v_n also reproduces EP correlations
  - Also see: j.physletb.2012.09.030 (Qui & Heinz) and j.nuclphysa.2013.02.025 (Teaney & Yan)

v_4-v_2 correlations : comparison to EP correlations

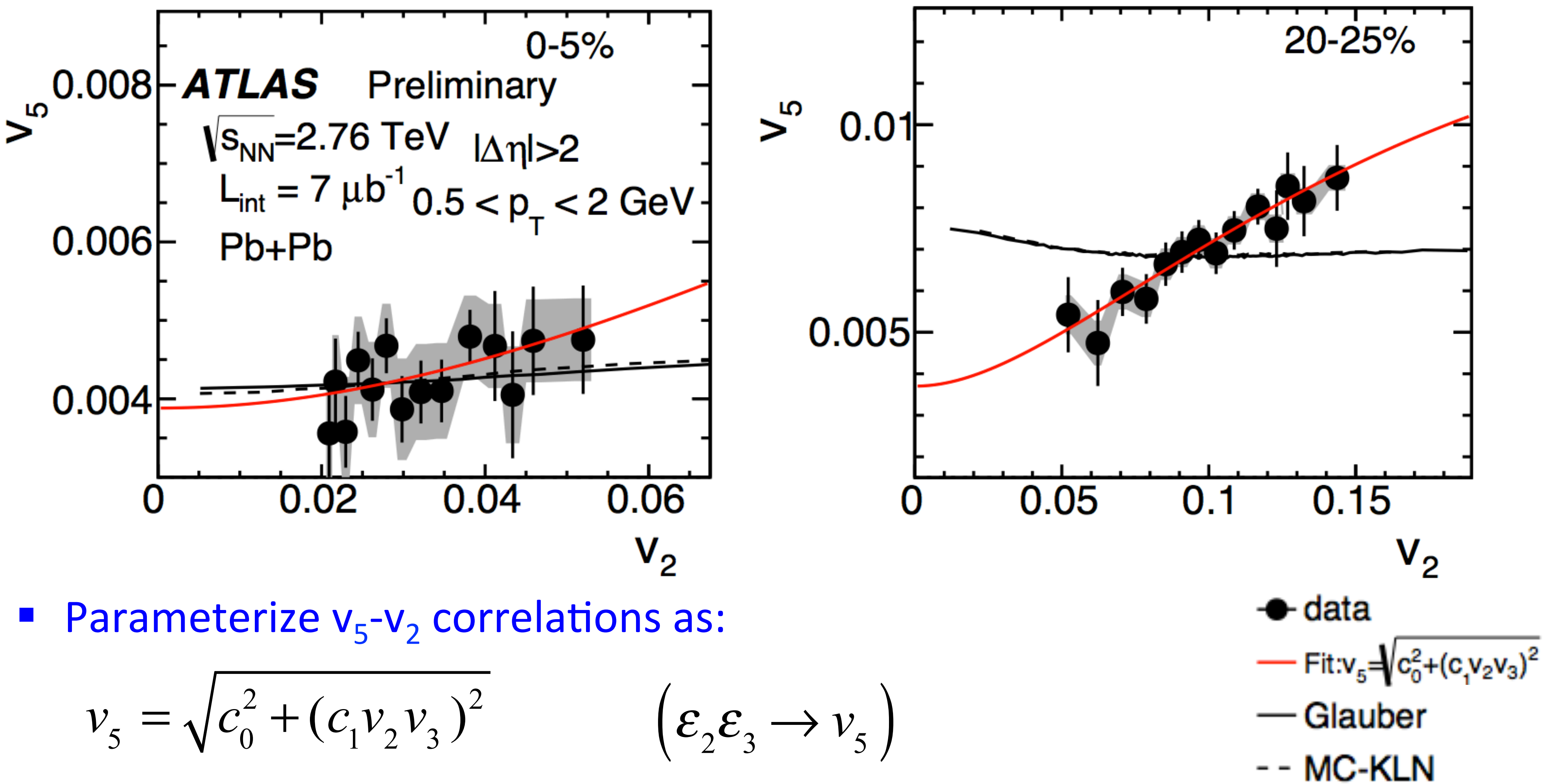


- The non-linear & linear components from EP correlations are obtained as:

$$v_4^{NL} = v_4 \langle \cos 4(\Phi_2 - \Phi_4) \rangle \,, \quad v_4^{L} = \sqrt{v_4^2 - (v_4^{NL})^2}$$

- The results from the two procedures compare quite well
- In most central cases almost all v_4 is uncorrelated with v_2
- Correlated component gradually increases and overtakes linear component as N_{part}~120

v_5-v_2 correlations : q_2-bins

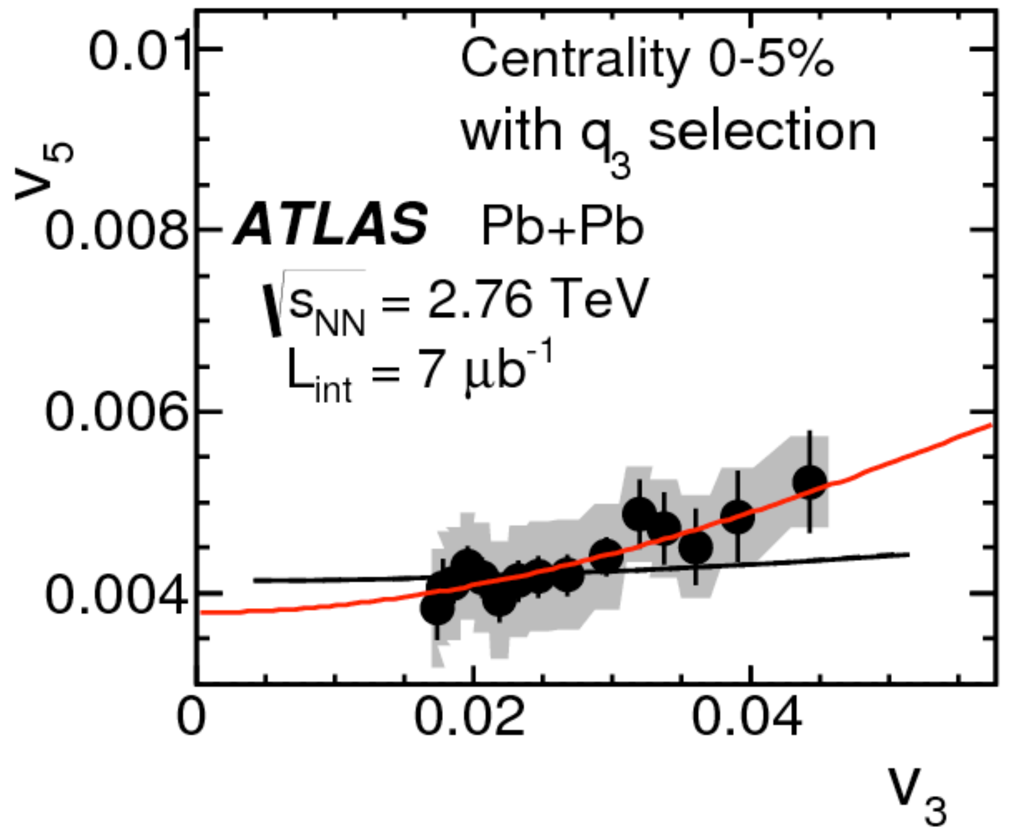


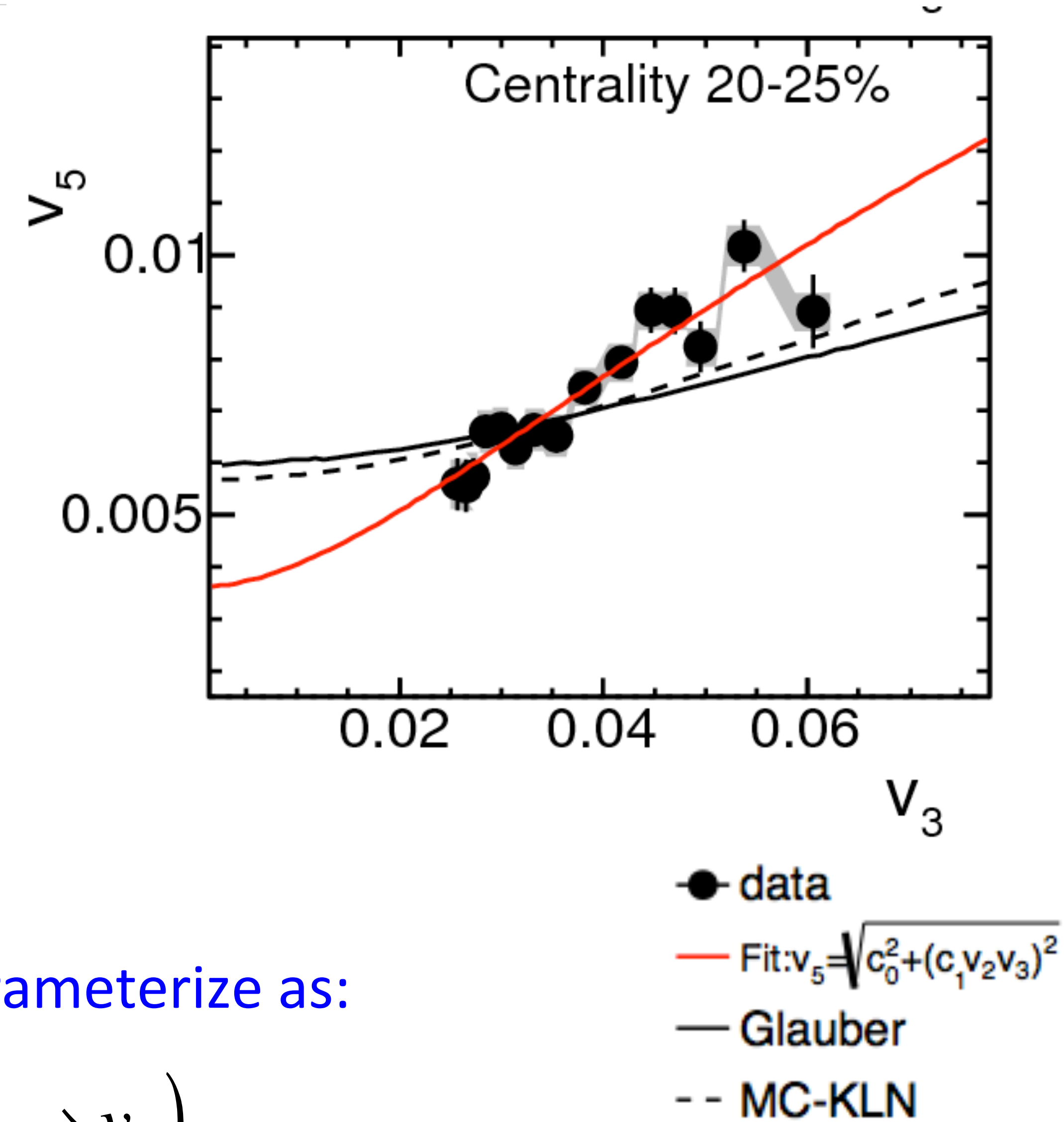
- Parameterize v_5-v_2 correlations as:

$$v_5 = \sqrt{c_0^2 + (c_1 v_2 v_3)^2} \qquad \left(\varepsilon_2\varepsilon_3 \rightarrow v_5\right)$$

- Fit v_5-v_2 correlation with above functional form to extract linear & non-linear components
- Comparison to Glauber & CGC models also shown, don't do a good job in describing data

v_5-v_3 correlations : q_3-bins



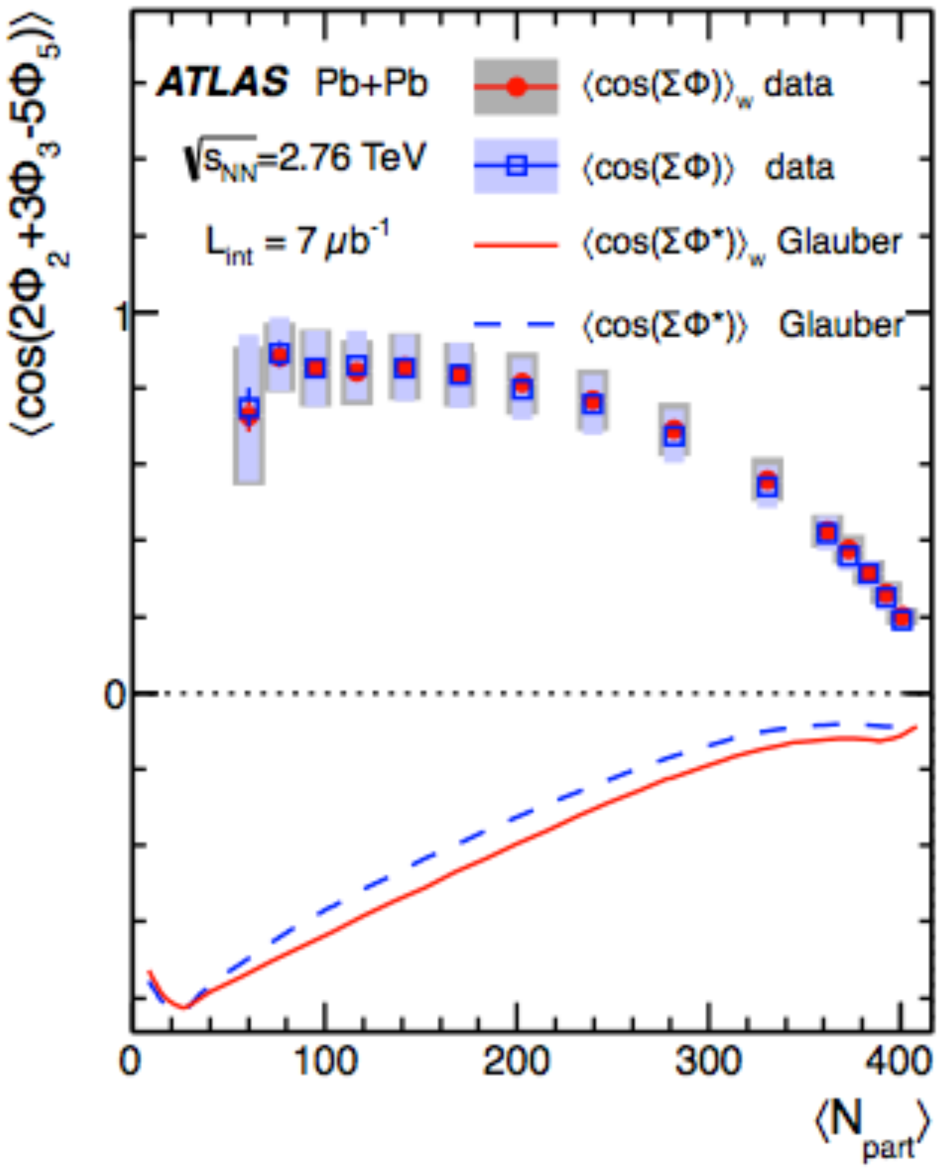


- Now measure v_5-v_3 correlations, Parameterize as:

$$v_5 = \sqrt{c_0^2 + (c_1 v_2 v_3)^2} \qquad \left(\varepsilon_2 \varepsilon_3 \rightarrow v_5\right)$$

- Fit v_5-v_2 correlation with above functional form to extract linear & non-linear components

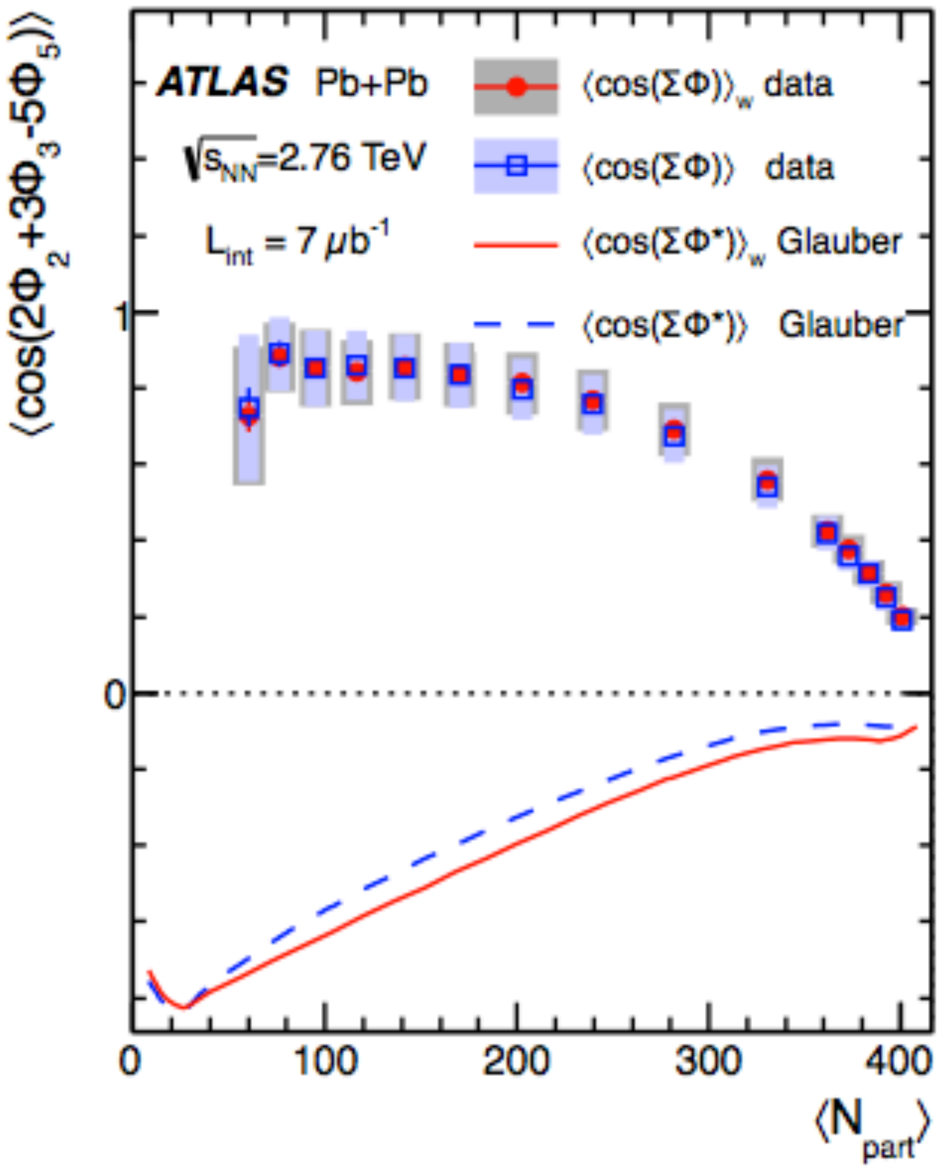
Three-plane : "2-3-5" correlation



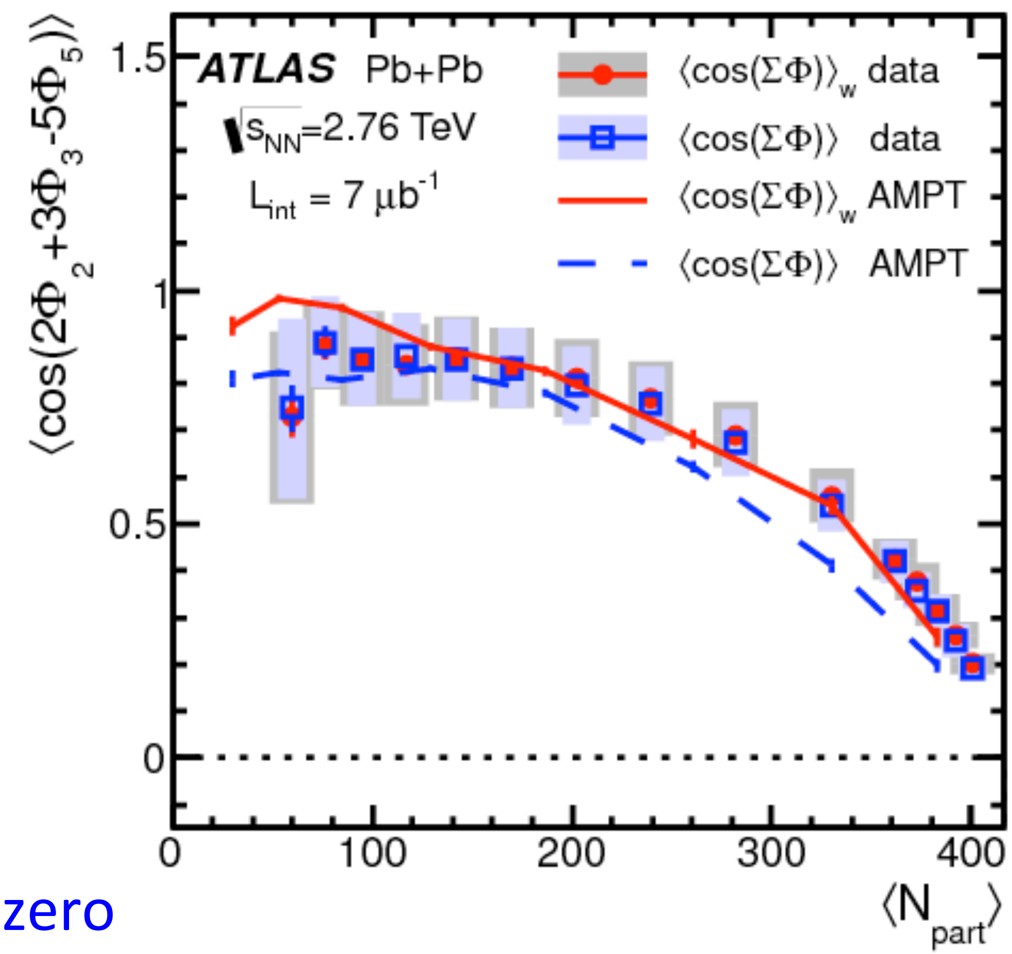
$$v_5 e^{i5\Phi_5} = \alpha_5 \varepsilon_5 e^{i5\Phi_5^*} + \beta_{2,3,5} v_2 e^{i2\Phi_2} v_3 e^{i3\Phi_3}$$

- $(2\Phi_2 + 3\Phi_3 - 5\Phi_5)$ correlation is non-zero
- Glauber geometry does not match the correlation

Three-plane : "2-3-5" correlation

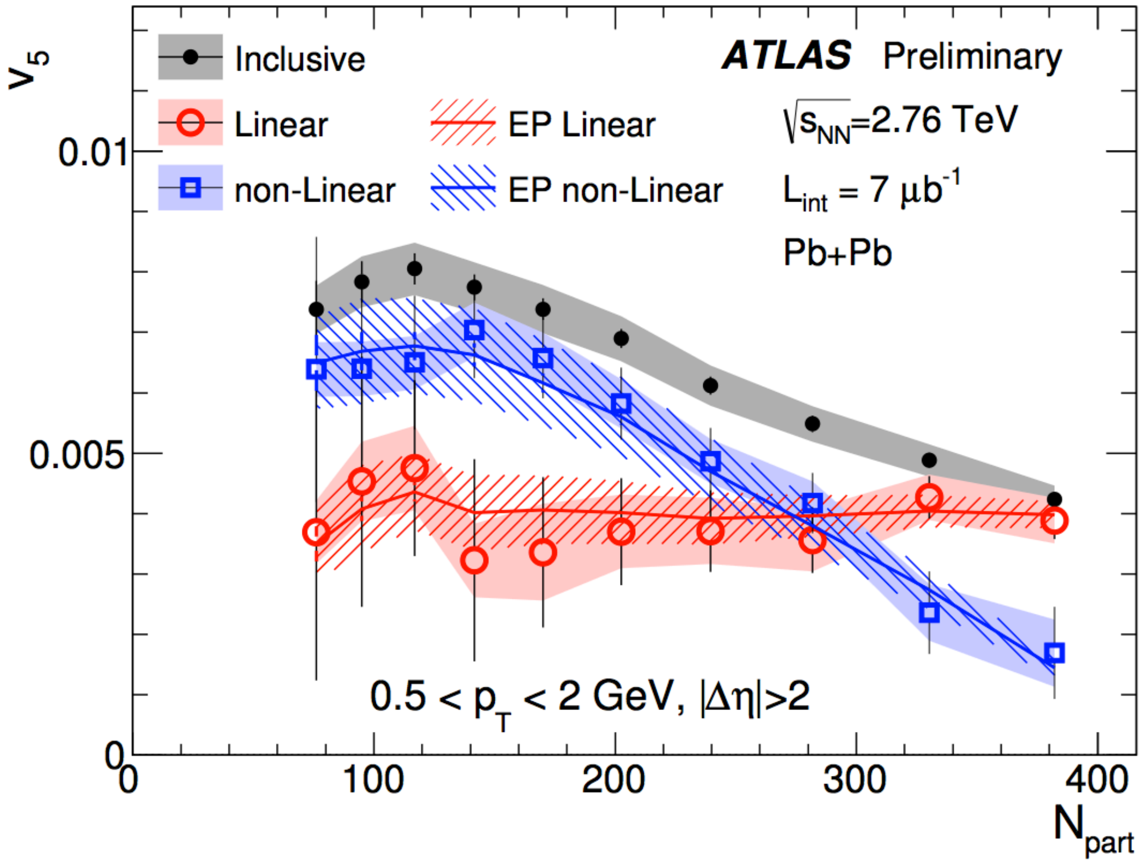


$$v_5 e^{i5\Phi_5} = \alpha_5 \varepsilon_5 e^{i5\Phi_5^*} + \beta_{2,3,5} v_2 e^{i2\Phi_2} v_3 e^{i3\Phi_3}$$



- $(2\Phi_2 + 3\Phi_3 - 5\Phi_5)$ correlation is non-zero
- Glauber geometry does not match the correlation

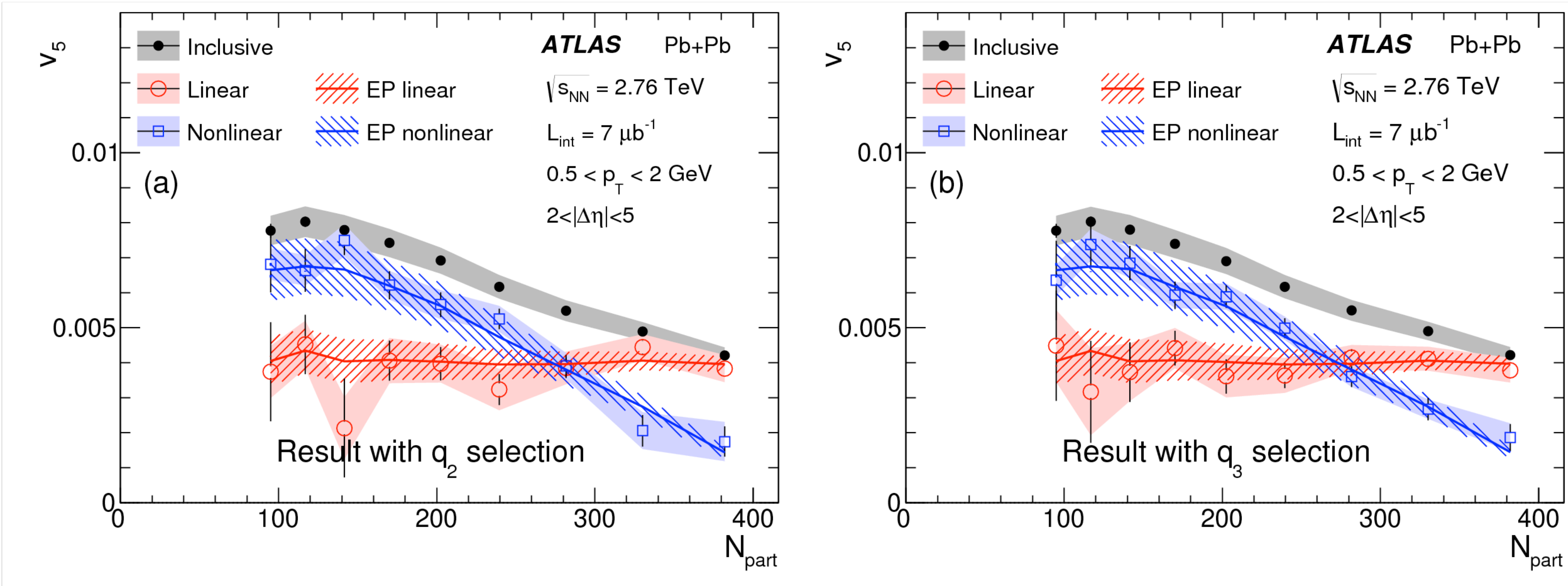
v_5-v_2 correlations : comparison to EP correlations



- Compare linear & non-linear components from this analysis to EP correlation results
- The non-linear & linear components from EP correlations are obtained as:

$$v_5^{NL} = v_5 \langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle , \quad v_5^{L} = \sqrt{v_5^2 - (v_5^{NL})^2}$$

v_5-$v_{2/3}$ correlations : comparison to EP correlations

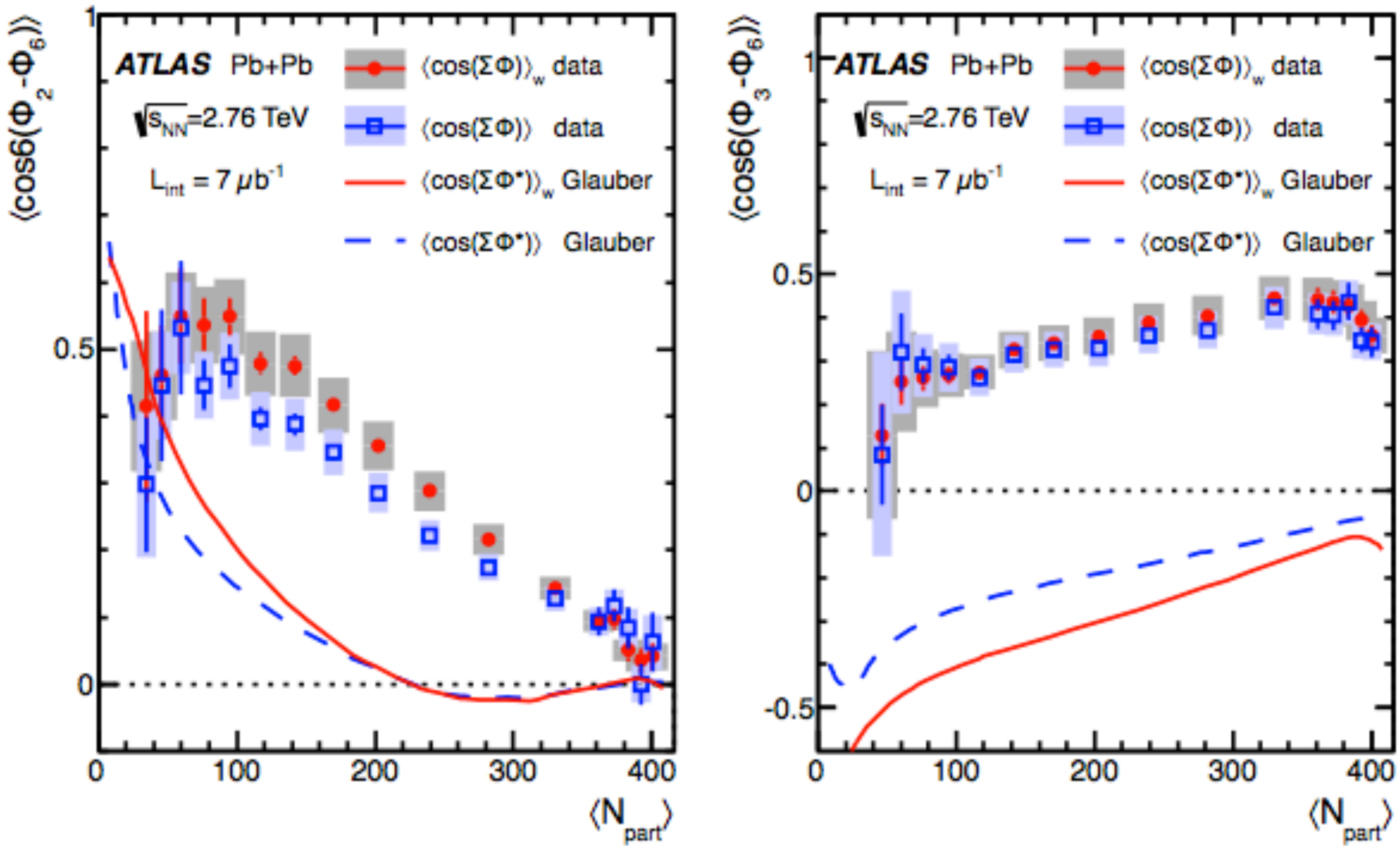


- Compare linear & non-linear components from this analysis to EP correlation results
- The non-linear & linear components from EP correlations are obtained as:

$$v_5^{NL} = v_5 \langle \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5) \rangle , \quad v_5^{L} = \sqrt{v_5^2 - (v_5^{NL})^2}$$

Non-linear response for v_6

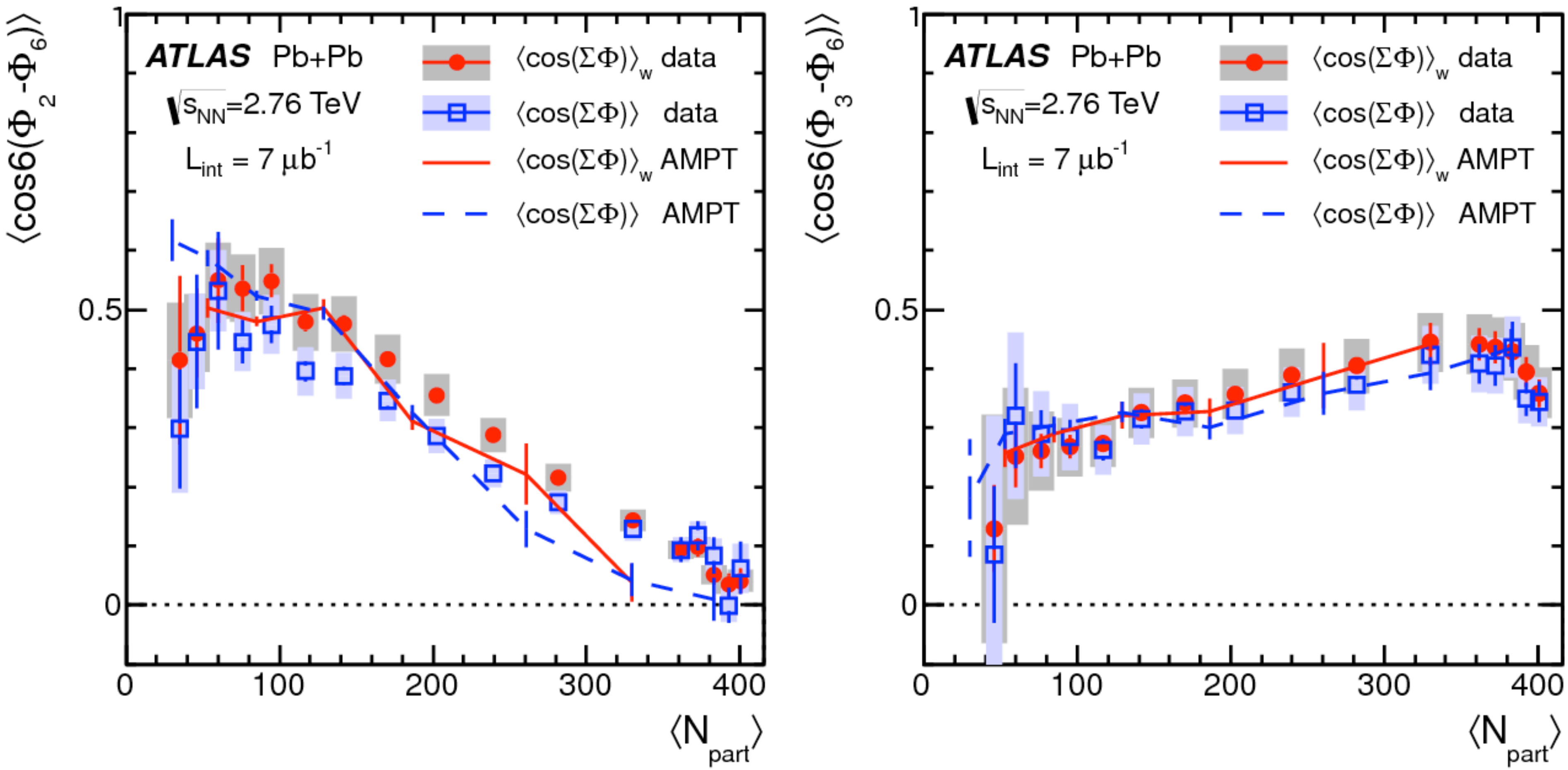
$$\left\langle \cos 6\left(\Phi_2 - \Phi_6\right)\right\rangle \qquad \left\langle \cos 6\left(\Phi_3 - \Phi_6\right)\right\rangle$$



- Φ_2 and Φ_3 weakly correlated, but both strongly correlated with Φ_6.
- They show opposite centrality dependence
  - v_6 dominated by non-linear contribution: v_2^3, v_3^2 ?

Non-linear response for v_6

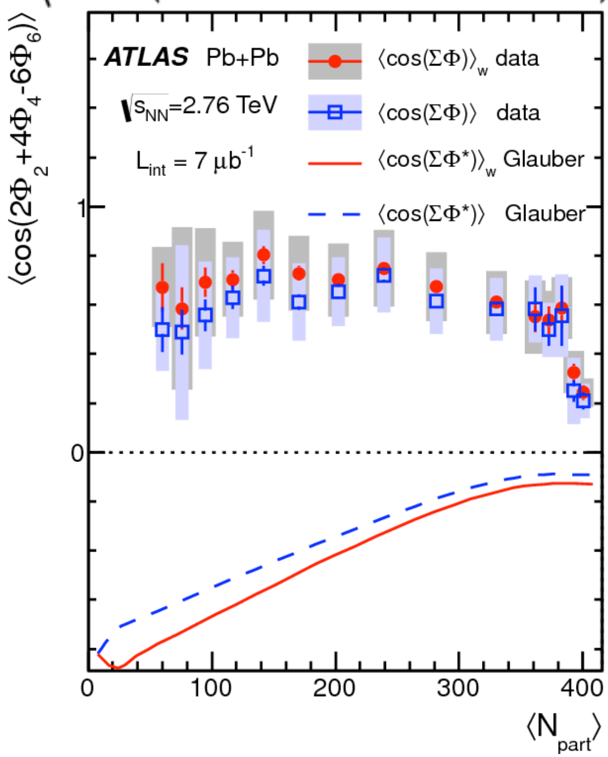
$\langle \cos 6(\Phi_2 - \Phi_6) \rangle$ $\langle \cos 6(\Phi_3 - \Phi_6) \rangle$



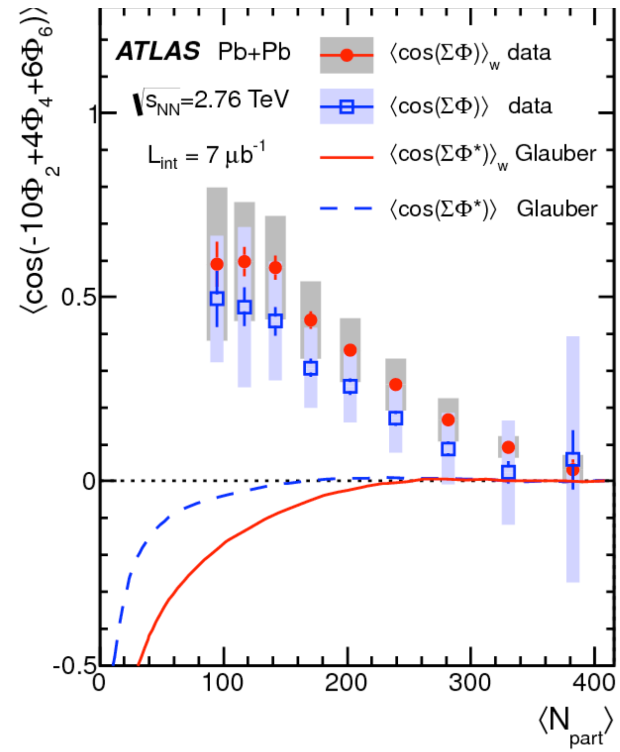
- Final state interactions reproduce the correlations
  - R. S. Bhalerao, J.-Y. Ollitrault, and S. Pal, Phys. Rev.C 88, 024909

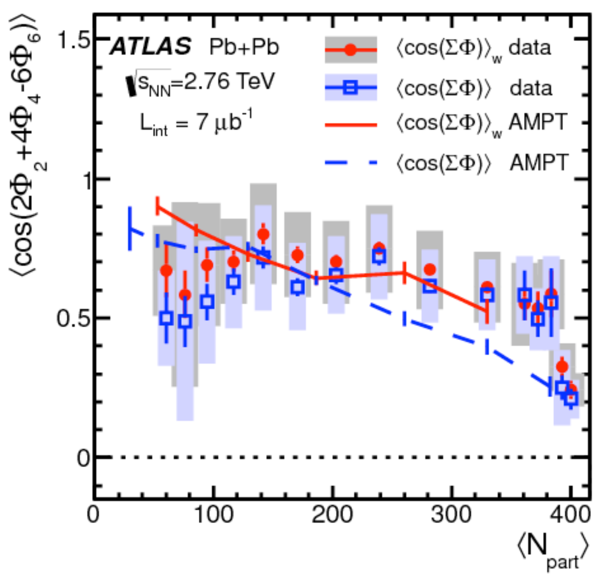
Three-plane : "2-4-6" correlation

$$\left\langle \cos\left(2\Phi_2 + 4\Phi_4 - 6\Phi_6\right)\right\rangle$$



$$\left\langle \cos\left(-10\Phi_2 + 4\Phi_4 + 6\Phi_6\right)\right\rangle$$

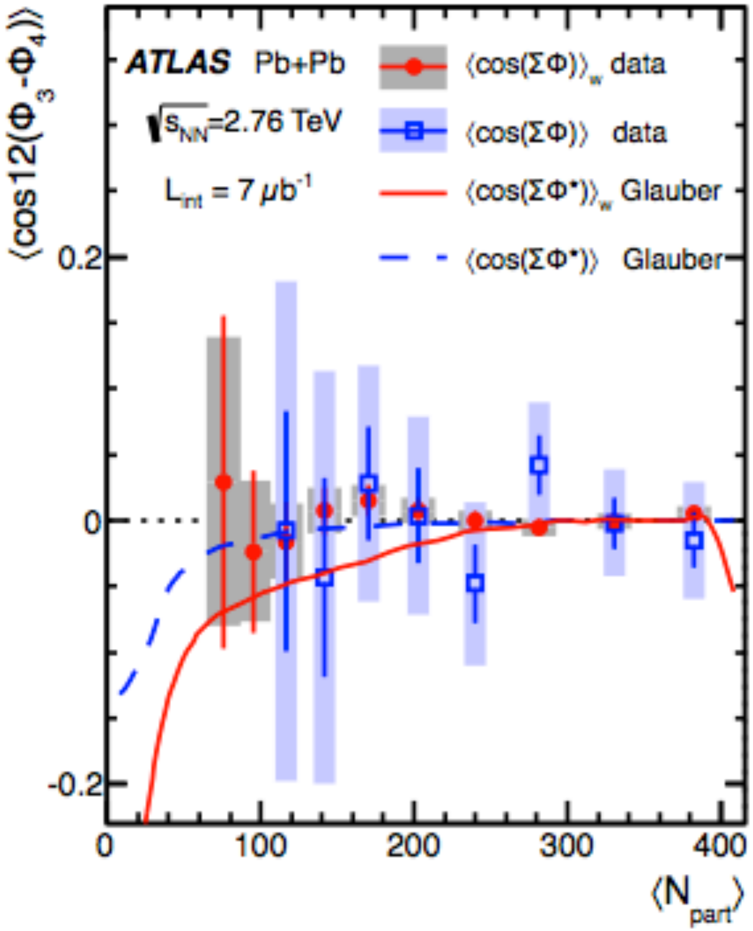




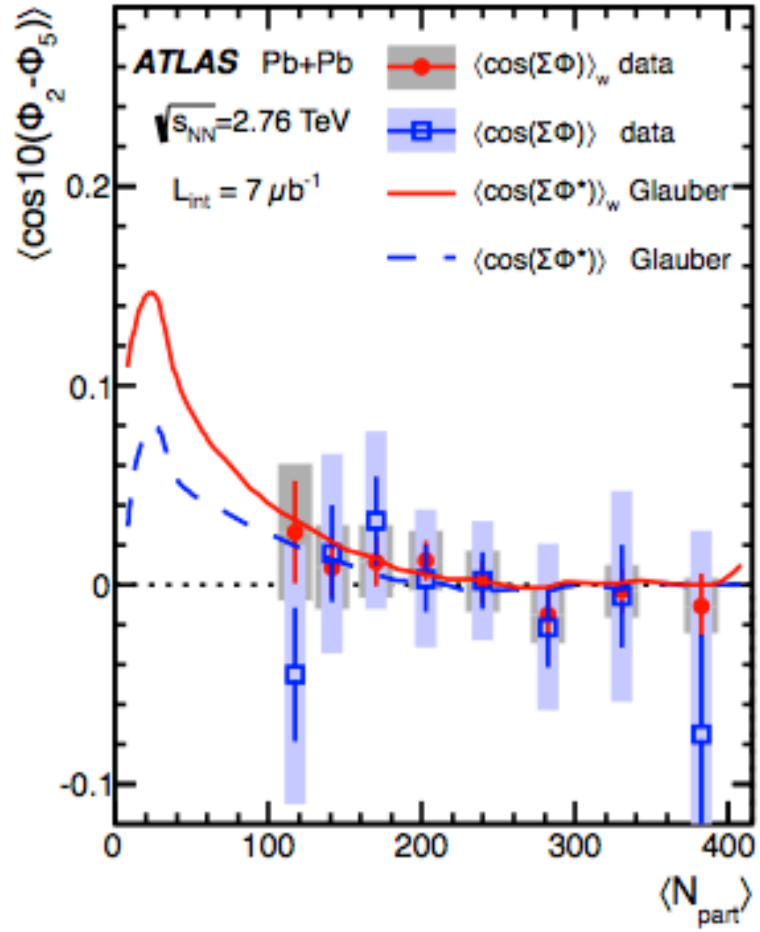
AMPT

Φ_3 vs Φ_4 and Φ_2 vs Φ_5

$$\left\langle \cos 12\left(\Phi_3 - \Phi_4\right)\right\rangle$$



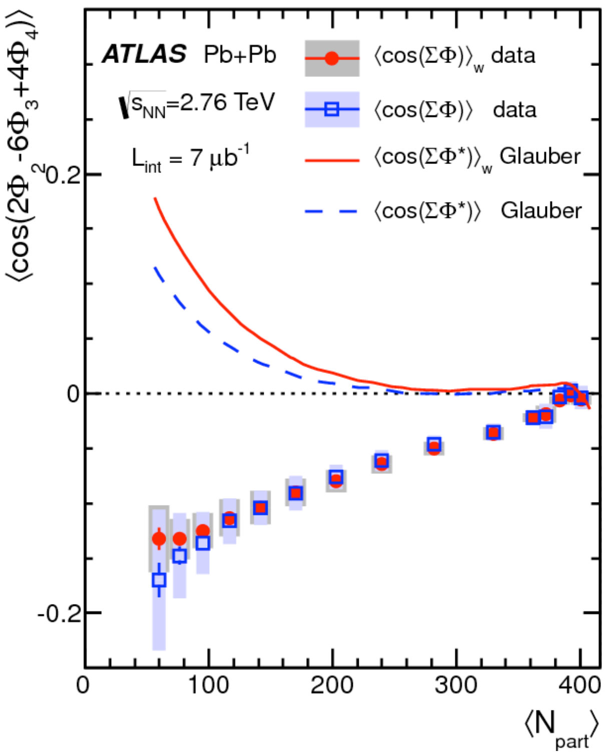
$$\left\langle \cos 10\left(\Phi_2 - \Phi_5\right)\right\rangle$$



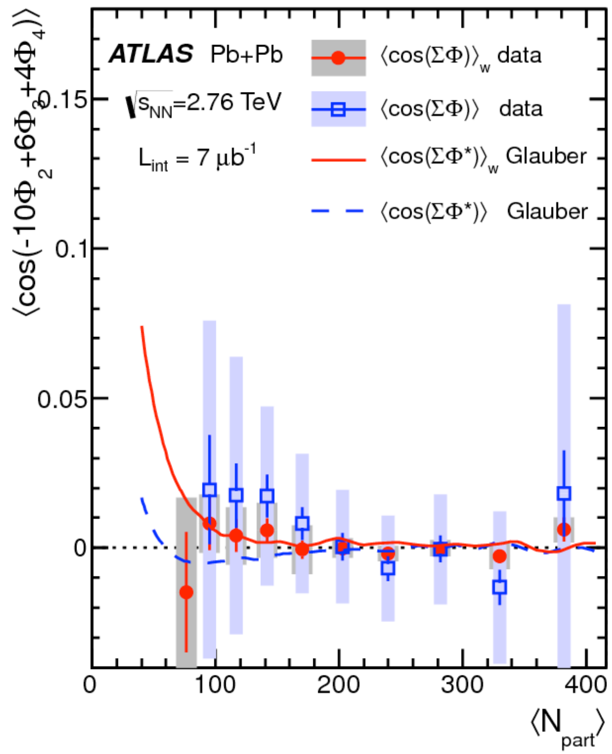
correlations are weak (< few %)

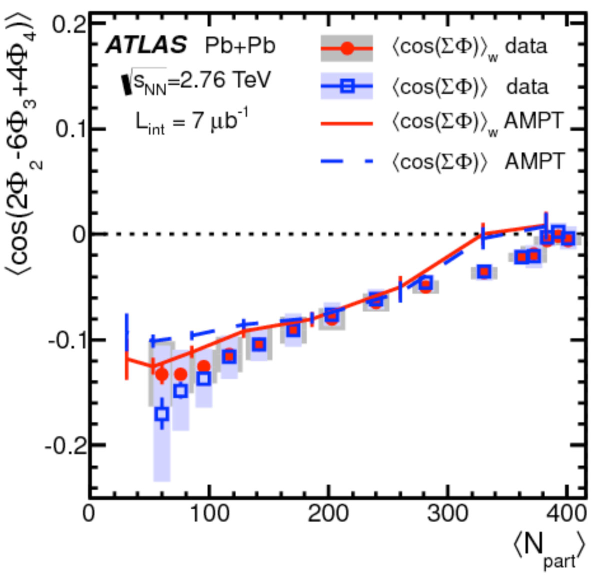
Three-plane : "2-3-4" correlation

$$\left\langle \cos\left(2\Phi_2 - 6\Phi_3 + 4\Phi_4\right)\right\rangle$$



$$\left\langle \cos\left(-10\Phi_2 + 6\Phi_3 + 4\Phi_4\right)\right\rangle$$

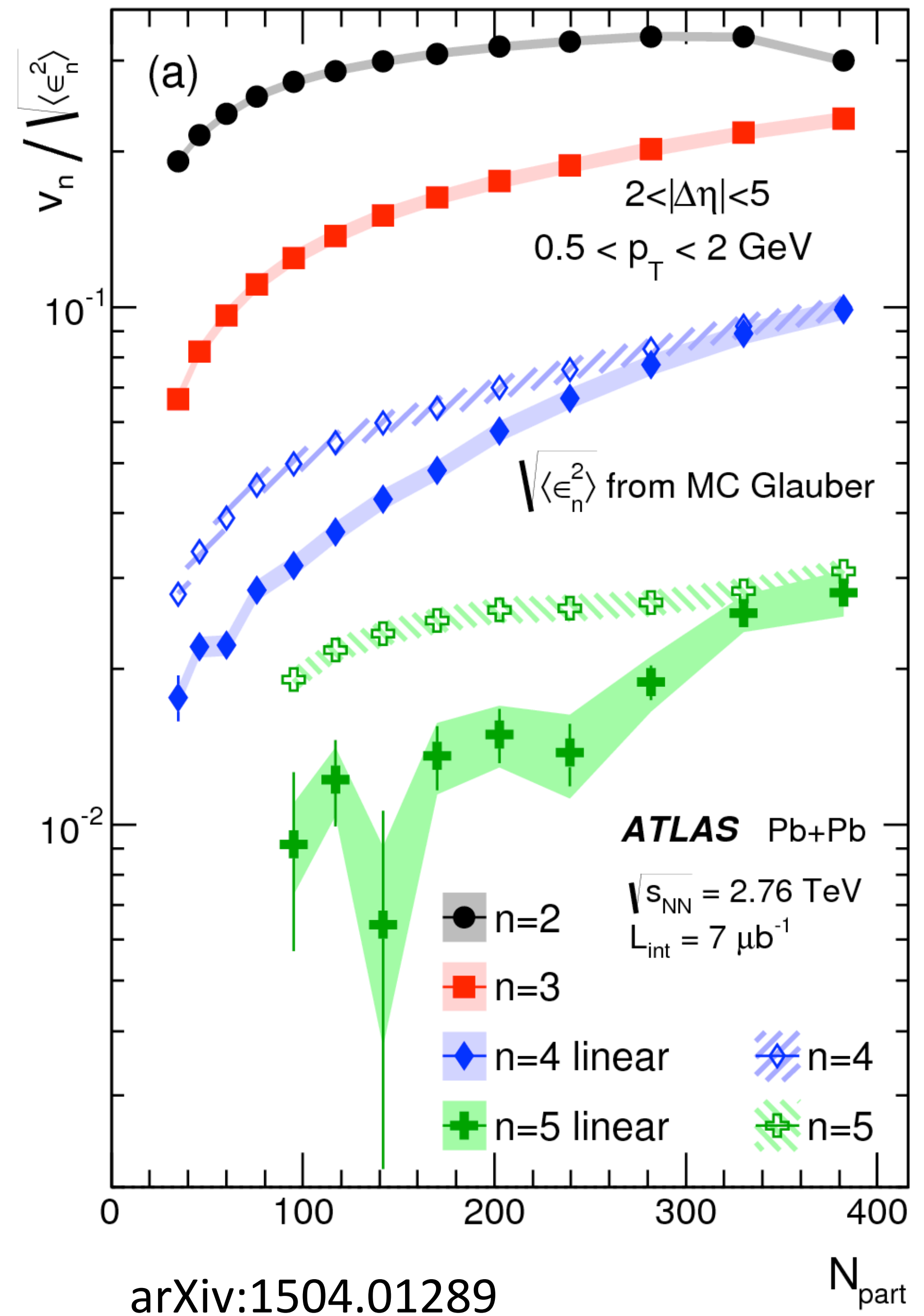




AMPT

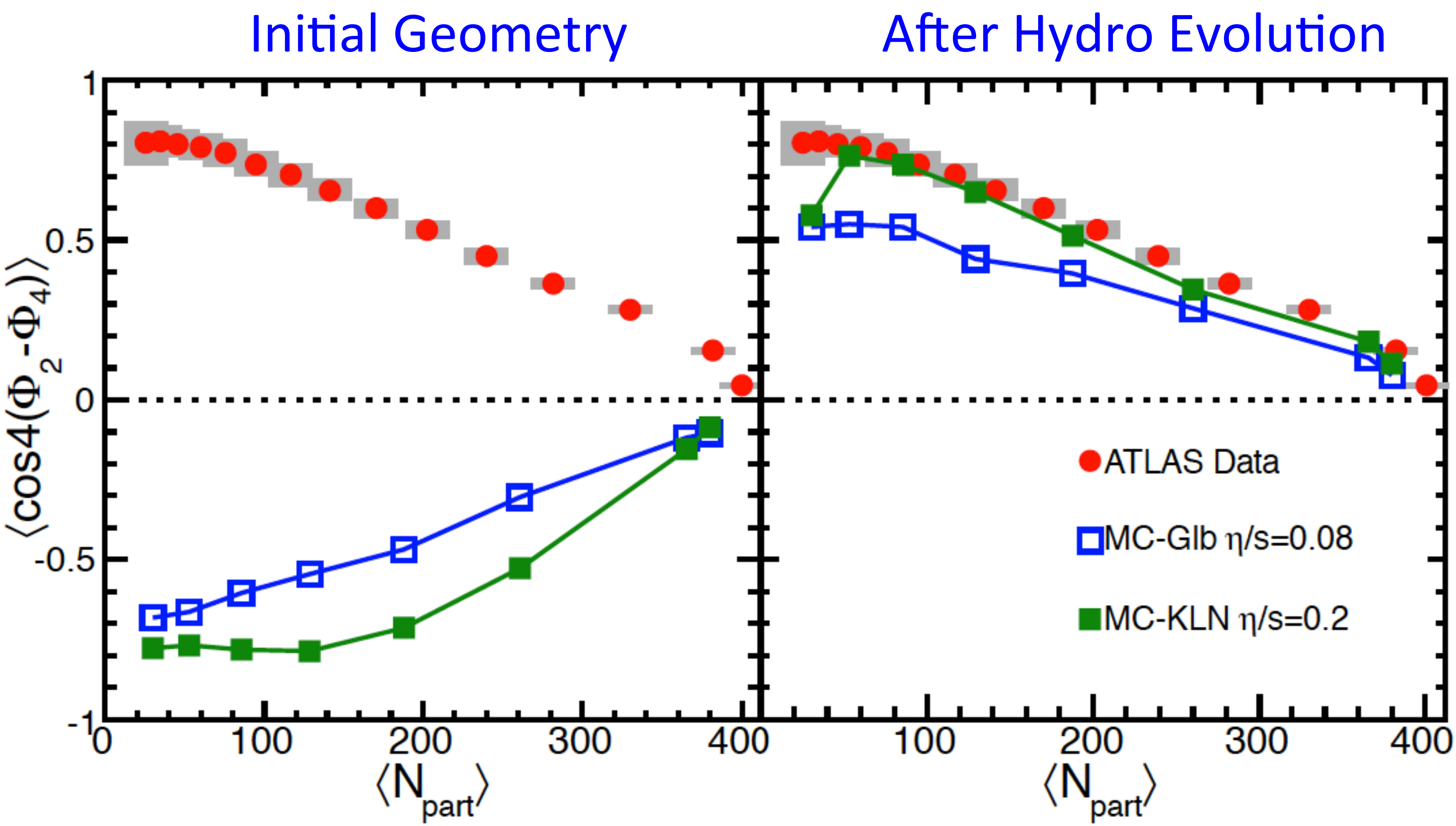
ε_n scaling of linear components

- The v_n/rms-ε_n ratios are shown as a function of centrality
- For v_4 & v_5, the ratio is shown for the linear component as well as the total v_n.
- The linear component show greater variation
- indicates larger viscous dampening for higher harmonics, with decreasing centrality.



arXiv:1504.01289

Flow-correlations also constrain η/s, initial geometry



j.physletb.2012.09.030: Qui & Heinz

Alternative parameterization of initial geometry

- Typically initial geometry in Heavy-Ion collisions is quantified by the eccentricities ε_n:

$$\varepsilon_n e^{in\Phi_n} = -\frac{\left\langle r^n e^{in\phi} \right\rangle}{\left\langle r^n \right\rangle}$$

- In a recent paper (arXiv:1206.1905) Teaney and Yan have pointed out that it might be better to quantify the initial geometry by cumulants c_n

- The cumulants are related to the eccentricities by:

$$C_2 e^{i2\Phi_2} \equiv -\frac{\langle z^2 \rangle}{\langle r^2 \rangle},$$

$$z = re^{i\phi}$$

$$C_3 e^{i3\Phi_3} \equiv -\frac{\langle z^3 \rangle}{\langle r^3 \rangle},$$

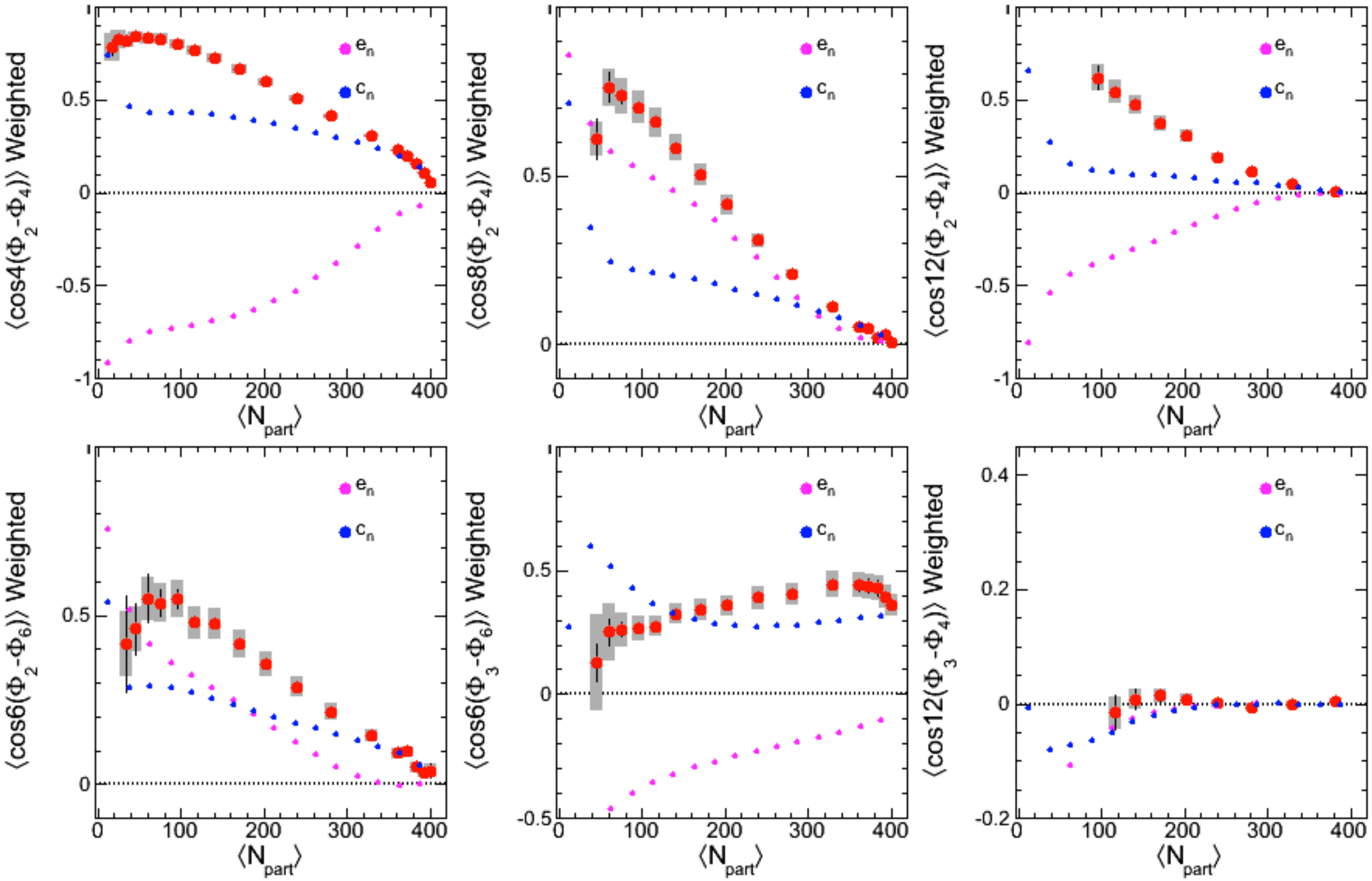
$$C_4 e^{i4\Phi_4} \equiv -\frac{1}{\langle r^4 \rangle}\left[\langle z^4 \rangle - 3\langle z^2 \rangle^2\right],$$

$$C_5 e^{i5\Phi_5} \equiv -\frac{1}{\langle r^5 \rangle}\left[\langle z^5 \rangle - 10\langle z^2 \rangle\langle z^3 \rangle\right],$$

$$C_6 e^{i6\Phi_6} = -\frac{1}{\langle r^6 \rangle}\left[\langle z^6 \rangle - 15\langle z^4 \rangle\langle z^2 \rangle - 10\langle z^3 \rangle^2 + 30\langle z^2 \rangle^3\right]$$

- Is this parameterization better?

Correlations In initial geometry



See also arXiv:1312.3689 Teaney & Yan

Compare correlation between cumulants to the ATLAS EP correlations

1. Do much better job than the correlations between the ε_n
2. Indicative that when we define initial geometry in terms of ε_n, we have to take into consideration a large degree on non-linear response in generation of the v_n

$$v_n e^{i\Phi_n} \propto \varepsilon_n e^{i\tilde{\Phi}_n} + \text{significant non-linear contribution from } \varepsilon_m (m < n)$$

$$v_n e^{i\Phi_n} \propto c_n e^{i\tilde{\Phi}_n} + \text{small non-linear contribution from } c_m (m < n)$$

Summary

- Measurements:
  - Event-plane correlations
  - Correlations between v_2/v_3 and v_m, m=2-5.
- $v_n(p_T^a)$-$v_n(p_T^b)$ correlations indicate viscous effects controlled by system size
  - Not system shape!!!
- See small anti-correlation between magnitudes of v_2 & v_3
  - Initial geometry effect, reasonably weak described by CGC & Glauber models
- See strong correlation between v_4-v_2 and v_5-v_2.
  - Indicate non-linear response to initial geometry (not described by initial geometry models)
  - Extracted linear & non-linear contributions by two component fits
  - Correlated with v_2 incase of v_4-v_2 correlation
  - Correlated with both v_3 and v_2 incase of v_5-v_2 correlation
- Results show good agreement with independent EP correlation results
- Dependence of the linear components on the rms-ε_n were also studied
  - Stronger damping seen for higher order harmonics as expected from hydrodynamics
- v_n-v_m and EP correlations are new flow observables
  - Have much potential in improving our understanding of HI collisions.