Principal Component Analysis and Subleading Flow

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A. Mazeliauskas and D. Teaney, PRC 91, (2015) 044902
R.S. Bhalerao, J.Y. Ollitrault, S. Pal, and D. Teaney, PRL 114, (2015) 152301
(also see Ollitrault's talk on July 8)

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Outline

PCA introduction

Hydro results for 3^{rd} harmonic

- Components
- Factorization breaking
- Initial geometry
- Predictors
- Linear response

Results for $n = 0, 1, 2$ harmonics

Summary

Motivation

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- *Aim to understand event-by-event particle flow in heavy ion collisions.*
  - integrate over y and/or $p_T \Rightarrow$ can use boost invariant models.
  - expand φ in Fourier harmonics $\Rightarrow$ only lowest harmonics matter

$$\frac{\mathrm{d}N}{\mathrm{d}\mathbf{p}} = V_0(p) + \sum_{n=1}^{\infty} V_n(p) e^{-in\varphi} + \mathrm{H.c.}$$

- Are there any good basis for expansion in p_T and η?

  Principal component analysis gives optimal data-driven basis.

Two-particle correlations

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- Measure pair correlations $\Leftarrow$ determined by single particle distribution

$$\left\langle \frac{\mathrm{d}N_{\text{pairs}}}{\mathrm{d}\mathbf{p}_1 \mathrm{d}\mathbf{p}_2} \right\rangle = \left\langle \frac{\mathrm{d}N}{\mathrm{d}\mathbf{p}_1} \frac{\mathrm{d}N}{\mathrm{d}\mathbf{p}_2} \right\rangle + \mathcal{O}\left(N\right) \ .$$

- Expand pair correlations in Fourier series

$$\left\langle \frac{\mathrm{d}N_{\text{pairs}}}{\mathrm{d}\mathbf{p}_1 \mathrm{d}\mathbf{p}_2} \right\rangle = \sum_n V_{n\Delta}(p_1, p_2) e^{-in(\varphi_1 - \varphi_2)} \, .$$

- $V_{n\Delta}(p_1, p_2)$ is equal to covariance matrix of $V_n(p)$

$$\boxed{V_{n\Delta}(p_1, p_2) = \langle V_n(p_1) V_n^*(p_2) \rangle} \, .$$

$\langle V_n(p_1) V_n^*(p_2) \rangle$ contains full two-particle correlation information.

Principal component analysis

- *Covariance matrix can be written as a sum of its eigenvectors*

$$\langle V_n(p_1)V_n^*(p_2)\rangle = \sum_a V_n^{(a)}(p_1) \times V_n^{(a)}(p_2).$$

- Eigenvectors are directions of maximized variance

$$\underbrace{V_n^{(a)}(p)}_{a^{\text{th}}\text{ principal component}} = \sqrt{\underbrace{\lambda_a}_{\text{eigenvalue}}} \times \underbrace{\psi_n^{(a)}(p)}_{\text{normalized eigenvectors}}$$

- After PCA, use principal components as basis for event-by-event flow

$$V_n(p) = \underbrace{\xi_1 V_n^{(1)}(p)}_{\text{leading flow}} + \underbrace{\xi_2 V_n^{(2)}(p)}_{\text{subleading flow}} + \ldots.$$

Hydrodynamics and PCA

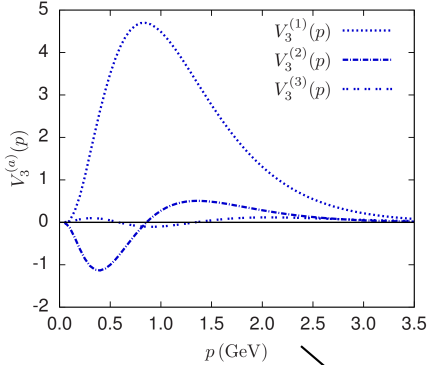
Find principal components of $\langle V_n(p_1)V_n^*(p_2)\rangle$ from hydro simulations.

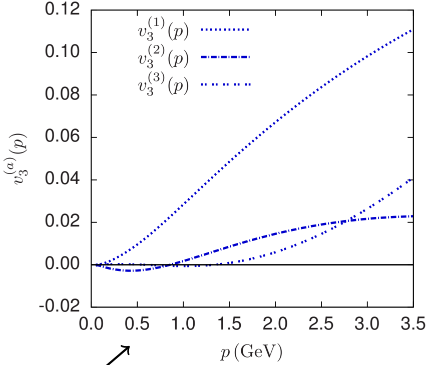
- 2+1, boost invariant hydro
- Pb-Pb Phobos MC Glauber initial conditions
- no resonance decays
- pion spectrum calculated at freeze-out
- $\eta/s = 0.08\hbar$
- 6000 events per centrality class (courtesy of Soumya Mohapatra)
- results shown for 0-5% centrality unless specified.

Start with triangular flow $V_3(p)$ – strong signal and driven entirely by fluctuations.

Subleading $V_3(p)$ flow

$$V_3(p) = \xi_1 V_3^{(1)}(p) + \xi_2 V_3^{(2)}(p) + \ldots, \quad \xi_a = |\xi_a| e^{i3\Phi_a}$$

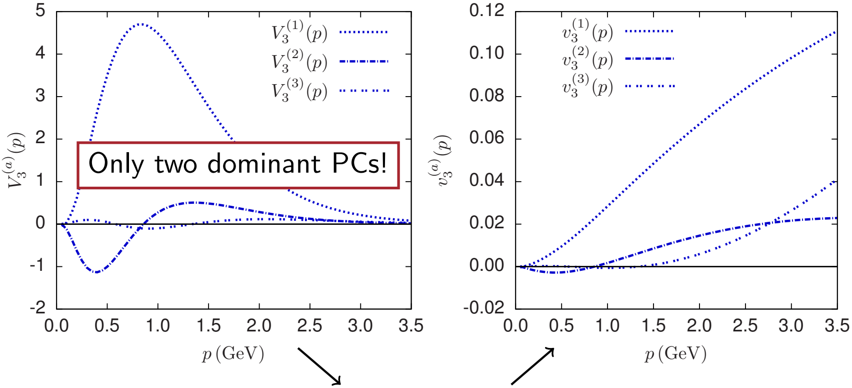




$$v_3^{(a)}(p) = \frac{V_3^{(a)}(p)}{\langle \mathrm{d}N/\mathrm{d}p \rangle}$$

Subleading $V_3(p)$ flow

$$V_3(p) = \xi_1 V_3^{(1)}(p) + \xi_2 V_3^{(2)}(p) + \ldots, \quad \xi_a = |\xi_a| e^{i3\Phi_a}$$

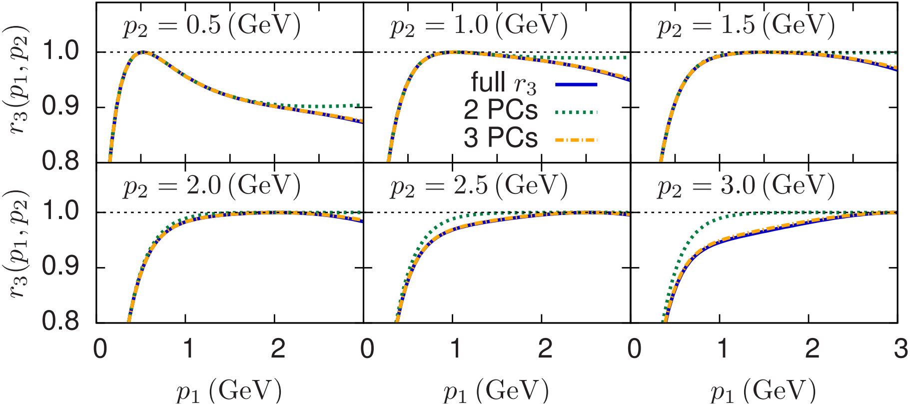


$$v_3^{(a)}(p) = \frac{V_3^{(a)}(p)}{\langle dN/dp \rangle}$$

Factorization breaking with PCs

$$\langle V_3(p_1)V_3^*(p_2)\rangle = V_3^{(1)}(p_1)V_3^{(1)}(p_2) + V_3^{(2)}(p_1)V_3^{(2)}(p_2) + \ldots$$

$$r_3(p_1,p_2) = \frac{\langle V_3(p_1)V_3^*(p_2)\rangle}{\sqrt{\langle |V_3(p_1)|^2\rangle \langle |V_3(p_2)|^2\rangle}} \leq 1$$

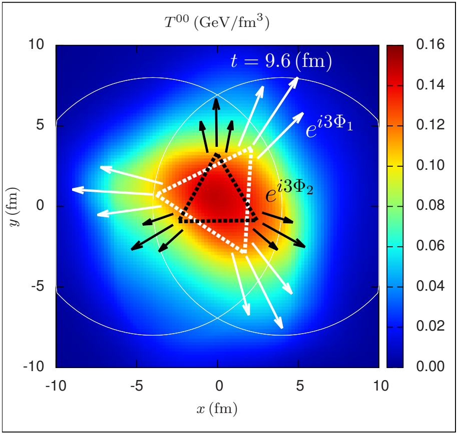


Two flows ⇒ two initial geometries

For each event project $V_3(p)$ to principal component basis

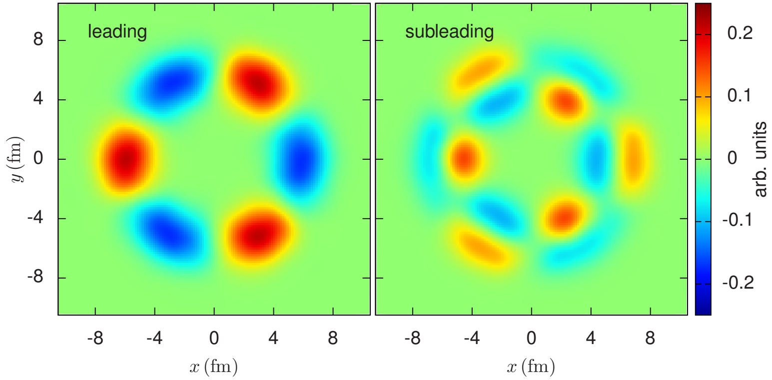
$$V_3(p) = |\xi_1| e^{i3\Phi_1} V_3^{(1)}(p) + |\xi_2| e^{i3\Phi_2} V_3^{(2)}(p)$$

Average initial entropy density $S(r,\phi)$ in subleading flow plane.



Geometry driving subleading flow

Average geometry $\langle S(r, \phi + \Phi_a) |\xi_a| \rangle \times r^3$ minus background



Geometry driving subleading flow

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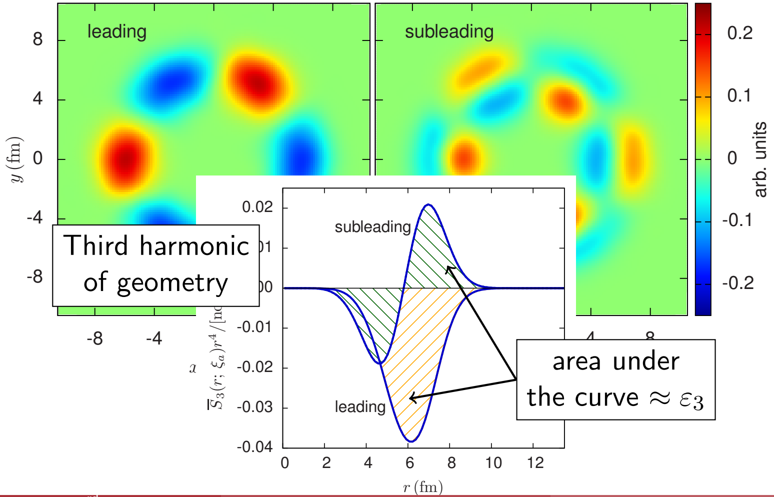
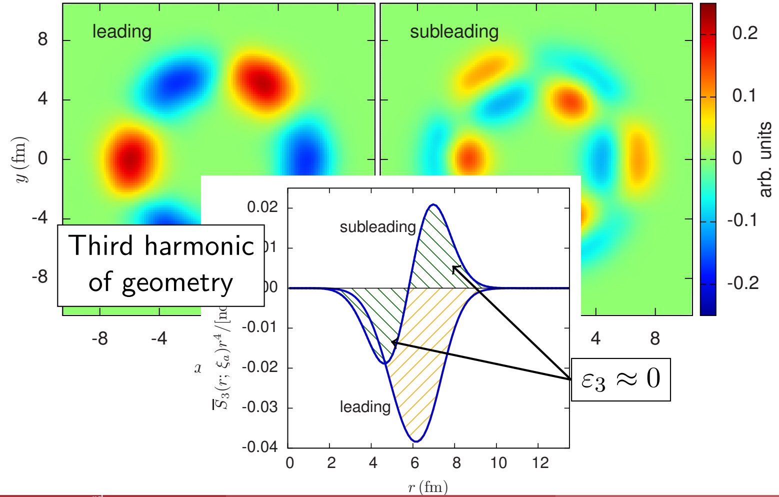
Average geometry $\langle S(r, \phi + \Phi_a)|\xi_a|\rangle \times r^3$ minus background



Geometry driving subleading flow

Average geometry $\langle S(r, \phi + \Phi_a)|\xi_a|\rangle \times r^3$ minus background



“ε_3” for subleading flow

Leading flow is well predicted by ε_3

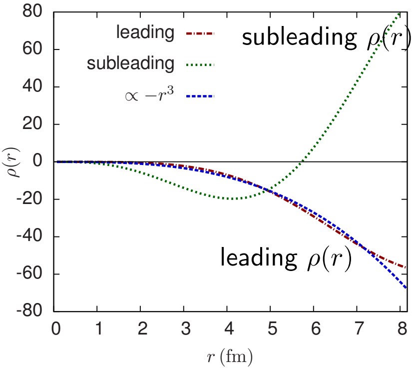
$$\varepsilon_3^{(1)} \propto \langle \underbrace{r^3}_{\text{radial weight}} e^{i3\phi} \rangle$$

Subleading flow needs custom radial weight

$$\varepsilon_3^{(2)} \propto \langle \rho(r)\, e^{i3\phi} \rangle$$

Use Bessel functions for $\rho(r)$

$$\rho(r) = \sum_i w_i J_3(k_i r),$$



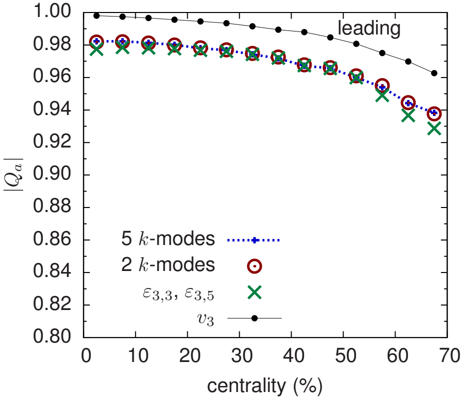
Choose $\rho(r)$ to maximize correlation with subleading flow ξ_2.

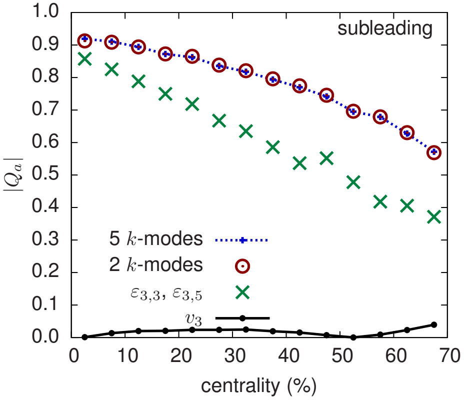
Tried: $k_1 R_{\text{rms}}/k_2 R_{\text{rms}} = J_{3,1}/J_{3,2}$ and 5 evenly spaced $k_i R_{\text{rms}}$.

Predicting subleading flow

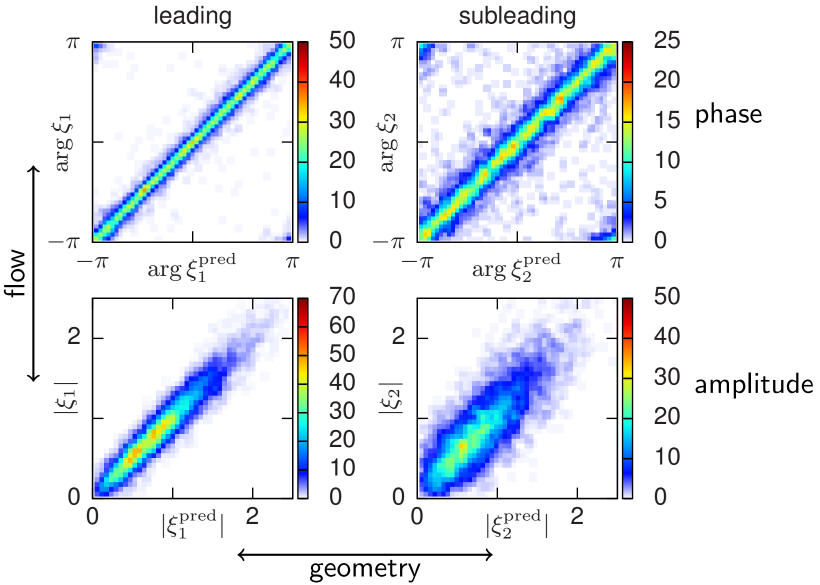
Correlation coefficient between flow and geometry

$$Q = \frac{\left\langle \varepsilon_3^{(2)} \xi_2^* \right\rangle}{\sqrt{\left\langle |\varepsilon_3^{(2)}|^2 \right\rangle} \sqrt{\left\langle |\xi_2|^2 \right\rangle}} \leq 1 \qquad \begin{cases} Q = 1 & \text{Perfect correlation} \\ Q = 0 & \text{Uncorrelated} \end{cases}$$





Event-by-event correlations



Single-shot vs event averaged response

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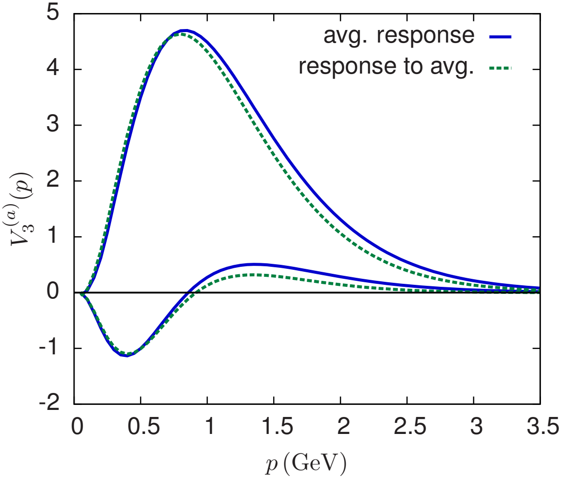
- evolve smooth initial geometry with radially excited eccentricity
- compare with event averaged subleading flow



Partial summary

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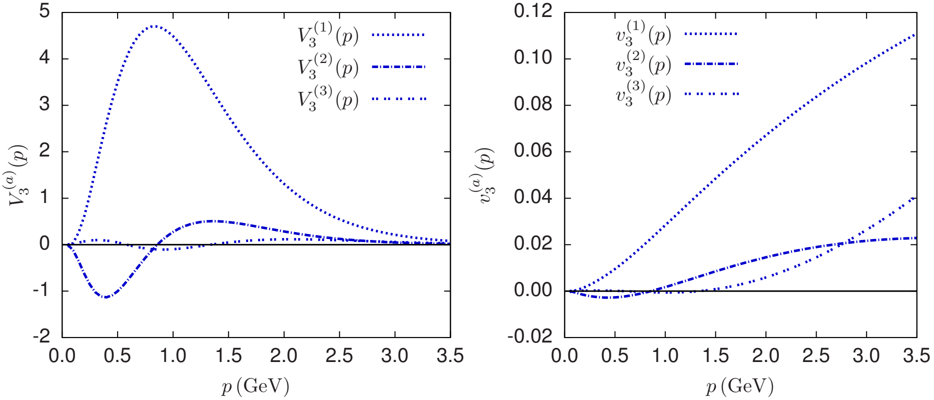
1. PCA is a systematic way of analyzing two particle correlations.
2. 2-3 principal components contain all information of r_n matrix.
3. Subleading flow originates from radial excitation in geometry.

Now compare $n = 3$ case with $n = 0, 1, 2$

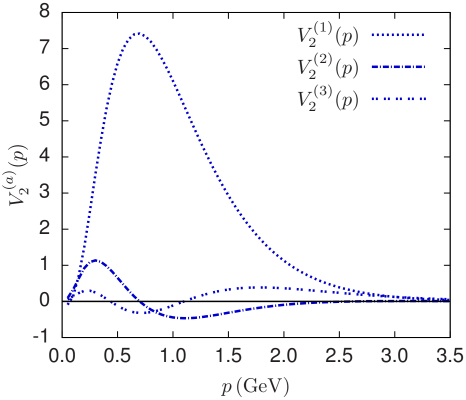
Principal components for $n = 0, 1, 2, 3$

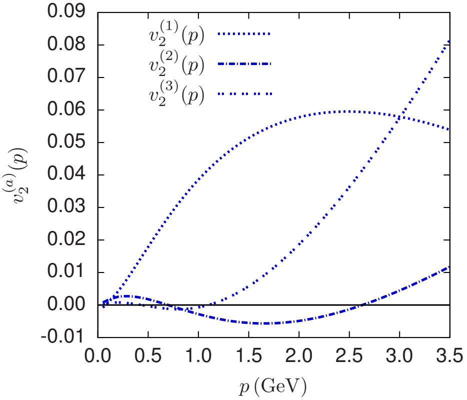
$n = 3$



Principal components for $n = 0, 1, 2, 3$

$n = 2$





Principal components for $n = 0, 1, 2, 3$

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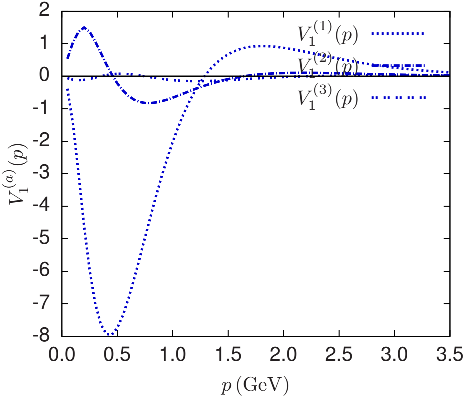
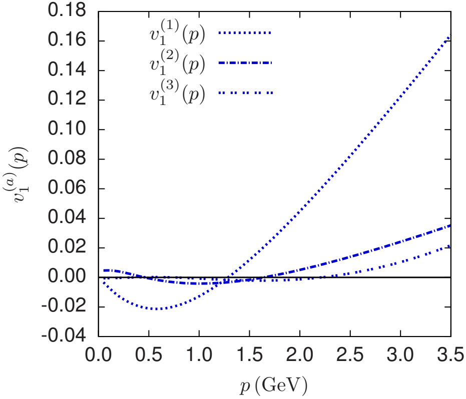
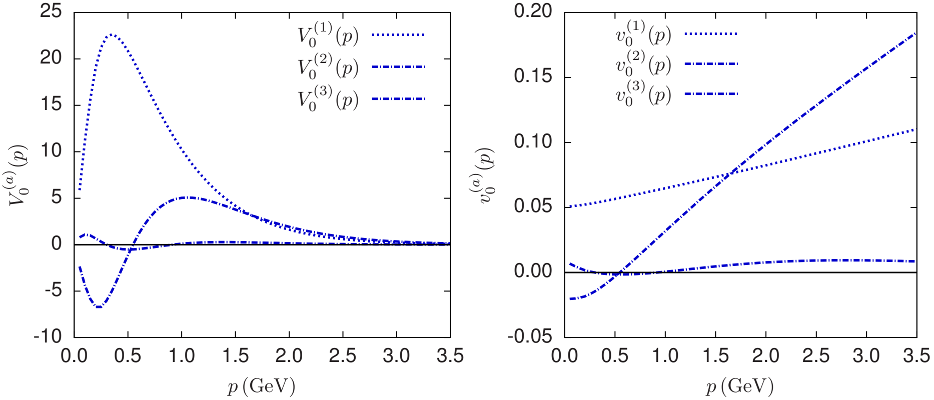
$n = 1$





Principal components for $n = 0, 1, 2, 3$

$n = 0$



Initial geometry for $n = 0, 1, 2, 3$

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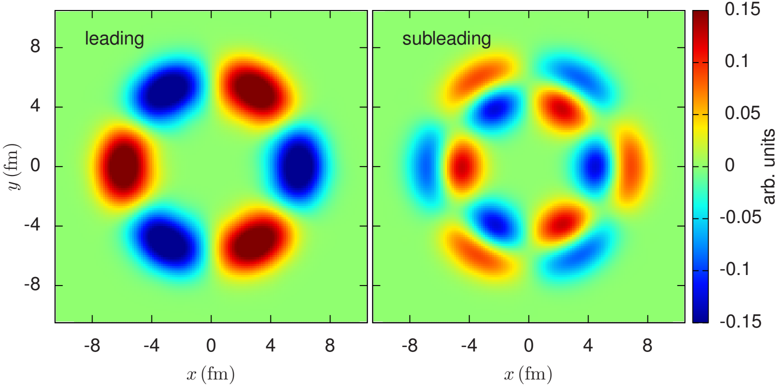
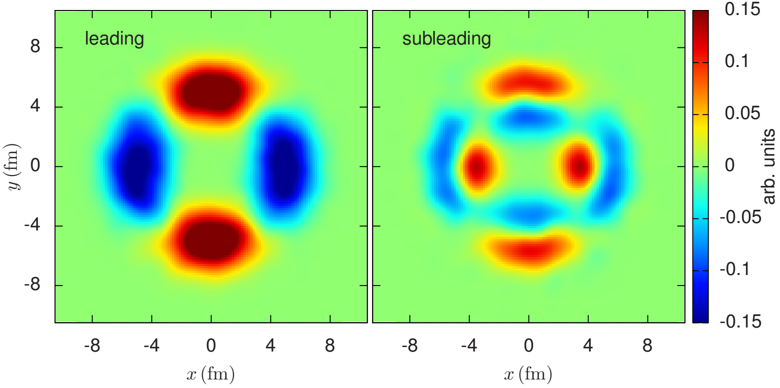
$n = 3$



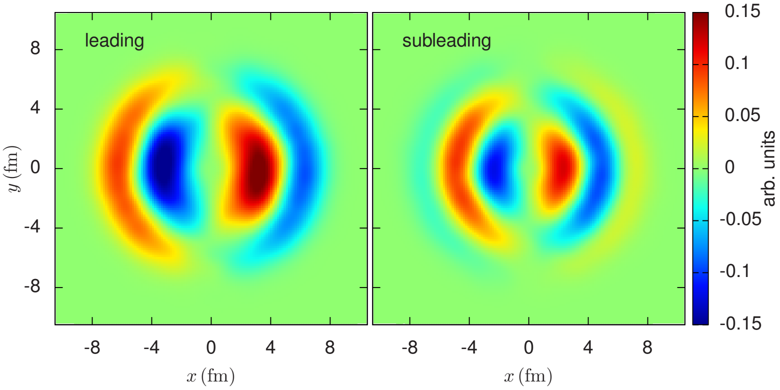
Initial geometry for $n = 0, 1, 2, 3$

$n = 2$



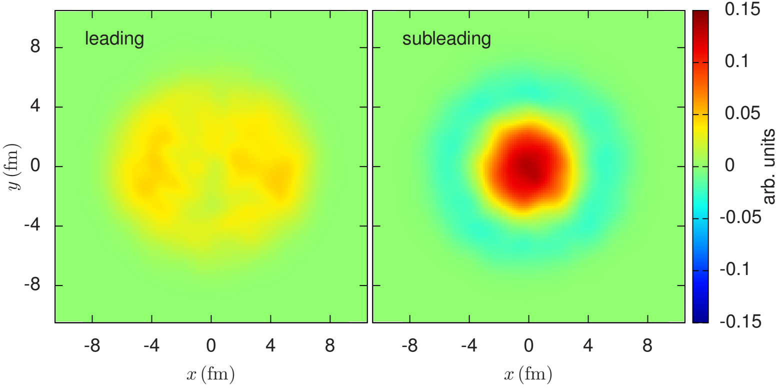
Initial geometry for $n = 0, 1, 2, 3$

$n = 1$



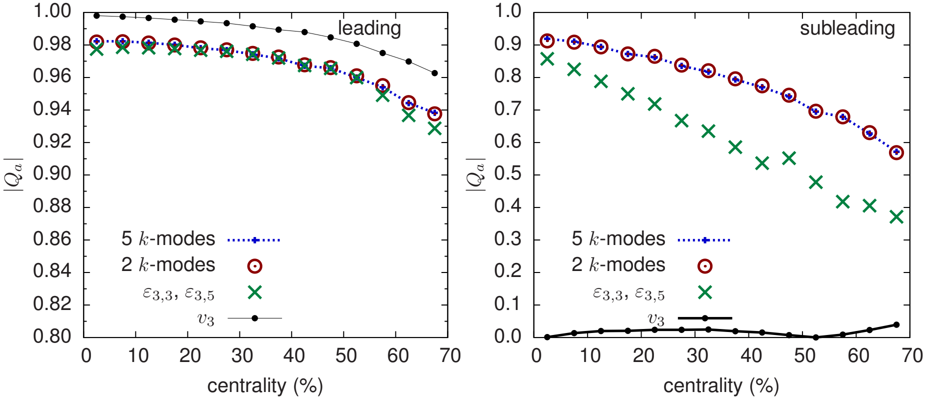
Initial geometry for $n = 0, 1, 2, 3$

$n = 0$



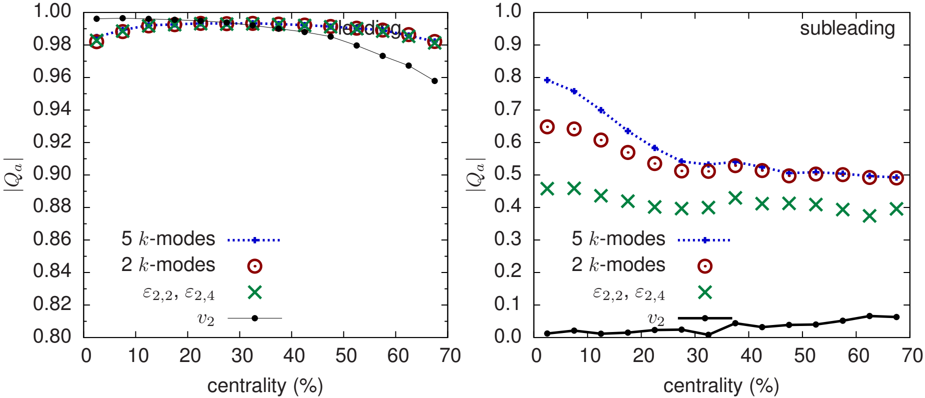
Correlations for $n = 0, 1, 2, 3$

$n = 3$



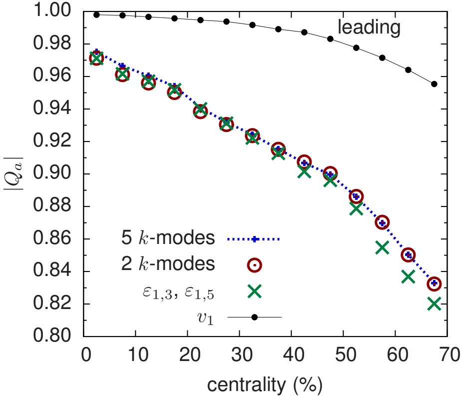
Correlations for $n = 0, 1, 2, 3$

$$n = 2$$



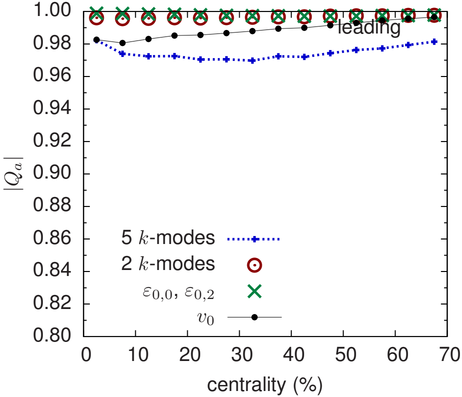
Correlations for $n = 0, 1, 2, 3$

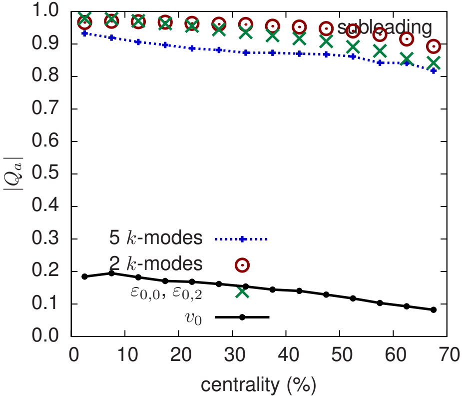
$n = 1$



Correlations for $n = 0, 1, 2, 3$

$n = 0$





Ordering of scaled eigenvalues

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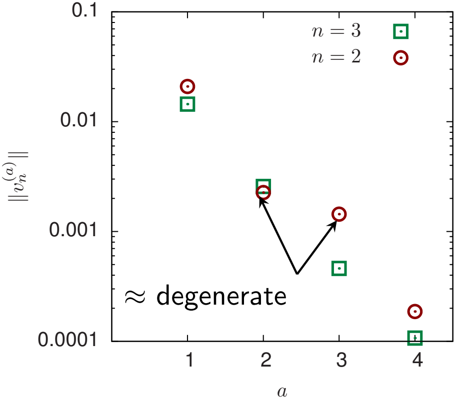
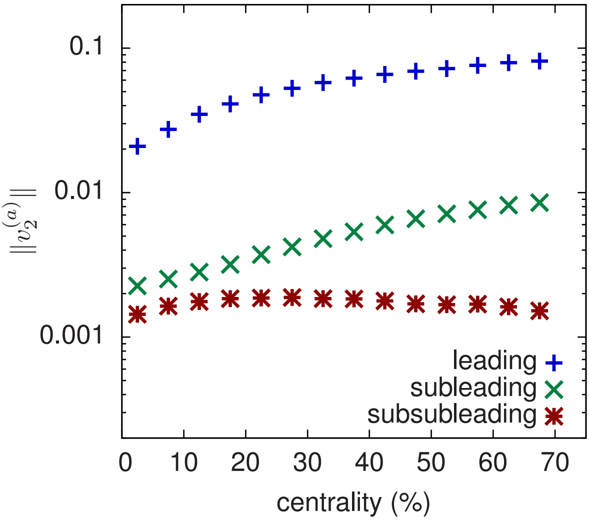
All harmonics except v_2 have two dominant principal components.

$$\|v_n^{(a)}\|^2 \equiv \frac{\int \left(V_n^{(a)}(p_T)\right)^2 \mathrm{d}p_T}{\int \langle \mathrm{d}N/\mathrm{d}p_T\rangle^2 \, \mathrm{d}p_T} = \frac{\lambda_a}{\int \langle \mathrm{d}N/\mathrm{d}p_T\rangle^2 \, \mathrm{d}p_T}$$



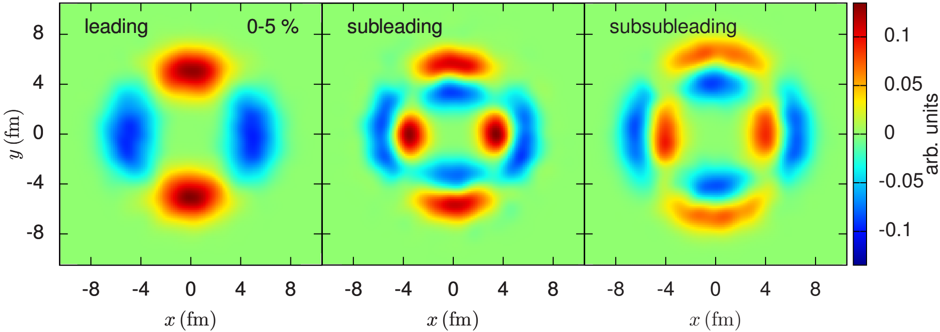
Sub-sub-leading flow of v_2

Central v_2 has contributions from three principal components.



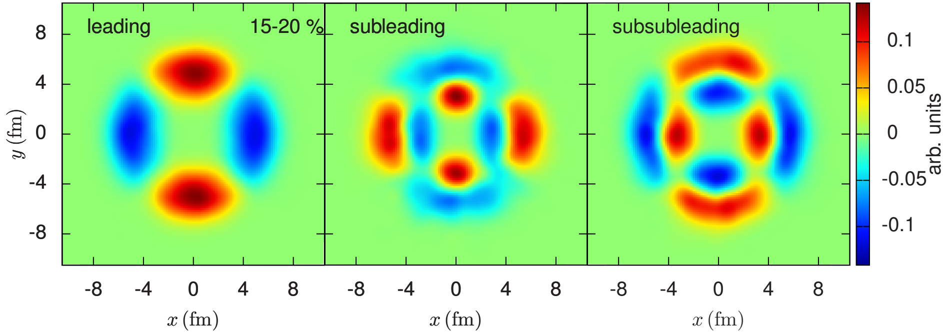
Evolution of initial geometry for v_2

Central

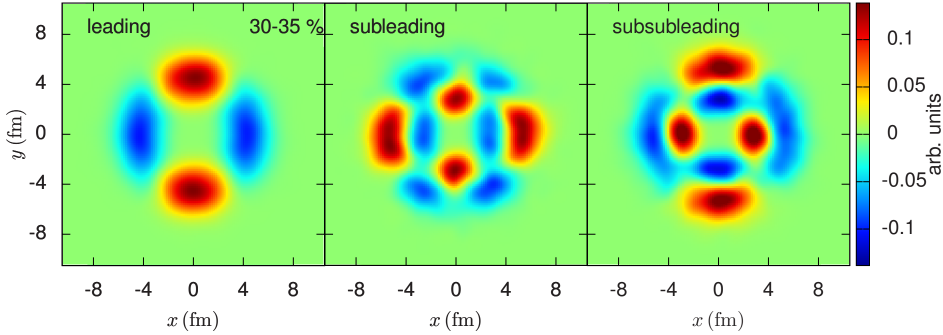


Evolution of initial geometry for v_2

Near central



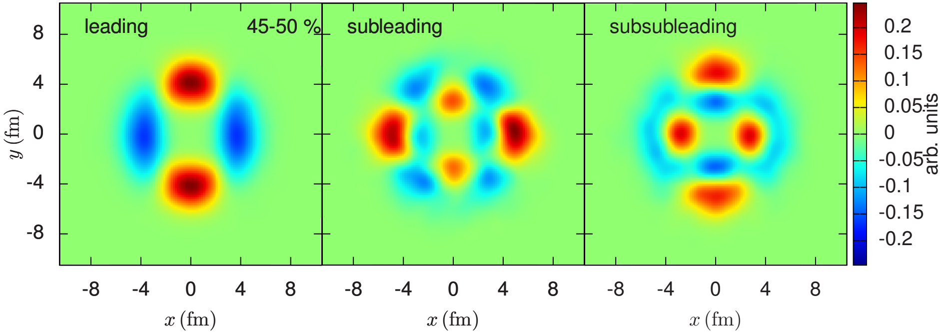
Mid-central



leading
30-35 %
subleading
subsubleading
8
4
0
-4
-8
y (fm)
-8
-4
0
4
8
x (fm)
x (fm)
x (fm)
0.1
0.05
0
-0.05
-0.1
arb. units

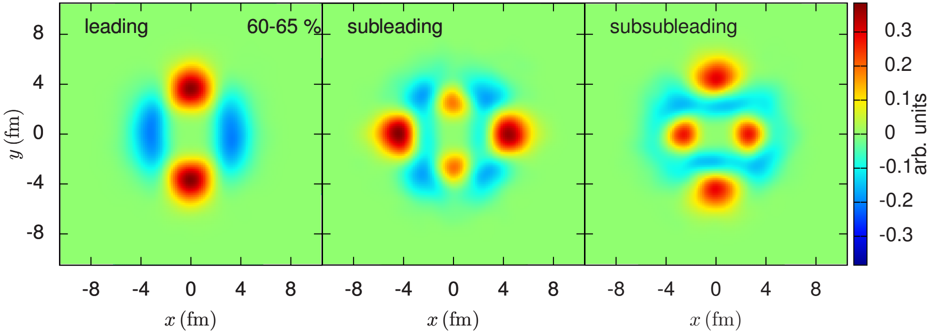
Evolution of initial geometry for v_2

Mid-peripheral



Evolution of initial geometry for v_2

Peripheral



Summary

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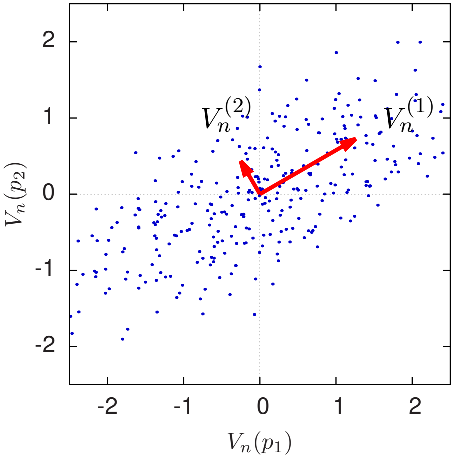
1. All harmonics except v_2 can be successfully described by just two PCs.
2. Subleading flow correlates well with radially excitations in geometry.
3. Subleading flow is strongly affected by average geometry for $n = 2$.

Thank you!

Backup

Mock example

$$V_n(p) = \underbrace{\xi_1 V_n^{(1)}(p)}_{\text{leading flow}} + \underbrace{\xi_2 V_n^{(2)}(p)}_{\text{subleading flow}} + \ldots .$$



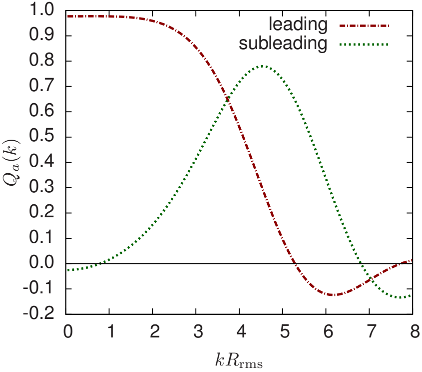
ξ_i – projection to PCs

$$\langle \xi_1 \xi_2^* \rangle = \delta_{ij}$$

Single term predictor

Correlate flow with Fourier components of geometry
$S_3(k) = \int_0^\infty r\mathrm{d}r J(kr) S_3(r)$

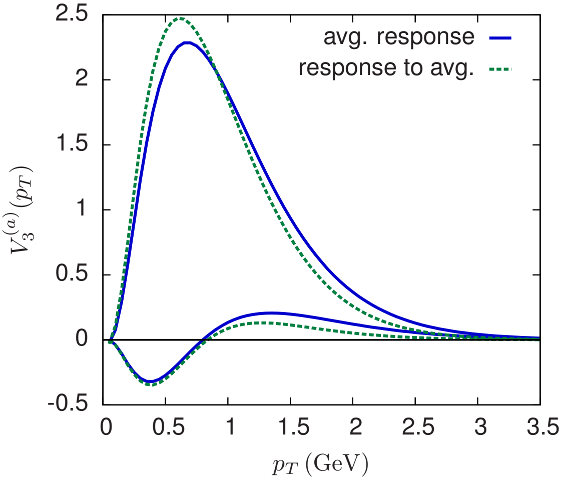
$$Q = \frac{\langle S_3(k)\xi_a^* \rangle}{\sqrt{\langle |S_3(k)|^2 \rangle}\sqrt{\langle |\xi_a|^2 \rangle}}$$



Single-shot vs event averaged response

40-45% centrality

- evolve smooth initial geometry with radially excited eccentricity
- compare with event averaged subleading flow



Single-shot evolution subleading flow

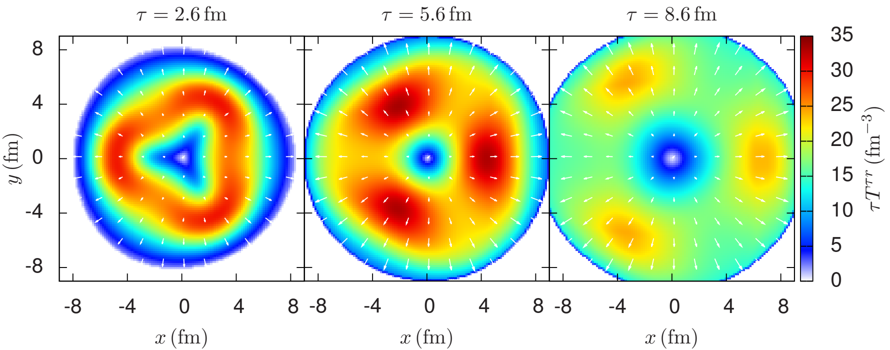


Figure: Hydrodynamic evolution of the subleading triangular flow. The color contours indicate the radial momentum density per rapidity, $T^{\tau r} = \tau(e+p)u^\tau u^r$, while the arrows indicate the radial flow velocity.

Centrality and viscosity dependence

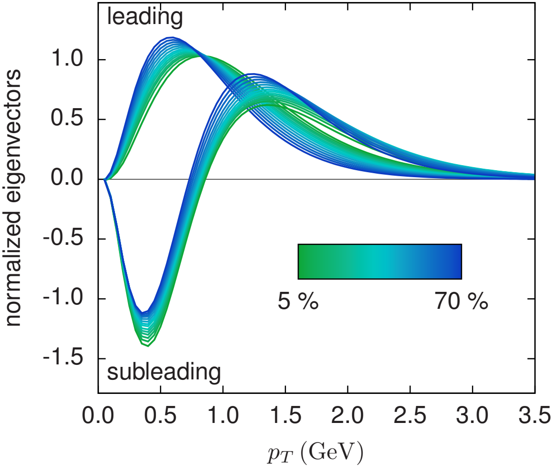


Figure: Centrality dependence of flow eigenvectors $\psi^a(p_T)$.

Centrality and viscosity dependence

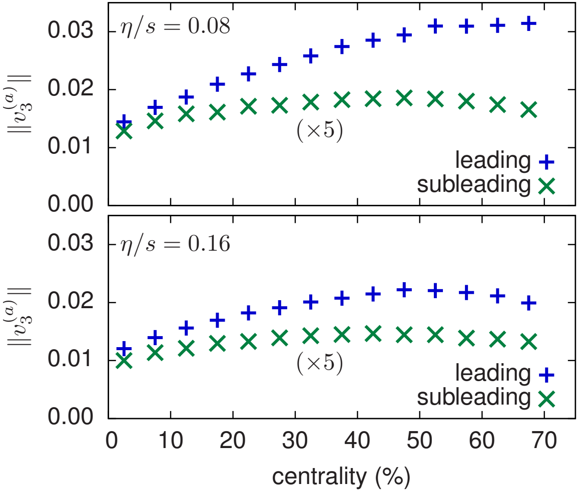


Figure: Centrality and viscosity dependence of scaled eigenvalues $\|v_3^{(a)}\|$. (The subleading flow has been magnified 5 times to bring to scale with leading flow.)

Averaged geometry in predicted event plane