Initial baryon number fluctuations and its hydrodynamic propagation on a Bjorken background

Mauricio Martinez Guerrero

In collaboration with Stefan Floerchinger

arXiv:1507.05569

Correlations and Fluctuations in p+A and A+A collisions
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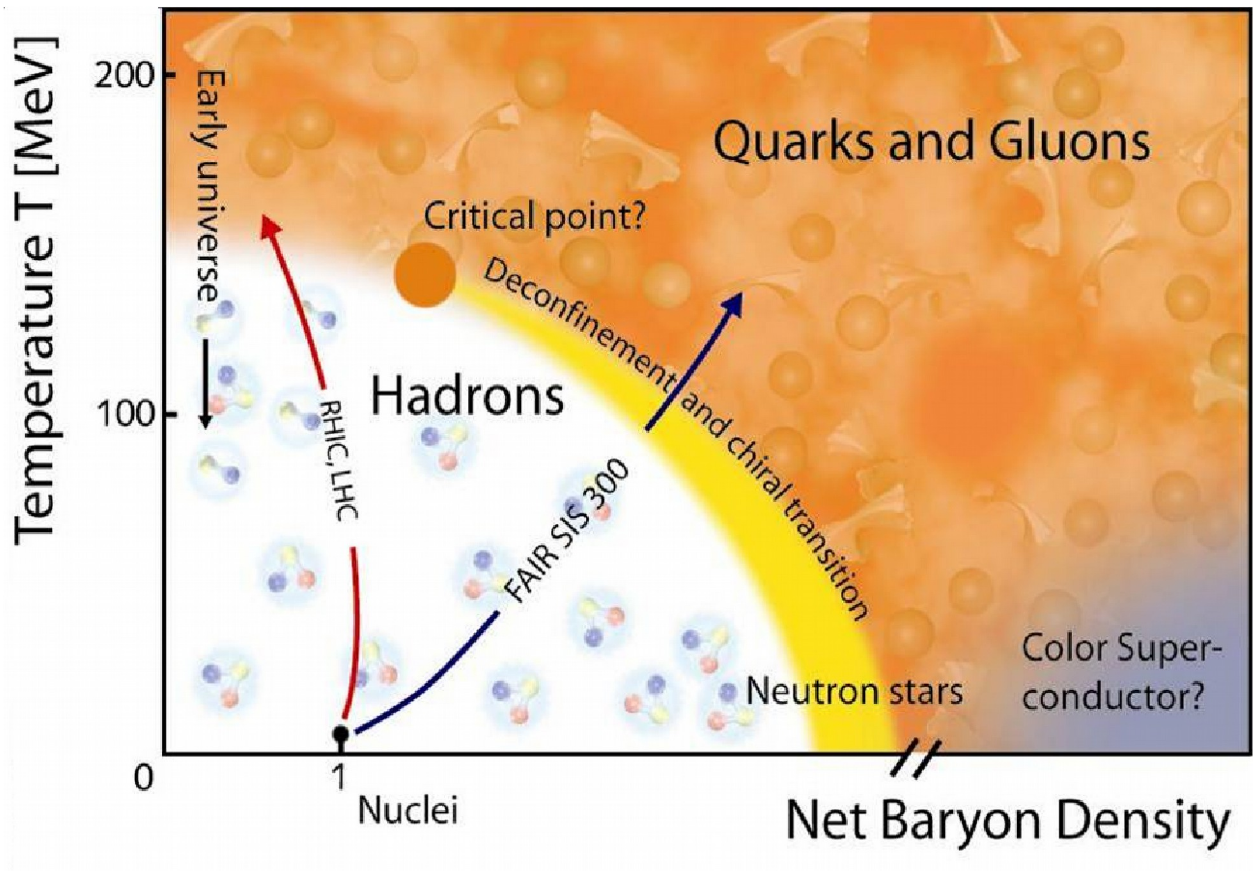
Motivation

QCD phase diagram



The critical point can be found experimentally by studying fluctuation observables as a function of the collision energy

- Event by event fluctuations of various particle multiplicities are sensitive to fluctuations
- Non gaussian cumulants are sensitive to these critical fluctuations

Sources of density fluctuations

Different sources of fluctuations in Relativistic Heavy Ion Collisions

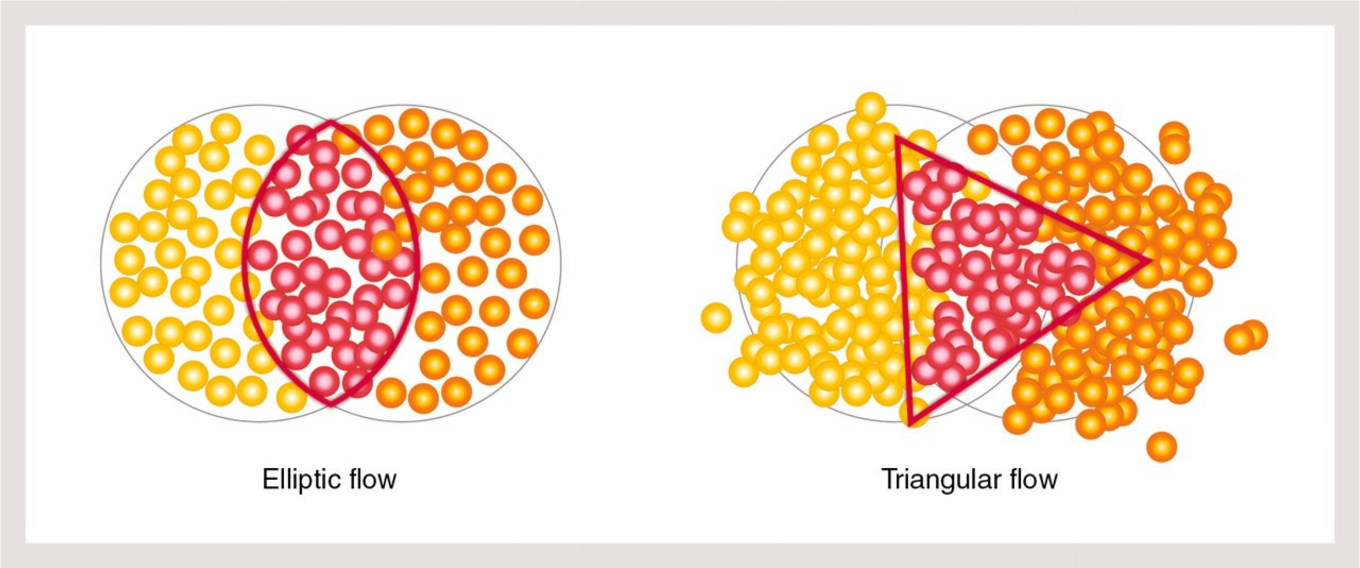
- Initial State Fluctuations
- Hydrodynamical fluctuations
- Fluctuations induced by hard processes
- Freeze-out fluctuations

Sources of density fluctuations

Different sources of fluctuations in Relativistic Heavy Ion Collisions

- Initial State Fluctuations — In this talk!!!
- Hydrodynamical fluctuations
- Fluctuations induced by hard processes
- Freeze-out fluctuations

Initial State fluctuations



- Quantum fluctuations in the densities of the two colliding nuclei and fluctuation of the energy mechanism.
- They appear as event by event fluctuations in the energy density and flow velocity distributions.
- Phenomenological studies indicate these fluctuations are responsible for the angular correlations of particle emissions observed in experiments.
- The power spectrum of the final state angular correlations may provide info of the transport coefficients.
- Initial state correlations have been connected with the ridge phenomenon.

In this talk: baryon density fluctuations

Outline

- Relativistic fluid dynamics at finite baryon density
- Boost invariant Bjorken flow with finite baryon density
- Fluctuations around Bjorken flow
- Two point correlation function for baryonic particles

Relativistic Fluid dynamics at finite baryon density

Hydrodynamics with finite density

For a system with a conserved charge (e.g. baryonic number)

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + (p + \pi_{\rm bulk})\Delta^{\mu\nu} + \pi^{\mu\nu}\,,$$
$$N^\mu = n\, u^\mu + \nu^\mu .$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

ϵ	energy density
u^μ	fluid velocity
p	pressure
$\pi^{\mu\nu}$	shear viscous tensor
π_{bulk}	bulk pressure
n	particle density
ν^μ	particle diffusion current

Hydrodynamics with finite density

For a system with a conserved charge (e.g. baryonic number)

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + (p + \pi_{\rm bulk})\Delta^{\mu\nu} + \pi^{\mu\nu}\,,$$
$$N^\mu = n\, u^\mu + \nu^\mu .$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

From the conservation laws, the equations of motion are (Landau frame)

$$D\epsilon + (\epsilon + p + \pi_{\rm bulk})\nabla_\mu u^\mu + \pi^{\mu\nu}\nabla_\mu u_\nu = 0,$$
$$(\epsilon + p + \pi_{\rm bulk})\, Du^\nu + \Delta^{\nu\mu}\,\partial_\mu(p + \pi_{\rm bulk}) + \Delta^\nu{}_\alpha \nabla_\mu \pi^{\mu\alpha} = 0,$$
$$Dn + n\nabla_\mu u^\mu + \nabla_\mu \nu^\mu = 0\,.$$

Evolution of the Dissipative currents

In addition to the EOS, one needs the evolution equations for the dissipative hydrodynamical fields. In the Navier-Stokes approach

$$\pi^{\mu\nu} = -2\eta\,\sigma^{\mu\nu} = -2\eta\left[\frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta} + \frac{1}{2}\Delta^{\mu\beta}\Delta^{\nu\alpha} - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\alpha\beta}\right]\nabla_\alpha u_\beta,$$

$$\pi_{\text{bulk}} = -\zeta\,\theta = -\zeta\,\nabla_\mu u^\mu,$$

$$\nu^\alpha = -\kappa\left[\frac{nT}{\epsilon+p}\right]^2 \iota^\alpha = -\kappa\left[\frac{nT}{\epsilon+p}\right]^2 \Delta^{\alpha\beta}\partial_\beta\left(\frac{\mu}{T}\right).$$

η	shear viscosity
ζ	bulk viscosity
κ	heat conductivity
μ	chemical potential
T	temperature

Estimates of the transport coefficients

Transport coefficient	Weakly-coupled QCD	Strongly-coupled theories
η	$k \frac{T^3}{g^4 \log(1/g)}$	$\frac{s(T,\mu)}{4\pi}$
ζ	$15\,\eta(T)\left(\frac{1}{3} - c_s^2(T)\right)^2$	$2\,\eta(T,\mu)\left(\frac{1}{3} - c_s^2(T,\mu)\right)$
κ	$\sim \mu^2/g^4$ for $\mu \gg T$ $\sim T^4/(g^4\mu^2)$for $\mu \ll T$	$8\pi^2\,\frac{T}{\mu^2}\eta(T,\mu)$

For practical purposes we use the strongly coupled relations for the transport coefficients

New calculations of the transport coefficients for strongly coupled systems with finite chemical potential.
See Rougemond and Noronha, arXiv:1507.06556

Equation of state

In the grand canonical ensemble one has P=P(T,μ)

In our work we make use of the ideal EOS one has

$$p(T,\mu) = \frac{1}{4!} a_1 T^4 + \frac{1}{4} a_2 T^2 \mu^2 + \frac{1}{4!} a_3 \mu^4$$

$$a_1 = \frac{8\pi^2}{15}\left(N_C^2 - 1 + \frac{7}{4} N_C N_F\right),$$

$$a_2 = \frac{2 N_C N_F}{27},$$

$$a_3 = \frac{2 N_C N_F}{81\pi^2}.$$

However, we can use more general equation of state from recent lattice data (BNL-Bielefeld collaboration, Wuppertal-Budapest) or analytical results from HTL (Strickland et. al, Vuorinen)

Equation of state

Since P=P(T,μ) the evolution equations for the temperature, fluid velocity and chemical potential are

$$\left[T\frac{\partial^2 p}{\partial T^2}+\mu\frac{\partial^2 p}{\partial T\partial\mu}\right]DT+\left[T\frac{\partial^2 p}{\partial T\partial\mu}+\mu\frac{\partial^2 p}{\partial\mu^2}\right]D\mu+(\epsilon+p)\,\theta-2\eta\,\sigma_{\alpha\beta}\sigma^{\alpha\beta}-\zeta\,\theta^2=0,$$

$$(\epsilon+p)\,Du^\nu+\Delta^{\nu\alpha}(s\,\partial_\alpha T+n\,\partial_\alpha\mu)-\Delta^\nu{}_\alpha\nabla_\beta\left(2\,\eta\,\sigma^{\alpha\beta}+\zeta\,\Delta^{\alpha\beta}\,\nabla_\gamma u^\gamma\right)=0\,,$$

$$\frac{\partial^2 p}{\partial T\partial\mu}DT+\frac{\partial^2 p}{\partial\mu^2}D\mu+n\,\theta+\nabla_\alpha\nu^\alpha=0\,.$$

Bjorken boost invariant solution with finite chemical potential

Evolution equations for the Bjorken flow

We use Bjorken model $T = T(\tau) \qquad \mu = \mu(\tau) \qquad u^\mu = (1,0,0,0)$

$$\tau = \sqrt{t^2 - z^2} \qquad \eta = \mathrm{arctanh}(z/t)$$

$$\partial_\tau T + \frac{-\frac{n}{\tau}\frac{\partial^2 p}{\partial T \partial \mu} + \frac{s}{\tau}\left(1 - \frac{4\eta/3+\zeta}{sT\tau}\right)\frac{\partial^2 p}{\partial \mu^2}}{\frac{\partial^2 p}{\partial T^2}\frac{\partial^2 p}{\partial \mu^2} - \left(\frac{\partial^2 p}{\partial T \partial \mu}\right)^2} = 0,$$

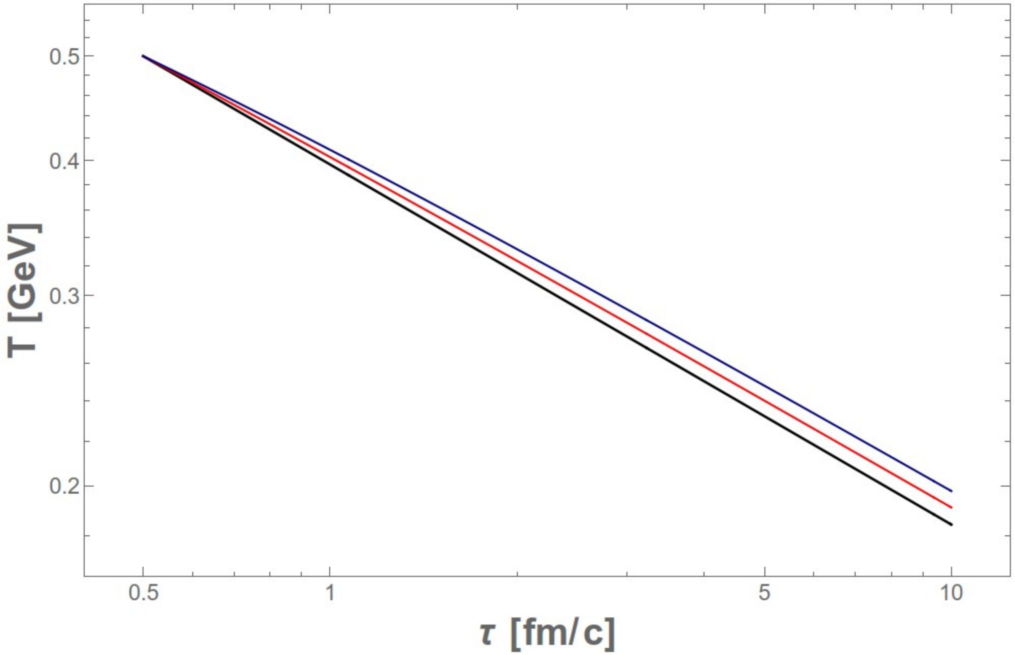
$$\partial_\tau \mu + \frac{\frac{n}{\tau}\frac{\partial^2 p}{\partial T^2} - \frac{s}{\tau}\left(1 - \frac{4\eta/3+\zeta}{sT\tau}\right)\frac{\partial^2 p}{\partial T \partial \mu}}{\frac{\partial^2 p}{\partial T^2}\frac{\partial^2 p}{\partial \mu^2} - \left(\frac{\partial^2 p}{\partial T \partial \mu}\right)^2} = 0.$$

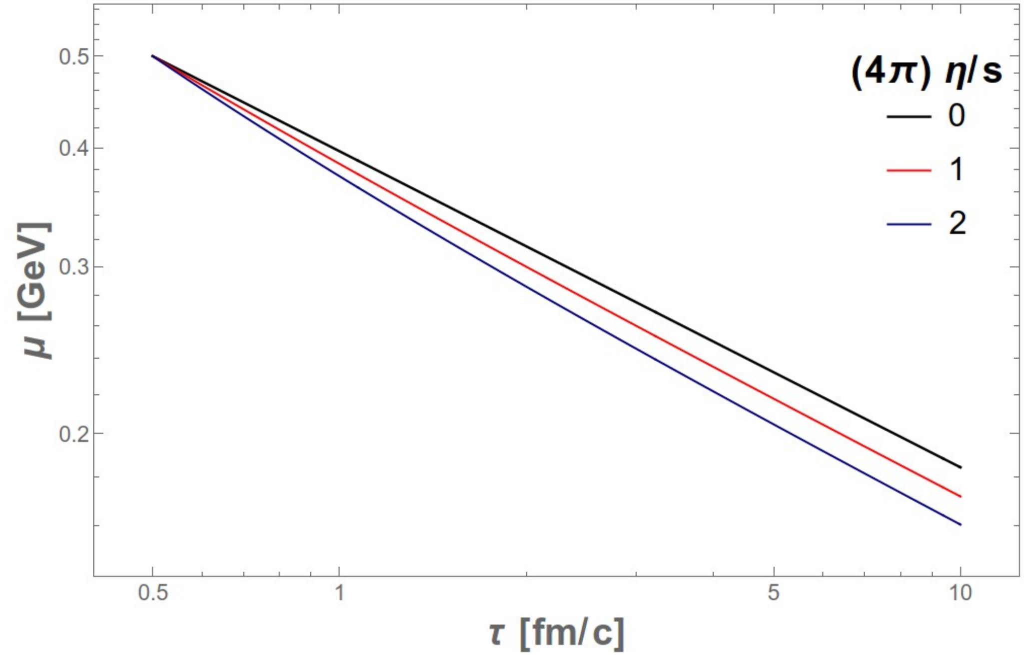
The size of the viscous corrections is determined by the parameter

$$\gamma = \frac{4\eta/3 + \zeta}{sT\tau}.$$

Numerical solutions

$\tau_0 = 0.5 \ \text{fm/c}$ $\mu_0 = 0.5 \ \text{GeV}$ $T_0 = 0.5 \ \text{GeV}$





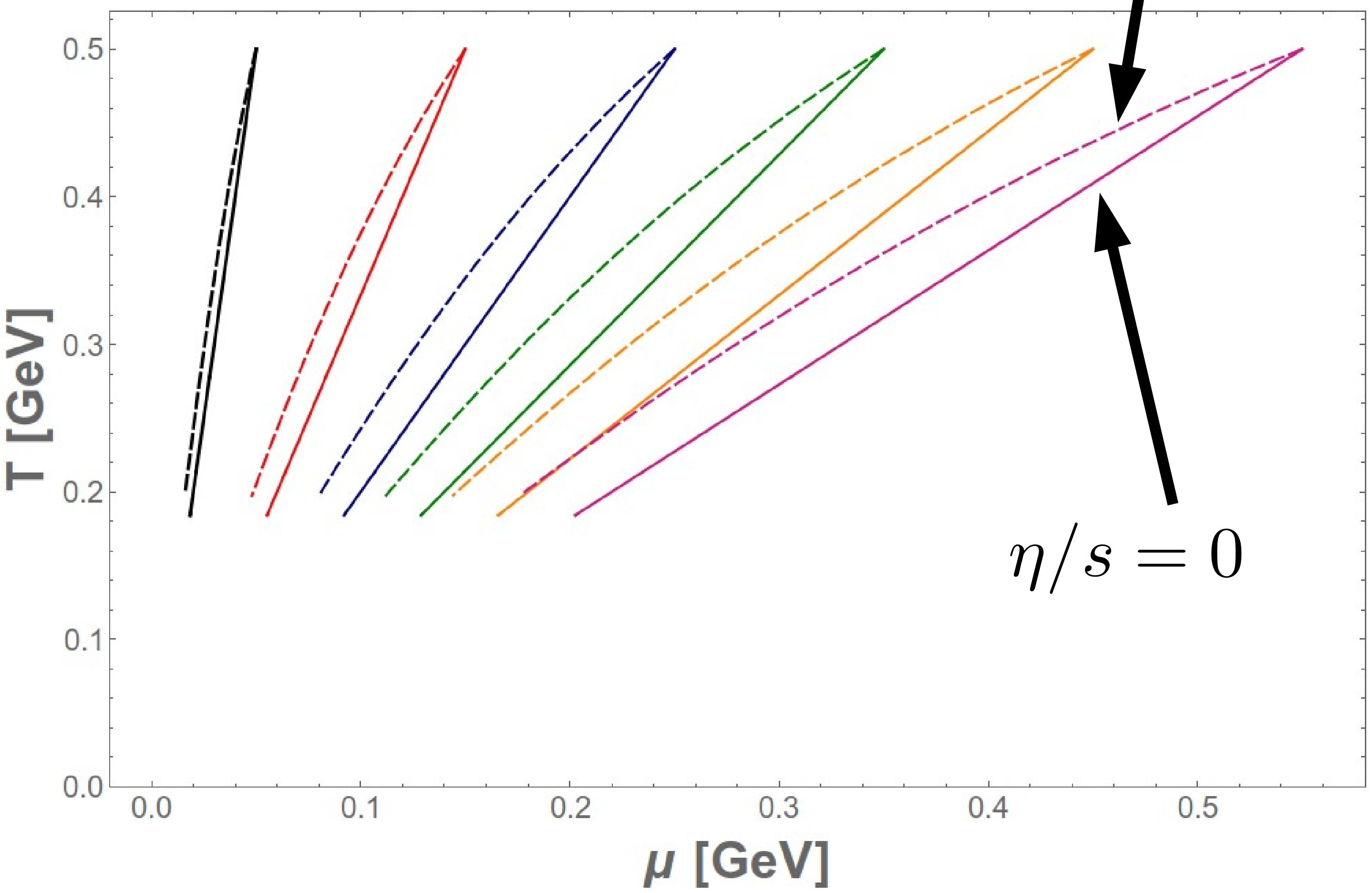
- Dissipative corrections play a role at early times
- At late times, the hydrodynamical fields decouple such that one recovers the Bjorken scaling

$$T(\tau) \mid_{\tau \gg \tau_0} \propto \tau^{-1/3},$$

$$\mu(\tau) \mid_{\tau \gg \tau_0} \propto \tau^{-1/3}$$

Trajectories in the T-μ plane

$\eta/s = 0.16$



$\eta/s = 0$

$\tau_0 = 0.5 \ \text{fm/c}$ $\tau_f = 10 \ \text{fm/c}$

Fluctuations around Bjorken flow

Background-fluctuation splitting

Consider small deviations around the evolving hydrodynamical fields

$$u^\mu = \bar{u}^\mu + \delta u^\mu, \qquad \epsilon = \bar{\epsilon} + \delta\epsilon, \qquad n = \bar{n} + \delta n,$$

Event average configuration which is smooth

Linear perturbation

For instance, for the Bjorken flow

$$u^\mu(\tau, r, \phi, \eta) = (1, \delta u^r(\tau, r, \phi, \eta), \delta u^\phi(\tau, r, \phi, \eta), \delta u^\eta(\tau, r, \phi, \eta)),$$
$$\epsilon(\tau, r, \phi, \eta) = \bar{\epsilon}(\tau) + \delta\epsilon(\tau, r, \phi, \eta)$$
$$n(\tau, r, \phi, \eta)) = \bar{n}(\tau) + \delta n(\tau, r, \phi, \eta)),$$

The linear equations of motion of the perturbations by replacing these expressions in the constitutive conservation laws and considering only terms linear in the fluctuations

Background-fluctuation splitting

Consider small deviations around the evolving hydrodynamical fields

$$u^\mu = \bar{u}^\mu + \delta u^\mu, \qquad \epsilon = \bar{\epsilon} + \delta\epsilon, \qquad n = \bar{n} + \delta n,$$

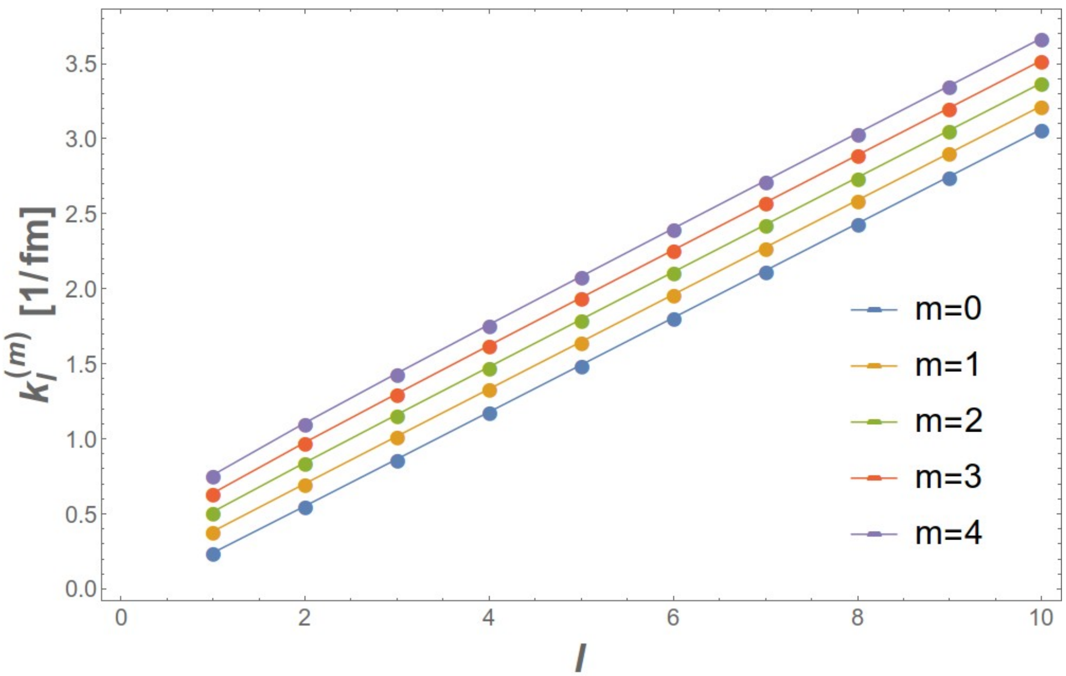
Event average configuration which is smooth

Linear perturbation

For instance, for the Bjorken flow, the equation of the perturbation of the energy density is

$$\partial_\tau \delta\epsilon + \left[\frac{1}{\tau} + \frac{1}{\tau}\left(\frac{\partial p}{\partial \epsilon}\right)_n - \frac{1}{\tau^2}\left(\frac{\partial \zeta}{\partial \epsilon}\right)_n - \frac{4}{3\tau^2}\left(\frac{\partial \eta}{\partial \epsilon}\right)_n\right]\delta\epsilon$$
$$+ \left[\frac{1}{\tau}\left(\frac{\partial p}{\partial n}\right)_\epsilon - \frac{1}{\tau^2}\left(\frac{\partial \zeta}{\partial n}\right)_\epsilon - \frac{4}{3\tau^2}\left(\frac{\partial \eta}{\partial n}\right)_\epsilon\right]\delta n$$
$$+ \left[\bar{\epsilon} + \bar{p} - \frac{2}{\tau}\bar{\zeta} + \frac{4}{3\tau}\bar{\eta}\right]\left(\partial_r \delta u^r + \frac{1}{r}\delta u^r + \partial_\phi \delta u^\phi + \partial_\eta \delta u^\eta\right) - \frac{4}{\tau}\bar{\eta}\,\partial_\eta \delta u^\eta = 0$$

Bessel Fourier decomposition

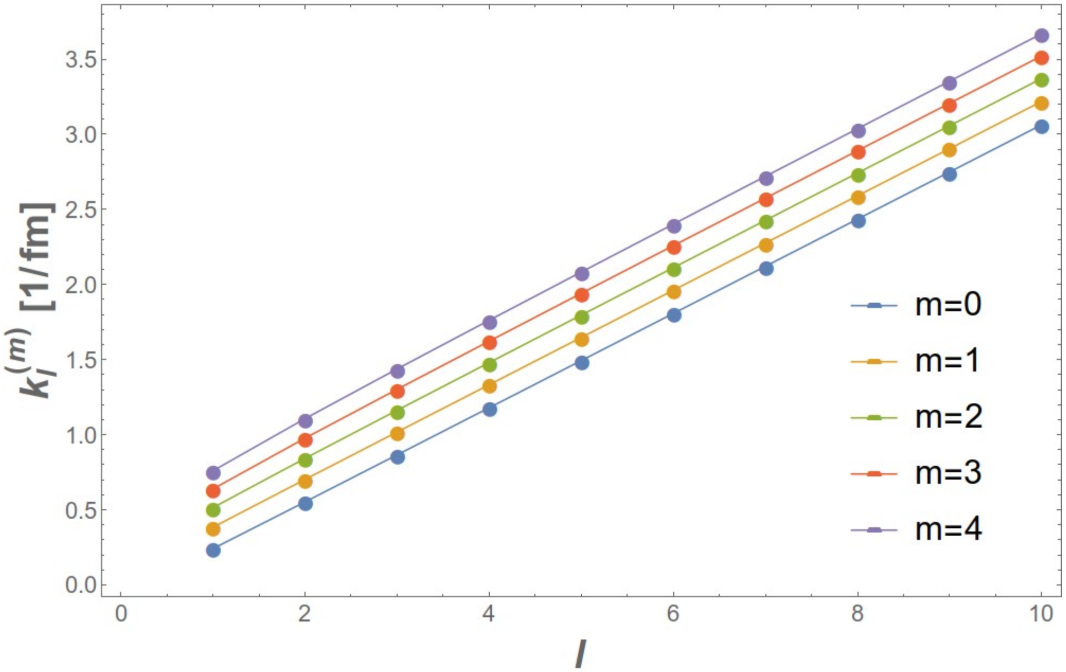


The fluctuating fields are decomposed as

$$\delta\epsilon(\tau, r, \phi, \eta) = \int_0^\infty dk\, k \sum_{m=-\infty}^{\infty} \int \frac{dq}{2\pi} \delta\epsilon(\tau, k, m, q)\, e^{i(m\phi+q\eta)} J_m(kr),$$

Similarly for δn and δu^η

Bessel Fourier decomposition



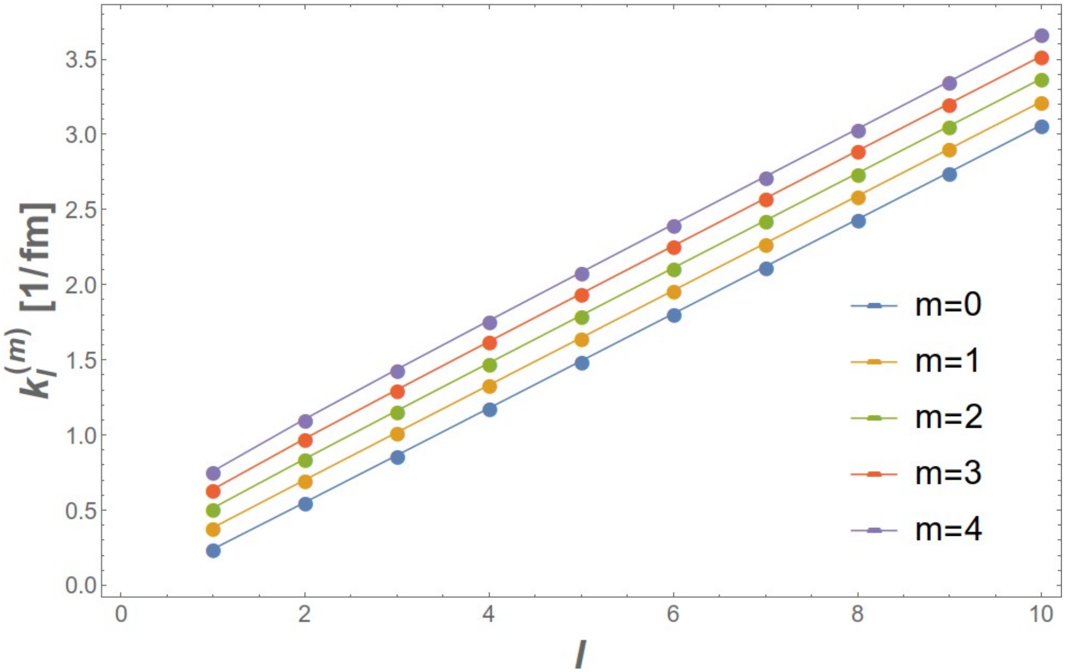
By introducing the polarizations

$$\delta u^r(\tau, r, \phi, \eta) = \frac{1}{\sqrt{2}}\left[\delta u^-(\tau, r, \phi, \eta) + \delta u^+(\tau, r, \phi, \eta)\right],$$

$$\delta u^\phi(\tau, r, \phi, \eta) = \frac{i}{r\sqrt{2}}\left[\delta u^-(\tau, r, \phi, \eta) - \delta u^+(\tau, r, \phi, \eta)\right],$$

$$\delta u^\pm(\tau, r, \phi, \eta) = \int_0^\infty dk\, k \sum_{m=-\infty}^{\infty} \int \frac{dq}{2\pi}\, \delta u^\pm(\tau, k, m, q)\, e^{i(m\phi + q\eta)}\, J_{m\pm1}(kr),$$

Bessel Fourier decomposition



For instance, for the Bjorken flow $\delta\epsilon(\tau,k,m,q)$ is

$$\partial_\tau \delta\epsilon + \left[\frac{1}{\tau} + \frac{1}{\tau}\left(\frac{\partial p}{\partial \epsilon}\right)_n - \frac{1}{\tau^2}\left(\frac{\partial \zeta}{\partial \epsilon}\right)_n - \frac{4}{3\tau^2}\left(\frac{\partial \eta}{\partial \epsilon}\right)_n\right]\delta\epsilon$$
$$+ \left[\frac{1}{\tau}\left(\frac{\partial p}{\partial n}\right)_\epsilon - \frac{1}{\tau^2}\left(\frac{\partial \zeta}{\partial n}\right)_\epsilon - \frac{4}{3\tau^2}\left(\frac{\partial \eta}{\partial n}\right)_\epsilon\right]\delta n$$
$$+ \left[\bar{\epsilon} + \bar{p} - \frac{2}{\tau}\bar{\zeta} + \frac{4}{3\tau}\bar{\eta}\right]\left(\frac{k}{\sqrt{2}}\left(\delta u^+ - \delta u^-\right) + iq\,\delta u^\eta\right) - \frac{4}{\tau}\bar{\eta}\, iq\,\delta u^\eta = 0.$$

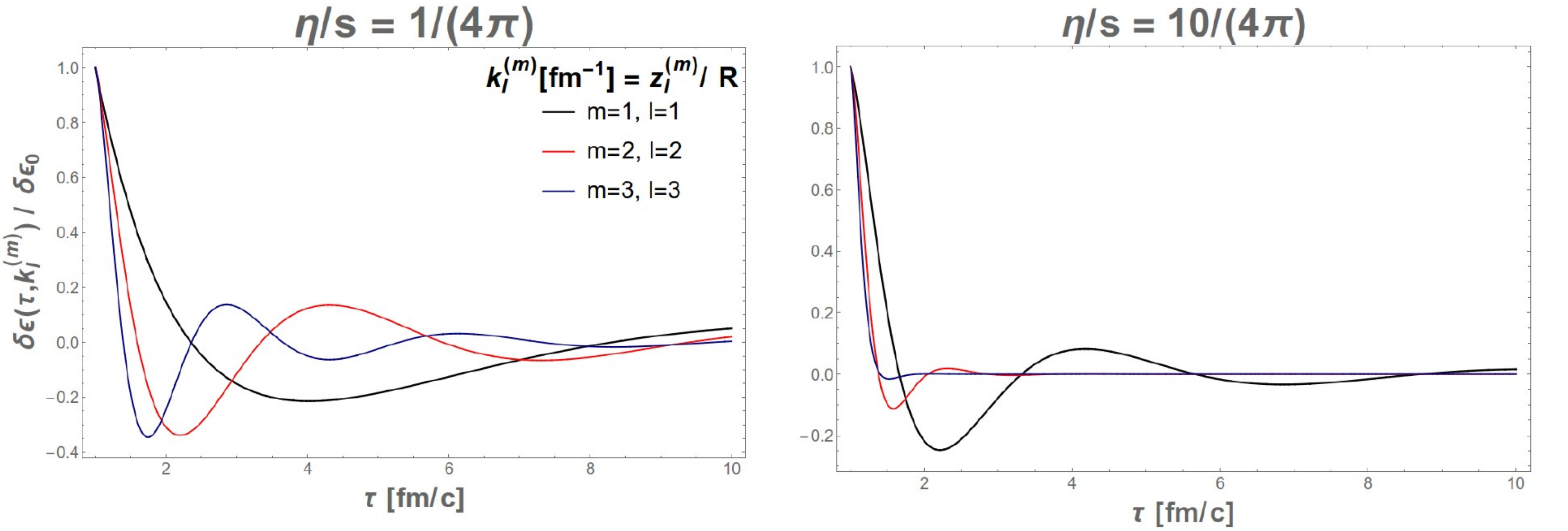
Exact boost symmetry (q=0)

The dynamics of the system depends only on
$\delta\epsilon(\tau,k,m)$, $\delta n(\tau,k,m)$ and $\Delta^- = u^+ - u^-$

$$\partial_\tau \delta\epsilon + \left[\frac{1}{\tau} + \frac{1}{\tau}\left(\frac{\partial p}{\partial \epsilon}\right)_n - \frac{1}{\tau^2}\left(\frac{\partial \zeta}{\partial \epsilon}\right)_n - \frac{4}{3\tau^2}\left(\frac{\partial \eta}{\partial \epsilon}\right)_n\right]\delta\epsilon$$
$$+ \left[\frac{1}{\tau}\left(\frac{\partial p}{\partial n}\right)_\epsilon - \frac{1}{\tau^2}\left(\frac{\partial \zeta}{\partial n}\right)_\epsilon - \frac{4}{3\tau^2}\left(\frac{\partial \eta}{\partial n}\right)_\epsilon\right]\delta n$$
$$+ \left[\bar\epsilon + \bar p - \frac{2}{\tau}\bar\zeta + \frac{4}{3\tau}\bar\eta\right]\left(\frac{k}{\sqrt{2}}\left(\delta u^+ - \delta u^-\right) + iq\,\delta u^\eta\right) - \frac{4}{\tau}\bar\eta\, iq\,\delta u^\eta = 0.$$

In addition, the propagation of the modes are determined by the values of the transport coefficients

Exact boost symmetry (q=0)



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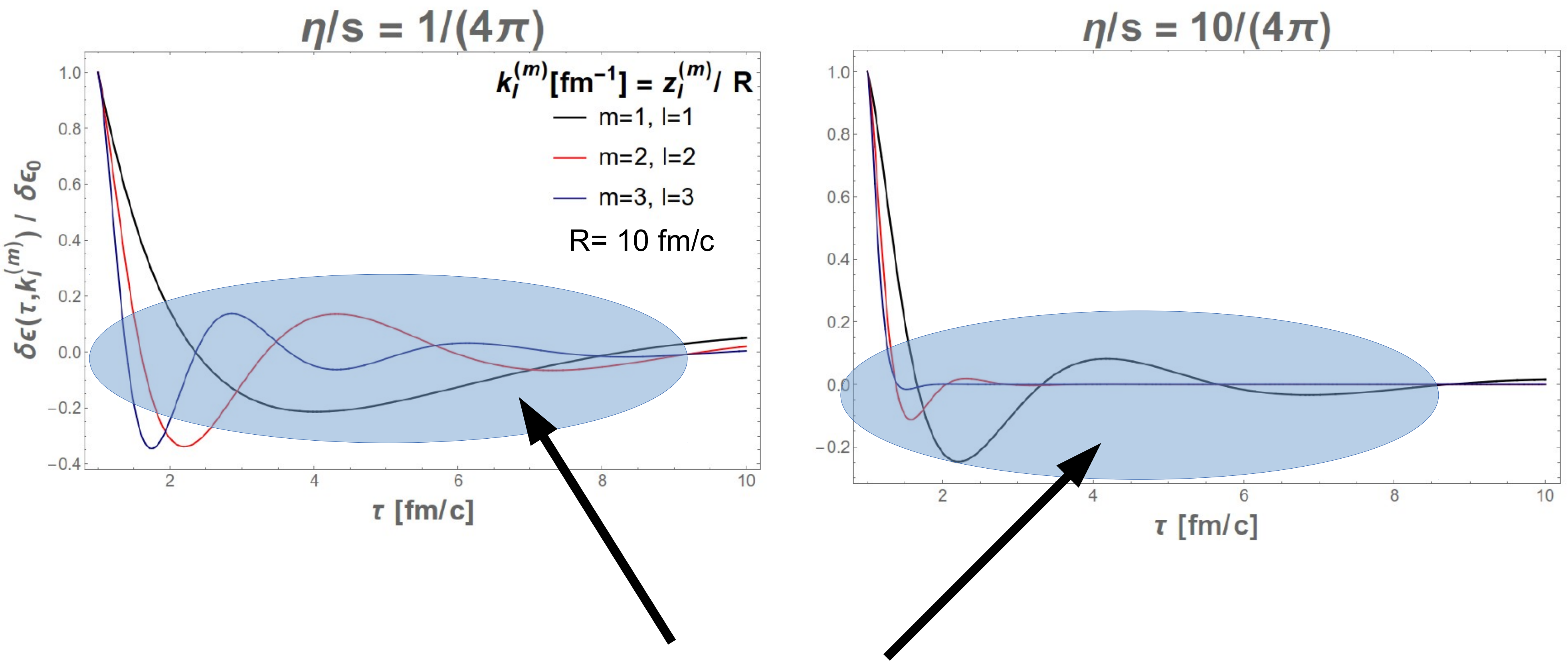
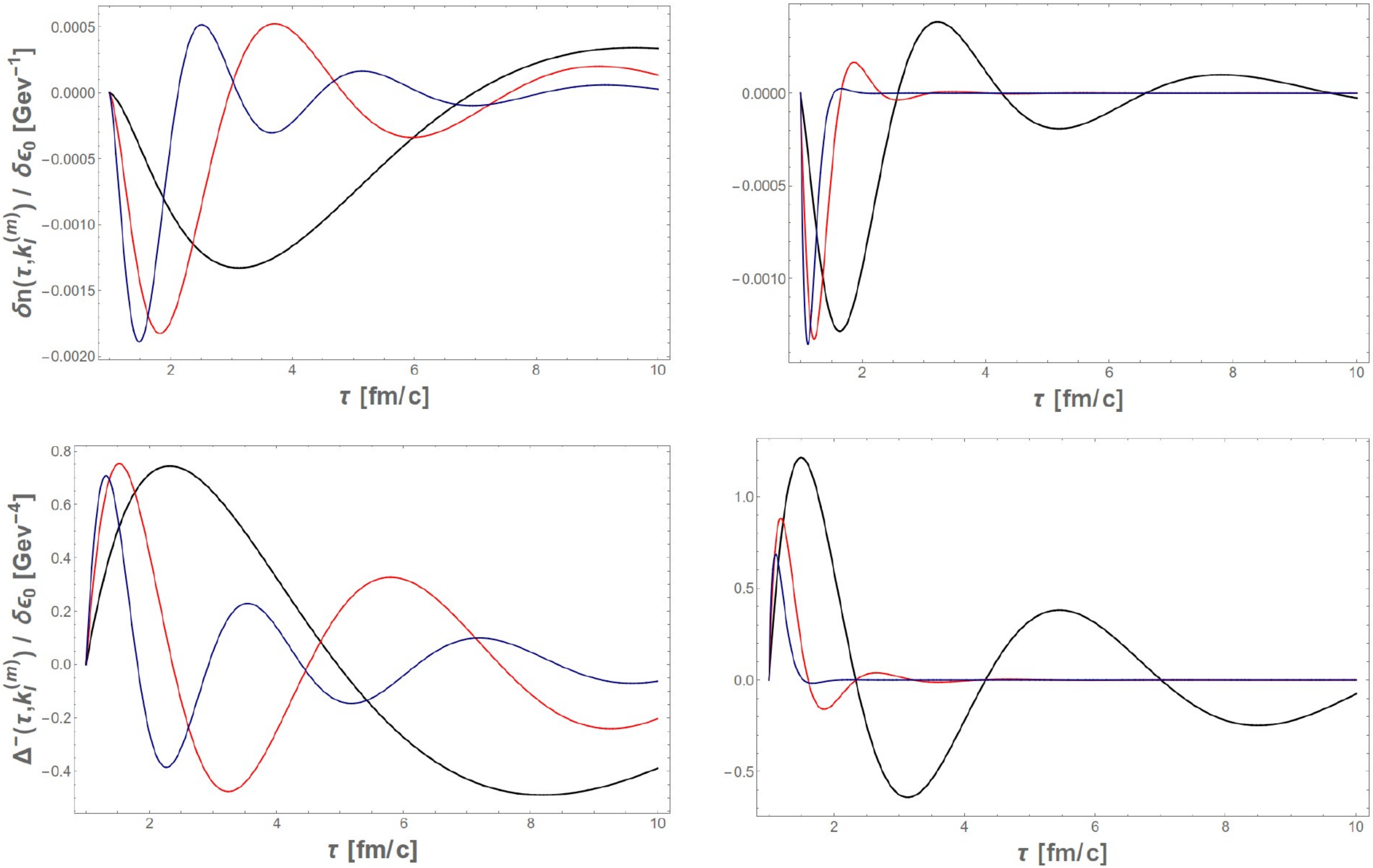


- The oscillation frequency depends on the wave number k
- As one increases η/s the amplitude of the perturbation decrease

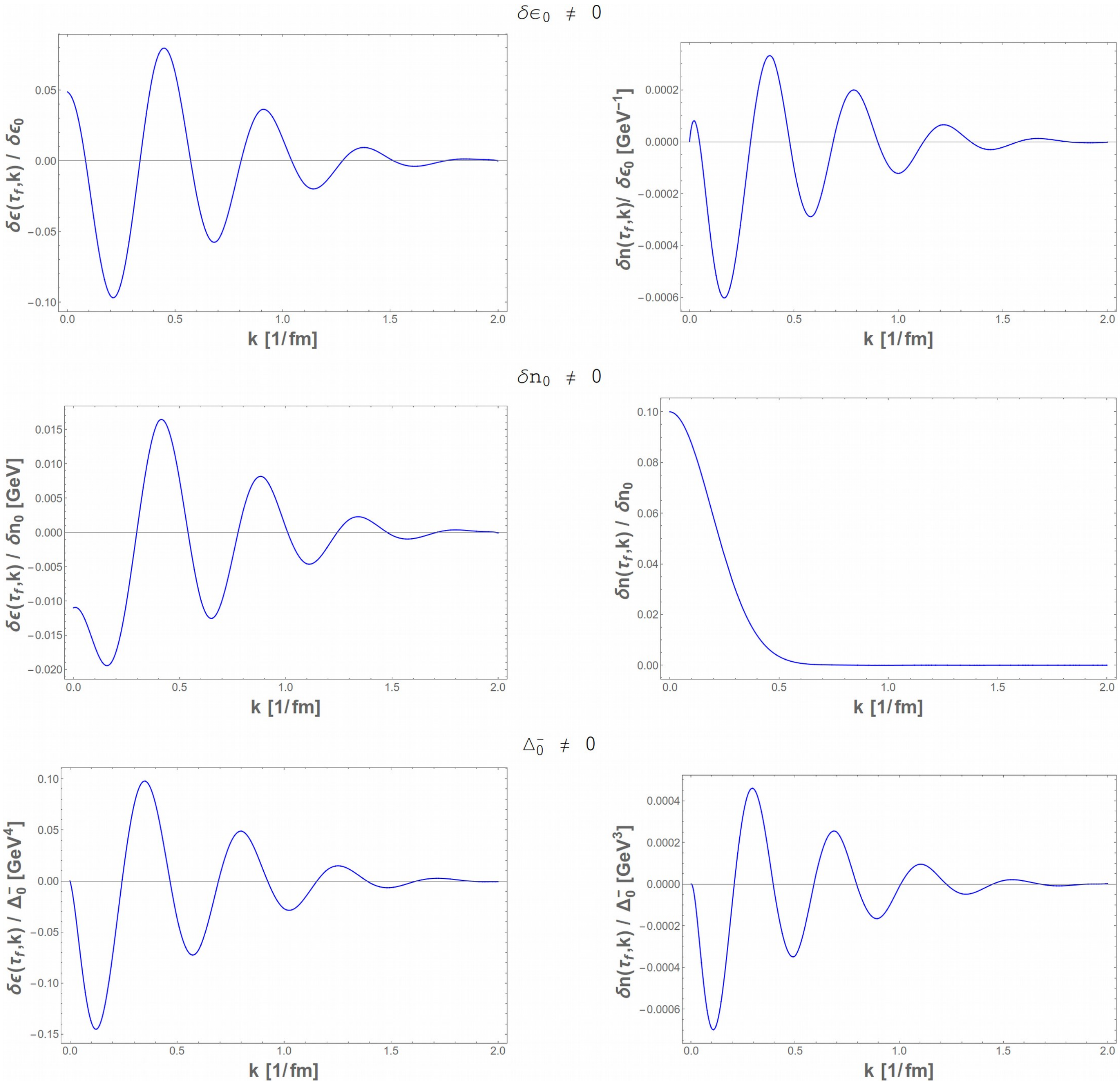
Exact boost symmetry (q=0)



A sound wave type initial condition induces perturbation in the other fluctuating fields due to mode by mode coupling and the non zero background-fluctuating coupling

Exact boost symmetry (q=0)

$$\frac{\eta}{s} = \frac{1}{4\pi}$$



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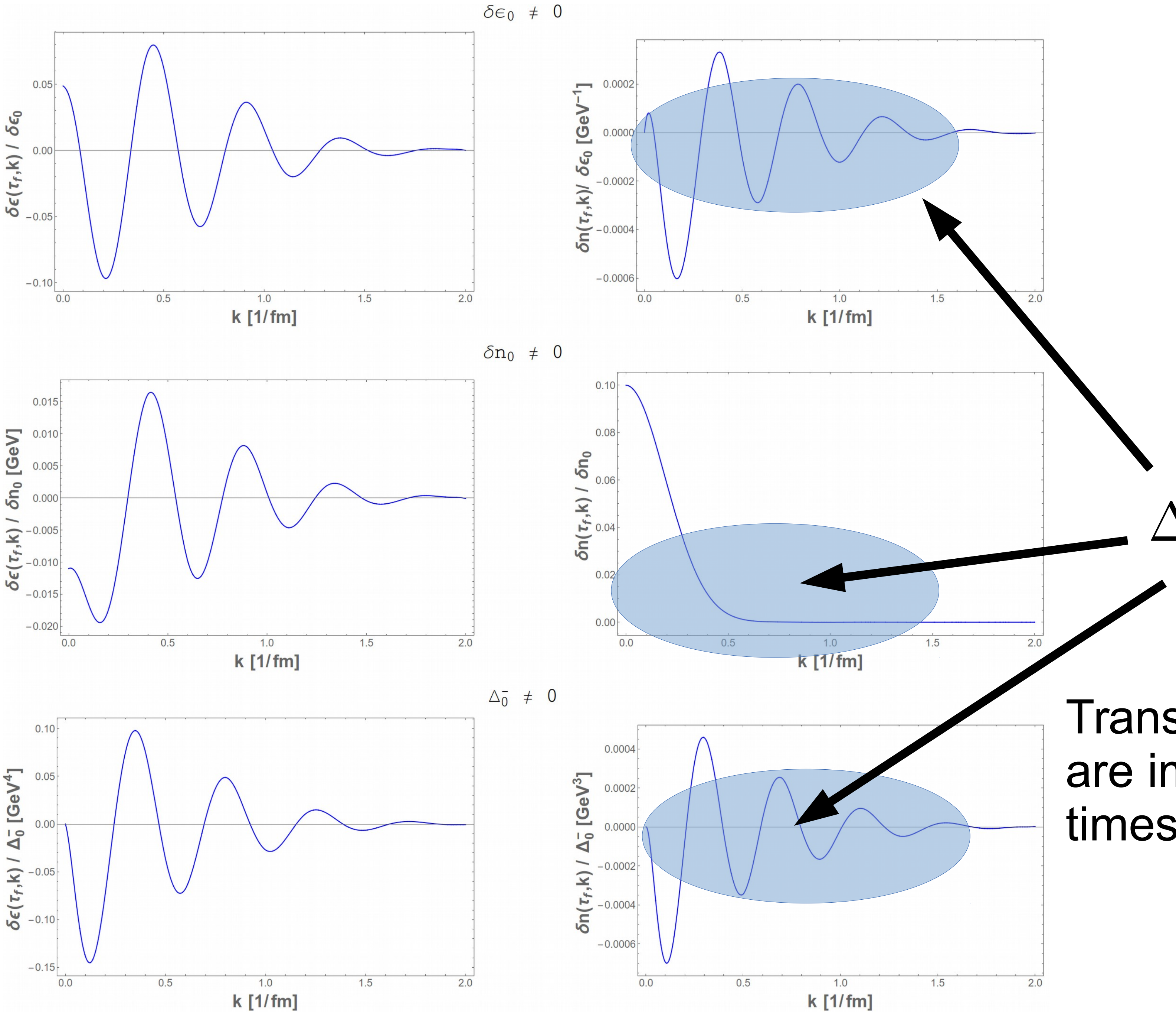


$$\Delta r \sim (\Delta k)^{-1}$$

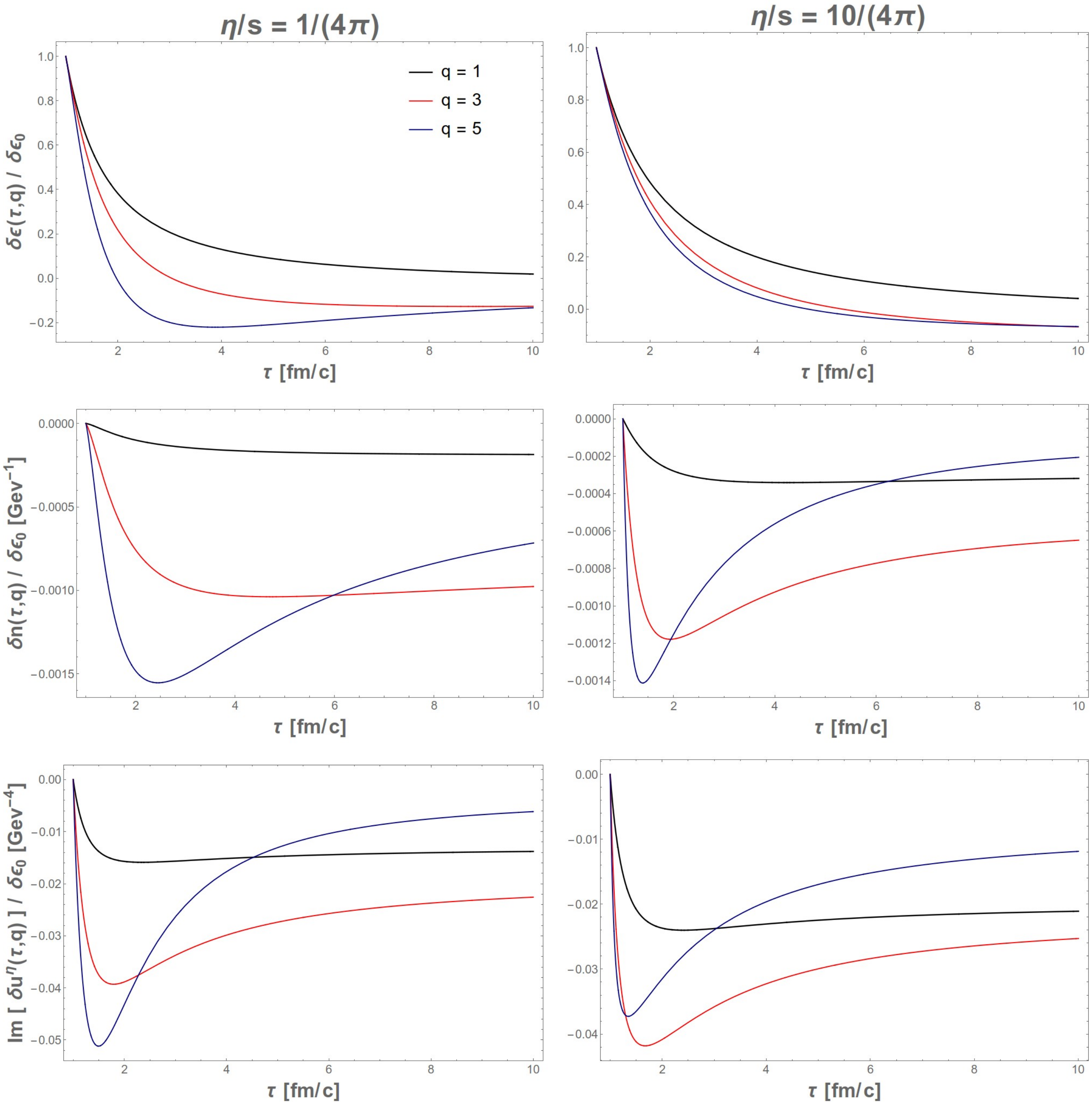
Transverse modes are important at late times

Propagation of longitudinal modes

When the system has exact transverse translation and rotation symmetry (k=0), the dynamics of the system depends only on $\delta\epsilon$, δn and u^η

$$\partial_\tau \delta\epsilon + \left[\frac{1}{\tau} + \frac{1}{\tau}\left(\frac{\partial p}{\partial \epsilon}\right)_n - \frac{1}{\tau^2}\left(\frac{\partial \zeta}{\partial \epsilon}\right)_n - \frac{4}{3\tau^2}\left(\frac{\partial \eta}{\partial \epsilon}\right)_n\right]\delta\epsilon$$
$$+ \left[\frac{1}{\tau}\left(\frac{\partial p}{\partial n}\right)_\epsilon - \frac{1}{\tau^2}\left(\frac{\partial \zeta}{\partial n}\right)_\epsilon - \frac{4}{3\tau^2}\left(\frac{\partial \eta}{\partial n}\right)_\epsilon\right]\delta n + \left[\bar\epsilon + \bar p - \frac{2}{\tau}\bar\zeta - \frac{8}{3\tau}\bar\eta\right] iq\, \delta u^\eta = 0,$$

Propagation of longitudinal modes



Propagation of longitudinal modes

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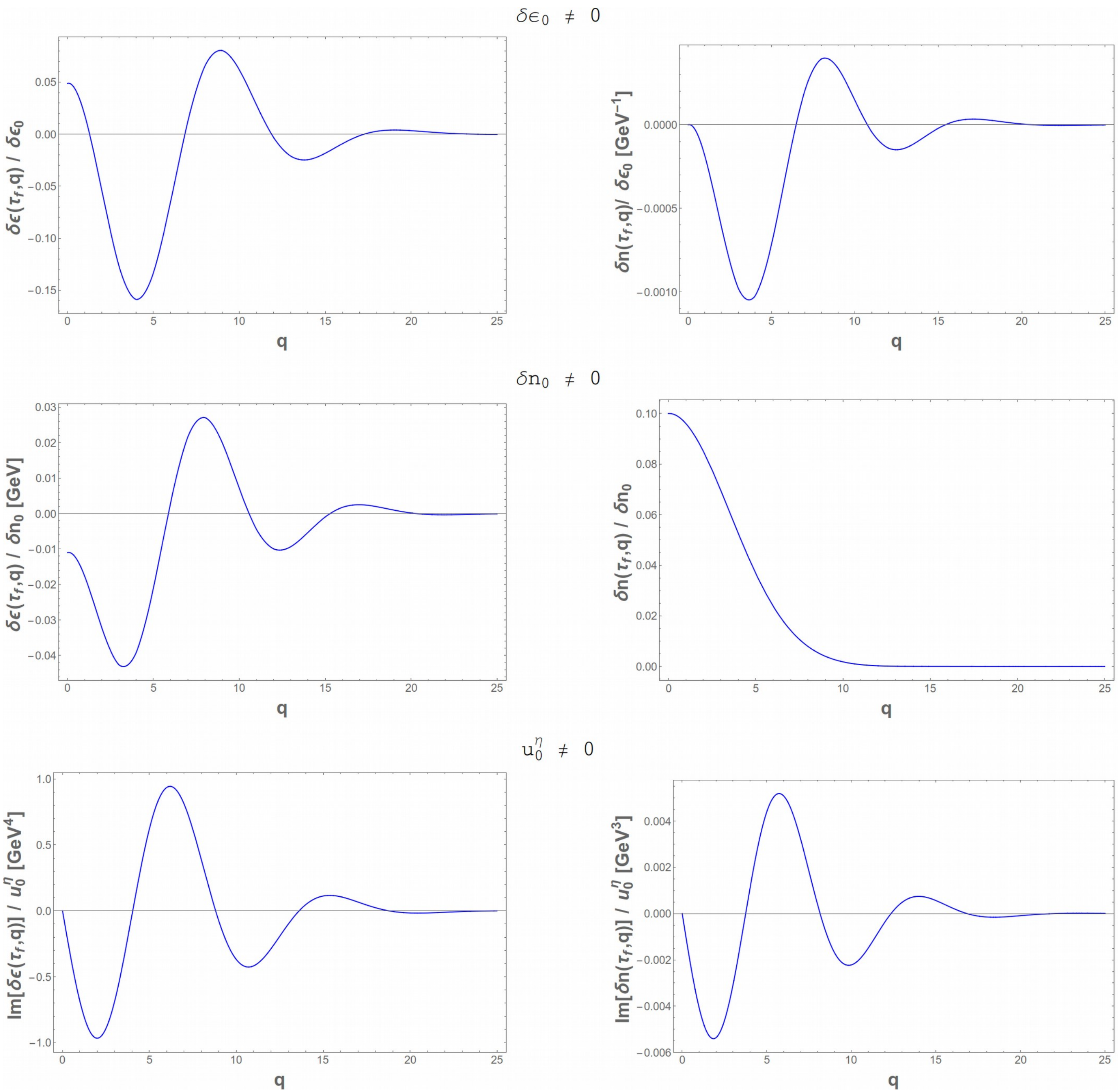

Propagation of longitudinal modes

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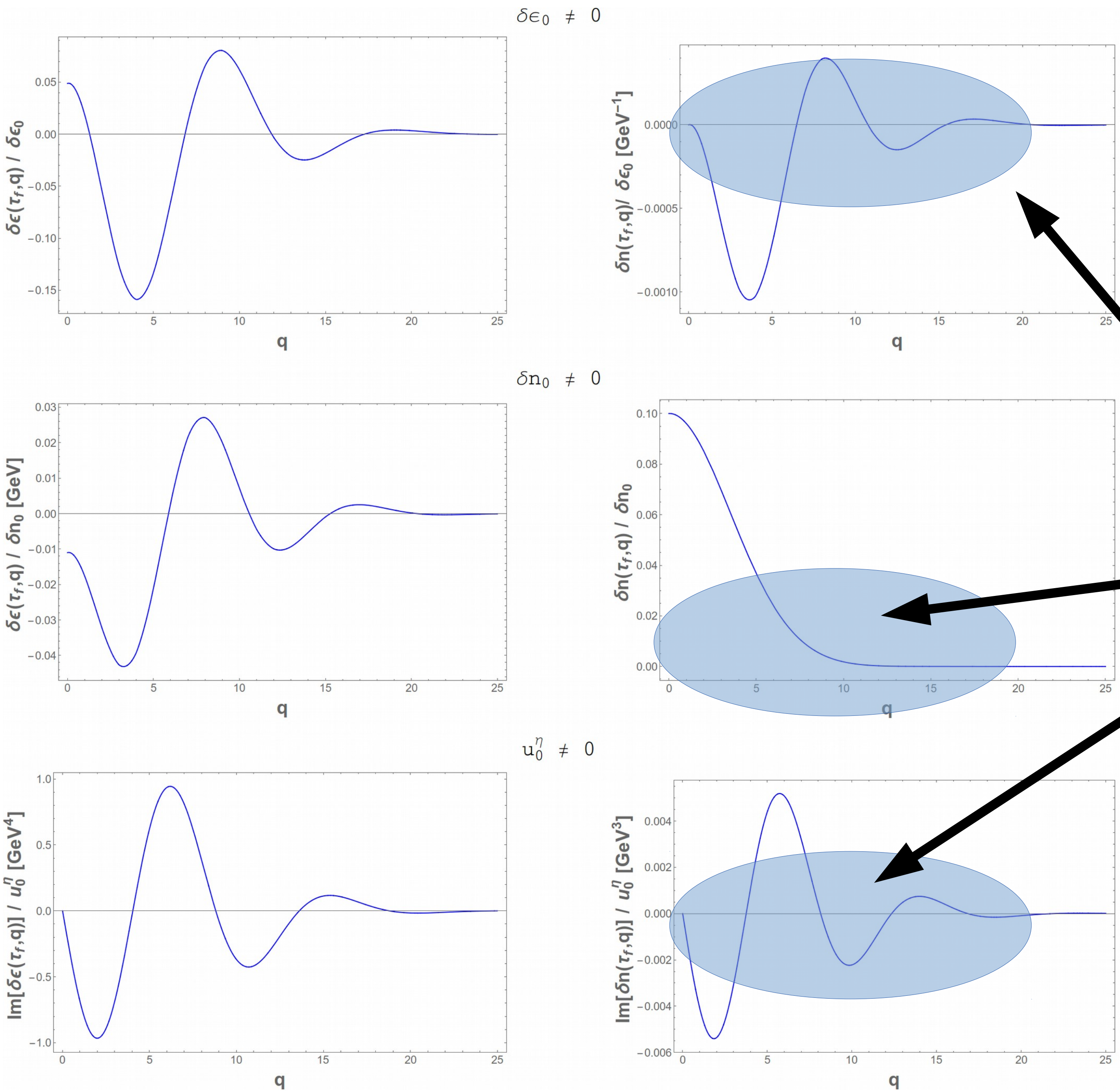

$$\Delta\eta \sim (\Delta q)^{-1}$$

Longitudinal modes are important at early times

The two point correlation function for baryonic particles

Defining the correlation function

In the case of small values of the baryon density
$\Rrightarrow$ the equation of motion for the perturbation of the baryon density decouples

$$\delta n(\tau,k,m,q) = \left(\frac{\tau_0}{\tau}\right) \exp\left[-k^2 I_1(\tau,\tau_0) - q^2 I_2(\tau,\tau_0)\right] \delta n(\tau_0,k,m,q),$$

$$I_1(\tau,\tau_0) = \int_{\tau_0}^{\tau} d\tau' \, \bar{\kappa} \left[\frac{\bar{n}\bar{T}}{\bar{\epsilon}+\bar{p}}\right]^2 \left(\frac{\partial(\mu/T)}{\partial n}\right)_{\epsilon},$$

$$I_2(\tau,\tau_0) = \int_{\tau_0}^{\tau} d\tau' \, \frac{1}{\tau'^2} \, \bar{\kappa} \left[\frac{\bar{n}\bar{T}}{\bar{\epsilon}+\bar{p}}\right]^2 \left(\frac{\partial(\mu/T)}{\partial n}\right)_{\epsilon},$$

Defining the correlation function

The initial "granularity" of the baryon density can be written as

$$\delta n(\tau_0, r, \phi, \eta) = \sum_{m=-\infty}^{\infty} \sum_{l=1}^{\infty} \int \frac{dq}{2\pi} \, \delta n_l^{(m)}(q) \, e^{im\phi + iq\eta} J_m\left(z_l^{(m)} \rho(r)\right)$$

An event by event ensemble is characterized by the weights $\delta n_l^{(m)}(q)$. They can be correlated

$$\langle \delta n_{l_1}^{(m_1)}(q_1) \, \delta n_{l_2}^{(m_2)}(q_2) \rangle = 2\pi \delta(q_1 + q_2) \delta_{m_1+m_2,0} \, C^{(m)}_{\delta n \delta n; l_1, l_2}(q).$$

Defining the correlation function

$$C_{\text{Baryon}}(\phi_1-\phi_2,\eta_1-\eta_2)=\langle n_{\text{Baryons}}(\phi_1,\eta_1)n_{\text{Baryons}}(\phi_2,\eta_2)\rangle_c,$$

In the Fourier space

$$C_{\text{Baryon}}(\phi_1-\phi_2,\eta_1-\eta_2)=\sum_{m=-\infty}^{\infty}\int\frac{dq}{2\pi}\,\tilde{C}_{\text{Baryon}}(m,q)\,e^{im(\phi_1-\phi_2)+iq(\eta_1-\eta_2)}.$$

At the freeze out the baryon number distribution is proportional to the weights within the linear response

$$n^{(m)}_{\text{Baryons}}(q)=\sum_{l}S_{\text{Baryons};(m)l}(q)\delta n^{(m)}_{l}(q)$$

$$\Rightarrow\ \tilde{C}_{\text{Baryon}}(m,q)=\sum_{l_1,l_2=1}^{\infty}S_{\text{Baryon};(m)l_1}(q)\,S_{\text{Baryon};(-m)l_2}(-q)\,C^{(m)}_{\delta n\delta n;l_1,l_2}(q).$$

Defining the correlation function

$$\tilde{C}_{\text{Baryon}}(m,q) = \sum_{l_1,l_2=1}^{\infty} S_{\text{Baryon};(m)l_1}(q)\, S_{\text{Baryon};(-m)l_2}(-q)\, C^{(m)}_{\delta n \delta n; l_1, l_2}(q).$$

$$\tilde{C}_{\text{Baryon}}(m,q) \approx \exp(-2m^2 I_1' - 2q^2 I_2')\tilde{C}_{\text{Baryon}}(m,q)$$

$$I_1' \approx \int_{\tau_0}^{\tau_f} d\tau \, \frac{1}{R^2} \, \bar{\kappa} \left[\frac{\bar{n}\bar{T}}{\bar{\epsilon}+\bar{p}}\right]^2 \left(\frac{\partial(\mu/T)}{\partial n}\right)_{\epsilon},$$

→ Relevant at late times

$$I_2' \approx \int_{\tau_0}^{\tau_f} d\tau \, \frac{1}{\tau^2} \, \bar{\kappa} \left[\frac{\bar{n}\bar{T}}{\bar{\epsilon}+\bar{p}}\right]^2 \left(\frac{\partial(\mu/T)}{\partial n}\right)_{\epsilon}.$$

→ Relevant at early times

Baryon ridge?

Conclusions

- We study the background solutions of the fluid dynamical equations for a globally conserved baryon number.
- We derive and solve the evolution equations for perturbations around the background solution.
- There are characteristic differences in the dependencies of the perturbations on longitudinal and transverse modes.
- Information of baryon number perturbations is accessible via two point correlation function of baryonic particles as a function of the difference of azimuthal angles and rapidities.

Outlook and Perspectives

- A more realistic transverse expansion is needed to study the implications of baryon number fluctuations at the freeze-out surface.
- We need a better description of the initial of the event by event fluctuations in baryon number density at the initial time when hydrodynamics becomes valid.
- How to generate initial baryon number fluctuations?

  1. URQMD initial conditions (Huovinen et. al.)

  2. Modified Monte Carlo Glauber ???

  3. CGC? Quark correlations might provide a source for the initial baryon number fluctuations. Work in progress with M. Sievert and D. Wertepny
- Study non linear evolution of the perturbations