

# COLLECTIVE BEHAVIOR IN SMALL COLLISION SYSTEMS

Matthew Luzum

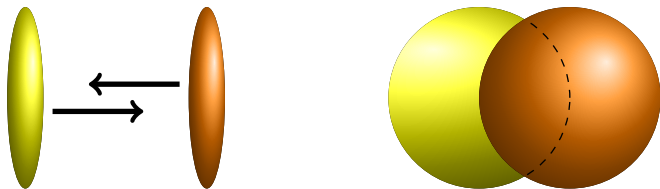
I. Kozlov, ML, G. Denicol, S. Jeon, C. Gale; arXiv:1405.3976

Universidade de Santiago de Compostela

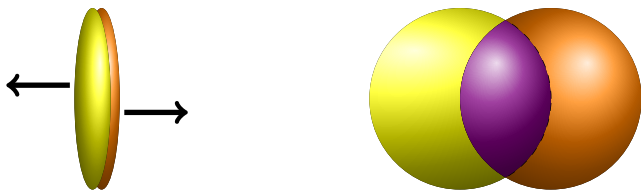
July 15, 2015

- 1 Introduction
  - Perfect fluidity in nucleus-nucleus collisions
  - Similar observation in proton-nucleus collisions
- 2 Hydrodynamic calculations
  - Comparison to existing data
- 3 Proposal for new measurement  $r_n$ 
  - Transverse momentum structure of pair correlation
  - Predictions
  - Comparison to subsequent measurements

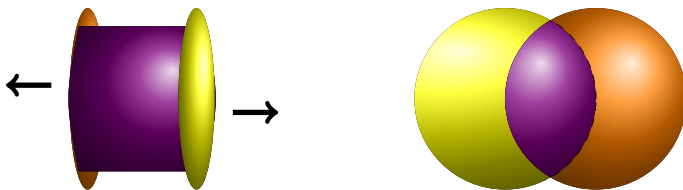
# WHAT WE SEE IN A-A COLLISIONS: PAIR CORRELATION



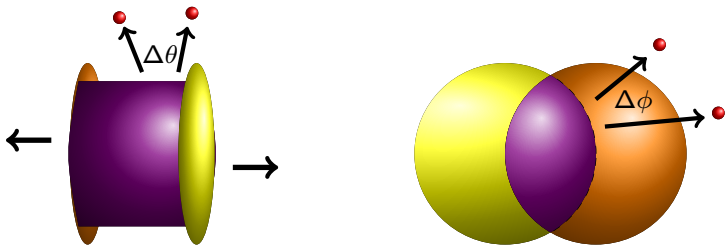
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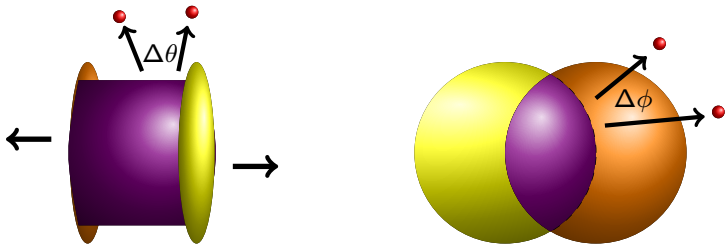
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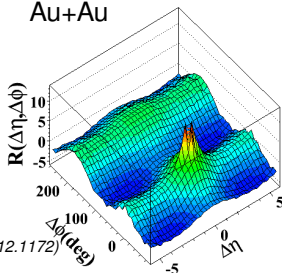


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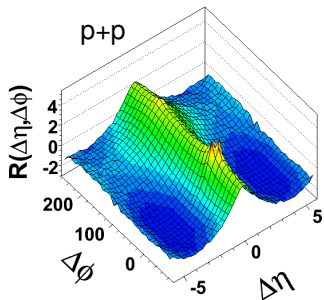
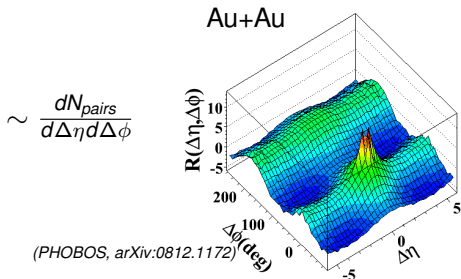
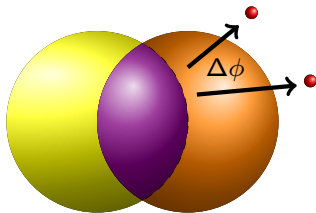
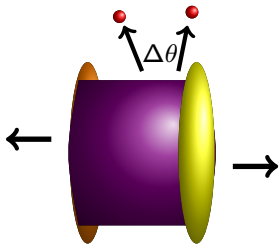
Au+Au

$$\sim \frac{dN_{pairs}}{d\Delta\eta d\Delta\phi}$$



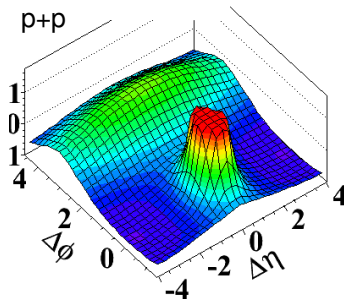
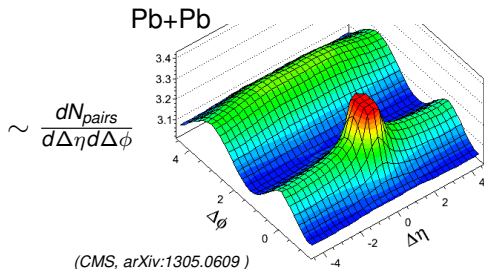
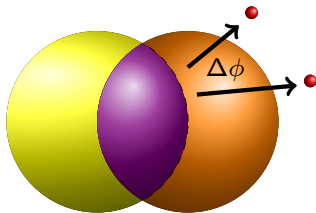
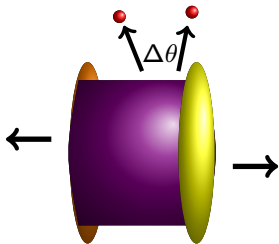
(PHOBOS, arXiv:0812.1172)

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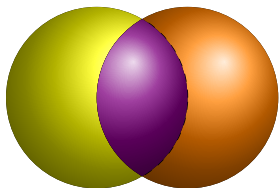


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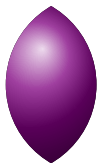
# INTERPRETATION: FLOW

- Hydro picture: system thermalizes and expands as fluid
- Particles emitted independently at end of evolution



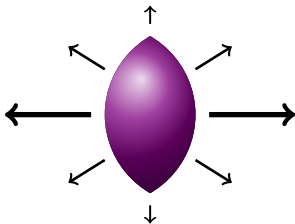
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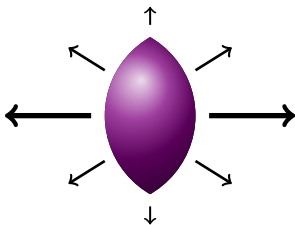
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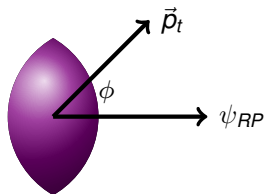
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# INTERPRETATION: FLOW

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$$\frac{2\pi}{N} \frac{dN}{d\phi} \simeq 1 + 2v_2 \cos 2\phi$$



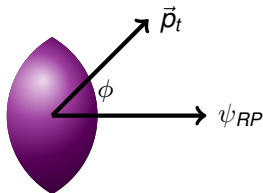
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- $\implies$  pair distribution is product of single-particle distributions

$$\frac{dN_{\text{pairs}}}{d^3p^a d^3p^b} = \frac{dN}{d^3p^a} \times \frac{dN}{d^3p^b}$$



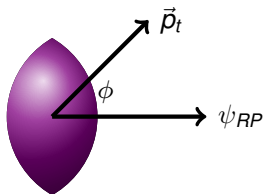
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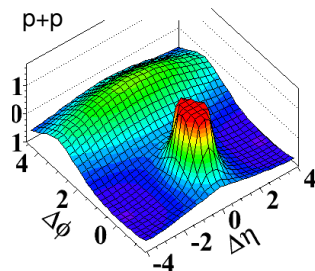
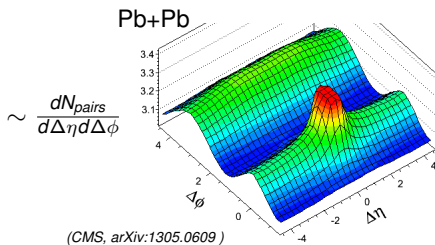
- $\implies$  pair distribution is product of single-particle distributions

$$\frac{dN_{\text{pairs}}}{d\Delta\phi} \stackrel{(\text{flow})}{\propto} 1 + 2v_2^2 \cos 2(\Delta\phi)$$

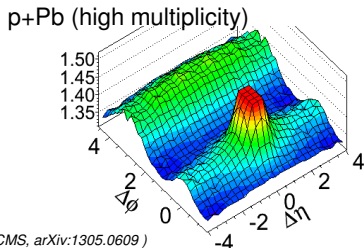
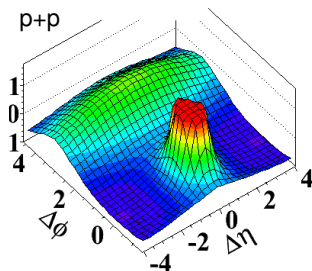
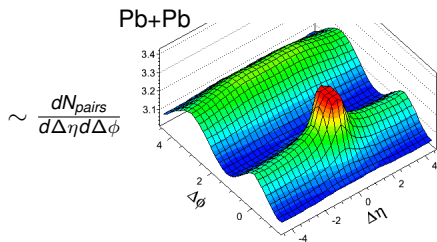




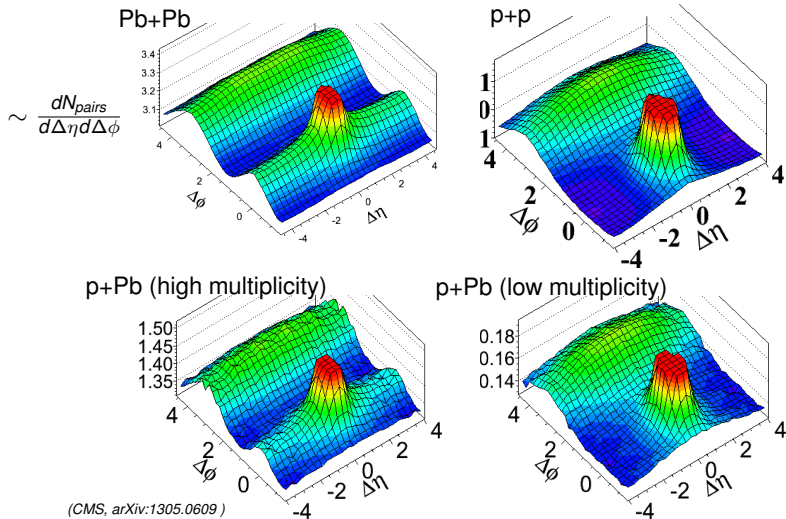
# FLOW-LIKE CORRELATIONS IN P-A COLLISIONS



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# FLOW IN HEAVY-ION COLLISIONS

- Validity of hydrodynamics requires separation of scales
- $\implies$  should break down when system size becomes too small

## QUESTIONS

- Do the observed pA correlations indicate collective behavior?
- Can proton-nucleus collision behave as a fluid?

# FLOW IN HEAVY-ION COLLISIONS

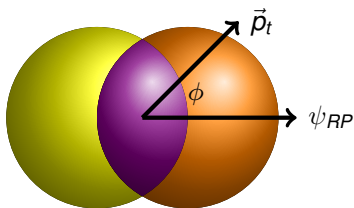
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MEASURING  $v_n$ 

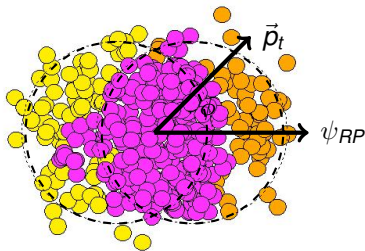
$$\frac{2\pi}{N} \frac{dN}{d\phi} \simeq 1 + 2v_2 \cos 2\phi$$



# MEASURING $v_n$

Fluctuations are important!

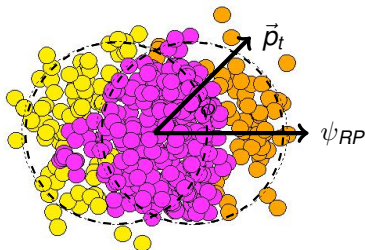
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# MEASURING $v_n$

Fluctuations are important!

$$\frac{2\pi}{N} \frac{dN}{d\phi} = 1 + \sum_{n=1}^{\infty} 2v_n \cos n(\phi - \psi_n)$$



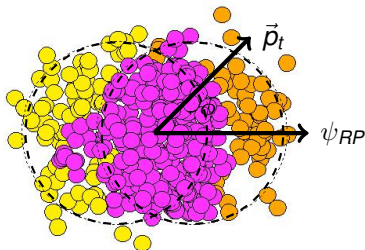


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$$\left\langle \left\langle e^{in(\phi_1 - \phi_2)} \right\rangle_{\text{pairs}} \right\rangle = v_n \{2\}^2$$

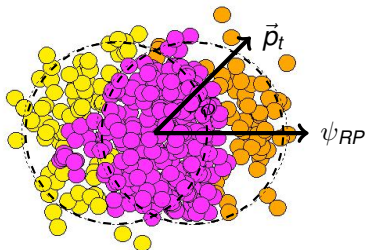


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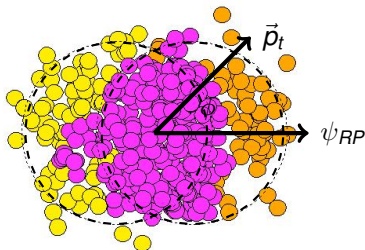
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$$\left\langle \left\langle e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \right\rangle_{\text{quad.}} \right\rangle \stackrel{(\text{flow})}{=} \left\langle v_n^4 \right\rangle$$



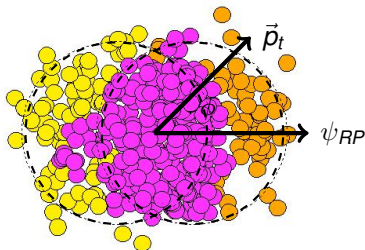
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$$\left\langle \left\langle e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \right\rangle_{\text{quad.}} \right\rangle - 2v_n \{2\}^2 = -v_n \{4\}^4 \stackrel{(\text{flow})}{=} \left\langle v_n^4 \right\rangle - 2 \left\langle v_n^2 \right\rangle^2$$

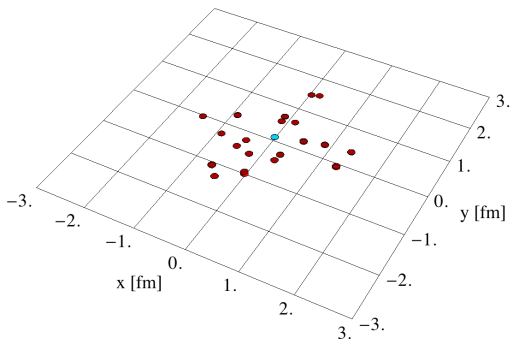


# STRATEGY

- 1 Explore plausibility:
  - Perform hydrodynamic calculations of p-Pb collisions
  - Look for generic trends
  - Is it possible to naturally describe (simultaneously) existing data with a reasonable model?
- 2 Look for “smoking gun”:
  - Try to find more rigorous test of collectivity
  - Can we kill flow as a possible explanation for data?
  - Make predictions (and compare to subsequent measurements)

# DETAILS OF HYDRODYNAMIC CALCULATIONS

Start with traditional Glauber participant model:



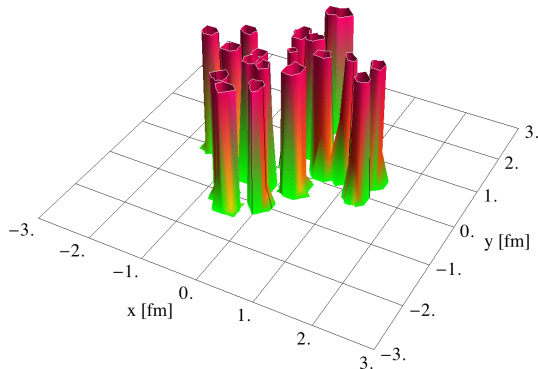
(Animation from Igor Kozlov)

# DETAILS OF HYDRODYNAMIC CALCULATIONS

Start with traditional Glauber participant model:

Each 'participant' contributes entropy as transverse Gaussian

$$\sigma = 0.05 \text{ fm}$$



(Animation from Igor Kozlov)

MATT LUZUM (USC)

FLOW IN PA

07/15/2015

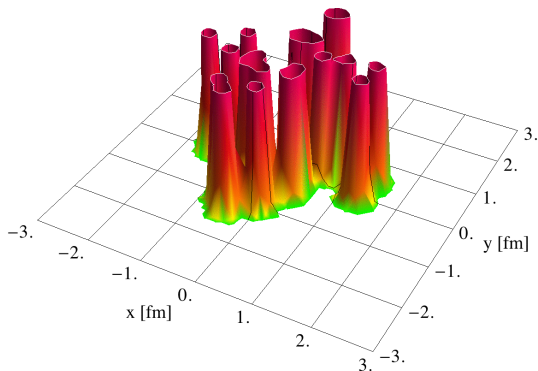
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# DETAILS OF HYDRODYNAMIC CALCULATIONS

Start with traditional Glauber participant model:

Each 'participant' contributes entropy as transverse Gaussian

$$\sigma = 0.10 \text{ fm}$$



(Animation from Igor Kozlov)

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FLOW IN PA

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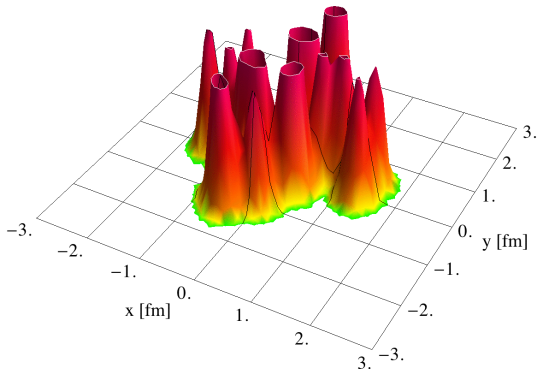


# DETAILS OF HYDRODYNAMIC CALCULATIONS

Start with traditional Glauber participant model:

Each 'participant' contributes entropy as transverse Gaussian

$$\sigma = 0.15 \text{ fm}$$



(Animation from Igor Kozlov)

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FLOW IN PA

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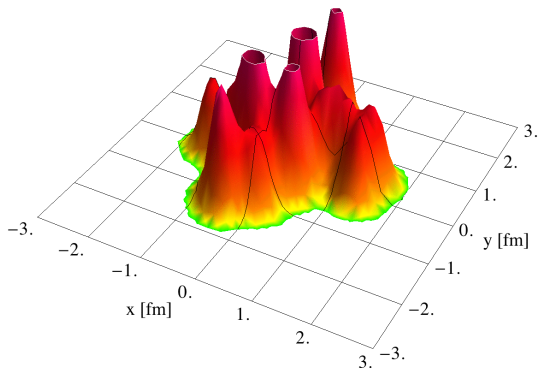
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# DETAILS OF HYDRODYNAMIC CALCULATIONS

Start with traditional Glauber participant model:

Each 'participant' contributes entropy as transverse Gaussian

$$\sigma = 0.20 \text{ fm}$$



(Animation from Igor Kozlov)

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FLOW IN PA

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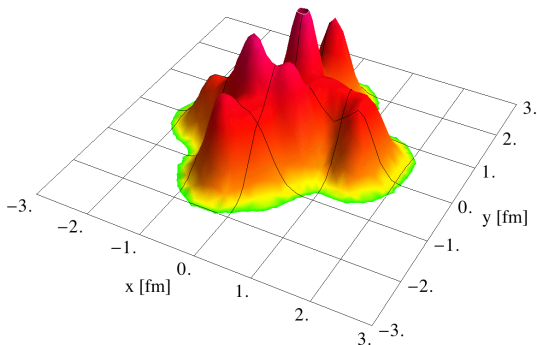
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# DETAILS OF HYDRODYNAMIC CALCULATIONS

Start with traditional Glauber participant model:

Each 'participant' contributes entropy as transverse Gaussian

$$\sigma = 0.25 \text{ fm}$$



(Animation from Igor Kozlov)

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FLOW IN PA

07/15/2015

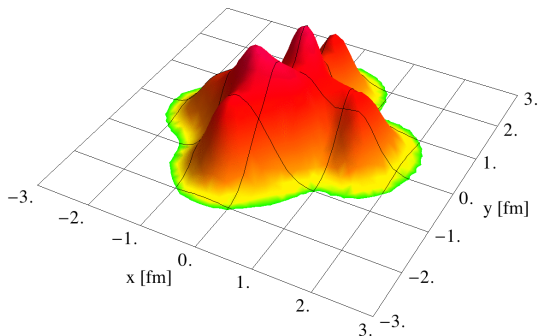
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# DETAILS OF HYDRODYNAMIC CALCULATIONS

Start with traditional Glauber participant model:

Each 'participant' contributes entropy as transverse Gaussian

$$\sigma = 0.30 \text{ fm}$$



(Animation from Igor Kozlov)

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FLOW IN PA

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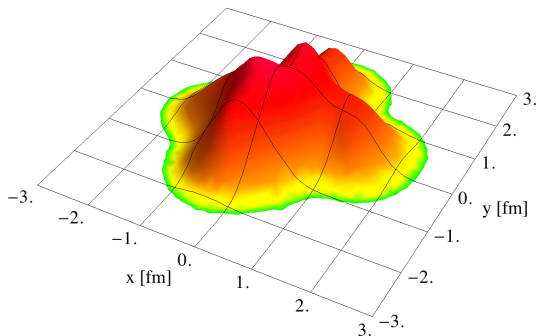
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# DETAILS OF HYDRODYNAMIC CALCULATIONS

Start with traditional Glauber participant model:

Each 'participant' contributes entropy as transverse Gaussian

$$\sigma = 0.35 \text{ fm}$$



(Animation from Igor Kozlov)

MATT LUZUM (USC)

FLOW IN PA

07/15/2015

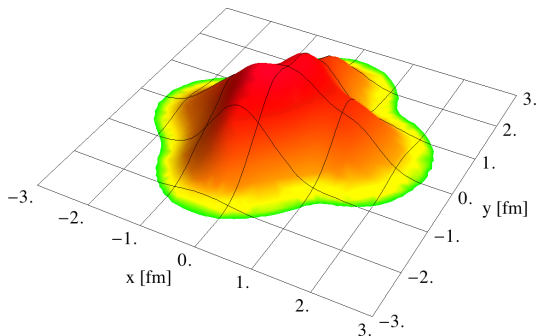
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# DETAILS OF HYDRODYNAMIC CALCULATIONS

Start with traditional Glauber participant model:

Each 'participant' contributes entropy as transverse Gaussian

$$\sigma = 0.40 \text{ fm}$$



(Animation from Igor Kozlov)

MATT LUZUM (USC)

FLOW IN PA

07/15/2015

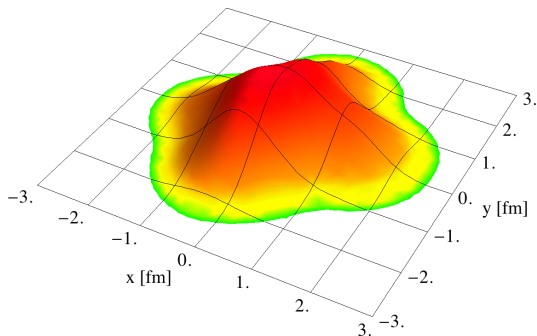
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# DETAILS OF HYDRODYNAMIC CALCULATIONS

Start with traditional Glauber participant model:

Each 'participant' contributes entropy as transverse Gaussian

$$\sigma = 0.45 \text{ fm}$$



(Animation from Igor Kozlov)

MATT LUZUM (USC)

FLOW IN PA

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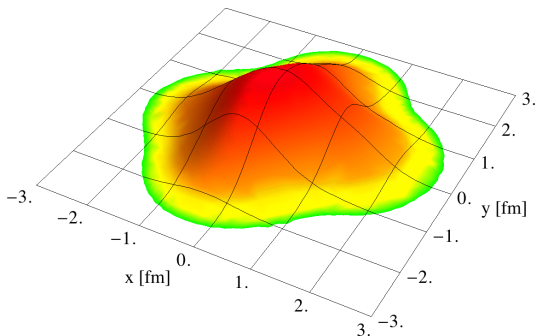
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# DETAILS OF HYDRODYNAMIC CALCULATIONS

Start with traditional Glauber participant model:

Each 'participant' contributes entropy as transverse Gaussian

$$\sigma = 0.50 \text{ fm}$$



(Animation from Igor Kozlov)

MATT LUZUM (USC)

FLOW IN PA

07/15/2015

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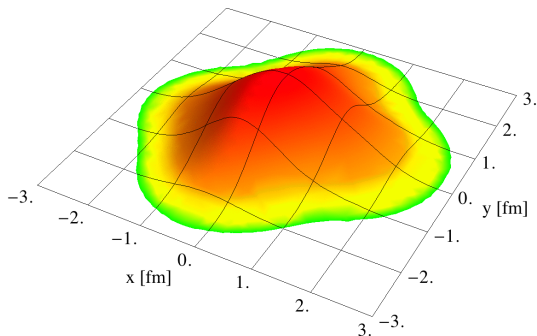


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Start with traditional Glauber participant model:

Each 'participant' contributes entropy as transverse Gaussian

$$\sigma = 0.55 \text{ fm}$$



(Animation from Igor Kozlov)

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FLOW IN PA

07/15/2015

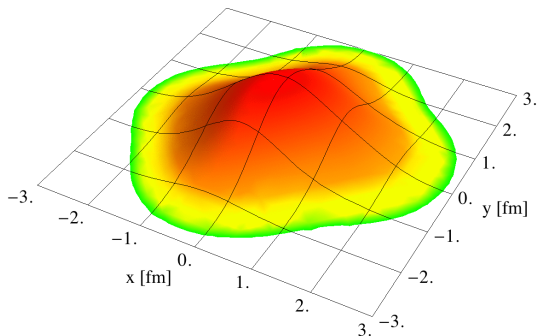
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# DETAILS OF HYDRODYNAMIC CALCULATIONS

Start with traditional Glauber participant model:

Each 'participant' contributes entropy as transverse Gaussian

$$\sigma = 0.60 \text{ fm}$$



(Animation from Igor Kozlov)

MATT LUZUM (USC)

FLOW IN PA

07/15/2015

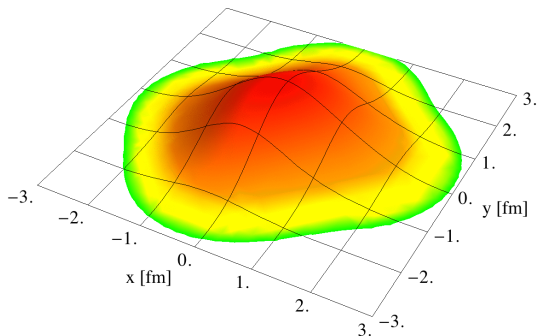
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# DETAILS OF HYDRODYNAMIC CALCULATIONS

Start with traditional Glauber participant model:

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$$\sigma = 0.65 \text{ fm}$$



(Animation from Igor Kozlov)

MATT LUZUM (USC)

FLOW IN PA

07/15/2015

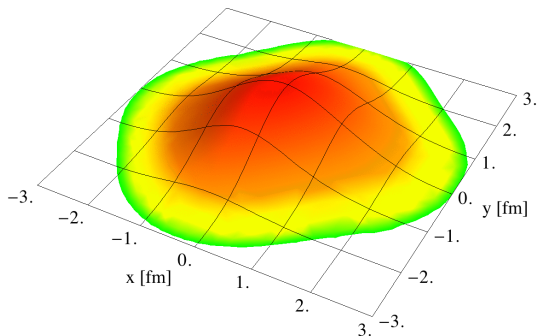
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# DETAILS OF HYDRODYNAMIC CALCULATIONS

Start with traditional Glauber participant model:

Each 'participant' contributes entropy as transverse Gaussian

$$\sigma = 0.70 \text{ fm}$$



(Animation from Igor Kozlov)

MATT LUZUM (USC)

FLOW IN PA

07/15/2015

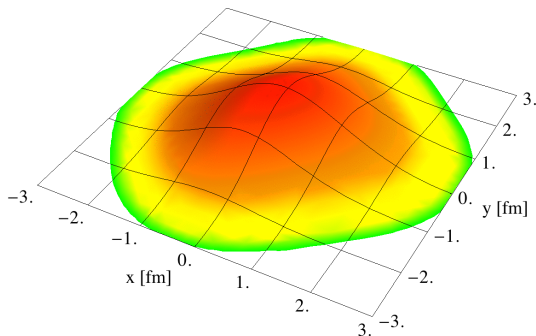
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# DETAILS OF HYDRODYNAMIC CALCULATIONS

Start with traditional Glauber participant model:

Each 'participant' contributes entropy as transverse Gaussian

$$\sigma = 0.75 \text{ fm}$$



(Animation from Igor Kozlov)

MATT LUZUM (USC)

FLOW IN PA

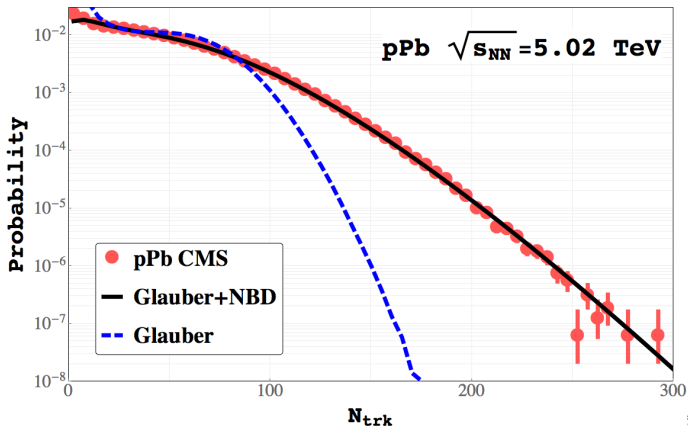
07/15/2015

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# DETAILS OF HYDRODYNAMIC CALCULATIONS

## GLAUBER + NBD

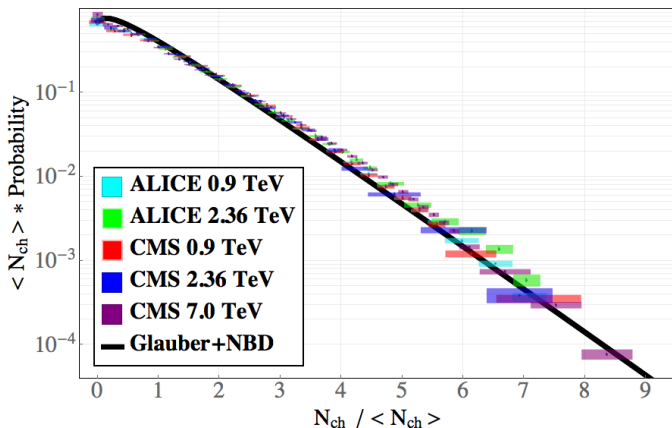
Entropy contribution of each participant chosen according to Negative Binomial Distribution



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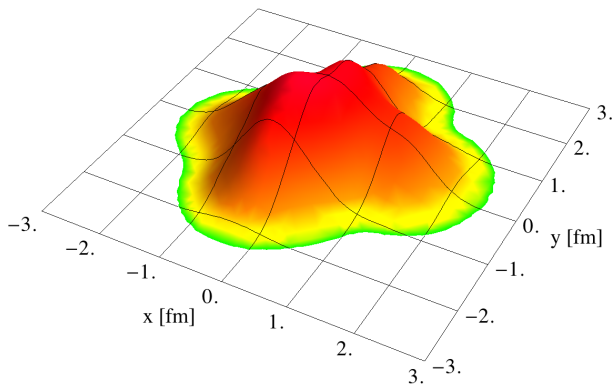
Entropy contribution of each participant chosen according to Negative Binomial Distribution



## DETAILS OF HYDRODYNAMIC CALCULATIONS

Without NBD:

$$\sigma = 0.40 \text{ fm}$$

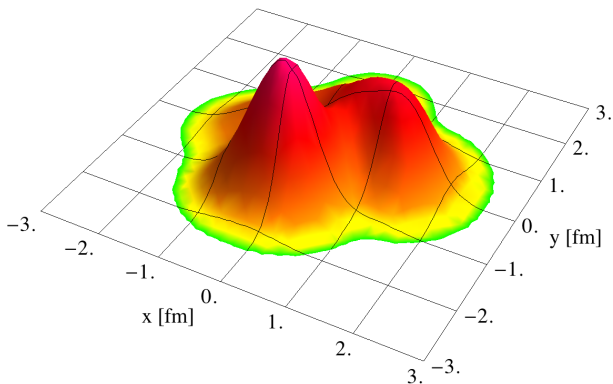




## DETAILS OF HYDRODYNAMIC CALCULATIONS

With NBD:

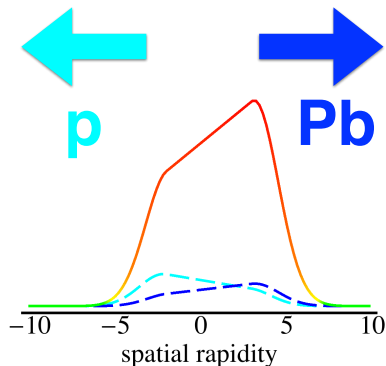
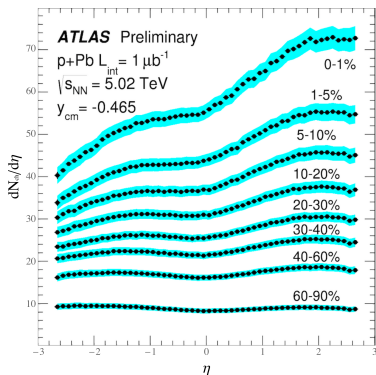
$$\sigma = 0.40 \text{ fm}$$



# DETAILS OF HYDRODYNAMIC CALCULATIONS

## GLAUBER + RAPIDITY DEPENDENCE

Choose asymmetric contribution from each participant



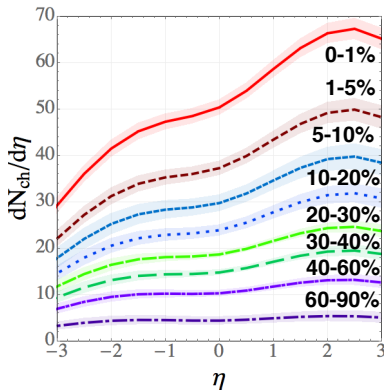
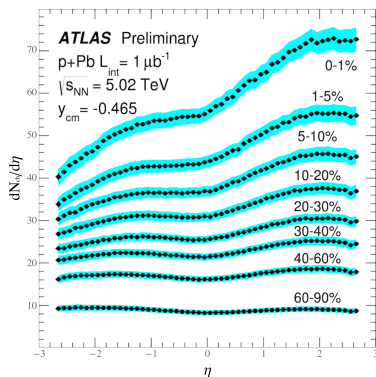
(ATLAS arXiv:1403.5738)

◀ ◻ ▶ ◀ (Bozek, Wyskiel arXiv:1002.4999) ◻ ▶

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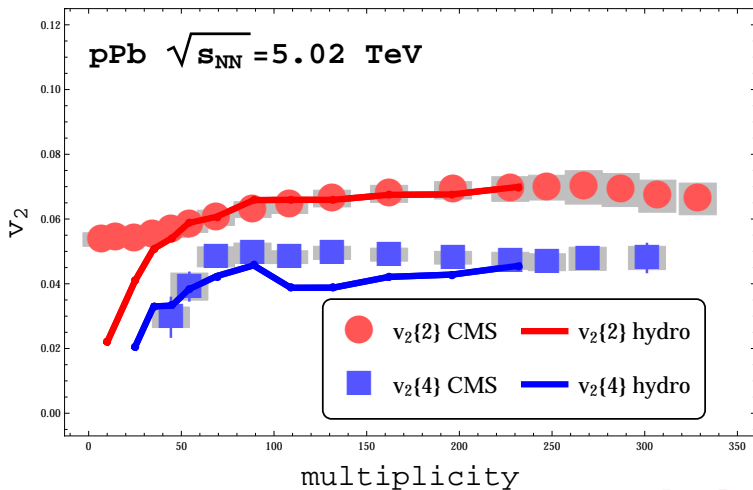


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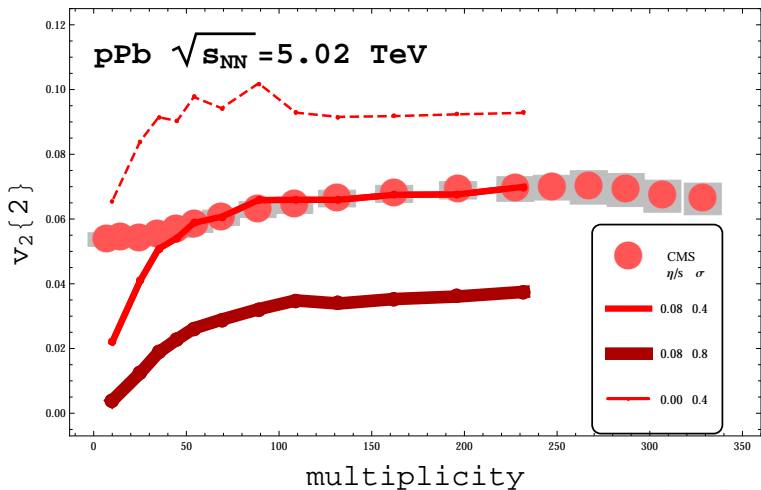
RESULTS:  $v_2$ 

$$\sigma = 0.4 \text{ fm}, \eta/s = 0.08$$



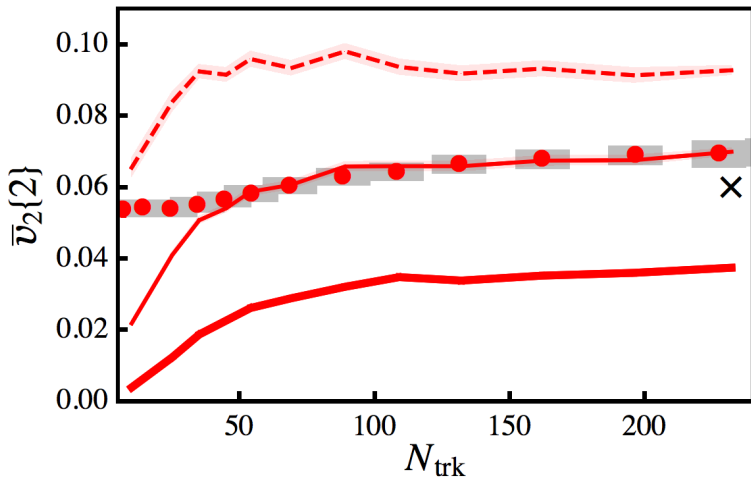
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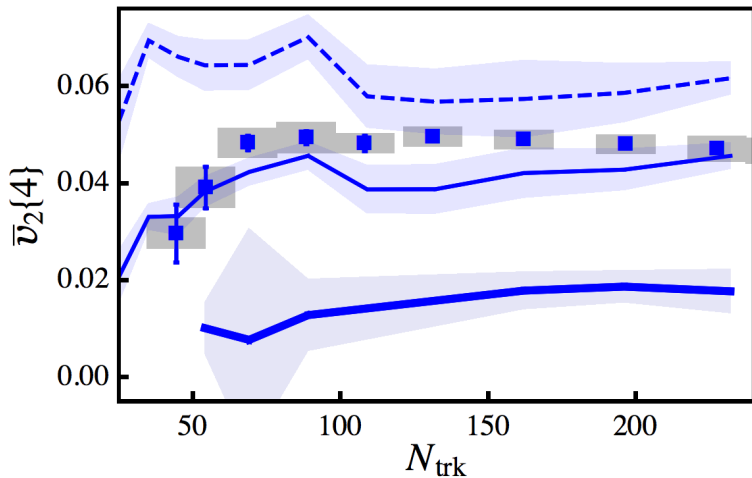
$$\sigma = 0.4\text{--}0.8 \text{ fm}, \eta/s = 0.0\text{--}0.08$$



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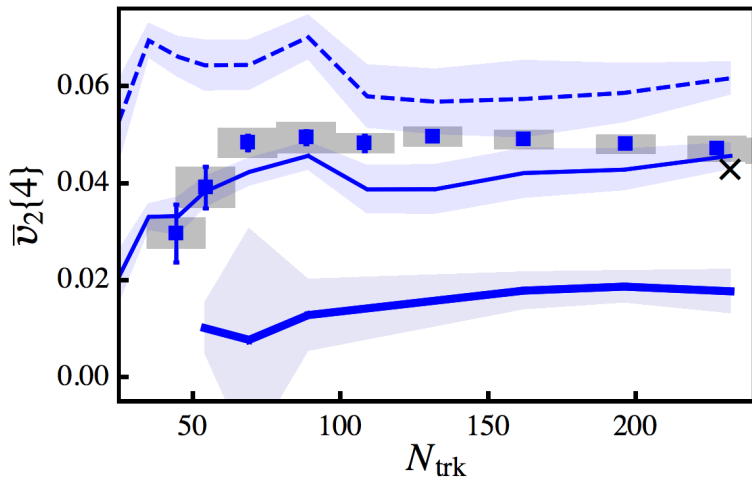
$\sigma = 0.4\text{--}0.8$  fm,  $\eta/s = 0.0\text{--}0.08$ , bulk viscosity



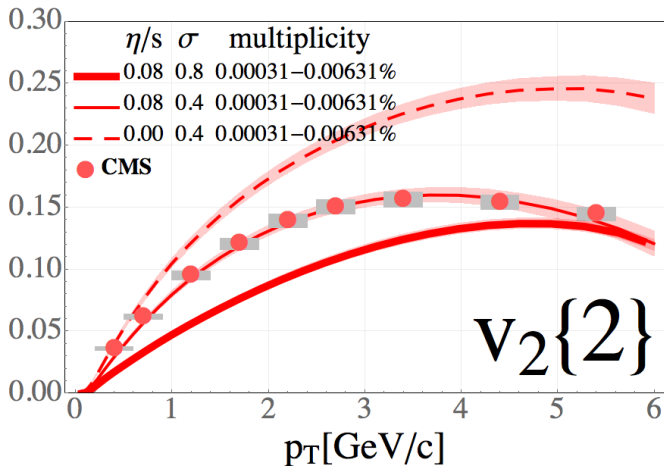
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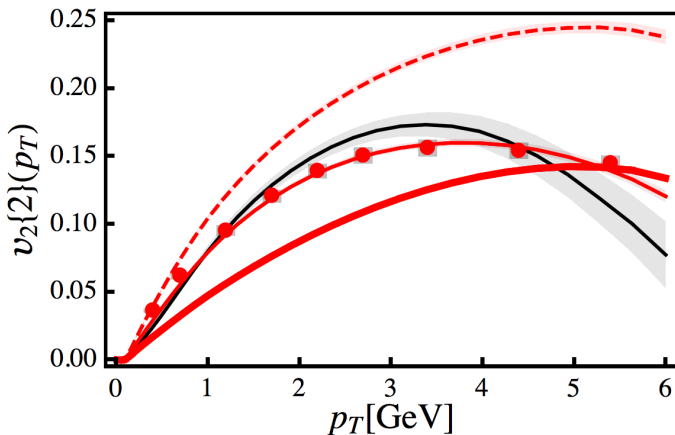
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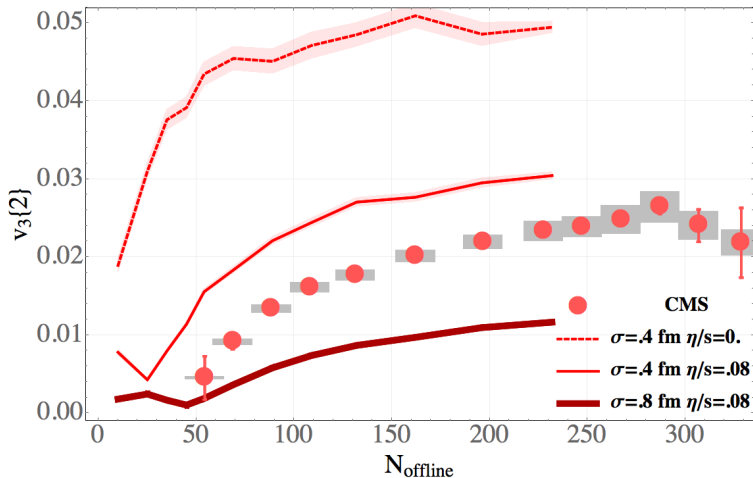
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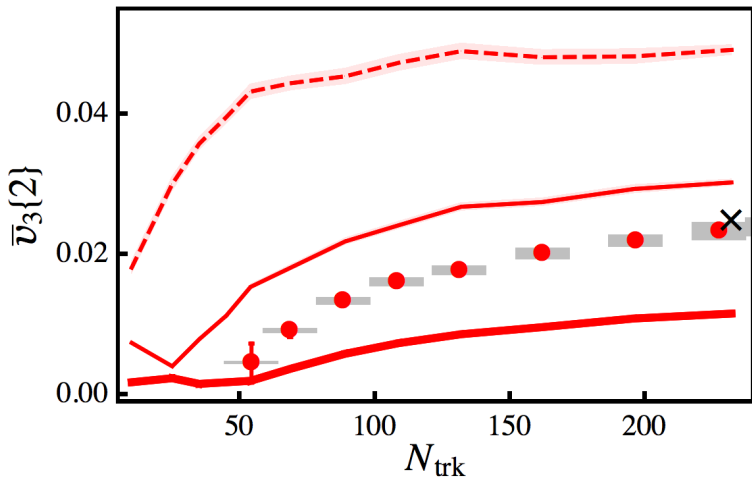
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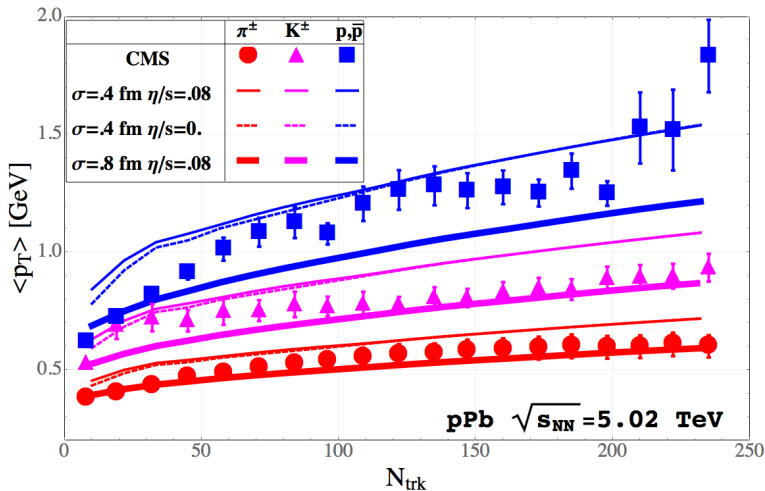
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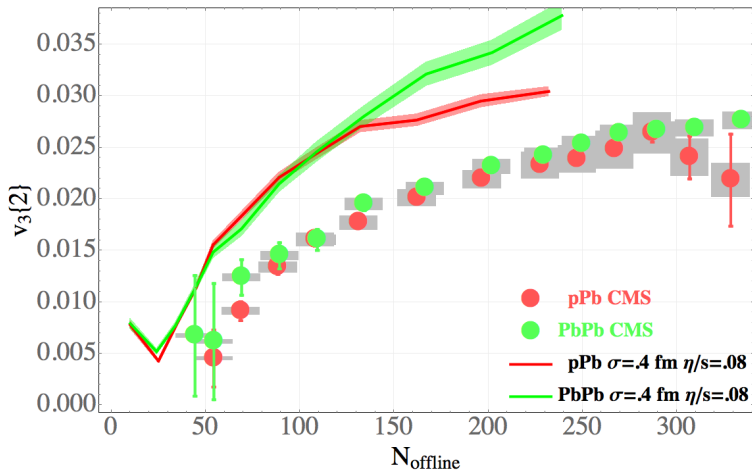
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RESULTS:  $v_3\{2\}$  $v_3\{2\}$  the same in p-Pb and Pb-Pb:

# HOW DO WE UNDERSTAND $v_3$ IN pA VS. AA?

We compare collisions with the same multiplicity.

A naive expectation:

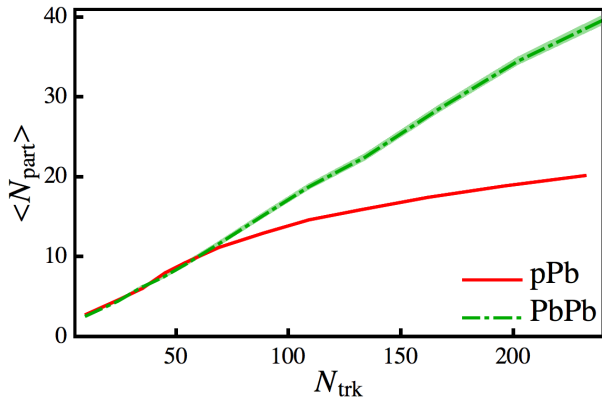
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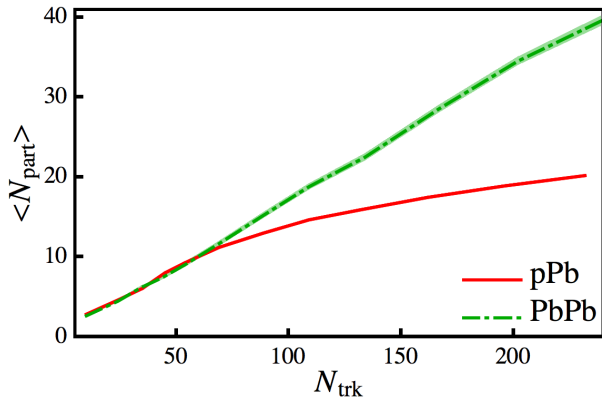


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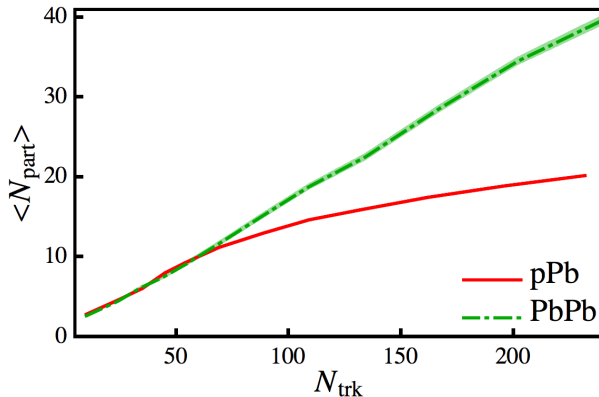


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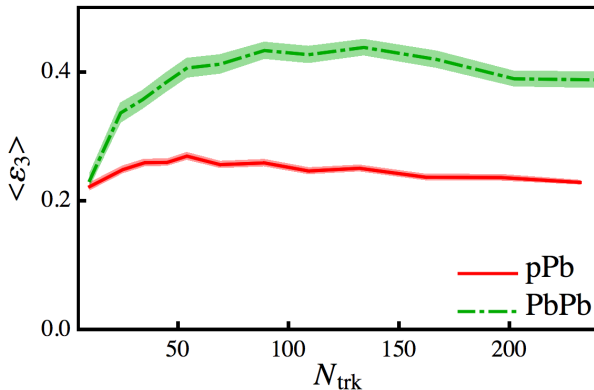


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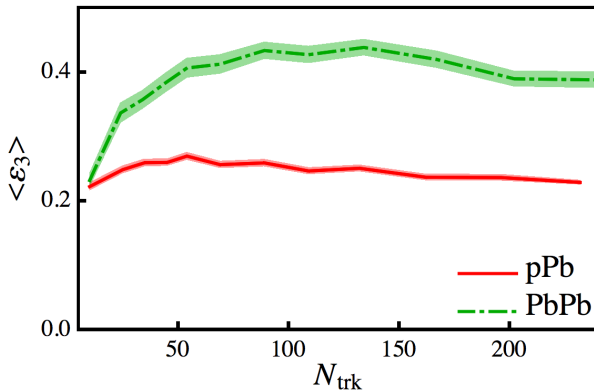


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# MOMENTUM STRUCTURE OF PAIR CORRELATION

Two-particle correlation is a multidimensional matrix:

$$\langle \cos n(\phi^a - \phi^b) \rangle = f(\mathbf{p}_T^a, \eta^a, \mathbf{p}_T^b, \eta^b)$$

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$$\frac{dN_{pairs}}{d^3p^a d^3p^b} = \frac{dN}{d^3p^a} \times \frac{dN}{d^3p^b} + C(p^a, p^b)$$

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# FLOW INEQUALITIES

This expression directly implies a set of inequalities:

$$V_{n\Delta}(p_T^a, p_T^b) \equiv \langle V_n^a V_n^{b*} \rangle = \langle v_n^a v_n^b \cos n(\Psi_n^a - \Psi_n^b) \rangle$$

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- If inequalities broken  $\implies$  unmistakable signal of non-flow
- If first inequality is satisfied, we can define the ratio

$$r_n \equiv \frac{V_{n\Delta}(p_T^a, p_T^b)}{\sqrt{V_{n\Delta}(p_T^a, p_T^a) V_{n\Delta}(p_T^b, p_T^b)}}$$

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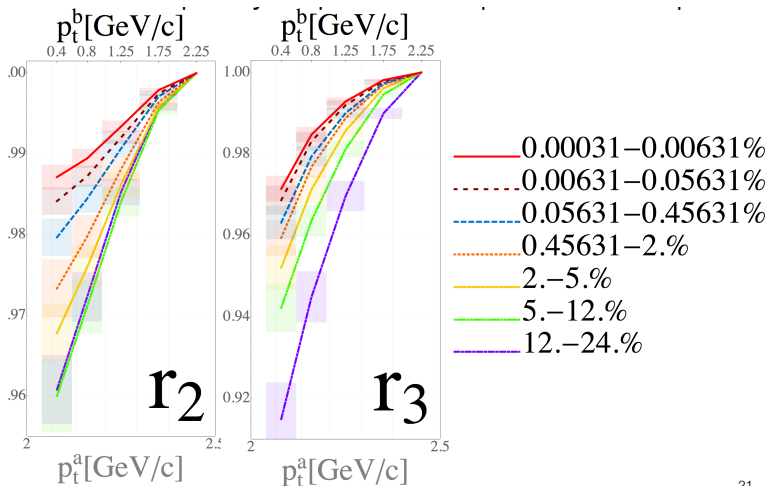
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# PREDICTIONS: CENTRALITY DEPENDENCE

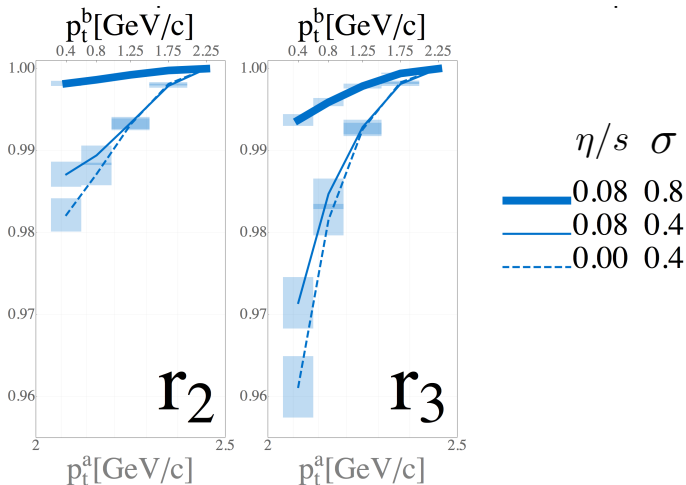
$r_n$  close to 1, closer with increasing multiplicity



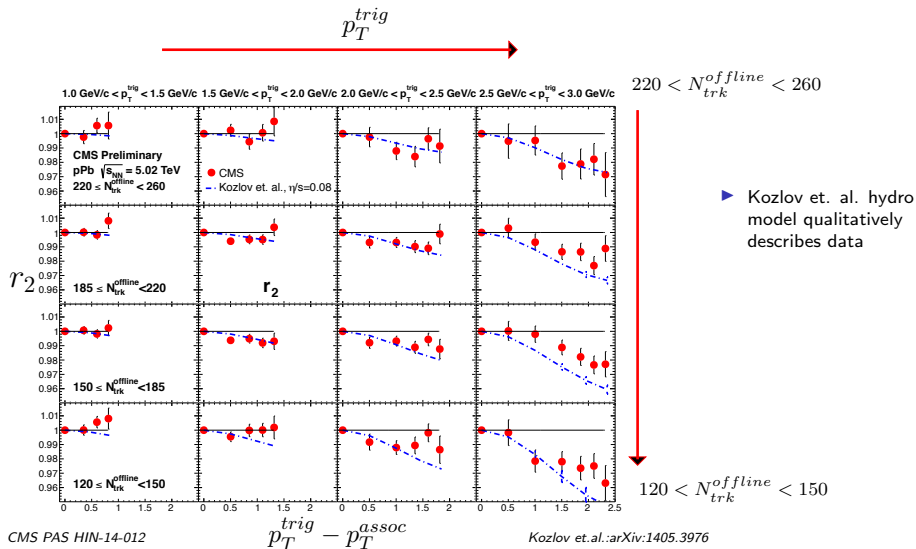
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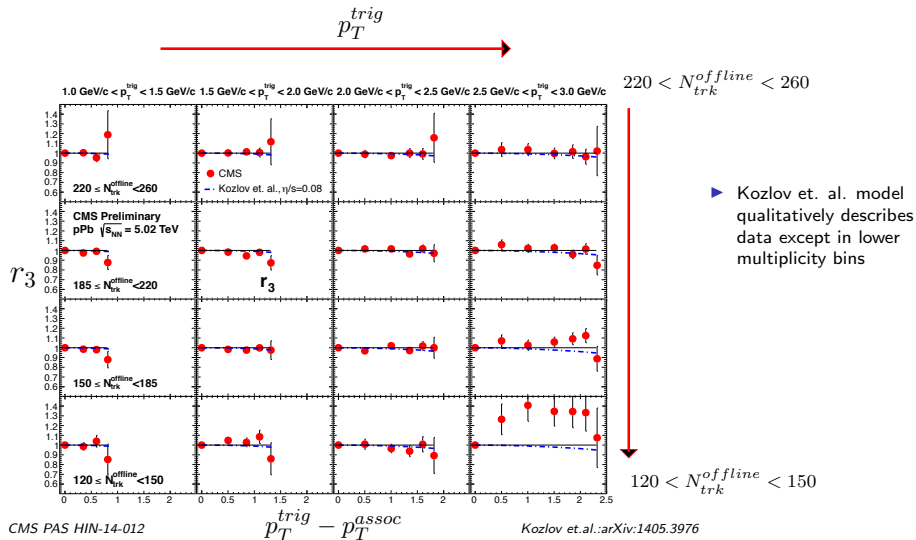
$r_n$  insensitive to viscosity, sensitive to granularity



## PRELIMINARY DATA: D. DEVETAK, QM14



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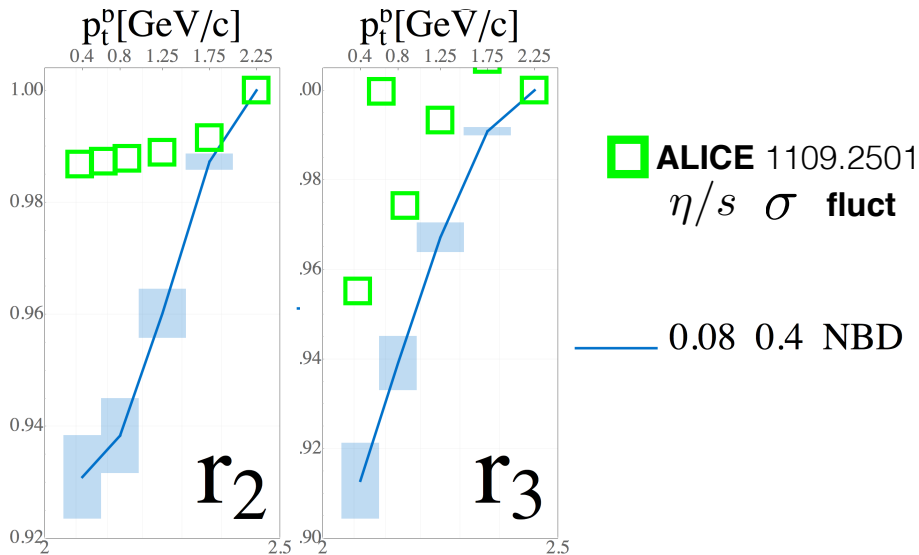




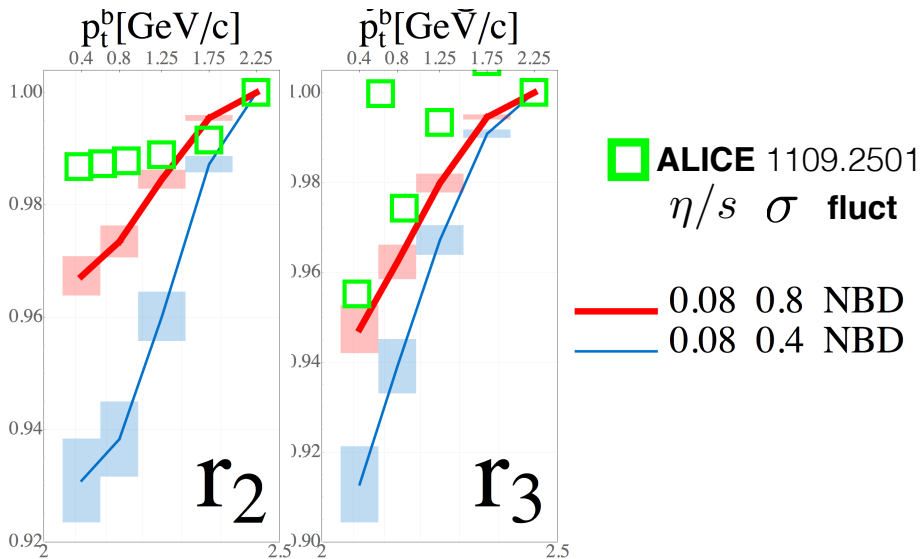
# INTERIM CONCLUSIONS

- Experimental data for  $r_n$  agree with hydrodynamic predictions
- $r_n$  indicates the breakdown of flow dominance ( $p_T > 2.5$  GeV +  $N_{ch} < 150$ )

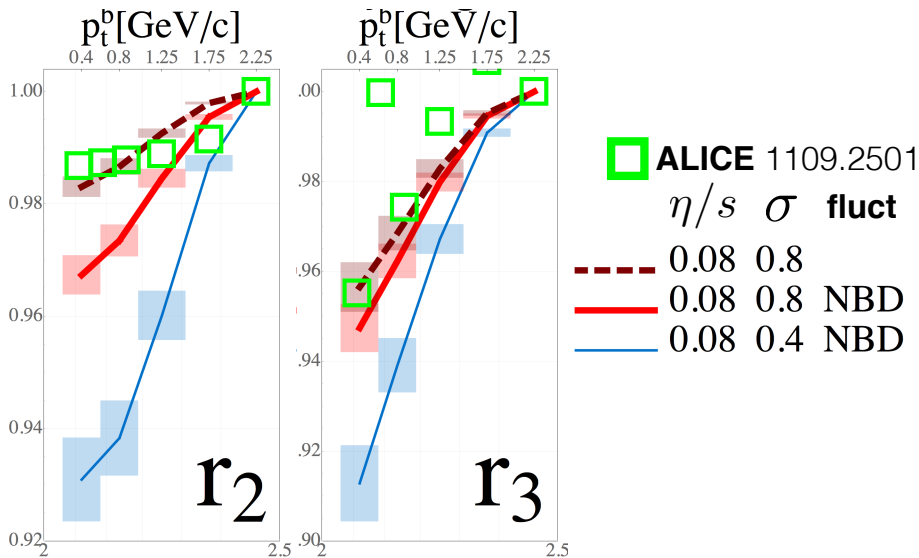
## REVISITING NUCLEUS-NUCLEUS COLLISIONS



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## PUZZLE

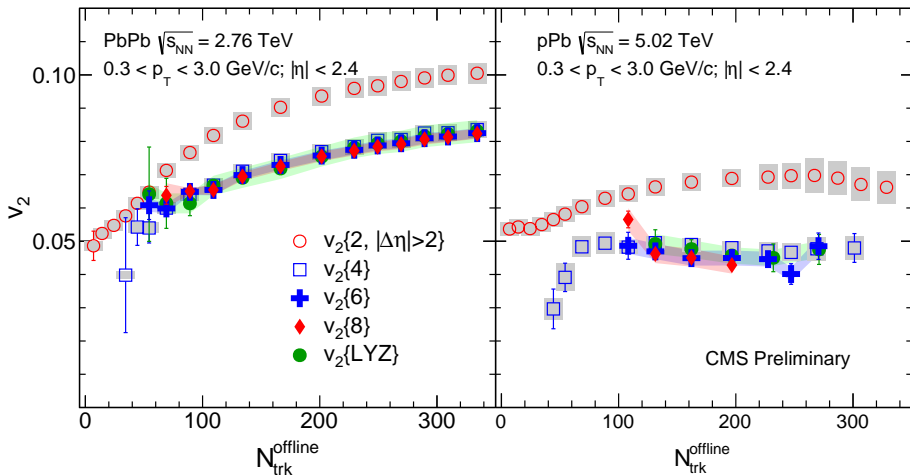
Including  $r_n$  adds tension to simultaneous description of p-Pb and Pb-Pb

# CONCLUSIONS

- Hydrodynamic calculations can describe many observables in high multiplicity p-Pb collisions
- Preliminary data for  $r_n$  agree with hydrodynamic predictions
- $r_n$  observable provides a useful new handle on physics. E.g.,
  - to indicate where hydrodynamic description breaks down
  - to probe aspects of initial condition not probed by other observables (granularity)

# EXTRA SLIDES

## HIGHER CUMULANTS





## RESULTS: P-PB VS PB-PB

