Two-particle correlations and Entanglement Entropy in CGC

Michael Lublinsky
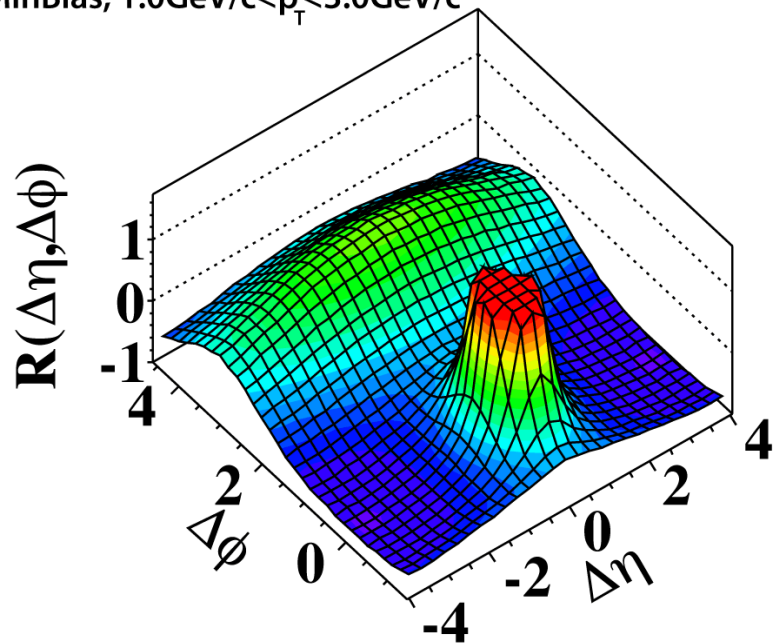
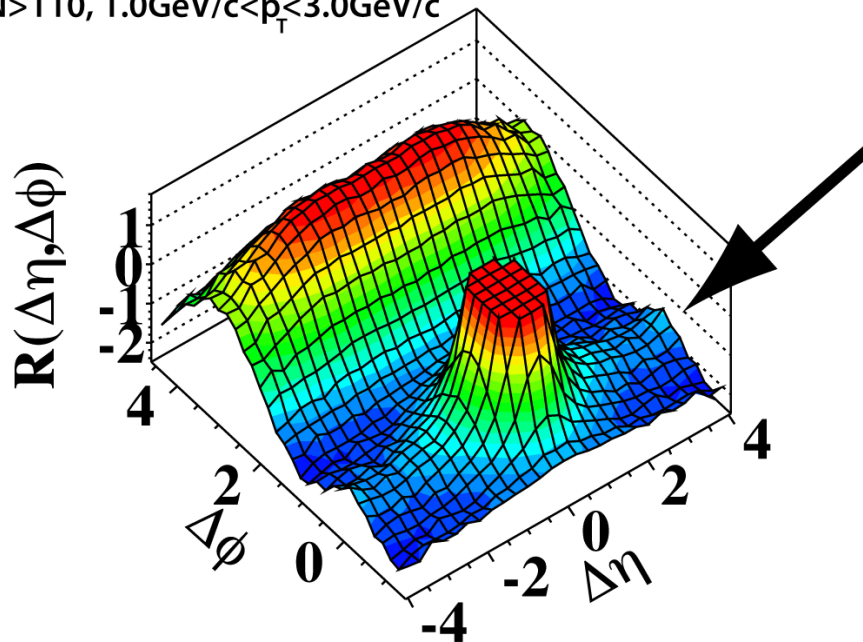**Ben-Gurion University of the Negev**

Israel

"RIDGE" - ANGULAR CORRELATIONS

Two particle correlations in $p-p$: long range in rapidity, near-side angular correlations

CMS 2010, $\sqrt{s}$=7TeV
MinBias, 1.0GeV/c<p_T<3.0GeV/c
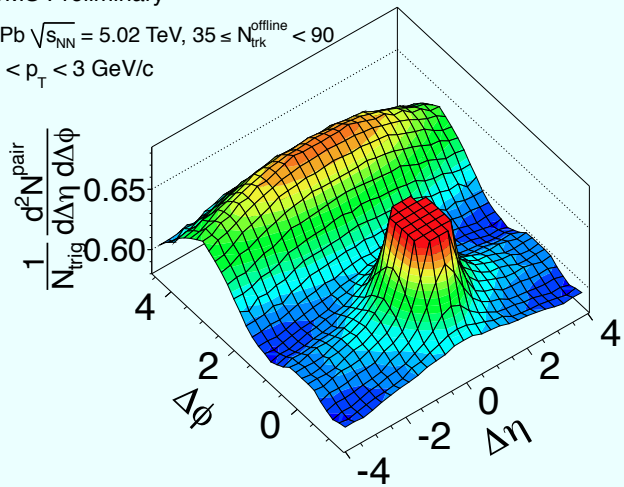
N>110, 1.0GeV/c<p_T<3.0GeV/c

"High multiplicity" collisions with over a hundred charged particles produced
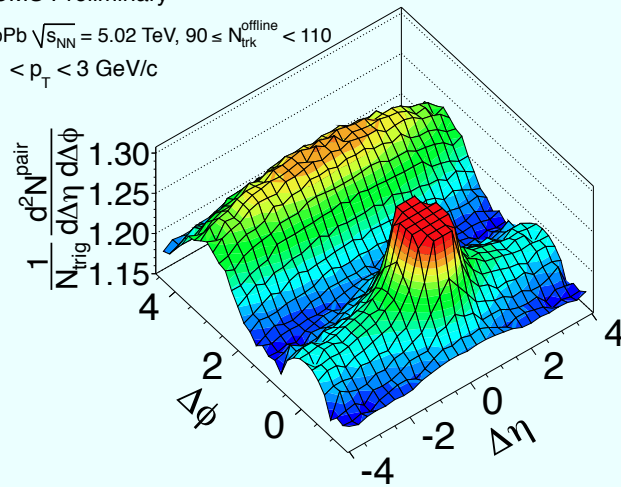
Multiplicity dependence of the ridge in pPb

CMS Preliminary

pPb $\sqrt{s_{NN}}$ = 5.02 TeV, 35 ≤ $N_{trk}^{offline}$ < 90

1 < p_T < 3 GeV/c

$\frac{1}{N_{trig}}\frac{d^2N^{pair}}{d\Delta\eta\,d\Delta\phi}$

0.65
0.60

$\Delta\phi$

$\Delta\eta$

CMS Preliminary

pPb $\sqrt{s_{NN}}$ = 5.02 TeV, 90 ≤ $N_{trk}^{offline}$ < 110

1 < p_T < 3 GeV/c

$\frac{1}{N_{trig}}\frac{d^2N^{pair}}{d\Delta\eta\,d\Delta\phi}$

1.30
1.25
1.20
1.15

$\Delta\phi$

$\Delta\eta$

CMS pPb $\sqrt{s_{NN}}$ = 5.02 TeV, $N_{trk}^{offline}$ ≥ 110

(b)

1 < p_T < 3 GeV/c

$\frac{1}{N_{trig}}\frac{d^2N^{pair}}{d\Delta\eta\,d\Delta\phi}$

1.8
1.7
1.6

$\Delta\phi$

$\Delta\eta$
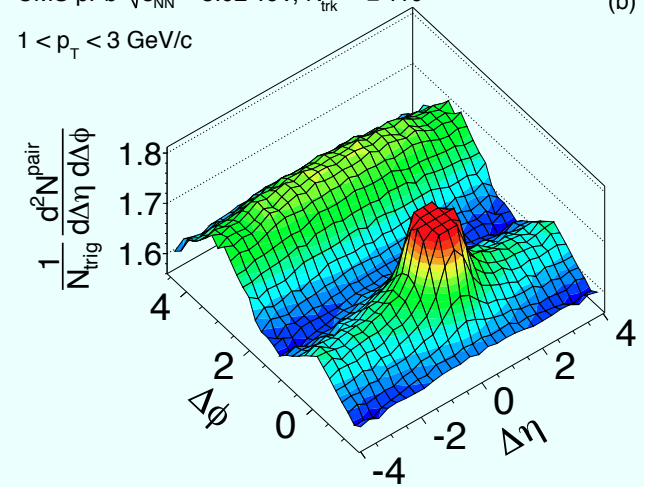
- **Origin of angular collimation?**
  Could be many. For sure explosive "wind" from hydro would lead to some.

- **Origin of long range rapidity correlations?**
  Causality: correlations exist in early stage of the collision (like in cosmology)

- **Do we see a sign of universality between $p - p$ and $p - A$ and $A - A$?**
  Hopefully Yes! High energy QCD implies this universality. In all experiments the effect emerges only when high densities are involved (color glass condensate (CGC))

- **Do we see a collective phenomenon (QGP?) in $p - p$ or $p - A$?**
  Many indications, but we are not sure yet ...

Our Goal

To discuss some general features of gluon production at high energy.

We need to compute correlations in two-gluon inclusive production rate

$$\left[\frac{d^2N}{d^2p\,d\eta\,d^2k\,d\xi} - \frac{dN}{d^2k\,d\xi}\frac{dN}{d^2p\,d\eta} \right] \bigg/ \frac{dN}{d^2k\,d\xi}\frac{dN}{d^2p\,d\eta}$$

We don't know how to compute dense on dense

We do know quite a lot about dilute on dense (DIS)
For DIS, we do have QCD-derived formulae for multi-gluon production, including high energy evolution between produced gluons.

Here I talk about only one source for the observed phenomena, INITIAL CONDITIONS, as follows from quite general QCD-based considerations, but I have no quantitative results.

High Energy Scattering: CGC-type approach

Target	Projectile
$\langle T|$ $\rightarrow$	$\leftarrow$ $|P\rangle$

S-matrix:

$$\mathbf{S(Y)} \;=\; \langle \mathbf{T} \, \langle \, \mathbf{P}| \; \hat{\mathbf{S}}(\rho^{\mathbf{t}}, \, \rho^{\mathbf{p}}) \; |\mathbf{P}\rangle \, \mathbf{T}\rangle$$

CGC-type averaging

$$\mathbf{S(Y)} \;=\; \int \mathbf{D}\rho^{\mathbf{p}} \, \mathbf{D}\rho^{\mathbf{t}} \; \mathbf{S}[\rho^{\mathbf{p}}, \, \rho^{\mathbf{t}}] \; \mathbf{W}^{\mathbf{p}}_{\mathbf{Y}-\mathbf{Y_0}}[\rho^{\mathbf{p}}] \; \mathbf{W}^{\mathbf{t}}_{\mathbf{Y_0}}[\rho^{\mathbf{t}}]$$

$\mathbf{W}^{\mathbf{p,t}}$ **are probability distributions, subject to high energy evolution equations**

For any other observable $\mathcal{O}$

$$\langle \mathcal{O}\rangle_{\mathbf{P,T}} \;=\; \int \mathbf{D}\rho^{\mathbf{p}} \, \mathbf{D}\rho^{\mathbf{t}} \; \mathcal{O}_{\mathbf{Y_0}}[\rho^{\mathbf{p}}, \, \rho^{\mathbf{t}}] \; \mathbf{W}^{\mathbf{p}}_{\mathbf{Y}-\mathbf{Y_0}}[\rho^{\mathbf{p}}] \; \mathbf{W}^{\mathbf{t}}_{\mathbf{Y_0}}[\rho^{\mathbf{t}}]$$

High energy evolution of light cone wave function
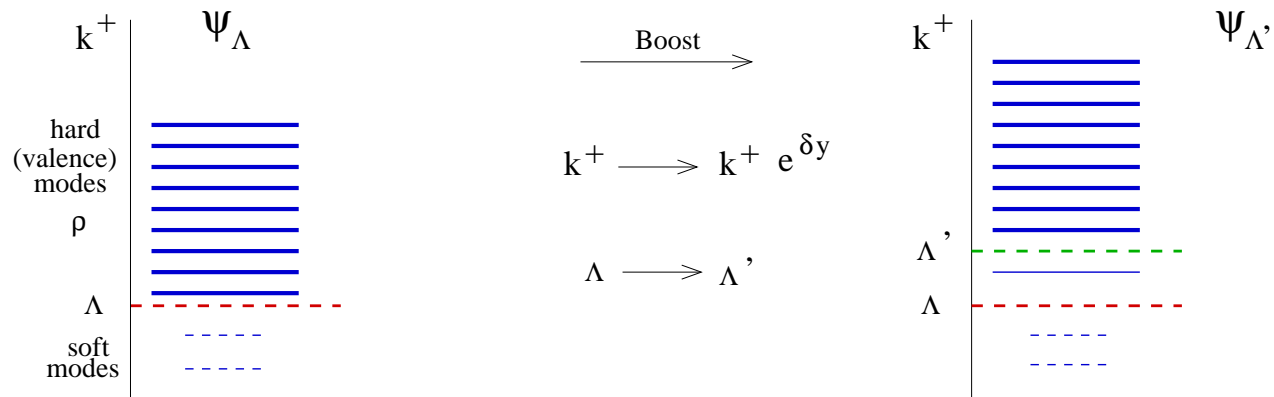
Hard particles with $k^+ > \Lambda$ scatter off the target. In the eikonal approximation, the scattering amplitude is independent of k^+. Hard (valence) modes are described by the valence density $\rho(x_\perp)$. Note rapidity independence!

Soft modes are not many. They do not contribute much to the scattering amplitude.

The boost opens a window above Λ with the width $\sim \delta y$. The window is populated by soft modes, which became hard after the boost. These newly created hard modes do scatter off the target.

In the dilute limit $\rho \sim 1$; gluon emission $\sim \alpha_s \rho$, LO = one gluon, NLO = 2 gluons

In the dense limit $\rho \sim 1/\alpha_s$, we have $\alpha_s \rho \sim 1$, and the number of gluons in the window can be very large.

Denote soft glue creation and annihilation operators as $\mathbf{a}$ and $\mathbf{a}^\dagger$.

$$\mathbf{H_{QCD}} = \mathbf{H}(\rho, \, \mathbf{a}, \, \mathbf{a}^\dagger)$$

Hadron wave function in the soft gluon Fock space

$$|\mathbf{\Psi}\rangle_{\mathbf{Y_0}} = |v\rangle \; = \; |\rho\rangle_{\text{valence}} \, \otimes \, |\mathbf{0_a}\rangle_{\text{soft}}$$

The evolved wave function

$$|\mathbf{\Psi}\rangle_{\mathbf{Y}} \; = \; \mathbf{\Omega_Y}(\rho, \, \mathbf{a}) \, |\mathbf{\Psi}\rangle_{\mathbf{Y_0}} \, ;$$

or equivalently

$$\mathbf{\Omega}^\dagger \, \mathbf{H} \, \mathbf{\Omega} \; = \; \mathbf{H}_{\text{diagonal}}$$

The major challenge is to find $\mathbf{\Omega}$ that does the job

Gluon coherent field operator in the dilute limit

$$\Omega_Y(\rho \to 0) \equiv C_Y = \mathrm{Exp}\left\{ i \int d^2z\, b_i^a(z) \int_{e^{Y_0}\Lambda}^{e^Y\Lambda} \frac{dk^+}{\pi^{1/2}|k^+|^{1/2}} \left[a_i^a(k^+, z) + a_i^{\dagger a}(k^+, z)) \right] \right\}$$

ρ_v^p
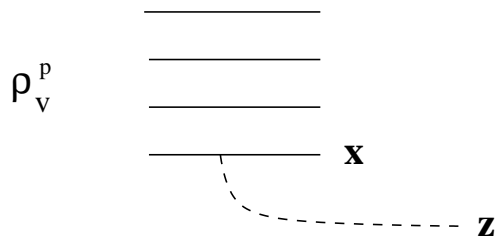
Linear evolution means $\delta\rho \propto \rho^p$

Emission amplitude is given by the Weizsaker-Williams field

$$b_i^a(z) = \frac{g}{2\pi} \int d^2x\, \frac{(x - z)_i}{(x - z)^2}\, \rho^a(x)$$

The operator C dresses the valence charges by a cloud of the WW gluons

In the dense regime: $\quad \Omega(\rho \sim 1/\alpha_s) = C\,B$ $\qquad$ **B is a Bogolyubov operator**

$$B = \exp[\,\Lambda(\rho)\,(a^2 + a^{\dagger 2}) + \cdots]$$

B defines quasiparticles above the WW background

Altinoluk, Kovner, ML, Peressutti, Wiedemann (2007-2009)

Given the evolution of the hadronic wave function we can calculate evolution of an arbitrary observable $\hat{\mathcal{O}}(\rho)$

The evolution of the expectation value

$$\frac{d\,\langle v|\hat{\mathcal{O}}|v\rangle}{d\,Y} = \lim_{Y\to Y_0} \frac{\langle v|\Omega_Y^\dagger\,\hat{\mathcal{O}}[\rho+\delta\rho]\,\Omega_Y|v\rangle \;-\; \langle v|\hat{\mathcal{O}}[\rho]|v\rangle}{Y \;-\; Y_0} = -\int D\rho\,W[\rho]\,H^{HE}[\rho]\,\mathcal{O}[\rho]$$

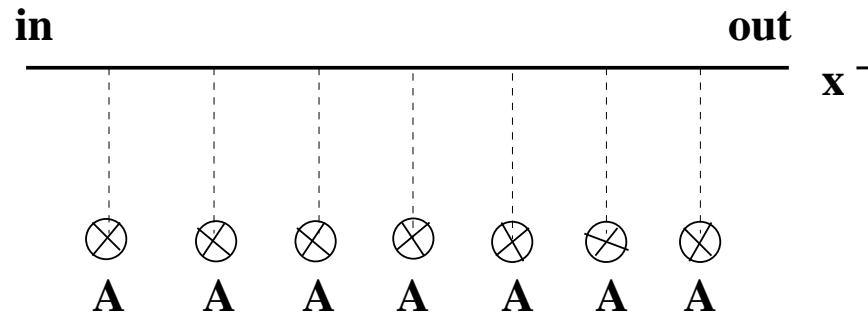**Charge density due to newly produced gluon**

$$\delta\rho^{\mathbf{a}}(\mathbf{x}) = \int_{e^{\mathbf{Y_0}}\Lambda}^{e^{\mathbf{Y}}\Lambda} d\mathbf{k}^+ \, \mathbf{a}_{\mathbf{i}}^{\dagger\mathbf{b}}(\mathbf{k}^+,\mathbf{x})\,\mathbf{T}_{\mathbf{bc}}^{\mathbf{a}}\,\mathbf{a}_{\mathbf{i}}^{\mathbf{c}}(\mathbf{k}^+,\mathbf{x})$$

$\mathbf{H^{KLWMIJ}} = \mathbf{H^{HE}}(\rho \to 0)$ **A. Kovner and M.L., Phys.Rev.D71:085004, 2005**

$\mathbf{H^{JIMWLK}} = \mathbf{H^{HE}}(\rho \to \infty)$ - **Jalilian Marian, Iancu, McLerran, Leonidov, Kovner (1997-2002)**

Balitsky-Kovchegov (BK) is the large $\mathbf{N_c}$ **version of JIMWLK**

Eikonal scattering approximation

in — — — — — — — — — — — — — — — **out**
x^-

Eikonal scattering is a color rotation
Eikonal factor does not depend on rapidity

In the light cone gauge $(A^+ = 0)$ the large target field component is $A^- = \alpha^t$.

$$S(x) = \mathcal{P} \exp \left\{ i \int dx^+ \, T^a \, \alpha_t^a(x, x^+) \right\}.$$

$$"\Delta" \alpha^t = \rho^t \quad (\mathbf{YM})$$

$$|in\rangle = |z, b\rangle ; \qquad |out\rangle = |z, a\rangle ; \qquad |out\rangle = S\,|in\rangle$$

Semi-inclusive reactions

The wave function coming into the collision region at time $t = 0$

$$|\Psi_{\text{in}}\rangle = \Omega_{\mathbf{Y}} |\rho, 0_{\text{a}}\rangle .$$
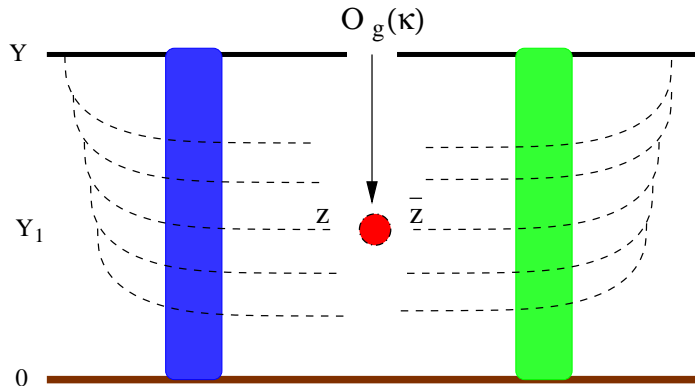
The system emerges from the collision region with the wave function

$$|\Psi_{\text{out}}\rangle = \hat{\mathbf{S}} \, \Omega_{\mathbf{Y}} \, |\rho, 0_{\text{a}}\rangle .$$

The system keeps evolving after the collision to the asymptotic time $t \to +\infty$, at which point the measurement of an observable $\hat{\mathcal{O}}$ is made

$$\langle \hat{\mathcal{O}} \rangle_{\mathbf{P,T}} = \langle \, \langle 0_{\text{a}} | \Omega_{\mathbf{Y}}^\dagger \, (1 - \hat{\mathbf{S}}^\dagger) \, \Omega_{\mathbf{Y}} \, \hat{\mathcal{O}} \, \Omega_{\mathbf{Y}}^\dagger \, (1 - \hat{\mathbf{S}}) \, \Omega_{\mathbf{Y}} \, |0_{\text{a}}\rangle \, \rangle_{\mathbf{P,T}}$$

Single inclusive gluon production

The observable

$$\hat{\mathcal{O}}_g \sim a_i^{\dagger\,a}(\mathbf{k})\, a_i^a(\mathbf{k})$$

$$\frac{dN}{d^2k\, d\eta} = \langle\, \sigma(\mathbf{k})\,\rangle_{\mathbf{P,T}}$$

After soft gluon averaged using dilute projectile approximation $(\Omega \to C)$

$$\sigma(\mathbf{k}) = \int_{z,\bar{z},x_1,\bar{x}_1} e^{ik(z-\bar{z})} \frac{(\bar{z}-\bar{x}_1)_i}{(\bar{z}-\bar{x}_1)^2} \frac{(x_1-z)_i}{(x_1-z)^2} \left\{ \rho(x_1)[S^\dagger(x_1) - S^\dagger(z)][S(\bar{x}_1) - S(z)]\rho(\bar{x}_1) \right\}$$

Long range rapidity correlations come for free with boost invariance

Incoming $|P\rangle$ is approximately boost invariant: exactly the same gluon distribution at Y_1 and Y_2.
What happens at Y_1, happens also at Y_2: If it is probable to produce a gluon at Y_1, it is also probable to produce a gluon at Y_2.

But exactly by the same logic there must be angular correlations:
Gluons scatter on exactly the same target
If the first gluon is most likely to be scattered to the right, the second gluon at the same impact parameter will be also scattered to the right

Eikonal scattering is rapidity independent!
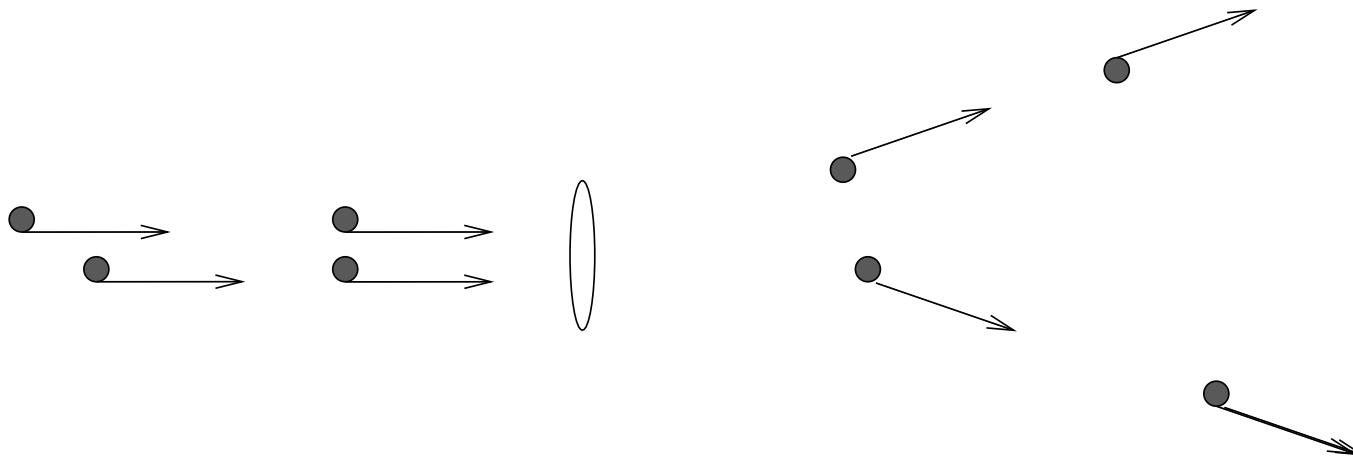
Double inclusive gluon production

$$\mathcal{O} = a^\dagger(\mathbf{k}) \, a(\mathbf{k}) \, a^\dagger(\mathbf{p}) \, a(\mathbf{p})$$

$$\frac{dN}{d^2p \, d^2k \, d\eta \, d\xi} = \sigma_4 = \langle \sigma(\mathbf{k}) \, \sigma(\mathbf{p}) \rangle_{P,T}$$

Configuration by configuration
(for fixed configuration of projectile charges ρ and fixed target fields S)

$\sigma(\mathbf{k})$ **is a real function of $\mathbf{k}$, which has a maximum at some value $\mathbf{k} = \mathbf{q_0}$. Then the two gluon production probability configuration by configuration has a maximum at**

$$\mathbf{k} = \mathbf{p} = \mathbf{q_0} \simeq \mathbf{Q_s}$$

The value of $\mathbf{q_0}$ depends on configuration, but the fact that $\mathbf{k} \simeq \mathbf{p}$ does not.

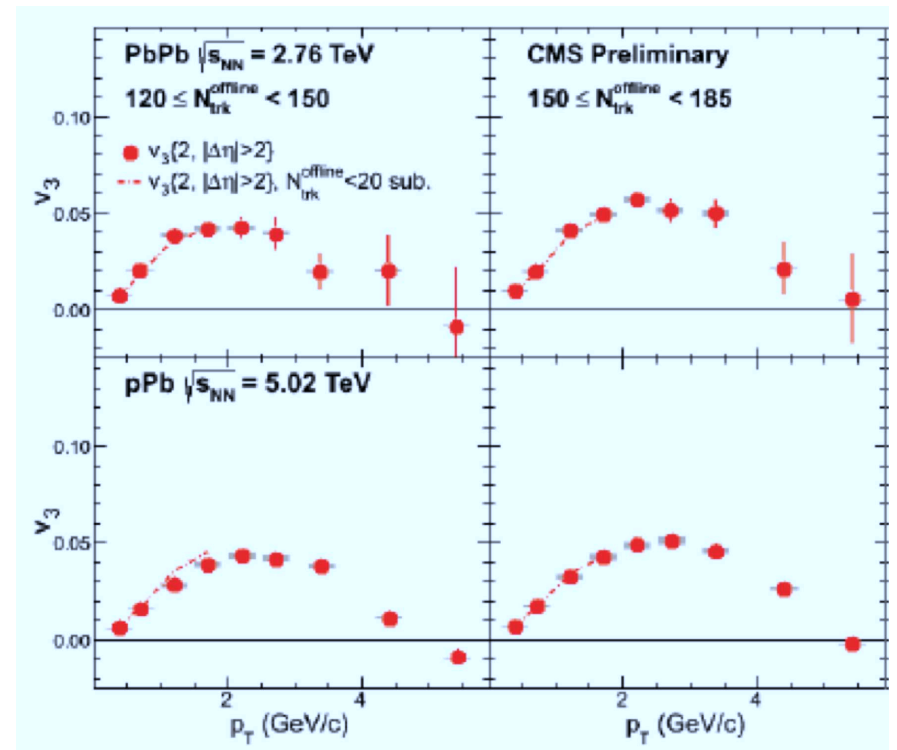**This is the near side correlation!**

Is the maximum of σ_1 unique?

No, σ_1 is symmetric under $k \to -k$ and thus has two maxima at q_0 and $-q_0$

This means that σ^4 has a symmetry $k, p \to -k, p$ and therefore has maxima at relative angles $\phi = 0$ and $\phi = \pi$

The maximum at $\phi = \pi$ is very difficult to distinguish experimentally.

**After all there is some asymmetry between 0 and π angles**

The v_3 story:

Projectile/Target averaging

We have to specify $\mathbf{W^p}$ and $\mathbf{W^t}$ by modeling them at some initial rapidities and then evolve to a desired one with H^{HE}.

● **"Glasma graphs"** A. Dumitru, F. Gelis, J. Jalilian-Marian, T. Lappi: Phys.Lett. B697 (2011) 21 (arXiv:1009.5295), followed by quite successful quantitative effort to describe all data by K. Dusling and R. Venugopalan, Phys.Rev.Lett. 108 (2012) 262001

McLerran-Venugopalan model:

$$\mathbf{W}^{\mathrm{MV}}[\rho] = \mathcal{N} \exp\left[- \int_{\mathbf{k}} \frac{1}{2\mu^2(\mathbf{k})} \rho(\mathbf{k})\rho(-\mathbf{k}) \right]$$
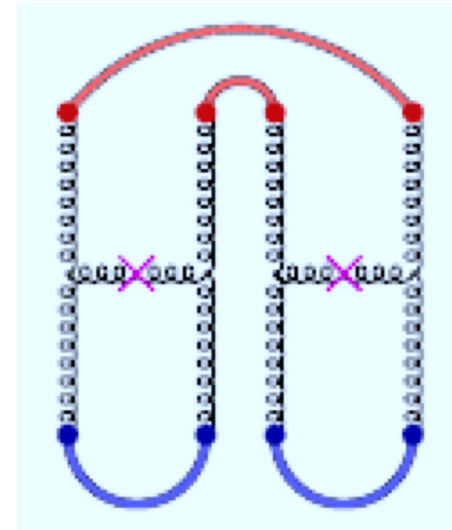
The resulting correlations are N_c suppressed. The physics of the "Glasma graphs" is Bose enhancement of gluons in the hadronic wave function

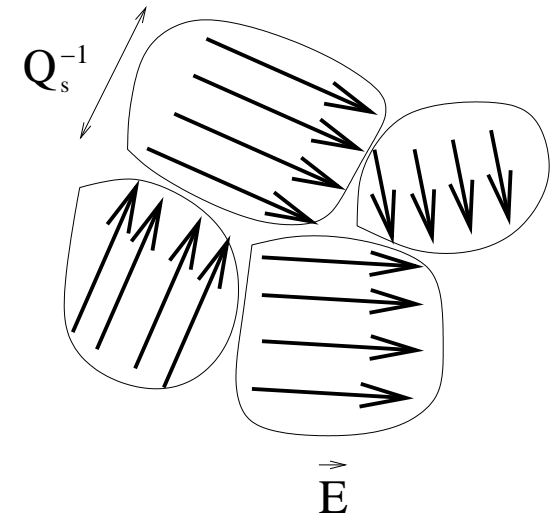T. Altinoluk, N. Armesto, G. Beuf, A. Kovner and ML, arXiv:1503.07126.

- **Local anisotropy** A. Kovner and M. L: PRD83 (2011) 034017; IJMP E Vol. 22 (2013) 1330001

$$\mathbf{tr}[S_x^\dagger S_y]/N_c \;=\; \mathbf{Exp}[-(\vec{r}\vec{E}(\vec{b}))^2]\;;$$

$$\vec{E}(\vec{b}) \;=\; \sum \vec{E}_0(\vec{b})\,e^{-d^2 Q_s^2}\;; \qquad\qquad E_0 \;=\; Q_s$$

$\vec{r} = \vec{x} - \vec{y}$ **is a vector of the dipole moment.**

$\langle \mathbf{tr}[S_x^\dagger S_y]\,\mathbf{tr}[S_u^\dagger S_v]\rangle$ **has non-trivial angular correlations and the effect is a leading** N_c.
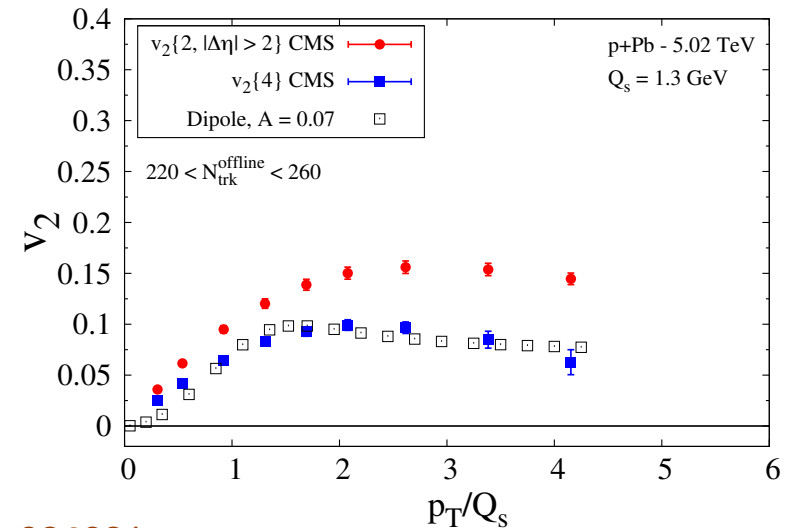
A. Dumitru, A. Giannini, NPA 933 (2014) 212

A. Dumitru, L. McLerran and V. Skokov, PLB 743, 134 (2015)

A. Dumitru and V. Skokov, PRD 91, 7, 074006 (2015)

A. Dumitru, A. V. Giannini and V. Skokov, arXiv:1503.03897,

V. Skokov, PRD 91, 5, 054014 (2015)

T. Lappi, PLB 744, 315 (2015)

- **Density variation** E. Levin a A. Rezaeian, PRD84 (2011) 034031

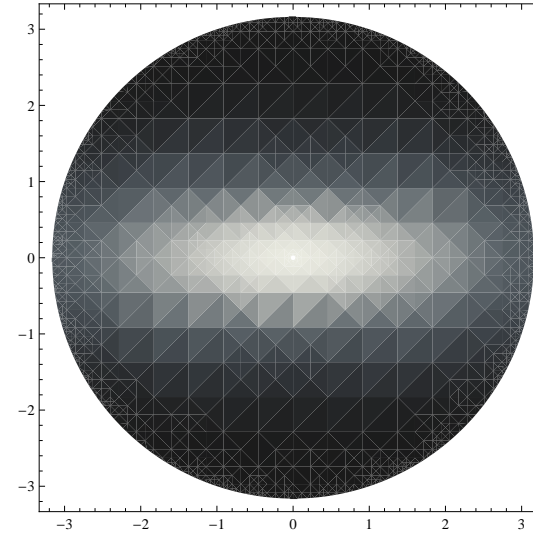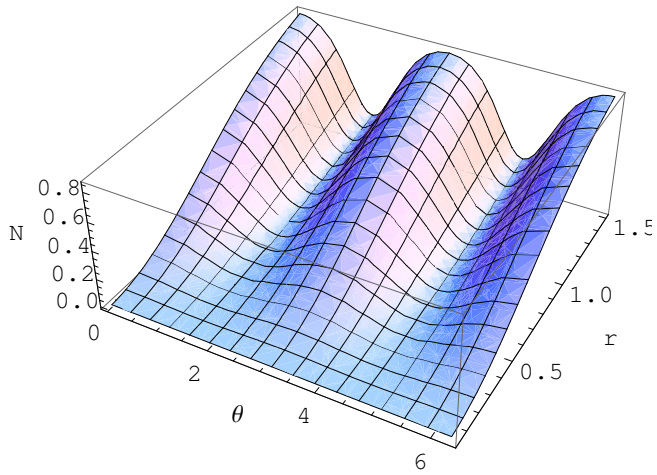**BKe for imaginary part of the dipole scattering amplitude** $N(\vec{r}) = 1 - tr[S_x^\dagger S_y]/N_c$

$$\partial_Y N(\vec{r}) = \frac{C_F \alpha_s}{2\pi} \int d^2\vec{r}' \frac{\vec{r}^2}{\vec{r}'^2 (\vec{r} - \vec{r}')^2} [N(\vec{r}') + N(\vec{r} - \vec{r}') - N(\vec{r}) - N(\vec{r}')\, N(\vec{r} - \vec{r}')]$$

Anisotropic initial conditions at some initial rapidity $Y_0 = \ln 10^2$.

$$N(Y_0, \vec{r}) = 1 - Exp[-a\, r^2\, xg^{LOCTEQ6}(x_0, 4/r^2)\, F(\theta)]; \qquad a = \frac{\alpha_s(r^2)\,\pi}{2\, N_c\, R^2}$$

$$F(\theta) = 1 - A + 2\, A\, \cos^2(\theta) \qquad\qquad A = 3/4$$

$\mathbf{W}[\delta] = \mathbf{1/2\pi}$, constant for any δ ranging from 0 to 2π.

$$\langle \mathbf{F} \rangle_\delta \; = \; \int_0^{2\pi} \mathbf{d}\delta \, \mathbf{F}(\theta + \delta) \, \mathbf{W}[\delta] \; = \; \mathbf{1}$$

We are interested in the two-dipole correlator $\langle \mathbf{N}(\mathbf{Y}, \mathbf{r_1}, \theta_1, \delta) \, \mathbf{N}(\mathbf{Y}, \mathbf{r_2}, \theta_2, \delta) \rangle_\delta$.

Single configuration solution
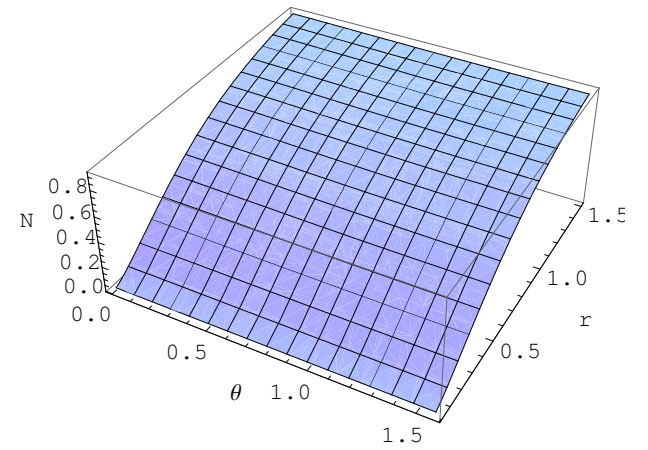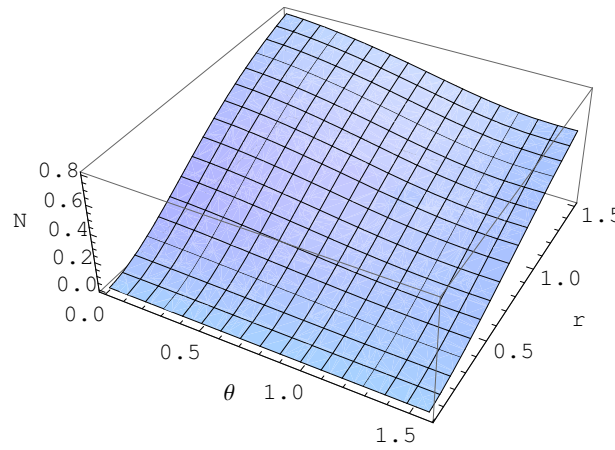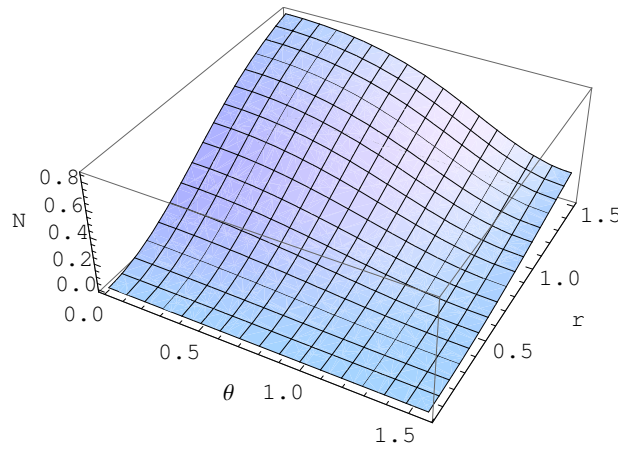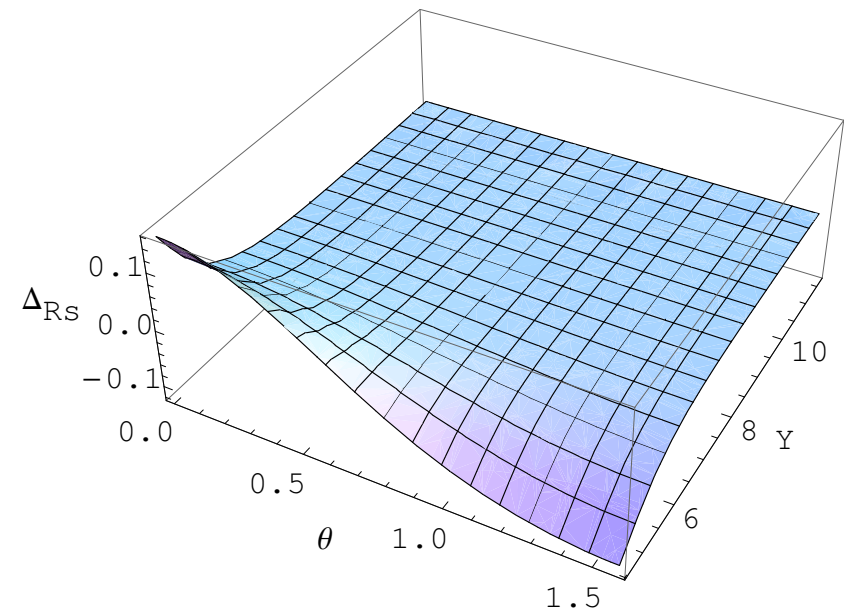
the saturation scale $\quad N(Y, R_S, \theta) = 1/2$
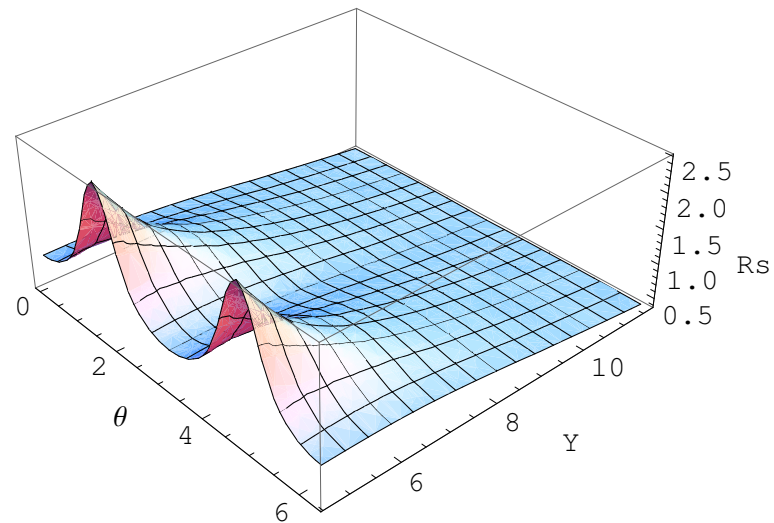
Very fast isotropization!

Two quantities of interest: correlator of two saturation scales $\langle \mathbf{R_s}(\theta_1)\mathbf{R_s}(\theta_2)\rangle_\delta$ **and**

$$\boldsymbol{\Delta}_{\mathbf{R_s}}(\mathbf{Y}, \mathbf{r}, \theta) \equiv \frac{\langle \mathbf{R_s}(\mathbf{Y}, \theta_1, \delta)\, \mathbf{R_s}(\mathbf{Y}, \theta_2, \delta)\rangle_\delta \;-\; \langle \mathbf{R_s}(\mathbf{Y}, \theta_1, \delta)\rangle_\delta\, \langle \mathbf{R_s}(\mathbf{Y}, \theta_2, \delta)\rangle_\delta}{\langle \mathbf{R_s}(\mathbf{Y}, \theta_1, \delta)\rangle_\delta^2}\,, \qquad \theta = \theta_1 - \theta_2$$
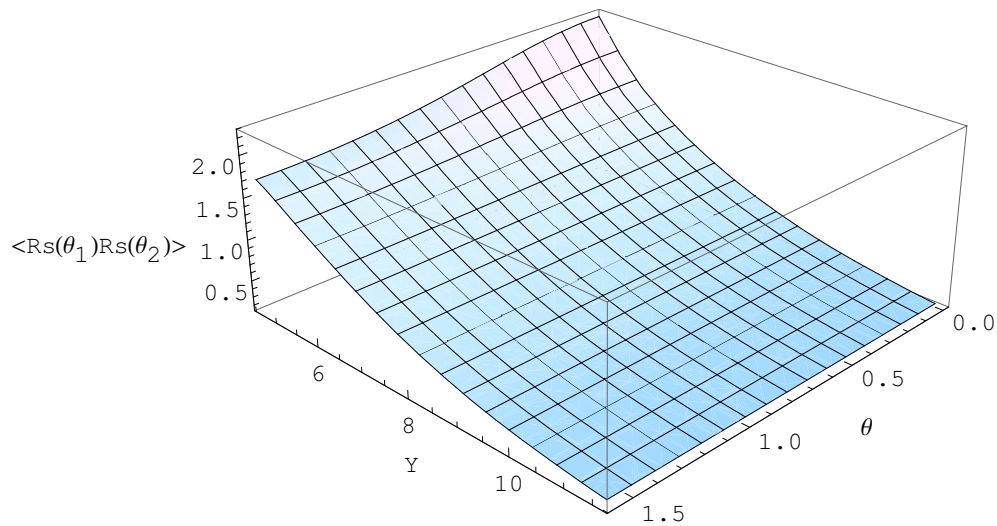
Angular correlations $\langle \mathbf{N}(\mathbf{Y}, \mathbf{r}, \theta_1)\mathbf{N}(\mathbf{Y}, \mathbf{r}, \theta_2)\rangle_\delta$
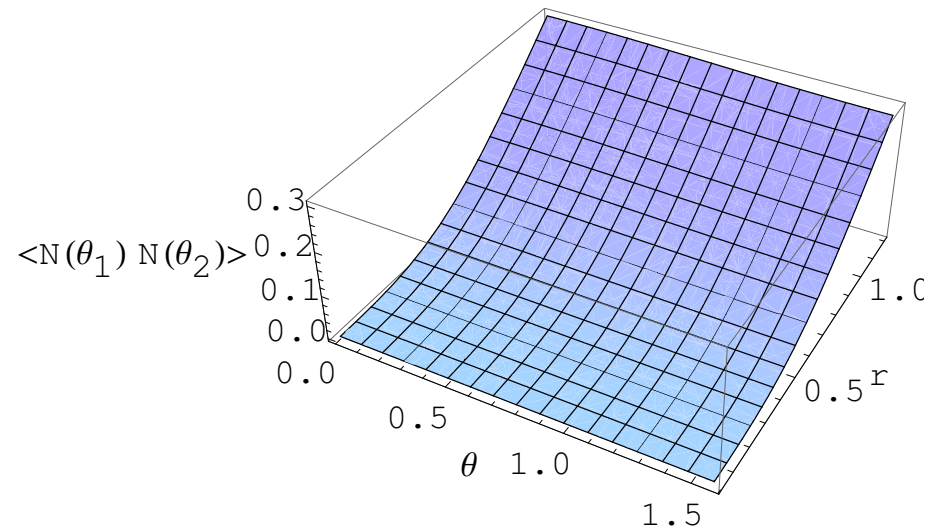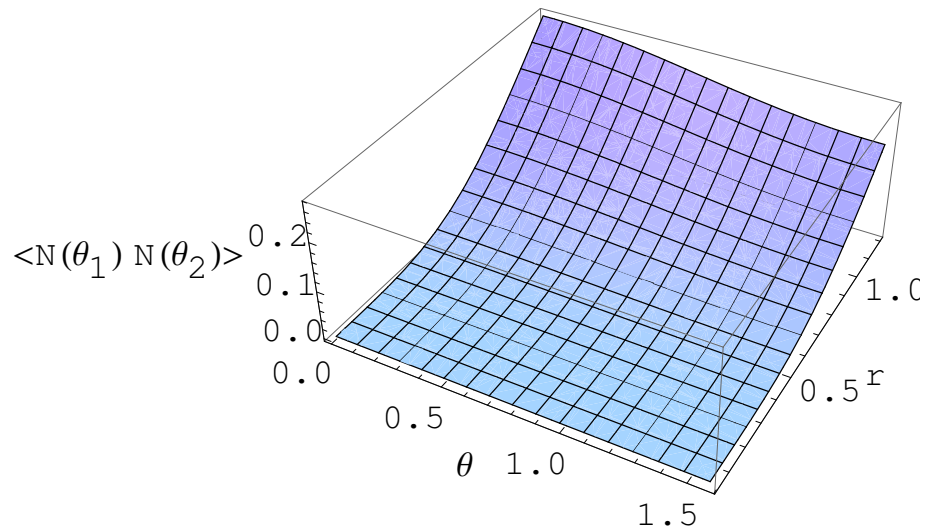
Again fast anizotropization

$$\text{Ang. correlations} \sim e^{-\lambda Y}, \qquad \lambda \simeq 0.6$$

Presumably related to the second BFKL eigenvalue

$$\omega_{n=0} = 4\ln 2\,\bar{\alpha}_s; \qquad\qquad \omega_{n=2} = 4\,(\ln 2 - 1)\,\bar{\alpha}_s$$

CONCLUSIONS

- **Gluon production at high energy leads naturally to rapidity correlations and angular correlations. There just have to be many gluons so that more than one is produced at fixed impact parameter (within $\Delta b \sim 1/Q_s$)**

- **Within the "projectile" dipole model, we find an exponentially fast isotropization with the exponent $\lambda_A \simeq 0.6$.**

- **Observed correlations must arise dynamically. Pomeron loops are needed**

Quite an opposite result

A. Dumitru, A. V. Giannini and V. Skokov, arXiv:1503.03897

$$\frac{N(r, \theta, Y)}{\langle N(r, Y) \rangle} = 1 + \sum_n A_n(r, Y) \cos(2n\theta)$$
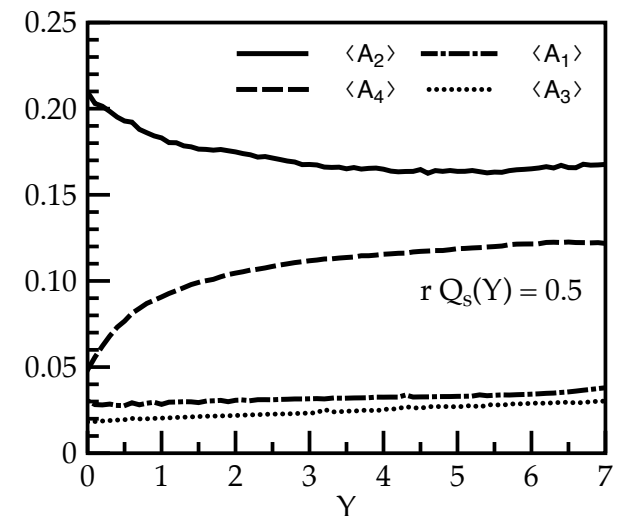
Density Matrix of soft modes

The wave function coming into the collision region at time $t = 0$

$$|\Psi_{\mathbf{in}}\rangle \ = \ \Omega_{\mathbf{Y}} \, |\rho, \mathbf{0_a}\rangle \,.$$

Standard CGC formalism: first integrate out the soft modes and then average over ρ

Q: Can we learn something if we do in the opposite order?   A: probably Yes.

Define the density matrix of soft modes

$$\hat{\rho} \ = \ \int D\rho \, W[\rho] \, |\Psi_{in}\rangle \, \langle \Psi_{in}|$$

"Glasma mix approximation": $\Omega = C$ and $W = W^{MV}$ (Gaussian),
$\hat{\rho}$ is computable analytically

T. Altinoluk, N. Armesto, G. Beuf, A. Kovner and ML, arXiv:1503.07126

Entanglement Entropy

Initial wave function: Entanglement Entropy of soft modes

$$\sigma^{\mathbf{E}} = -\mathbf{tr}[\hat{\rho} \ln \hat{\rho}]$$

Define:

$$\mathbf{M_{ij}} \equiv \mathbf{g^2} \int_{\mathbf{u,v}} \mu^2(\mathbf{u,v}) \frac{(\mathbf{x-u})_\mathbf{i}}{(\mathbf{x-u})^2} \frac{(\mathbf{y-v})_\mathbf{j}}{(\mathbf{y-v})^2} \delta^{\mathbf{ab}}$$

The result

$$\sigma^{\mathbf{E}} = \frac{1}{2} \mathbf{tr} \left\{ \ln \frac{\mathbf{M}}{\pi} + \sqrt{1 + \frac{4\mathbf{M}}{\pi}} \ln \left[1 + \frac{\pi}{2\mathbf{M}} \left(1 + \sqrt{1 + \frac{4\mathbf{M}}{\pi}} \right) \right] \right\}$$

Translationally invariant limit ($\mu = const$):

$$\mathbf{M_{ij}^{ab}(p)} = \mathbf{g^2}\mu^2\frac{\mathbf{p_i p_j}}{\mathbf{p^4}}\delta^{\mathbf{ab}}$$

For small M, or the UV contribution

$$\sigma_{UV}^E = tr\left[\frac{M}{\pi}\ln\frac{\pi e}{M}\right] = -\frac{N_c^2 - 1}{\pi}S\int_{p^2 > \frac{Q_s^2}{g^2}}\frac{d^2p}{(2\pi)^2}\frac{Q_s^2}{g^2 p^2}\ln\frac{Q_s^2}{eg^2\,p^2}$$

where $Q_s^2 = \frac{g^4}{\pi}\mu^2$ In all σ^E is formally UV divergent

$$\sigma_{UV}^E = \frac{Q_s^2}{4\pi g^2}(N_c^2 - 1)S\left[\ln^2\frac{g^2\Lambda^2}{Q_s^2} + \ln\frac{g^2\Lambda^2}{Q_s^2}\right]$$

The large M, IR contribution is

$$\sigma_{IR}^E \simeq \frac{1}{2}tr[\ln\frac{e^2 M}{\pi}] = \frac{N_c^2 - 1}{2}S\int_{p_2 < \frac{Q_s^2}{g^2}}\frac{d^2p}{(2\pi)^2}\ln\frac{e^2\,Q_s^2}{g^2 p^2} = \frac{3(N_c^2 - 1)}{8\pi g^2}SQ_s^2$$

Entropy of the system of produced particles is formally very similar

$$\sigma^{\mathbf{P}} = \frac{1}{2}\langle\mathbf{tr}\left\{\ln\frac{\mathbf{M^P}}{\pi} + \sqrt{1 + \frac{4\mathbf{M^P}}{\pi}}\,\ln\left[1 + \frac{\pi}{2\mathbf{M^P}}\left(1 + \sqrt{1 + \frac{4\mathbf{M^P}}{\pi}}\right)\right]\right\}\rangle_{\mathbf{T}}$$

with

$$\mathbf{M^P} \equiv \mathbf{g^2}\int_{\mathbf{u,v}}\mu^{\mathbf{2}}(\mathbf{u,v})\frac{(\mathbf{x-u})_{\mathbf{i}}}{(\mathbf{x-u})^{\mathbf{2}}}\frac{(\mathbf{y-v})_{\mathbf{j}}}{(\mathbf{y-v})^{\mathbf{2}}}\,[(\mathbf{S(u)-S(x))(S^\dagger(v)-S^\dagger(y)})]^{\mathbf{ab}}$$

The T-averaging is very complicated

Let expand σ^P around $\bar{\mathbf{M}} \equiv \langle\mathbf{M^P}\rangle_{\mathbf{T}}$ (dilute projectile limit)

$$\bar{\mathbf{M}} = \mathbf{g^2}\delta^{\mathbf{ab}}\int_{\mathbf{u,v}}\mu^{\mathbf{2}}(\mathbf{u,v})\frac{(\mathbf{x-u})_{\mathbf{i}}}{(\mathbf{x-u})^{\mathbf{2}}}\frac{(\mathbf{y-v})_{\mathbf{j}}}{(\mathbf{y-v})^{\mathbf{2}}}\,\mathbf{tr}[(\mathbf{S(u)-S(x))(S^\dagger(v)-S^\dagger(y)})]/\mathbf{N_c^2}$$

$\bar{M}$ **is** *almost* **single inclusive gluon, but it is not summed over** ij

$$\sigma^{\mathbf{P}} = \mathbf{tr}\left[\frac{\bar{\mathbf{M}}}{\pi} \ln \frac{\pi e}{\bar{\mathbf{M}}}\right] - \frac{1}{2\pi} \mathbf{tr}\left[\left\{\langle(\mathbf{M}^{\mathbf{P}} - \bar{\mathbf{M}})\ (\mathbf{M}^{\mathbf{P}} - \bar{\mathbf{M}})\rangle_{\mathbf{T}}\right\}\bar{\mathbf{M}}^{-1}\right] \ldots.$$

First term is *almost* $-n \ln n$**, where** n **is a multiplicity per unit rapidity** $(dN/d\eta)$

Second term - *almost* **correlated part of double inclusive gluon production.**

Correlations between gluons decrease entropy of the produced state.

We can naturally define temperature through: $\quad \mathbf{T}^{-1} = \frac{\mathrm{d}\sigma}{\mathrm{d}\mathbf{E}_{\mathbf{T}}}$

Keeping only mean field term in the entropy: $\quad \mathbf{T} = \frac{\pi}{2}\langle\mathbf{k_T}\rangle.$

Towards correlations in symmetric collisions.

T. Altinoluk, A. Kovner, E. Levin, ML, JHEP 1404 (2014) 075

$$\frac{dN}{d^2p\,d^2k\,d\eta\,d\xi} = \langle \sigma(\mathbf{k})\ \sigma(\mathbf{p})\rangle_{P,T}$$

$$\sigma(\mathbf{k}) = \int_{\mathbf{z},\bar{\mathbf{z}},\mathbf{x_1},\bar{\mathbf{x}_1}} e^{i\mathbf{k}(\mathbf{z}-\bar{\mathbf{z}})} \vec{\mathbf{f}}(\bar{\mathbf{z}}-\bar{\mathbf{x}_1}) \cdot \vec{\mathbf{f}}(\mathbf{x_1}-\mathbf{z}) \left\{ \rho(\mathbf{x_1})[\mathbf{S}^\dagger(\mathbf{x_1}) - \mathbf{S}^\dagger(\mathbf{z})][\mathbf{S}(\bar{\mathbf{x}_1}) - \mathbf{S}(\mathbf{z})]\rho(\bar{\mathbf{x}_1}) \right\}$$

Identify target Pomeron $P_A^T(x,y) \equiv 1 - \langle S(x)S^\dagger(y)\rangle_T/N_c$

and projectile Pomeron as $P_A^P(x,y) \sim \frac{1}{\nabla^2}(x-\bar{x})\frac{1}{\nabla^2}(y-\bar{y})\,\langle\rho(\bar{x})\rho(\bar{y})\rangle_P$

after color projection algebra and some little massage

$$\frac{d\sigma}{d\eta \, dk^2 \, d\xi \, dp^2} \sim \frac{1}{k^2}\frac{1}{p^2}\int_{x,y,u,v} \cos k(x-y)\cos p(u-v)$$

$$\times \left\{ \frac{1}{4}\frac{\partial}{\partial(ij\bar{i}\bar{j})}[\bar{P}_A^T(x,y)\bar{P}_A^T(u,v)]\Delta^{ijkl}\Delta^{\bar{i}\bar{j}\bar{k}\bar{l}}\frac{\partial}{\partial(kl\bar{k}\bar{l})}[\bar{P}_A^P(x,y)\bar{P}_A^P(u,v)] \right.$$

$$\left. -\frac{8}{N_c^2}\frac{\partial}{\partial(ij\bar{i}\bar{j})}[\bar{N}_{xy}^T\bar{N}_{uv}^T\bar{Q}_{yuvx}^T]\Delta^{ijkl}\Delta^{\bar{i}\bar{j}\bar{k}\bar{l}}\frac{\partial}{\partial(kl\bar{k}\bar{l})}[\bar{N}_{yx}^P\bar{N}_{vu}^P\bar{Q}_{xvuy}^P] \right\}$$

where we have defined

$$\frac{\partial}{\partial(ijkl)} \equiv \frac{\partial}{\partial x_i}\frac{\partial}{\partial y_j}\frac{\partial}{\partial u_k}\frac{\partial}{\partial v_l}$$

$$\Delta^{ijkl} \equiv \delta^{ij}\delta^{kl} + \delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk}$$

Here $Q^T(yuvx) = tr[S(y)S^\dagger(u)S(v)S^\dagger(y)]$ **(quadrupole/B-Reggeon)**

The expression is manifestly symmetric with respect to target/projectile.