Initial State Correlations

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[with T. Altinoluk, N. Armesto, G. Beuf, and M. Lublinsky]

Initial state ("saturation") mechanism(s)

Local anisotropy A.K., M. Lublinsky : Phys.Rev. D83 (2011) 034017 (arXiv:1012.3398); Int. J. Mod. Phys. E Vol. 22 (2013) 1330001 (arXiv:1211.1928)

Density variation E. Levin a A. Rezaeian: Phys.Rev. D84 (2011) 034031 (arXiv:1105.3275)

"Glasma graphs" Dumitru, Gelis, Jalilian-Marian, Lappi: Phys.Lett. B697 (2011) 21 (arXiv:1009.5295)
Followed by serious (and successful) quantitative effort to describe data: Dusling and Venugopalan Phys.Rev.Lett. 108 (2012) 262001 (arXiv:1201.2658); arXiv:1302.7018

Q: What is the physics of "Glasma graphs"?

A: Bose enhancement of gluons in the hadronic wave function.

Bose Enhancement.

Consider bosonic state with occupation numbers $n^i(p)$:

$$|\{n^i(p)\}\rangle \equiv \prod_{i,p} \frac{1}{\sqrt{n^i(p)!}} (a_i^\dagger(p))^{n^i(p)} |0\rangle, \qquad i = 1, \ldots, N$$

Mean particle density:

$$n \equiv \langle a^{\dagger i}(x) a^i(x) \rangle = \sum_{i,p} n^i(p)$$

Density-density correlation?

$$D(x, y) \equiv \langle a^{\dagger i}(x) a^{\dagger j}(y) a^i(x) a^j(y) \rangle$$

Calculate in the momentum space:

$$\langle a^{\dagger i}(p) a^{\dagger j}(q) a^i(l) a^j(m) \rangle = \delta(p - l)\delta(q - m) \sum_i n^i(p) \sum_j n^j(q)$$

$$+\delta(p - m)\delta(q - l) \sum_i n^i(p) n^i(q)$$

So that:

$$D(x, y) = n^2 + \sum_i \left| \int \frac{d^3p}{(2\pi)^3} \, e^{ip(x-y)} n^i(p) \right|^2 ; \qquad D(p, k) = \left[\sum_i n_i(p) \right] \left[\sum_j n_j(k \right] + \delta(k - p) \sum_i (n_i(p))^2$$

Bose Enhancement in CGC?

Bose enhancement is pretty generic, but not present in classical states.
Coherent state:

$$|b(x)\rangle \equiv \exp\{i \int d^3x\, b^i(x)(a^i(x) + a^{\dagger i}(x))\} |0\rangle$$

A trivial calculation gives

$$\langle b(x)|a^{\dagger i}(x)a^i(x)|b(x)\rangle = b^i(x)b^i(x)$$

$$\langle b(x)|a^{\dagger i}(x)a^{\dagger j}(y)a^i(x)a^j(y)|b(x)\rangle = b^i(x)b^i(x)b^j(y)b^j(y)$$

so

$$D(x,y) = n(x)n(y)$$

Q: CGC is "classical fields", can they produce Bose Enhancement?
A: Yes on both counts.

First things first.

Double inclusive gluon production via “Glasma Graphs”:

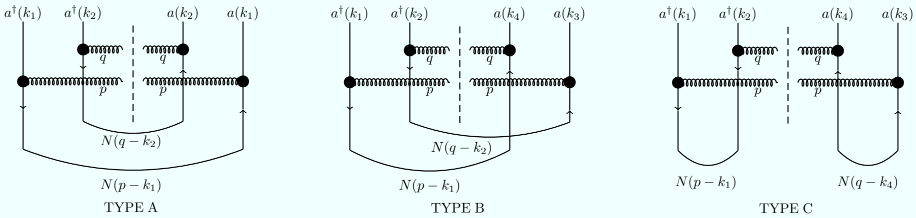

Figure: Glasma graphs for two gluon inclusive production before averaging over the incoming projectile state.

$N(k) = -\int d^2x e^{i\vec{k}\vec{x}} \langle \frac{1}{N_c} tr[S^\dagger(x)S(0)]\rangle_{\text{Target}}$ - the (adjoint) dipole scattering probability.

Gluon Production

$$\text{Type A} \propto \int_{k_1,k_2} \langle in|a^{\dagger i}_a(k_1)a^{\dagger j}_b(k_2)a^i_a(k_1)a^j_b(k_2)|in\rangle_{\text{Projectile}}\ N(p-k_1)N(q-k_2)$$

$N(k)$ - probability of momentum transfer k from the target.

Type B+Type C="upside down" Type A + "suppressed"

IMPORTANT! k - is transverse momentum only.

CGC is boost invariant: $a^i_a(k) \equiv \frac{1}{\sqrt{Y}} \int_{|\eta<Y/2|} \frac{d\eta}{2\pi}\ a^i_a(\eta, k)$

$$[a^i_a(k), a^{\dagger j}_b(p)] = (2\pi)^2\delta_{ab}\delta^{ij}\delta^{(2)}(k-p)$$

The $|in\rangle$ state.

The wave function for the soft field is classical at fixed valence color charge density:

$$|in\rangle_\rho = \exp\left\{ i \int_k b^i_a(k) \left[a^{\dagger i}_a(k) + a^i_a(-k) \right] \right\} |0\rangle,$$

Weizsäcker-Williams field $b^i_a(k) = g\rho_a(k)\frac{ik^i}{k^2}$.

Projectile is a distribution of color charge density configurations (in transverse plain) $\rho_a(\vec{x})$.

Averaging over the $|in\rangle$ state $\equiv$ averaging over the configurations of $\rho_a(\vec{x})$ with some weight function(al) $W[\rho]$.

For any observable O: $\langle O\rangle_{\rm projectile} = \int D\rho W[\rho] O[\rho]$

We will work with the "McLerran-Venugopalan model"

$$W[\rho] = N \exp\{-\int_k \frac{1}{2\mu^2(k)} \rho_a(k)\rho_a(-k)\}$$

The density matrix.

Thus the full hadronic wave function is not "classical".

This defines the density matrix (operator) on the soft gluon Hilbert space:

$$\hat{\rho} = \mathcal{N} \int D[\rho]\; e^{-\int_k \frac{1}{2\mu^2(k)}\rho_a(k)\rho_a(-k)}\, e^{i\int_q b^i_b(q)\phi^i_b(-q)}|0\rangle\langle 0|\; e^{-i\int_p b^j_c(p)\phi^j_c(-p)}$$

With MV model can integrate over ρ explicitly:

$$\hat{\rho} = e^{-\int_k \frac{g^2\mu^2(k)}{2k^4} k^i k^j\, \phi^i_b(k)\,\phi^j_b(-k)} \left\{ \sum_{n=0}^{+\infty} \frac{1}{n!} \left[\prod_{m=1}^{n} \int_{p_m} \frac{g^2\mu^2(p_m)}{p_m^4}\, p_m^{i_m}\, \phi^{i_m}_{a_m}(p_m) \right] |0\rangle \right.$$

$$\left. \times \langle 0| \left[\prod_{m=1}^{n} p_m^{j_m} \phi^{j_m}_{a_m}(-p_m) \right] \right\} e^{-\int_{k'} \frac{g^2\mu^2(k')}{2k'^4} k'^{i'} k'^{j'}\, \phi^{i'}_c(k')\,\phi^{j'}_c(-k')}$$

with

$$\phi^i_a(k) = a^i_a(k) + a^{\dagger i}_a(-k)$$

The Enhancement.

Easy to show that correlators in this $\hat{\rho}$ Wick factorize in terms of two basic elements:

$$tr[\hat{\rho}a_a^{\dagger i}(k)a_b^j(p)] = (2\pi)^2\delta_{ab}\,\delta^{(2)}(k-p)\,g^2\mu^2(p)\,\frac{p^i p^j}{p^4}$$

$$tr[\hat{\rho}a_a^i(k)a_b^j(p)] = tr[\hat{\rho}a_a^{\dagger i}(k)a_b^{\dagger j}(p)] = -(2\pi)^2\delta_{ab}\,\delta^{(2)}(k+p)\,g^2\mu^2(p)\,\frac{p^i p^j}{p^4}$$

So that:

$$tr[\hat{\rho}a_a^{\dagger i}(k_1)a_b^{\dagger j}(k_2)a_a^i(k_1)a_b^j(k_2)] = S^2(N_c^2-1)^2\left\{\frac{g^4\mu^2(k_1)\mu^2(k_2)}{k_1^2k_2^2}\right.$$
$$\left.+\frac{1}{S(N_c^2-1)}\left[\delta^{(2)}(k_1-k_2)+\delta^{(2)}(k_1+k_2)\right]\frac{g^4\mu^4(k_1)}{k_1^4}\right\}$$

The first term is the "classical" square of the density.

The last term is a *bona fide* Bose enhancement contribution.

Correlated production.

The main ("only") source of correlated production in the "glasma graph" calculation is Bose enhancement in the **initial** wave function.

Initial state Bose enhancement $\rightarrow$ correlation in the final state.

Say projectile has saturation momentum Q_s, and $|k_1, k_2| \sim Q_s$: the momentum transfer in the scattering is $< Q_s$, and $N(p - k_i)$ does not have large effect.
Initial correlations are reflected in the final state (final state interactions aside!).

PART TWO: The "other" Bose correlations: HBT

Another interesting feature of the calculation: HBT correlations for particles widely separated in rapidity.

Consider the following contraction of the graphs on the projectile side

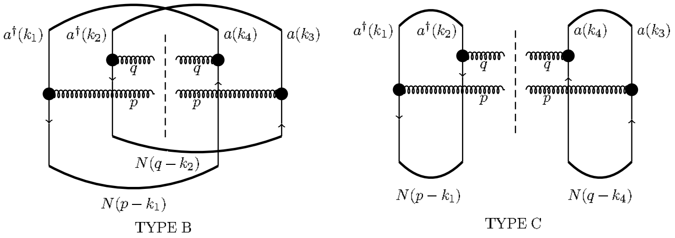

Figure: Glasma graphs for "initial state" HBT.

For translationally invariant averaging get $\delta^2(\vec{p} \pm \vec{q})$. In fact it is smeared over the inverse proton radius; $|\vec{p} \pm \vec{q}| \sim R^{-1}$.

The emitter.

Usual HBT is local in transverse and longitudinal momentum.

Here emission is directly from the initial moment of interaction: the emitter is localized in time and in longitudinal coordinate.

Only exist where the projectile and target charge densities overlap in space-time:

$$\rho_P \propto \delta(x^+); \quad \rho_T \propto \delta(x^-)$$

The emission function

$$S(x, K) \propto \int d^4 y e^{iK^\mu y_\mu} < J(x + \frac{y}{2}) J(x - \frac{y}{2}) > \propto \delta(x^-)\delta(x^+)$$

This is rapidity independent!

Transverse space structure.

In transverse space: the radius of the source is the radius of the proton R.

Randomization of sources due to color decoherence.
Only areas $\sim Q_T^2$ are correlated in color - outside color is decorrelated by the scattering.

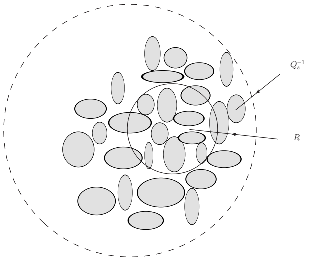

Figure: Transverse structure of the emitter.

Rapidity independent emitter of area R^2 consisting of the number of coherent sources $N \sim R^2 Q_T^2$.

HBT or Bose Enhancement?

In general correlation function has two pieces:

$$C(p,q) = \frac{\frac{dN}{dpdq}}{\frac{dN}{dp}\frac{dN}{dq}} = 1 + C_{BE}(p,q) + C_{HBT}(p,q)$$

Both $C_{BE}(p,q)$ and $C_{HBT}(p,q)$ are rapidity independent.

C_{HBT} is unsuppressed when the number of sources is large $R^2 Q_T^2 \gg 1$, but gives a narrow peak $\sim e^{-(\vec{p}-\vec{q})^2 R^2}$ (and $\sim e^{-(\vec{p}+\vec{q})^2 R^2}$).

C_{BE} - the coherent (or nonfactorizable) contribution is suppressed $\sim 1/R^2 Q_T^2$ but is "wide" in momentum space $\sim e^{-(\vec{p}-\vec{q})^2/Q_s^2}$ (and $\sim e^{-(\vec{p}+\vec{q})^2/Q_s^2}$).

For $N \sim \mathrm{several}$, both should be visible. Measure correations with better bin resolution $\Delta \sim 300 - 400 Mev$ and one should see that the ridge has structure. **Of course, provided final state interactions don't destroy the signal** - admittedly this HBT signal is fragile.

Still an interesting possibility: measure proton radius in HBT at $\delta\eta \gg 1$, this is certainly a direct initial state effect!

PART THREE: Density matrix and Entropy.

Now that we have the density matrix, we can do things with it.

$$\hat{\rho} = e^{-\int_k \frac{g^2\mu^2(k)}{2k^4} k^i k^j \phi_b^i(k)\phi_b^j(-k)} \left\{ \sum_{n=0}^{+\infty} \frac{1}{n!} \left[\prod_{m=1}^{n} \int_{p_m} \frac{g^2\mu^2(p_m)}{p_m^4} p_m^{i_m} \phi_{a_m}^{i_m}(p_m) \right] |0\rangle \right.$$

$$\left. \times \langle 0| \left[\prod_{m=1}^{n} p_m^{j_m} \phi_{a_m}^{j_m}(-p_m) \right] \right\} e^{-\int_{k'} \frac{g^2\mu^2(k')}{2k'^4} k'^{i'} k'^{j'} \phi_c^{i'}(k')\phi_c^{j'}(-k')}$$

with

$$\phi_a^i(k) = a_a^i(k) + a_a^{\dagger i}(-k); \qquad a_a^i(k) = \frac{1}{\sqrt{Y}} \int_{|\eta < Y/2|} \frac{d\eta}{2\pi} a_a^i(\eta, k)$$

Immediate thought: let us calculate entropy MV model!.

Initial wave function: Entanglement entropy of soft modes

$$\sigma^E = tr[\hat{\rho} \ln \hat{\rho}]$$

How to calculate ln?

The standard "replica trick":

$$\ln \hat{\rho} = \lim_{\epsilon \to 0} \frac{1}{\epsilon} (\hat{\rho}^{\epsilon} - 1)$$

Calculate ρ^N and take $N \to 0$. N copies of the field - replicas, do the job.
Define:

$$M_{ij} \equiv g^2 \int_{u,v} \mu^2(u,v) \frac{(x-u)_i}{(x-u)^2} \frac{(y-v)_j}{(y-v)^2} \delta^{ab}$$

The result

$$\sigma^E = \frac{1}{2} tr \left\{ \ln \frac{M}{\pi} + \sqrt{1 + \frac{4M}{\pi}} \ln \left[1 + \frac{\pi}{2M} \left(1 + \sqrt{1 + \frac{4M}{\pi}} \right) \right] \right\}$$

Calculating σ^E

Translationally invariant limit (and original MV model):

$$M^{ab}_{ij}(p) = g^2\mu^2 \frac{p_i p_j}{p^4}\delta^{ab}$$

For small M, or the UV contribution

$$\sigma^E_{UV} = tr\left[\frac{M}{\pi}\ln\frac{\pi e}{M}\right] = -\frac{N_c^2-1}{\pi} S\int_{p^2>\frac{Q_s^2}{g^2}}\frac{d^2p}{(2\pi)^2}\frac{Q_s^2}{g^2p^2}\ln\frac{Q_s^2}{eg^2\,p^2}$$

where $Q_s^2 = \frac{g^4}{\pi}\mu^2$ In all σ^E is formally UV divergent

$$\sigma^E_{UV} = \frac{Q_s^2}{4\pi g^2}(N_c^2-1)S\left[\ln^2\frac{g^2\Lambda^2}{Q_s^2}+\ln\frac{g^2\Lambda^2}{Q_s^2}\right]$$

The large M, IR contribution is

$$\sigma^E_{IR} \simeq \frac{1}{2}tr[\ln\frac{e^2M}{\pi}] = \frac{N_c^2-1}{2}S\int_{p_2<\frac{Q_s^2}{g^2}}\frac{d^2p}{(2\pi)^2}\ln\frac{e^2\,Q_s^2}{g^2p^2} = \frac{3(N_c^2-1)}{8\pi g^2}SQ_s^2$$

Properties of σ^E.

$$\sigma \approx \sigma^E_{UV} + \sigma_{IR} = \frac{SQ_s^2}{4\pi g^2}(N_c^2 - 1)\left[\ln^2 \frac{g^2\Lambda^2}{Q_s^2} + \ln \frac{g^2\Lambda^2}{Q_s^2} + \frac{3}{2}\right]$$

UV divergent: the divergence is cutoff physically at $\Lambda \sim Me^{Y_0} \gg M$, where eikonal approximation breaks down.

σ^E is not extensive in rapidity: ony one longitudinal mode (rapidity independent) is entangled with valence degrees of freedom.

Similar to "topological entropy": insensitive to boundary region between the modes.

But not quite what we would like to know.

Entropy of the produced system.

Long story short: entropy of the system of produced particles is formally very similar

$$\sigma^P = \frac{1}{2}\langle tr\left\{\ln\frac{M^P}{\pi} + \sqrt{1+\frac{4M^P}{\pi}}\ln\left[1+\frac{\pi}{2M^P}\left(1+\sqrt{1+\frac{4M^P}{\pi}}\right)\right]\right\}\rangle_T$$

with

$$M^P \equiv g^2\int_{u,v}\mu^2(u,v)\frac{(x-u)_i}{(x-u)^2}\frac{(y-v)_j}{(y-v)^2}\left[(S(u)-S(x))(S^\dagger(v)-S^\dagger(y))\right]^{ab}$$

Here $\langle\ldots\rangle_T$ is average over the target.

$$\langle M^P\rangle_T = \delta^{ab}\frac{Q_P^2\pi}{g^2}\int_z\frac{(x-z)_i}{(x-z)^2}\frac{(y-z)_j}{(y-z)^2}[P_A(x,y)+1-P_A(x,z)-P_A(z,y)]$$

P_A - S-matrix of an adjoint dipole on the target
Q_p - saturation momentum of the **projectile**.

Entropy and inclusive gluons production.

Expand σ^P around $\bar{M}$ (dilute projectile limit):

$$\sigma^P = tr\left[\frac{\bar{M}}{\pi}\ln\frac{\pi e}{\bar{M}}\right] - \frac{1}{2\pi}\, tr\left[\left\{\langle(M^P - \bar{M})\,(M^P - \bar{M})\rangle_T\right\}\bar{M}^{-1}\right]....$$

$\bar{M}$ is *almost* single inclusive gluon.

Second term - *almost* **correlated part** of double inclusive gluon production.

Correlations between gluons decrease entropy of the produced state.

We can naturally define **temperature** through: $T^{-1} = \frac{d\sigma}{dE_T}$

Keeping only mean field term in the entropy: $T = \frac{\pi}{2} < k_T >$.

Summary

Initial state has some pretty interesting and easily understandabe physics.

Perhaps it is seen in p-p (p-A) ridge?

If it is not, it would be nice to understand whether it can be seen elsewhere. What you can calculate, you should be able to measure!