Event-by-event distributions of HBT radii – how to access their moments, and what do do with them

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References:
C. Plumberg and U. Heinz, PRC 91 (2015) 054905;
and arXiv: 1507.xxxxx (to be announced on Monday)

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Overview

Motivation: Two questions

- HBT radii, which measure the size and shape of homogeneity regions of the emission function of a heavy-ion collision, fluctuate from event to event.
- Due to limited number of pairs per event, they cannot be reconstructed experimentally with any kind of precision event by event.
- Experimental HBT analyses sum over pairs from large ensembles of events before constructing the correlation function – **What kind of average or moment of the event-by-event distribution of HBT radii do they determine?**
- For flow variables, different measures are available that determine different moments of the flow probability distribution; from these the mean flow and the variance of the flow distribution can be determined. Both contain important information on the initial fluctuation spectrum and the transport properties of the medium created in heavy-ion collisions. **Can something similar be done for HBT radii? If yes, how will this information complement that extracted from flow measurements?**

Overview

Different ways to average the HBT correlator

For a single event:

$$C(\vec{p}_1,\vec{p}_2) \equiv \frac{E_{p_1}E_{p_2}\frac{d^6N}{d^3p_1 d^3p_2}}{\left(E_{p_1}\frac{d^3N}{d^3p_1}\right)\left(E_{p_2}\frac{d^3N}{d^3p_2}\right)} \approx 1 + \left|\frac{\int d^4x\, S(x,K)\, e^{iq\cdot x}}{\int d^4x\, S(x,K)}\right|^2 = 1+\lambda(\vec{K})\exp\left(-\sum_{i,j=o,s,l} R^2_{ij}(\vec{K})q_iq_j\right)$$

HBT radii from Gaussian fit to C or (approximately) from space-time variances of S:

$$R^2_{ij}(\vec{K}) = \langle(\tilde{x}_i - \beta_i\tilde{t})(\tilde{x}_j - \beta_j\tilde{t})\rangle_S$$

where $\vec{\beta} = \vec{K}/E_K$ is the pair velocity.

For an ensemble of events:

1. Average over initial conditions and evolve only once with hydro to obtain "single-shot averaged emission function" $S_{\rm ssh}(x,K)$:

$$R^2_{ij}(\vec{K}) = \langle(\tilde{x}_i - \beta_i\tilde{t})(\tilde{x}_j - \beta_j\tilde{t})\rangle_{\rm ssh}$$

2. Run hydro event-by-event with fluctuating initial conditions and compute average emission functions $\bar{S}(x,K)$ at the end ("ensemble-averaged emission function"):

$$\bar{R}^2_{ij}(\vec{K}) = \langle(\tilde{x}_i - \beta_i\tilde{t})(\tilde{x}_j - \beta_j\tilde{t})\rangle_{\bar{S}}$$

Different ways to average the HBT correlator

3. Average over signal pairs in numerator and mixed pairs in denominator over the entire ensemble before constructing the correlator ("ensemble-averaged correlation function") – this is what is done in experiment:

$$C_{\rm avg}(\vec{p}_1,\vec{p}_2) \equiv \frac{\left\langle E_{p_1} E_{p_2} \frac{d^6 N}{d^3 p_1 d^3 p_2} \right\rangle}{\left\langle E_{p_1} \frac{d^3 N}{d^3 p_1} \right\rangle \left\langle E_{p_2} \frac{d^3 N}{d^3 p_2} \right\rangle} \approx 1 + \frac{\left\langle \left| \int d^4x\, S(x,K)\, e^{iq\cdot x} \right|^2 \right\rangle}{\left| \int d^4x\, \bar{S}(x,K) \right|^2}$$

This yields the pair-multiplicity weighted average

$$R^2_{\langle ij \rangle}(\vec{K}) = \frac{\left\langle N^2(\vec{K}) R^2_{ij}(\vec{K}) \right\rangle}{\left\langle N^2(\vec{K}) \right\rangle}$$

where $N(\vec{K}) \equiv E_K(d^3N/d^3K)$ is the pair yield at momentum $\vec{K}$.

4. Construct correlation function for each individual fluctuating event after hydro evolution, extract the HBT radii event by event, and compute their "direct ensemble average" over $N_{\rm ev}$ events:

$$\left\langle R^2_{ij} \right\rangle (\vec{K}) \equiv \frac{1}{N_{\rm ev}} \sum_{i=1}^{N_{\rm ev}} \left(R^2_{ij} \right)^{(k)} (\vec{K})$$

Different ways to perform the azimuthal average

If the analysis is not done differentially in the azimuthal angle of the pair emission ("azimuthally sensitive HBT (asHBT) analysis"), there are two more options of performing the azimuthal average:

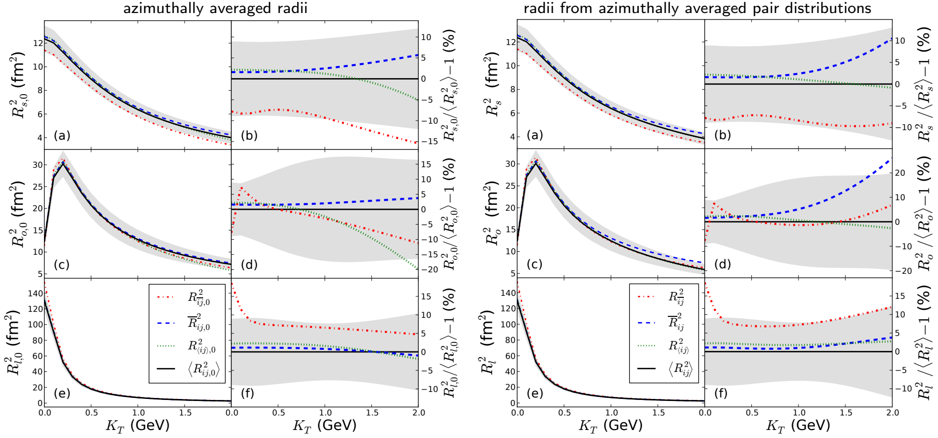
1. Average the ensemble-averaged HBT radii over the emission angle:

$$\left\langle (R^2_{ij,0})_{\rm any} \right\rangle \equiv \left\langle \left\langle (R^2_{ij})_{\rm any}(K_T, \Phi_K) \right\rangle_{\rm ev} \right\rangle_{\Phi_K}$$

Different ways to perform the azimuthal average

2. Average the numerator and denominator of the correlation function over emission angle Φ_K *before* contructing the correlator and extracting the HBT radii (this is what is done in experiment): This gives
   - single-shot hydro: $R^2_{ij}(K_T) \equiv \frac{\left\langle N^2_{\mathrm{ssh}}(K_T,\Phi_K) R^2_{ij}(K_T,\Phi_K)\right\rangle_{\Phi_K}}{\left\langle N^2_{\mathrm{ssh}}(K_T,\Phi_K)\right\rangle_{\Phi_K}}$
   - averaged emission function: $\bar{R}^2_{ij}(K_T) \equiv \frac{\left\langle \langle N(K_T,\Phi_K)\rangle^2_{\mathrm{ev}} \bar{R}^2_{ij}(K_T,\Phi_K)\right\rangle_{\Phi_K}}{\left\langle \langle N(K_T,\Phi_K)\rangle^2_{\mathrm{ev}}\right\rangle_{\Phi_K}}$
   - averaged correlation function:
     $$R^2_{\langle ij\rangle}(K_T) = \frac{\left\langle \left\langle N^2(K_T,\Phi_K) R^2_{ij}(K_T,\Phi_K)\right\rangle_{\mathrm{ev}}\right\rangle_{\Phi_K}}{\left\langle \left\langle N^2(K_T,\Phi_K)\right\rangle_{\mathrm{ev}}\right\rangle_{\Phi_K}}, = \frac{\left\langle R^2_{ij}(K_T) \left\langle N^2(K_T,\Phi_K)\right\rangle_{\Phi_K}\right\rangle_{\mathrm{ev}}}{\left\langle \left\langle N^2(K_T,\Phi_K)\right\rangle_{\Phi_K}\right\rangle_{\mathrm{ev}}}$$
   - direct ensemble average: $\left\langle R^2_{ij}\right\rangle \equiv \left\langle R^2_{ij}(K_T)\right\rangle_{\mathrm{ev}} \equiv \left\langle \frac{\left\langle N^2(K_T,\Phi_K) R^2_{ij}(K_T,\Phi_K)\right\rangle_{\Phi_K}}{\left\langle N^2(K_T,\Phi_K)\right\rangle_{\Phi_K}}\right\rangle_{\mathrm{ev}}$

Comparison of different definitions for average HBT radii



The conceptually interesting mean squared HBT radii ("direct ensemble average", solid black lines) are most closely represented by the experimental "average correlation function" and (next best) "average emission function" methods. The only difference between the mean radii and the experimentally extracted ones is a pair multiplicity weight in the experimental radii.

Estimating the true mean and variance of HBT radii distributions from measurements of ensemble-averaged correlation functions: 1. Estimating the mean

Estimating the mean of an obervable $\mathcal{O}$, defined by $\langle\mathcal{O}\rangle = \lim_{N\to\infty}\langle\mathcal{O}\rangle_N = \lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}\mathcal{O}_k$

when experiment gives us only weighted averages $\langle w\mathcal{O}\rangle_N \equiv \sum_{k=1}^{N} w_k^{(N)}\mathcal{O}_k$:

1. Order events by their weights (assumed to be measurable event-by-event).
2. Distribute all N events into n_b bins of size $n = N/n_b$ according to increasing weight. If N and n_b are large, the weight fluctuations in each bin will be small.
3. Form re-weighted bin averages of each bin $\langle w\mathcal{O}\rangle_n^{(\ell)} \equiv \frac{1}{\sum_{k\in(\ell)} w_k^{(N)}}\sum_{k\in(\ell)} w_k^{(N)}\mathcal{O}_k$.

   These are of the type that can be measured experimentally.
4. Compute $\langle\mathcal{O}\rangle_{N,\mathrm{est}} = \frac{1}{n_b}\sum_{\ell=1}^{n_b}\langle w\mathcal{O}\rangle_n^{(\ell)}$. This estimator becomes exact and equal to the true mean $\langle\mathcal{O}\rangle$ in the limit $N, n_b \to \infty$. For $N = 10,000$, its accuracy is better than 1% for $n_b \geq 10$.

2. Estimating the variance

To estimating the variance $\sigma^2_{\mathcal{O}} = \lim_{N\to\infty} \sigma^2_{\mathcal{O},N} = \lim_{N\to\infty} \frac{1}{N-1} \sum_{i=k}^{N} \left(\mathcal{O}_k^2 - \langle \mathcal{O} \rangle_N^2 \right)$, distribute the N events randomly into n_b bins of size $n = N/n_b$ and compute the bin averages $\langle \mathcal{O} \rangle_{k,\ell}$. Go through this random subdivision M times. The indices on the bin averages label the bin (ℓ) and the binning iteration k. Now compute

$$\sigma^2_{\mathcal{O},N,\mathrm{est}} \equiv \frac{N}{M n_b (n_b - 1)} \sum_{k=1}^{M} \sum_{\ell=1}^{n_b} \left(\langle \mathcal{O} \rangle_{k,\ell}^2 - \langle \mathcal{O} \rangle_N^2 \right)$$

As the number of binning iterations M approaches the maximal possible number of ways that produce different results,

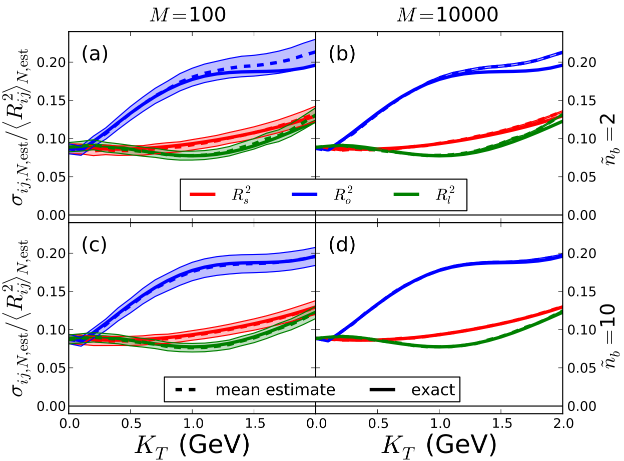
$$M_{\max}(N, n_b) \equiv \binom{N}{n} \binom{N-n}{n} \cdots \binom{2n}{n} = \frac{N!}{(n!)^{n_b}} = \frac{N!}{((N/n_b)!)^{n_b}}$$

this estimator exactly reproduces the variance $\sigma^2_{\mathcal{O},N}$. But $M_{\max}$ is astronomical. Fortunately, quite moderate M values already produce excellent estimators.

The bin means needed in the definition of the variance estimator are computed with the algorithm explained before, by subdividing the n events into $\tilde{n}_b$ sub-bins according to increasing weights.

An example

To test the method, we generated $N = 5,000$ fluctuating hydro events and applied the algorithm to estimate the variances of their HBT radii:



Overview

Conclusions

1. The means and variances of the (squared) HBT radii contain interesting information about the initial fluctuations and transport properties characterizing the medium created in relativistic heavy-ion collisions. Look at PRC 91 (2015) 054905 for a study of their sensitivity to the temperature dependence of η/s.

2. The means and variances of the (squared) HBT radii can be extracted from ensemble-averaged HBT measurements by the algorithm reported here.

The End