Pre-equilibrium evolution effects on heavy-ion collision observables

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In collaboration with Jia Liu and Chun Shen

Reference:
J. Liu, C. Shen and U. Heinz, PRC 91 (2015) 064906

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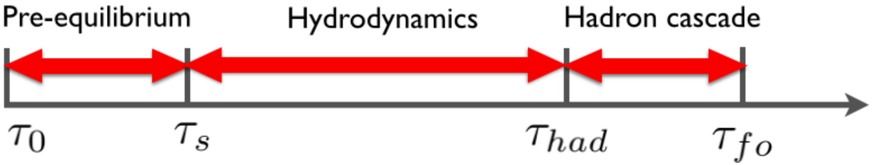
1. Motivation
2. Free-streaming and Landau matching
   - Free-streaming
   - Landau matching
3. Hydrodynamic evolution
   - Hydrodynamic initial conditions after free-streaming
   - Effects of free-streaming on hydro evolution
4. Constraining τ_s from data
   - A technical issue: how to treat the non-thermalized halo
5. Parameter optimization
6. Conclusions

Motivation

- Relativistic viscous hydrodynamics has become the workhorse of dynamical modeling of ultra-relativistic heavy-ion collisions
- Hydrodynamics does not become valid until the medium has reached a certain degree of local momentum isotropization
- In an inhomogeneous system, collective flow (i.e. space-momentum correlations) begin, however, to develop already before hydrodynamics becomes valid.
- The hydrodynamic stage thus starts with a non-vanishing pre-equilibrium flow.
- **Goal: To perform a systematic study of pre-equilibrium flow effects on heavy-ion collision observables.**

Weak vs. strong coupling



- Here: model pre-equilibrium stage by kinetic theory (no mean fields, no plasma instabilities)
- **Weak coupling:** very few collisions, long thermalization time: $\tau_s \gg \tau_0$
- **Strong coupling:** frequent collisions, very rapid thermalization: $\tau_s \approx \tau_0$
- $\Longrightarrow$ Use τ_s to parametrize the rate of approach to hydrodynamic behavior;
  model the period $\tau_0 < \tau < \tau_s$ by free-streaming massless degrees of freedom (extreme weak-coupling limit)

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Collisionless BE: $p^\mu \partial_\mu f(x,p) = 0.$

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To match to VISH2+1, impose longitudinal boost invariance:

$$f(\boldsymbol{x}_\perp, \eta_s, \tau;\, \boldsymbol{p}_\perp, y) = \frac{\delta(y-\eta_s)}{\tau m_\perp \cosh(y-\eta_s)} \tilde{f}(\boldsymbol{x}_\perp, \tau;\, \boldsymbol{p}_\perp, y).$$

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Assume massless degrees of freedom. Analytic soln. of collisionless BE:

$$f(\boldsymbol{x}_\perp, \eta_s, \tau_s; \boldsymbol{p}_\perp, y) = f(\boldsymbol{x}_\perp - (\tau_s - \tau_0)\hat{\boldsymbol{p}}_\perp, \eta_s, \tau_0; \boldsymbol{p}_\perp, y).$$

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Energy-momentum tensor $T^{\mu\nu}(x) = \frac{g}{(2\pi)^3} \int \frac{d^3p}{E} p^\mu p^\nu f(x,p)$

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Energy-momentum tensor $T^{\mu\nu}(x) = \frac{g}{(2\pi)^3} \int \frac{d^3p}{E} p^\mu p^\nu f(x,p)$

Need it only at midrapidity $\eta_s = 0$ (boost invariance):

$$T^{\mu\nu}(\boldsymbol{x}_\perp, \eta_s{=}0, \tau) = \frac{1}{\tau} \int_{-\pi}^{\pi} d\phi_p\, \hat{p}^\mu \hat{p}^\nu F(\boldsymbol{x}_\perp, \tau; \phi_p),$$

where

$$F(\boldsymbol{x}_\perp, \tau; \phi_p) = F_0(\boldsymbol{x}_\perp - (\tau-\tau_0)\hat{\boldsymbol{p}}_\perp) = \frac{g}{(2\pi)^3} \int_0^\infty p_\perp^2 dp_\perp \tilde{f}(\boldsymbol{x}_\perp - (\tau-\tau_0)\hat{\boldsymbol{p}}_\perp, \tau_0; p_\perp, 0)$$

is independent of how $\tilde{f}$ depends on the magnitude of $p_\perp$, and $F_0(\boldsymbol{x}_\perp)$ is the spatial distribution function at $\tau = \tau_0$.

Landau matching I:

Hydrodynamic form of energy-momentum tensor:

$$T^{\mu\nu}_{\rm hyd} = e u^\mu u^\nu - (\mathcal{P} + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

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Project with LRF spatial projector $\Delta_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$ to get bulk viscous pressure Π:

$$\Pi = -\frac{1}{3}\mathrm{Tr}(\Delta_{\mu\nu} T^{\mu\nu}) - \mathcal{P}$$

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Use double projector $\Delta^{\mu\nu}_{\alpha\beta} \equiv \frac{1}{2}\left(\Delta^\mu_\alpha\Delta^\nu_\beta + \Delta^\mu_\beta\Delta^\nu_\alpha\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}$ to get $\pi^{\mu\nu}$:

$$\pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta}T^{\alpha\beta}$$

Alternatively $\pi^{\mu\nu} = T^{\mu\nu} - e u^\mu u^\nu + (\mathcal{P}+\Pi)\Delta^{\mu\nu}$.

Landau matching II: Entropy generation and bulk pressure

Free-streaming preserves entropy (collisionless!)
After matching to hydro, entropy density is given by EOS from $s = \partial\mathcal{P}/\partial T$

$\Longrightarrow$ s jumps (increases) after Landau matching.

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For successful phenomenology, normalize entropy density profile after Landau matching such that, upon completion of the dynamical evolution, it correctly reproduces the observed final multiplicity dN_{ch}/dy.

In spite of the entropy jump at τ_s, the normalization of the entropy density profile after Landau matching has a one-to-one relation with the normalization of the initial distribution function. Since the entropy jump depends on τ_s, so does the initial normalization. As τ_s is varied, the normalization is adjusted to preserve dN_{ch}/dy.

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Since $m = = 0$, $\Pi = 0$ in the free-streaming stage. Since EOS from latttice QCD breaks conformal symmetry, $\Pi \neq 0$ after Landau matching. We here set $\zeta = 0$ and let Π evolve back to zero with IS EOM over a short relaxation time τ_Π

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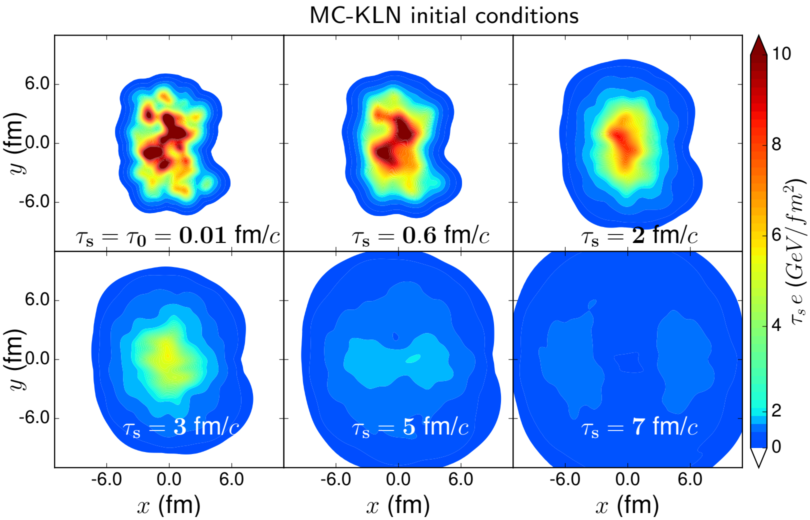
Initial LRF energy density profile after free-streaming:



Initial radial flow after free-streaming:

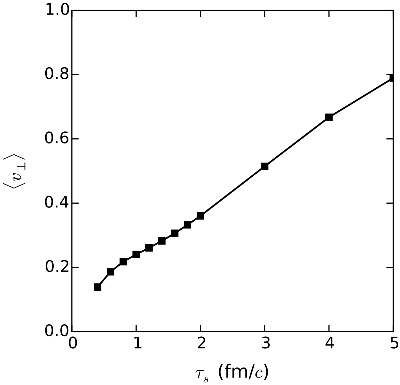
$$\{v_\perp\} = \frac{\int d^2 r_\perp \gamma(\boldsymbol{r}_\perp)\, v_\perp(\boldsymbol{r}_\perp)\, e(\boldsymbol{r})_\perp}{\int d^2 r_\perp \gamma(\boldsymbol{r}_\perp)\, e(\boldsymbol{r}_\perp)},$$

$$\langle v_\perp \rangle = \frac{1}{N_{events}} \sum_{i=1}^{N_{events}} \{v_\perp\}^{(i)}.$$

Rises initially very quickly, reaching 25% of speed of light after 1 fm/c

Continues to grow over the next 5 fm/c at an approximate rate $\langle a_\perp \rangle \approx \frac{d\langle v_\perp \rangle}{d\tau_s} = 0.13\, c^2/\text{fm}$.

Free-streaming should yield an upper limit for this radial pre-equilibrium flow.



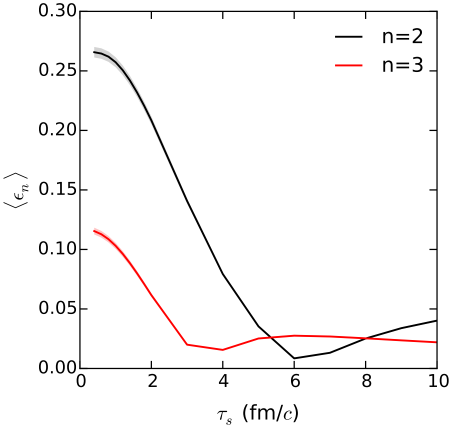
Initial eccentricities after free-streaming:

Eccentricities drive anisotropic flow:

$$\mathcal{E}_n(\tau_s) = \epsilon_n(\tau_s) e^{in\Phi_n(\tau_s)}$$
$$= -\frac{\int_{\tau_s} d^3\sigma_\mu(x)\, T^{\mu\nu}_{\rm hyd}(x)\, u_\nu(x)\, r_\perp^n\, e^{in\phi}}{\int_{\tau_s} d^3\sigma_\mu(x)\, T^{\mu\nu}_{\rm hyd}(x)\, u_\nu(x)\, r_\perp^n}$$
$$= -\frac{\int d^2 r_\perp \gamma(\boldsymbol{r}_\perp)\, e(\boldsymbol{r}_\perp)\, r_\perp^n\, e^{in\phi}}{\int d^2 r_\perp \gamma(\boldsymbol{r}_\perp)\, e(\boldsymbol{r}_\perp)\, r_\perp^n},$$

$$(n > 1)$$

Note, this counts only contributions from fluid elements, not from cells that are already frozen out after Landau matching!



Initial shear stress I:

Inverse Reynold number measures importance of first-order viscous stress relative to ideal hydro pressure:

$$R^{-1} = \frac{\sqrt{\pi^{\mu\nu}\pi_{\mu\nu}}}{-\Delta^{\mu\nu}T_{\mu\nu}/3} = \frac{\sqrt{\pi^{\mu\nu}\pi_{\mu\nu}}}{\mathcal{P}+\Pi}$$

[For conformal systems, $\mathcal{P}=e/3$ and $\Pi=0$.]
Initial value can be calculated from free-streamed distribution function:

$$\pi^{\mu\nu}\pi_{\mu\nu} = \int \frac{g\, d^3p}{(2\pi)^3 p^0} \int \frac{g\, d^3p}{(2\pi)^3 p'^0} \left[(\boldsymbol{p}\cdot\boldsymbol{p}')^2 - \frac{1}{3}\boldsymbol{p}^2\boldsymbol{p}'^2 \right] f(p) f(p')$$

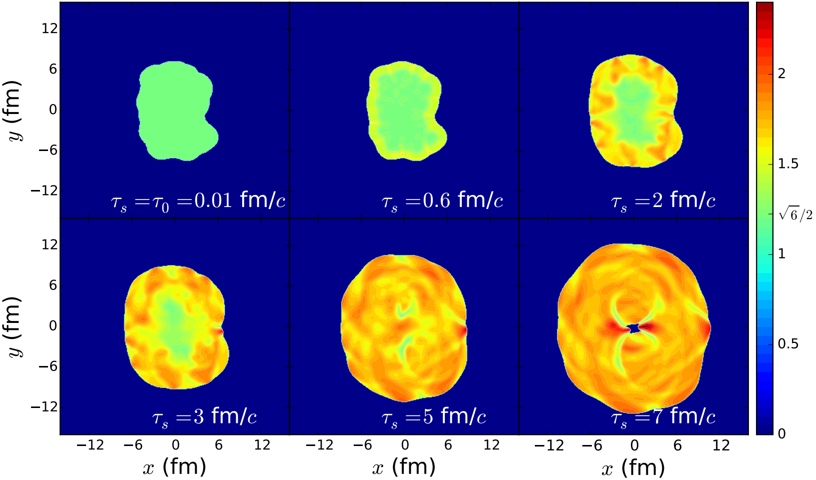
The value at τ_0 (before onset of free-streaming) can be worked out exactly:

$$\pi^{\mu\nu}\pi_{\mu\nu}\Big|_{\tau_0} = \frac{2\pi^2}{3} C^2, \qquad \mathcal{P}+\Pi = \frac{2\pi}{3} C,$$

where $C \equiv \frac{g}{\tau_0} \int \frac{1}{(2\pi)^3} p_\perp^2\, dp_\perp\, \tilde{f}(p_\perp)$.

Hence, $\boldsymbol{R^{-1}\big|_{\tau_0,\eta_s=0} = \sqrt{3/2} \approx 1.225}$.

Initial shear stress II: Inverse shear Reynolds number after Landau matching



Effects of pre-flow on final radial flow

Initial conditions rescaled such that

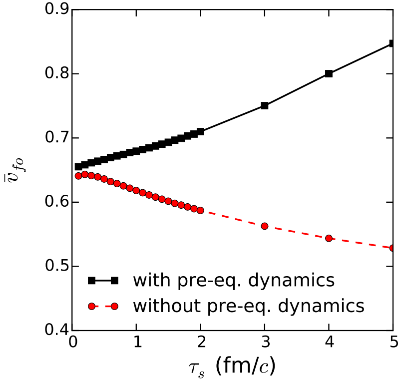
$$\frac{dE}{dy}\Big|_{\Sigma_{\rm fo}} = \frac{dE}{d\eta_s}\Big|_{\Sigma_{\rm fo}} = \int_{\Sigma_{\rm fo}} T^{0\mu}(x)\, \frac{d^3\sigma_\mu(x)}{d\eta_s}$$

is held fixed when varying τ_s (see discussion below).

For $T_{\rm dec} = {\rm const.} = 120\,{\rm MeV}$

$$v_{\rm fo} \equiv \frac{\int_{\Sigma_{\rm fo}} u^\mu d^3\sigma_\mu\, v_\perp\, e}{\int_{\Sigma_{\rm fo}} u^\mu d^3\sigma_\mu\, e} = \frac{\int_{\Sigma_{\rm fo}} u^\mu d^3\sigma_\mu\, v_\perp}{\int_{\Sigma_{\rm fo}} u^\mu d^3\sigma_\mu}$$

$v_{\rm fo}$ controls slope if final hadron p_T spectra.



Strong effect from pre-flow on final radial flow for all values of τ_s!

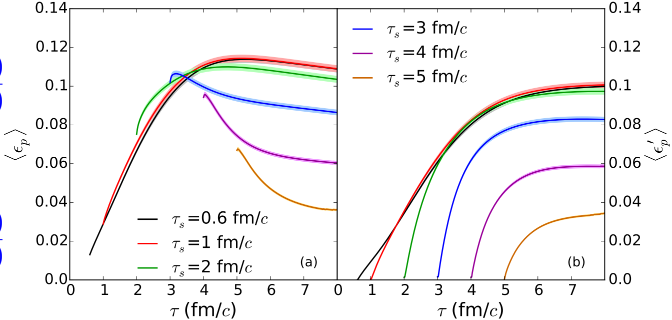
Effects of pre-flow on final flow anisotropy

Total momentum anisotropy

$$\epsilon'_p = \frac{\int d^2r_\perp (T^{xx} - T^{yy})}{\int d^2r_\perp (T^{xx} + T^{yy})}$$

Momentum anisotropy from collective flow

$$\epsilon_p = \frac{\int d^2r_\perp (T^{xx}_{\rm id} - T^{yy}_{\rm id})}{\int d^2r_\perp (T^{xx}_{\rm id} + T^{yy}_{\rm id})}$$



Calculated by rotating for each event $T^{\mu\nu}$ in transverse plane to maximize ϵ'_p or ϵ_p, respectively.

Not much effect from preflow on final momentum anisotropy unless $\tau_s > 2\,\mathrm{fm}/c$.

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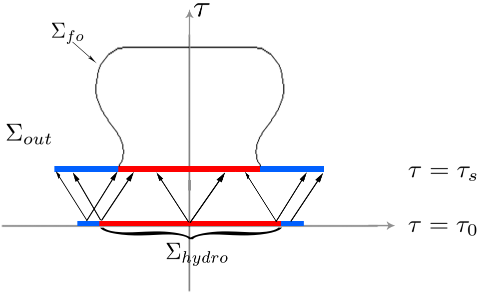
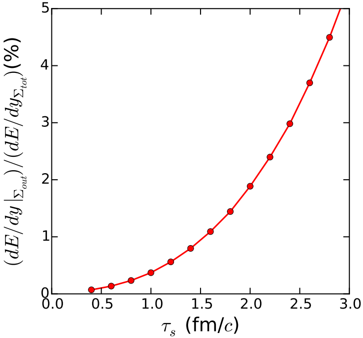
Avoiding particle loss from the non-thermalized halo





For large τ_s we have a large halo of partons that never become part of the fluid since the density is too low $\Longrightarrow$ big problem, since we don't know how to convert them correctly to final hadrons, and we need hadron spectra to compute v_n

But we know how to account for their energy!

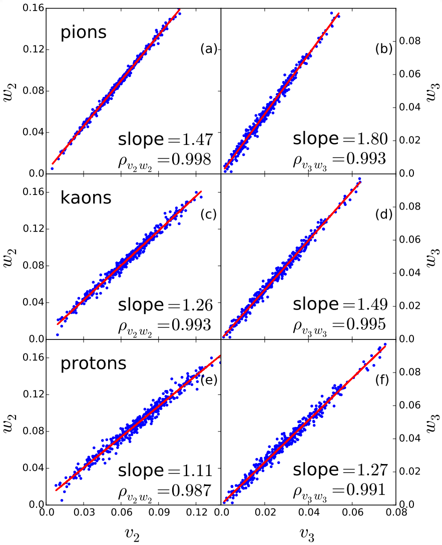
Way out: use energy flow rather than particle flow to define anisotropic flow coefficients!

Anisotropic energy flow coefficients w_n as proxy for v_n

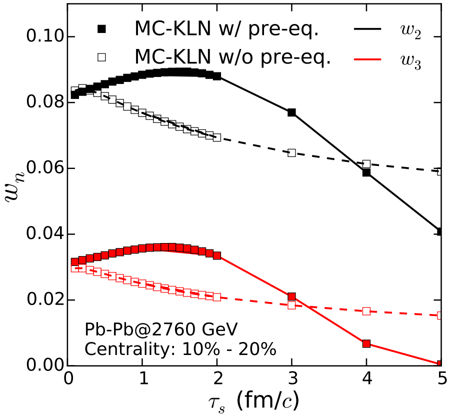
- $dE/dyd\phi$ receives contributions from $\Sigma_{\rm out}$ (non-thermalized parton halo) and $\Sigma_{\rm fo}$ (thermal emission from the liquid at freeze-out)
- **Anisotropic energy flow coefficients:**

$$w_n e^{in\bar{\Psi}_n} = \frac{\int \frac{dE}{dyd\phi_p} e^{in\phi_p}\, d\phi_p}{\int \frac{dE}{dyd\phi_p}\, d\phi_p}$$

- Plot shows that, for small τ_s when surface loss through $\Sigma_{\rm out}$ can be neglected, $v_n^{\pi,K,p}$ are all tightly linearly correlated with w_n for $n = 2, 3$.
- $\Rightarrow$ **use $w_{2,3}$ as proxy for $v_{2,3}$ for all τ_s**

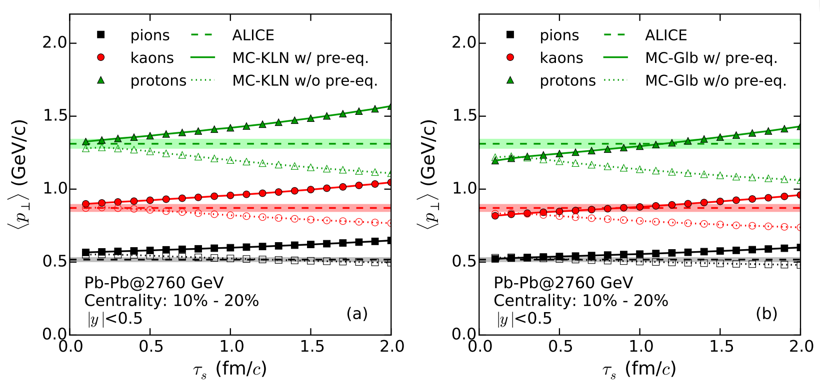


Constraints from elliptic and triangular flow



- Without pre-flow, $w_{2,3}$ drop quickly with increasing τ_s, due to decreasing initial eccentricity $\Longrightarrow$
  old lore: *"Large anisotropic flow requires fast thermalization"*
- With pre-flow, $w_{2,3}$ do not begin to decrease appreciably until $\tau_s > 2\,\mathrm{fm}/c$ $\Longrightarrow$
  new lore: *"Large anisotropic flow almost independent of τ_s unless $\tau_s > 2\,\mathrm{fm}/c$"*
- For very weakly interacting pre-equilibrium stage, pre-flow more than compensates for the decrease in eccentricity during the first $2\,\mathrm{fm}/c$

Constraints from $p_\perp$-spectra



- Without pre-flow, $\bar{v}_{\rm fo}$ and $\langle p_\perp \rangle$ *decrease* with increasing $\tau_s \Longrightarrow$ old lore: *"Without fast thermalization not enough radial flow"*
- With pre-flow, $\bar{v}_{\rm fo}$ and $\langle p_\perp \rangle$ *increase* with increasing $\tau_s \Longrightarrow$ new lore: *"Without fast thermalization too much radial flow"*
- $\Longrightarrow$ **Once pre-flow is properly accounted for, $\langle p_\perp \rangle$ yields tighter constraint on τ_s than anisotropic flow** (especially for MC-KLN initial conditions)

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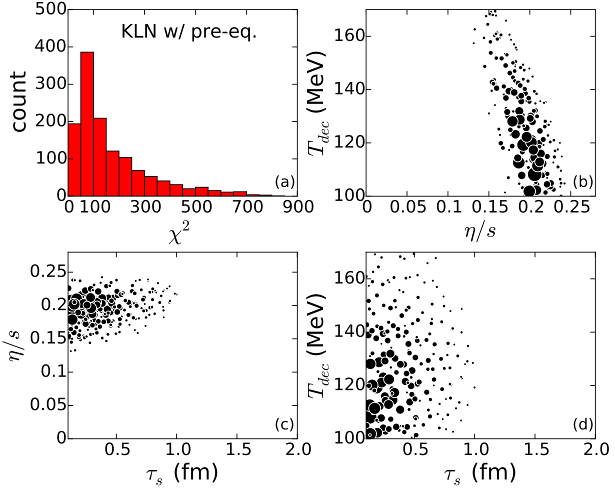
Parameter optimization

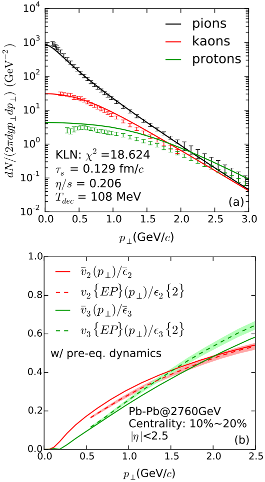
Vary three parameters simultaneously that all have strong influence on spectra and anisotropic flow: τ_s, η/s, $T_{\rm dec}$.

Fit five observables measured in 2.76 A TeV Pb+Pb collisions at 10-20% centrality: $\langle v_2^{\rm ch}\rangle$, $\langle v_3^{\rm ch}\rangle$, $\langle p_\perp\rangle_{\pi^+}$, $\langle p_\perp\rangle_{K^+}$, $\langle p_\perp\rangle_p$.

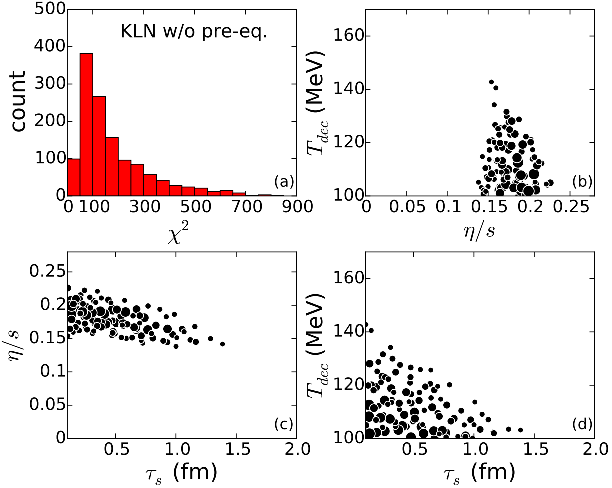
$\langle v_2^{\rm ch}\rangle$	0.0782 ± 0.0019
$\langle v_3^{\rm ch}\rangle$	0.0316 ± 0.0008
$\langle p_\perp\rangle_{\pi^+}$ (GeV/c)	0.517 ± 0.017
$\langle p_\perp\rangle_{K^+}$ (GeV/c)	0.871 ± 0.027
$\langle p_\perp\rangle_p$ (GeV/c)	1.311 ± 0.034

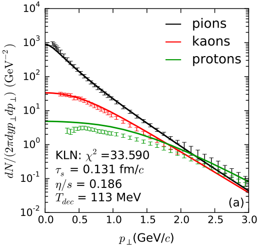
Parameter optimization: MC-KLN ICs with pre-flow

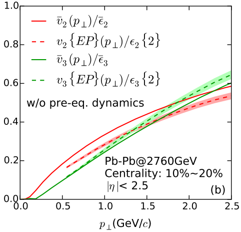




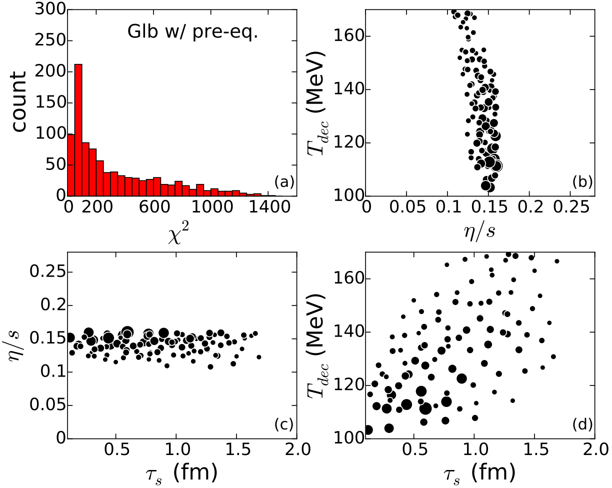
Parameter optimization: MC-KLN ICs without pre-flow

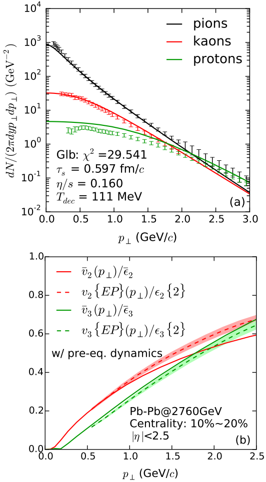




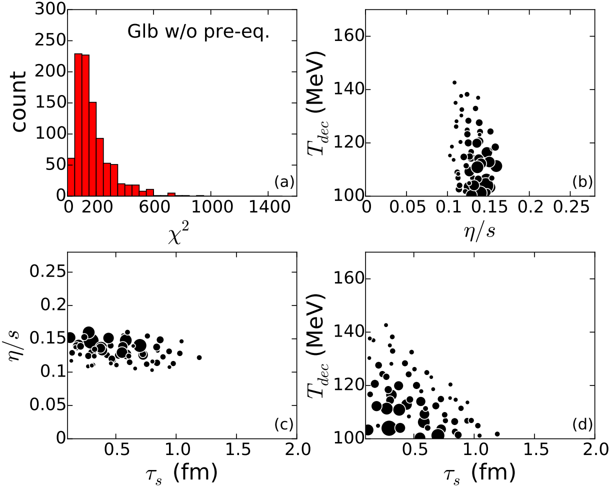


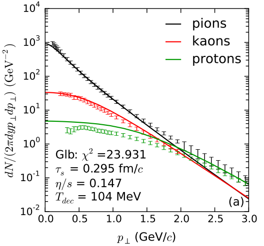
Parameter optimization: MC-Glauber ICs with pre-flow

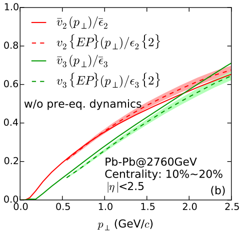




Parameter optimization: MC-Glauber ICs without pre-flow







Optimal parameter sets

Model	Pre-eq.	τ_s (fm/c)	η/s	T_{dec} (MeV)
MC-KLN	Yes	0.129	0.206	108
MC-KLN	No	0.131	0.186	113
MC-Glb	Yes	0.597	0.160	111
MC-Glb	No	0.295	0.147	104

Best-fit parameters

Model Pre-eq.	MC-KLN Yes	MC-KLN No	MC-Glb Yes	MC-Glb No
$\bar{v}_2^{\mathrm{ch}}$	0.083	0.088	0.070	0.071
$\bar{v}_3^{\mathrm{ch}}$	0.030	0.030	0.034	0.034
$\langle p_\perp \rangle_{\pi^+}$ (GeV/c)	0.550	0.545	0.539	0.518
$\langle p_\perp \rangle_{K^+}$ (GeV/c)	0.900	0.877	0.869	0.852
$\langle p_\perp \rangle_p$ (GeV/c)	1.349	1.302	1.293	1.279
χ^2	18.624	33.590	29.541	23.931

Model predictions for the five observables using the best-fit parameters.

Overview

1 Motivation

2 Free-streaming and Landau matching
- Free-streaming
- Landau matching

3 Hydrodynamic evolution
- Hydrodynamic initial conditions after free-streaming
- Effects of free-streaming on hydro evolution

4 Constraining τ_s from data
- A technical issue: how to treat the non-thermalized halo

5 Parameter optimization

6 Conclusions

Conclusions

- Inclusion of pre-equilibrium evolution in hydrodynamical modeling of heavy-ion collisions **is important**.
- **Qualitatively different conclusions** as to how long the pre-equilibrium stage can last are obtained from comparisons with data when its effects on flow development are included.
- **Mean transverse momenta** of hadrons (radial flow) **provide tighter upper limits on** the duration τ_s of the pre-equilibrium stage **than anisotropic flow coefficients**.
- When pre-flow is properly accounted for, **estimates for η/s tend to slightly increase, more time is allowed for the system to approach thermal equilibrium, and the sensitivity to the choice of decoupling temperature decreases**.

The End