Viscous Effects on the Mapping of the Initial to Final State in Heavy Ion Collisions

F.Gardim, J.Noronha-Hostler, M.Luzum, FG, PRC91 (2015) 034902,
F.Gardim, FG, M.Luzum, J.-Y.Ollitrault, PRC85 (2012) 024908.

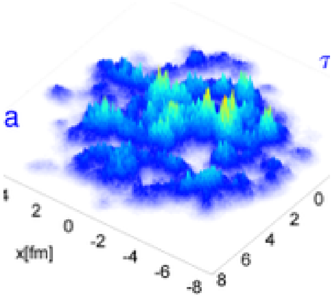
Frédérique Grassi
Universidade de São Paulo-Brazil

INT 2015

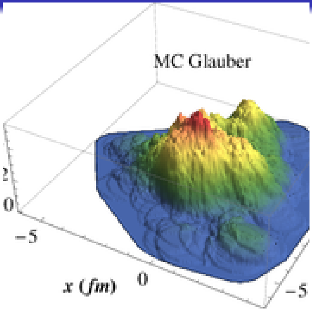
Outline

Examples of Initial Conditions

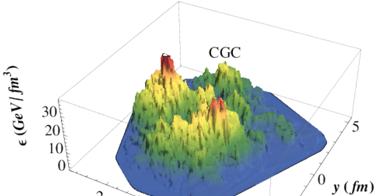
Energy Density profile

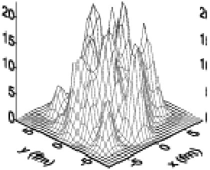


Schenke, Tribedy, Venugopalan,Phys.Rev.Lett. 108 (2012) 252301



Drescher,Nara, Phys. Rev. C 75, 034905 (2007); ,76, 041903 (2007).

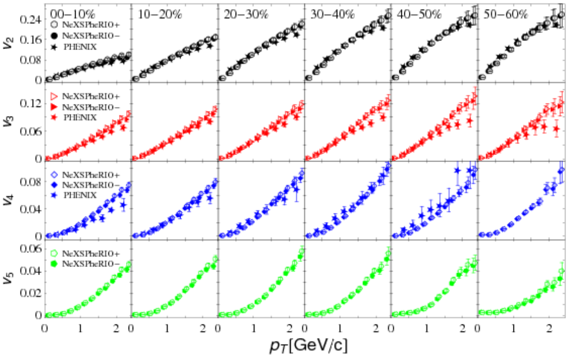




Drescher,Hladik,Ostapchenko,Pierog, Werner, Phys.Rept. 350, 93 (2001)

Hydrodynamics prediction for flow harmonics

NeXSPheRIO (ideal hydro) fits the flow harmonics well



Gardim, Grassi, Luzum, Ollitrault, Phys.Rev.Lett. 109 (2012) 202302

So do models with shear viscosity and shear+bulk viscosities.

Objective

Study the mapping between initial conditions and flow harmonics.

Many works on the subject:

- H. Petersen, G. Y. Qin, S. A. Bass and B. Muller, Phys.Rev. C82 , 041901 (2010)
- B. Schenke, S. Jeon and C. Gale, Phys. Rev. Lett.106, 042301 (2011)
- Z. Qiu and U. W. Heinz, Phys. Rev. C84 , 024911 (2011)
- Z. Qiu, C. Shen and U. Heinz, Phys. Lett. B707, 151(2012)
- B. Alver and G. Roland, Phys. Rev. C81, 054905 (2010) [Erratum-ibid. C82 , 039903 (2010)]
- B. H. Alver, C. Gombeaud, M. Luzum and J. Y. Ollitrault, Phys. Rev. C82, 034913 (2010)
- M. Luzum and H. Petersen, J. Phys. G41, 063102 (2014)
- F. G. Gardim, F. Grassi, M. Luzum and J. -Y. Ollitrault, Phys. Rev. C85, 024908 (2012)
- H. Niemi, G. S. Denicol, H. Holopainen and P. Huovinen, Phys. Rev. C87, 054901 (2013)
- ...

Here: systematic study.

Characterizing the initial state

Use the cumulant expansion of Teaney & Yan PRC 83, 064904 (2011):

$$\rho(\mathbf{x}) \sim \text{energy or entropy density}$$

$$\longrightarrow \quad \rho(\mathbf{k}) = \int d^2x \rho(\mathbf{x}) e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$\longrightarrow \quad W(\mathbf{k}) \equiv \ln \rho(\mathbf{k}) = \sum_{n=-\infty}^{\infty} \sum_{m \geq |n|} W_{n,m} k^m e^{-in\phi_k}$$

Useful properties:

- n specifies the rotation symmetry and only $n \geq 0$ matters (because $W_{-n,m} \propto W^*_{n,m}$),
- $m - n$ is even and $m \geq n$,
- Hydro sensitive only to large scale $\Rightarrow$ truncate the sum $m \leq m_{max}$.

Examples ($\{\}$= weighted average over $\rho(\mathbf{x})$):
$W_{1,1} = 0$, $W_{0,2} \propto \{r^2\}$, $W_{2,2} \propto \{r^2 e^{i2\phi}\}$, $W_{1,3} \propto \{r^3 e^{i\phi}\}$ etc.
So: $\epsilon_2 = |2W_{2,2}/W_{0,2}|$, $\epsilon_1 \propto |W_{1,3}|$ etc.

Find a best estimator $V_{est,n}$

Step 1: Find $V_{est,n}$ computed from the intial state properties, estimator for
$V_n = v_n e^{in\Psi_n}$, complex flow coefficient, obtained from hydro.

Since $W_{n\neq 0,m}$ small, write (including only cumulants leading to same rotation properties as V_n):

$$V_{est,n} = \sum_{m=n}^{m_{max}} k_{n,m} W_{n,m} + \sum_{l=1}^{m_{max}} \sum_{m=l}^{m_{max}} \sum_{m'=|n-l|}^{m_{max}} K_{l,m,m'} W_{l,m} W_{n-l,m'} + O(W^3).$$

It is common to use the eccentricities to build estimators but they are not cumulants. $\varepsilon_{n,m} e^{i\Phi_{n,m}} \equiv -\frac{\{r^m e^{in\phi}\}}{\{r^m\}}$ (We can revert to them in the end.)

Step 2: Make the best estimator

For simplicity, suppose $V_{est,n}$ has only one term and re-write

$$V_{est,n} = k_{n,m} W_{n,m} = k\, \mathcal{V}_{est,n}$$

with k a real scaling coefficient and $\mathcal{V}_{est,n}$ unscaled.

Define the error vector $V_n \equiv V_{est,n} + \mathcal{E}_n = k\, \mathcal{V}_{est,n} + \mathcal{E}_n$

Minimize $\langle |\mathcal{E}_n|^2 \rangle$ in a given centrality bin

$$\Rightarrow \begin{cases} k = \dfrac{\mathrm{Re}\left\langle V_n \mathcal{V}^*_{est,n} \right\rangle}{\left\langle |\mathcal{V}_{est,n}|^2 \right\rangle} \\ \left\langle |\mathcal{E}_n|^2 \right\rangle = \left\langle |V_n|^2 \right\rangle - \left\langle |V_{est,n}|^2 \right\rangle . \end{cases}$$

From this last equation, we define the quality of the estimator:

$$Q_n^2 \equiv \frac{\left\langle |V_{est,n}|^2 \right\rangle}{\left\langle |V_n|^2 \right\rangle} = 1 - \frac{\left\langle |\mathcal{E}_n|^2 \right\rangle}{\left\langle |V_n|^2 \right\rangle}$$

The closer Q_n to 1, the better the estimator.

Effect of viscosity on fluid evolution

Based on J.Noronha-Hostler et al. PRC88(2013)044916, PRC90(2014)034907

Equations of Motion

Conservation of Energy and Momentum

$$D_\mu T^{\mu\nu} = 0,$$

The energy-moment tensor contains a bulk viscous pressure Π and shear stress tensor $\pi^{\mu\nu}$

$$T^{\mu\nu} = (\epsilon + p + \Pi)\, u^\mu u^\nu - (p + \Pi)\, g^{\mu\nu} + \pi^{\mu\nu}$$

Using memory function method and minimal IS description

$\tau_\Pi u^\mu D_\mu \Pi + \Pi = -(\zeta + \tau_\Pi \Pi)\, D_\mu u^\mu$

$\tau_\pi \Delta^{\mu\nu\lambda\rho} u^\alpha D_\alpha \pi_{\lambda\rho} + \pi^{\mu\nu} = \eta\sigma^{\mu\nu} - \tau_\pi \pi^{\mu\nu} D_\alpha u^\alpha$ (standard notations)

$\rightarrow$ 4 transport coefficients ζ, η, τ_Π, τ_π, which depend on T.

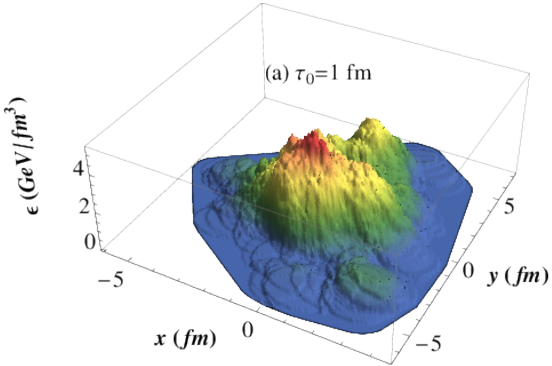
$\pi^{\mu\nu}_{Navier-Stokes} = \eta\sigma^{\mu\nu}$: prevents deformations of fluid cell.

$\Pi_{Navier-Stokes} = -\zeta D_\mu u^\mu$: negative pressure, slowing expansion

Effect of viscosity on fluid evolution

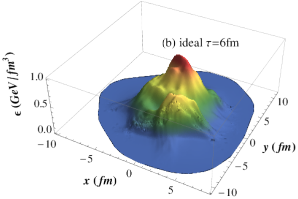
Initial Conditions:

- MC-Glauber: energy density $= cn_{coll}(\vec{r})$ (c adjusted to get midrapidity multiplicity)
- $\tau_0 = 1$ *fm* (tested)

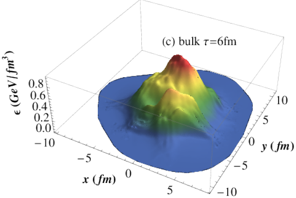


Effect of viscosity on fluid evolution

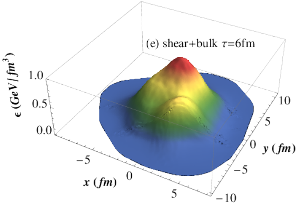
Ideal after $\tau = 5.6fm$



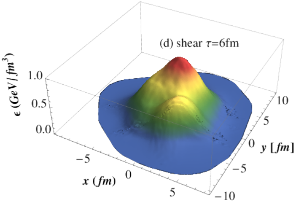
Bulk after $\tau = 5.6fm$



Shear+Bulk after $\tau = 5.6fm$



Shear after $\tau = 5.6fm$



- Viscosity attenuates other forces $\rightarrow$ smearing of granularity.
- Shear dominates, bulk barely affects expansion ($\zeta/s << \eta/s$)

Effect of viscosity on particle emission

Compute observables with Cooper-Frye formula:

Particle spectra: $E\frac{d^3N}{dp^3} = \int_{f.o.} f(x,p) p^\mu d\sigma_\mu$

$f = f_{eq} + \delta f_{shear} + \delta f_{bulk}$

δf_{shear}

Common ansatz: $\delta f_{shear} \sim \pi_{\mu\nu} p^\mu p^\nu / [(\epsilon + p) T^2]$.

Navier-Stokes limit, $\delta f_{shear} \propto (\eta/s) p^2$

$\rightarrow$ stronger effect for larger η/s and p.

δf_{bulk}

Using method of moments as in Denicol, Niemi NPA904-905 (2013) 369c

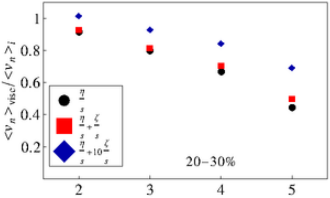
$\delta f_{bulk}^{(\pi)} = f_{eq} \times \Pi \times [B_0^{(\pi)} + D_0^{(\pi)} u.p + E_0^{(\pi)} (u.p)^2]$

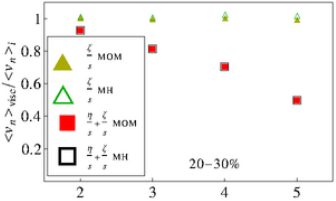
where

$B_0^{(\pi)} = -65.85\ fm^4$, $D_0^{(\pi)} = 171,27\ fm^4/GeV$, $E_0^{(\pi)} = -63.05\ fm^4/GeV^2$

Effect of viscosity on particle emission

Integrated V_n's

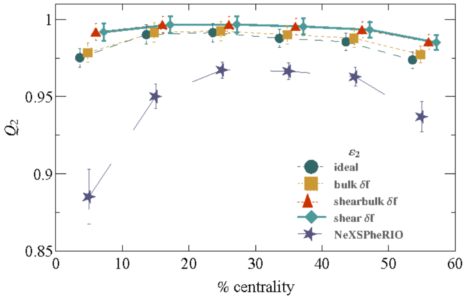




- sensitive to value of viscosities
- not to choice of δf_{bulk} (though the $V_n(p_T)$'s are).

v_2 from ε_2

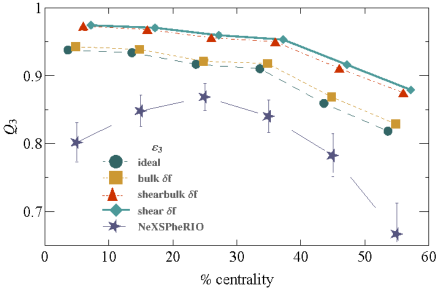
$$V_2 \sim V_{est,2} = k_{2,2} W_{2,2} + O(W^3) \sim k\, \varepsilon_2 e^{i2\Phi_2} \text{ if } m_{max} = 2.$$

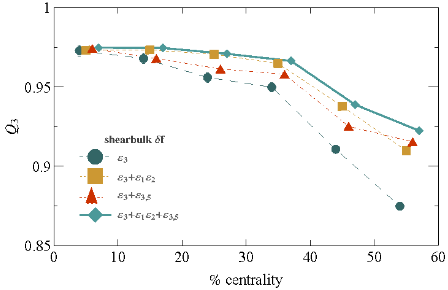


- $Q_2 \sim 1$: $v_2 \propto \varepsilon_2$ and $\Psi_2 \sim \Phi_2$
- Shear viscosity improves the estimator
- NeXSPheRIO "$\sim$ worst-case scenario": ideal + initial tranverse flow and rapidity-dependent fluctuations (not treated in this approach).

v_3 from ε_3

$$V_{est,3} = k_{3,3} W_{3,3} + k_{3,5} W_{3,5} + K_{132} W_{1,3} W_{2,2} + O(W^2).$$



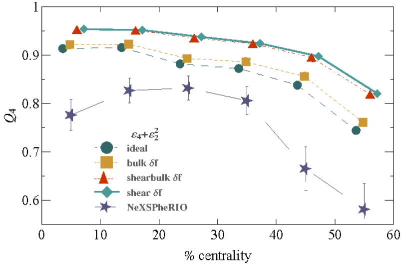


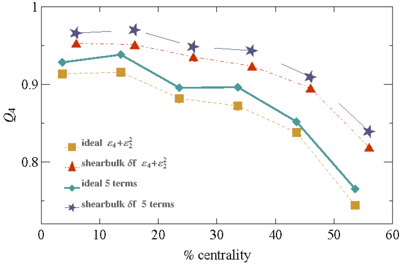
- ε_3 and v_3 are strong correlated
- Shear viscosity improves estimator
- $\varepsilon_1\varepsilon_2$ and $\varepsilon_{3,5}$ help for peripheral collisions
- NeXSPheRIO again below.

Obs.: To get unscaled cumulants, we shoud use $W_{0,2}^{3/2}$ but it makes little difference to use $\{r^3\}$ and revert to eccentricities.

v_4 from ε_4, ε_2^2 and $\Phi_2 - \Phi_4$

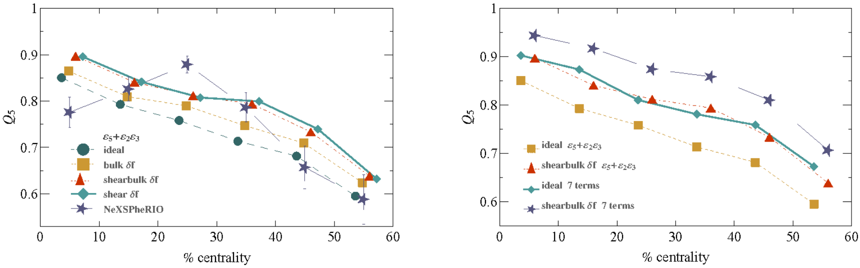
$V_{est,4} = k_{4,4} W_{4,4} + K_{2,2,2} W_{2,2} W_{2,2} + O(W^2)$: both terms important





- Well predicted with two terms (better if there is viscosity).
- $v_4^2 = |V_4|^2 = k^2\epsilon_4^2 + k'^2\epsilon_2^4 + 2kk'\epsilon_4\epsilon_2^2 cos4(\Phi_2 - \Phi_4)$
- Small improvement with more terms (5): $V_{est,4} = k_{4,4} W_{4,4} + K_{2,2,2} W_{2,2} W_{2,2} + K_{2,4,2} W_{2,4} W_{2,2} + K_{2,4,4} W_{2,4} W_{2,4} + K_{1,3,3} W_{1,3} W_{3,3} + O(W^2)$

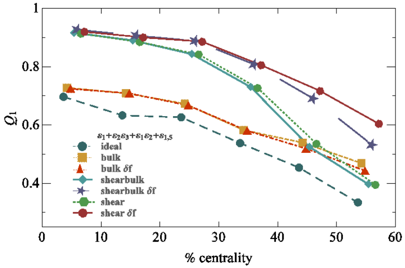
v_5 from ε_5, $\varepsilon_2\varepsilon_3$ and $\Phi_5 - (2\Phi_2 + 3\phi_3)$

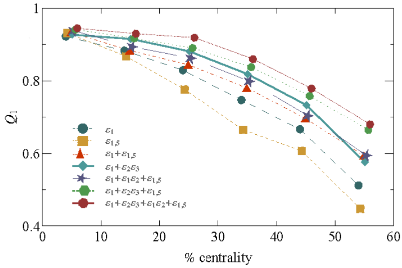


- Two terms OK for central collisions, not peripheral.
- Improvement with more terms (7)

$$\varepsilon_5 + \varepsilon_3\varepsilon_2 + \varepsilon_{3,5}\varepsilon_2 + \varepsilon_3\varepsilon_{2,4} + \varepsilon_{3,5}\varepsilon_{2,4} + \varepsilon_1\varepsilon_4 + \varepsilon_{1,5}\varepsilon_4$$

v_1





- v_1 predicted by ε_1 for central collisions.
- Both higher order terms (sensitive to smaller scale) and non-linear terms necessary for non-central collisions.
- Large effect of viscosities.
- $v_1(p_T)$ changes sign. Smaller p_T range improves estimator

Conclusion

- η/s and ζ/s improve the mapping of the initial state onto the final flow harmonics.
  Correlation between v_n and ϵ_n larger than for ideal hydro.
- NeXSPheRIO results generally below v-USPhydro.
  Central collisions more sensitive to this.
  (NeXSPheRIO has non-zero initial transverse flow and rapidity-dependant fluctuations.)
- More higher order eccentricities and non-linear terms are necessary for peripheral collisions (what about pA?)