Fluctuating Hydrodynamics Confronts the Rapidity Dependence of Transverse Momentum Fluctuations

George Moschelli and Rajendra Pokharel — Lawrence Technological University
Sean Gavin — Wayne State University

I. Motivation: impact of viscosity on fluctuations and correlations

II. Hydrodynamics modes: fluctuations and dissipation

a. Viscous diffusion of transverse shear modes

b. 1st and 2nd order hydrodynamics

III. Contributions to correlation measurements

Work in progress!

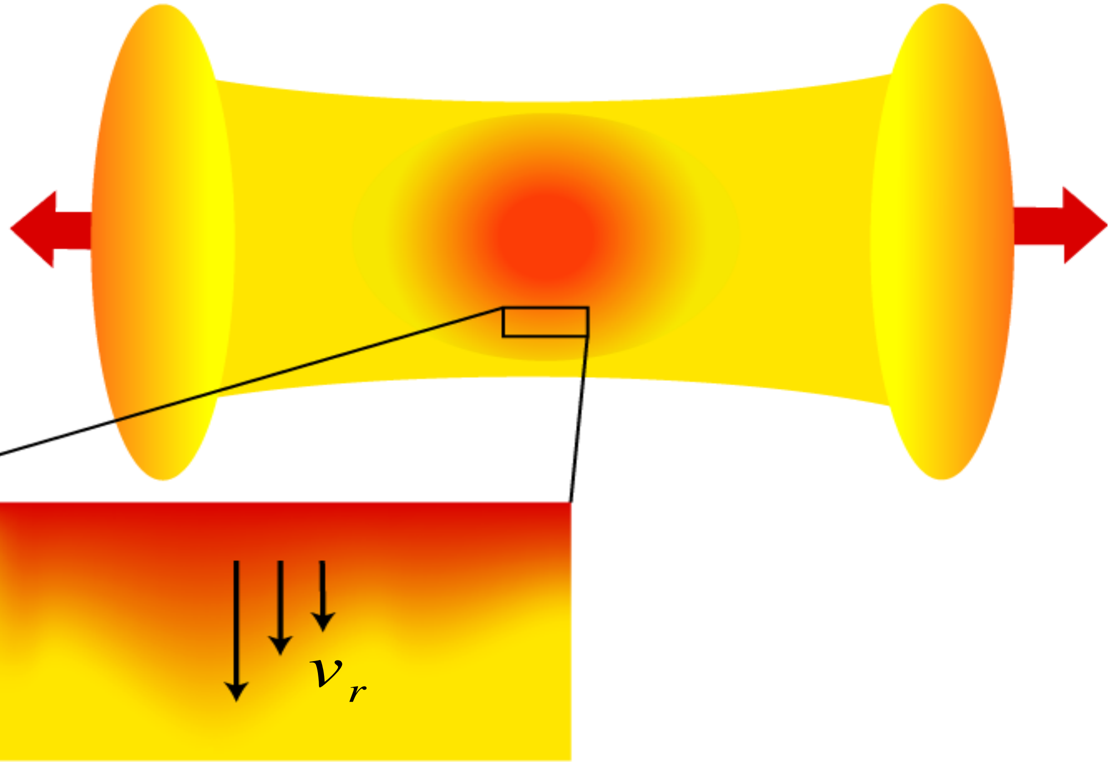
Transverse Momentum Fluctuations

small variations in transverse flow in each event

viscous friction as fluid elements flow past one another

shear viscosity drives velocity toward the average

$$T_{zr} = -\eta\, \partial v_r / \partial z$$



damping of transverse flow fluctuations $\Rightarrow$ viscosity

viscosity: SG & Abdel-Aziz, PRL 97 (2006) 162302
baryon diffusion: SG & Abdel-Aziz, PR C70 (2004) 034905
ϕ correlations (CME): Pratt, Schlichting, SG, PR C84 (2011) 024909

Momentum in Fluctuating Hydrodynamics

momentum current – small fluctuations $M_i \equiv T_{0i} - \langle T_{0i} \rangle \approx (e+p)v_i \approx sTv_i$

momentum conservation
– linearized Navier-Stokes

$$\partial_t M_i + \nabla_i p = \frac{\eta/3+\zeta}{sT} \nabla_i (\vec{\nabla} \cdot \vec{M}) + \frac{\eta}{sT} \nabla^2 M_i$$

Helmholtz decomposition: $\vec{M} \equiv \vec{g}_L + \vec{g}$

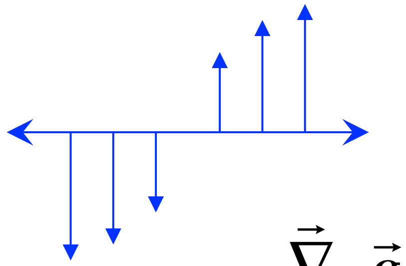
"longitudinal" mode: $\vec{\nabla} \times \vec{g}_L = 0$

"transverse" modes: $\vec{\nabla} \cdot \vec{g} = 0$

Hydrodynamic Modes

transverse modes: **viscous diffusion**

$$\partial_t \vec{g} = \nu \nabla^2 \vec{g}, \qquad \nu = \eta / Ts$$

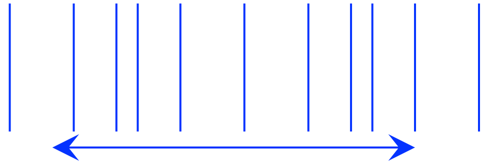


$$\vec{\nabla} \cdot \vec{g} = 0$$

- no transverse ‘sound waves’
- vorticity $\vec{\omega} \propto \vec{\nabla} \times \vec{g}$

longitudinal modes

$$\partial_t \vec{g}_L + \vec{\nabla} p = \frac{\frac{4}{3}\eta + \zeta}{sT} \vec{\nabla}(\vec{\nabla} \cdot \vec{g}_L)$$



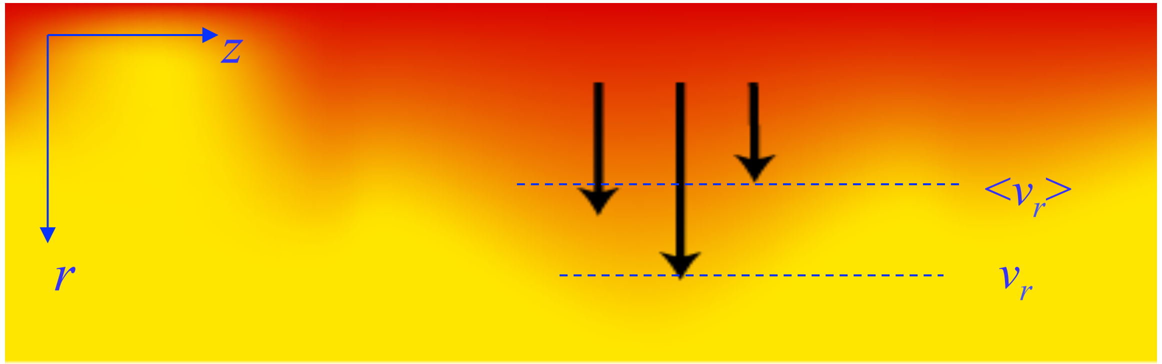
$$\vec{\nabla} \times \vec{g}_L = 0$$

longitudinal modes + energy and baryon conservation imply:

sound waves – compression waves, damped by viscosity

thermal diffusion – heat flow relative to baryons

Transverse Flow Fluctuations



transverse velocity fluctuations ➔ vorticity
➔ "transverse" shear modes

$$T_{0i} - \langle T_{0i} \rangle \approx g_i \qquad T_{ji}^{diss} \approx -\eta \nabla_j v_i = -\nu \nabla_j g_i + \text{Langevin noise}$$

diffusion equation for momentum current

$$\frac{\partial}{\partial t} g_r = \nu \nabla^2 \left(g_r + \text{noise} \right)$$

correlation function measures deviation of fluctuations from mean

$$r = \langle g_r(x_1) g_r(x_2) \rangle - \langle g_r(x_1) \rangle \langle g_r(x_2) \rangle$$

SG & Abdel-Aziz, PRL 97 (2006) 162302

Rapidity Dependence of Transverse Momentum Correlations

momentum flux density correlation function

$$r = \langle g_r(x_1) g_r(x_2) \rangle - \langle g_r(x_1) \rangle \langle g_r(x_2) \rangle$$

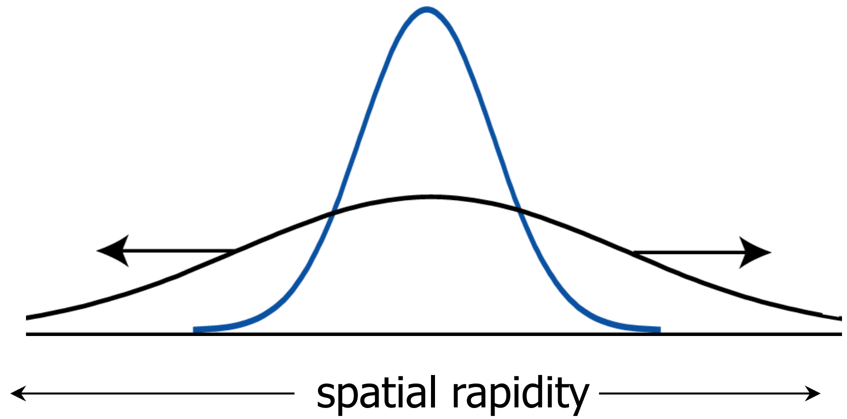
$\Delta r = r - r_{eq}$ **satisfies diffusion equation**

Gardiner, Handbook of Stochastic Methods, (Springer, 2002)

fluctuations **diffuse** **through volume, driving** $r \rightarrow r_{eq}$

width in relative spatial rapidity grows $y = \sinh^{-1} z / \tau$
from initial value σ_0

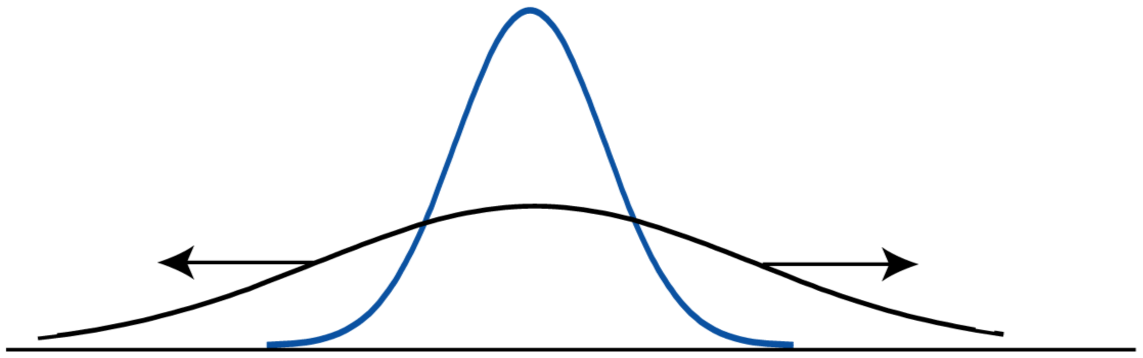
$$\sigma^2 = \sigma_0{}^2 + 4\frac{\eta}{Ts}\left(\frac{1}{\tau_0} - \frac{1}{\tau}\right)$$



Diffusion vs. Wave Motion

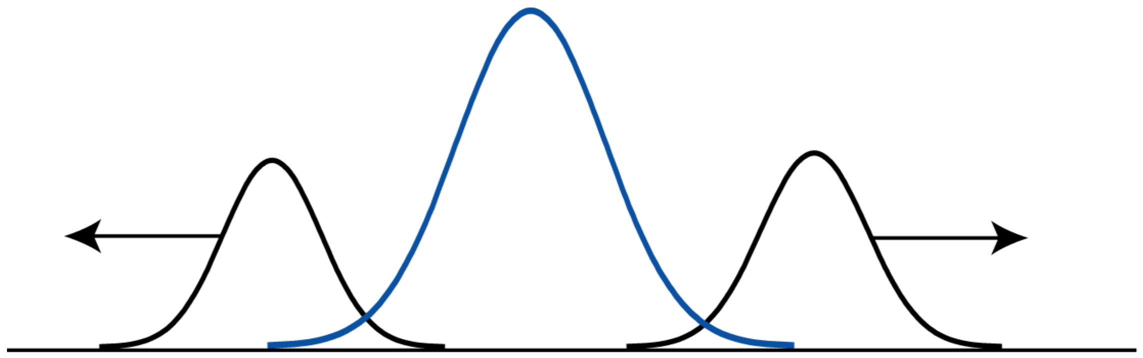
Diffusion (1^{st} Order)

- Gaussian peak spreads
- tails violate causality



Wave propagation – e.g. sound waves

- peak splits into left and right traveling pulses
- propagation speed c_s



2^{nd} Order Viscous Diffusion

causal transport equation:

- transverse modes
- derived from linearized Israel-Stewart hydro equations

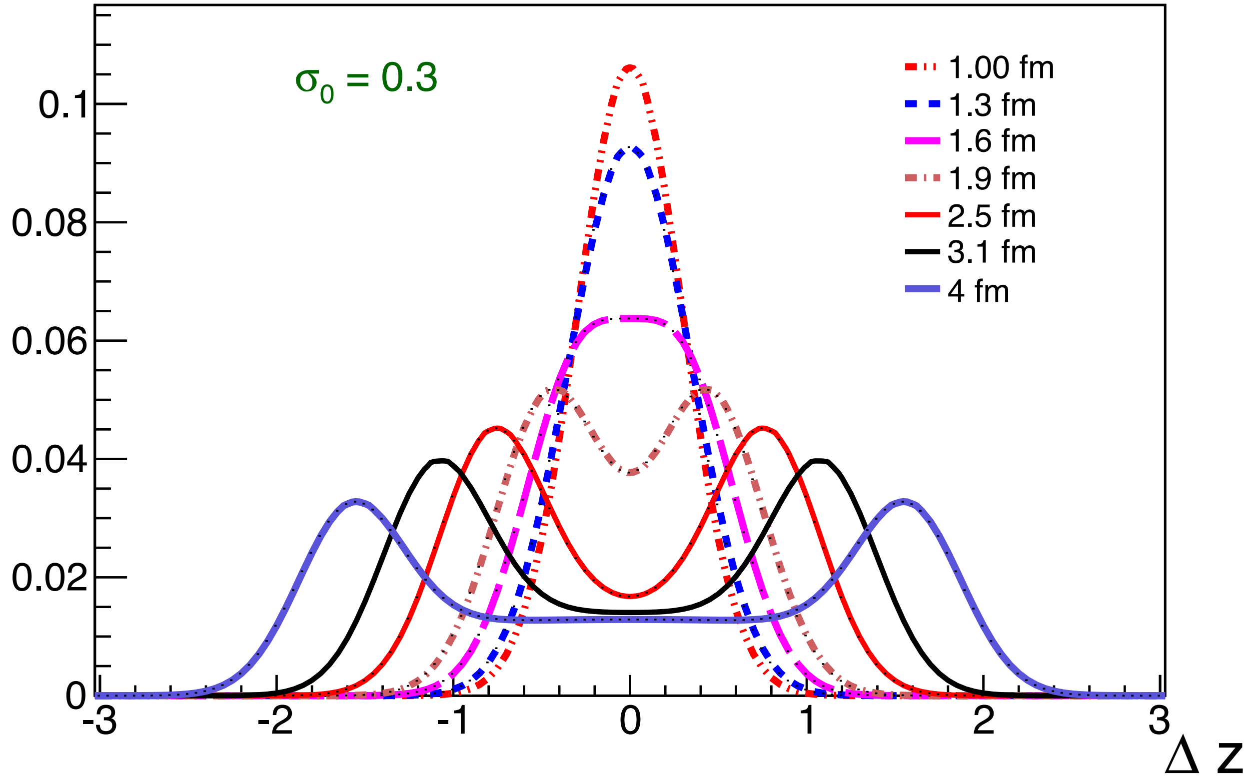
$$\left(\tau_\pi \frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial t} - \nu\left(\nabla_1^2 + \nabla_2^2\right)\right)\Delta r = 0$$

relaxation time $\tau_\pi \sim$ (mean free path)/(thermal speed)

coordinate space:

- wave-fronts traveling at speed $= (\nu/\tau_\pi)^{1/2}$
- diffusion-like behavior in between
- no peak at $\Delta z = 0$

$$\Delta r = r - r_{eq}$$

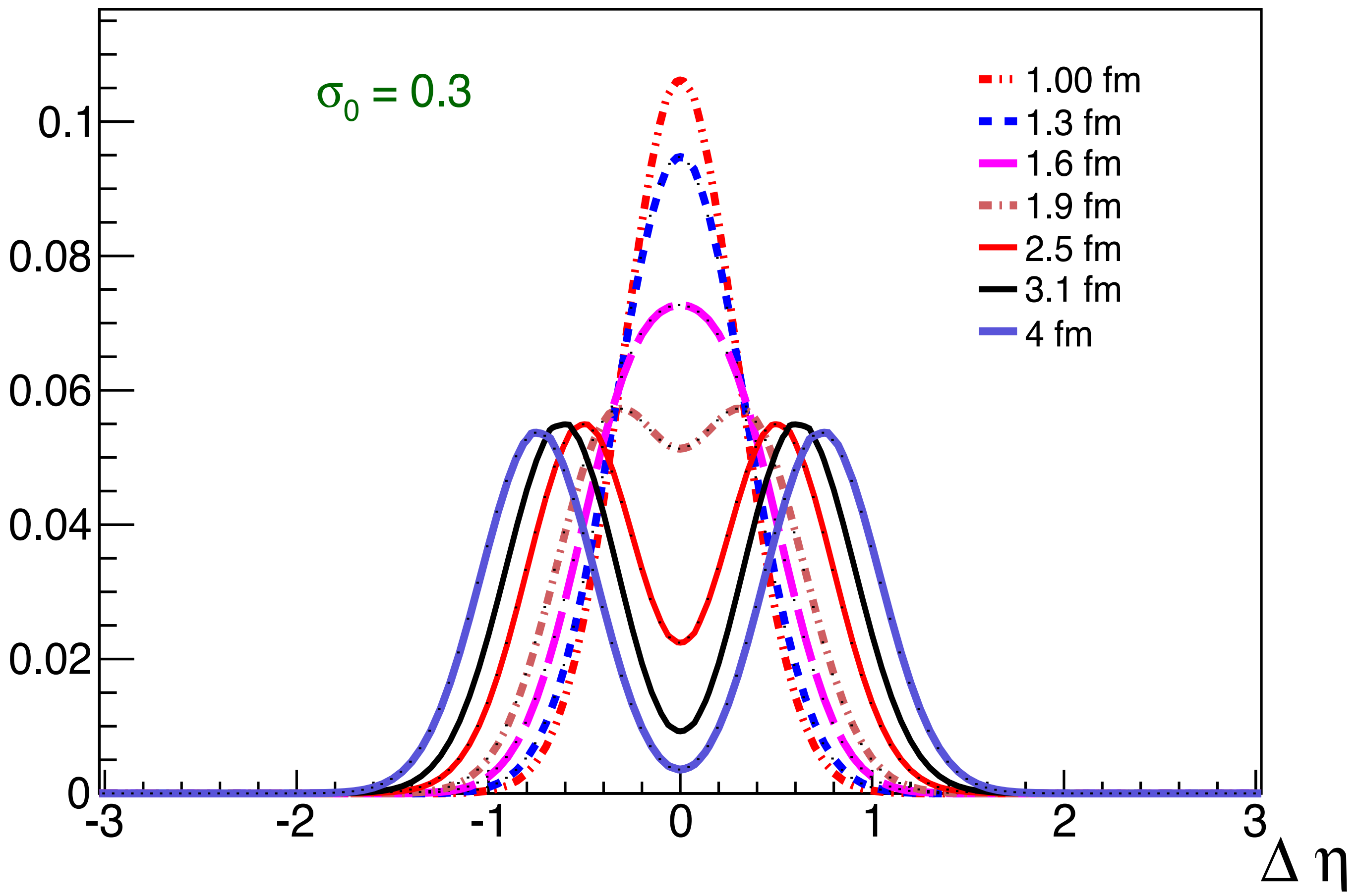


2^{nd} Order Viscous Diffusion in Rapidity

$$\left(\tau_\pi \frac{\partial^2}{\partial \tau^2} + \frac{\partial}{\partial \tau} - \frac{\nu}{\tau^2}\left(\frac{\partial^2}{\partial \eta_1^2} + \frac{\partial^2}{\partial \eta_2^2}\right)\right)\Delta r = 0$$

spatial rapidity

- rapidity separation of fronts saturates
  $\Delta\eta \sim \Delta z/\tau$
- profile depends on initial width σ_0



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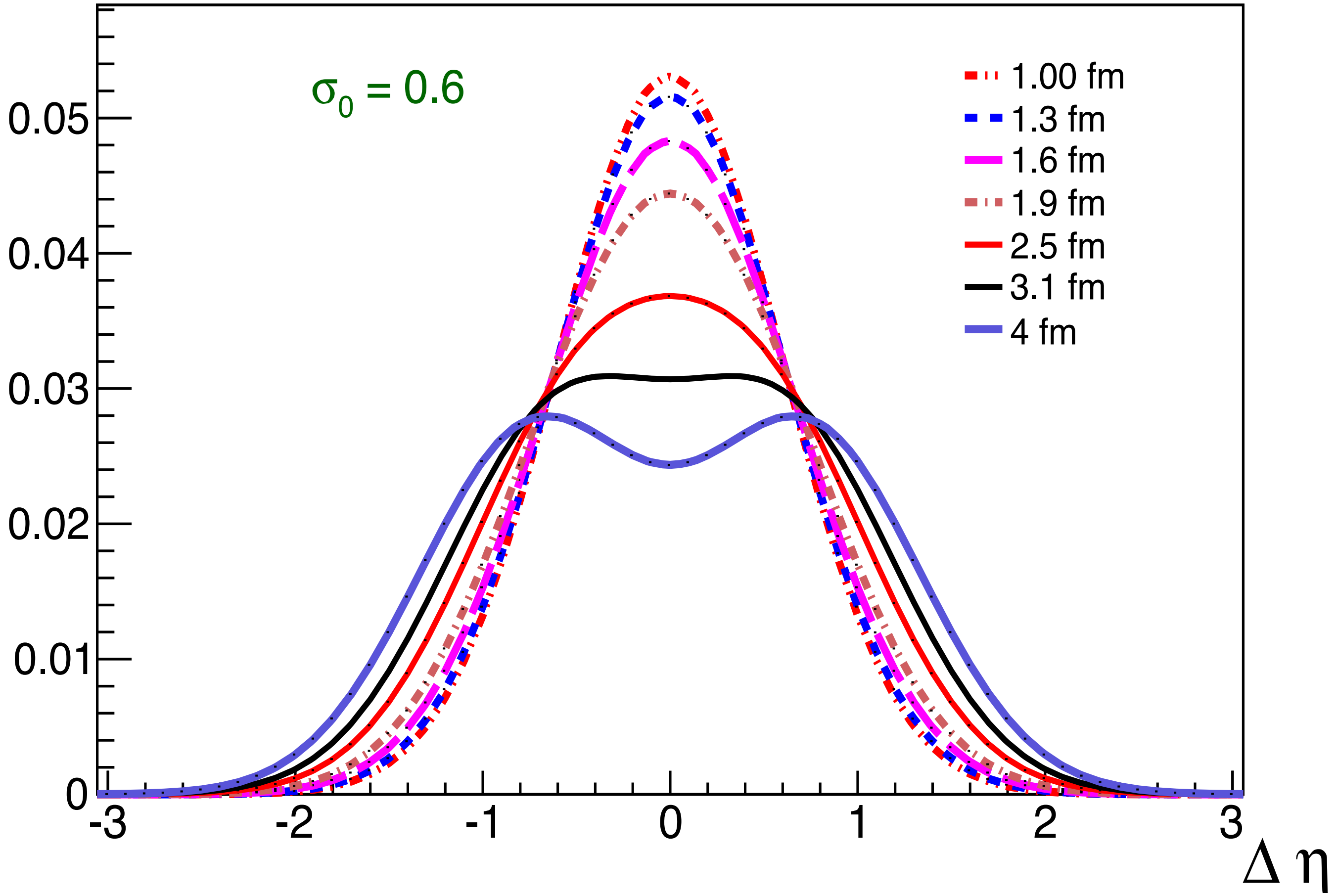
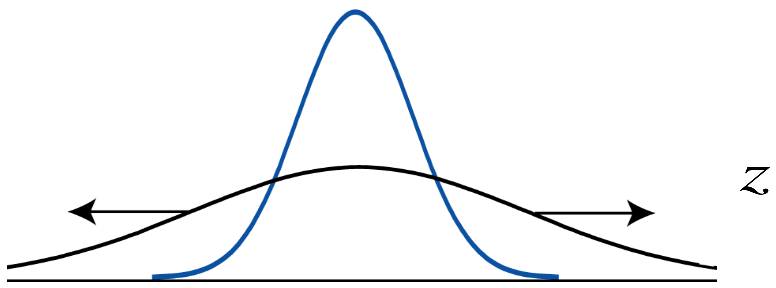
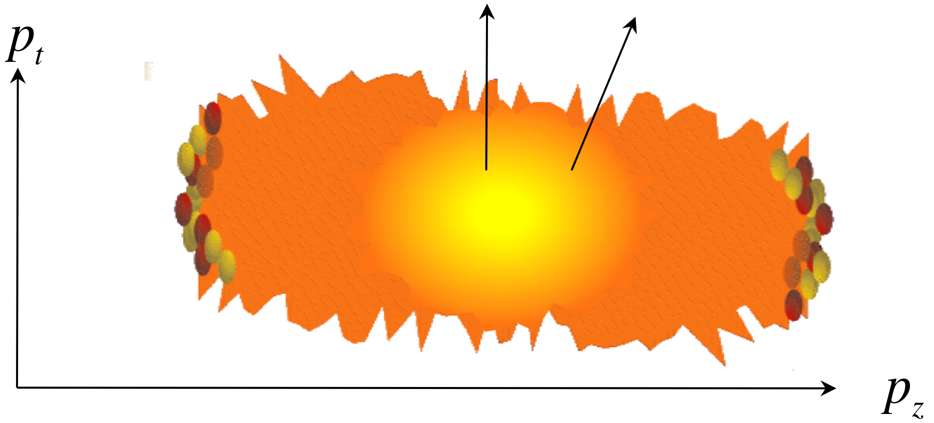


Measuring the Correlations

correlation function $r = \langle g_t(x_1) g_t(x_2) \rangle - \langle g_t(x_1) \rangle \langle g_t(x_2) \rangle$





observable: $C = \frac{1}{\langle N \rangle^2} \left\langle \sum_{\text{pairs}} p_{ti} p_{tj} \right\rangle - \langle p_t \rangle^2 \quad = \frac{1}{\langle N \rangle^2} \int (r - r_{eq}) \, dx_1 \, dx_2$

Abdel-Aziz & S.G., PRL 97 (2006) 162302; PR C70 (2004) 034905
Pratt, Schlichting, SG, Phys. Rev. C 84 (2011) 024909

p_t Covariance Measured

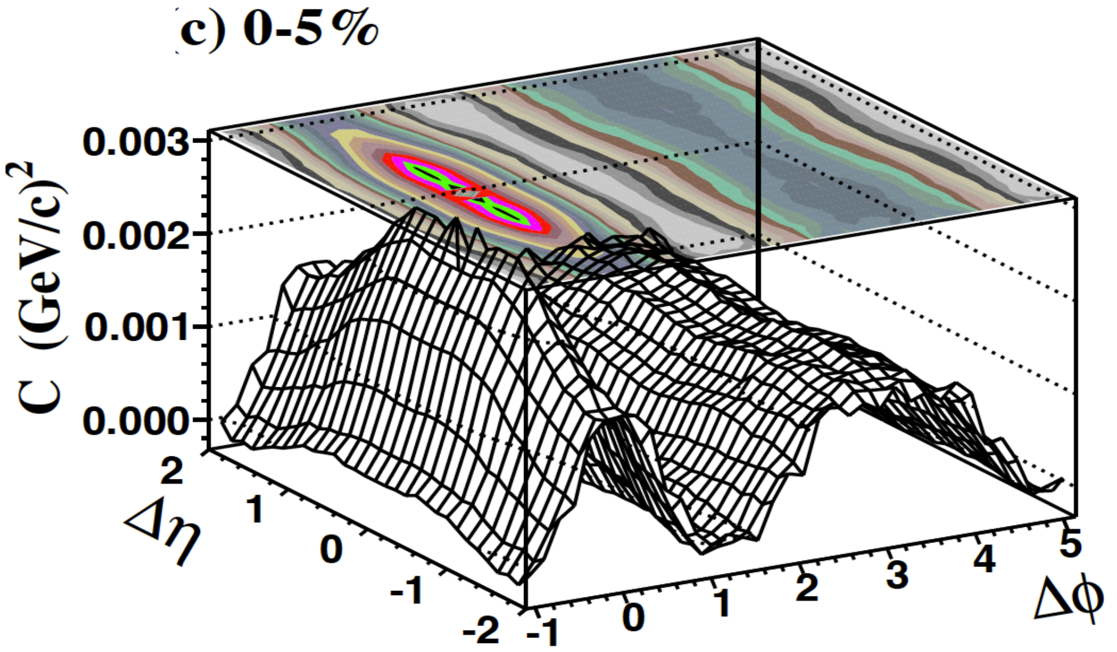
measured: rapidity width of near side peak

- fit peak + constant offset
- offset is ridge, i.e., long range rapidity correlations
- report rms width of the peak

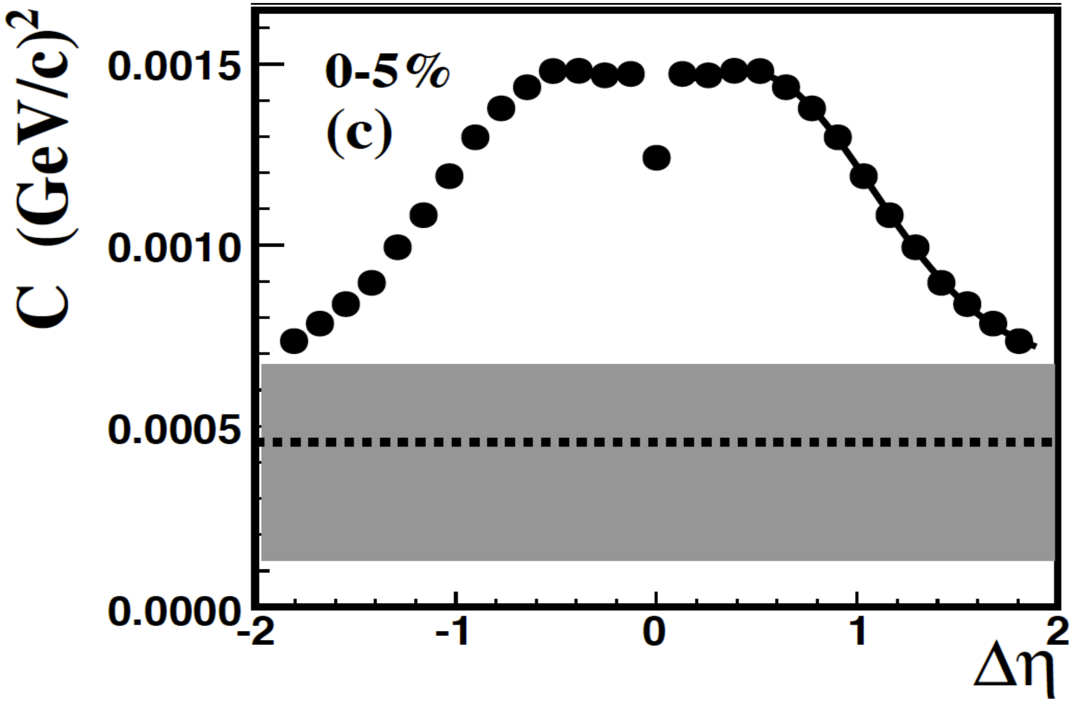
find: width increases in central collisions

$$\sigma_{central} = 1.0 \pm 0.2$$

$$\sigma_{peripheral} = 0.54 \pm 0.02$$



STAR data, Phys. Lett. B704 (2011) 467



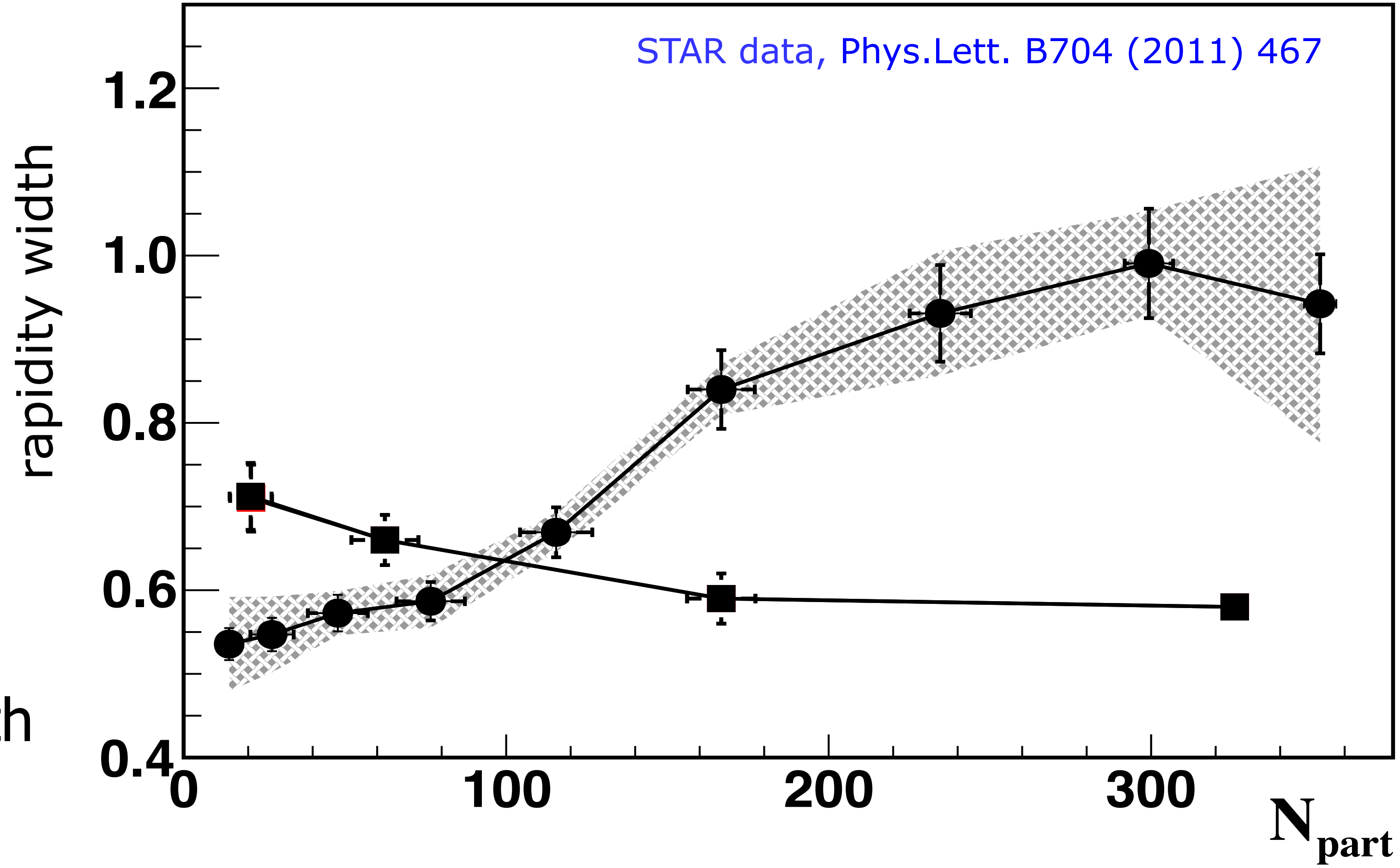
Rapidity Width Increases in Central Collisions

Central vs. peripheral increase consistent with $\eta/s = 0.17 \pm 0.08$

NeXSPheRIO calculations **fail**

Sharma et al., Phys.Rev. C84 (2011) 054915

ideal fluctuating hydro doesn't explain measured growth of width



2^{nd} Order Viscous Diffusion

Pokharel, Moschelli, S.G. in preparation

causal transport equation:

$$\left(\tau_\pi \frac{\partial^2}{\partial \tau^2} + \frac{\partial}{\partial \tau} - \frac{\eta}{sT}\left(\nabla_1^2 + \nabla_2^2\right)\right)\Delta r_g = 0$$

relaxation time $\tau_\pi \sim$ (mean free path)/(thermal speed)

kinetic theory $\tau_\pi = \beta(\eta / sT)$ $\beta \approx 5$

temperature vs time:

entropy production:
TdS/dt = viscous heating

$$\frac{ds}{d\tau} + \frac{s}{\tau} = \frac{\Phi}{T\tau}$$

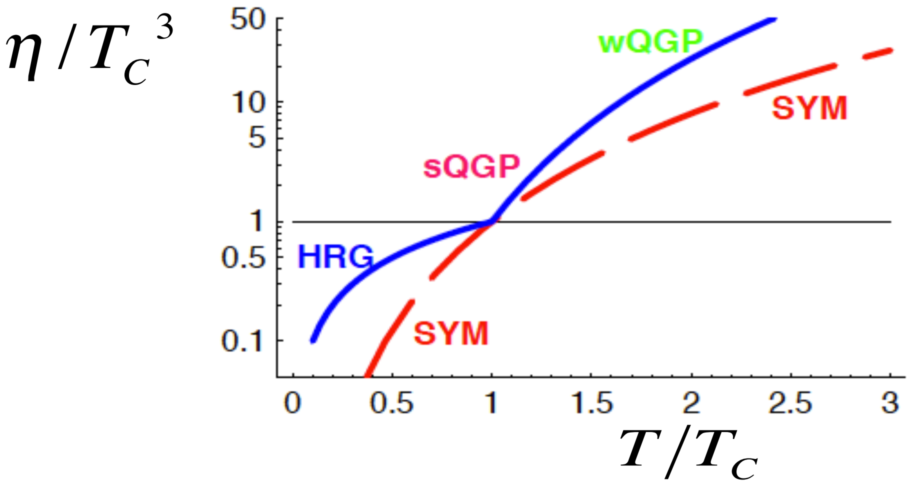
relaxation equation: causality delays heating

$$\frac{d\Phi}{d\tau} = -\frac{1}{\tau_\pi}\left(\Phi - \frac{4\eta}{3\tau}\right) - \left[\frac{1}{\tau} + \frac{\eta T}{\tau_\pi}\frac{d}{d\tau}\left(\frac{\tau_\pi}{\eta T}\right)\right]\frac{\Phi}{2}$$

Minimum Viscosity Near T_c

sQGP viscosity Hirano & Gyulassy

- pQCD at high T; hadron gas at low T
- limit at T = T_C: $\eta / s = 1/4\pi$

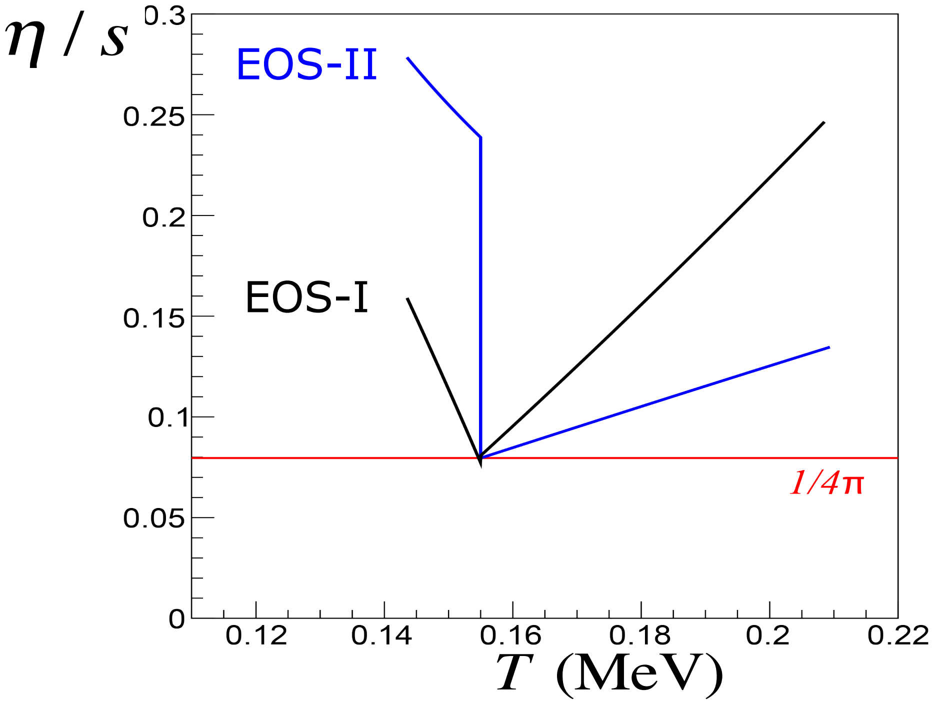


EOS-I – Niemi, Denicol et al.

- Lattice – HotQCD Collaboration
- Lattice viscosity T > TC – Nakamura & Sakai
- Hagadorn HG – Noronha-Hostler et al.

EOS-II – Hirano & Gyulassy

- Bag Model EOS
- QGP viscosity T > T_C – Danielewicz & Gyulassy
- Pion gas T < T_C – Gavin



Time Dependence of Correlation Profile

Compute contribution from early time diffusion in rapidity

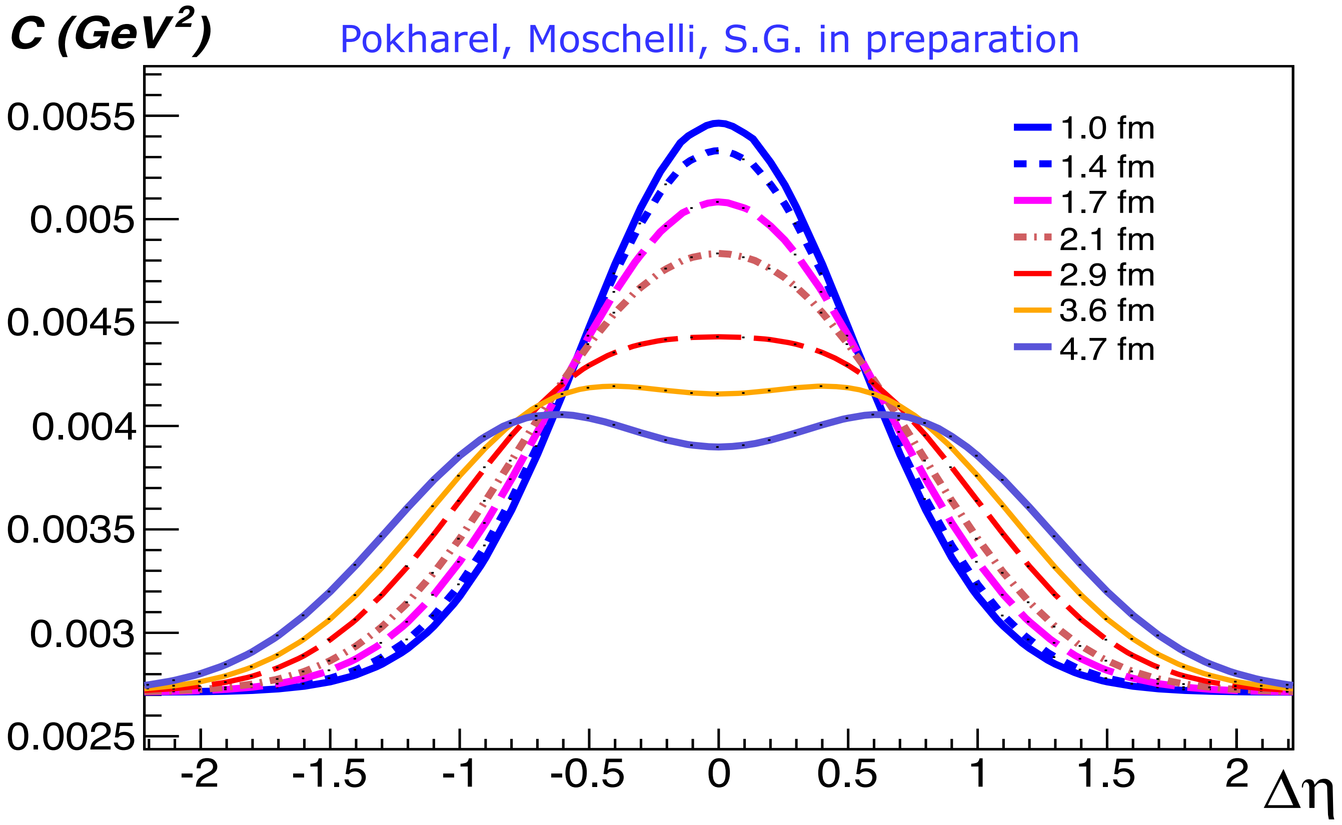
$$\left(\tau_\pi \frac{\partial^2}{\partial \tau^2} + \frac{\partial}{\partial \tau} - \frac{\nu}{\tau^2}\left(\frac{\partial^2}{\partial \eta_1^2} + \frac{\partial^2}{\partial \eta_2^2}\right)\right)\Delta r = 0$$

Gaussian initial profile, fixed width

$$\sigma_0 \approx \sigma_{peripheral}$$

Separate peaks?

depends on EOS through $\nu(\tau)$



Rapidity Dependence of Covariance vs. Centrality

C. Pruneau, M. Sharma (STAR)
private communications

freeze out time

$$\tau_F - \tau_0 \propto (R - R_0)^2$$

2^{nd} Order Diffusion, EOS I

$d\Delta r/d\tau|_0 = 0$, $\tau_0 = 0.91$ fm

$\tau_F(b=0) = 12.7$ fm

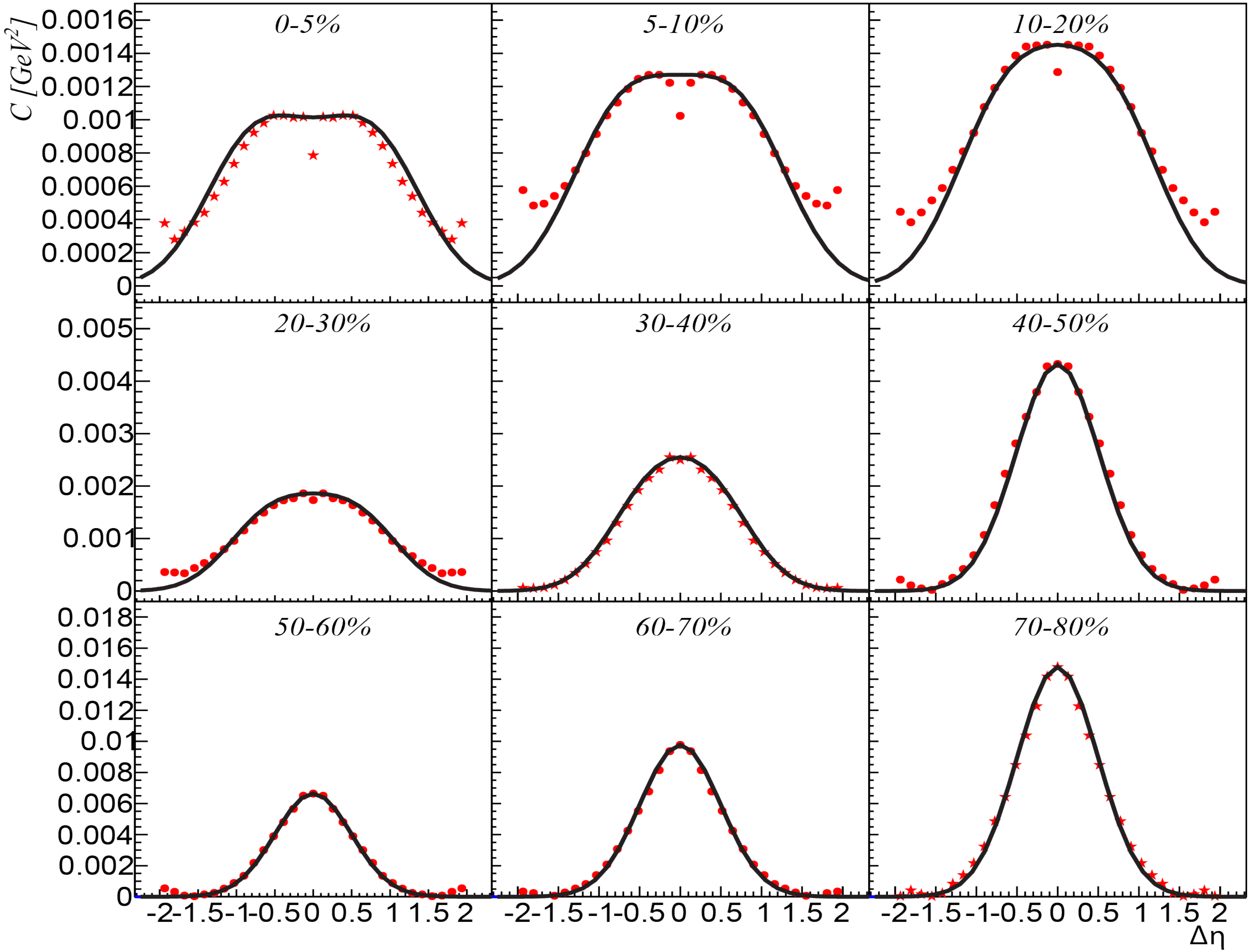
$T_C = 155$ MeV

$T_F = 143$ MeV

$T_0(b=0) = 209$ MeV

$\sigma_0 = 0.50$

Important: tails inflate extracted widths



Rapidity Width of Momentum Covariance

freeze out time

$$\tau_F - \tau_0 \propto (R - R_0)^2$$

2^{nd} Order Diffusion, EOS I

$d\Delta r/d\tau|_0 = 0$, $\tau_0 = 0.91$ fm

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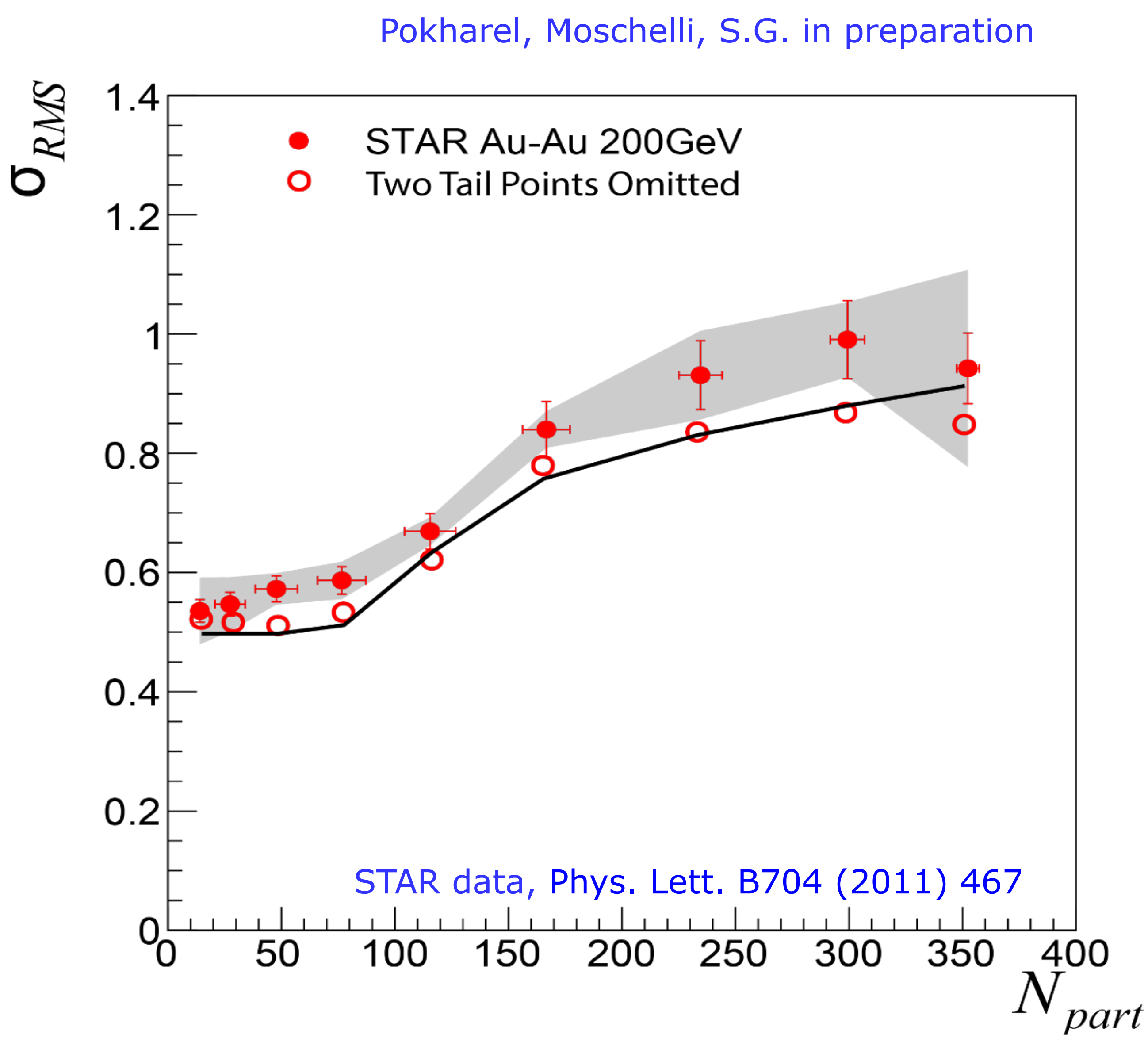
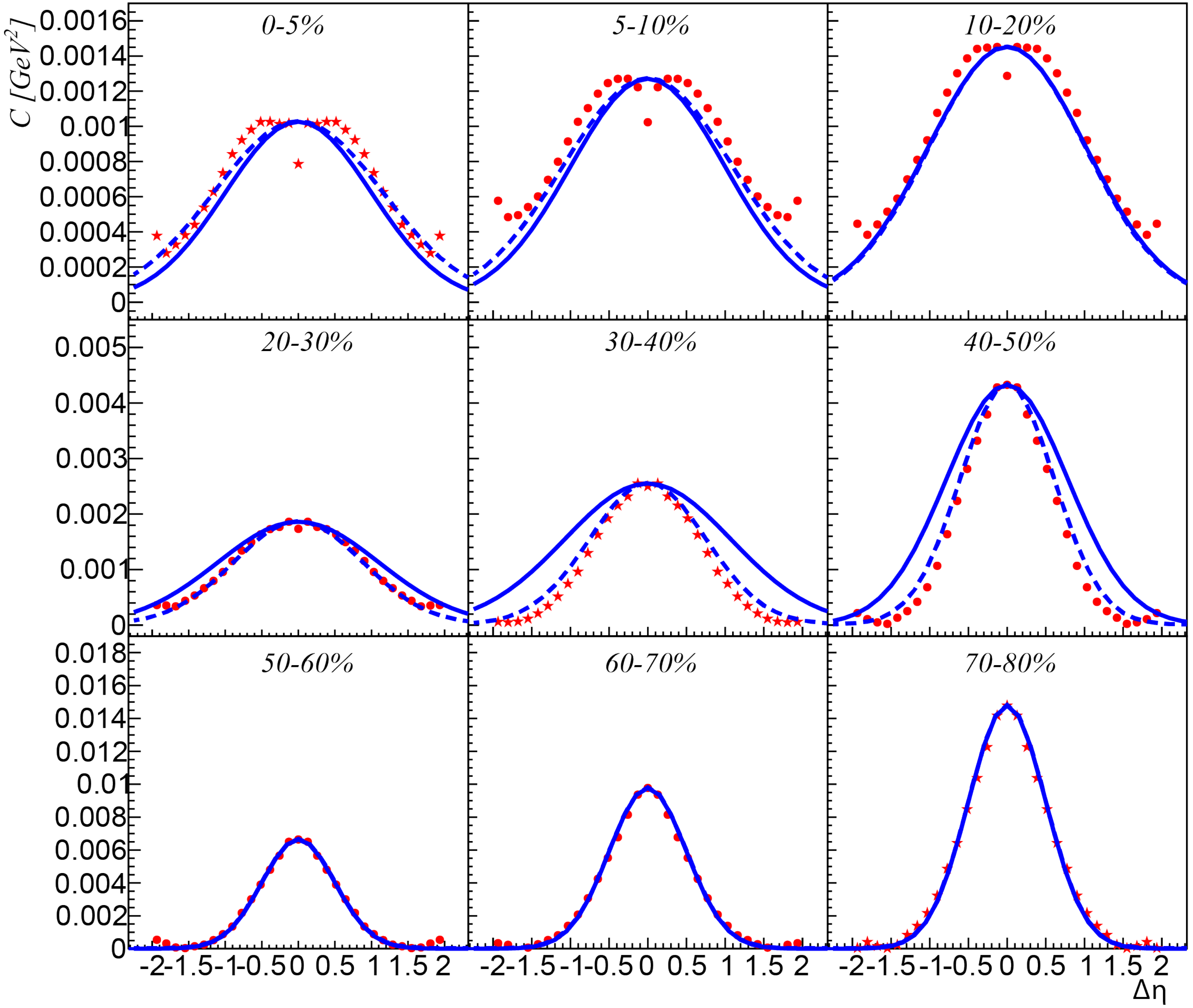
$T_0(b=0) = 209$ MeV

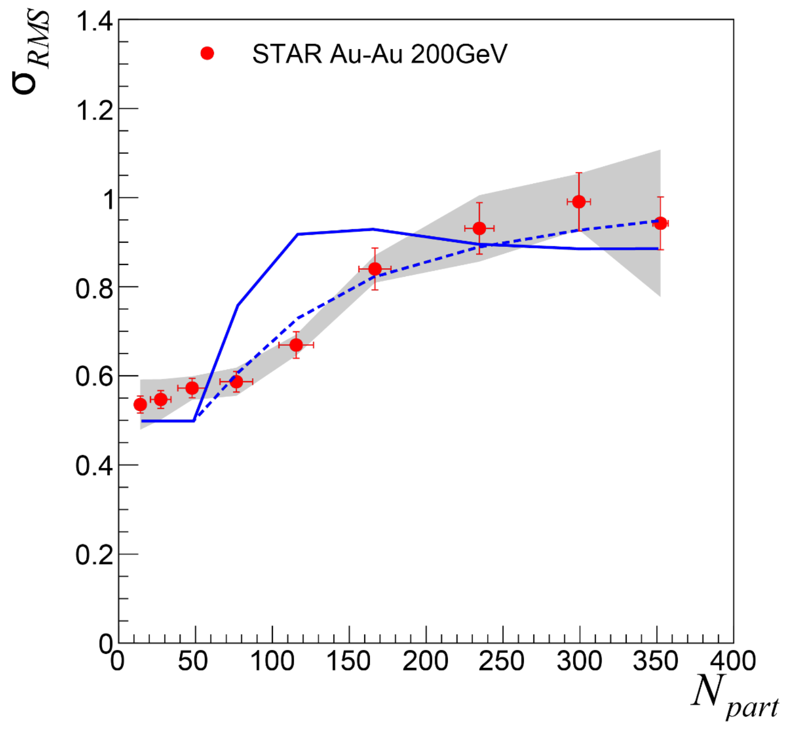
$\sigma_0 = 0.50$

Important: reported widths include tails



1^{st} Order Diffusion **Can't** Describe Rapidity Shape

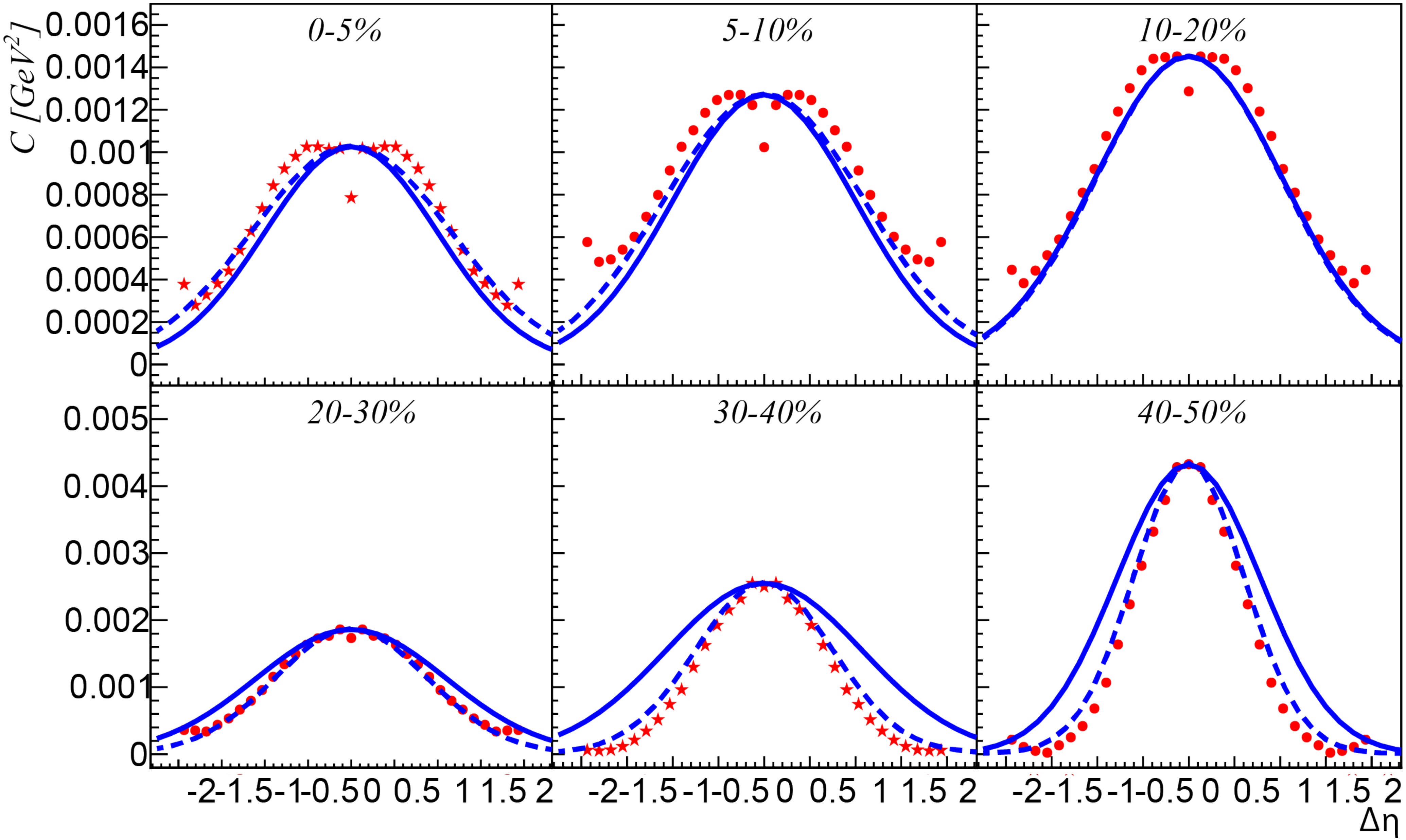




Solid Blue
EOS I
1^{st} Order Diffusion
t_0 = 0.6667 fm
t_{Fc} = 9 fm
T_C = 0.155 GeV
T_F = 0.1434 GeV

Dashed Blue
EOS II
1^{st} Order Diffusion
t_0 = 0.7 fm
t_{Fc} = 9 fm
T_C = 0.155 GeV
T_F = 0.150 GeV

1^{st} Order Diffusion: Gaussian **Doesn't** Describe Shape



Solid: EOS I

t_0 = 0.6667 fm

t_{Fc} = 9 fm

T_C = 0.155 GeV

T_F = 0.1434 GeV

Dashed: EOS II

t_0 = 0.7 fm

t_{Fc} = 9 fm

T_C = 0.155 GeV

T_F = 0.150 GeV

2^{nd} Order Diffusion: Initial Conditions

Gaussian initial distribution Δr at $\tau = \tau_0$, width σ_0

Moschelli, Pokharel, S.G. in preparation

Initial derivative possibilities:

Near equilibrium

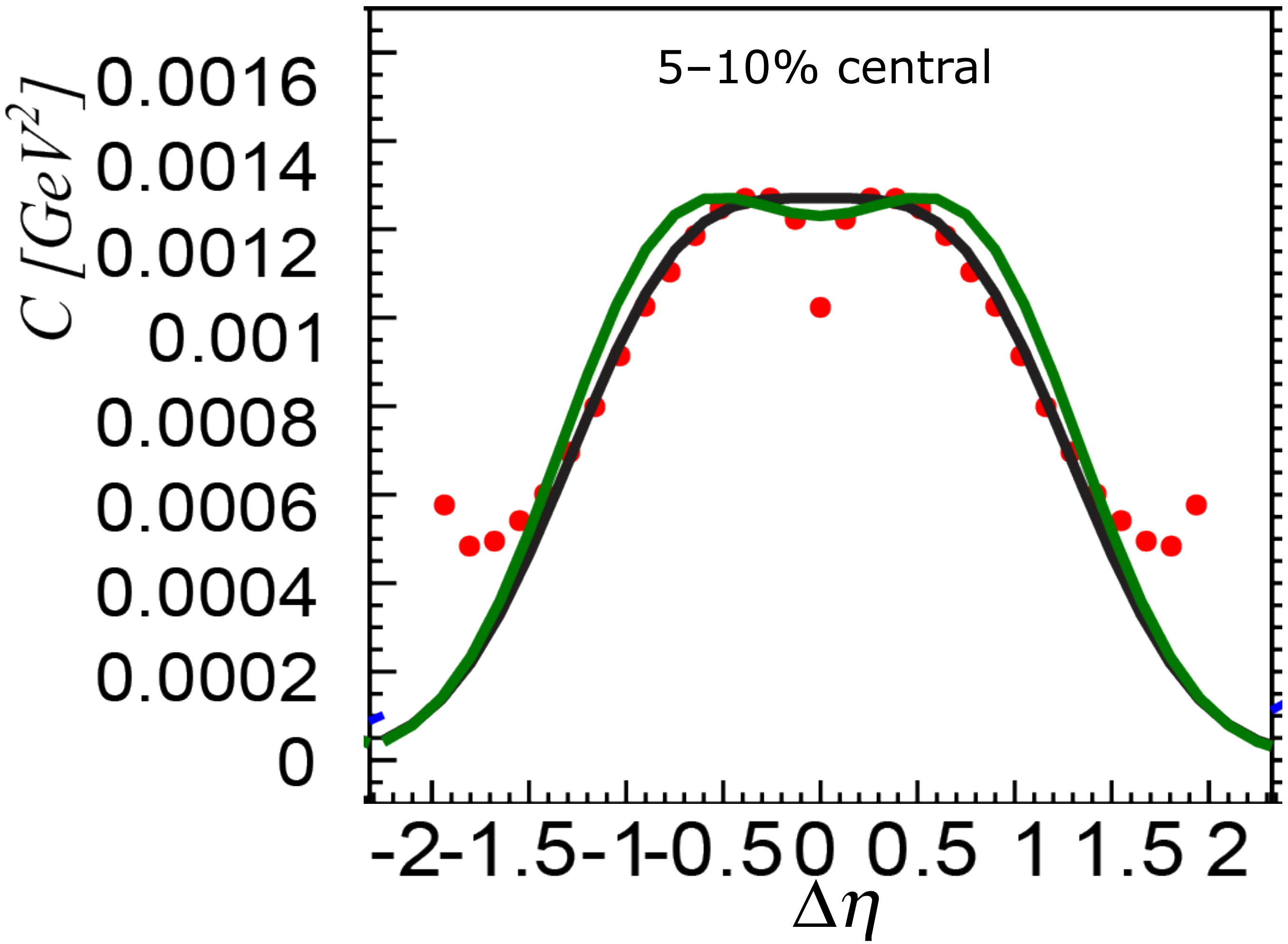
$$\left.\frac{\partial \Delta r}{\partial \tau}\right|_{\tau=\tau_0} = \frac{\nu}{\tau^2}\left(\frac{\partial^2}{\partial \eta_1^2} + \frac{\partial^2}{\partial \eta_2^2}\right)\Delta r$$

- fits valley better

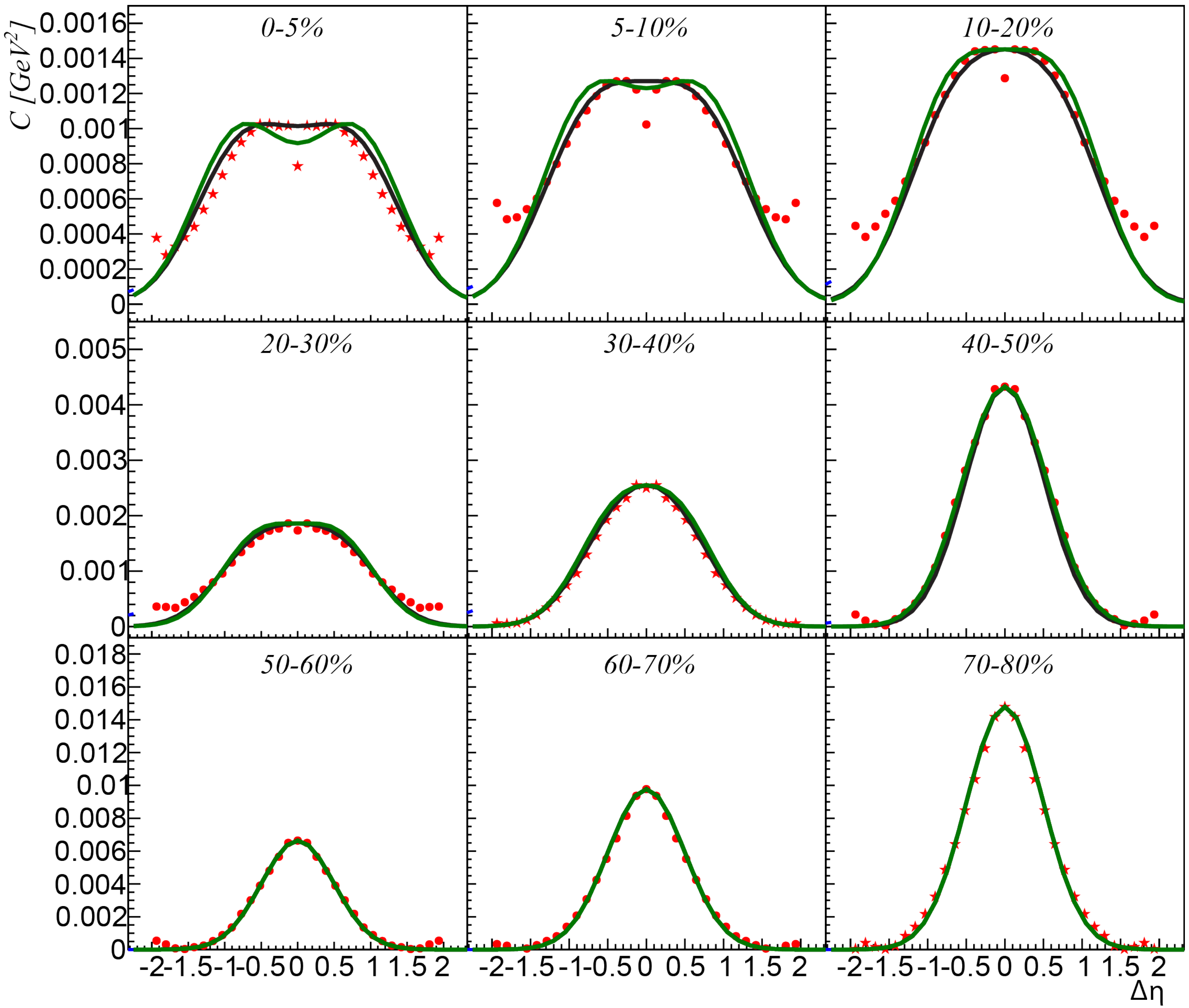
Nonequilibrium:

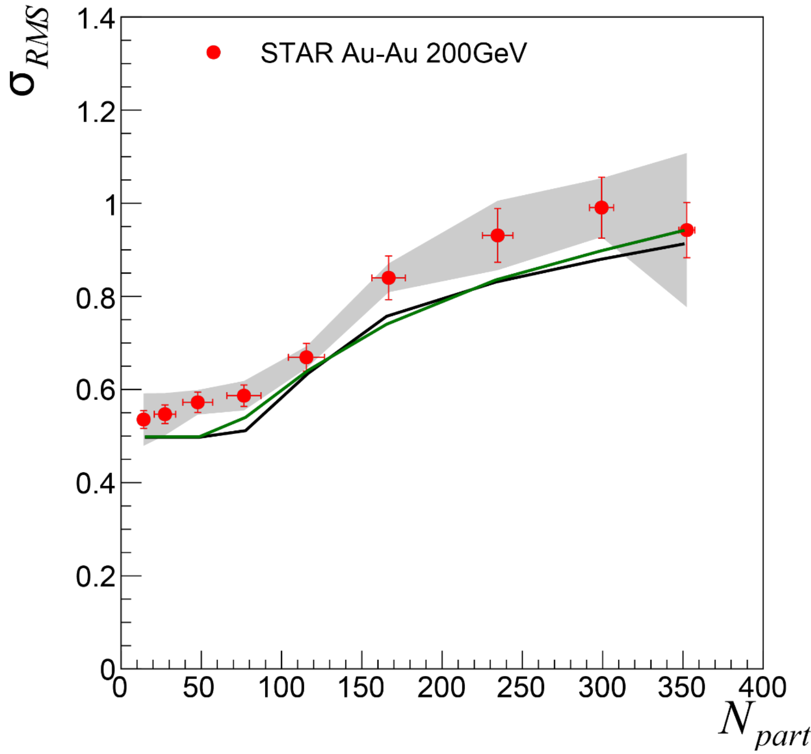
$$\left.\frac{\partial \Delta r}{\partial \tau}\right|_{\tau=\tau_0} = 0$$

- better overall shape



Near Equilibrium Initial Conditions Enhance Valley

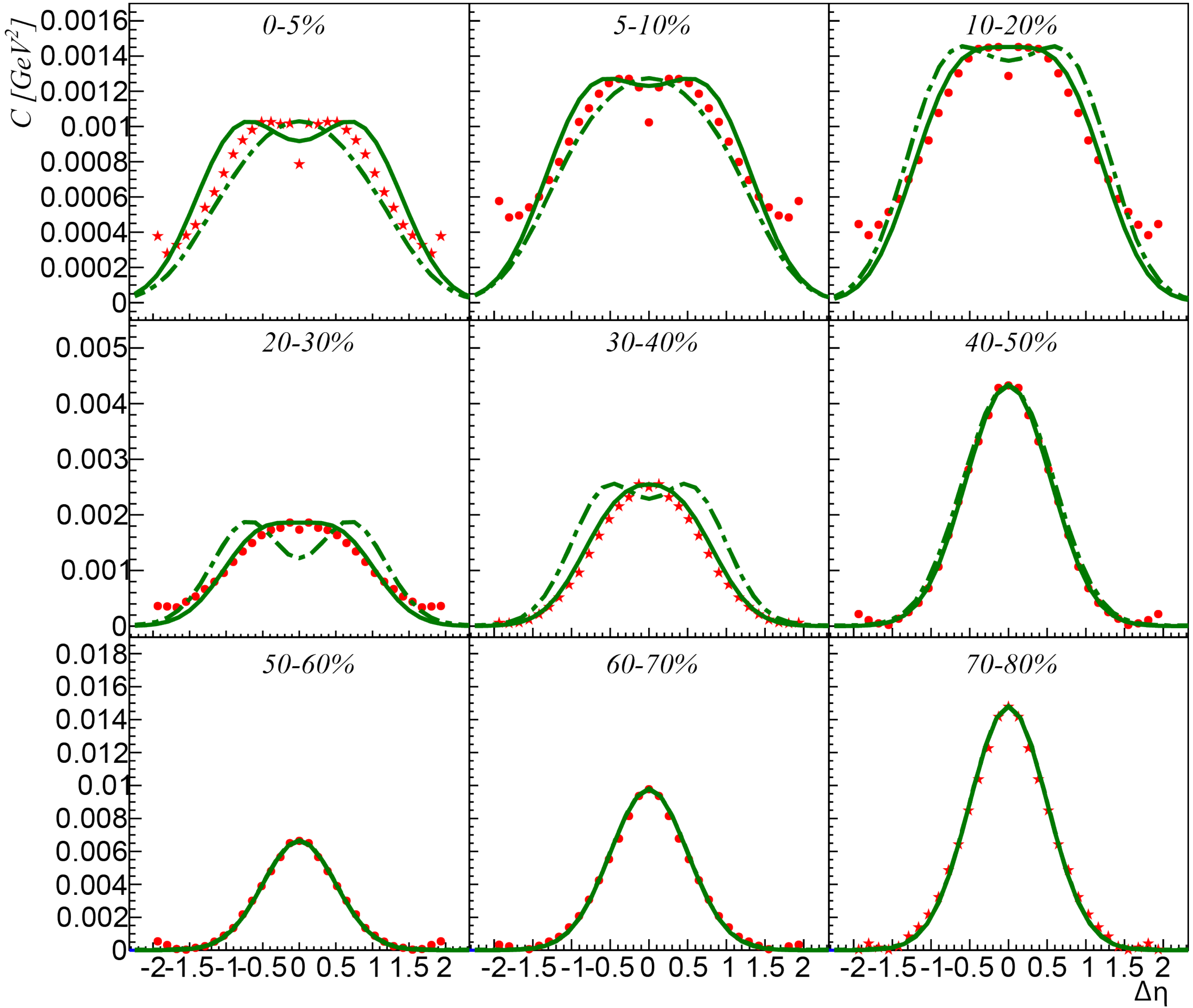


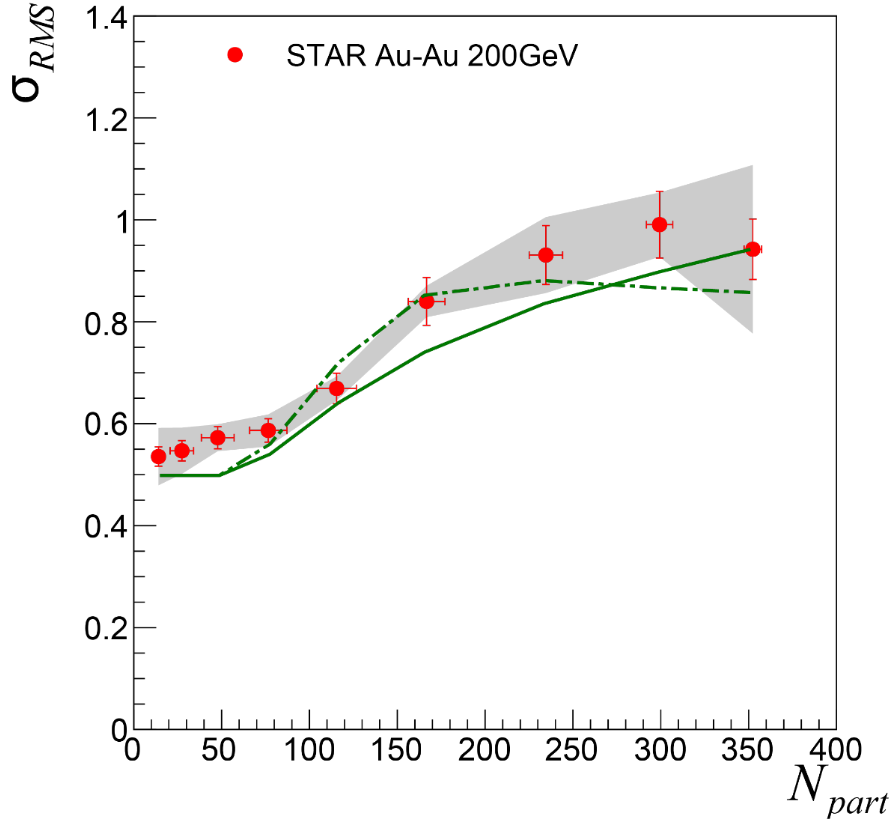


Solid Black
EOS I
2nd Order Diffusion
nonequilibrium IC
τ_0 = 0.911 fm
τ_{Fc} = 12.667 fm
T_F = 0.1434 GeV
T_0 central = 0.209 GeV

Solid Green
EOS I
2nd Order Diffusion
near equilibrium IC
τ_0 = 1.1 fm
τ_{Fc} = 9 fm
T_F = 0.1434 GeV
T_0 central = 0.191 GeV

1^{st} Order Phase Transition Inconsistent

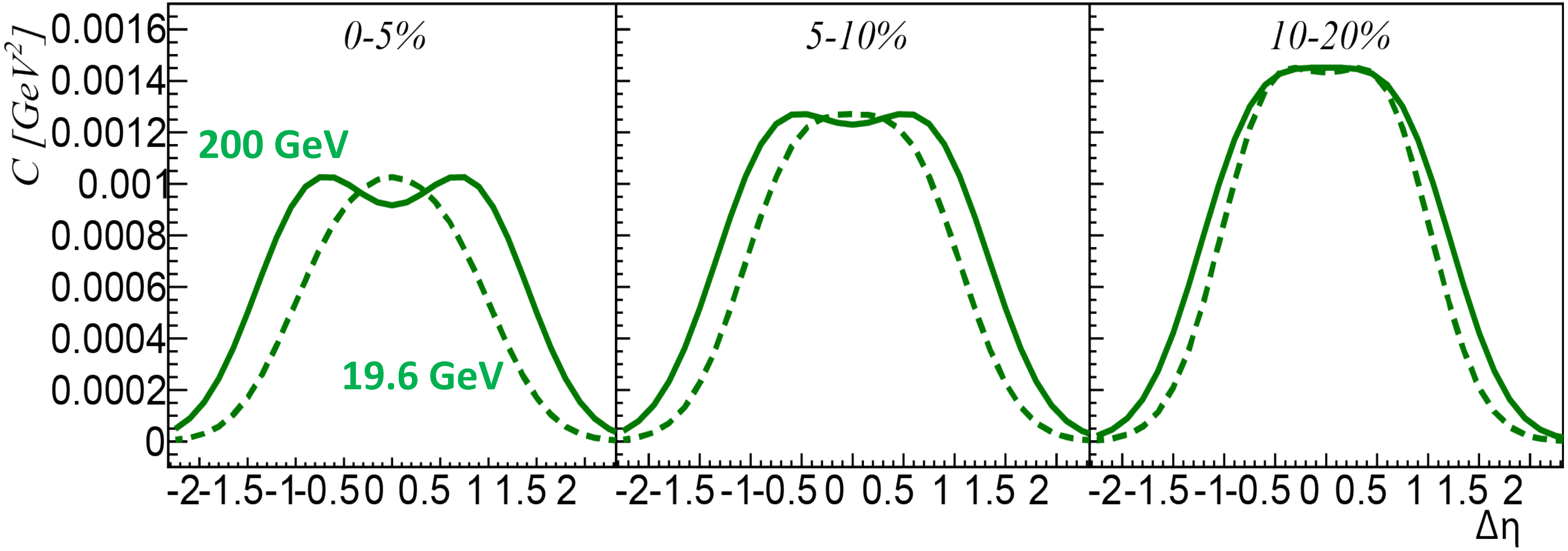


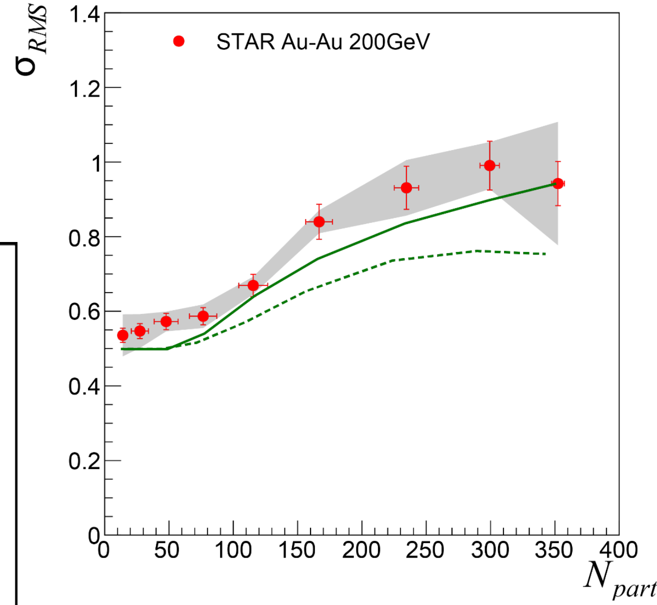


Solid
EOS I (lattice)
2^{nd} Order Diffusion
τ_0 = 1.1 fm
τ_{Fc} = 9 fm
T_C = 0.155 GeV
T_F = 0.1434 GeV
T_0 central = 0.191 GeV

Dot-Dashed
EOS II (first order)
2^{nd} Order Diffusion
t_0 = 1.1 fm
t_{Fc} = 9 fm
T_C = 0.155 GeV
T_F = 0.150 GeV
s_0 = 0.49874

Prediction: Valley Disappears at Low Energy





Dashed Green
19.6 GeV, EOS I
2^{nd} Order Diffusion
t_0 = 2 fm
t_{Fc} = 8 fm
T_F = 0.130 GeV

Solid Green
200 GeV, EOS I
2^{nd} Order Diffusion
t_0 = 1.1 fm
t_{Fc} = 9 fm
T_F = 0.1434 GeV

Beam Energy Scan prediction – expect less prominent valley at lower energy

Summary: rapidity dependence of p_t correlations

Hydro formulation: longitudinal and transverse modes

- Sound waves, shear modes, and heat modes
- Diffusive transverse shear modes important for rapidity dependence of p_t correlations
- 1^{st} and 2^{nd} order viscous fluctuating hydro description of shear modes

Causality shapes the rapidity dependence of correlations

Open Questions

- Influence of sound and heat modes on observables
- Charge balancing, resonances, jets, HBT

Covariance Measures Momentum Flux

covariance
$$C = \frac{1}{\langle N \rangle^2} \left\langle \sum_{\text{pairs } i \neq j} p_{ti} p_{tj} \right\rangle - \langle p_t \rangle^2$$

unrestricted sum:
$$\sum_{\text{all} i,j} p_{ti} p_{tj} = \int p_{t1} p_{t2} dn_1 dn_2$$

$$dn = f(x,p)dpdx$$
$$g_t(x) = \int dp \; p_t \Delta f(x,p)$$

$$= \int dx_1 dx_2 \left(\int dp_1 \, p_{t1} f_1 \right) \left(\int dp_2 p_{t2} f_2 \right)$$
$$= \langle N \rangle^2 \langle p_t \rangle^2 + \int g(x_1) g(x_2) dx_1 \, dx_2$$

correlation function:
$$r_g = \langle g_t(x_1) g_t(x_2) \rangle - \langle g_t(x_1) \rangle \langle g_t(x_2) \rangle$$

$$\int r_g dx_1 dx_2 = \left\langle \sum p_{ti} p_{tj} \right\rangle - \langle N \rangle^2 \langle p_t \rangle^2 = \left\langle \sum p_{ti}{}^2 \right\rangle + \langle N \rangle^2 C$$

$C = 0$ in equilibrium
$$C = \frac{1}{\langle N \rangle^2} \int (r_g - r_{g,eq}) dx_1 \, dx_2$$

Experimental Fit