Finite size of hadrons and Bose-Einstein correlations in *pp* collisions at 7 TeV

Wojciech Florkowski

Institute of Nuclear Physics, Kraków
& Jan Kochanowski University, Kielce, Poland

INT Program INT-15-2b, Correlations and Fluctuations in p+A and A+A Collisions
July 31, 2015

based on recent work with A. Bialas and K. Zalewski, **Phys.Lett. B748 (2015) 9**

Prologue

Simple model of Bose-Einstein correlations in hadron production

$$N(p_1,p_2) = \int dx_1\, dx_2\, w(x_1,x_2) |\Psi_{12}|^2 \tag{1}$$

For independent production and Bose particles

$$w(x_1,x_2) = w(x_1) w(x_2), \quad \Psi_{12} = \frac{1}{2\pi} \frac{1}{\sqrt{2}} \left(e^{i x_1 p_1} e^{i x_2 p_2} + e^{i x_2 p_1} e^{i x_1 p_2} \right) \tag{2}$$

$$|\Psi_{12}|^2 = \frac{1}{(2\pi)^2} \left(1 + \frac{1}{2} \left(e^{iQ(x_2-x_1)} + e^{-iQ(x_2-x_1)} \right) \right), \quad Q = p_1 - p_2. \tag{3}$$

Finally, we get

$$N(p_1,p_2) = \int \frac{dx_1}{2\pi} \frac{dx_2}{2\pi} w(x_1) w(x_2) + \int \frac{dx_1}{2\pi} \frac{dx_2}{2\pi} w(x_1) w(x_2) e^{iQ(x_2-x_1)} \tag{4}$$

and

$$2 \geq \frac{N(p_1,p_2)}{N(p_1) N(p_2)} \geq 1, \quad N(p) = \int \frac{dx}{2\pi} w(x) \tag{5}$$

1.1 Motivation

It has been recently pointed out by A. Bialas and K. Zalewski, **Phys.Lett. B727 (2013) 182**, that since hadrons are not point-like objects, they cannot be uncorrelated. Indeed, being composite, hadrons cannot occupy too close space-time points (because at small distance the constituents of hadrons mix and there are no separate hadrons to interfere).

Consequently, since the HBT experiment measures the quantum interference between wave functions of hadrons, it cannot see hadrons which are too close to each other. Therefore, the distribution function of the pair of hadrons must vanish at small distances between them.

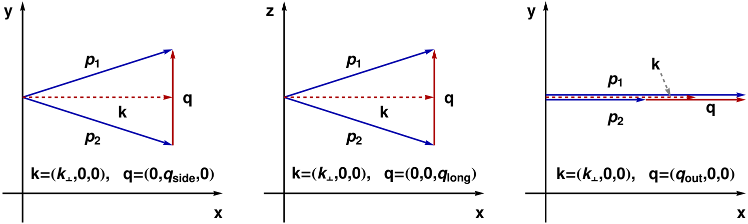
This implies a correlation in space-time. As this correlation is the necessary consequence of the composite structure of hadrons it is interesting to investigate to what extent it modifies the accepted ideas about the quantum interference which are, usually, derived under the assumption that such correlations can be neglected.

It was already shown, see again **Phys.Lett. B727 (2013) 182**, that such space-time correlations may be responsible for the observation that the two-pion Bose-Einstein correlation function takes values below unity, at variance with the well-known theorem valid when the correlations are ignored.

In this talk the study of this phenomenon is continued, using the recently published analysis of the data on HBT radii, measured by the **ALICE** collaboration, **Phys.Rev. D84 (2011) 112004**.

This allows to estimate quantitatively the magnitude of the effect and to give predictions for its size in all three directions **side, long** and **out**, commonly used in discussion of the quantum interference.

Bertsch-Pratt parameterization (boost-invariant, cylindrically symmetric systems)
$k = P_{12} = (p_1 + p_2)/2, \quad Q = q = (p_1 - p_2)$



one can alternatively use the invariant variable: $Q = \sqrt{-(p_1 - p_2)^2} = \sqrt{M^2 - 4m_\pi^2}$.

$$R^2 = d \ln C(k_\perp, q)/dq^2|_{q^2=0}$$

1.2 A bit advanced formalism

Emission functions, Wigner functions, Cooper-Frye formula, etc.

$$w(p_1, p_2; x_1, x_2) = w(p_1, x_1) w(p_2, x_2) \tag{6}$$

and

$$N(p_1, p_2) = \int dx_1 dx_2 w(p_1, p_2; x_1, x_2) + \int dx_1 dx_2 \; e^{iQ(x_1 - x_2)} w(P_{12}, P_{12}; x_1, x_2) \tag{7}$$

where

$$P_{12} = (p_1 + p_2)/2, \quad Q = p_1 - p_2. \tag{8}$$

Consequently, the Bose-Einstein correlation function between the momenta of two identical particles

$$C(p_1, p_2) \equiv \frac{N(p_1, p_2)}{N(p_1)N(p_2)} \tag{9}$$

is given by

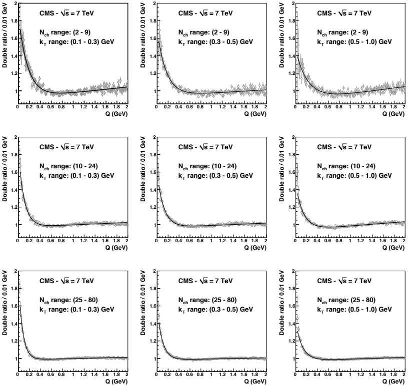
$$C(p_1, p_2) = 1 + \frac{\tilde{w}(P_{12}; Q)\tilde{w}(P_{12}; -Q)}{w(p_1)w(p_2)} = 1 + \frac{|\tilde{w}(P_{12}, Q)|^2}{w(p_1)w(p_2)} \geq 1. \tag{10}$$

Here

$$\tilde{w}(P_{12}; Q) = \int dx\ e^{iQx} w(P_{12}; x), \quad w(p) = \int dx\ w(p; x), \tag{11}$$

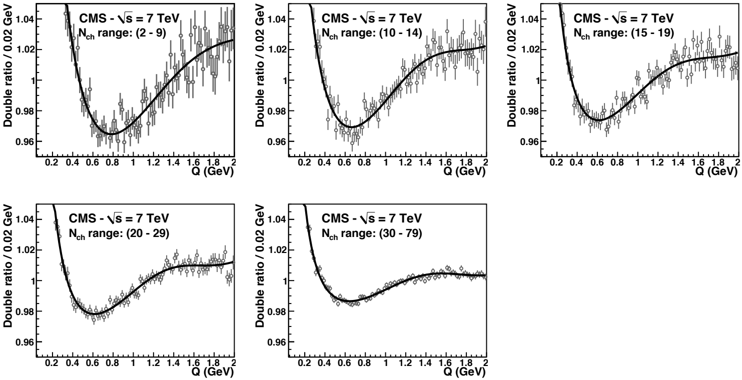
1.3 Dips in correlation functions

The data from the L3 collaboration and from the CMS colllaboration show that the correlation function $C(p_1, p_2)$ takes values below unity, contrary to Eq. (10).



Measurement of Bose-Einstein Correlations in pp Collisions at $\sqrt{s} = 0.9$ and 7 TeV

CMS Collaboration, JHEP 1105 (2011) 029



Measurement of Bose-Einstein Correlations in pp Collisions at $\sqrt{s}$ = 0.9 and 7 TeV

CMS Collaboration, JHEP 1105 (2011) 029

Thus the particles must be correlated and we propose that this effect is due to the composite nature of hadrons.

To implement these space-time correlations, we replace formula (6) for the two-particle source function by

$$W(p_1, p_2; x_1, x_2) = w(p_1; x_1) w(p_2; x_2)[1 - D(x_1 - x_2)], \tag{12}$$

where $D(x_1 - x_2)$ is the cut-off function that satisfies the constraint $D(x_1 - x_2 = 0) = 1$ and tends to 0 at larger distances (above, let us say, 1 fm).

Then, the HBT correlation function becomes

$$C(P_{12}, Q) = C_{\rm noncorr}(P_{12}, Q) - C_{\rm corr}(p_1, p_2), \tag{13}$$

The uncorrelated part $C_{\rm noncorr}(P_{12}, Q)$ is given by (10), while the correction due to space-time correlations reads

$$C_{\rm corr} = C^{(0)}_{\rm corr} + C^{(Q)}_{corr} \tag{14}$$

where

$$C^{(0)}_{\rm corr} = \frac{\int dx_1 dx_2 w(p_1; x_1) w(p_2; x_2) D(x_1 - x_2)}{w(p_1) w(p_2)}, \tag{15}$$

$$C^{(Q)}_{\rm corr} = \frac{\int dx_1 dx_2 e^{i(x_1 - x_2)Q} w(P_{12}; x_1) w(P_{12}; x_2) D(x_1 - x_2)}{w(p_1) w(p_2)}. \tag{16}$$

One sees that the contribution from the correlation part is negative. Moreover, since it obtains contributions from a small region of space-time, its dependence on Q is much less steep than that of the uncorrelated part. Consequently, at Q large enough $C(P_{12}, Q)$ may easily fall below one.

1.4 Halo model

To describe the actual measurements one has to take into account that particles produced very far from the center form a ”halo” and do not contribute to the HBT correlations. Thus we have

$$\hat{C}_{obs}(P_{12}, Q) = 1 - p^2 + p^2 C(P_{12}, Q) \tag{17}$$

where p^2 is the probability that both particles originate from the ”core”.

In the ALICE experiment $\hat{C}_{\rm obs}$ was, in addition, normalized to 1 at some Q_0 where the influence of quantum interference is expected to be negligible. Thus we finally have to consider the function

$$C_{\rm obs}(P_{12}, Q) = \frac{1 - p^2 + p^2 C(P_{12}, Q)}{1 - p^2 + p^2 C(P_{12}, Q_0)}. \tag{18}$$

Introducing the (measured) intercept parameter λ by the condition

$$1 + \lambda \equiv C_{\rm obs}(P_{12}, Q = 0) \tag{19}$$

one obtains

$$p^2 = \frac{\lambda}{C(P_{12}, Q = 0) - C(P_{12}, Q_0) + \lambda[1 - C(P_{12}, Q_0)]} \tag{20}$$

This allows to evaluate the measured correlation function in terms of the measured intercept parameter λ and the evaluated correlation function $C(P_{12}, Q)$.

Note that in absence of space-time correlations we have $C(P_{12}, Q = 0) = 2$, $C(P_{12}, Q_0) = 1$, and thus $p^2 = \lambda$, as is usually assumed.

2.1 Freeze-out geometry

To get an estimate of the magnitude of the effect we discuss, we use the popular blast-wave model. In this model, at freeze-out, hadrons are created at a fixed (longitudinal) proper time

$$\tau \equiv \sqrt{t^2 - z^2} = \tau_f. \tag{21}$$

The single-particle source function (in the longitudinal c.m.s. system) becomes

$$w(k,x) = k_0 \cosh \eta e^{-U \cosh \eta + V \cos \phi} f(r) r dr d\phi d\eta \tag{22}$$

where $k_0 = \sqrt{m^2 + k_\perp^2}$, whereas η, ϕ and r are space-time rapidity, azimuthal angle and transverse distance from the symmetry axis [1]. We have also introduced the notation

$$U = \beta k_0 \cosh \theta; \quad V = \beta k_\perp \sinh \theta, \tag{23}$$

with $T = 1/\beta$ being the freeze-out temperature. Finally, θ describes the transverse flow

$$\sinh \theta = \omega r, \tag{24}$$

with ω being a parameter. The function $f(r)$ describes the transverse profile of the source.

[1] All irrelevant constants are cancelled in the definition of $w(p,x)$.

It was shown in that the model is flexible enough to describe the HBT radii measured by the ALICE collaboration. The function $f(r)$ was taken in the form

$$f(r) \sim e^{-(r-R)^2/\delta^2} \tag{25}$$

corresponding to a "shell" of the width $\sqrt{2}\,\delta$ and radius R.

The SHELL structure is crucial for description of the data! Gubser flow geometry applicable?

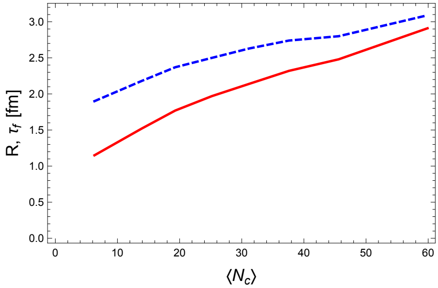
Thus the model contains 5 free parameters: T, ω, τ_f, R and δ,
which may depend on the multiplicity of the event.

The freeze-out temperature is set equal to $T = 100$ MeV, much poorer fits for larger values of T.

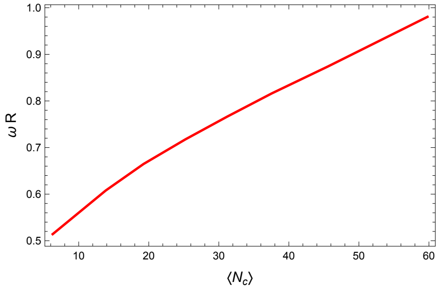
The measured average transverse momentum, taken from CMS, fixes (for each multiplicity bin) the relation between ωR and the ratio R/δ.

The width of the shell is taken as $\delta = 0.75$ fm.

Finally we have two free parameters: R and τ_f.



R, τ_f [fm]
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
10
20
30
40
50
60
$\langle N_c \rangle$



ω R
1.0
0.9
0.8
0.7
0.6
0.5
10
20
30
40
50
60
$\langle N_c \rangle$

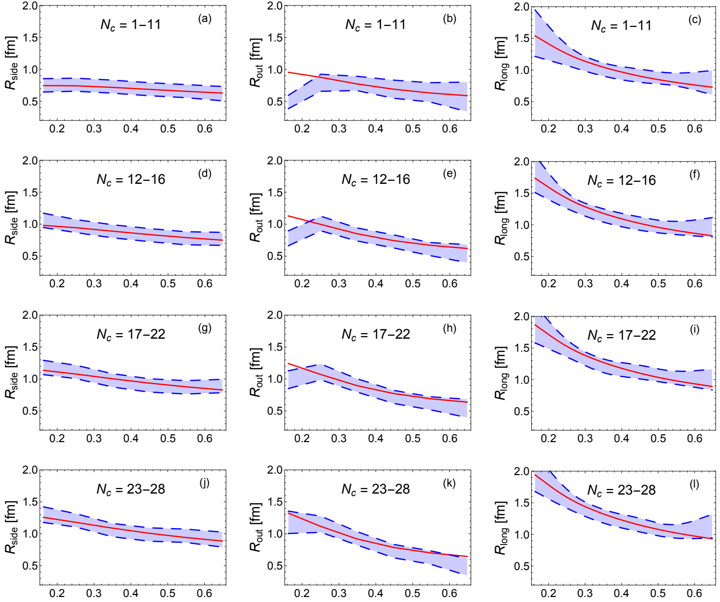
2.2 HBT radii (fitted without the cut-off)

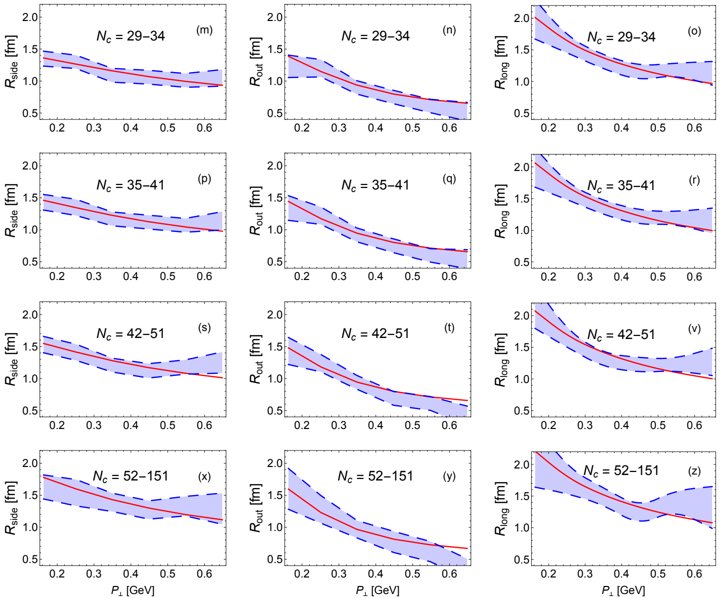
A. Bialas, WF. K.Zalewski, J.Phys. G42 (2015) 4, 045001
We use

mult. class	1–11	12–16	17–22	23–28	29–34	35–41	42–51	52–151
$\langle N_c \rangle$	6.3	13.9	19.3	25.2	31.2	37.6	45.6	59.9
R [fm]	1.15	1.52	1.77	1.97	2.14	2.32	2.49	2.91
τ_f [fm]	1.90	2.18	2.37	2.50	2.63	2.74	2.80	3.09
χ^2	0.96	1.90	2.89	4.06	5.88	5.45	11.63	8.48
$\chi^2_{\rm tot}$	18.77	11.08	6.34	4.86	6.65	5.79	11.72	9.7

Model parameters and χ^2 values for different multiplicity classes, # of degrees of freedom = 13, χ^2 excludes the first bin in $k_\perp$, $\chi^2_{\rm tot}$ includes all the bins

and we get...


N_c = 1–11 (a) (b) (c)
N_c = 12–16 (d) (e) (f)
N_c = 17–22 (g) (h) (i)
N_c = 23–28 (j) (k) (l)
R_{side} [fm]
R_{out} [fm]
R_{long} [fm]
0.5 1.0 1.5 2.0
0.2 0.3 0.4 0.5 0.6



N_c = 29–34 (m)
N_c = 29–34 (n)
N_c = 29–34 (o)
N_c = 35–41 (p)
N_c = 35–41 (q)
N_c = 35–41 (r)
N_c = 42–51 (s)
N_c = 42–51 (t)
N_c = 42–51 (v)
N_c = 52–151 (x)
N_c = 52–151 (y)
N_c = 52–151 (z)
R_{side} [fm]
R_{out} [fm]
R_{long} [fm]
$P_\perp$ [GeV]
0.5
1.0
1.5
2.0
0.2
0.3
0.4
0.5
0.6

HBT puzzle solved by the shell geometry!

Mult. class 23–28

$P_\perp$ [GeV]	$R_{\rm long}$[fm] model	$R_{\rm long}$[fm] exp.	$R_{\rm side}$[fm] model	$R_{\rm side}$[fm] exp.	$R_{\rm out}$[fm] model	$R_{\rm out}$[fm] exp.
0.163	1.94	1.99±0.31	1.26	1.30±0.12	1.32	1.18±0.17
0.251	1.58	1.56±0.15	1.18	1.21±0.10	1.12	1.15±0.13
0.349	1.33	1.29±0.11	1.09	1.06±0.10	0.92	0.93±0.10
0.448	1.15	1.15±0.11	1.01	0.99±0.10	0.79	0.73±0.10
0.548	1.03	1.05±0.11	0.94	0.97±0.10	0.70	0.63±0.10
0.648	0.93	1.13±0.19	0.89	0.91±0.12	0.65	0.48±0.13

2.3 Cut-off function

Since we treat particles as extended objects produced on the hyperbola (21), the longitudinal distance between the two hadrons located at the space-time rapidities $\eta_1,\ \eta_2$ should be calculated along this curve, which yields

$$d_{\|} = \int_{\eta_1}^{\eta_2} \sqrt{dz^2 - dt^2} = \tau_f(\eta_2 - \eta_1). \tag{26}$$

In the frame where $\eta_1 + \eta_2 = 0$ we also have $t_1 = t_2$ and thus the total distance between particles is

$$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + d_{\|}^2 \equiv d_{\perp}^2 + d_{\|}^2. \tag{27}$$

Since this expression is invariant under boost in the longitudinal direction, it is also valid in the LCMS system, and thus we finally have

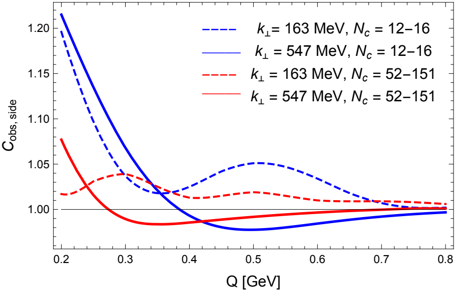
$$d^2(x_1, x_2) = r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\phi_1 - \phi_2) + \tau_f^2 (\eta_1 - \eta_2)^2. \tag{28}$$

The correlation functions were studied using a gaussian cut-off function

$$D(x_1, x_2) = e^{-d(x_1,x_2)^2/\Delta^2}, \tag{29}$$

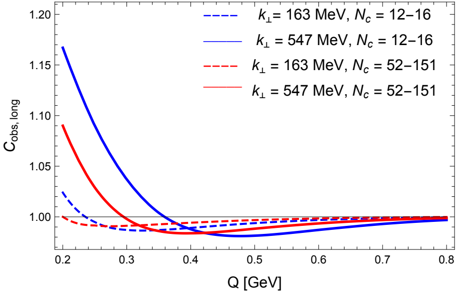
where Δ is a constant fixing the scale of the cut-off region.

3.1 Side



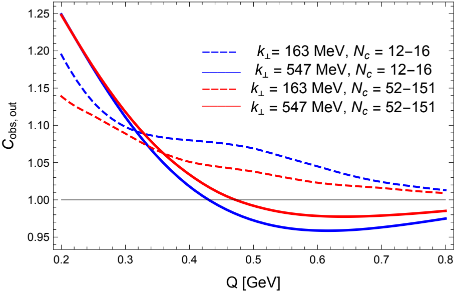
Correlation function $C_{\rm obs}$ for the *side* direction in the interval 0.2 GeV $\leq Q \leq$ 0.8 GeV (normalized to 1 at Q= 1GeV). The dashed lines describe the results for $k_\perp$ = 163 MeV and the two multiplicity classes: N_c = 12–16 and N_c = 52–151. The solid lines describe the results for $k_\perp$ = 547 MeV and the same two multiplicity classes.

3.2 Long

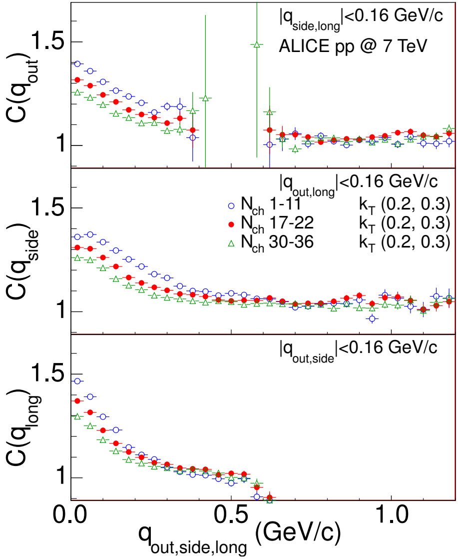


$k_\perp$= 163 MeV, N_c = 12–16
$k_\perp$ = 547 MeV, N_c = 12–16
$k_\perp$ = 163 MeV, N_c = 52–151
$k_\perp$ = 547 MeV, N_c = 52–151
$C_{obs, long}$
Q [GeV]
1.20
1.15
1.10
1.05
1.00
0.2
0.3
0.4
0.5
0.6
0.7
0.8

3.3 Out


$C_{obs, out}$
1.25
1.20
1.15
1.10
1.05
1.00
0.95
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Q [GeV]
$k_\perp$= 163 MeV, N_c = 12–16
$k_\perp$ = 547 MeV, N_c = 12–16
$k_\perp$ = 163 MeV, N_c = 52–151
$k_\perp$ = 547 MeV, N_c = 52–151

3.4 ALICE



|q_side,long|<0.16 GeV/c
ALICE pp @ 7 TeV
|q_out,long|<0.16 GeV/c
N_ch 1-11 k_T (0.2, 0.3)
N_ch 17-22 k_T (0.2, 0.3)
N_ch 30-36 k_T (0.2, 0.3)
|q_out,side|<0.16 GeV/c
C(q_out)
C(q_side)
C(q_long)
1.5
1
0.0
0.5
1.0
q_out,side,long (GeV/c)

4.1 Summary

(i) The space-time correlations induced by the finite size of hadrons lead to a rich structure of the HBT correlation functions, depending on (i) the measurement direction, (ii) multiplicity and (iii) the transverse momentum of the pair.

(ii) The difference between the *long* and the two other directions at small $k_\perp$ is particularly striking.

(iii) At large $k_\perp$ the minimum below 1 shows up in every direction. It is about twice deeper for *out* than for the *long* and *side* directions.

4.2 Closing remarks

(i) We have found that the modification of the HBT correlation functions are only marginally sensitive to the change of shape of the cut-off function $D(x_1 - x_2)$. This means that the effect we discuss is, in practice, described by a single parameter Δ.

(ii) We have been considering the space-time correlations in the source function of two pions, which are a necessary consequence of their composite nature. Naturally, there might be also other mechanisms contributing to these correlations (e.g. the final state interaction). In this case the parameter Δ should be considered as an effective cut-off distance which summarizes all contributions.

(iii) In our approach the cut-off function is taken independent of particle density. This approximation seems reasonable because particle density at freeze-out changes only by 10% in the range of multiplicities we consider. Moreover, the dominant effect of the changing particle density is expected to be a modification of the single-particle source functions of the two pions contributing to interference rather than of their space-time correlation described by $D(x_1 - x_2)$.

4.2 Closing remarks

(iv) It is interesting to speculate about the size of the effects we discuss in case of heavy ion collisions. Taking the source functions in the transverse direction in form of Gaussians (which is a reasonable approximation for heavy ion collisions) one expects that the corrections due to finite size of hadrons fall as $(\Delta/R)^2$ where R is the radius of the system. For PbPb collisions this gives factor $\sim 1/30$ compared to the results shown in this paper, implying that the expected effects are negligible. For smaller systems, as those created in p-Pb collisions, the effects are also expected to be smaller than in *pp*. Precise estimate would, however, require determination of the source functions.