Anisotropic gluon distributions of a nucleus

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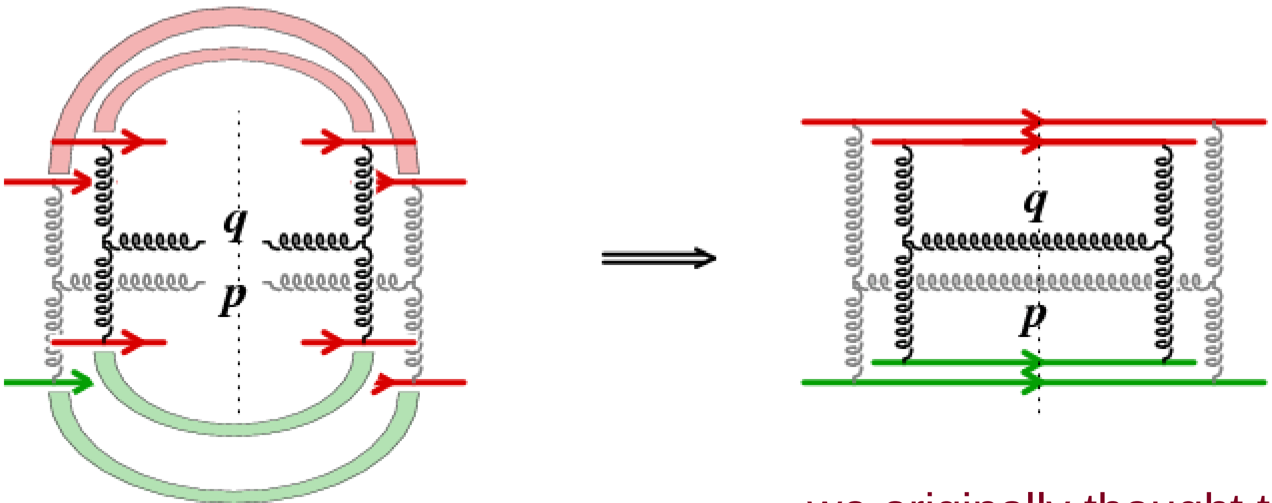
Correlations and Fluctuations
in p+A and A+A Collisions

- anisotropy of gluon distribution in CGC formalism
- v_2 anisotropy from TMD factorization,
  dijets in eA DIS

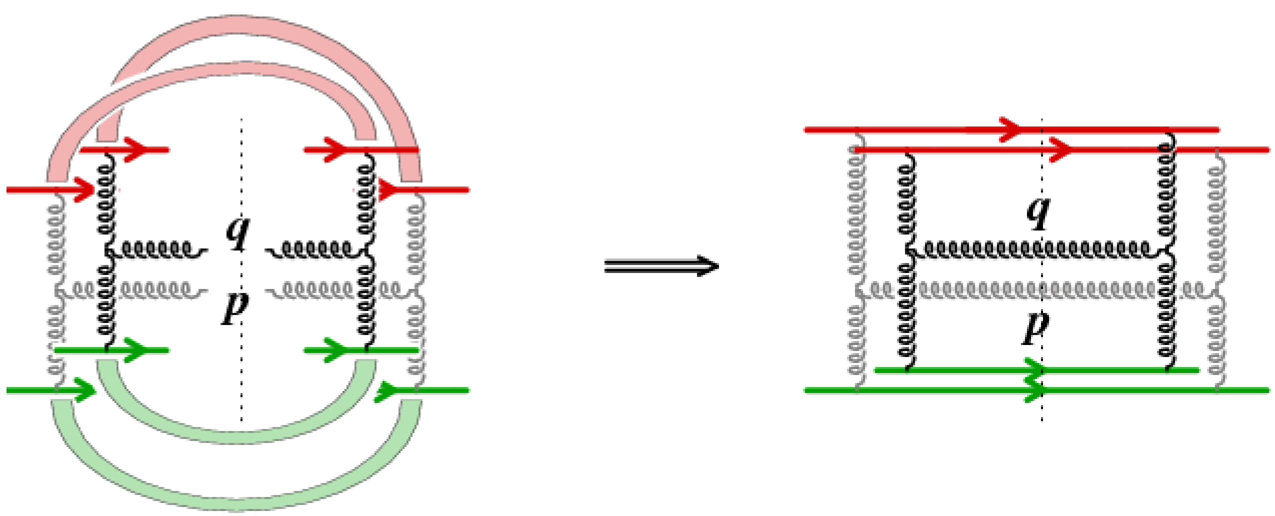
Independent production of two gluons:



“independent” production

we originally thought that this drops out: wrong
(pointed out by Kovner & Lublinsky)

Correlated two-gluon production / “glasma graphs”:

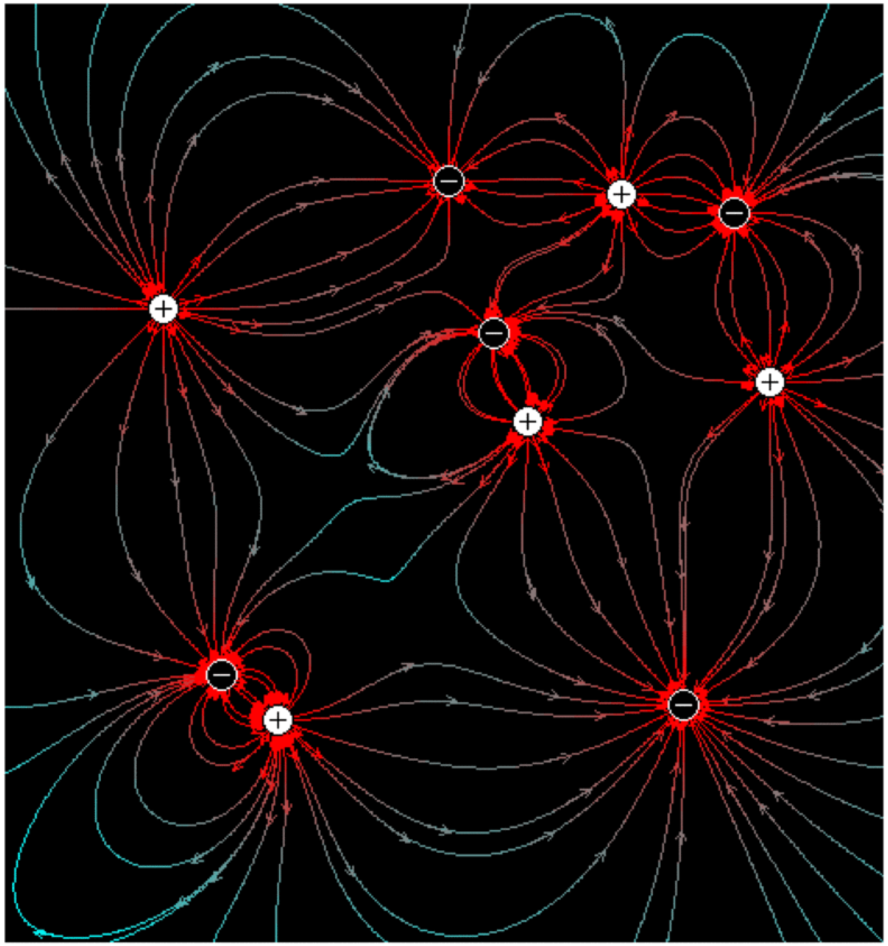


$$v_n\{2\}^2 \equiv c_n\{2\} = \left\langle e^{in(\phi_1-\phi_2)} \right\rangle$$
$$= \left\langle e^{in\phi_1} \right\rangle \left\langle e^{-in\phi_2} \right\rangle + \left\langle e^{in(\phi_1-\phi_2)} \right\rangle_{\mathrm{conn.}}$$

- factorized contribution not zero if target field breaks 2D rot. symmetry of 1-particle distrib. !

E-field of Abelian charges

- isotropic monopole fields
- dipole fields
- and some more "exotic" field line configurations, too



Java applet by M. J. McGuffin, Univ. of Toronto
(uses ~1/r Coulomb potential for d= 3)

McLerran-Venugopalan (MV) model

- "valence" quarks & gluons described as recoilless charges on the light-cone (slow fields)
- act as sources for soft (small-x) gluon field

MV model action: (C-even)

$$S_{\mathrm{MV}} = \int d^2x_\perp dx^- \, \frac{\rho^a \rho^a}{2\mu^2}$$

(dense sources)

more generally:

$$S_{\mathrm{eff}} = \int d^2x_\perp dx^- \left(\frac{\rho^a \rho^a}{2\mu^2} - \frac{d^{abc}\rho^a\rho^b\rho^c}{\kappa_3} + \frac{\delta^{ab}\delta^{cd}\rho^a\rho^b\rho^c\rho^d}{\kappa_4} + \cdots \right)$$

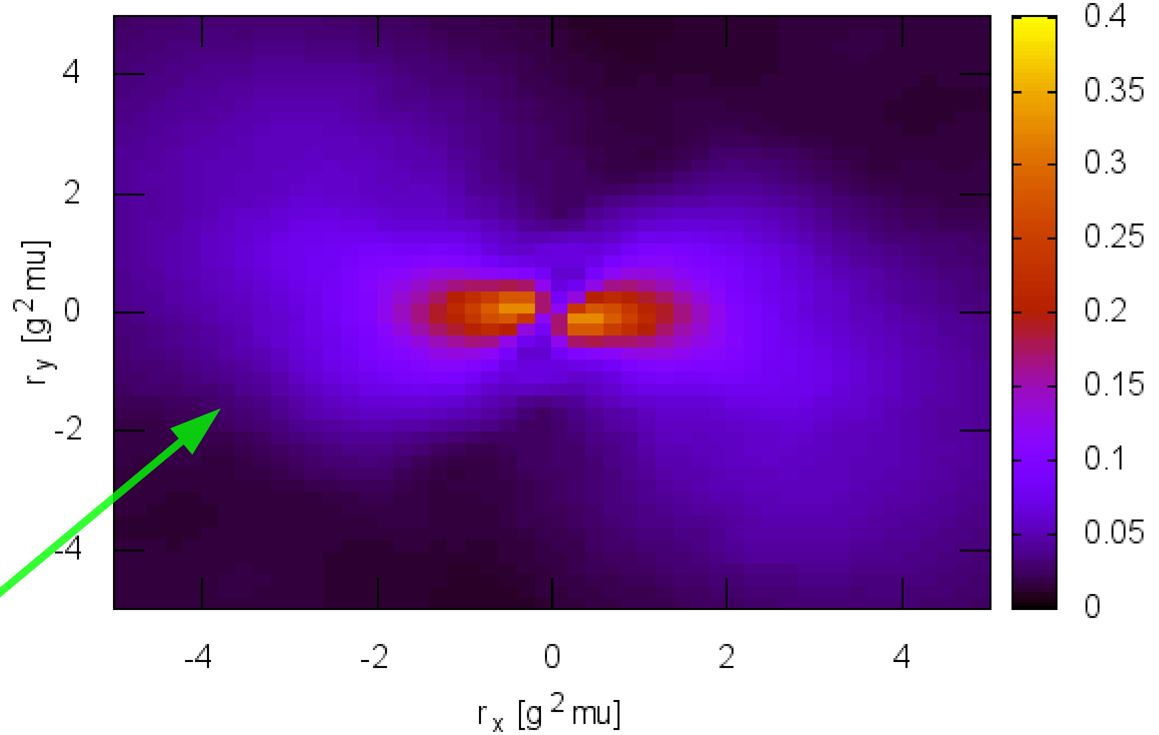
generate configuration $\rho^a(x_\perp, x^-)$ with weight $\exp(-S_{\mathrm{MV}}[\rho])$

soft field (in cov. gauge) $-\nabla_\perp^2 A^{+a} = g\,\rho^a \quad , \quad F^{+i} = -\partial^i A^+$

compute VEV of observable: $\langle O[A^+] \rangle$ ex: $O[A^+] = \frac{1}{N} \mathrm{tr}\, V_x V_y^\dagger$

E-field in MV model (at some fixed b), *single configuration*

A.D. & V. Skokov, arXiv:1411.6630

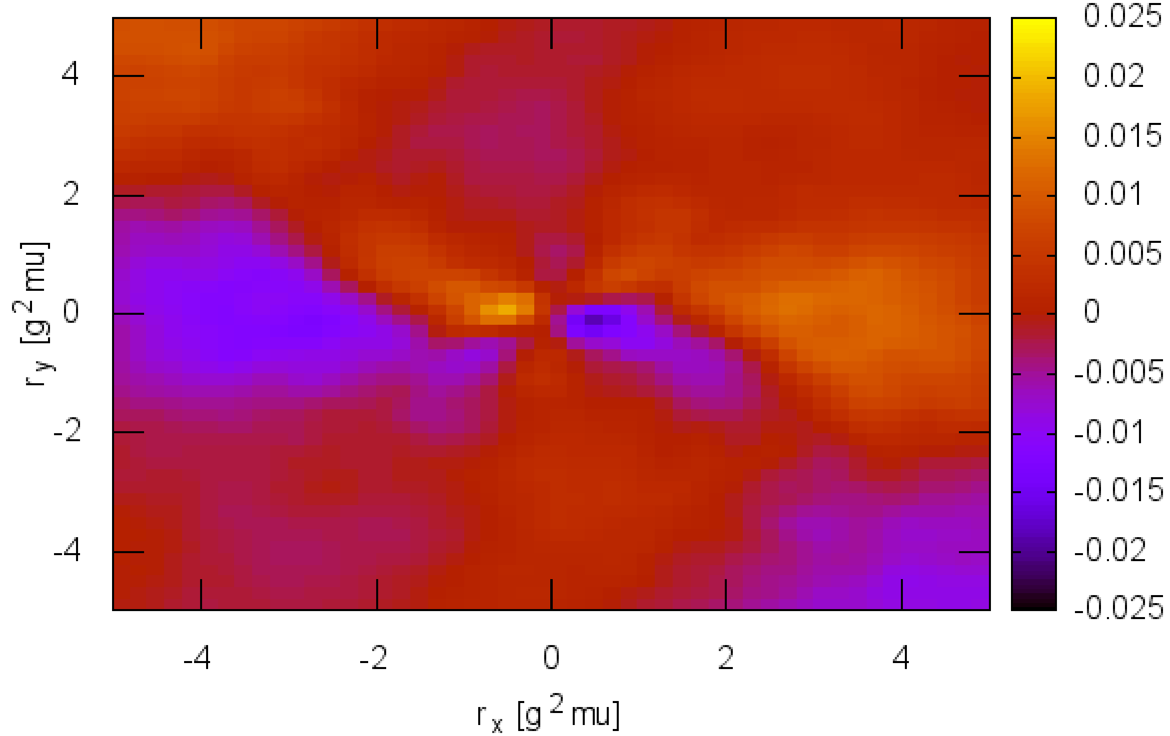


- clear cos(2φ) + other components

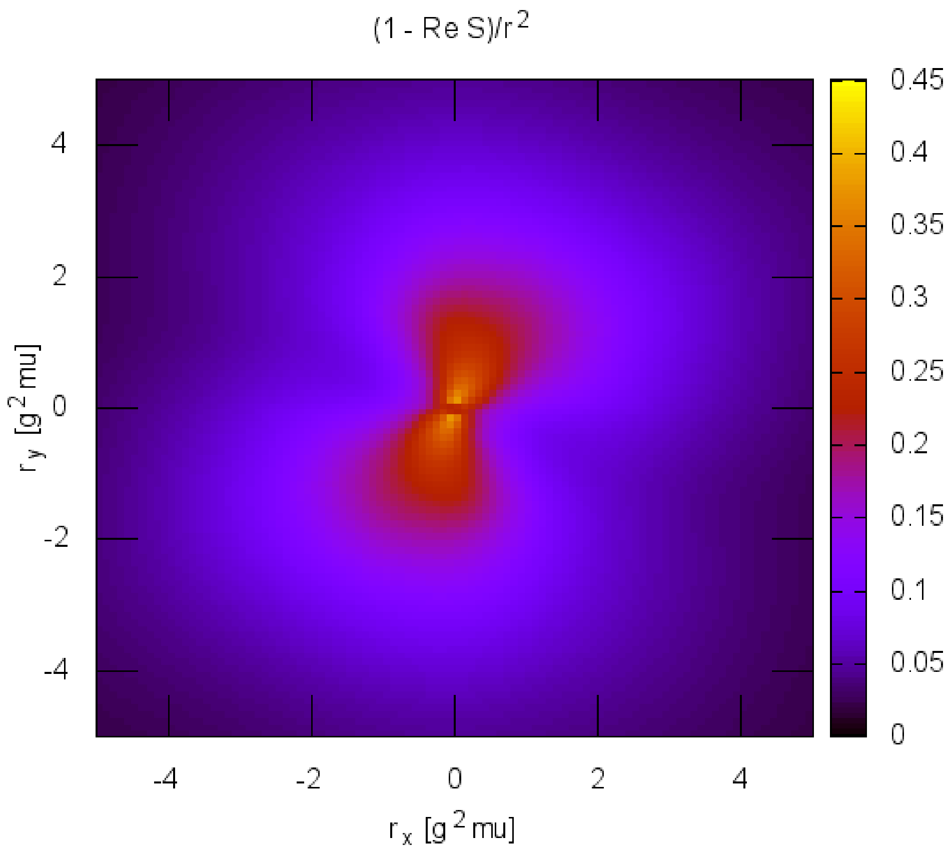
$$\frac{1}{r^2}\left[1-\frac{1}{N}\mathrm{Re}\,\mathrm{tr}\,V_{\vec{x}}V_{\vec{y}}^{\dagger}\right] \sim g^2\,\mathrm{tr}\left(\hat{r}\cdot\vec{E}(\vec{b})\right)^2 + \mathcal{O}\left((igr)^4\right)$$

$$\frac{1}{r^2}\frac{1}{N}\mathrm{Im}\ \mathrm{tr}\, V_{\vec{x}} V_{\vec{y}}^{\dagger}$$

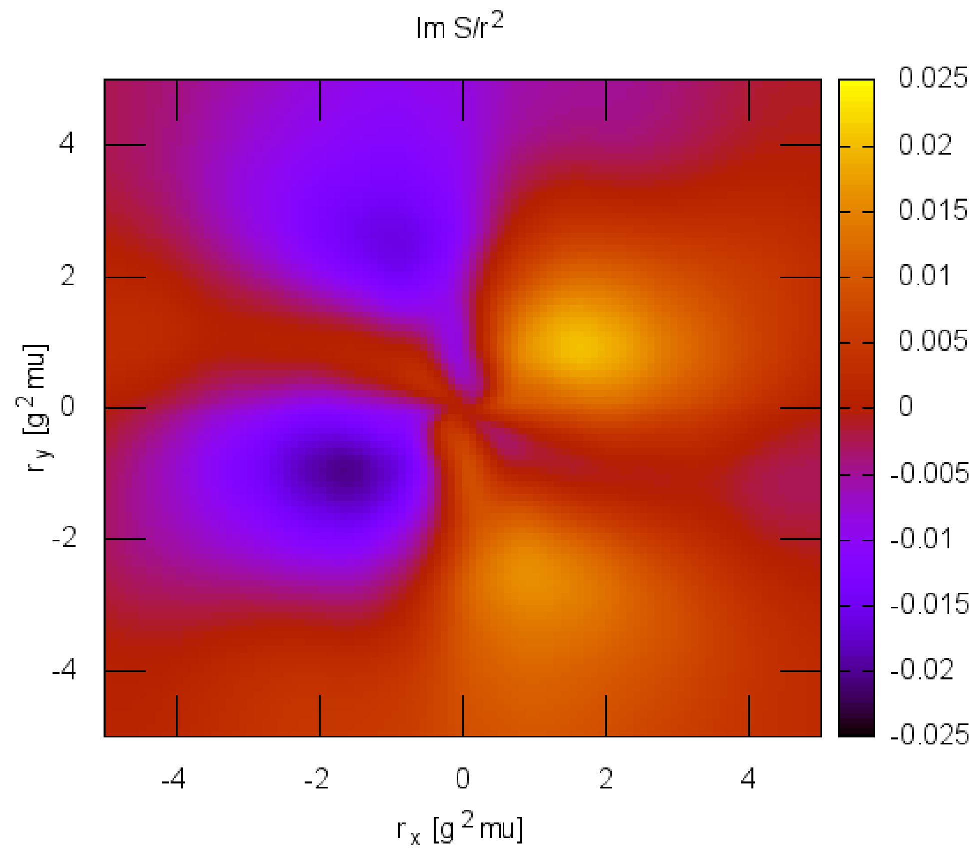
- cos(φ)
  + higher components



another nice configuration:



cos 2φ



cos 3φ

Extracting the amplitudes of cos(ϕ), cos(2ϕ), ...

$$1 - \mathrm{Re}\; S_\rho(\vec{r}) = \mathcal{N}(r)\left(1 + \sum_{n=1}^{\infty} A'_{2n}(r)\cos(2n\phi_r)\right)$$

dipole amplitude

$$\mathrm{Im}\; S_\rho(\vec{r}) = \mathcal{N}(r)\sum_{n=0}^{\infty} A'_{2n+1}(r)\cos((2n+1)\phi_r)$$

odderon

the A_n' contain a random global phase (angular zero mode: $\varphi \longrightarrow \varphi + \psi$)

[$\longrightarrow \langle A_n' \rangle = 0$]

which needs to be removed: $A_n = (\pi/2)\,|A_n'|$

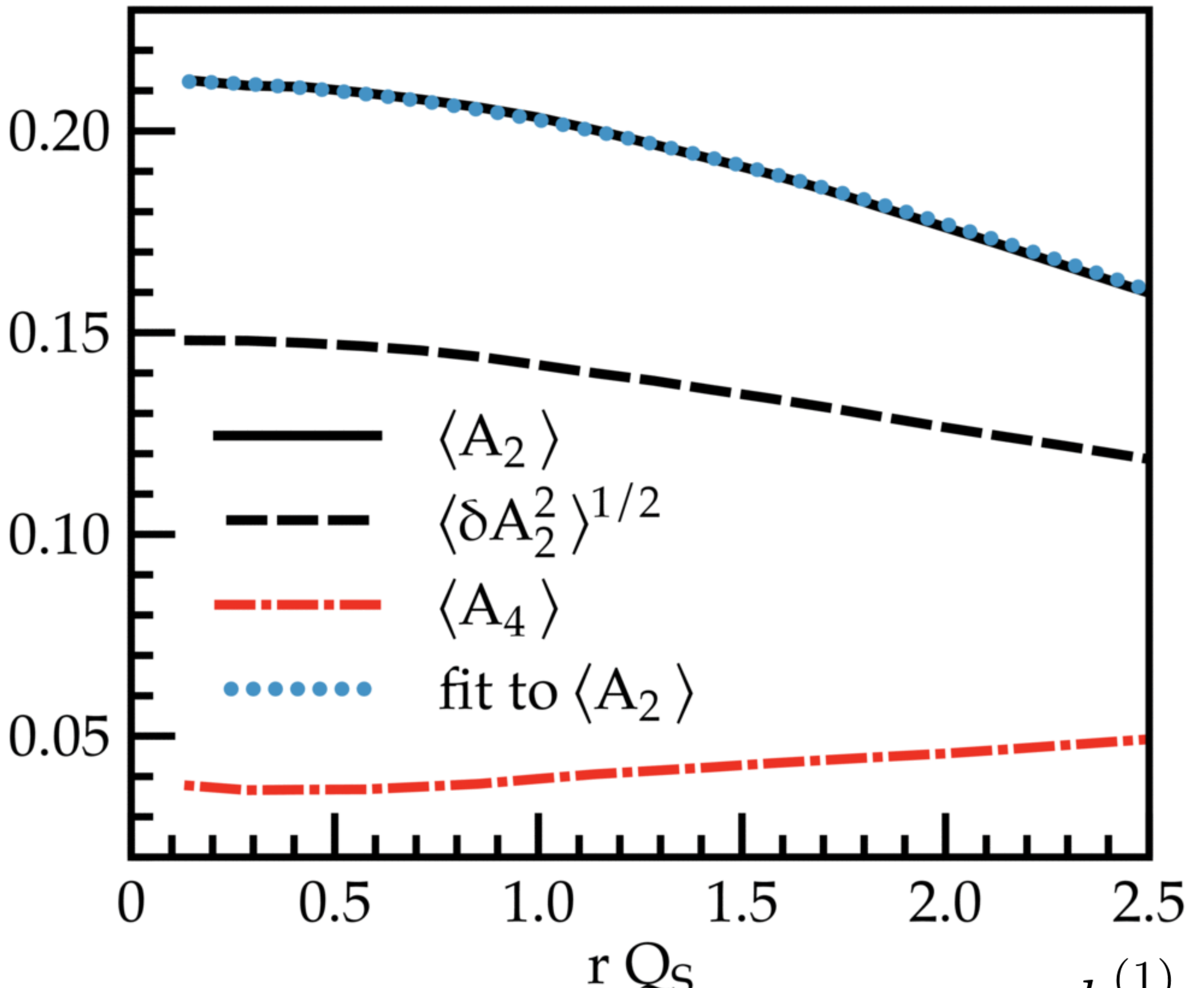
So, we shall compute $\langle A_n \rangle(r)$

analogy: SSB for complex scalar field φ: $V(\phi) = -\frac{1}{2}m^2\phi\phi^* + \frac{1}{4}\lambda(\phi\phi^*)^2 \;, \quad (m^2, \lambda > 0)$

U(1) circle of degenerate vacua: $\phi_0 = \sqrt{\frac{m^2}{\lambda}}\, e^{i\alpha}$ (α is random)

$$\langle\phi\rangle = 0 \;\;,\;\; \langle|\phi|^2\rangle \neq 0$$

Even harmonics



- <A_2> is largest amplitude
- $\sqrt{<A_2^2>}$ comparable to <A_2> : large fluctuations
- <A_4> ≠ 0 and ~ const at small r ?

$$\mathrm{Re}\, S(\vec{r},\vec{b}) - 1 = \frac{(ig)^2}{2N_c}\left(\vec{r}\cdot\vec{E}(\vec{b})\right)^2 + \frac{1}{2}\left[\frac{(ig)^2}{2N_c}\left(\vec{r}\cdot\vec{E}(\vec{b})\right)^2\right]^2 + \cdots$$

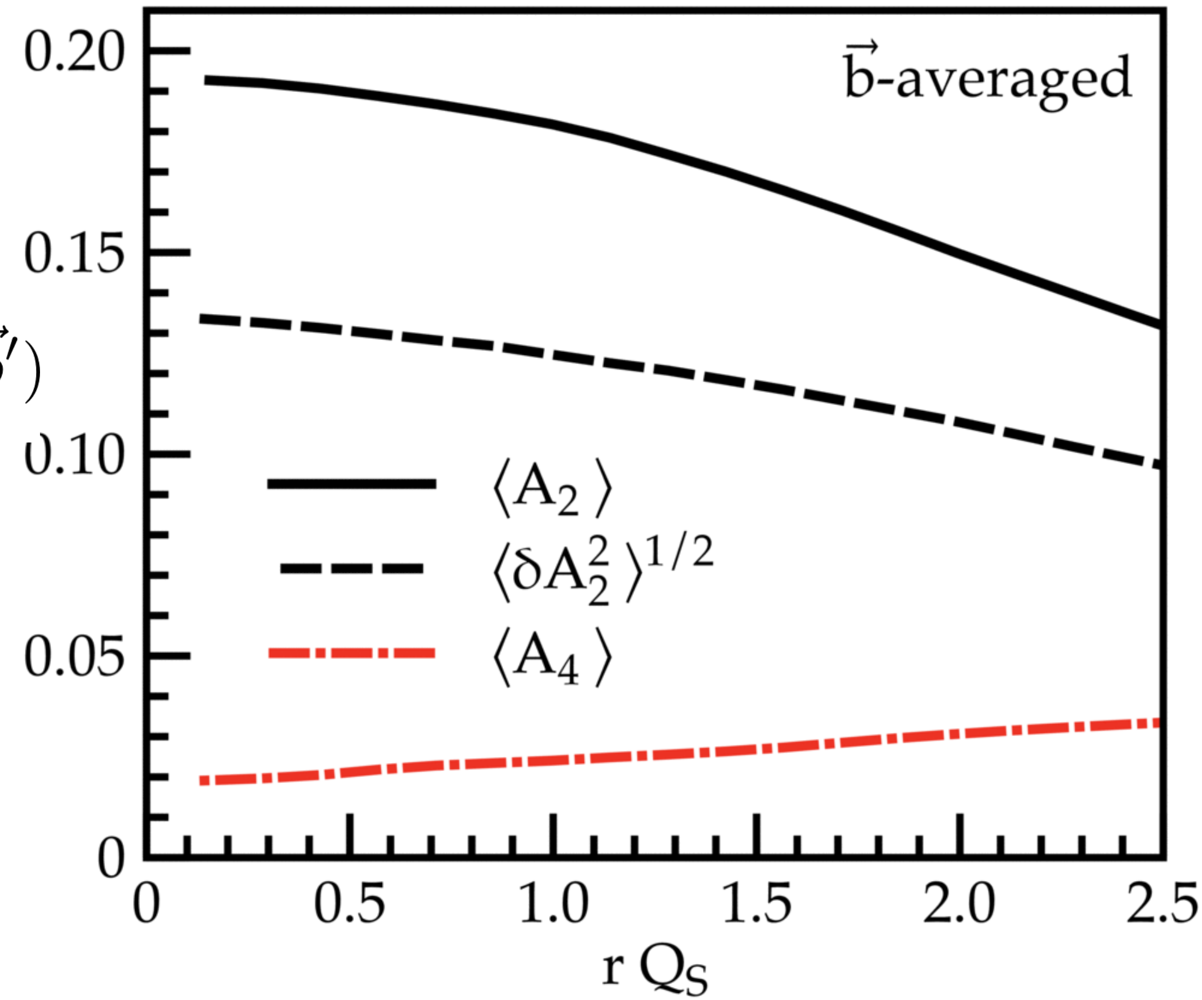
- fit of <A_2>(r) motivated by WW distribution of linearly polarized gluons Metz & Zhou (1105.1991)

$$xh_\perp^{(1)}(r^2) \propto \frac{1}{r^2Q_s^2}\left[1 - \exp\left(-\frac{r^2Q_s^2}{4}\right)\right]$$

[should've compared to $xh_\perp^{(2)}(r^2)$ by Dominguez et al, PRD2012;
at small r it behaves the same though;
that's the distribution of linearly polarized gluons from the dipole operator)

"b-smeared" fields:

$$\int \frac{d^2b'}{\pi r^2}\,\Theta\left(r-|\vec{b}-\vec{b}'|\right)\,D_\rho(\vec{r},\vec{b}')$$



- no difference to b=0 for small πr^2
- for large πr^2: A_2 & A_4 suppressed
  [not shown here, see arXiv:1411.6630 fig.5]
- $\longrightarrow$ $\vec{E}$-field correlation length (magnitude & direction) of order $1/Q_s$

Spontaneous breaking of rotational symmetry :

Kovner & Lublinsky:
PRD 84 (2011)

- $\vec{E}$ field "domains"

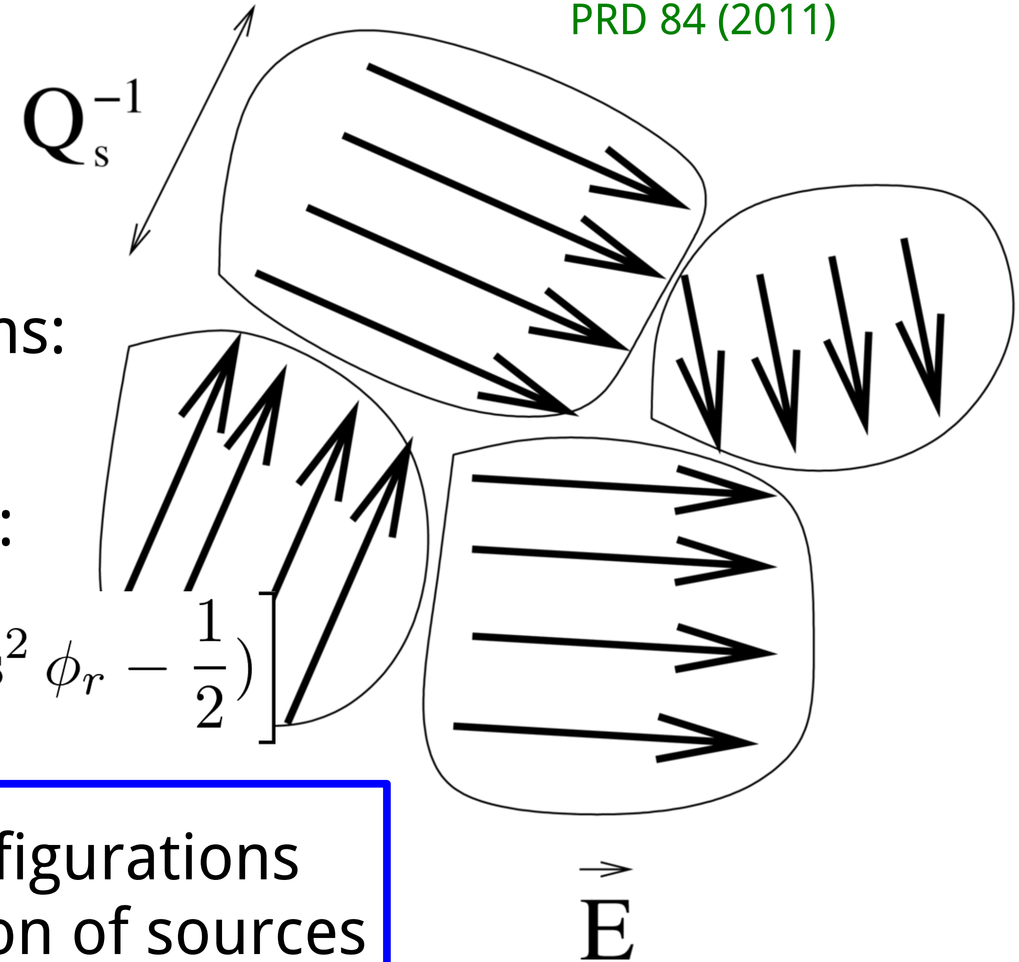
$$D(\vec{r}) = \left\langle e^{-\frac{1}{2N_c}\mathrm{tr}\,(g\,\vec{r}\cdot\vec{E})^2} \right\rangle$$

usually: avg over ALL configurations:

$$g^2\, r^i r^j\, \langle \mathrm{tr}\, E^i E^j \rangle \sim r^2\, Q_s^2$$

here: avg at fixed orientation of $\vec{E}$:

$$g^2\, r^i r^j\, \langle \mathrm{tr}\, E^i E^j \rangle \sim r^2\, Q_s^2 \left[1 + 2\mathcal{A}(\cos^2\phi_r - \frac{1}{2}) \right]$$



- $\mathcal{A} > 0$ arises for *individual* configurations due to *fluctuations* of distribution of sources

- $\longrightarrow$ Angular dependence of *single-particle* distribution
- finite correl. length / domain size is essentially equivalent to "saturation" of gluon distribution

One interesting example:

A.D., McLerran, Skokov,
arXiv:1410.4844

$$c_2\{4\} \equiv \left\langle e^{2i(\phi_1-\phi_2+\phi_3-\phi_4)}\right\rangle - 2\left\langle e^{2i(\phi_1-\phi_2)}\right\rangle \left\langle e^{2i(\phi_3-\phi_4)}\right\rangle$$

$$= -\frac{1}{N_D^3}\left[\mathcal{A}^4 - \frac{1}{4(N_c^2-1)^3}\right]$$

(around $c_2\{4\}$~0)

- disconnected ~ connected (resp. $c_2\{4\}$~0) when $\mathcal{A} \sim 1/N_c^{3/2}$
- 4-particle correlation dominated by disconnected part *before* 2-particle correlation ! (i.e. for smaller *A*)
- analogous to BBGKY / Dyson-Schwinger hierarchy of n-particle correlations
- Note: not to be compared to experiment, assumes all four pT are >> Qs! (→ numerical results by Schlichting)

In other words:

$$\left\langle e^{in(\phi_1-\phi_2)}\right\rangle = \mathcal{N}\int d^2b_1 d^2b_2 \int d\phi_1 d\phi_2\, e^{in(\phi_1-\phi_2)} \int d^2r_1 d^2r_2\, e^{i\vec{k}_1\cdot\vec{r}_1+i\vec{k}_2\cdot\vec{r}_2} \int \mathcal{D}\rho\, e^{-S[\rho]}\ \frac{1}{N}\mathrm{tr}\ V_{x_1}V_{y_1}^\dagger\ \frac{1}{N}\mathrm{tr}\ V_{x_2}V_{y_2}^\dagger$$

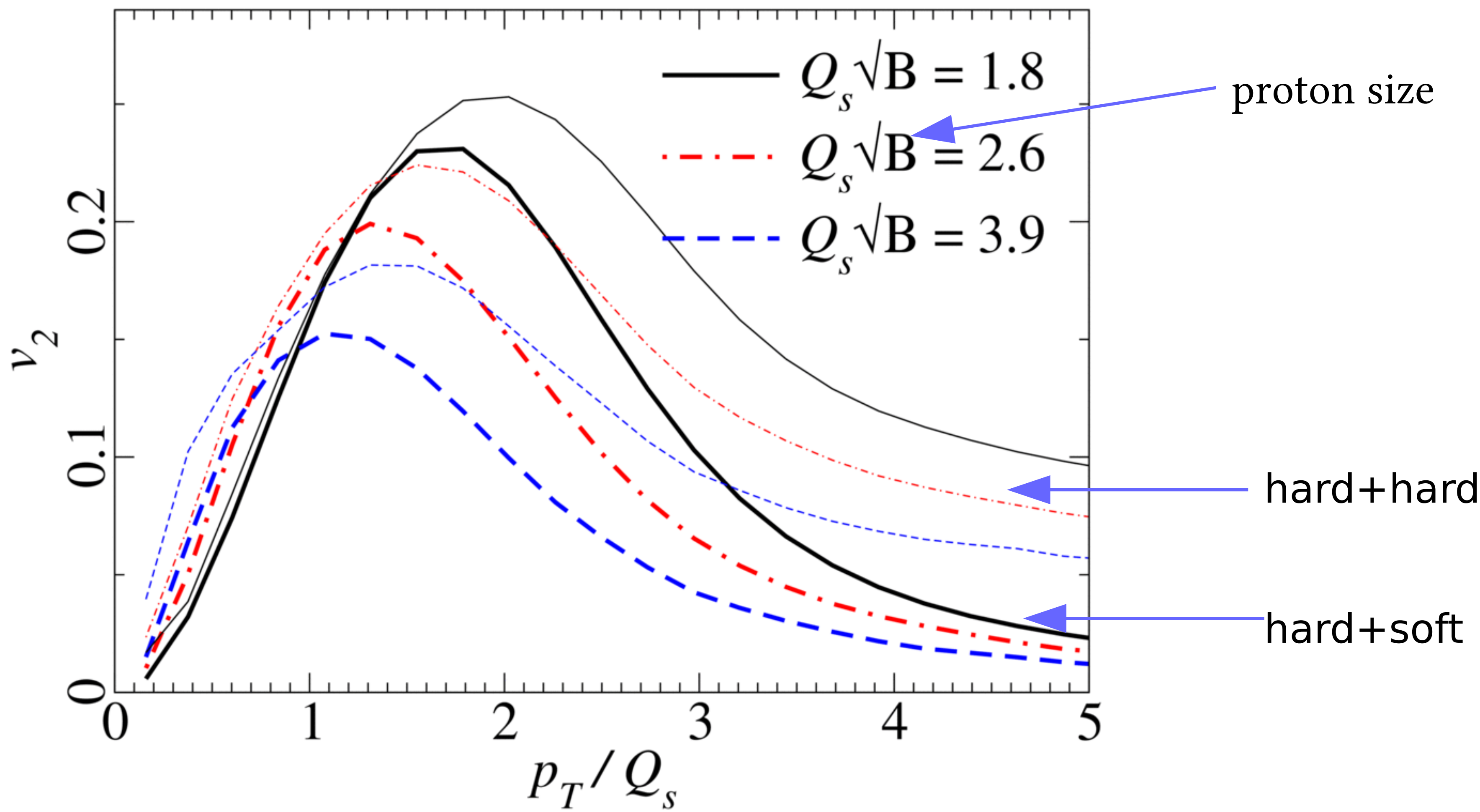
while $$\int d^2b_1 d^2b_2 \int d\phi_1 d\phi_2\, e^{in(\phi_1-\phi_2)} \int d^2r_1 d^2r_2\, e^{i\vec{k}_1\cdot\vec{r}_1+i\vec{k}_2\cdot\vec{r}_2} \left(\int \mathcal{D}\rho\, e^{-S[\rho]}\ \frac{1}{N}\mathrm{tr}\ V_{x_1}V_{y_1}^\dagger\right)\left(\int \mathcal{D}\rho\, e^{-S[\rho]}\ \frac{1}{N}\mathrm{tr}\ V_{x_2}V_{y_2}^\dagger\right) = 0 \qquad \text{(for n>0)}$$

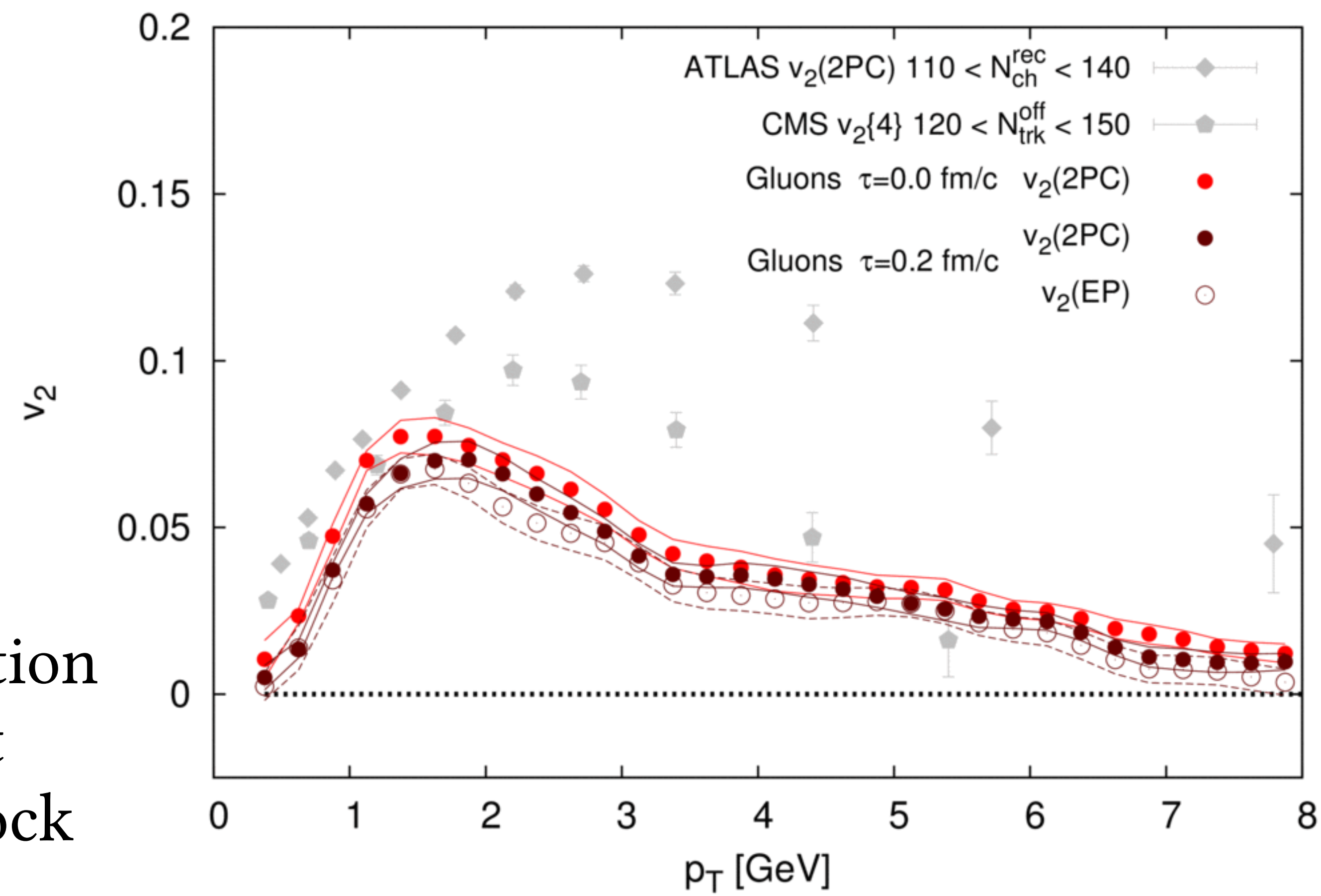
- contains a disconnected contribution from anisotropic 1-particle distribution :

$$\left\langle e^{in(\phi_1-\phi_2)}\right\rangle = \left\langle e^{in\phi_1}\right\rangle\left\langle e^{-in\phi_2}\right\rangle + \left\langle e^{in(\phi_1-\phi_2)}\right\rangle_{\mathrm{conn.}}$$

Recent numerical calculation by T. Lappi (for pA)

arXiv:1501.05505



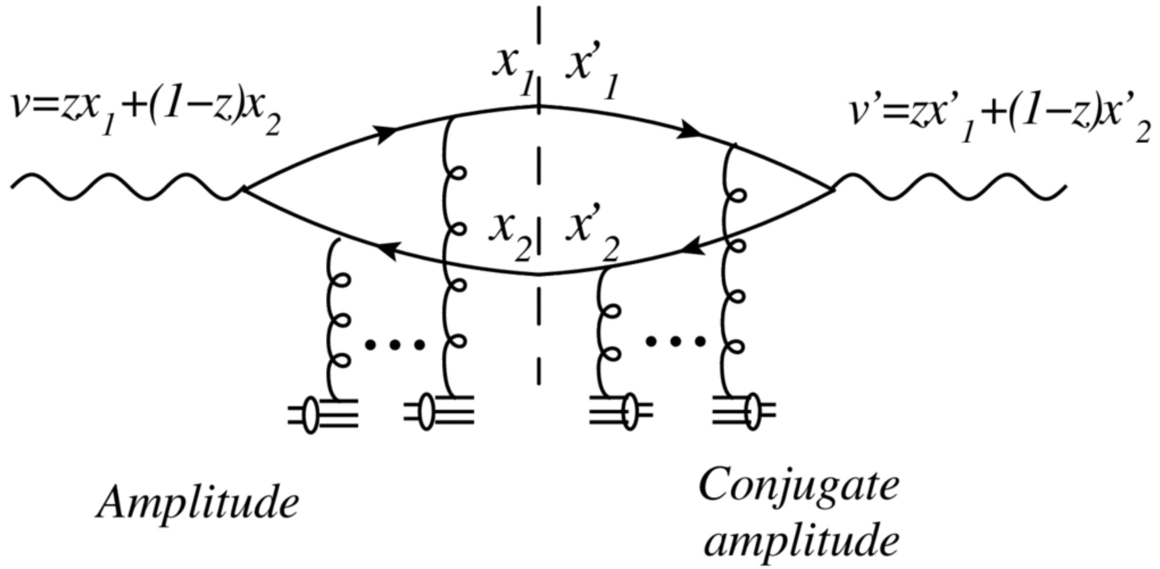


Schenke, Schlichting, Venugopalan: 1502.01331

- numerical computation in dense-dense limit (collision of two shock waves / CGCs)
- v2>0 instantly at τ=+0 !
- clearly not a rescattering effect but due to anisotropic gluon fields

Dijets in γ*A :

(Dominguez, Marquet, Xiao, Yuan, PRD 2011)



Dijet total tr. momentum:

$$\vec{P} = \frac{1}{2}\left(\vec{k}_1 - \vec{k}_2\right) \quad \text{or} \quad \widetilde{P} = (1-z)\vec{k}_1 - z\vec{k}_2$$

and net momentum (imbalance): $\vec{q} = \vec{k}_1 + \vec{k}_2$

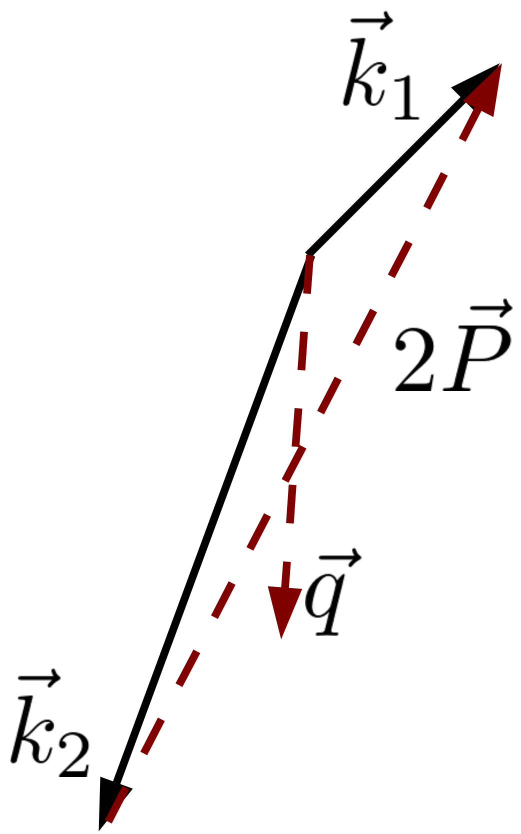
“correlation limit” $P \gg q$ involves only 2-point functions / UGDs, no quadrupole

Azimuthal anisotropy

(Dominguez, Qiu, Xiao, Yuan, PRD 2012)

$$d\sigma^{\gamma_L^* A \to q\bar{q}X} = e_q^2 \alpha \alpha_s z^2 (1-z)^2 \frac{8\epsilon_f^2 \widetilde{P}^2}{(\widetilde{P}^2 + \epsilon_f^2)^4} \left(xG^{(1)}(x,q) + \cos(2\phi)\, xh_\perp^{(1)}(x,q) \right)$$

ϕ = angle between $\vec{P}$ and $\vec{q}$



$\rightarrow$ rotate net transverse momentum vector q around and measure amplitude of cos(2ϕ) modulation

$$v_2(q,x) = \langle \cos 2\phi \rangle = \frac{1}{2} \frac{h_\perp^{(1)}(x,q)}{G^{(1)}(x,q)}$$

(if x independent of ϕ, true at z=1/2)

The distribution of linearly polarized gluons

(Metz, Zhou: PRD 2011; Dominguez, Qiu, Xiao, Yuan, PRD 2012)

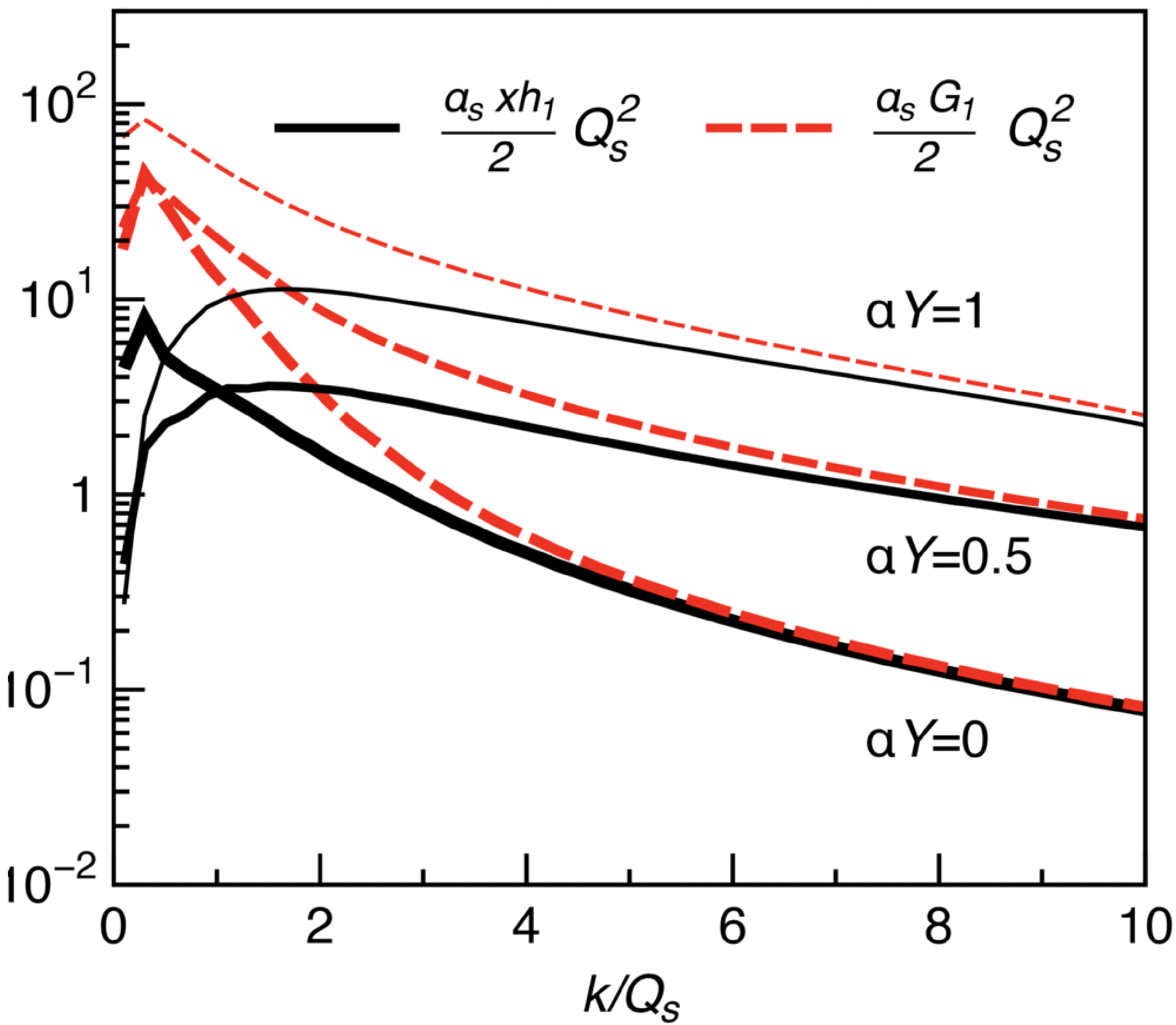
$$xG_{\perp}^{(1)}(x,k) = -\frac{2}{\alpha_s}\delta^{ij}\left\langle \mathrm{Tr}\left[E_i(\vec{k})E_j(-\vec{k})\right]\right\rangle$$

$$xh_{\perp}^{(1)}(x,k) = \frac{2}{\alpha_s}\left(\delta^{ij} - 2\frac{k^i k^j}{k^2}\right)\left\langle \mathrm{Tr}\left[E_i(\vec{k})E_j(-\vec{k})\right]\right\rangle$$

$$E_i(\vec{k}) = \int \frac{d^2y}{(2\pi)^2} e^{-i\vec{k}\cdot\vec{y}}\, U^{\dagger}(\vec{y})\partial_i U(\vec{y})$$

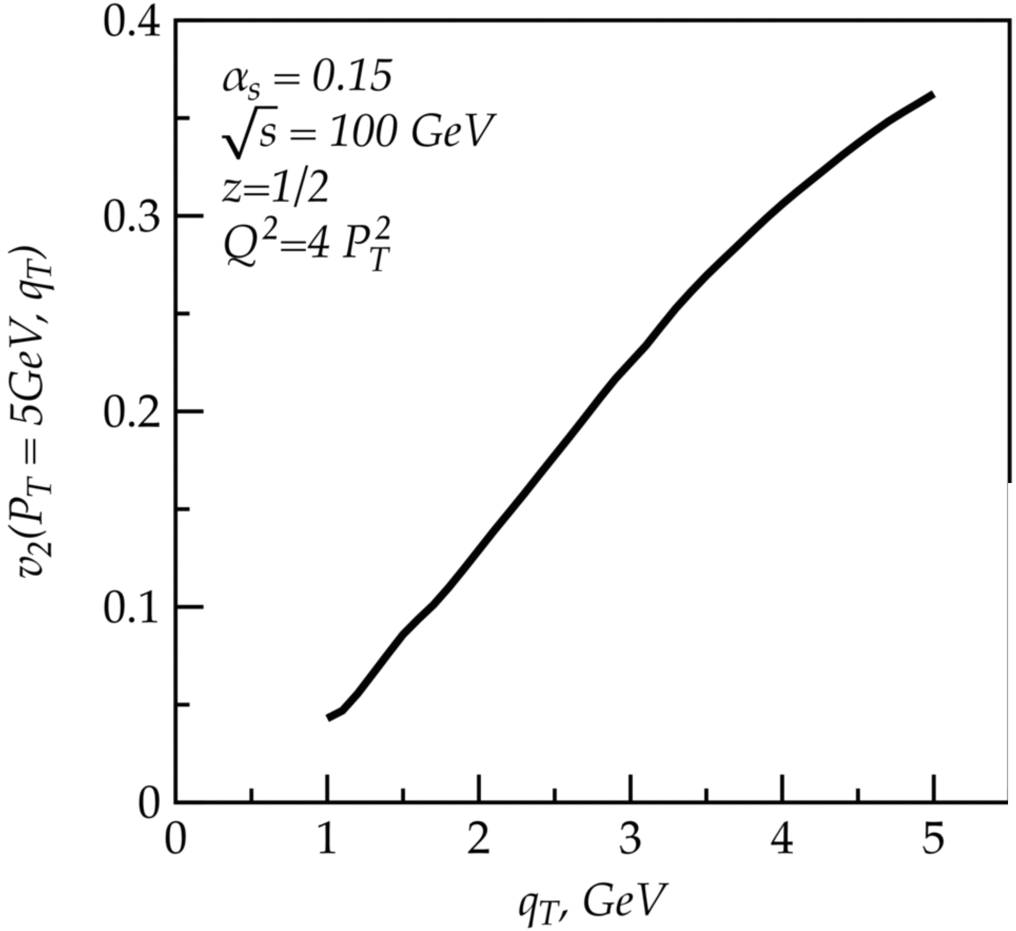
We have computed these functions at small x
by solving JIMWLK from MV model initial conditions

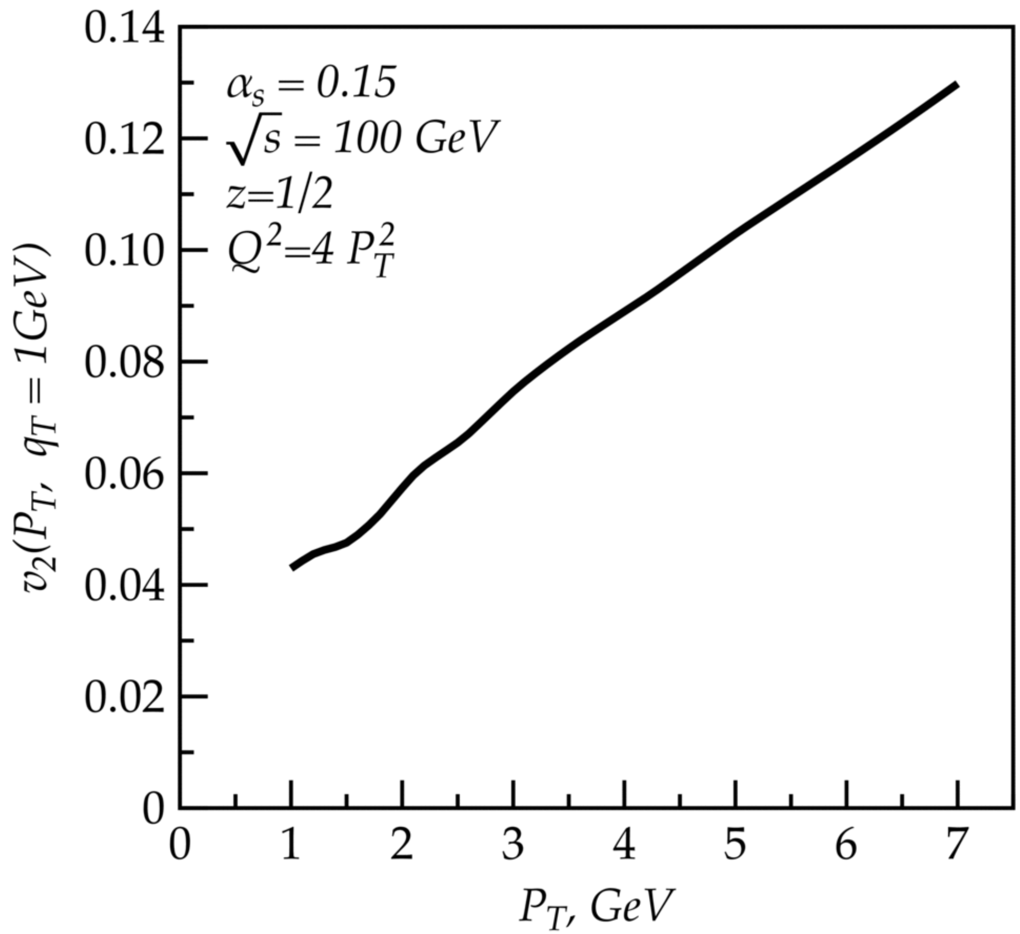
(A.D., T. Lappi, V. Skokov: in preparation)



- $h^{(1)}_{\perp}$ / $G^{(1)} \rightarrow$ 0 at low q
- but $h^{(1)}_{\perp}$ / $G^{(1)} \rightarrow$ 1 at high transv. momentum

Large cos(2ϕ) amplitudes...





Summary

- azimuthal anisotropies from short-distance QCD frameworks

  - CGC / classical fields: $$\langle E^i E^j \rangle \sim \delta^{ij} + 2\mathcal{A}\left(\hat{a}^i \hat{a}^j - \frac{1}{2}\delta^{ij}\right) + \cdots$$

  - TMD factorization: $h^{(1)}_{\perp}$ (dijet in eA); $h^{(2)}_{\perp}$ (DY, pA)

- semi-analytical estimates & numerical calculations predict substantial <u>initial-state anisotropies</u> in pA at "high" p_T

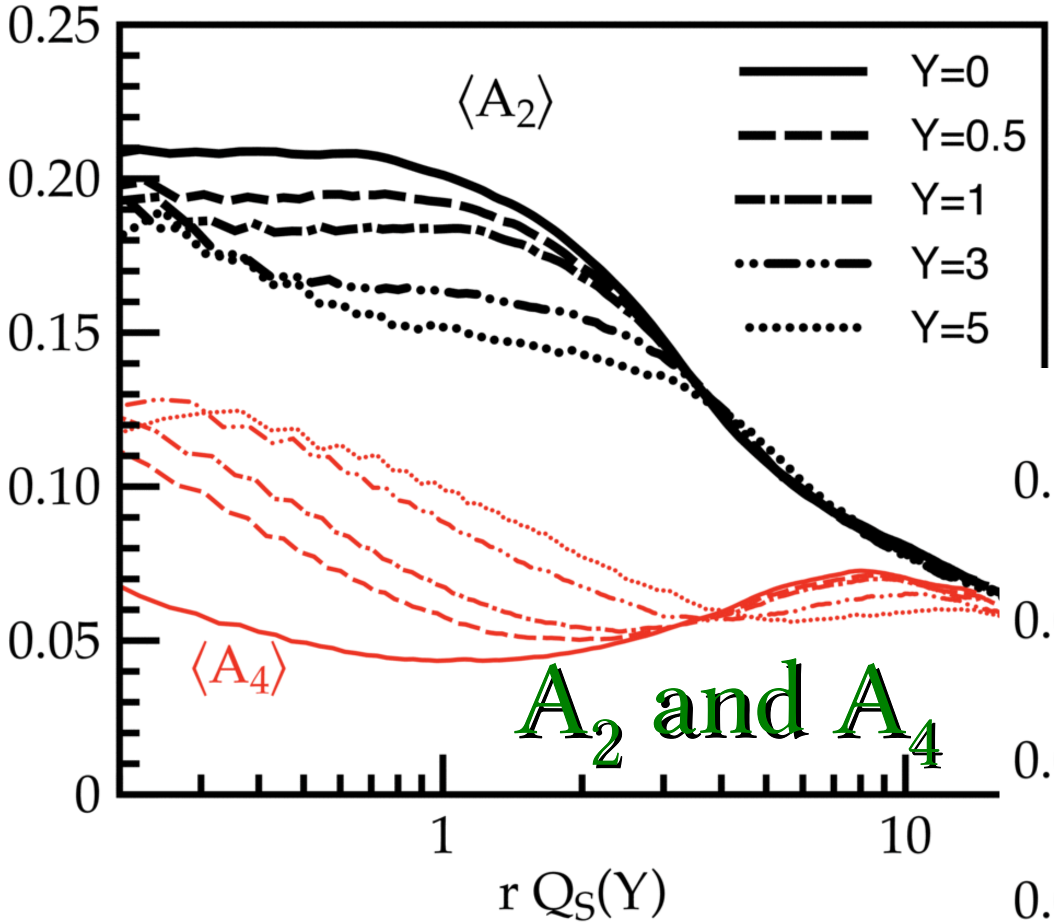
- also predict very substantial v_2 for $\gamma^* A \longrightarrow$ jet + jet + X

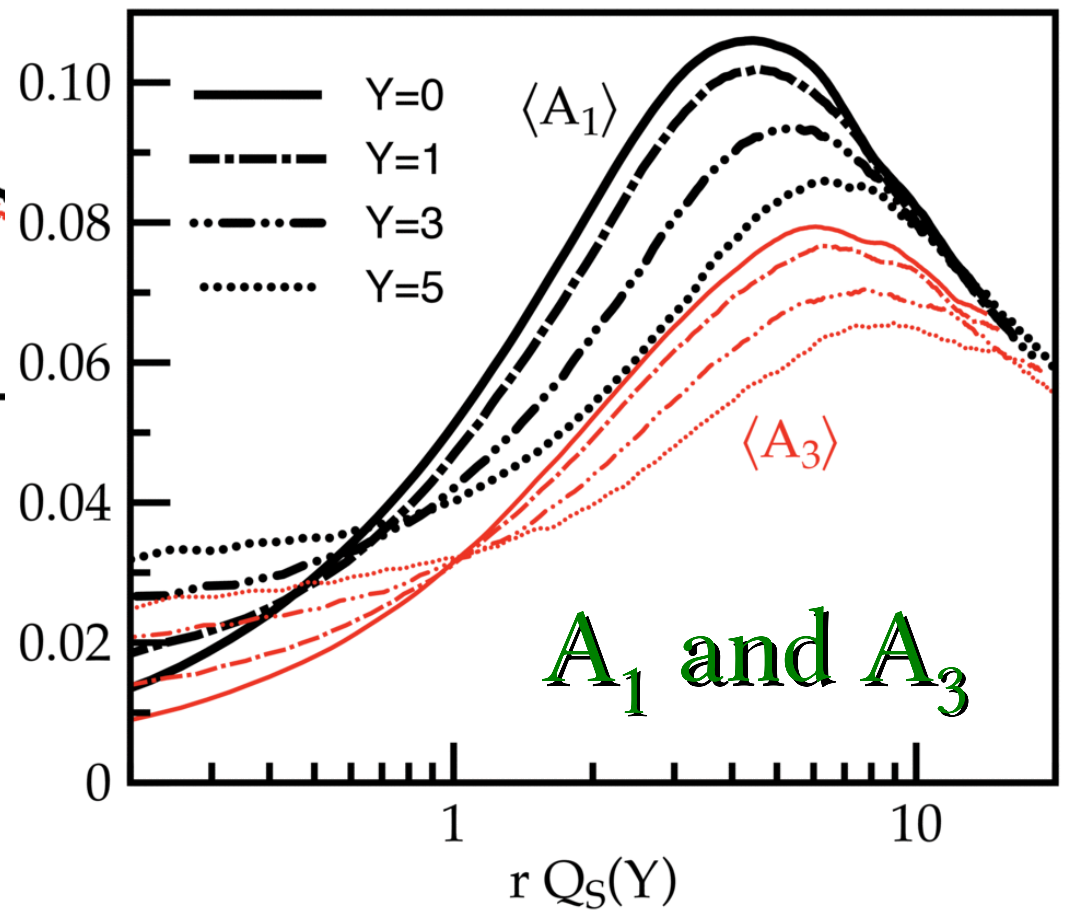
Backup Slides

JIMWLK evolution (impact parameter dependent !) and its effect on anisotropy amplitudes A_n

- α_s=0.14 fixed

  (details in A.D. & V. Skokov, 1411.6630)





Resummation of boost-invariant quantum fluctuations (JIMWLK):

classical ensemble at Y = log x_0/x = 0:

$$P[\rho] \sim e^{-S_{\rm cl}[\rho]}\ , \quad S_{\rm MV} \;=\; \int d^2x_\perp \, \frac{1}{2\mu^2}\rho^a\rho^a\ ,$$

$$V(x_\perp) = \mathcal{P}\exp\ ig^2\int dx^- \frac{1}{\nabla_\perp^2}\rho(x_\perp)$$

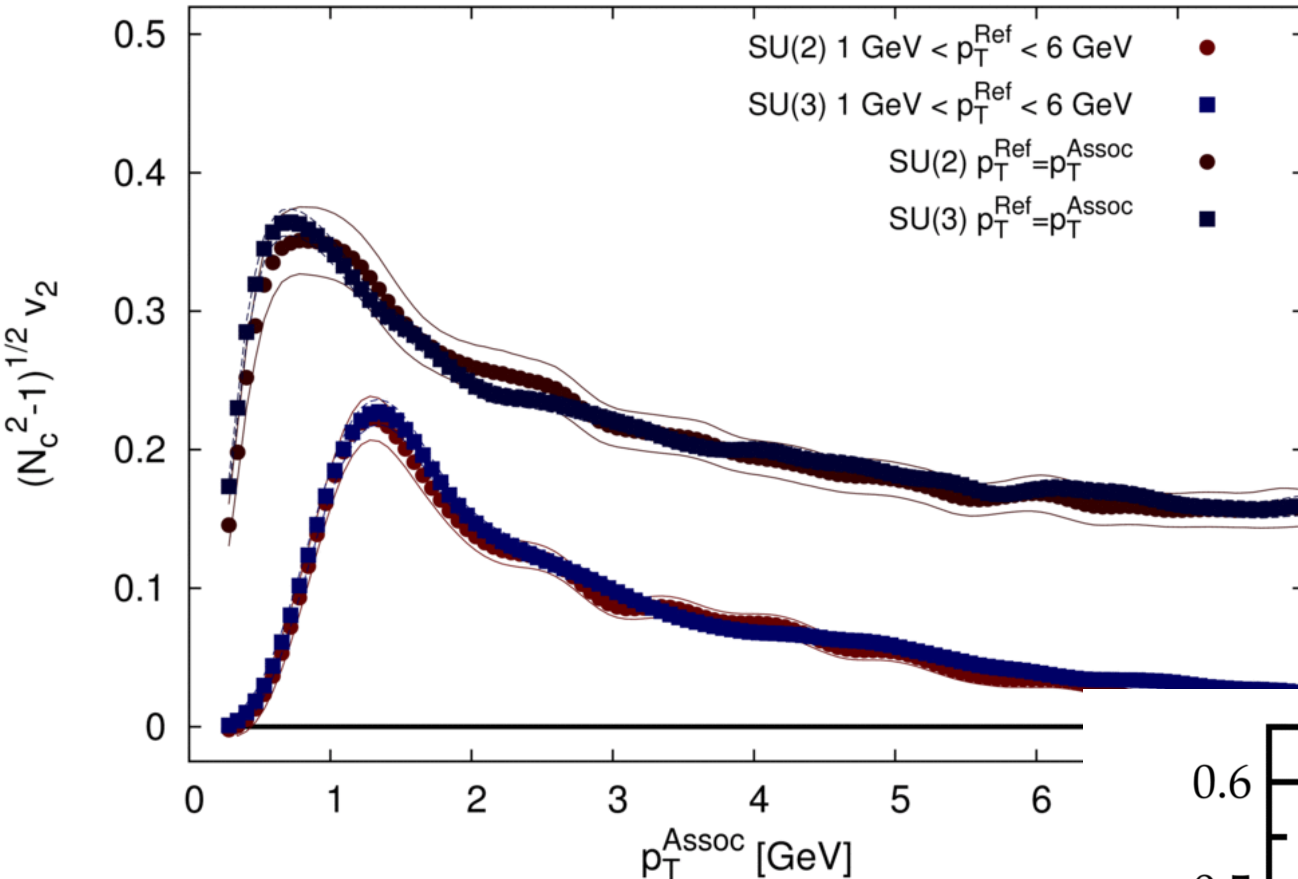
quantum evolution to Y>0: random walk in space of Wilson lines

$$\partial_Y V(x_\perp) = V(x_\perp)\ it^a \left\{ \int d^2y_\perp\ \varepsilon_k^{ab}(x_\perp, y_\perp)\ \xi_k^b(y_\perp) + \sigma^a(x_\perp)\right\}\ .$$

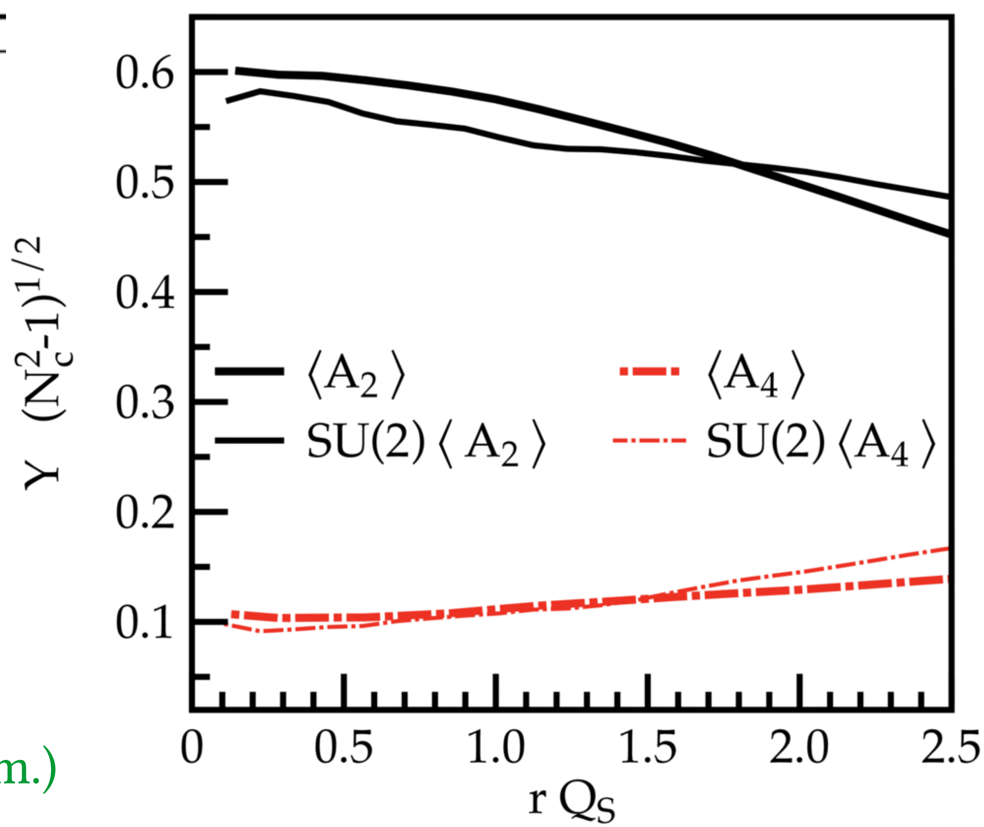
$$\varepsilon_k^{ab} = \left(\frac{\alpha_s}{\pi}\right)^{1/2}\ \frac{(x_\perp - y_\perp)_k}{(x_\perp - y_\perp)^2}\ \left[1 - U^\dagger(x_\perp)U(y_\perp)\right]^{ab}$$

$$\langle \xi_i^a(x_\perp)\ \xi_j^b(y_\perp)\rangle = \delta^{ab}\delta_{ij}\delta^{(2)}(x_\perp - y_\perp)$$

$$\sigma^a(x_\perp) = -i\frac{\alpha_{\rm s}}{2\pi^2}\int d^2z_\perp \, \frac{1}{(x_\perp - z_\perp)^2}{\rm tr}\ \left(T^aU^\dagger(x_\perp)\,U(z_\perp)\right)$$

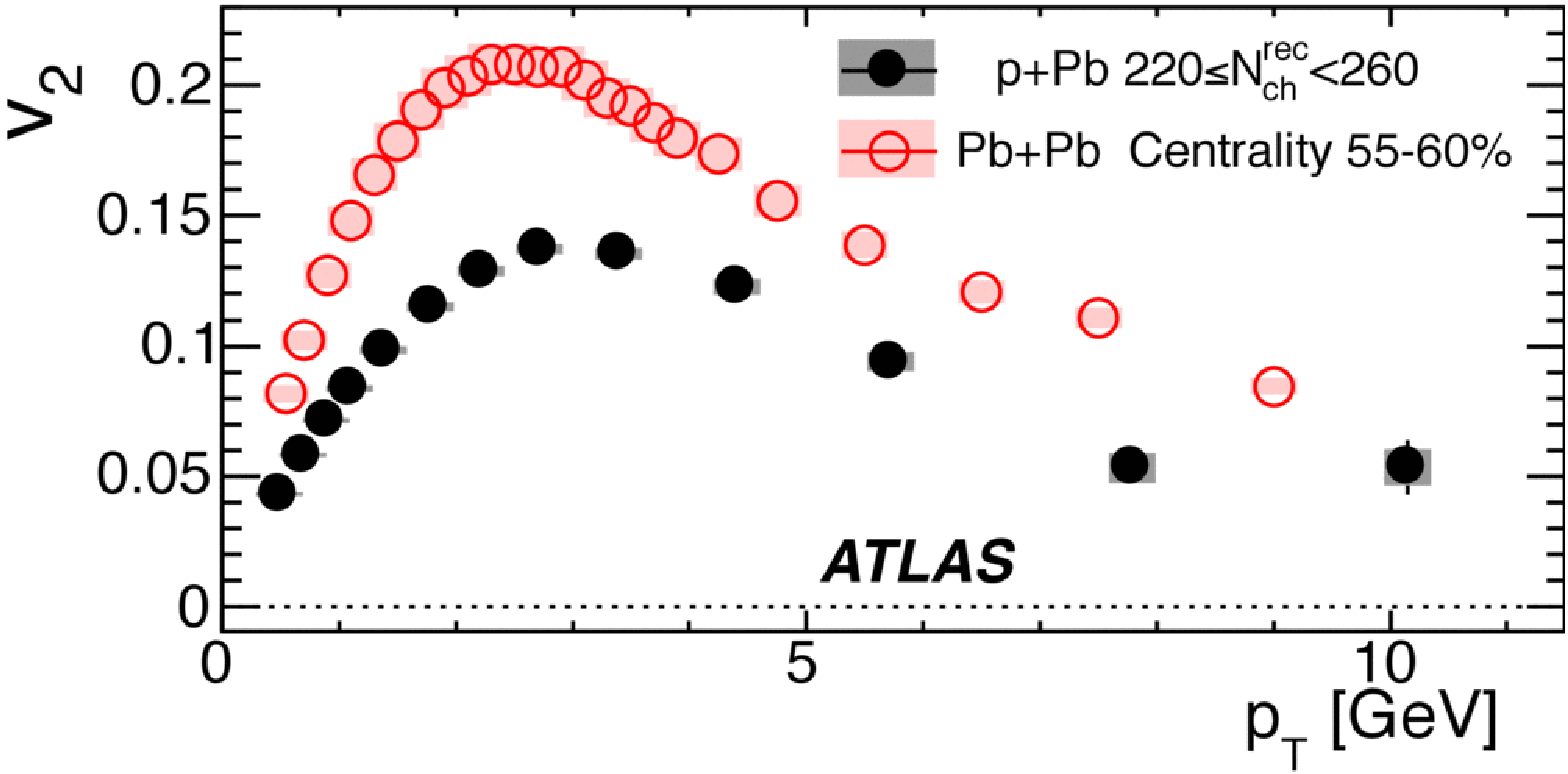


(S. Schlichting, priv. comm.)



(V. Skokov, priv. comm.)

p+Pb collisions at the LHC: significant anisotropy up to rather high p_T



- can we say anything from short distance QCD ?
- and carry that over to DIS on nuclei ?