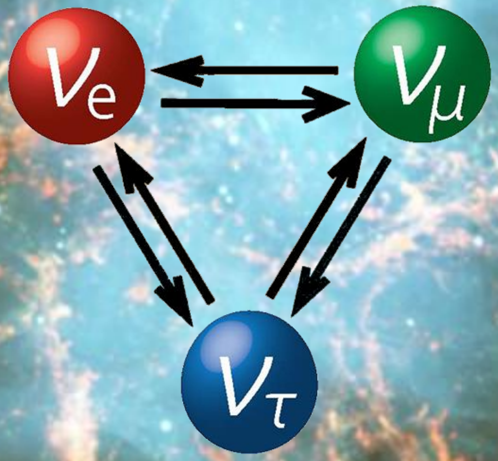
Neutrino Flavor Conversion in Supernovae



Georg Raffelt, Max-Planck-Institut für Physik, München

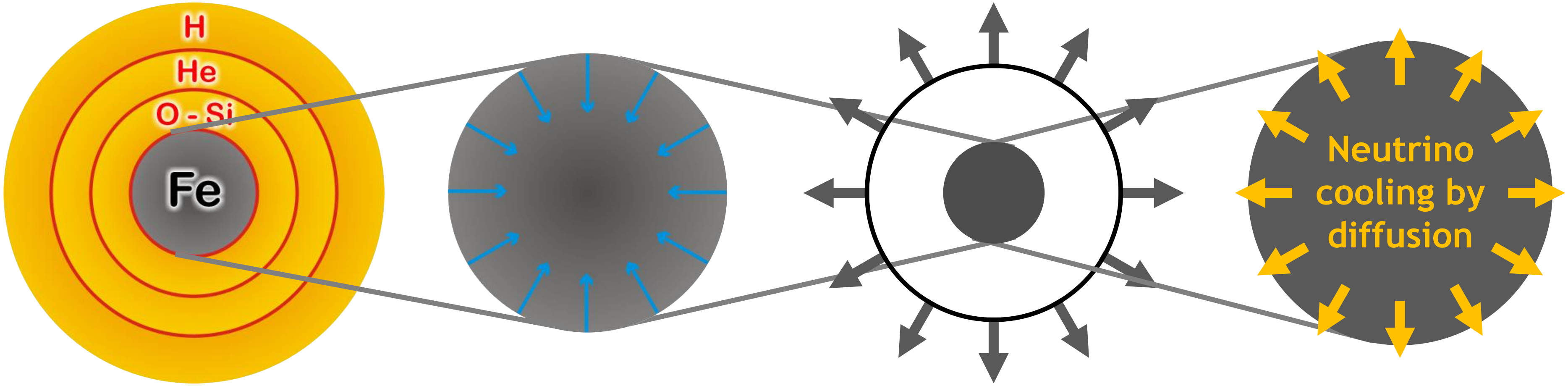
Core-Collapse Supernova Explosion

End state of a massive star
$M \gtrsim 6\text{–}8\ M_\odot$

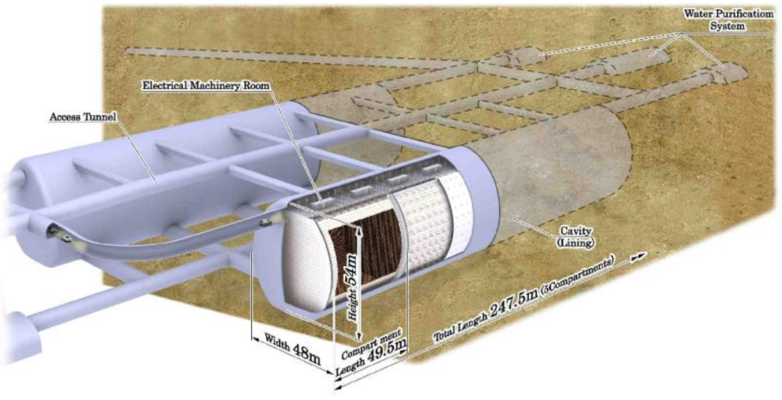
Collapse of degenerate core

Bounce at $\rho_{\rm nuc}$
Shock wave forms explodes the star

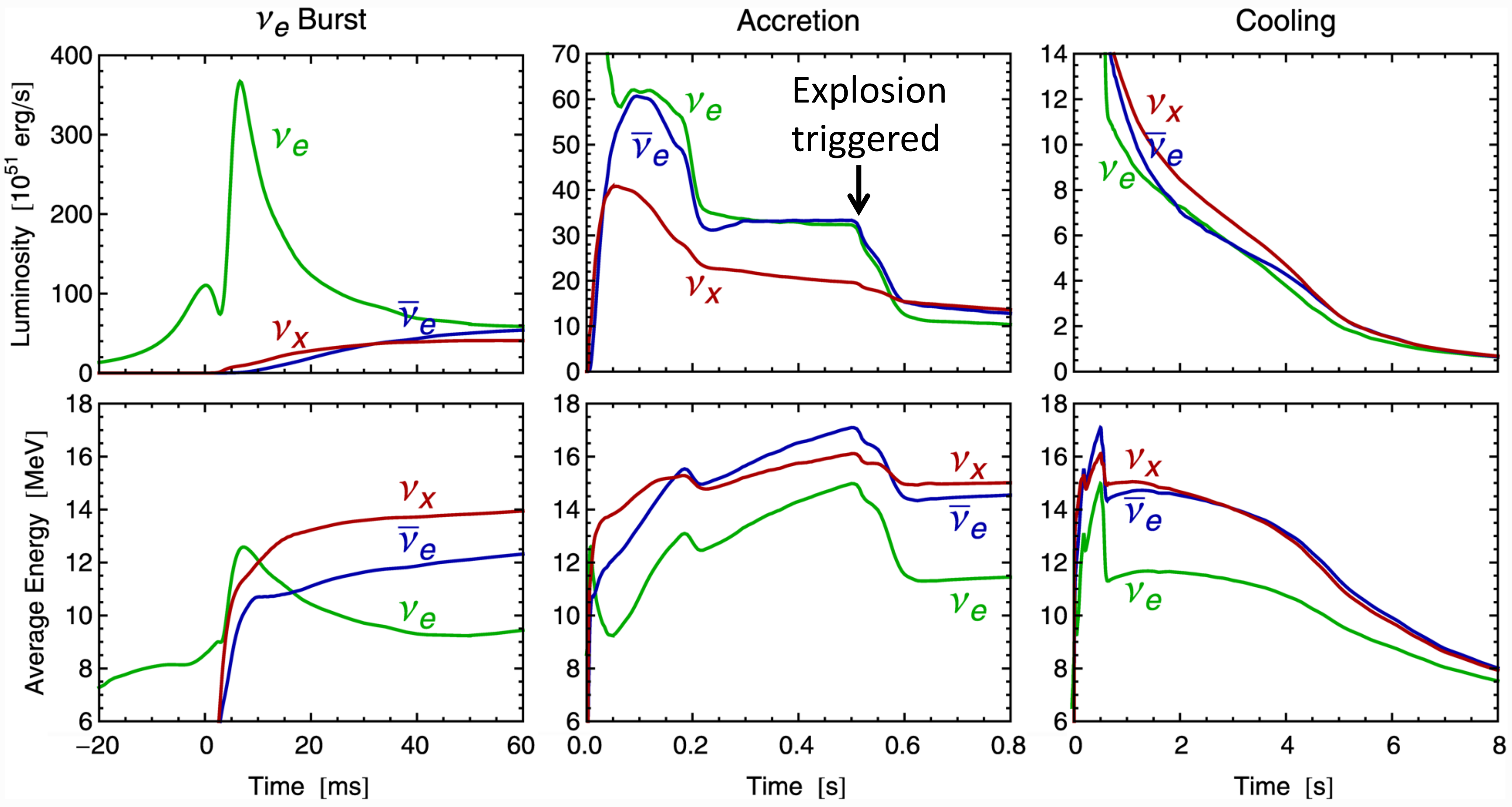
Grav. binding E
$\sim 3 \times 10^{53}$ erg
emitted as nus of all flavors



- Huge rate of low-E neutrinos (tens of MeV) over few seconds in large-volume detectors
- A few core-collapse SNe in our galaxy per century
- Once-in-a-lifetime opportunity



Three Phases of Neutrino Emission



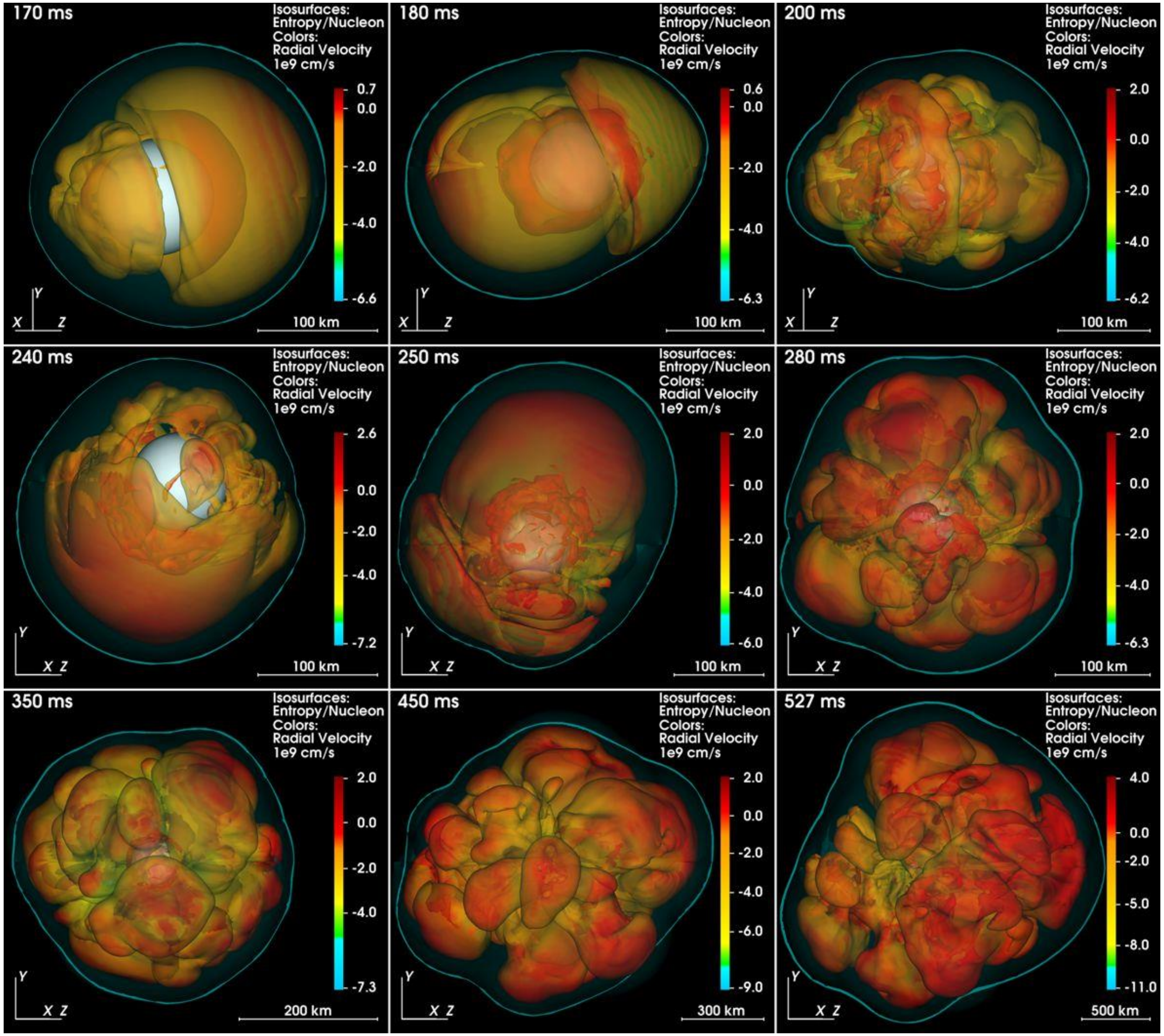
- Shock breakout
- De-leptonization of outer core layers

- Shock stalls ~ 150 km
- Neutrinos powered by infalling matter

Cooling on neutrino diffusion time scale

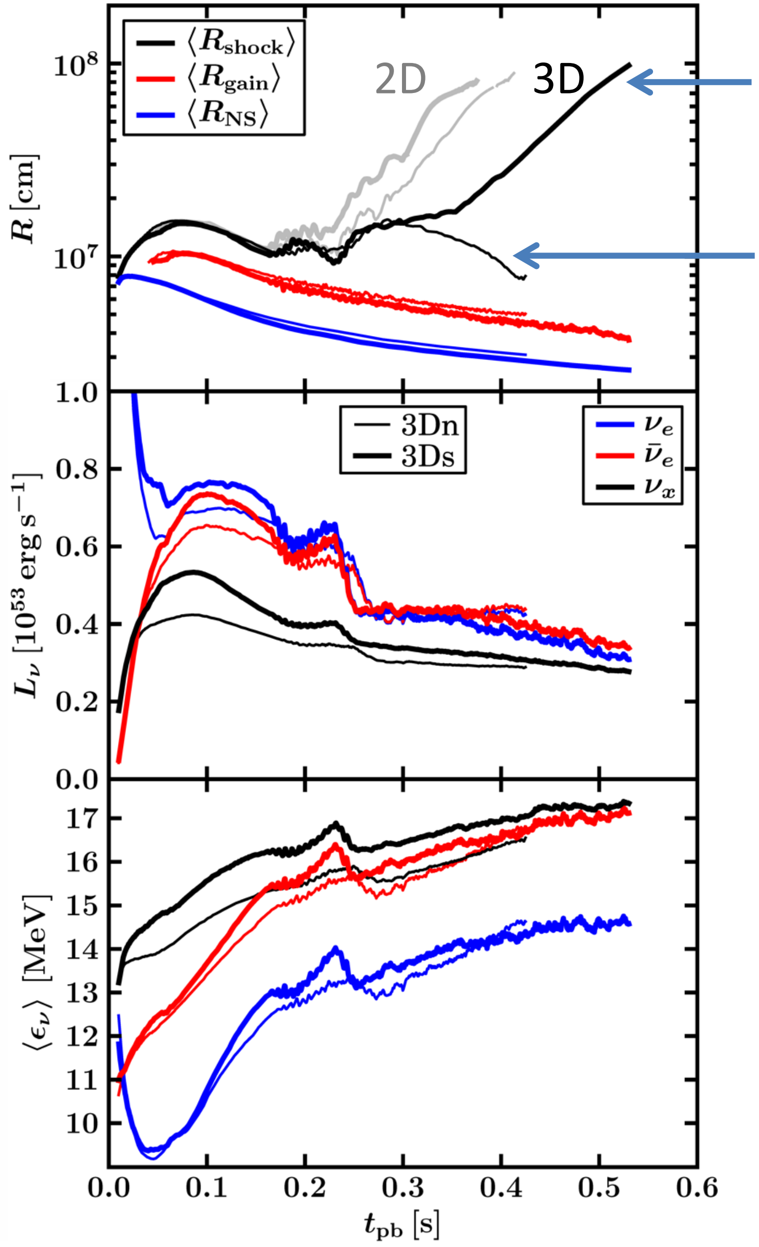
Spherically symmetric Garching model (25 $M_\odot$) with Boltzmann neutrino transport

Exploding 3D Garching Model (20 M_{SUN})



Melson, Janka, Bollig, Hanke, Marek & Müller, arXiv:1504.07631

Exploding 3D Garching Model (20 M_{SUN})

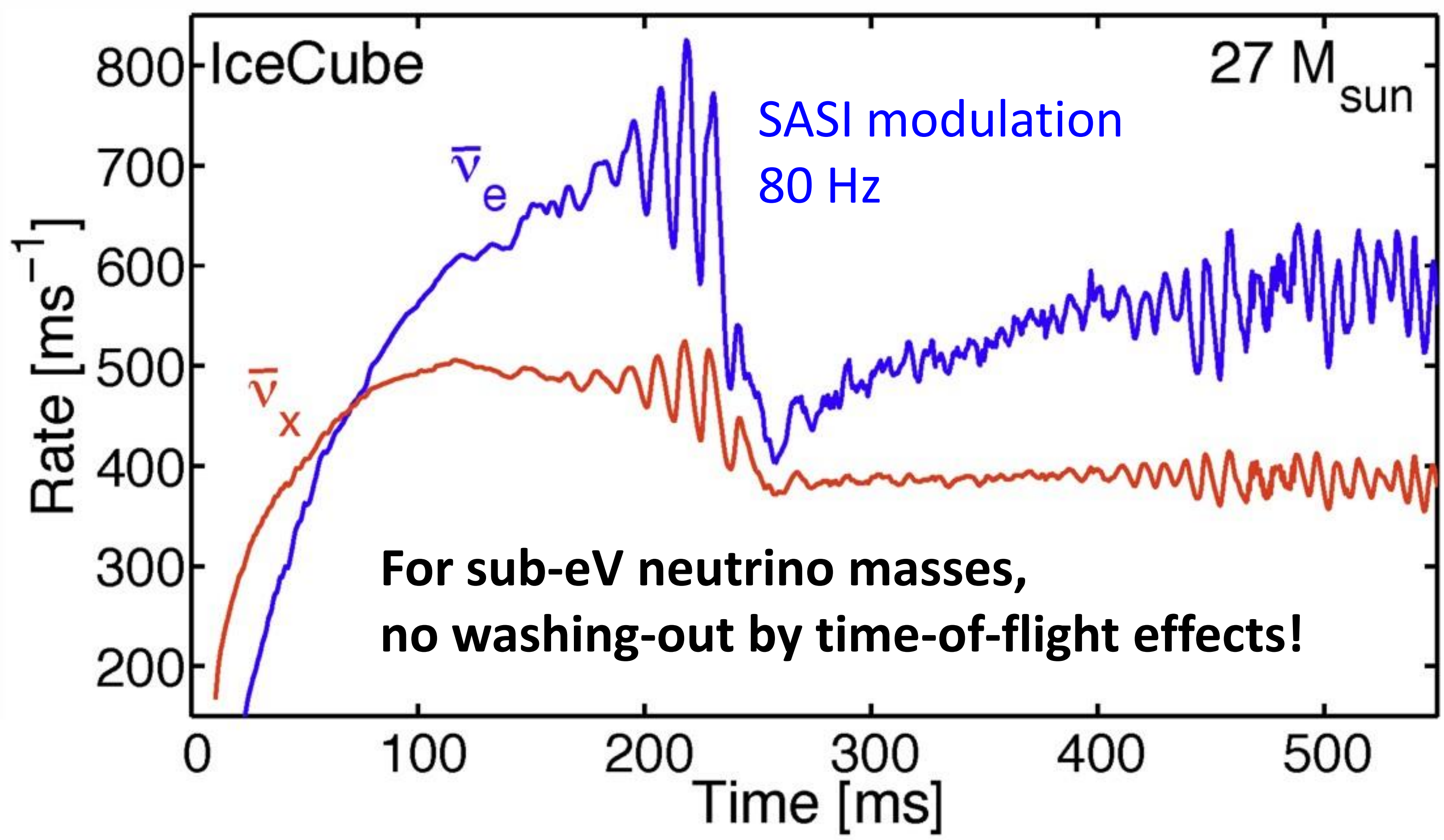


Neutrino opacity reduced (few 10%) by strange quark contribution to nucleon spin (thick lines)

“Standard” neutrino opacity (thin lines)

Melson, Janka, Bollig, Hanke, Marek & Müller, arXiv:1504.07631

Variability seen in Neutrinos (3D Model)



SASI modulation
80 Hz

For sub-eV neutrino masses,
no washing-out by time-of-flight effects!

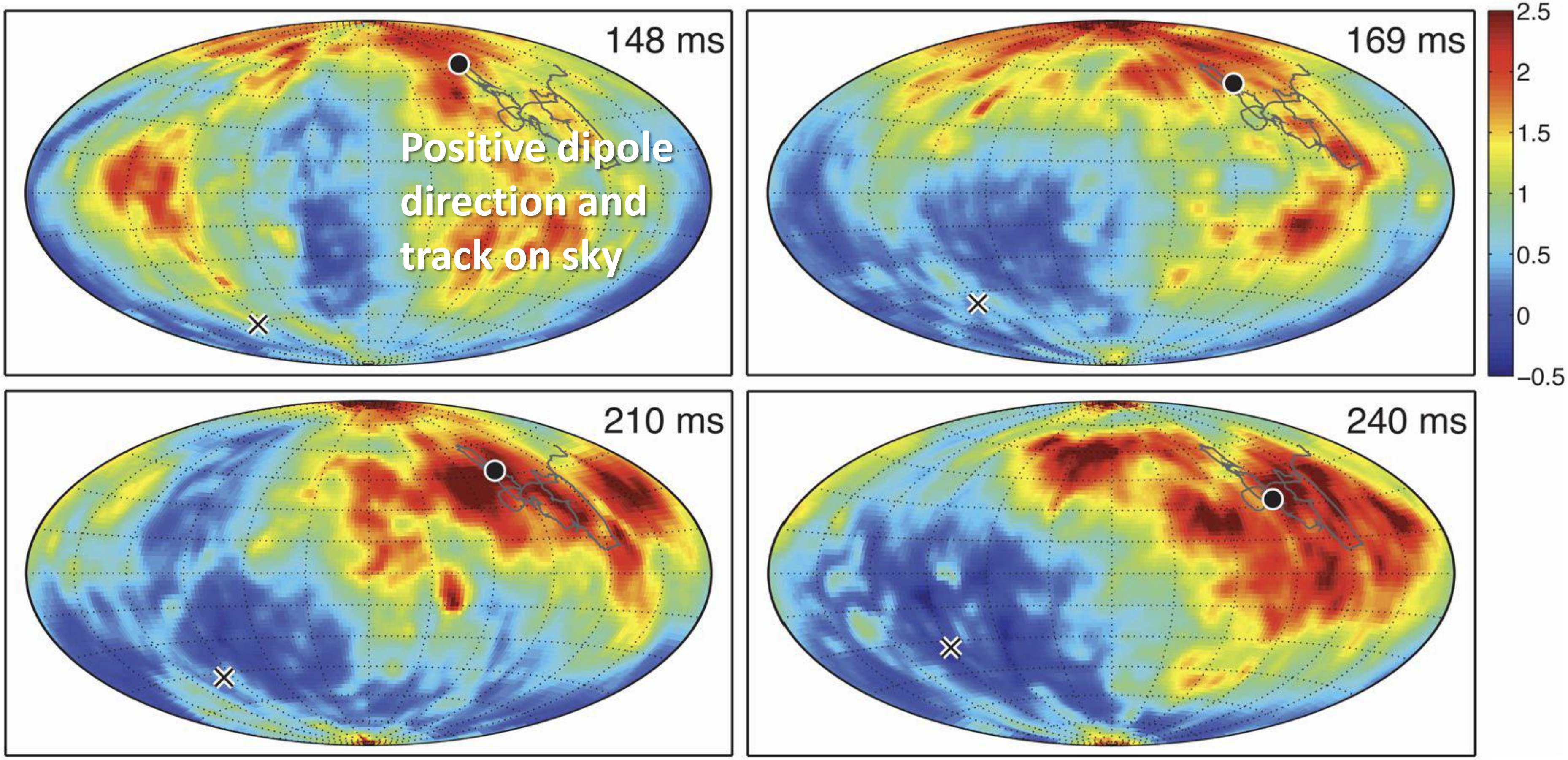
Tamborra, Hanke, Müller, Janka & Raffelt, arXiv:1307.7936
See also Lund, Marek, Lunardini, Janka & Raffelt, arXiv:1006.1889

Sky Map of Lepton-Number Flux (11.2 M_{SUN} Model)

Lepton-number flux ($\nu_e - \overline{\nu}_e$) relative to 4π average

Deleptonization flux into one hemisphere, roughly dipole distribution

(LESA — Lepton Emission Self-Sustained Asymmetry)



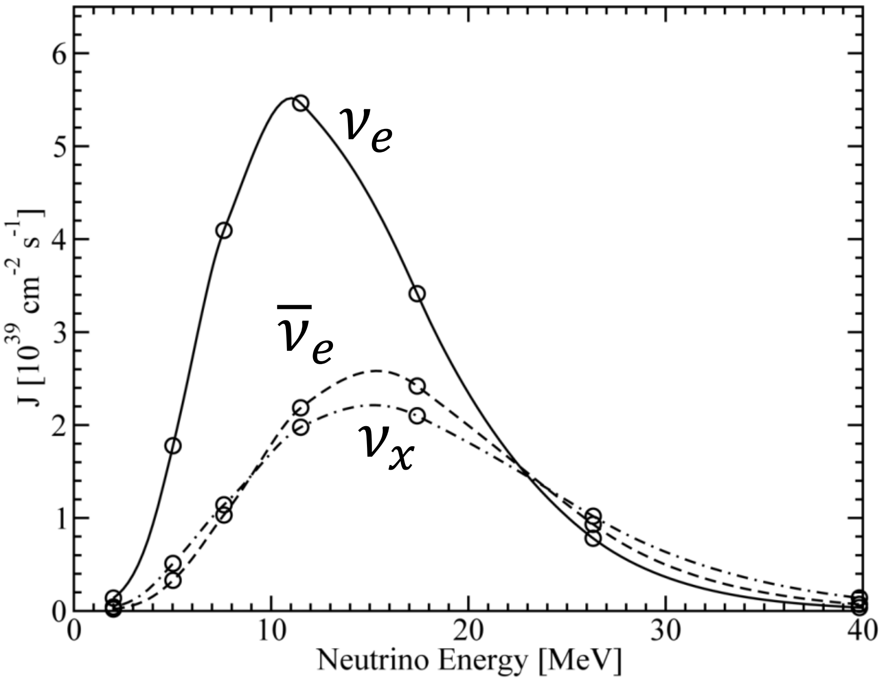
Positive dipole direction and track on sky

Tamborra, Hanke, Janka, Müller, Raffelt & Marek, arXiv:1402.5418

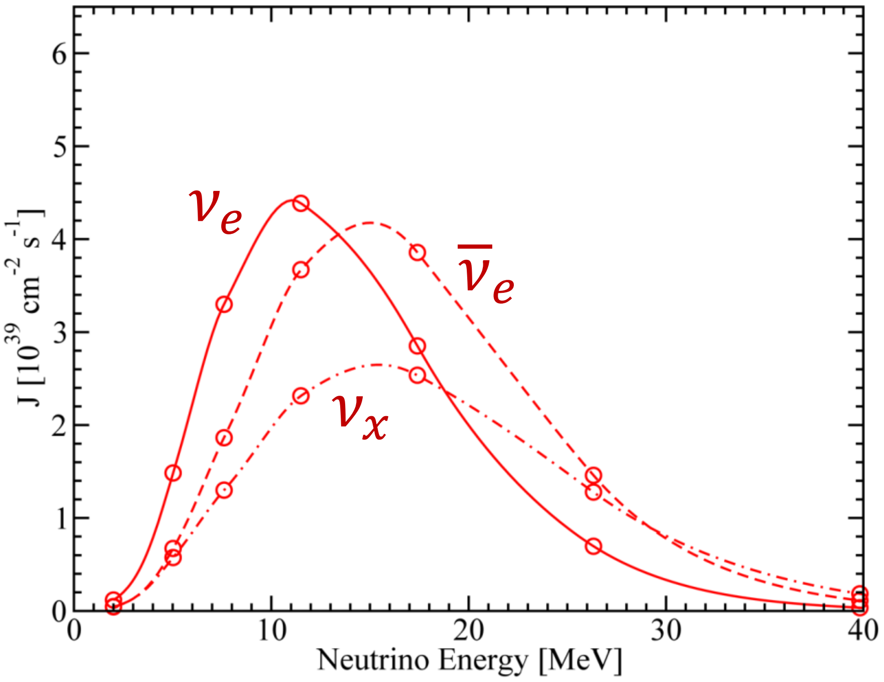
Spectra in the two Hemispheres

Neutrino flux spectra (11.2 M_{SUN} model at 210 ms) in opposite LESA directions

Direction of
maximum lepton-number flux



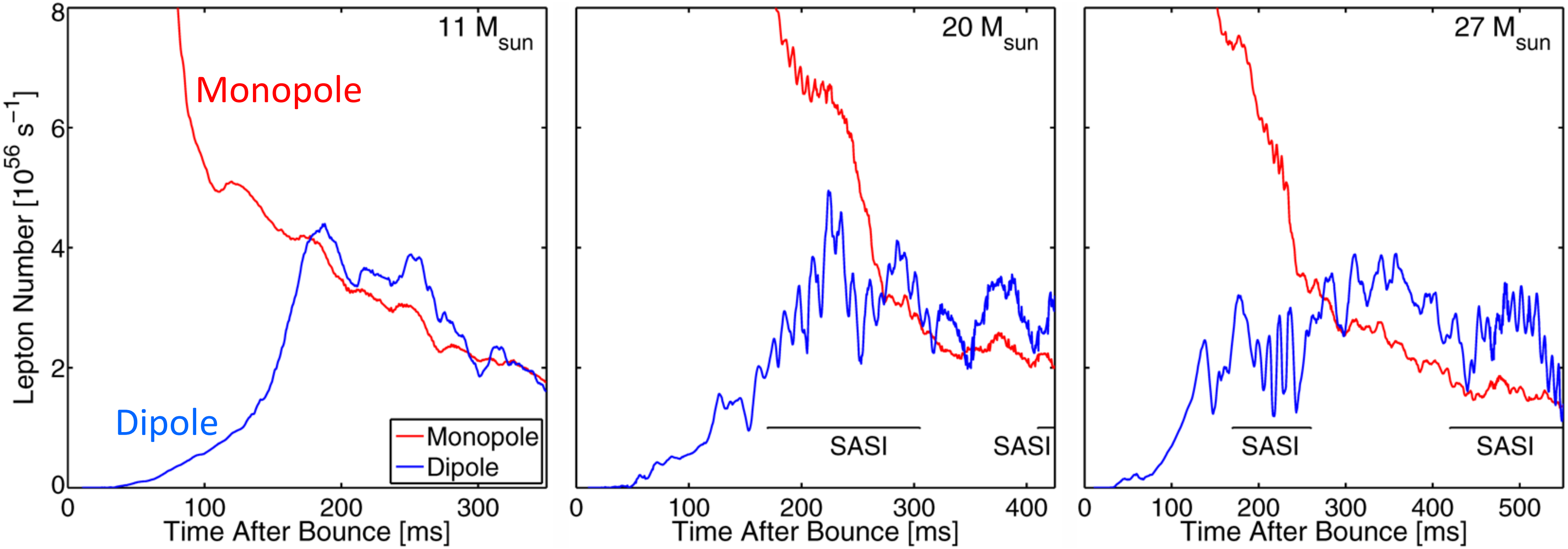
Direction of
minimum lepton-number flux



During accretion phase, flavor-dependent fluxes vary strongly with observer direction!

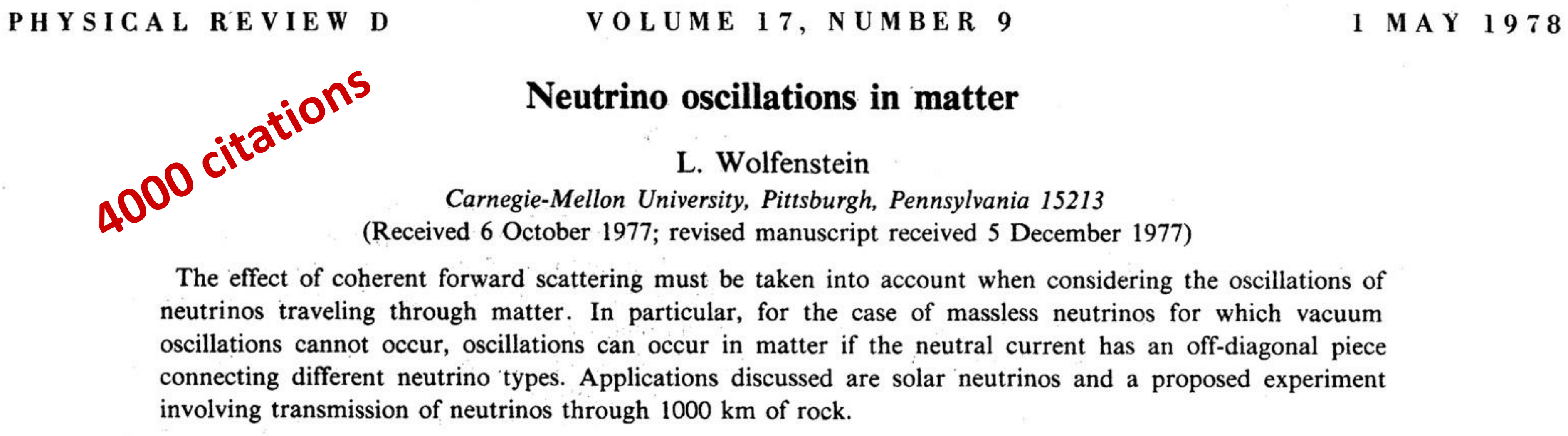
Growth of Lepton-Number Flux Dipole

Tamborra et al., arXiv:1402.5418



- Overall lepton-number flux (monopole) depends on accretion rate, varies between models
- Maximum dipole similar for different models
- Dipole persists (and even grows) during SASI activity
- SASI and LESA dipoles uncorrelated

4000 citations



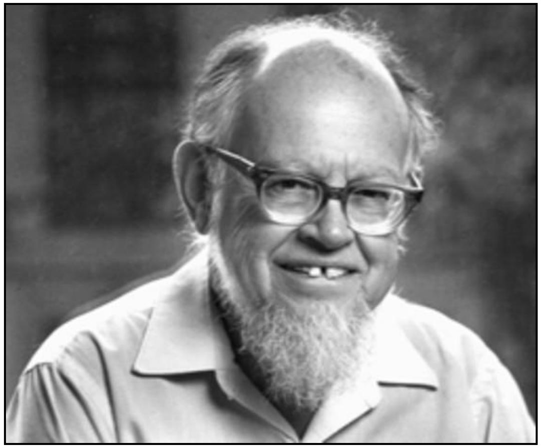
PHYSICAL REVIEW D VOLUME 17, NUMBER 9 1 MAY 1978

Neutrino oscillations in matter

L. Wolfenstein
Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213

(Received 6 October 1977; revised manuscript received 5 December 1977)

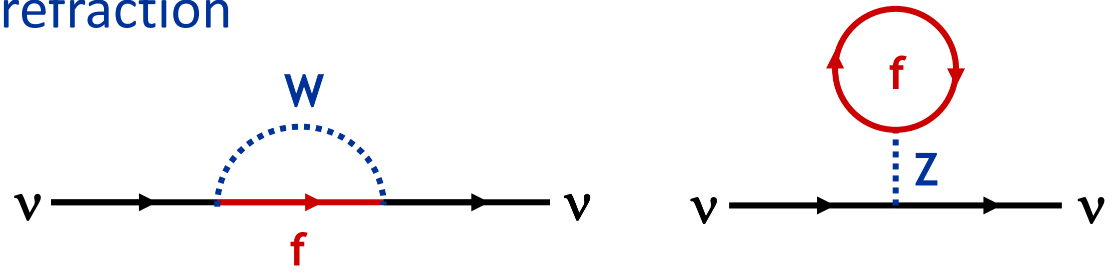
The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.



Lincoln Wolfenstein

10 Feb 1923–27 Mar 2015

Neutrinos in a medium suffer flavor-dependent refraction

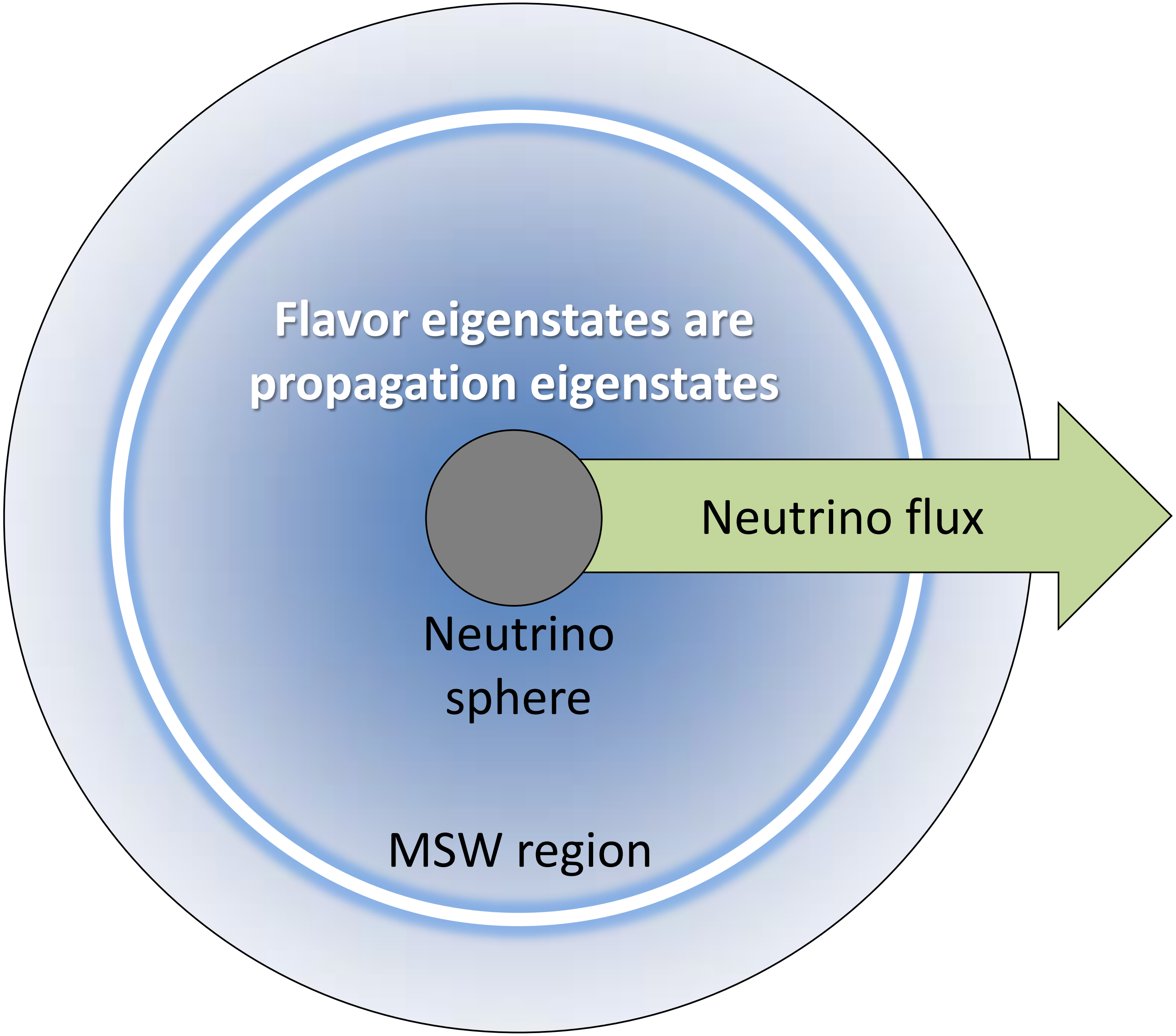


$$V_{\rm weak} = \sqrt{2} G_{\rm F} \times \begin{cases} N_{\rm e} - N_{\rm n}/2 & \text{for } \nu_{\rm e} \\ -N_{\rm n}/2 & \text{for } \nu_\mu \end{cases}$$

Typical density of Earth: 5 g/cm^3

$$\Delta V_{\rm weak} \approx 2 \times 10^{-13}\ {\rm eV} = 0.2\ {\rm peV}$$

Flavor Oscillations in Core-Collapse Supernovae



Flavor eigenstates are propagation eigenstates

Neutrino flux

Neutrino sphere

MSW region

SN Flavor Oscillations and Mass Hierarchy

- Mixing angle Θ_{13} has been measured to be “large”
- MSW conversion in SN envelope adiabatic
- Assume that collective flavor oscillations are not important

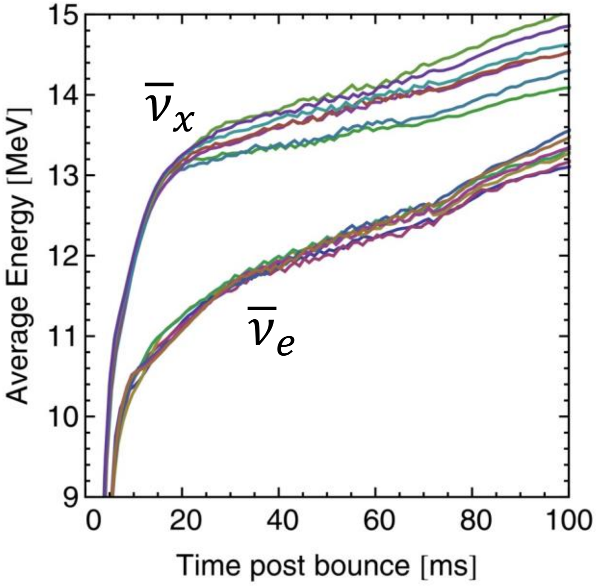
	Mass ordering	
	Normal (NH)	Inverted (IH)
ν_e survival prob.	0	$\sin^2\theta_{12} \approx 0.3$
$\overline{\nu}_e$ survival prob.	$\cos^2\theta_{12} \approx 0.7$	0
$\overline{\nu}_e$ Earth effects	Yes	No

- When are collective oscillations important?
- How to detect signatures of hierarchy?

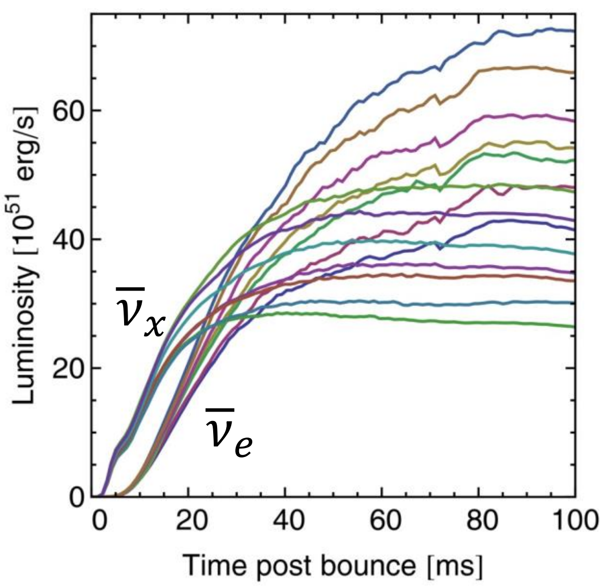
Early-Phase Signal in Anti-Neutrino Sector

Garching Models with M = 12–40 $M_\odot$

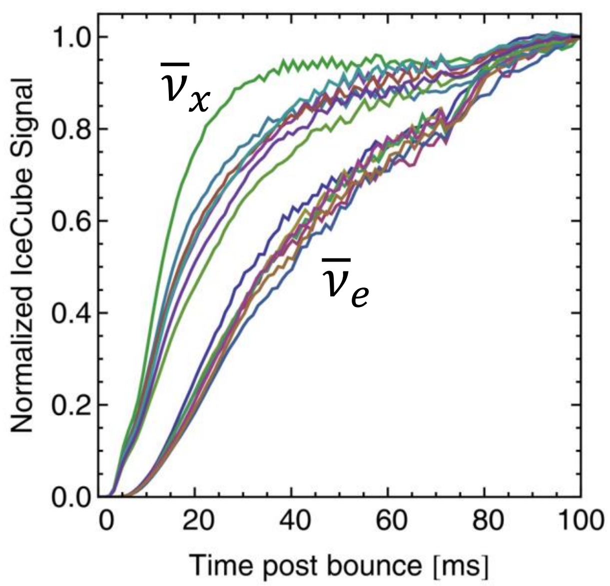
Average Energy



Luminosity



IceCube Signature

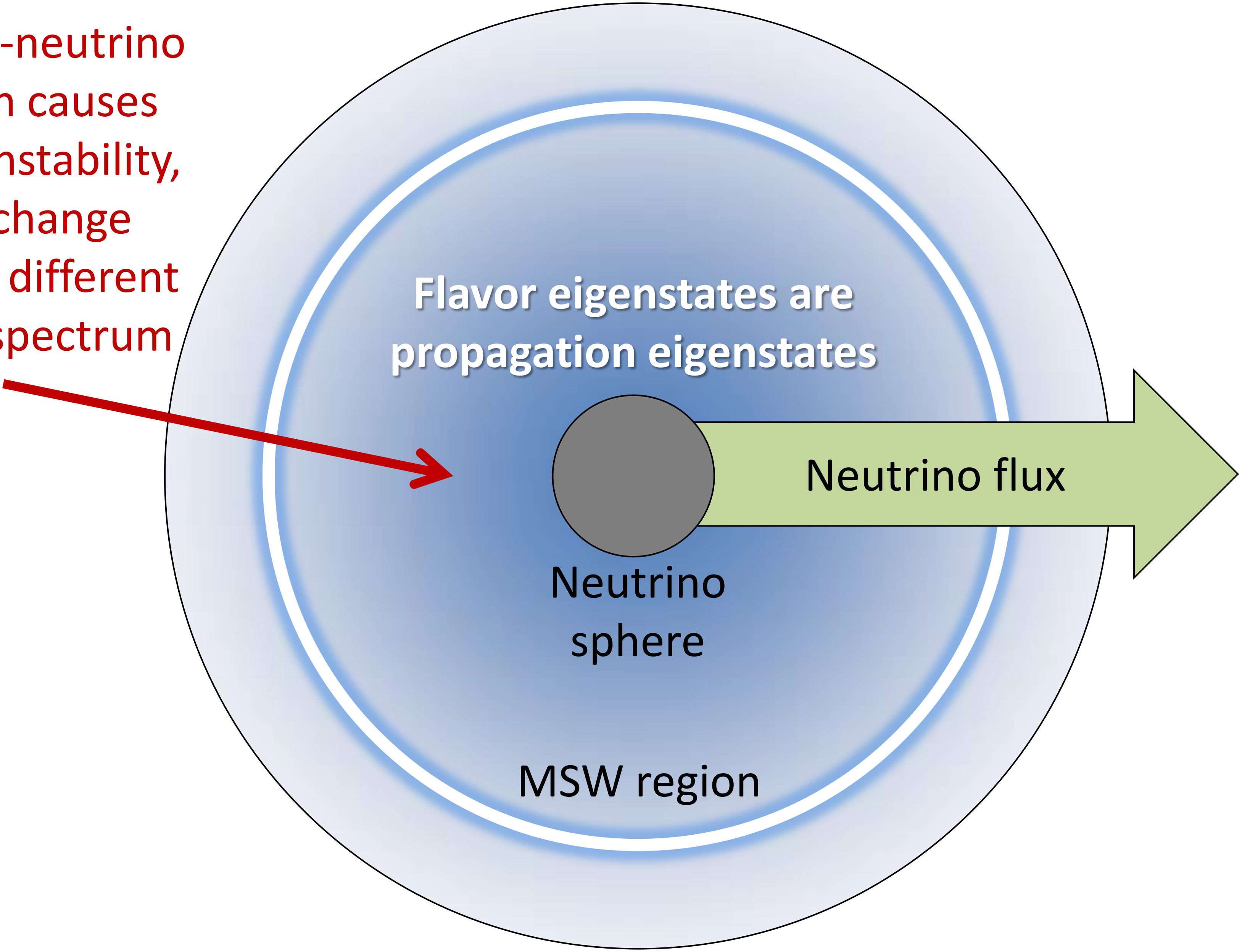


- In principle very sensitive to hierarchy, notably IceCube
- “Standard candle” to be confirmed by other than Garching models

Abbasi et al. (IceCube Collaboration) A&A 535 (2011) A109
Serpico, Chakraborty, Fischer, Hüdepohl, Janka & Mirizzi, arXiv:1111.4483

Flavor Oscillations in Core-Collapse Supernovae

Neutrino-neutrino refraction causes a flavor instability, flavor exchange between different parts of spectrum



Flavor eigenstates are propagation eigenstates

Neutrino flux

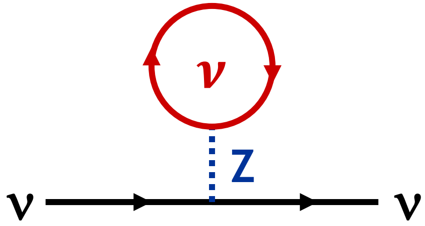
Neutrino sphere

MSW region

Flavor-Off-Diagonal Refractive Index

2-flavor neutrino evolution as an effective 2-level problem

$$\mathrm{i}\frac{\partial}{\partial t}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = H\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$



Effective mixing Hamiltonian

$$H = \frac{M^2}{2E} + \sqrt{2}G_\mathrm{F}\begin{pmatrix} N_e - \frac{N_n}{2} & 0 \\ 0 & -\frac{N_n}{2}\end{pmatrix} + \sqrt{2}G_\mathrm{F}\begin{pmatrix} N_{\nu_e} & N_{\langle\nu_e|\nu_\mu\rangle} \\ N_{\langle\nu_\mu|\nu_e\rangle} & N_{\nu_\mu}\end{pmatrix}$$

Mass term in flavor basis: causes vacuum oscillations

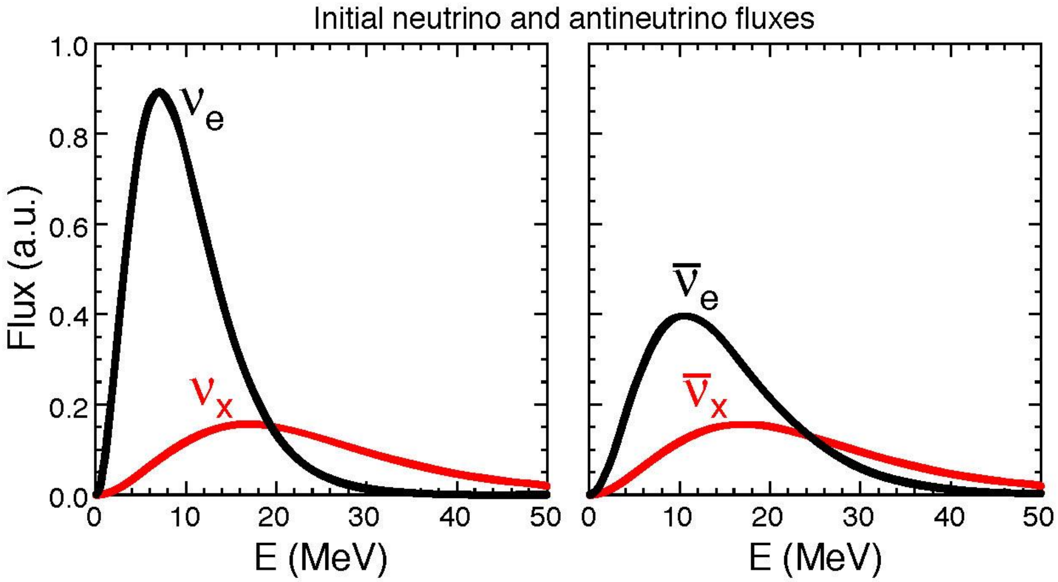
Wolfenstein's weak potential, causes MSW "resonant" conversion together with vacuum term

Flavor-off-diagonal potential, caused by flavor oscillations. (J.Pantaleone, PLB 287:128,1992)

Flavor oscillations feed back on the Hamiltonian: Nonlinear effects!

Spectral Split

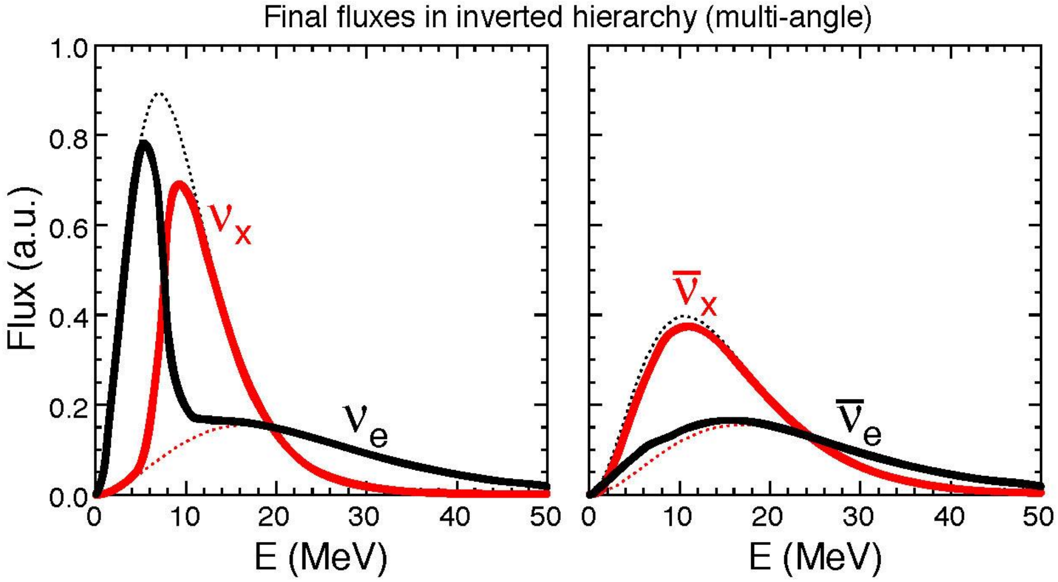
Initial fluxes at neutrino sphere


Initial neutrino and antineutrino fluxes

Figures from

Fogli, Lisi, Marrone & Mirizzi, arXiv:0707.1998

After collective trans-formation

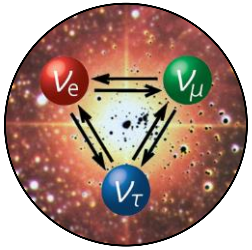

Final fluxes in inverted hierarchy (multi-angle)

Explanations in

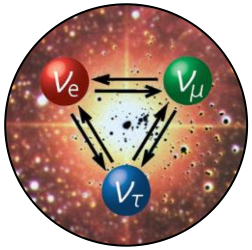
Raffelt & Smirnov arXiv:0705.1830 and 0709.4641

Duan, Fuller, Carlson & Qian arXiv:0706.4293 and 0707.0290

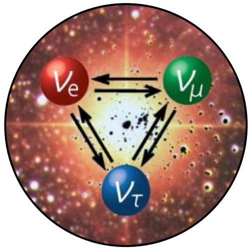
Self-Induced Flavor Conversion



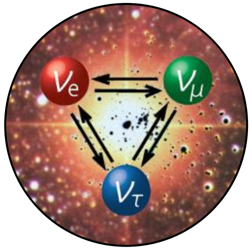
No net flavor conversion of ensemble
(in contrast to MSW conversion)



Flavor content exchanged
between different momentum modes
(or nus and anti-nus changing together)

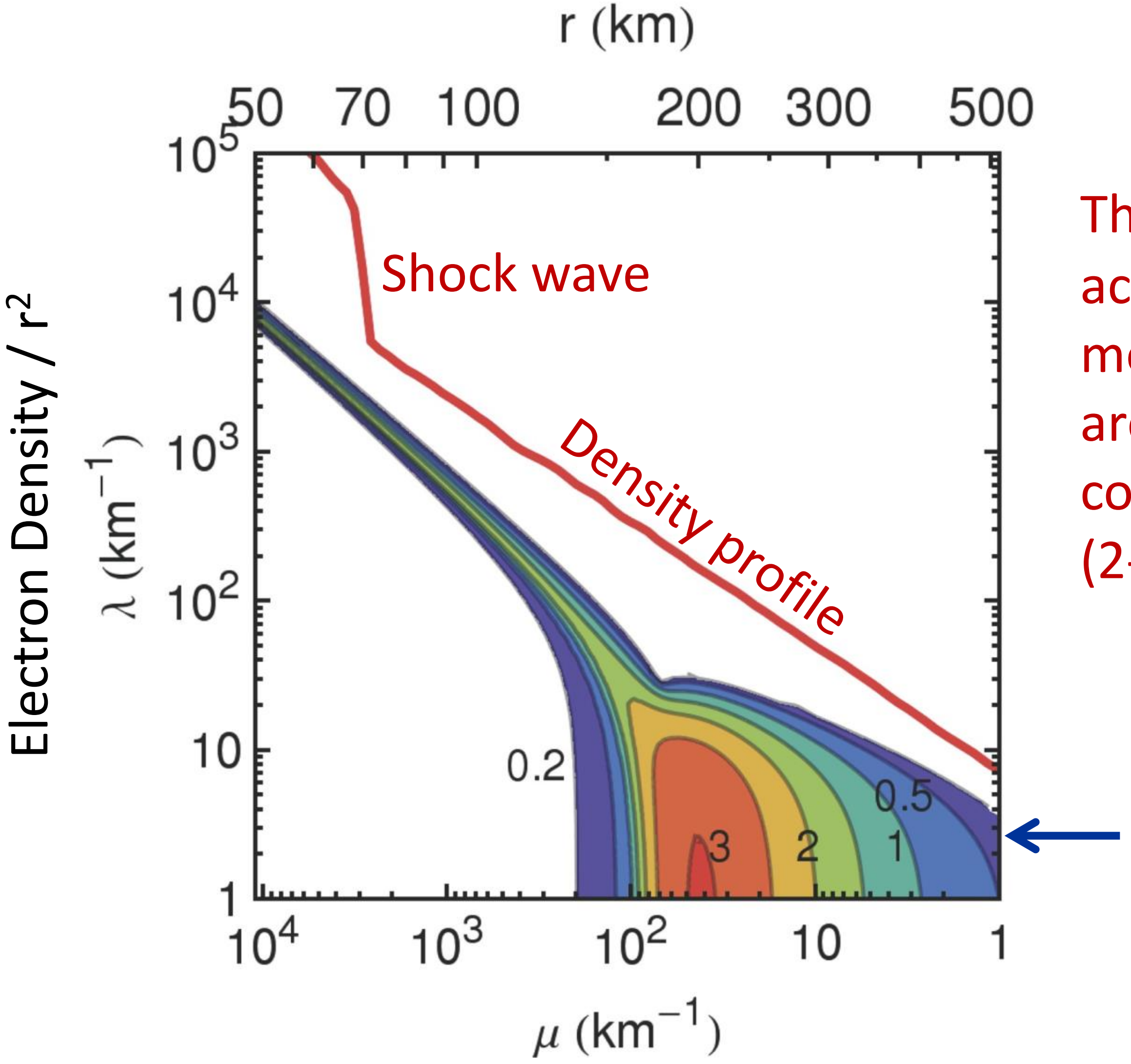


Interacting neutrino system: Coupled oscillators
- Collective harmonic oscillation modes
- Exponential run-away modes



Instability required to get started
- Exponentially growing off-diagonals in density matrix
- Linearized stability analysis to find growing modes

Multi-Angle Multi-Energy Stability Analysis



The studied 15 $M_\odot$ accretion-phase models (Garching) are stable against collective flavor conversion (2-flavor, inverted hierarchy)

Contours of growth rate $\kappa\ [\mathrm{km}^{-1}]$

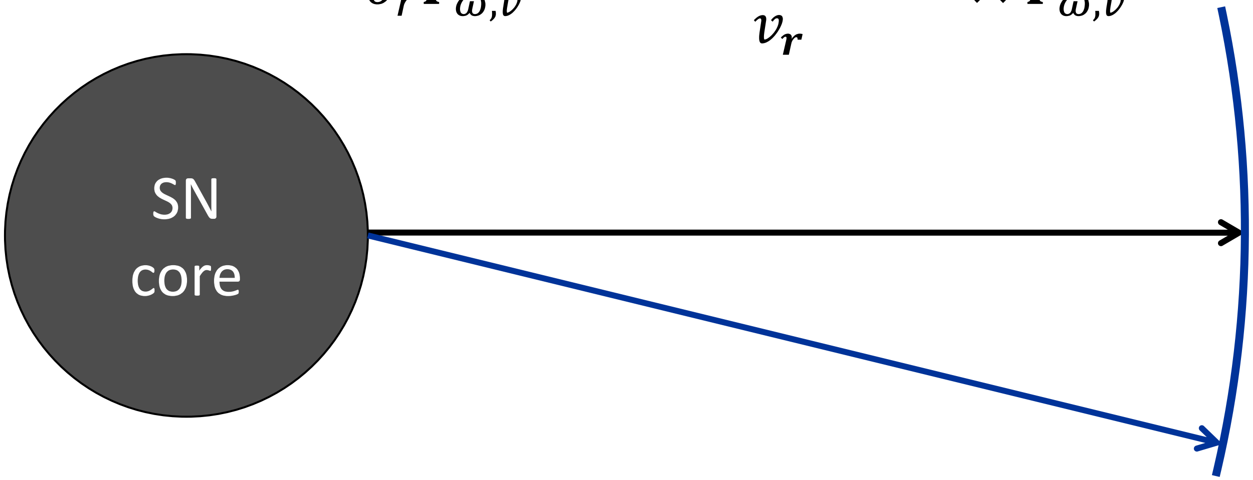
Sarikas, Raffelt, Hüdepohl & Janka, arXiv:1109.3601

Multi-Angle Matter Effect

Liouville form of oscillation equation

$$\underset{\substack{\uparrow\\ \text{Drops out for stationary solutions}}}{\dot{\mathbf{P}}_{\omega,\mathbf{v}}} + (\mathbf{v}\cdot\boldsymbol{\nabla}_r)\,\mathbf{P}_{\omega,\mathbf{v}} = (\omega\mathbf{B} + \underset{\substack{\uparrow\\ \sqrt{2}G_\mathrm{F}N_e}}{\lambda}\mathbf{L} + \underset{\substack{\uparrow\\ \sqrt{2}G_\mathrm{F}N_\nu}}{\mu}\mathbf{P}) \times \mathbf{P}_{\omega,\mathbf{v}}$$

$$\partial_r\mathbf{P}_{\omega,v} = \frac{\omega\mathbf{B} + \lambda\mathbf{L} + \mu\mathbf{P}}{v_r} \times \mathbf{P}_{\omega,v}$$



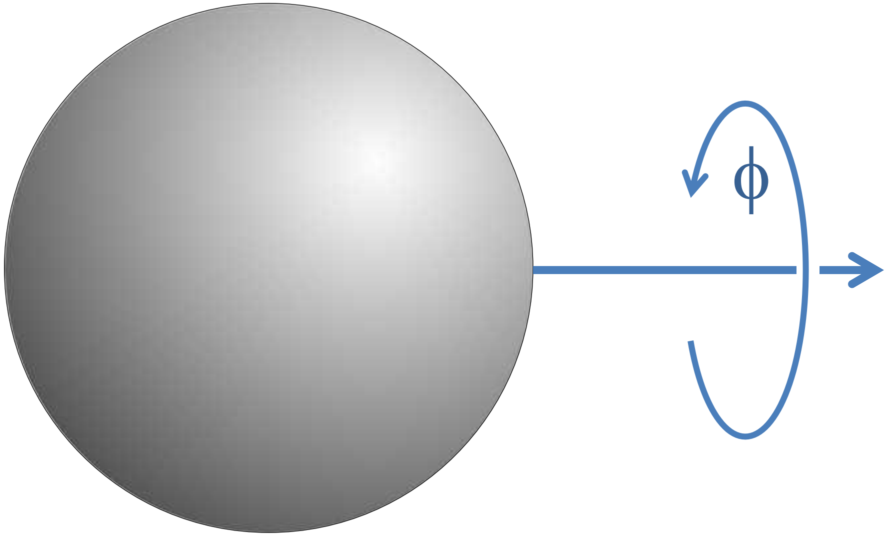
Longer path to
same radial distance:
Larger matter effect

Self-induced conversion suppressed for $N_e \gtrsim N_\nu$

Esteban-Pretel, Mirizzi, Pastor, Tomàs, Raffelt, Serpico & Sigl, arXiv:0807.0659

Symmetry Breaking in Collective Flavor Oscillations

Assume globally spherically symmetric neutrino emission from SN core
→ Axial symmetry in chosen direction

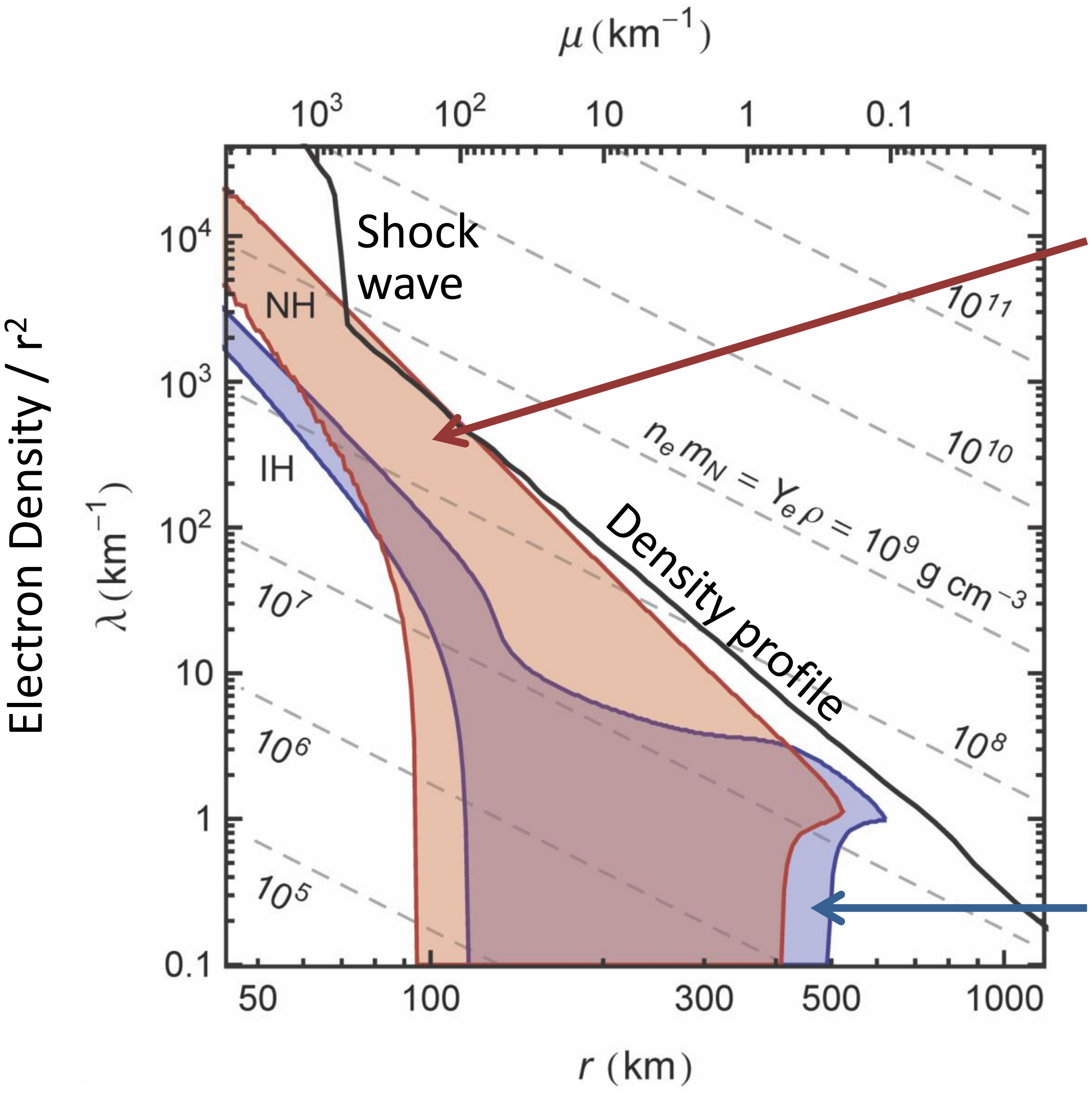


Self-induced neutrino flavor conversion in both hierarchies (unless suppressed by multi-angle matter effect)

- Axially symmetric solution:
  Conversion for inverted hierarchy
  (usual result)
- Spontaneous breaking of axial symmetry:
  Dipole solution ($\propto \cos\phi$ or $\sin\phi$)
  Conversion for normal hierarchy
  (Was missed by enforcing axial symmetry
  because of axially symmetric emission)

G. Raffelt, S. Sarikas & D. de Sousa Seixas
Axial symmetry breaking in self-induced flavor conversion of SN neutrino fluxes
PRL 111 (2013) 091101 [arXiv:1305.7140]

Instability Footprints

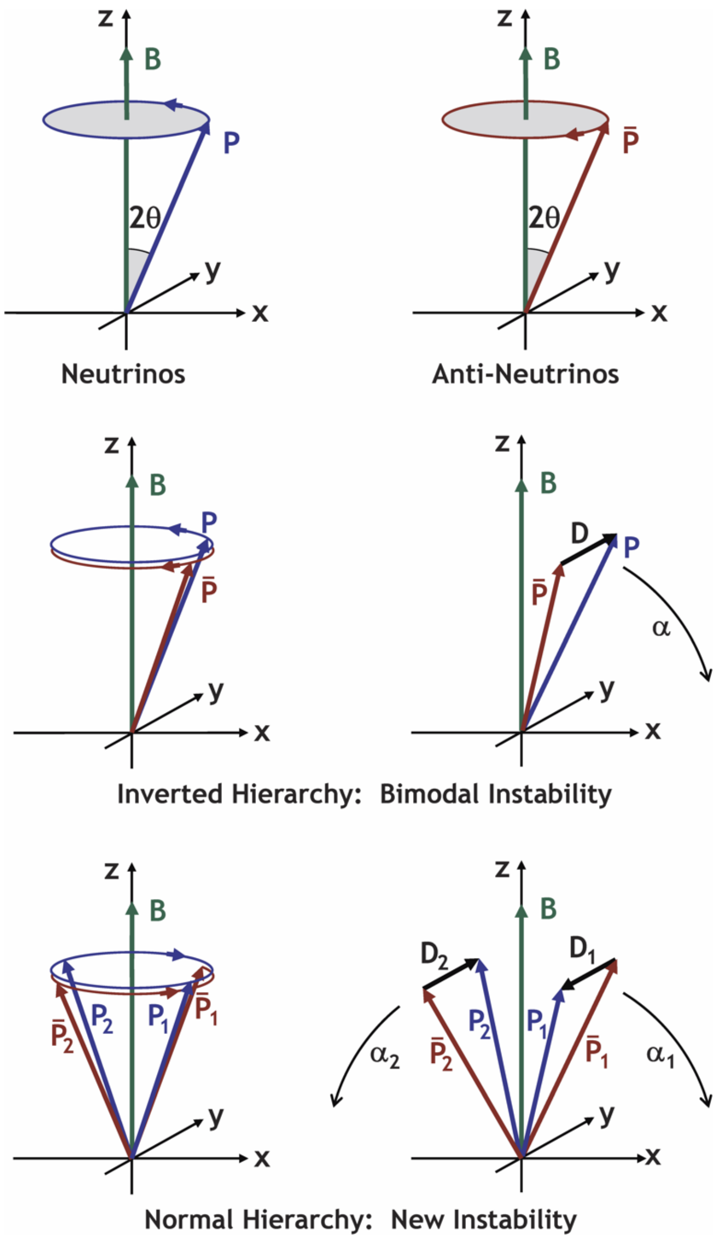


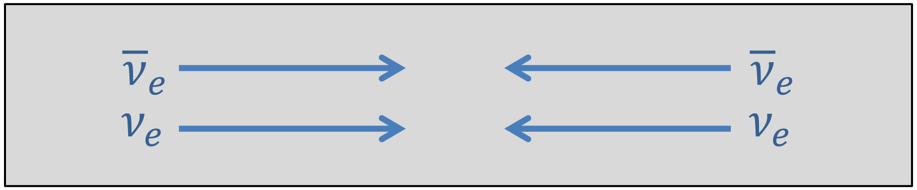
Axial-symmetry breaking (MAA) instability (normal ordering NH) is “more dangerous” to trigger self-induced flavor conversion

Traditional “bimodal” instability (inverted mass ordering IH)

Raffelt, Sarikas & Seixas, arXiv:1305.7140

Colliding Beam Model





Raffelt & Seixas, arXiv:1307.7625

Left- and right-moving neutrinos behave symmetrically

Instability for inverted mass ordering (IH)

Left-right symmetry breaking:

- Anti-symmetric mode for normal mass ordering (NH)
- Corresponds to axial symmetry breaking in SN case (MAA instability)

Symmetry Assumptions

Neutrino transport and flavor oscillations: 7D problem

$$\left(\partial_t + \vec{v}\cdot\vec{\nabla}_x + \vec{F}\cdot\vec{\nabla}_p\right)\rho(t,\vec{x},\vec{p}) = -i\left[H(t,\vec{x},\vec{p}),\rho(t,\vec{x},\vec{p})\right] + \mathcal{C}[\rho(t,\vec{x},\vec{p})]$$

Ignore external forces
(e.g. no grav. deflection)

Includes vacuum, matter,
nu-nu refraction

Ignore collision term:
Free streaming

- **Homogeneous, isotropic system evolving in time ("early universe")**
  or 1D homogeneous evolving in time ("colliding beams")

$$\partial_t\rho(t,E) = -i\left[H(t,E),\rho(t,E)\right]$$

- **Stationary, spherically symmetric, evolving with radius ("supernova")**

$$v_r\partial_r\rho(r,E,\theta) = -i\left[H(r,E,\theta),\rho(r,E,\theta)\right]$$

Zenith angle of nu momentum $\vec{p}$

Radial velocity depends on θ, leads to multi-angle matter effect

- **Ordinary differential equations in "time" or "radius" with maximal symmetries**
- **Misses dominant solutions (spontaneous symmetry breaking)**

Spatial Symmetry Breaking (SSB)

Oscillation equation with explicit transport term

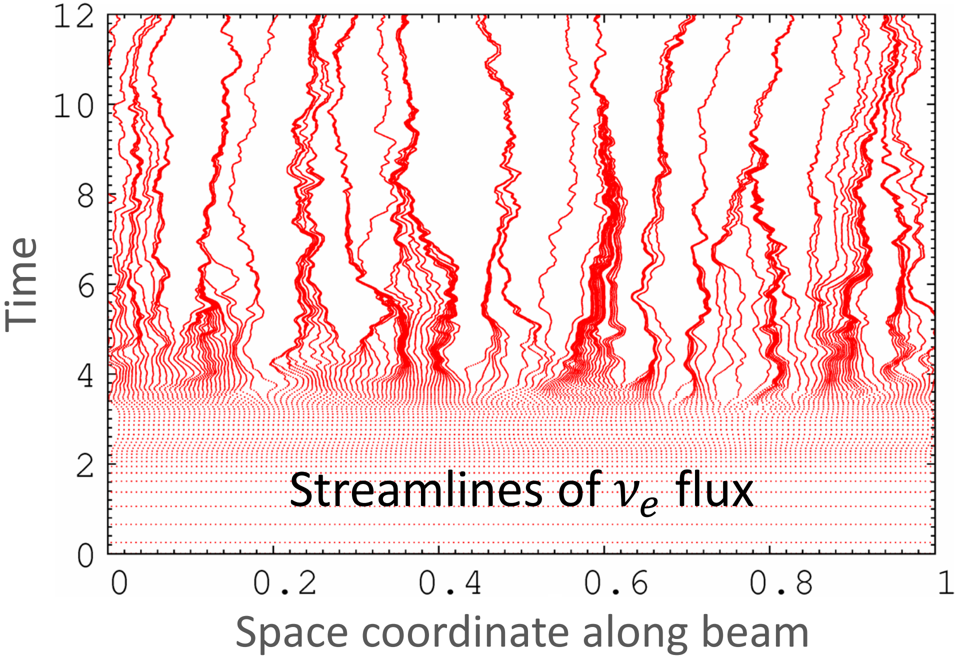
$$i\,\underbrace{\left(\partial_t + \vec{v}\cdot\vec{\nabla}_x\right)}_{}\ \rho(t,\vec{x},\vec{p}) = [H(t,\vec{x},\vec{p}),\rho(t,\vec{x},\vec{p})]$$

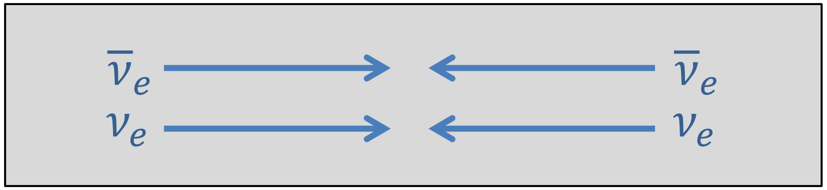
Without flavor oscillations: free streaming

Spatial Fourier transform (plane wave expansion)

$$\left(i\partial_t + \vec{v}\cdot\vec{k}\right)\ \rho\left(t,\vec{k},\vec{p}\right) = \int d^3\vec{x}\ e^{-i\,\vec{k}\cdot\vec{x}}\ [H(t,\vec{x},\vec{p}),\rho(t,\vec{x},\vec{p})]$$

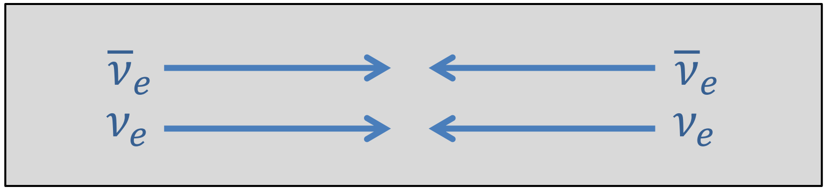
Interaction term couples different Fourier modes





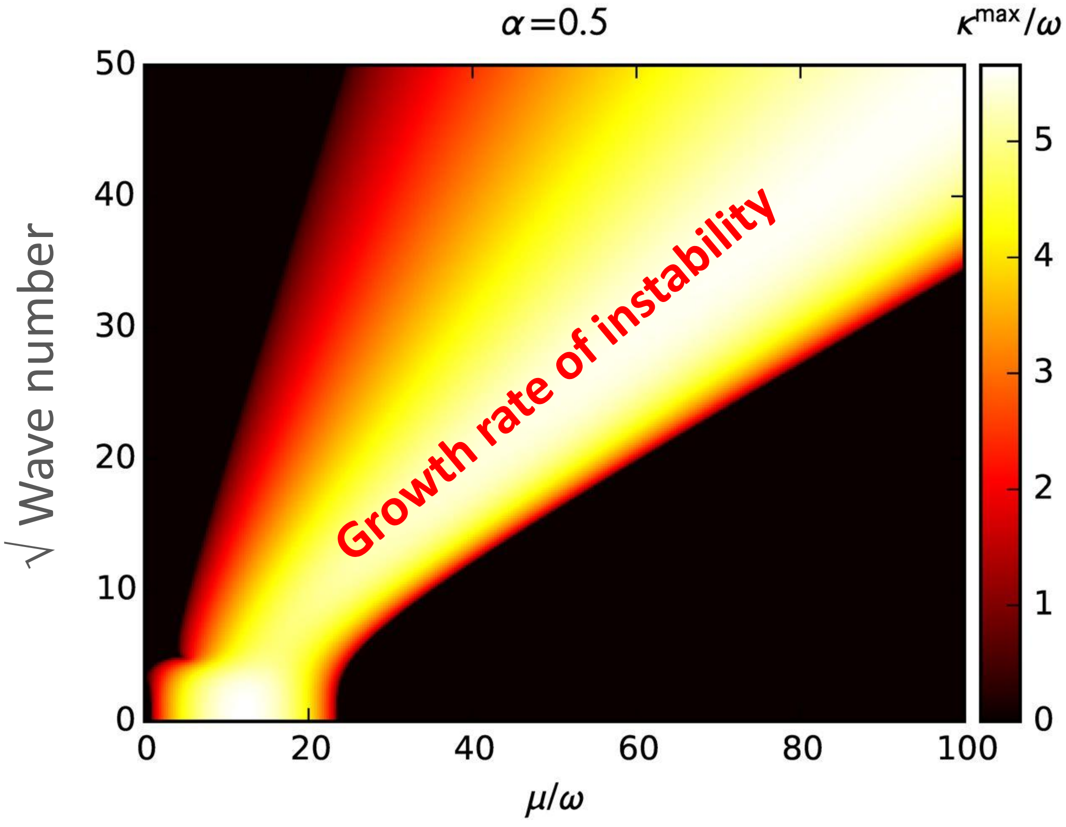
Mirizzi, Mangano & Saviano
arXiv:1503.03485

Spatial Symmetry Breaking (SSB)



Linearized stability analysis for colliding-beam model

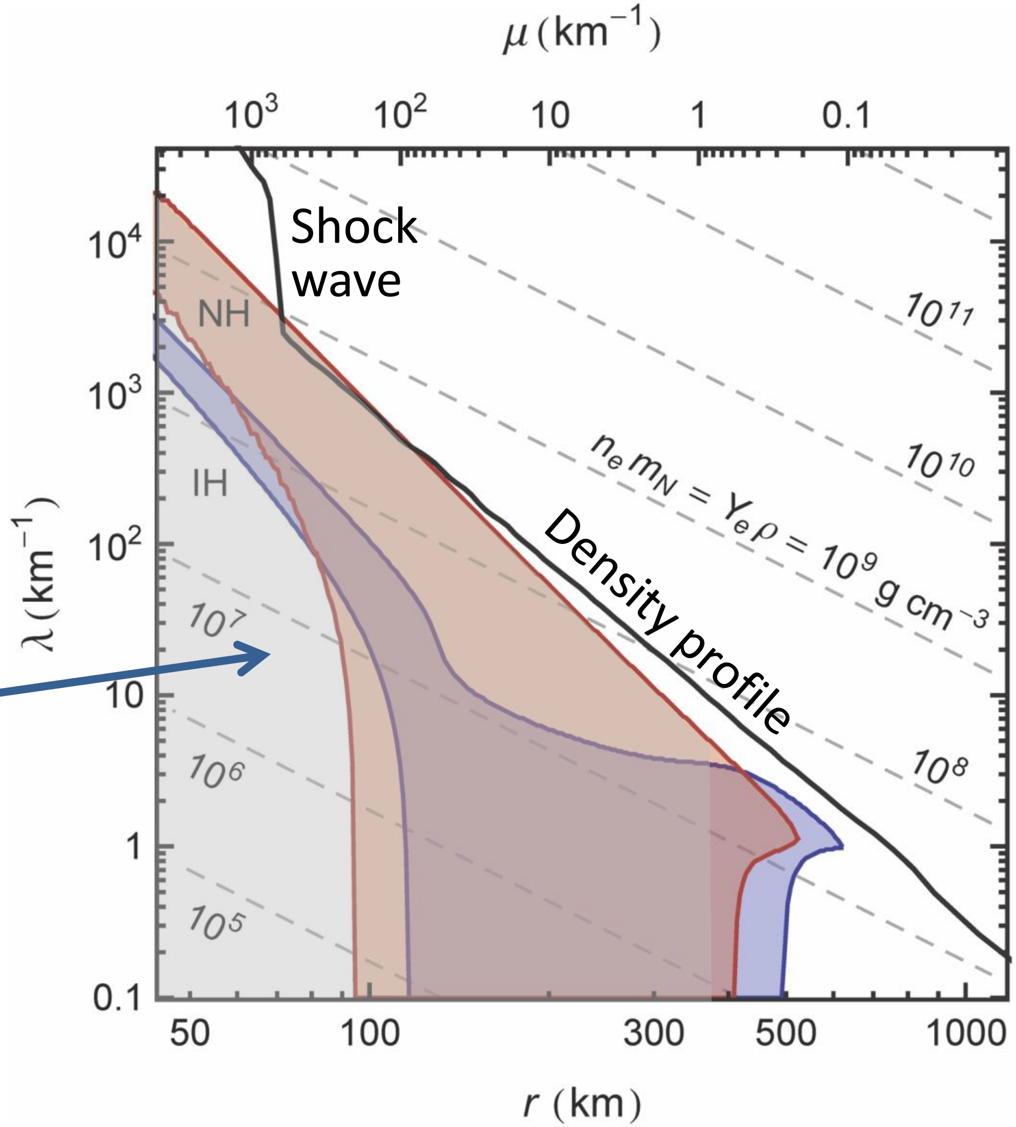
Duan & Shalgar, arXiv:1412.7097



Effective neutrino density

- Instability footprint shifted to larger neutrino density μ for larger wave number k
- For any neutrino density, unstable for some k-range
- No flavor-stable conditions exist for homogeneous neutrino gas (no “sleeping top” regime)

Small-Scale Instabilities



- Small-scale modes “fill in” the stability footprint for large neutrino density
- Largest-scale mode is “most dangerous” to cross SN density profile

Chakraborty, Hansen, Izaguirre & Raffelt, Work in progress (2015)

Space-Time-Dependent Problem in Supernova

- Neutrino momentum distribution not limited to “outward” direction
- Important “halo” flux even at large distance
- Large 3D effects

→ Inhomogeneous, anisotropic, non-stationary problem

Really no self-induced flavor conversion below shock-wave or even below nu-sphere?

- Investigations to date are simplified case studies
- May not represent real SNe

Status of Collective Flavor Conversion

- Self-induced flavor conversion is an instability in flavor space of the interacting neutrino ensemble
- Space-time dependent phenomenon (not simply stationary or homogeneous)
- Solutions do not respect symmetries of initial system Instabilities can occur on all scales

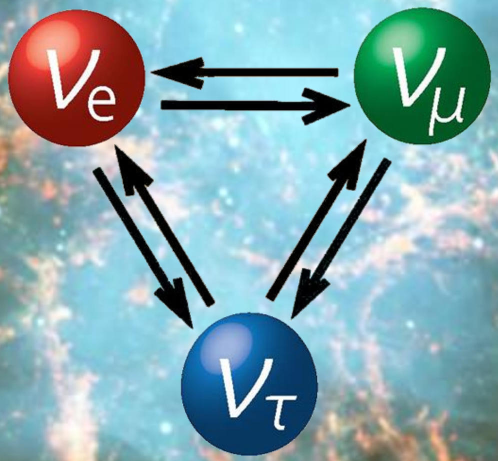
'If it's what I think it is, we've got some work ahead of us.'

Essentially back to the drawing board …

Literature on Spatial Symmetry Breaking (SSB)

1. Axial symmetry breaking in self-induced flavor conversion of SN neutrino fluxes
   Raffelt, Sarikas & Seixas, PRL 111 (2013) 091101
2. Neutrino flavor pendulum in both mass hierarchies
   Raffelt & Seixas, PRD 88 (2013) 045031
3. Chaotic flavor evolution in an interacting neutrino gas
   Hansen & Hannestad, PRD 90 (2014) 025009
4. Damping the neutrino flavor pendulum by breaking homogeneity
   Mangano, Mirizzi & Saviano, PRD 89 (2014) 073017
5. Spontaneous breaking of spatial symmetries in collective neutrino oscillations
   Duan & Shalgar, arXiv:1412.7097
6. Self-induced flavor instabilities of a dense neutrino stream in a 2D model
   Mirizzi, Mangano & Saviano, arXiv:1503.03485
7. Self-induced flavor-conversion anisotropy of supernova neutrinos
   Chakraborty, Hansen, Izaguirre & Raffelt, work in progress (2015)

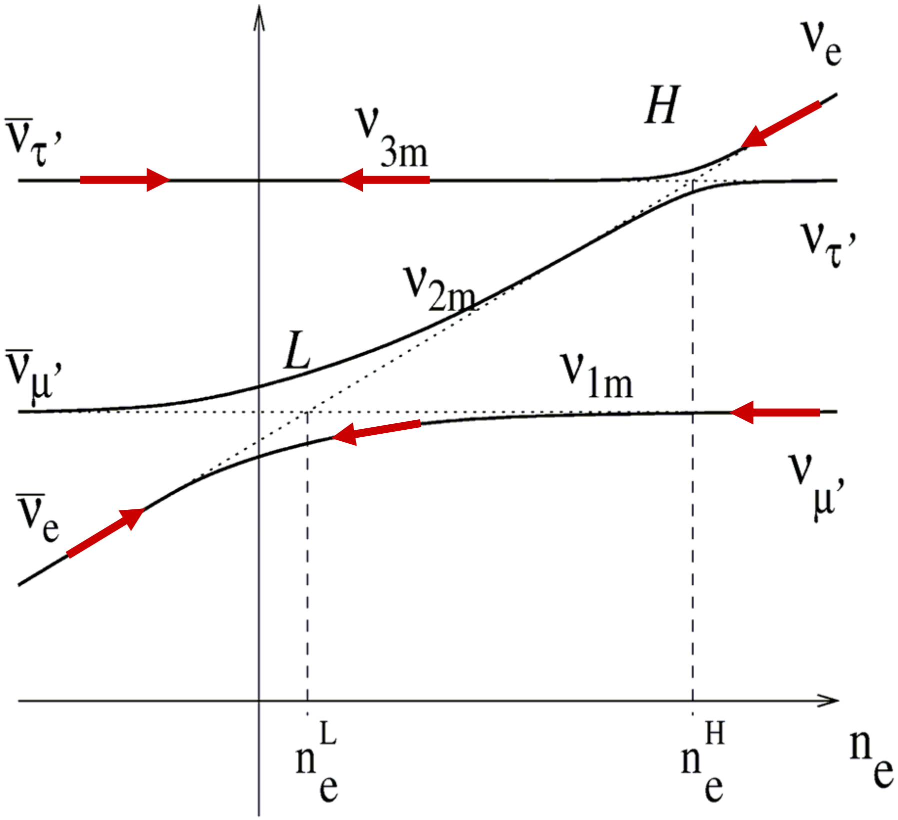
More theory progress is needed to understand flavor conversion of supernova neutrinos!



Backup

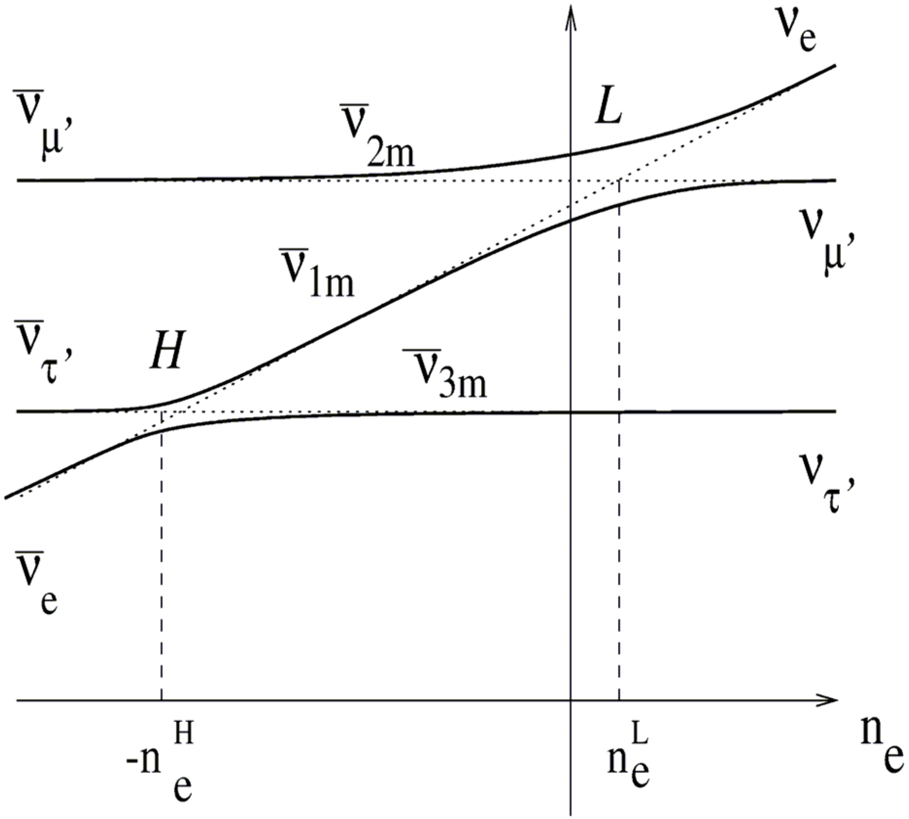
Three-Flavor Eigenvalue Diagram

Normal mass hierarchy



Vacuum

Inverted mass hierarchy



Vacuum

Dighe & Smirnov, Identifying the neutrino mass spectrum from a supernova neutrino burst, astro-ph/9907423

Three Ways to Describe Flavor Oscillations

Schrödinger equation in terms of "flavor spinor"

$$\mathrm{i}\partial_t \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = H \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{\Delta m^2}{2E} \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

Neutrino flavor density matrix

$$\rho = \begin{pmatrix} \langle \nu_e | \nu_e \rangle & \langle \nu_e | \nu_\mu \rangle \\ \langle \nu_\mu | \nu_e \rangle & \langle \nu_\mu | \nu_\mu \rangle \end{pmatrix}$$

Equivalent commutator form of Schrödinger equation

$$\mathrm{i}\partial_t \rho = [H, \rho]$$

Expand 2×2 Hermitean matrices in terms of Pauli matrices

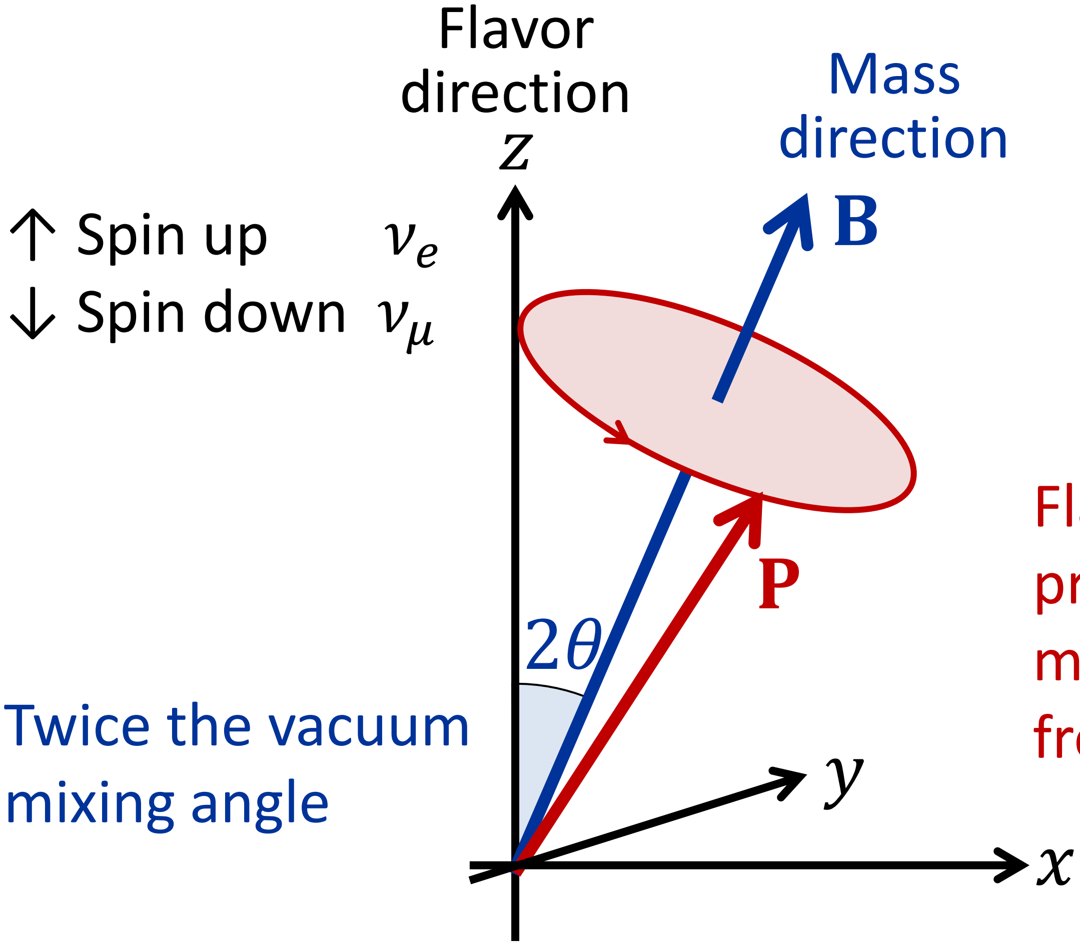
$\rho = \frac{1}{2}[\mathrm{Tr}(\rho) + \mathbf{P} \cdot \boldsymbol{\sigma}]$ and $H = \frac{\Delta m^2}{2E} \mathbf{B} \cdot \boldsymbol{\sigma}$ with $\mathbf{B} = (\sin 2\theta, 0, \cos 2\theta)$

Equivalent spin-precession form of equation of motion

$\dot{\mathbf{P}} = \omega \mathbf{B} \times \mathbf{P}$ with $\omega = \frac{\Delta m^2}{2E}$

P is "polarization vector" or "Bloch vector" or "flavor isospin vector"

Flavor Oscillation as Spin Precession



Twice the vacuum mixing angle

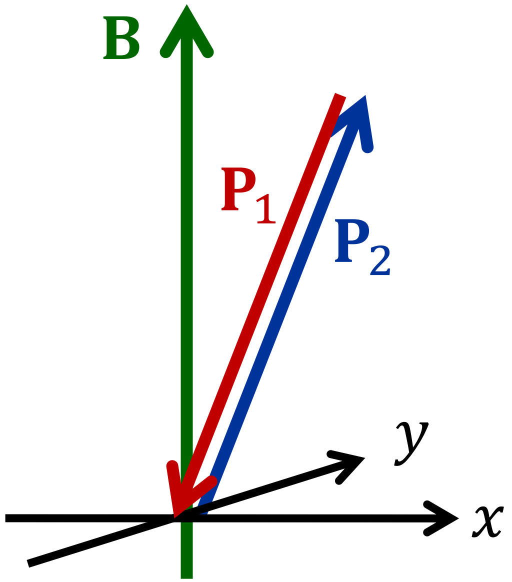
Flavor polarization vector precesses around the mass direction with frequency $\omega = \Delta m^2/2E$

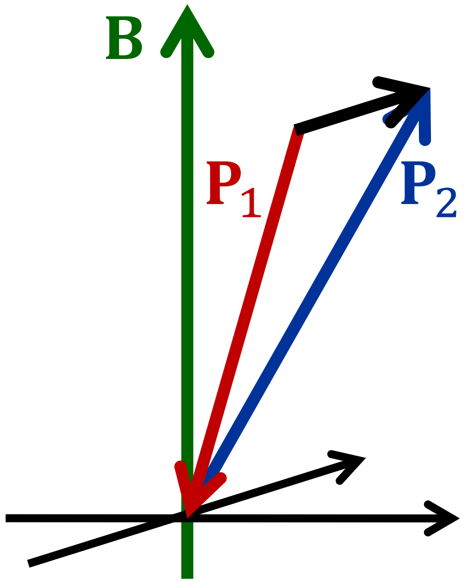
Instability in Flavor Space

Two-mode example in co-rotating frame, initially $\mathbf{P}_1 = \downarrow$, $\mathbf{P}_2 = \uparrow$ (flavor basis)

$$\dot{\mathbf{P}}_1 = [-\omega \mathbf{B} + \mu\,(\mathbf{P}_1 + \mathbf{P}_2)] \times \mathbf{P}_1$$

$$\dot{\mathbf{P}}_2 = [+\omega \mathbf{B} + \mu\,\underbrace{(\mathbf{P}_1 + \mathbf{P}_2)}_{\text{0 initially}}] \times \mathbf{P}_2$$





$\mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2$

Matter effect transverse to mass and flavor directions

Both $\mathbf{P}_1$ and $\mathbf{P}_2$ tilt around $\mathbf{P}$ if μ is large

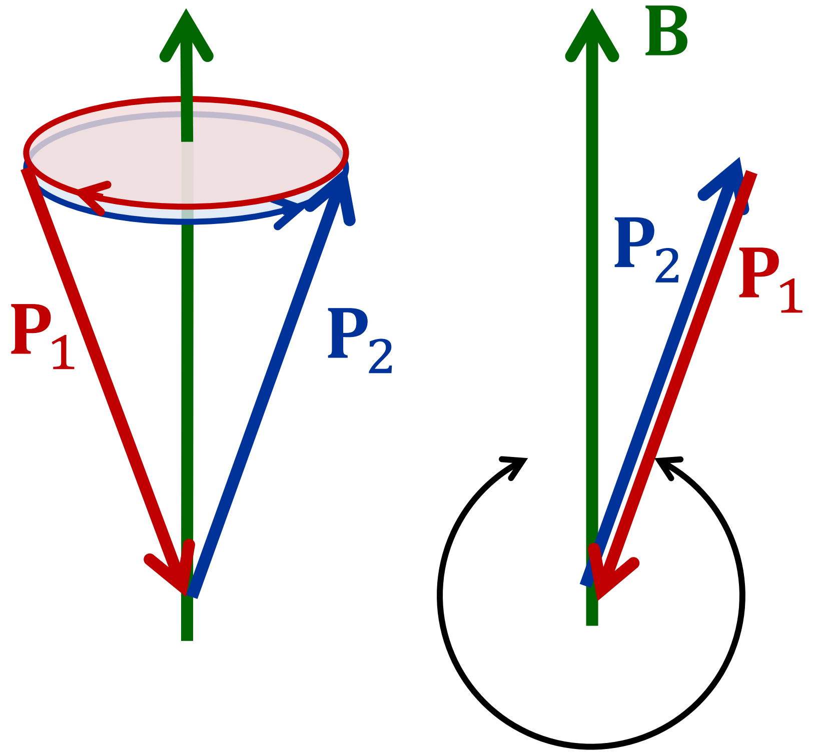
- Initially aligned in flavor direction and $\mathbf{P} = 0$
- Free precession $\pm\omega$

After a short time, transverse $\mathbf{P}$ develops by free precession

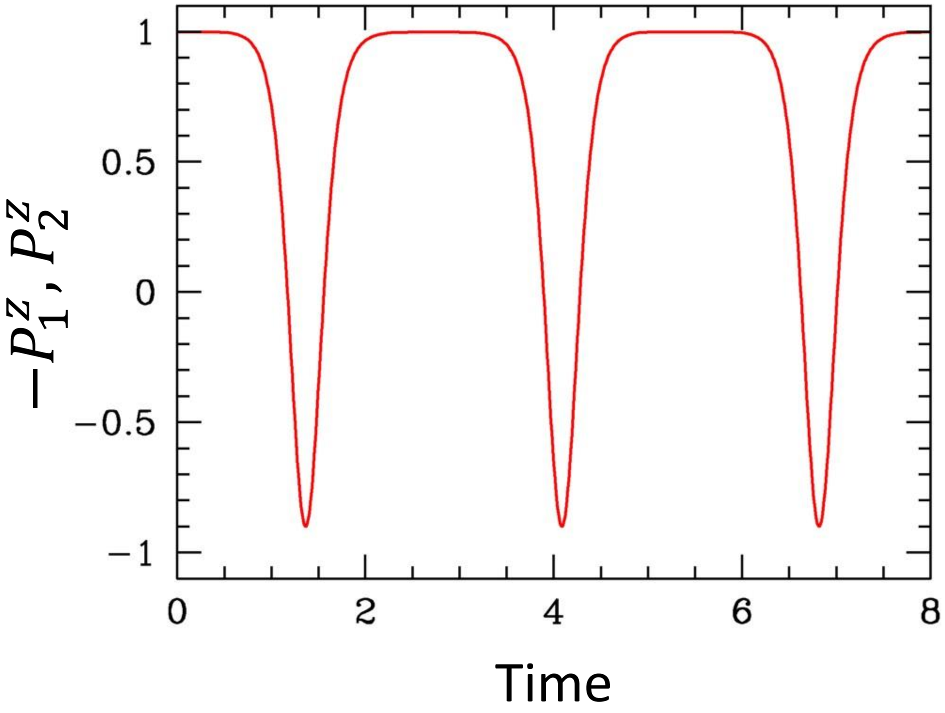
Two Spins with Opposite Initial Orientation

No interaction ($\mu = 0$)
Free precession in opposite directions

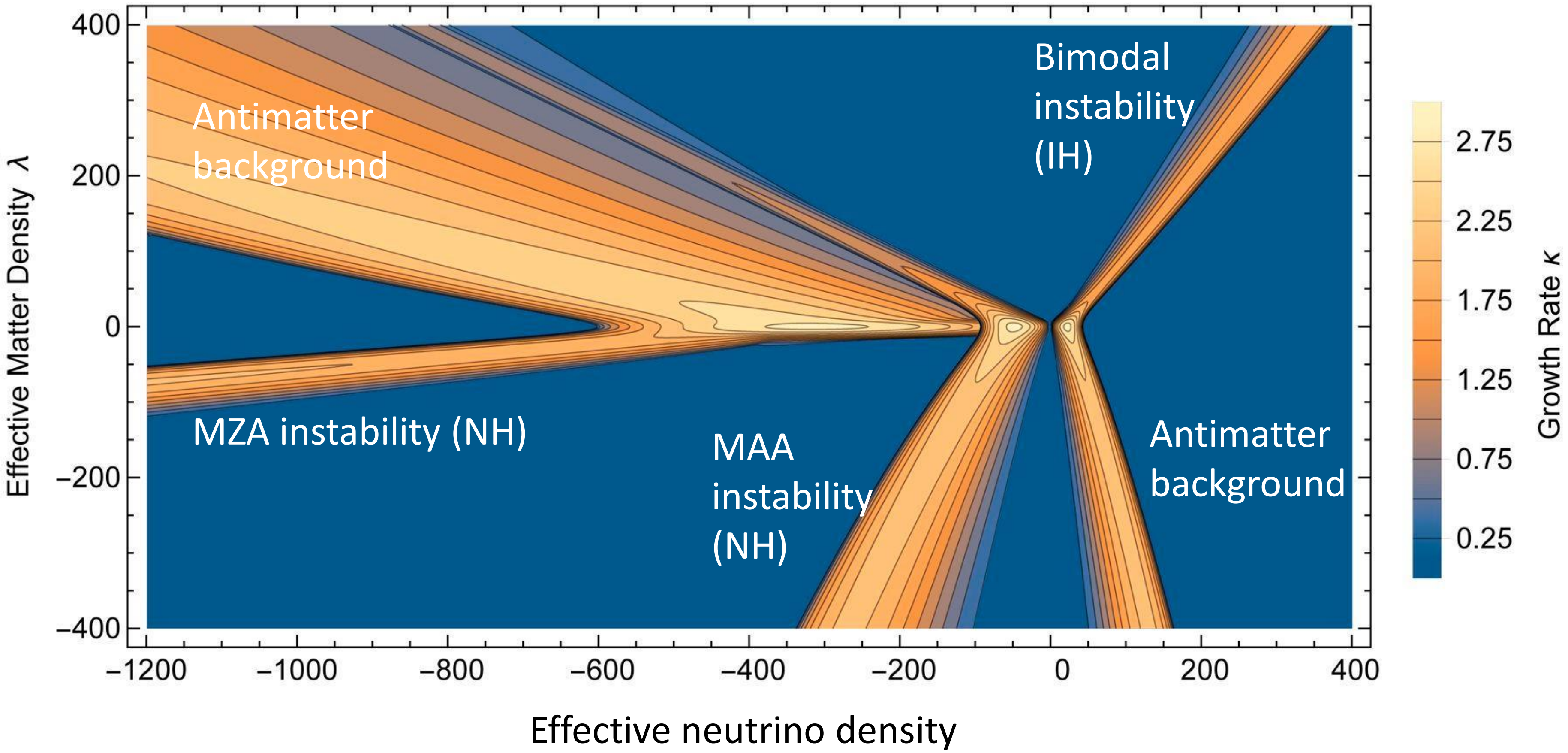
Even for very small mixing angle, large-amplitude flavor oscillations



Strong interaction
($\mu \to \infty$)
Pendular motion

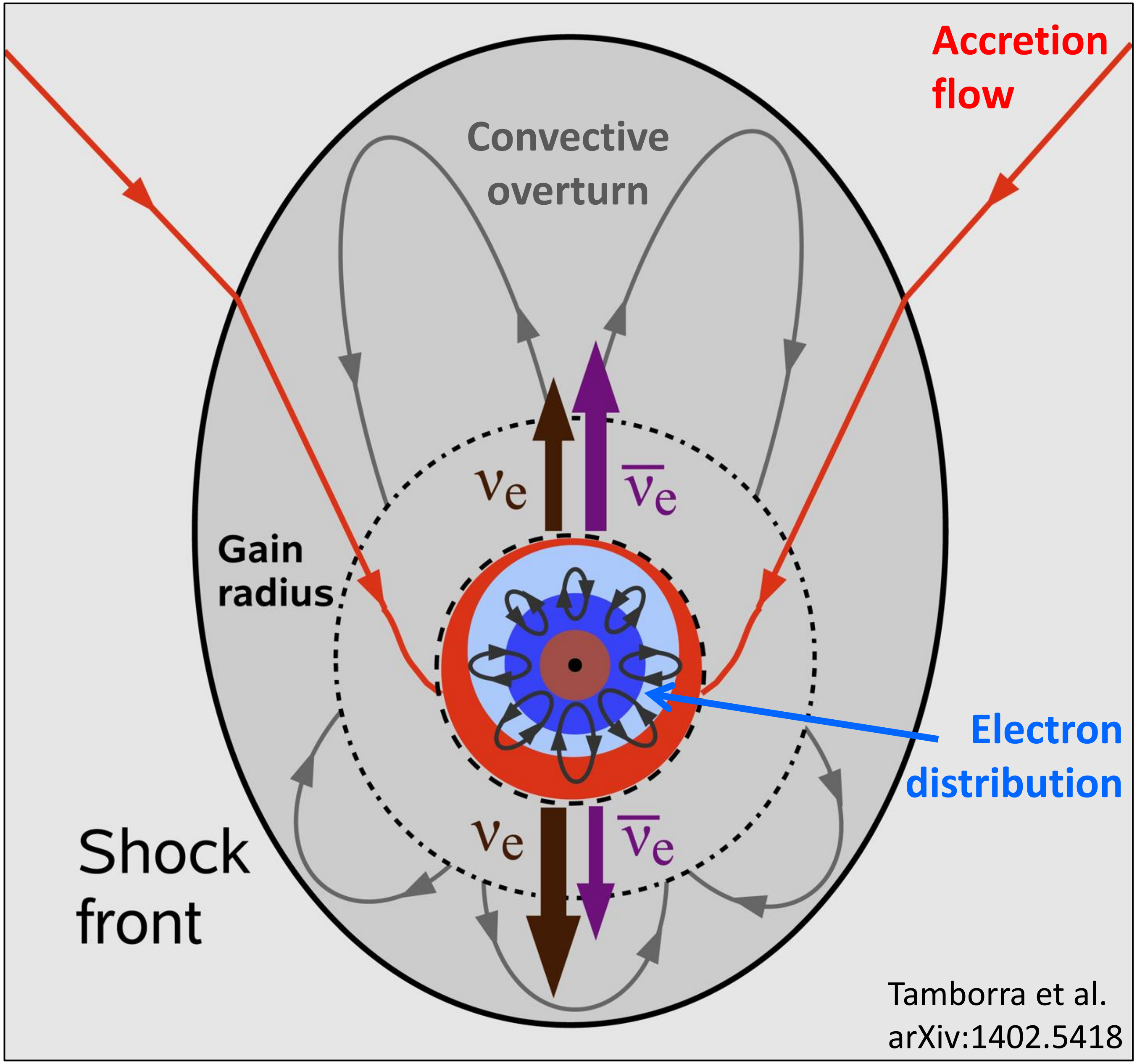


Multi-Angle Matter Effect in Supernovae



Chakraborty, Hansen, Izaguirre & Raffelt, Work in progress (2015)

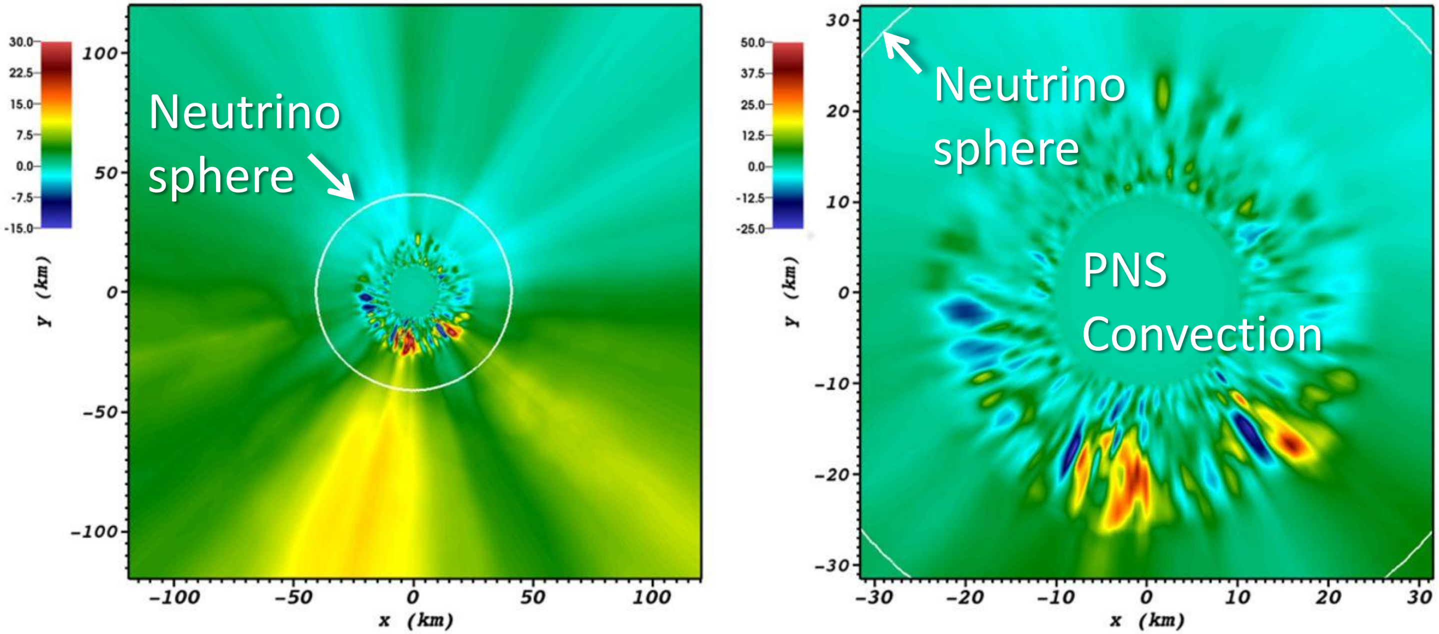
Schematic Theory of LESA



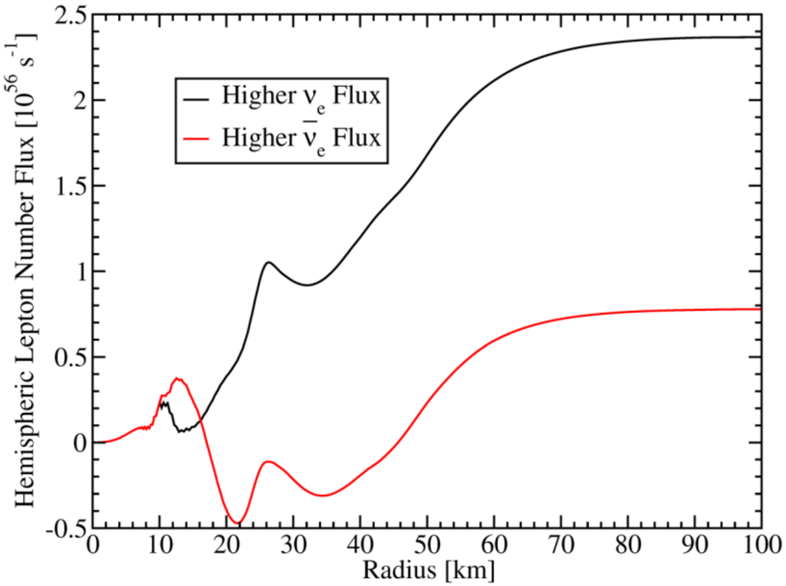
Tamborra et al. arXiv:1402.5418

Feedback loop consists of asymmetries in
- accretion rate
- lepton-number flux
- neutrino heating rate
- dipole deformation of shock front

LESA Dipole and PNS Convection



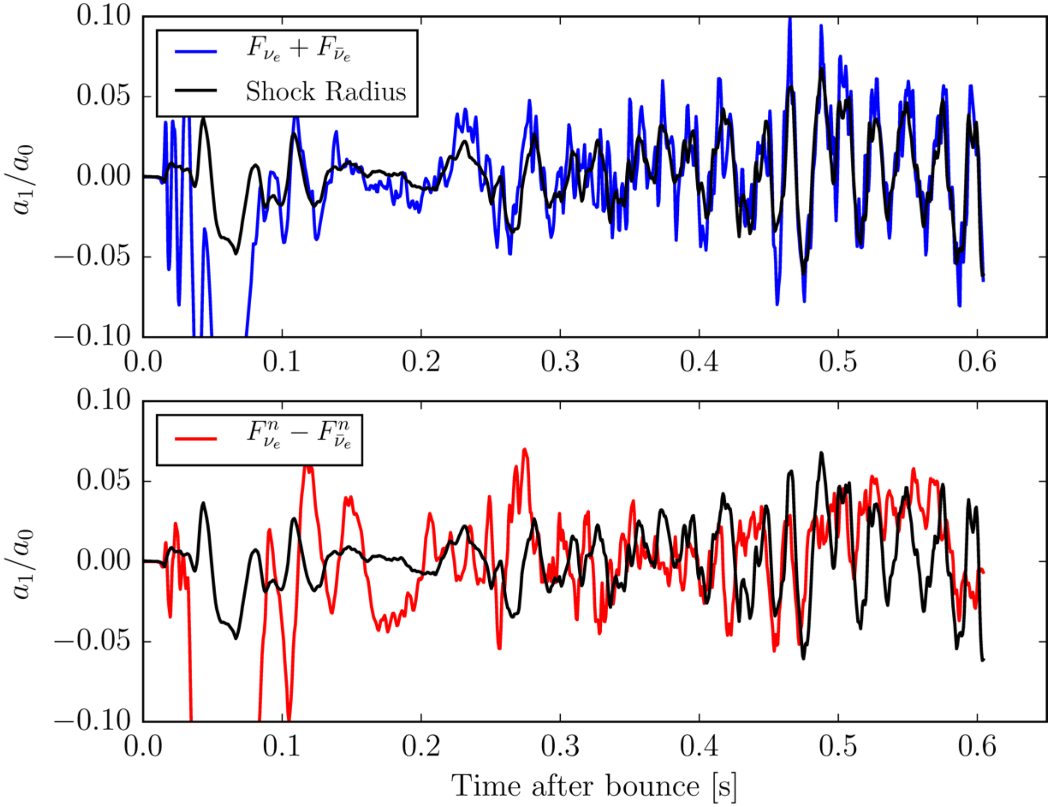
Color-coded lepton-number flux along radial rays ($11.2\ M_{SUN}$ model at 210 ms)



Lepton flux dipole builds up mostly below the neutrino sphere in a region of strong convection in the proto-neutron star (PNS)

Is the LESA Phenomenon Real?

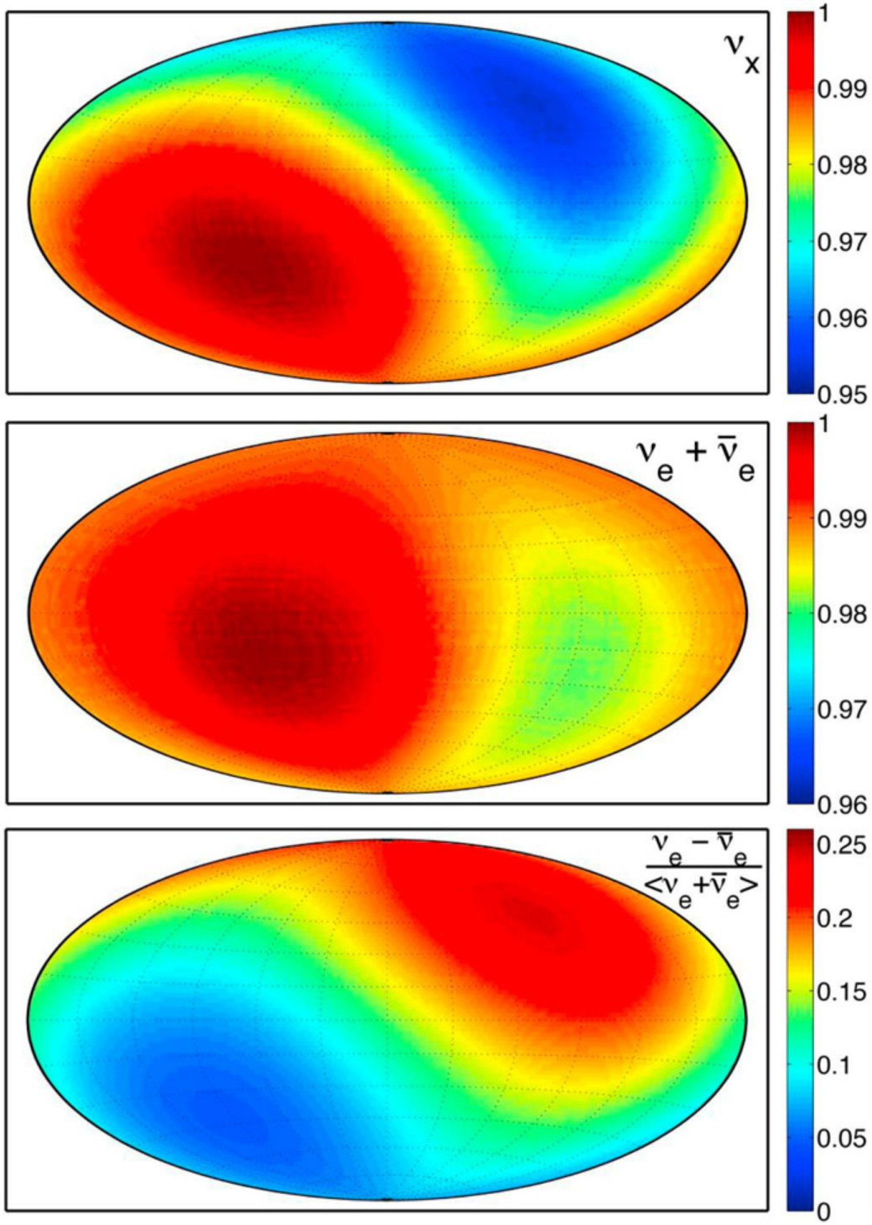
- Couch & O'Connor (2014) also find LESA in their 3D models
- Dolence, Burrows & Zhang (arXiv:1403.6115), 2D models: No LESA dipole at all



Red curve:
Lepton-number dipole × 5
No evidence for beyond-noise dipole evolution
(Fig.11 of arXiv:1403.6115)

Different method of neutrino radiative transfer, different interaction rates, and many other physics differences — needs to be understood

Sky Distribution of Number Fluxes (11.2 M_{SUN})



Neutrino number flux distribution for 11.2 M_{SUN} model integrated over 150–250 ms

Heavy-flavor neutrino fluxes (ν_x) nearly isotropic

Flux of $\nu_e + \bar{\nu}_e$ nearly isotropic

Lepton-number flux ($\nu_e - \bar{\nu}_e$) has strong dipole distribution

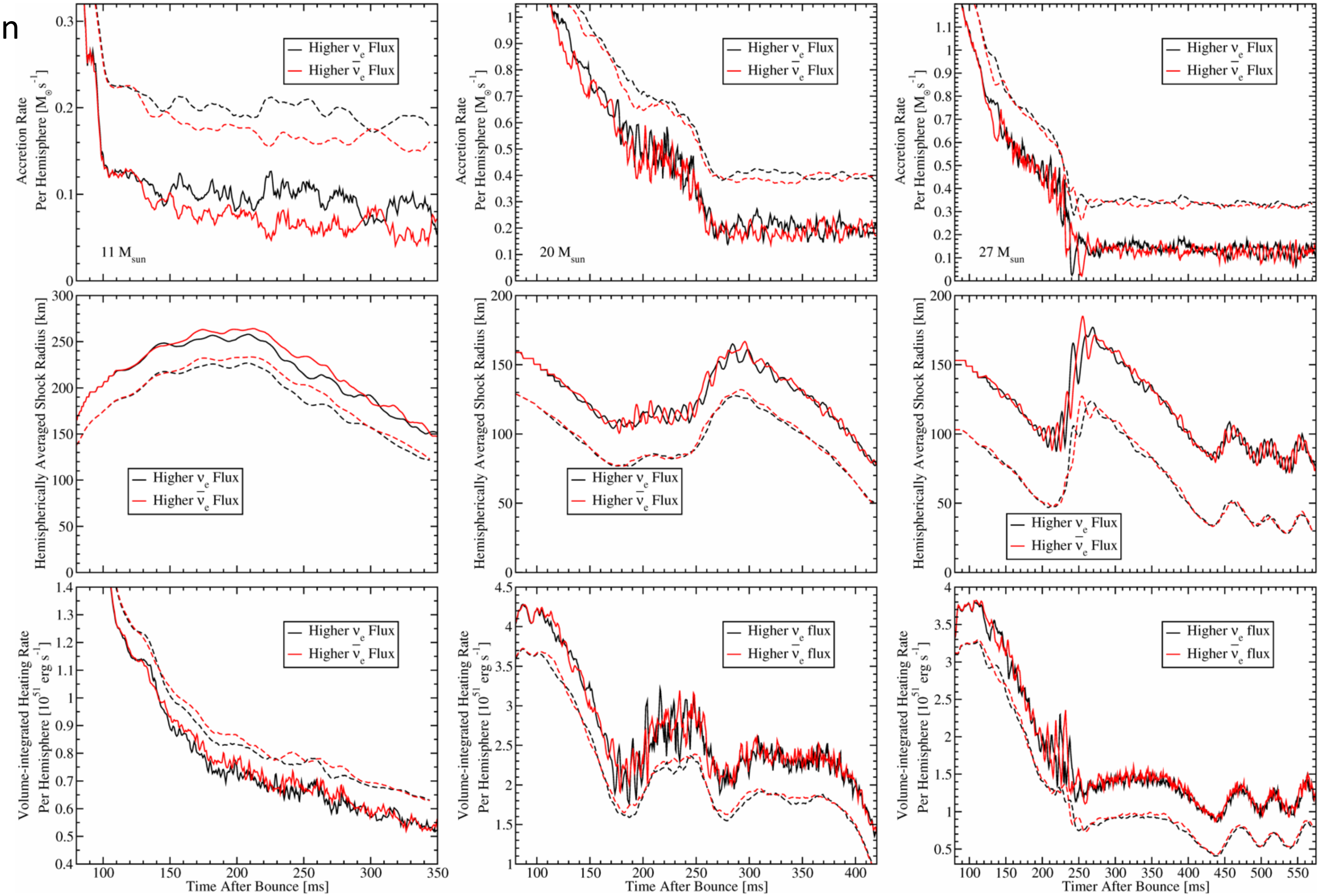
Asymmetries of Elements Relevant for LESA

$11.2\ M_{SUN}$ $20\ M_{SUN}$ $27\ M_{SUN}$

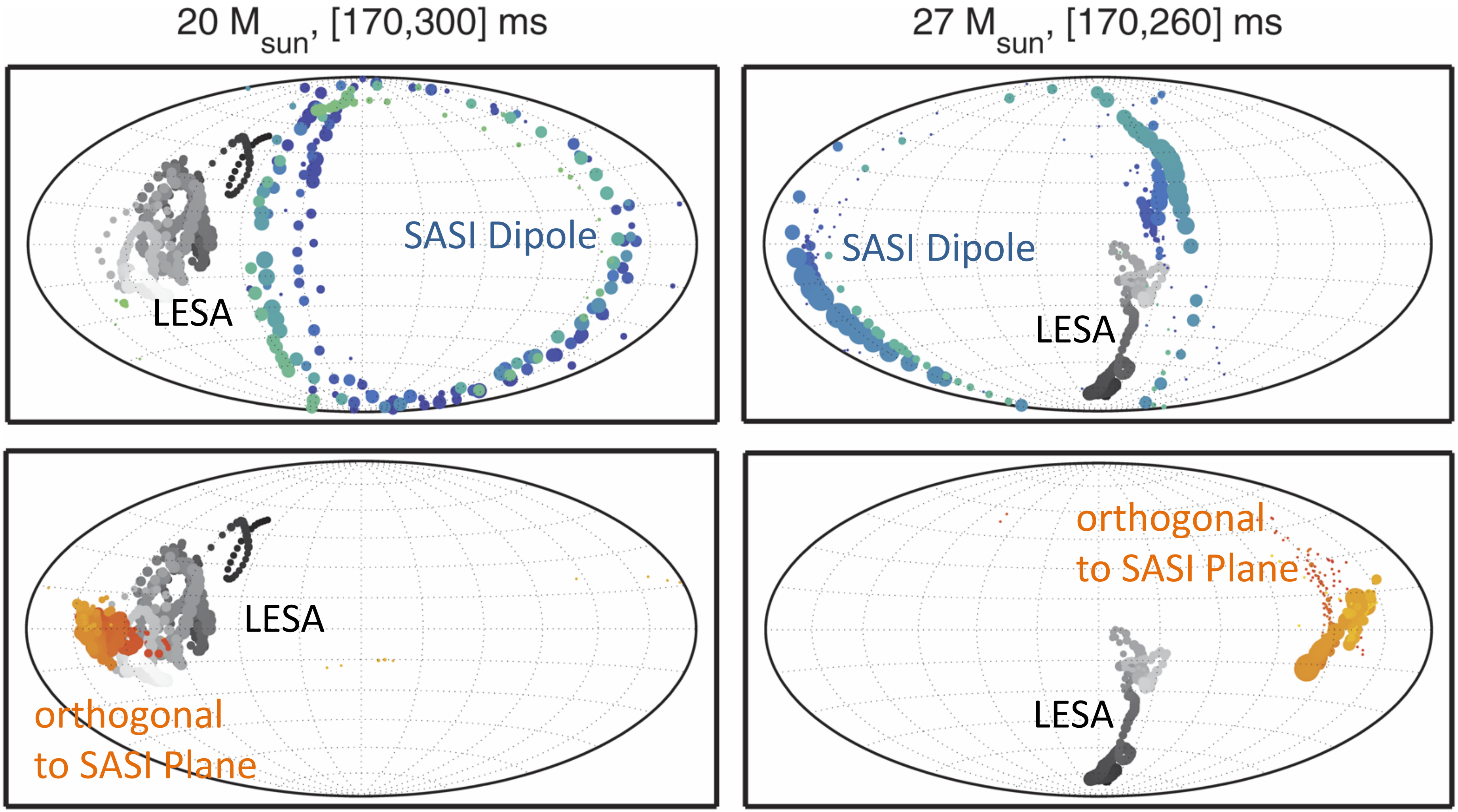
Accretion Rate

Shock Radius

Heating Rate



LESA vs. SASI Dipole Motions



No apparent directional correlation between SASI and LESA