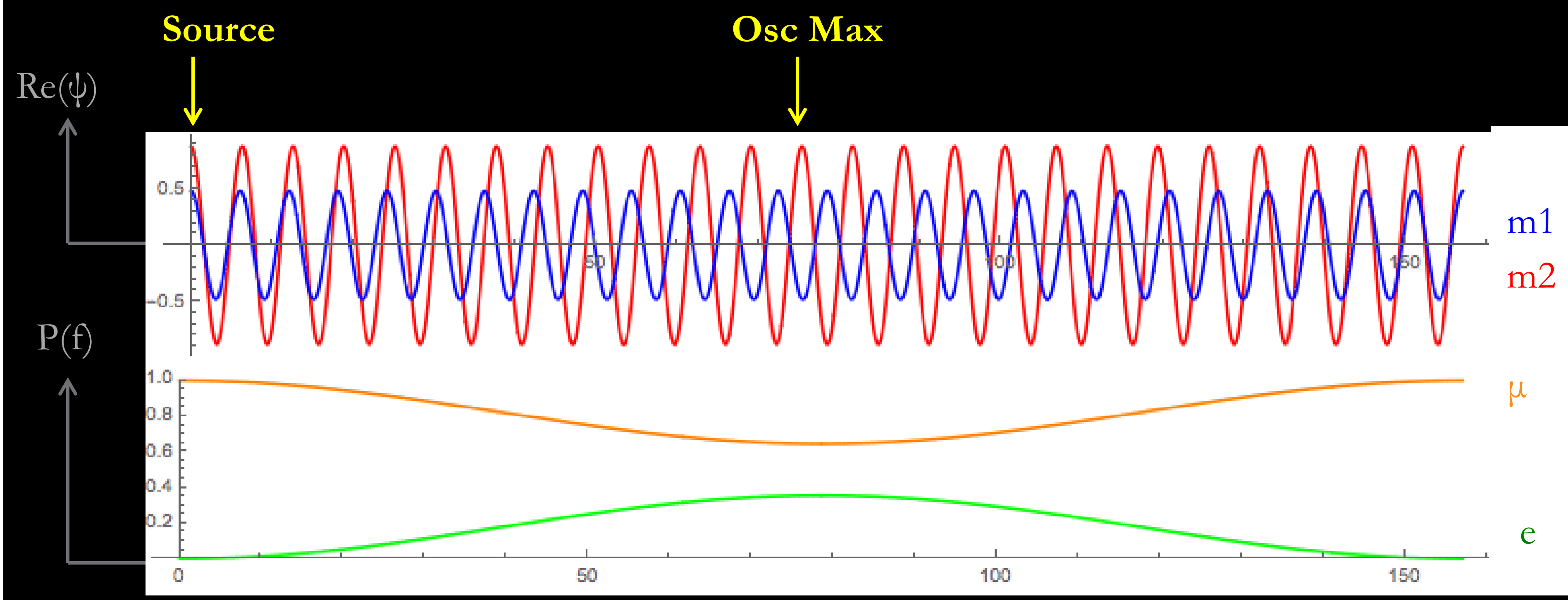
THE COHERENCE PROPERTIES OF NEUTRINO BEAMS

Phys.Rev. D91 (2015) 5, 053002 [B.J.P.J]

Ben Jones, MIT

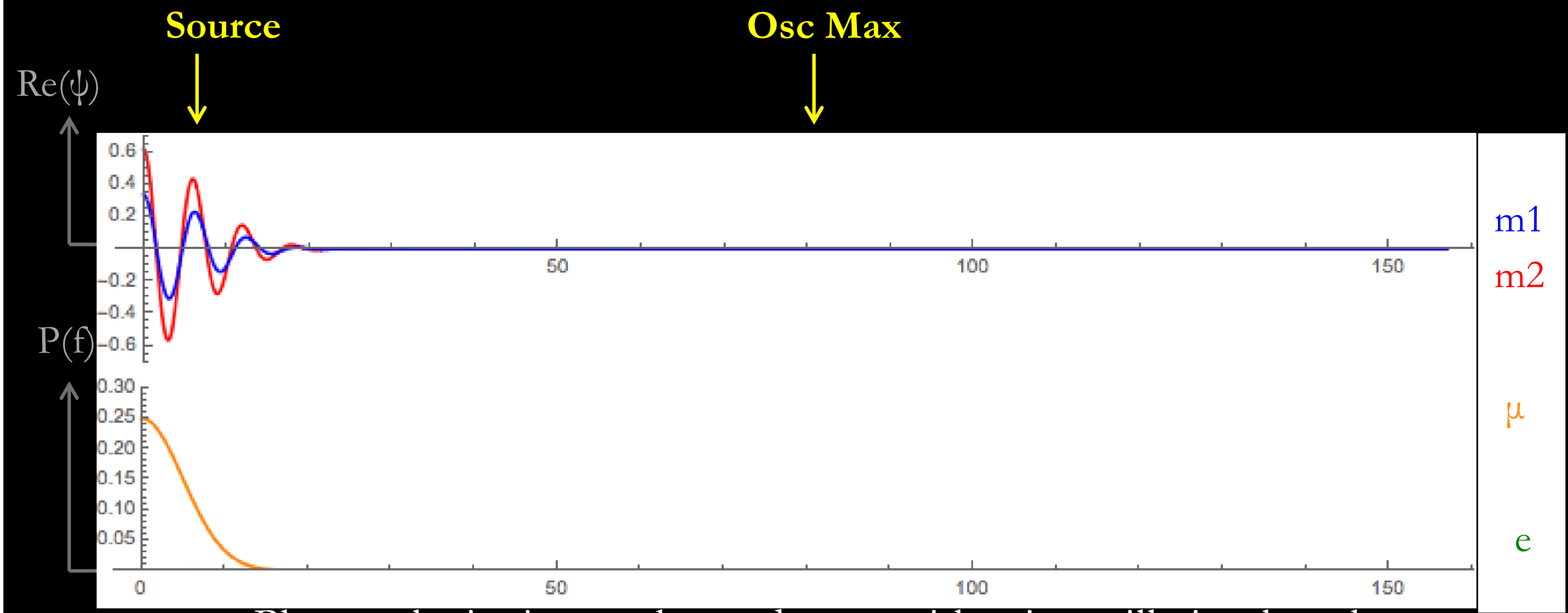
PLANE-WAVE PICTURE



PROPAGATION WITH WAVEPACKETS

No process can make a neutrino of perfectly defined momentum.

Source makes a wave-packet with some momentum and some position width



Phase velocity is not changed – so neither is oscillation length

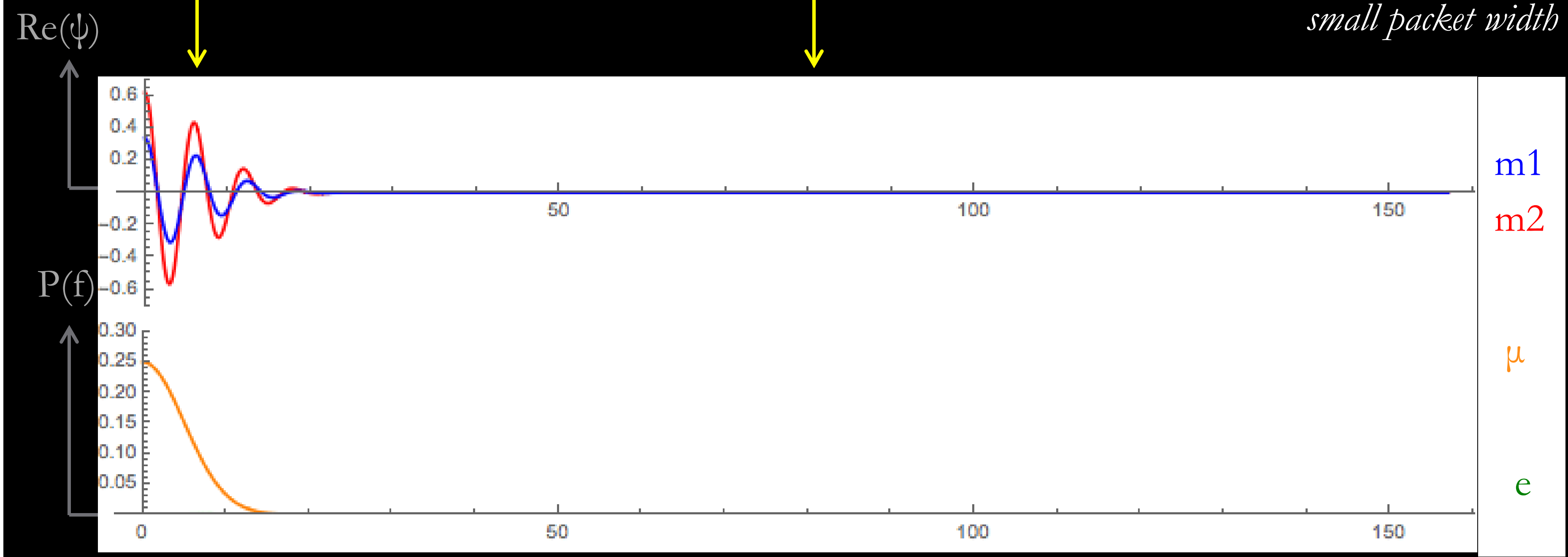
Wave-packet moves with group velocity of c for massless neutrinos

PROPAGATION WITH WAVEPACKETS

Wave-packets actually do separate. After separation, oscillation becomes incoherent (no L or E depedence)

Source **Osc Max**

Kicks in sooner with big Δm^2 or small packet width



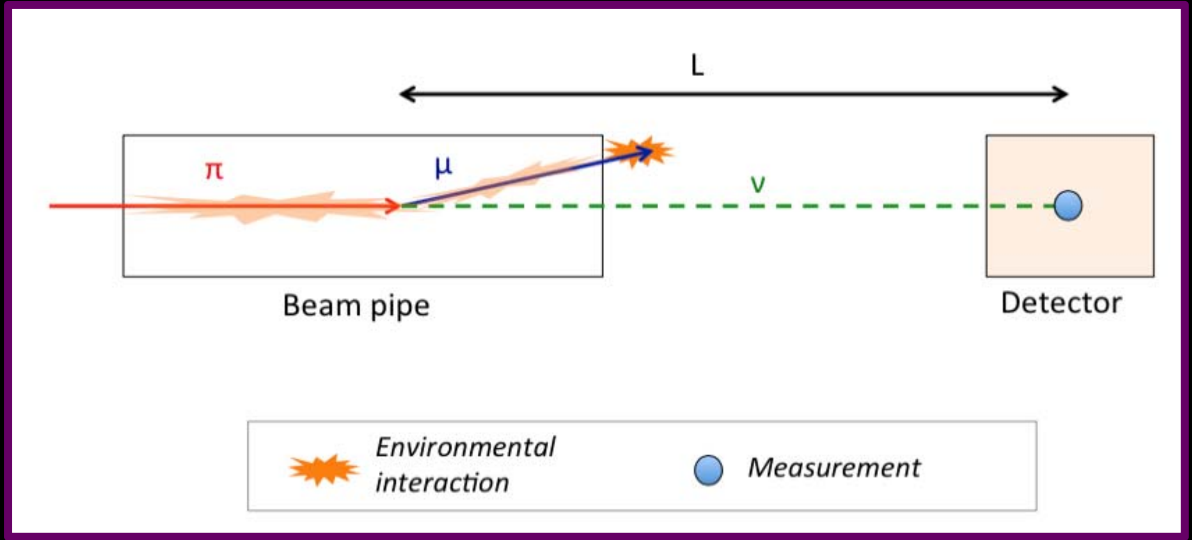
This is sometimes called "decoherence" but I'll call it "coherence loss"

IS IT IMPORTANT?

Could this affect sterile neutrino phenomenology?

To find out, We need to know the speed of separation (easy), and the wave-packet width (not so easy)

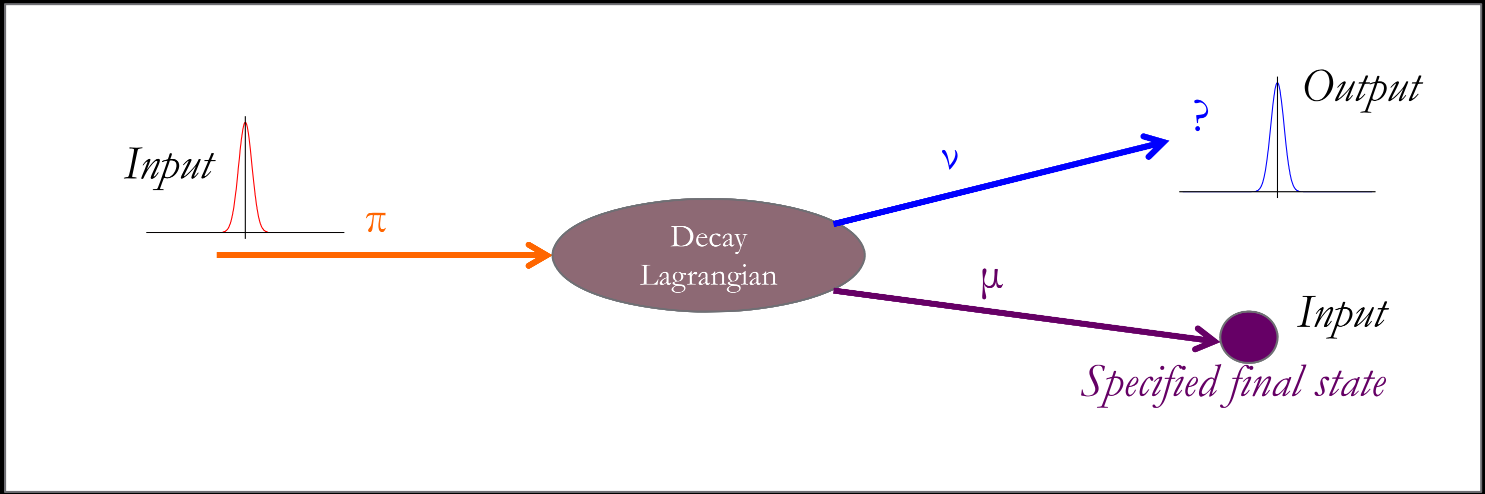
For the latter, we consider the production process.



EXTERNAL WAVE-PACKET PICTURE

Important progress was made by Beuthe, Akmedov +Smirnov :

They calculated neutrino state emerging from a pion of a specified width, alongside a specified detected muon



But …

now two unknown states, rather than one!

Beuthe arxiv:0109119

Akhmedov + Smirnov arxiv:0905.1903

EXTERNAL WAVE-PACKET PICTURE

Having done the calculation to go pion → neutrino,
to describe the incoming pion the traditional approach is to…

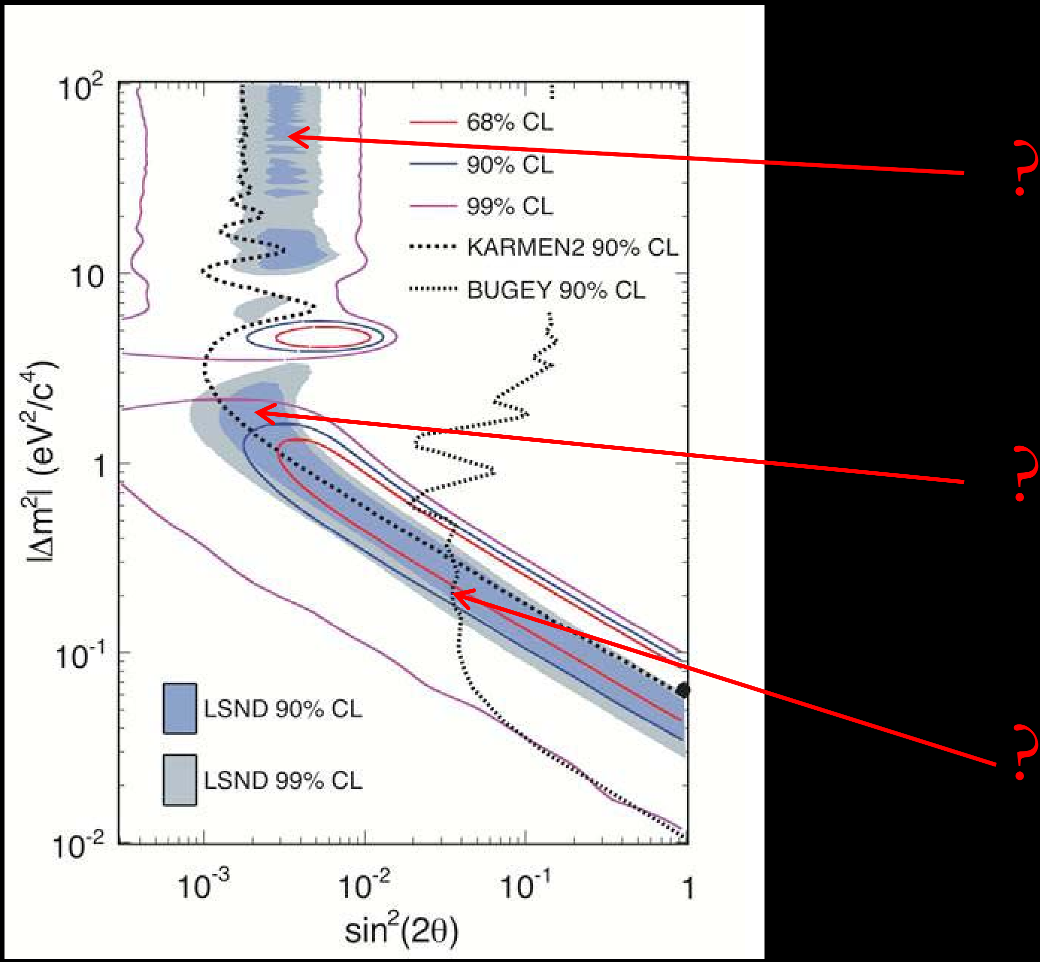
Wave our hands and make something up!

The width of the incoming pion wave-packet must be:

1. The inverse of its mass width
2. The mean-free path between collisions
3. Something to do with its form factor / physical size
4. The length of the decay pipe
5. Very small / big / … something?

For every complex problem, there is an answer that is clear, simple, elegant, and wrong (H.L.Mencken)

WHERE DOES IT HAPPEN?



The worrying thing is :

Putting aside sketchy guesses…

We haven't seen it for active neutrinos – that's all we know.

When should sterile neutrinos actually oscillate?

We have approx. no idea.

A RIGOROUS DERIVATION OF COHERENCE LENGTH?

The question of coherence length is a well-defined question with a real answer.

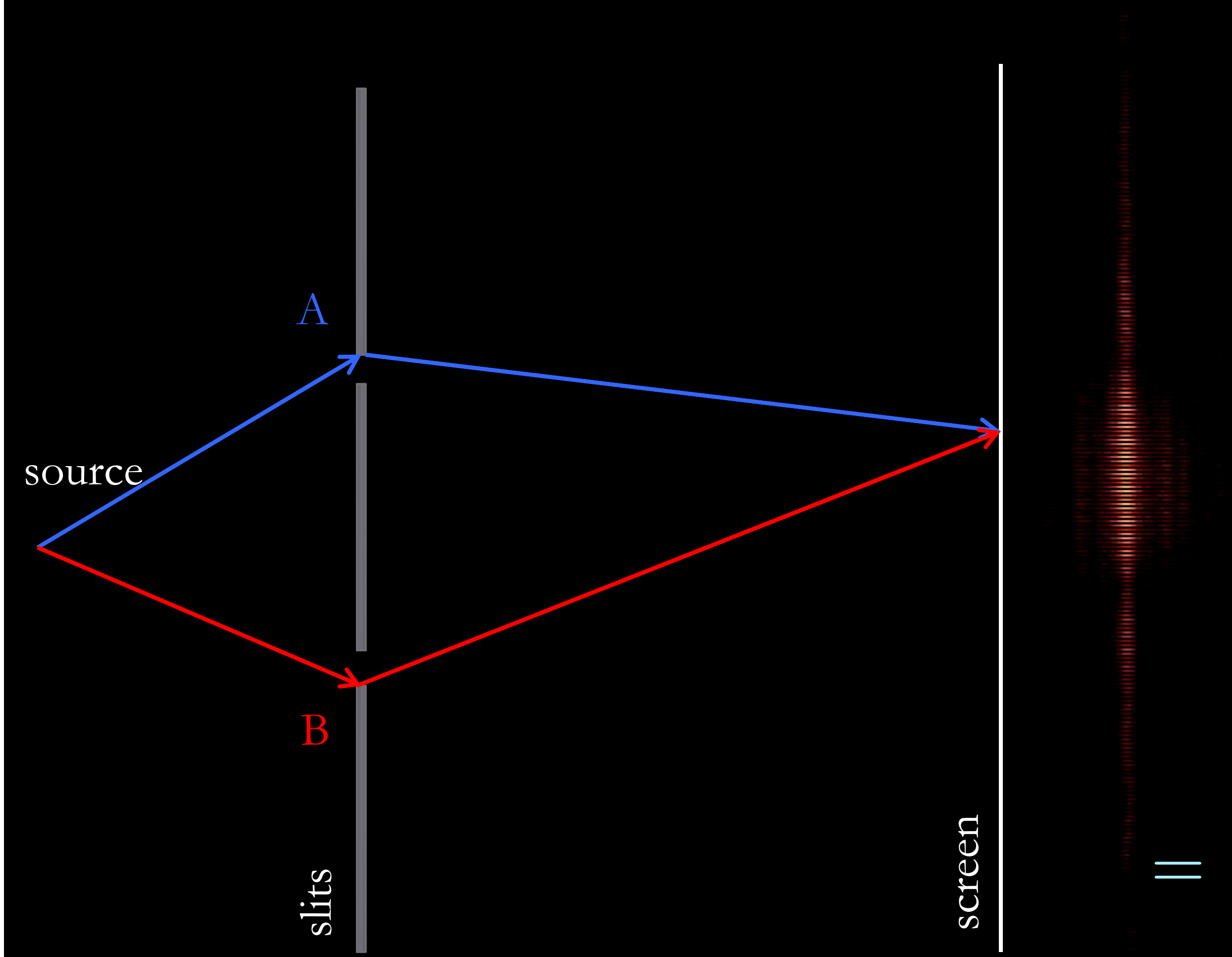
But the neutrino beam needs to be understood as an open quantum system in order to move forward.

Summary of the rest of the talk:

1) Re-formulating the problem in the right language
2) Deriving the equivalent of the pion width from its life-story
3) Calculating the coherence length for neutrino beams with different Δm^2

Resulting in a zero-free-parameter prediction of the coherence length, derived quantum mechanically.

DETOUR : ENVIRONMENTAL DECOHERENCE



Interference pattern:

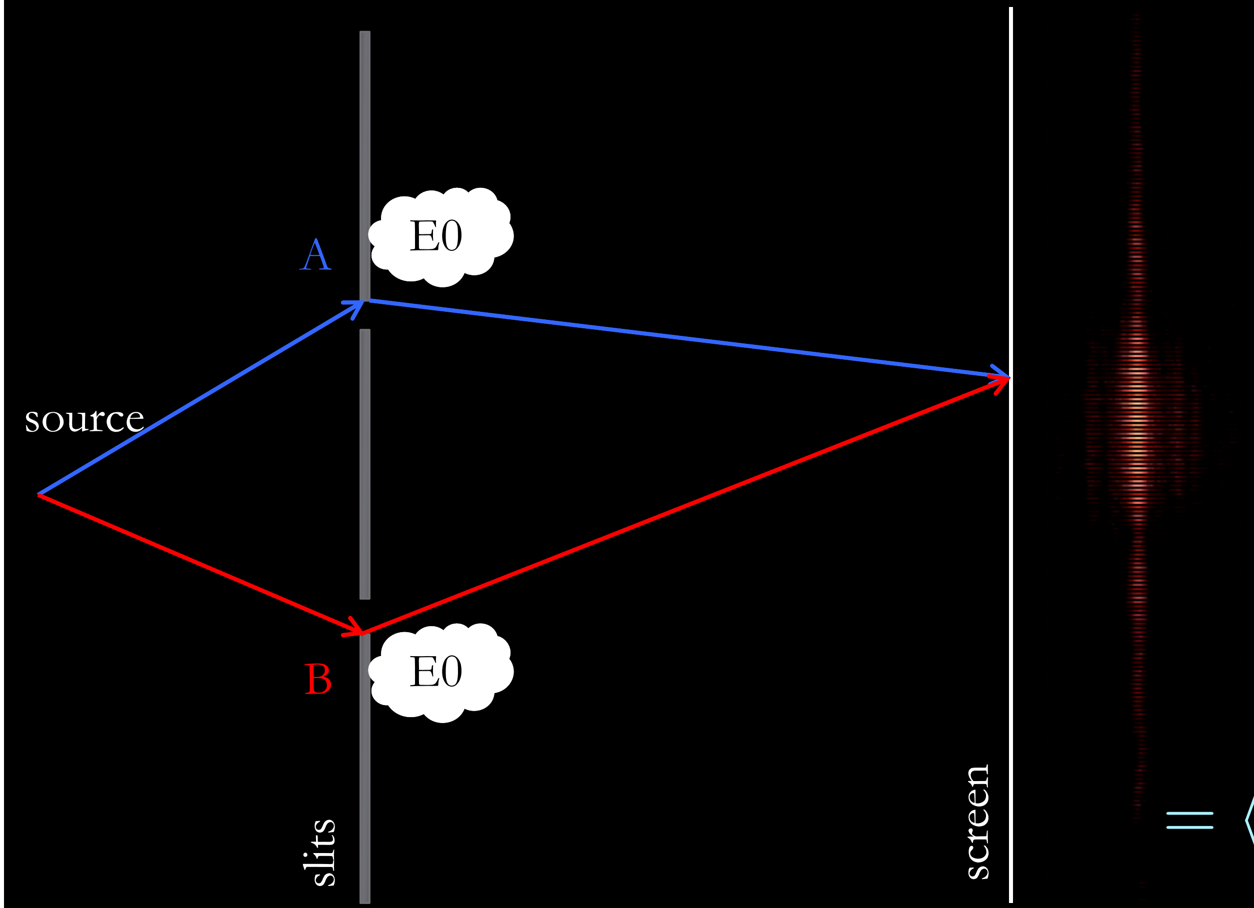
$$I = \langle \psi | \psi \rangle$$

$$= \langle \psi_A + \psi_B | \psi_A + \psi_B \rangle$$

$$= \langle \psi_A | \psi_A \rangle + \langle \psi_B | \psi_B \rangle + 2Re\langle \psi_A | \psi_B \rangle$$

YOUNGS TWO-SLIT EXPERIMENT

What if we add an environment?



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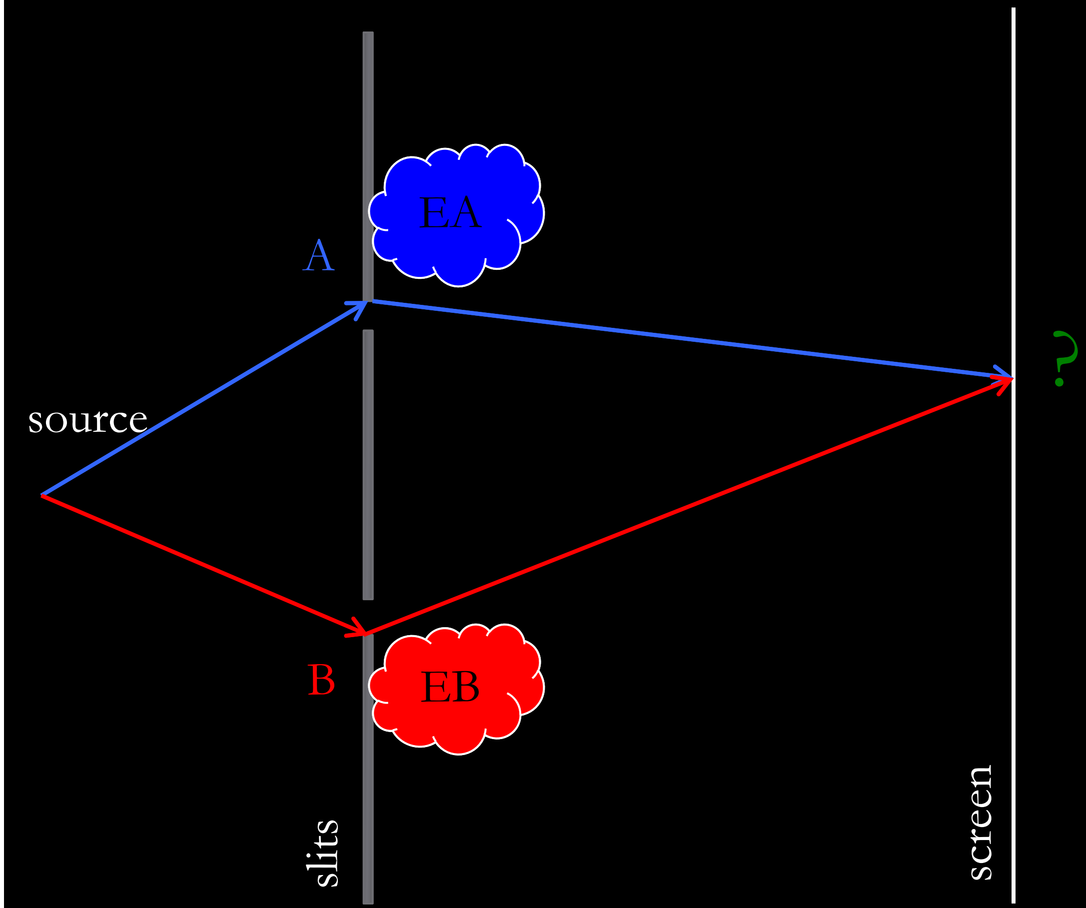
YOUNGS TWO-SLIT EXPERIMENT



Switch on the coupling.

The particle becomes entangled with the environment via its interactions

$$|\psi_A>|E_0>\to|\psi_A>|E_A>$$
$$|\psi_B>|E_0>\to|\psi_B>|E_B>$$

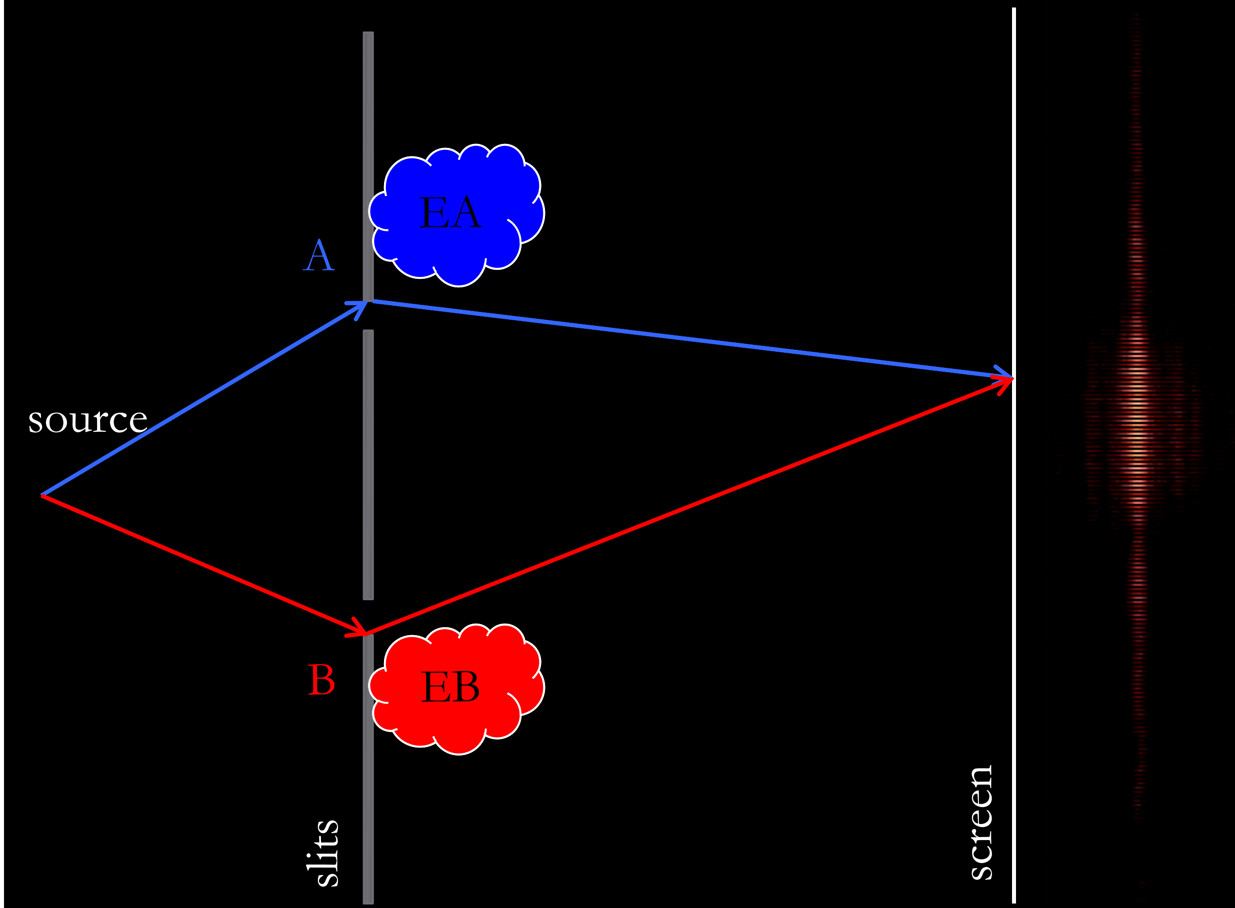
The interference pattern now depends on how much overlap there is between E_A, E_B

$$I=\langle\psi|\psi\rangle$$

$$=\langle\psi_A|\psi_A\rangle+\langle\psi_B|\psi_B\rangle+2Re\langle\psi_A|\psi_B\rangle\langle E_A|E_B\rangle$$

YOUNGS TWO-SLIT EXPERIMENT

Extreme cases :



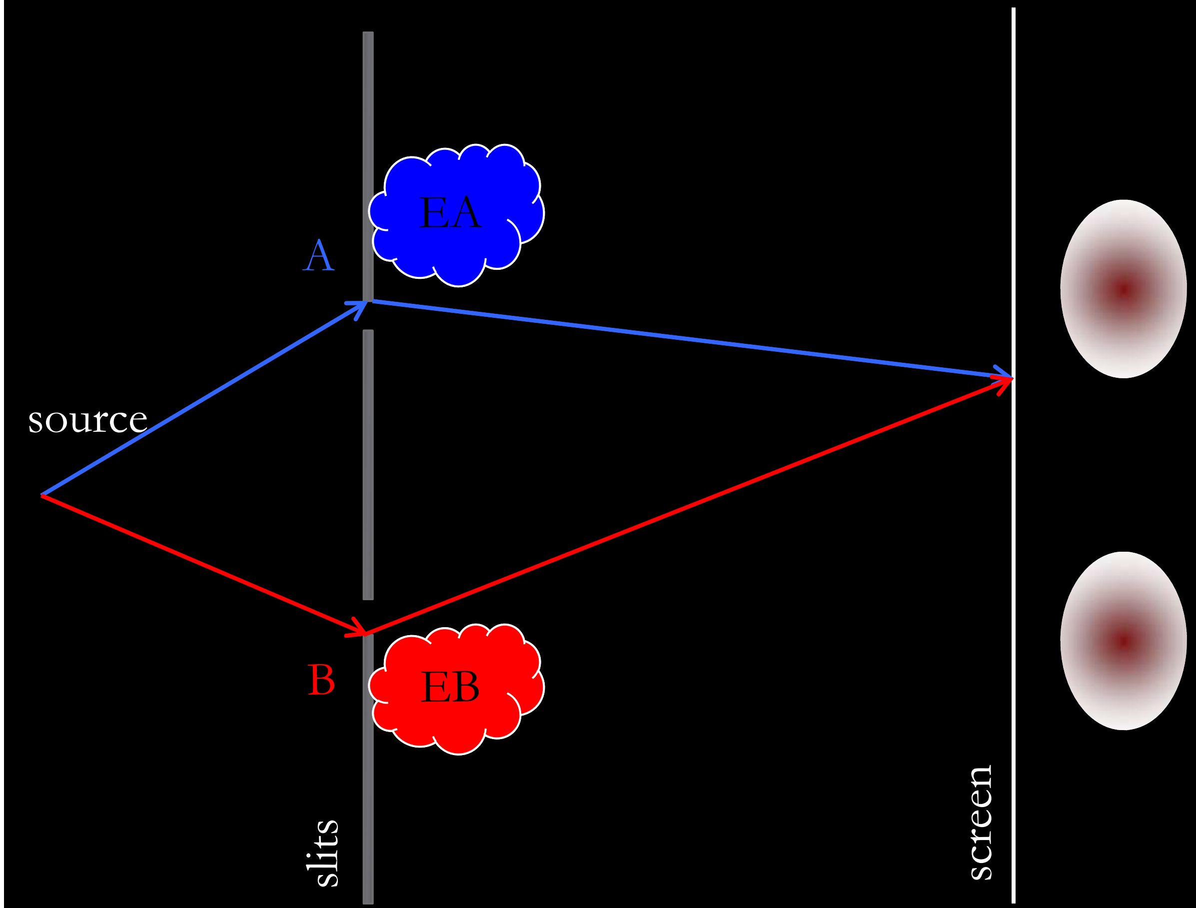
$$< E_A|E_B >= 1$$

Fully quantum-mechanical-looking particles

$$I = \langle \psi|\psi \rangle$$

$$= \langle \psi_A|\psi_A \rangle + \langle \psi_B|\psi_B \rangle + 2Re\langle \psi_A|\psi_B \rangle \langle E_A|E_B \rangle$$

YOUNGS TWO-SLIT EXPERIMENT



Extreme cases :

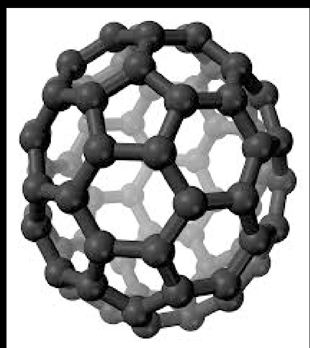
$$< E_A | E_B >= 0$$

Fully classical-looking particles

Diverging entanglements with the environment make quantum-looking systems into classical-looking ones

$$= \langle \psi_A | \psi_A \rangle + \langle \psi_B | \psi_B \rangle + 2Re \langle \psi_A | \psi_B \rangle \langle E_A | E_B \rangle$$

SEEING DECOHERENCE IN PRACTICE



e.g. Talbot Lau interferometry with C_{70} fullerenes

VOLUME 88, NUMBER 10 PHYSICAL REVIEW LETTERS 11 MARCH 2002

Matter-Wave Interferometer for Large Molecules

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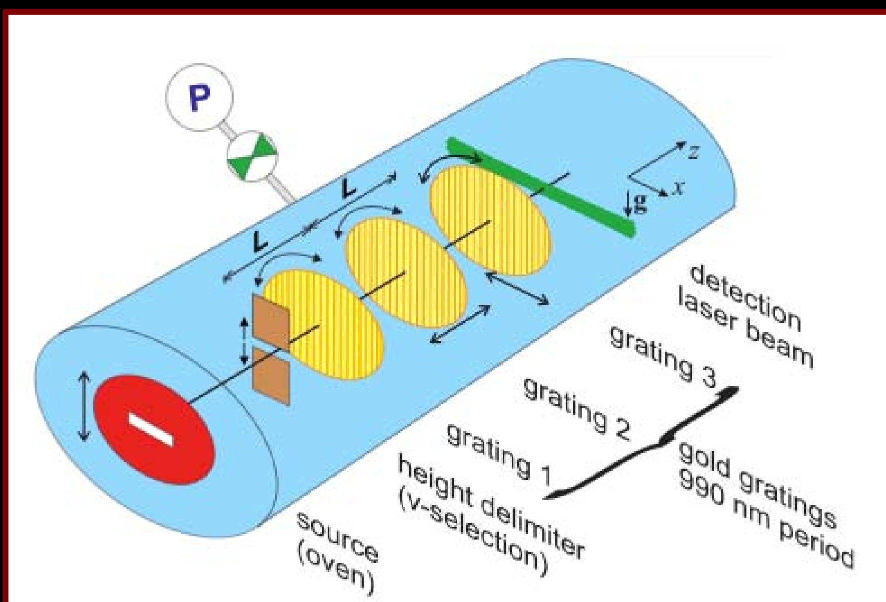
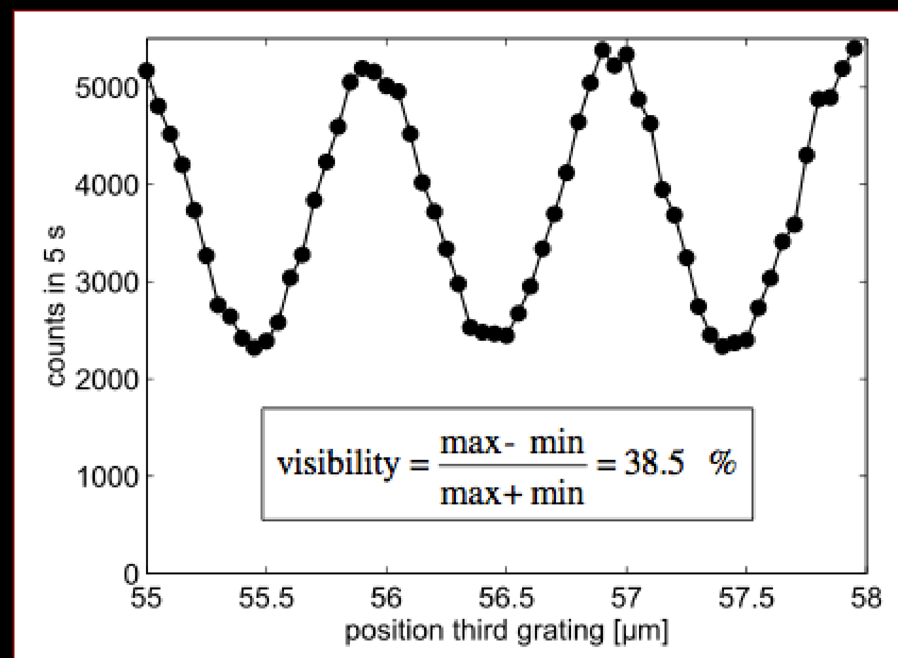
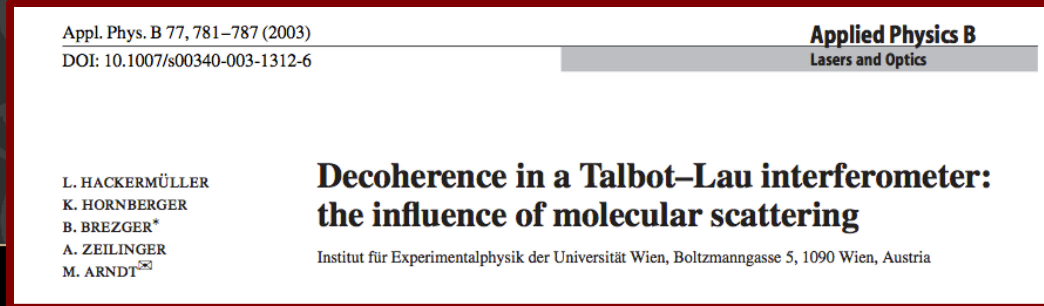
(Received 20 November 2001; published 26 February 2002)







Appl. Phys. B 77, 781–787 (2003)
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Applied Physics B
Lasers and Optics

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Decoherence in a Talbot–Lau interferometer: the influence of molecular scattering

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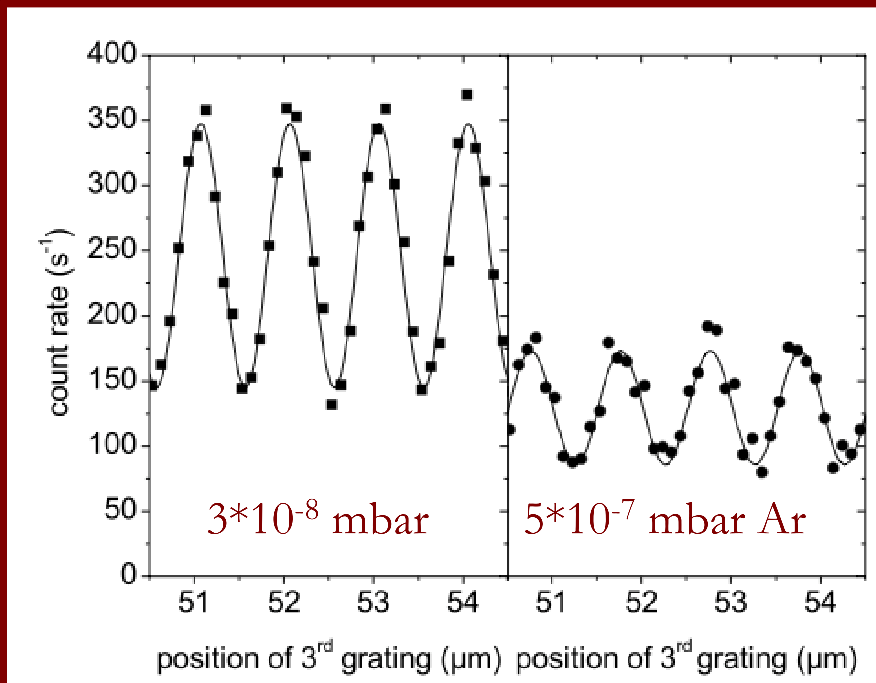
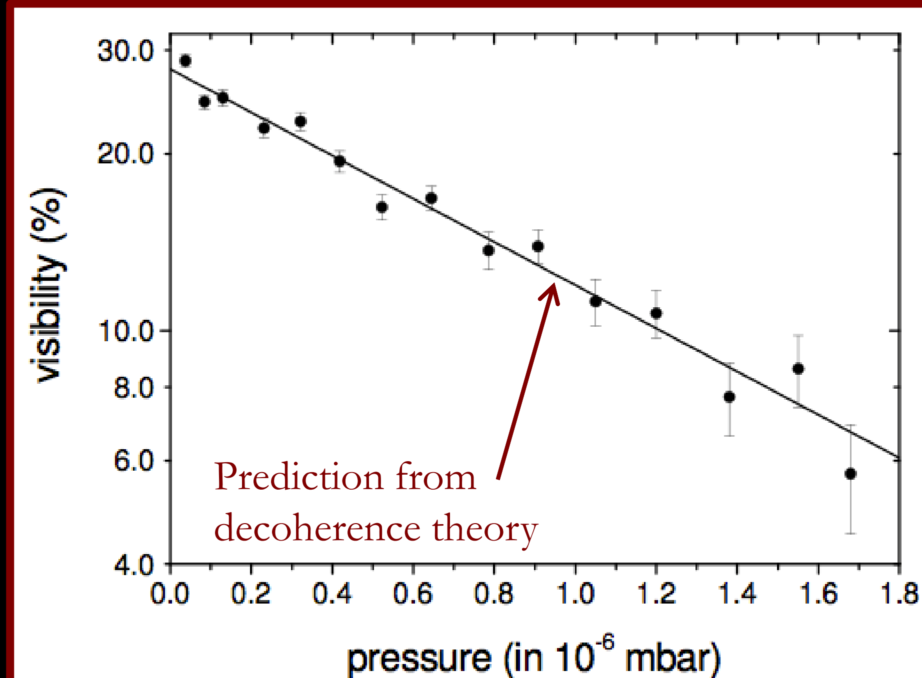
Environmental gasses are bled into the vacuum chamber. These cause scattering interactions.

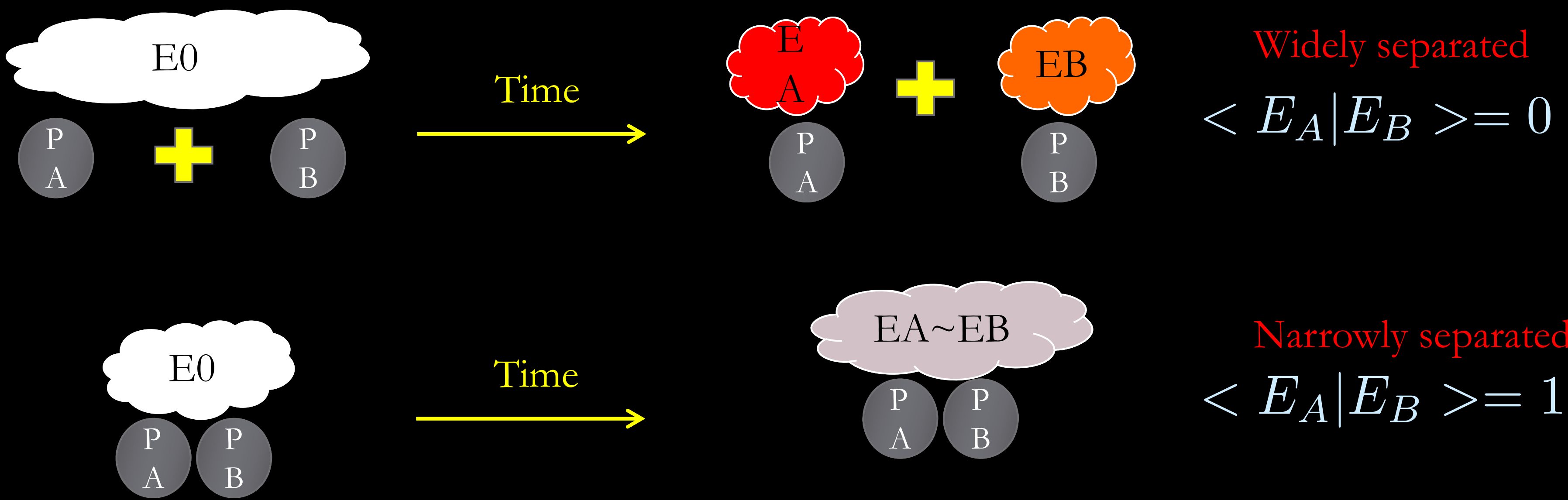
Entanglements generated with the environment encode "which way" information and suppress coherent superpositions.





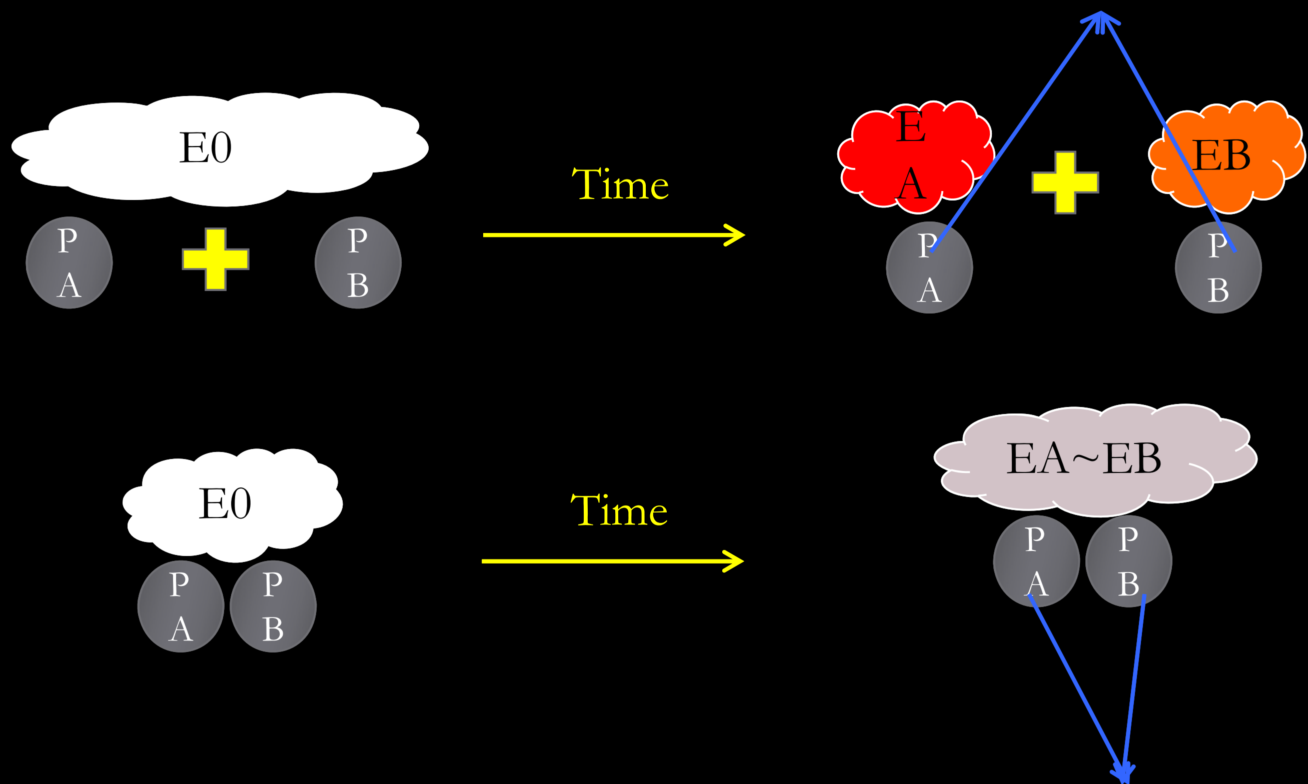
POSITION STATES OF A PARTICLE

Now consider a superposition of two particle position states, talking to an environment, which is "measuring" them with some resolution.



POSITION STATES OF A PARTICLE

$$= \langle \nu_A | \nu_A \rangle + \langle \nu_B | \nu_B \rangle + 2Re\langle \nu_A | \nu_B \rangle \langle E_A | E_B \rangle$$



Widely separated

$< E_A | E_B >= 0$

Narrowly separated

$< E_A | E_B >= 1$

"THE WAVEPACKET"

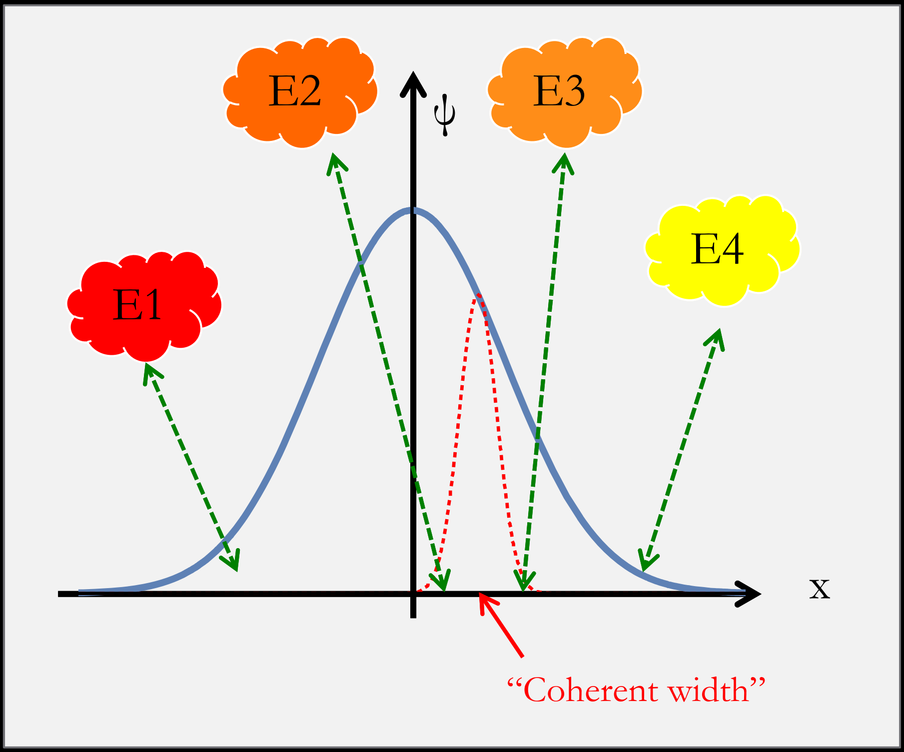
The "wave-packet" is a gross oversimplification of what is really happening.

The pion exists in a highly entangled quantum state with the environment

Each part of the pion wave-function entangles with a different environmental state

Some of those environmental states are so similar that the corresponding neutrinos can oscillate coherently

The width of that region plays the role of the "wave-packet" in the simplistic picture



With environmental entanglement the pion has both a total width and a coherent width

DENSITY MATRIX PICTURE OF PI DECAY

Start with some pion density matrix:

$$\rho_\pi(p_1, p_2)|p_1\rangle\langle p_2|$$

And let it decay:

$$\rho(p_{\nu 1}, p_{\mu 1}; p_{\nu 2}, p_{\mu 2})\ |p_{\nu 1}\rangle|p_{\mu 1}\rangle\langle p_{\nu 2}|\langle p_{\mu 2}|$$

REDUCING, TRACING, MEASURING….

Density matrix for entangled muon-neutrino system emerging from general pion state ϱ_π

$$\rho(t) = N^2 U_{\mu i} U^\dagger_{\mu j} \Theta_{ij}(t) |m_i\rangle\langle m_j|.$$

$$\Theta_{ij}(t) = \int dp_1 dp_2 \rho_\pi(p_1, p_2) e^{i(E_\pi(p_1) - E_\pi(p_2))t} \left(|p^i_\nu(p_1)\rangle\langle p^j_\nu(p_2)|\right)_\nu \left(|p^i_\mu(p_1)\rangle\langle p^j_\mu(p_2)|\right)_\mu .$$

Tracing out the muon and apply a flavor measurement operator at baseline L:

$$P(\nu_\alpha, L) = U_{\alpha j} U^\dagger_{\alpha i} U_{\mu i} U^\dagger_{\mu j} \int dp \rho_\pi(p - \delta^{ij}_\mu, p + \delta^{ij}_\mu) e^{-i\phi_{ij}}$$

$$\phi_{ij} = \frac{m^2_{ij} p}{M^2} \left(\frac{E}{p} L - t \right) . \qquad (6)$$

THE GENERALIZED OSCILLATION PROBABILITY

$$P(\nu_\alpha, L) = U_{\alpha j} U^\dagger_{\alpha i} U_{\mu i} U^\dagger_{\mu j} \int dp \rho_\pi (p - \delta^{ij}_\mu, p + \delta^{ij}_\mu) e^{-i\phi_{ij}}$$

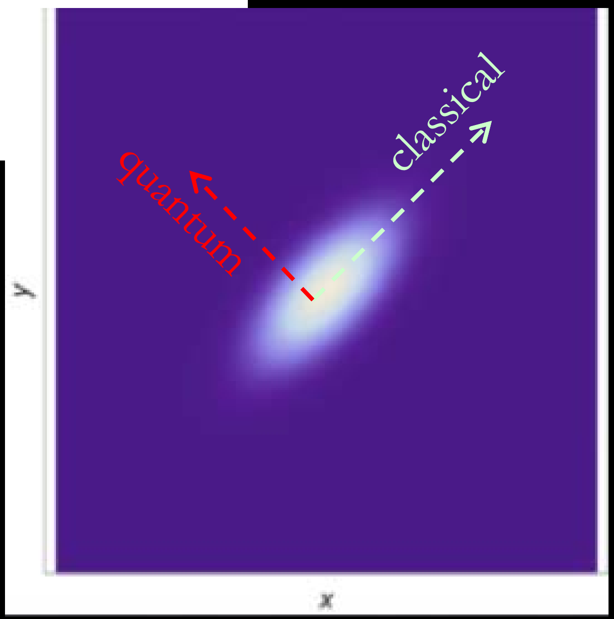
$$\phi_{ij} = \frac{m^2_{ij} p}{M^2} \left(\frac{E}{p} L - t \right). \qquad (6)$$

THE GENERALIZED OSCILLATION PROBABILITY

What about this? Initial pion state…

$$P(\nu_\alpha, L) = U_{\alpha j} U^\dagger_{\alpha i} U_{\mu i} U^\dagger_{\mu j} \int dp \rho_\pi (p - \delta^{ij}_\mu, p + \delta^{ij}_\mu) e^{-i\phi_{ij}}$$

$$\phi_{ij} = \frac{m^2_{ij} p}{M^2} \left(\frac{E}{p} L - t \right).$$



We should now put in whatever initial pion state we have in the experiment.

It will have some diagonal (classical) and some off-diagonal (coherent) width.

Lets take a 2D Gaussian ansatz to see how it behaves.

KINEMATICS

Oscillations require i≠j terms to be non-zero.

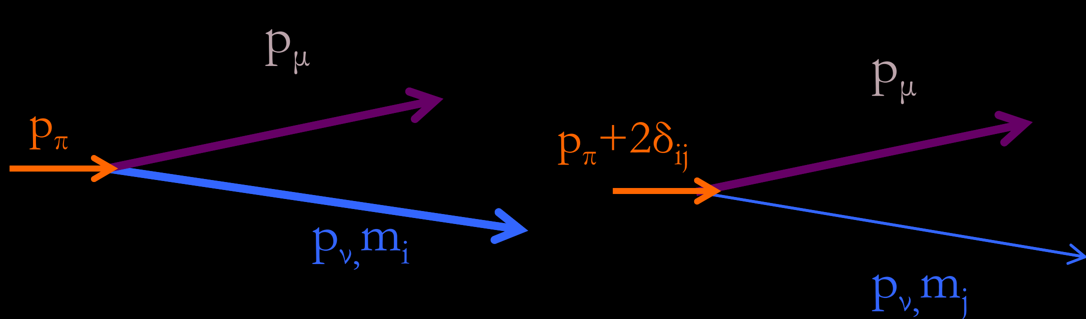
Can only happen if the pion density matrix is coherently wide enough (δ):

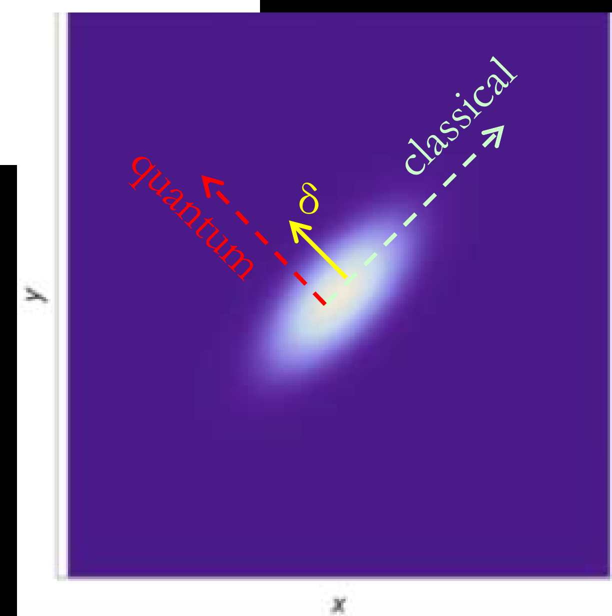
And whats this thing?

$$P(\nu_\alpha, L) = U_{\alpha j} U^\dagger_{\alpha i} U_{\mu i} U^\dagger_{\mu j} \int dp \rho_\pi (p - \delta^{ij}_\mu, p + \delta^{ij}_\mu) e^{-i\phi_{ij}}$$

$$\phi_{ij} = \frac{m^2_{ij} p}{M^2} \left(\frac{E}{p} L - t \right).$$

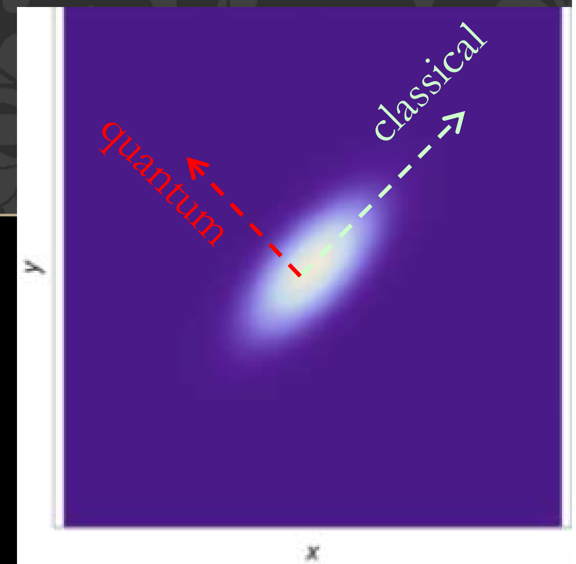
How should we understand δ?





THE OSCILLATION PROBABILITY

Substitute in a representative pion density matrix with Gaussian on and off diagonal widths:



$$P(\nu_\alpha, L) = N U_{\alpha j} U^\dagger_{\alpha i} U_{\mu i} U^\dagger_{\mu j} \left[e^{-i \frac{m_{ij}^2}{2p_0} \frac{m_\pi^2}{m_\pi^2 - m_\mu^2} L} \right] \left[e^{-\frac{p_0^2 + m_\pi^2}{8\left(m_\pi^2 - m_\mu^2\right)^2} \left(m_{ij}^2\right)^2 \sigma_{diag}^2} \right] \left[e^{-\left(\frac{m_{ij}^2}{2p_0^2} \frac{m_\pi^2}{m_\pi^2 - m_\mu^2}\right)^2 \left(\frac{L^2}{2\sigma_{od}^2}\right)} \right]$$

Standard oscillation

Classical coherence condition

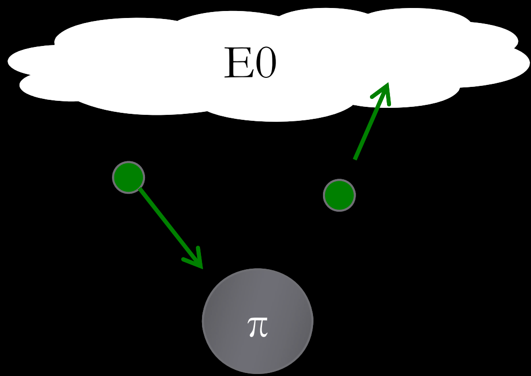
Know location of source to within 1 osc length

Quantum coherence condition

Need WP's not to separate

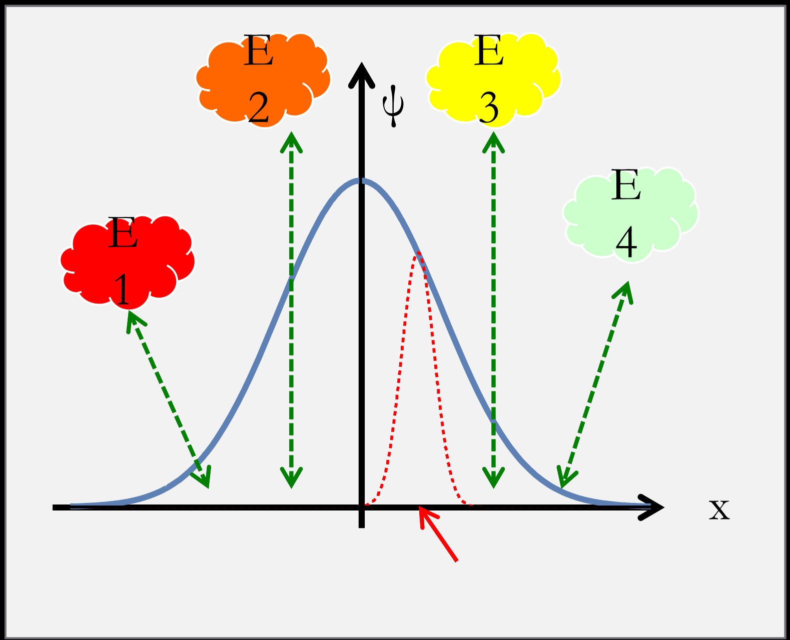
WHICH BRINGS US BACK TO…

What is the coherent pion with?



Repeated bombardment of scatterers encodes information about the pion into the environment.

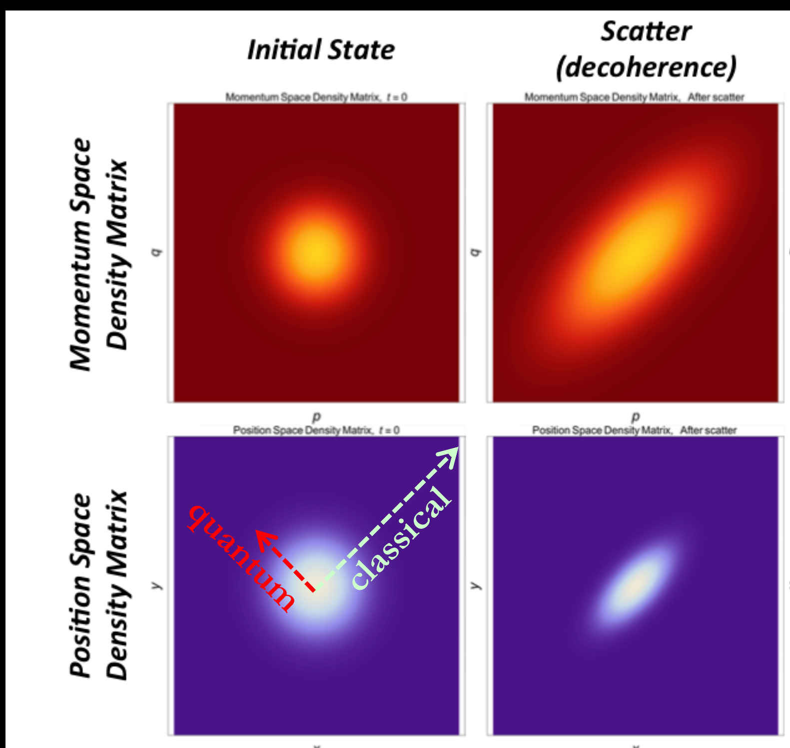
How to describe this hugely entangled state?



With environmental entanglement the pion has both a total width and a coherent width

OPEN QUANTUM SYSTEMS

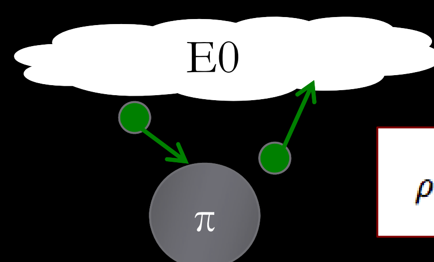
Under some loose assumptions, we can describe what happens to the pion reduced density matrix without considering the full system DM



Foundations of Physics Letters, Vol. 6, No. 6, p. 571-590 (1993)

APPARENT WAVE FUNCTION COLLAPSE CAUSED BY SCATTERING

Max Tegmark

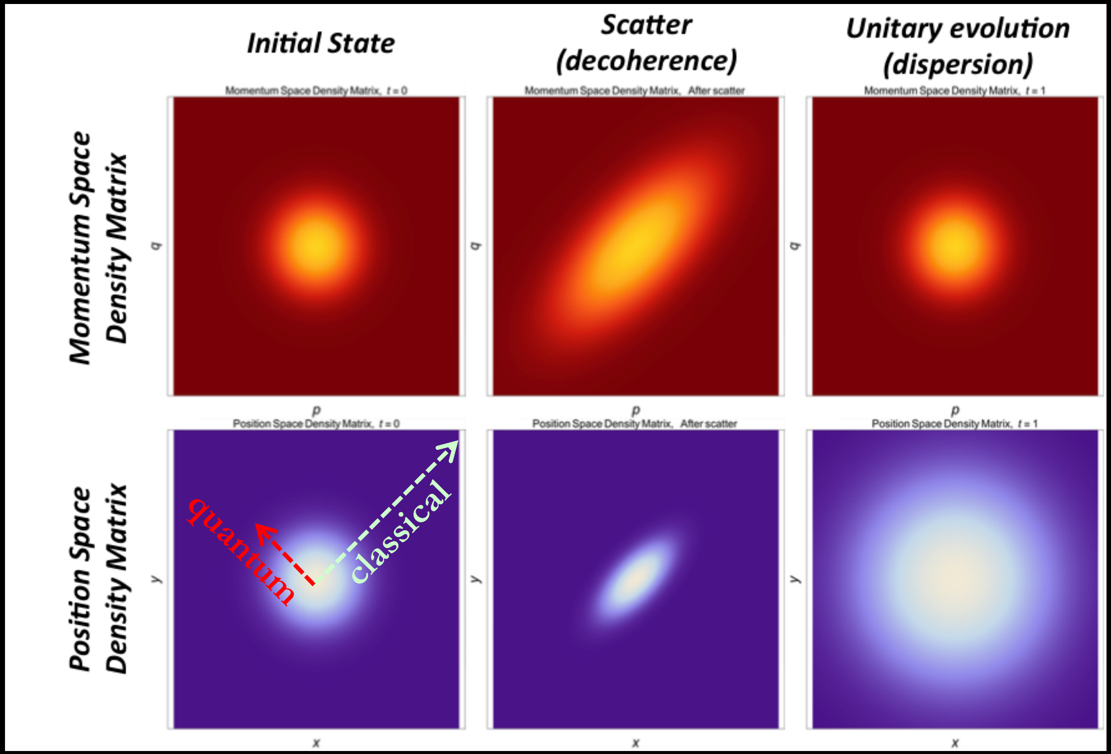


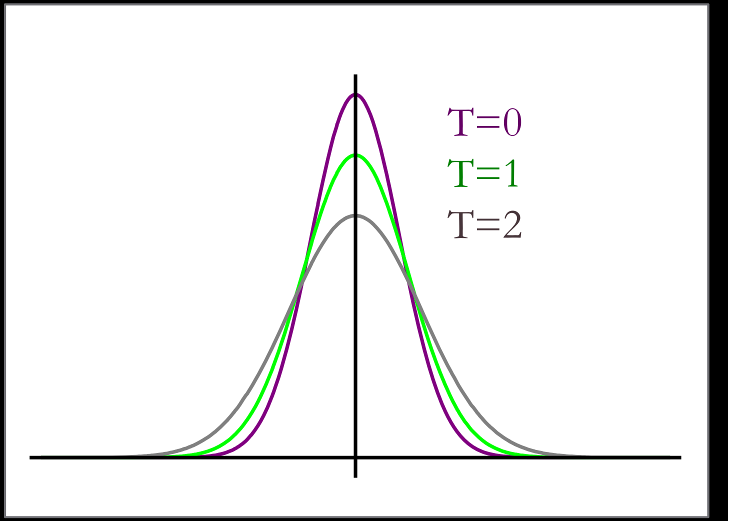
$$\rho_\pi(x_1, x_2) \to \rho_\pi(x_1, x_2)\widetilde{P_q}(x_2 - x_1),$$

FT of mom transfer prob dist

OPEN QUANTUM SYSTEMS

Between scatters the state evolves according to the free Schrodinger equation
This leads to dispersion (broadening) of both coherent and incoherent widths





COHERENCE LENGTHS

Competition of unitary evolution and collapse define a stable coherent width:

Cause of apparent wave function collapse	Free electron	$10\mu m$ dust	Bowling ball
300K air at 1 atm pressure	10^{-6} m	10^{-17} m	10^{-21} m
300K air in lab vacuum	10^{7} m	10^{-13} m	10^{-18} m
Sunlight on earth	10^{9} m	10^{-12} m	10^{-17} m
300K photons	10^{4} m	10^{-12} m	10^{-16} m
Background radioactivity	n/a	10^{-11} m	10^{-15} m
Quantum gravity	10^{4} m	10^{-9} m	10^{-15} m
GRW effect	10^{19} m	10^{-9} m	10^{-15} m
Cosmic microwave background	10^{10} m	10^{-8} m	10^{-14} m
Solar neutrinos	n/a	n/a	10^{-13} m

Tegmark, arxiv:9310032

To calculate this width for a relativistic pion, we need:

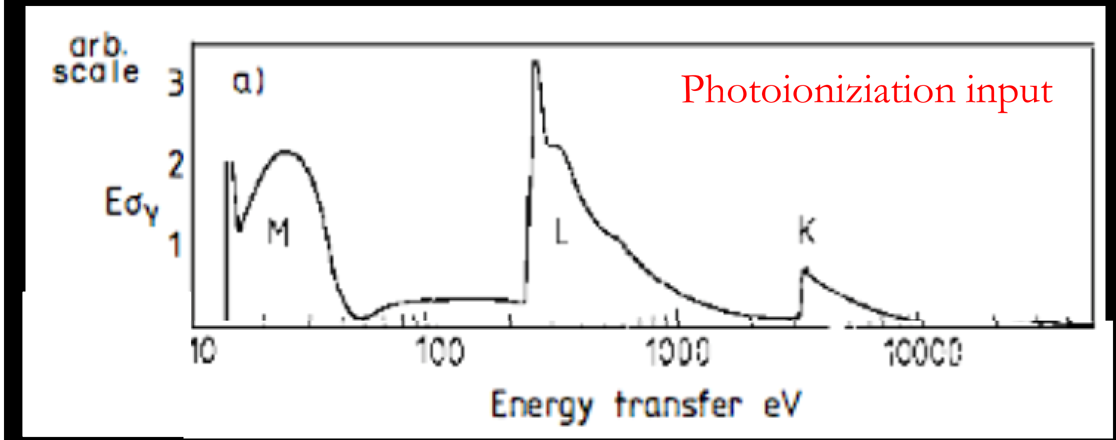
1) Scattering momentum transfer probability distribution + rate
2) Method of calculating convergent width in a relativistic system

MOMENTUM TRANSFERS

Semi-classical model – consider energy losses in a continuum with some complex refractive index and re-interpret in terms of photon exchanges with electrons

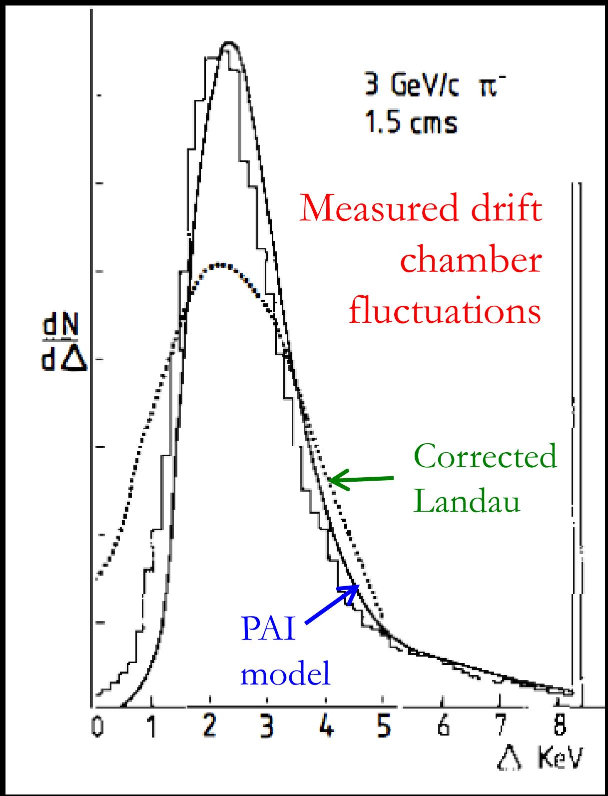
Derived from photoionization cross section, in a somewhat complicated way

We need photoionization spectrum of beam-pipe gas as input:

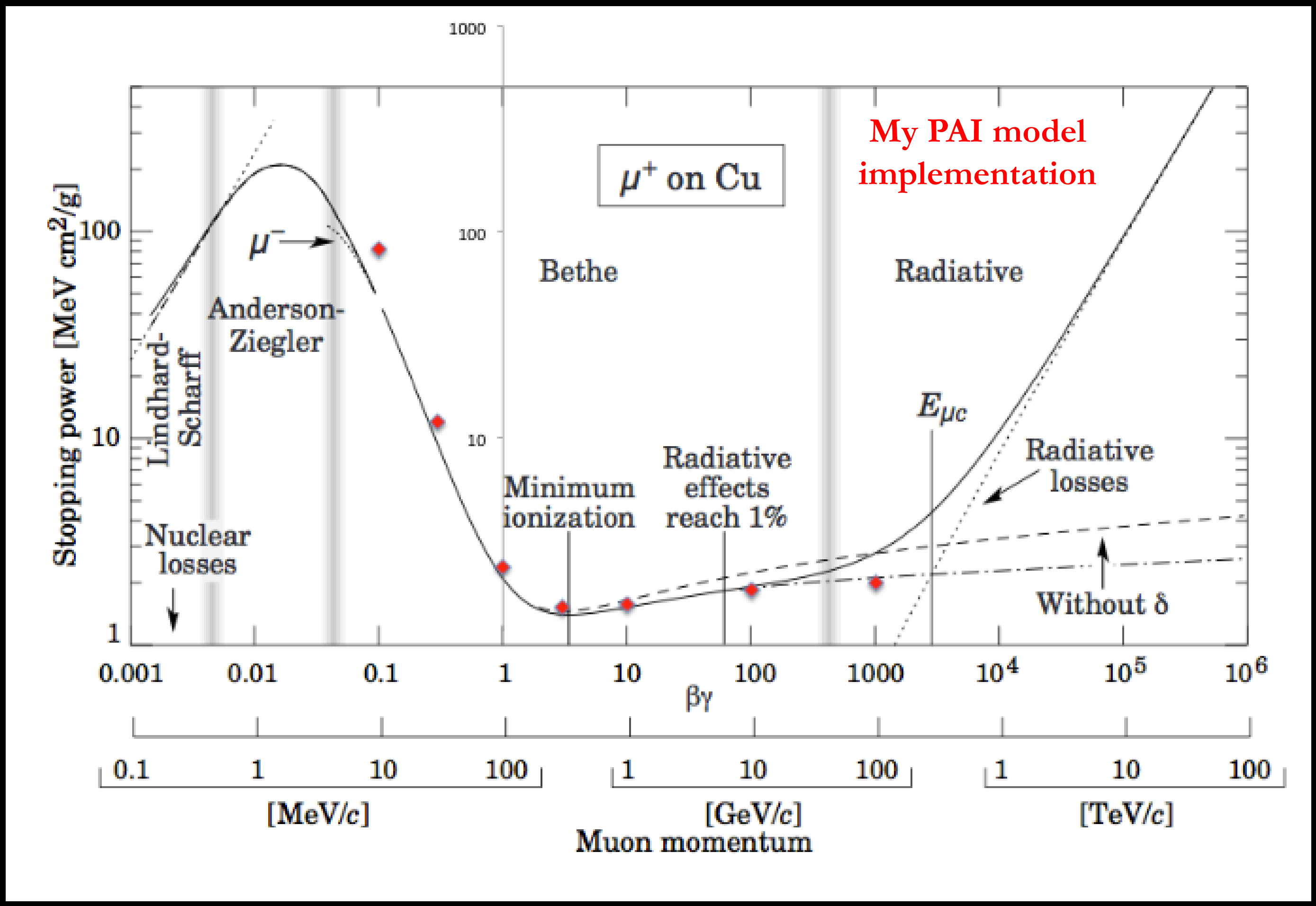


PAI Model:

Allison and Cobb, Ann.Rev.Nucl.Part.Sci 1980. 30:253-98

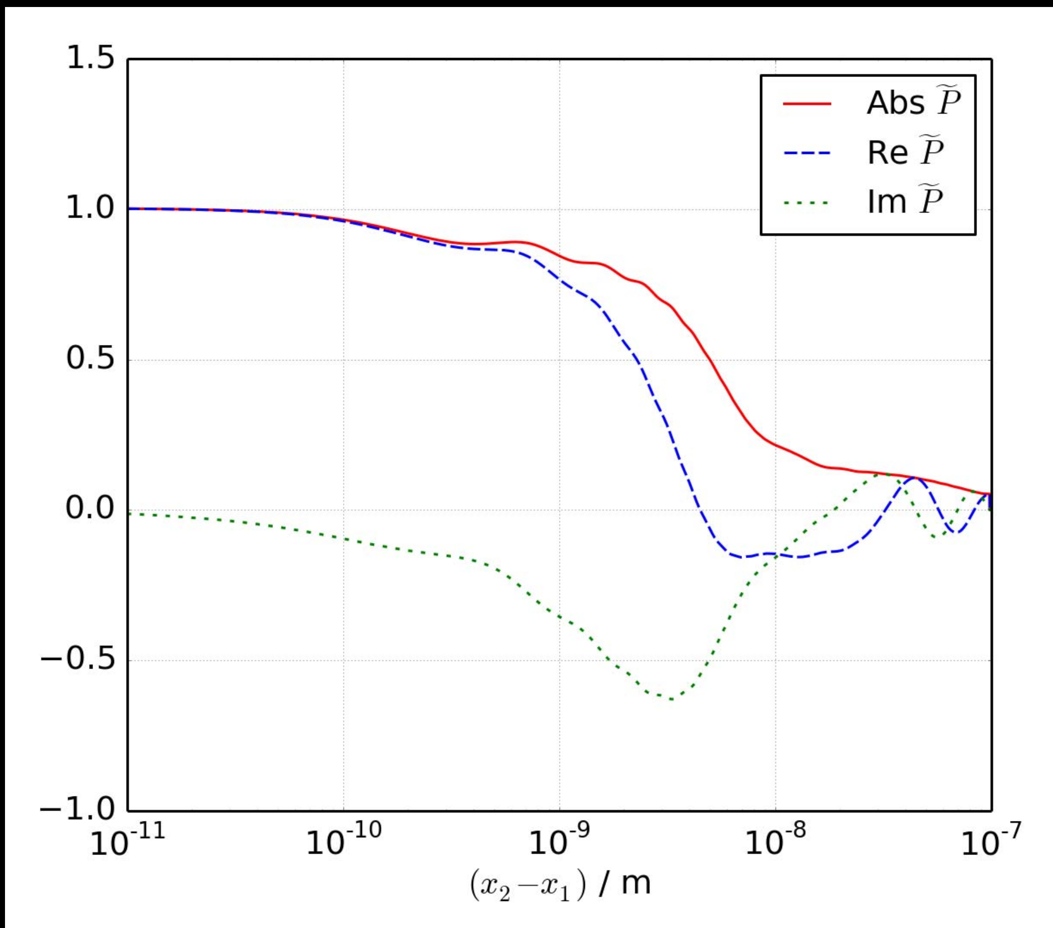


PAI SANITY CHECK

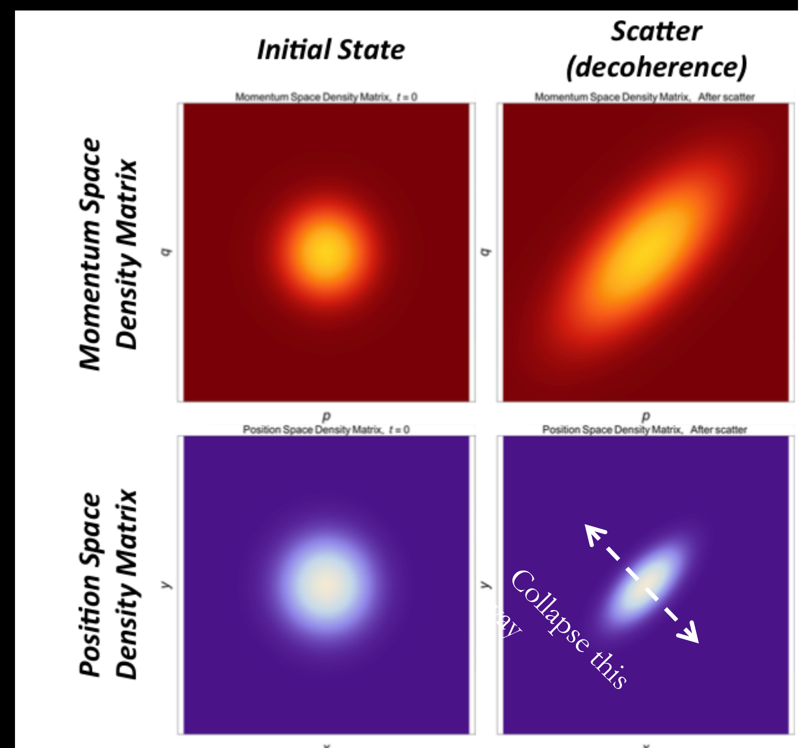


← Standard plot of energy losses from the PDG

THE DECOHERENCE FUNCTION



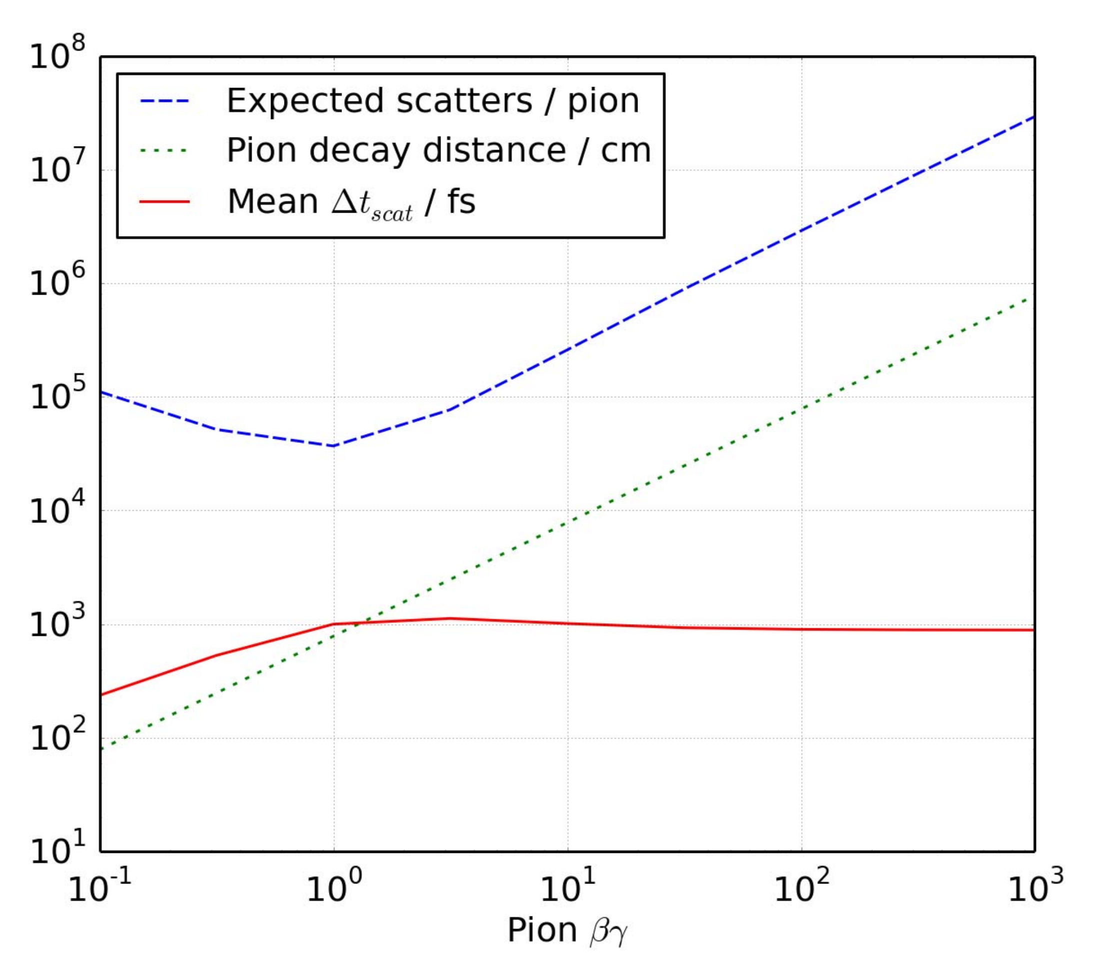
$$\rho_\pi(x_1, x_2) \to \rho_\pi(x_1, x_2)\widetilde{P_q}(x_2 - x_1),$$



SCATTERING RATE

Predicted by PAI model.

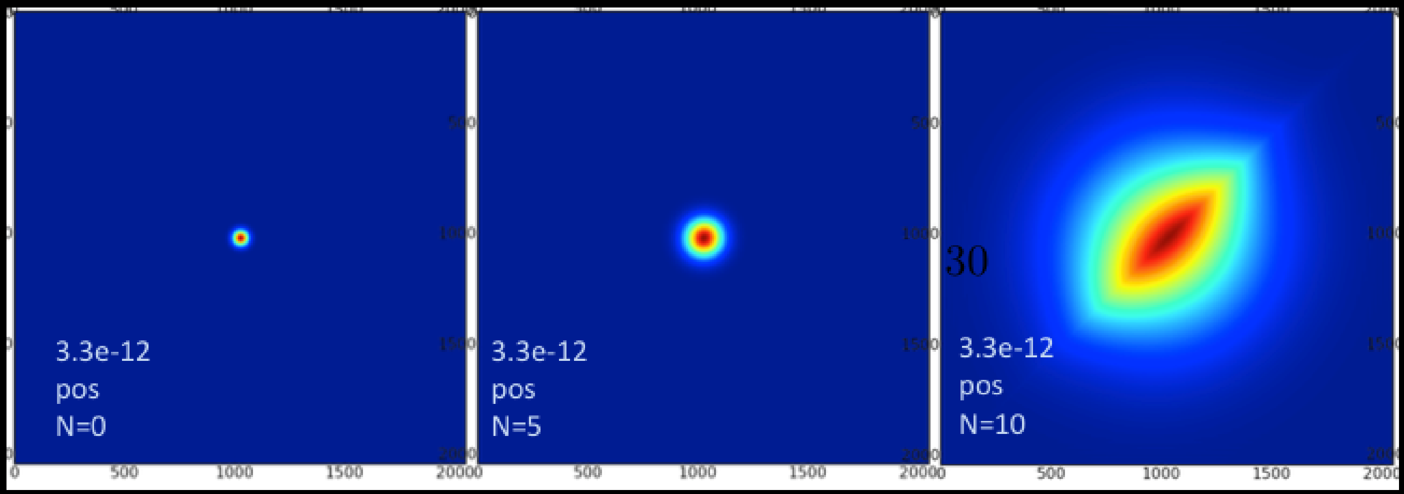
Another input we need to model the wavefunction collapse.



SIMULATING DYNAMICAL COLLAPSE

These ingredients are used to construct a dynamical wave-function collapse MC simulation of the pion evolution:

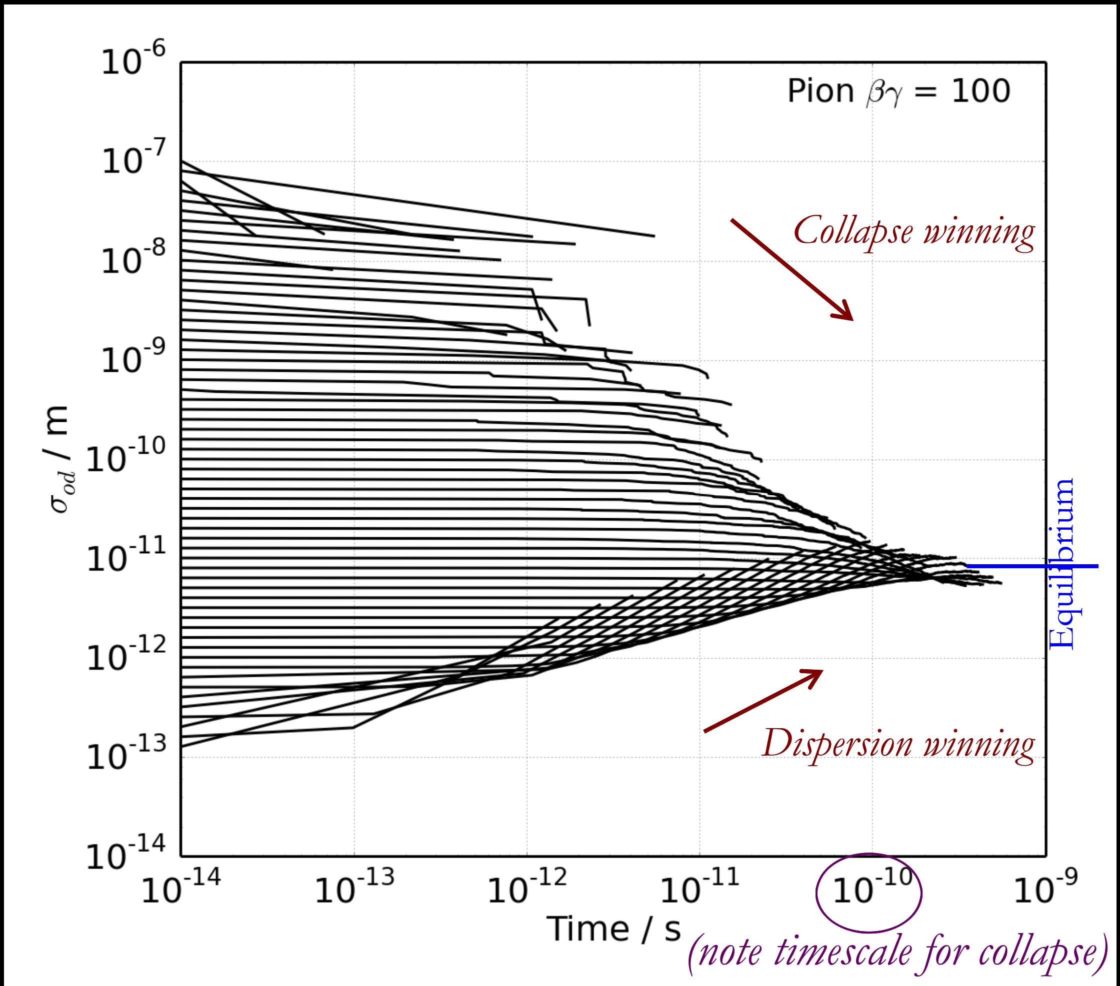
Start with very narrow pure state density matrix



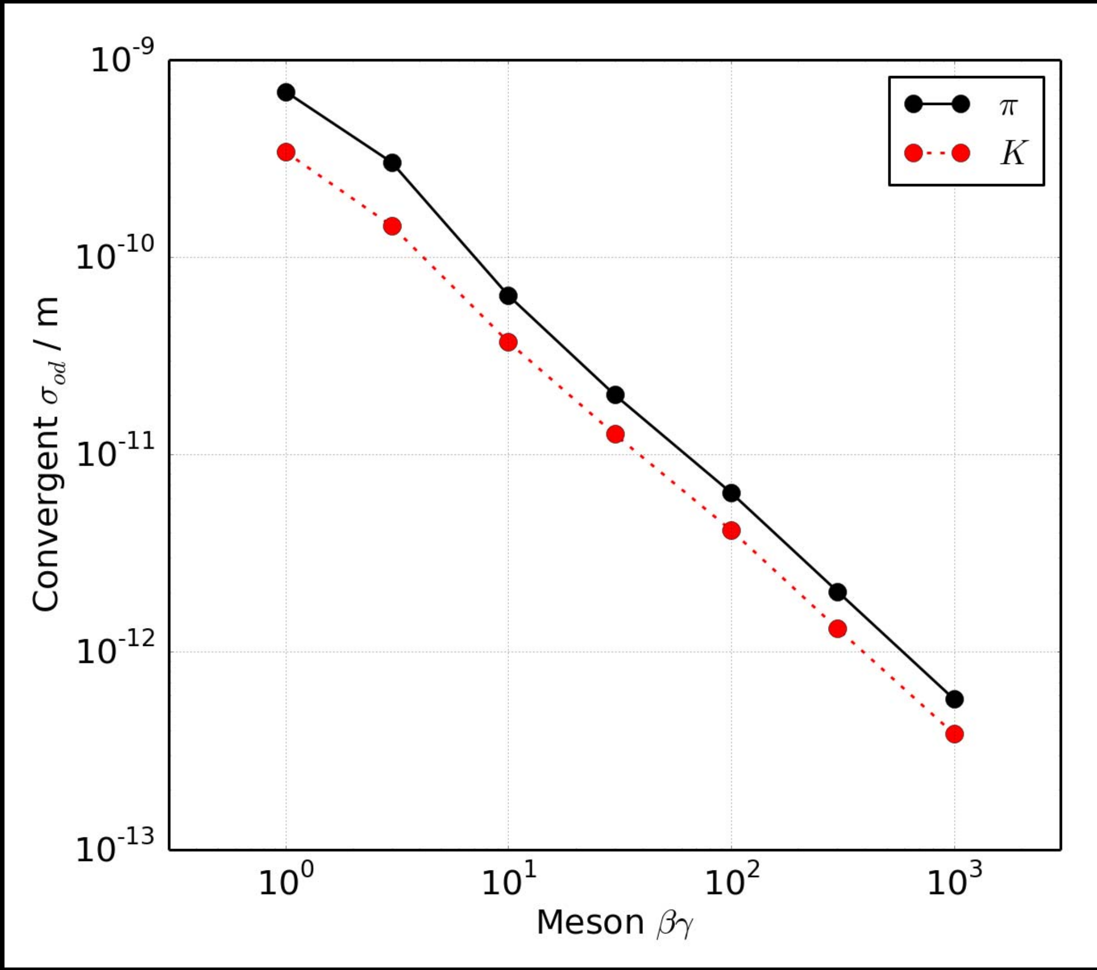
No longer fits on grid, simulation halts

Time *evolve* / *scatter* / *evolve* / *scatter* / *evolve* / *scatter*…

SIMULATING DYNAMICAL COLLAPSE



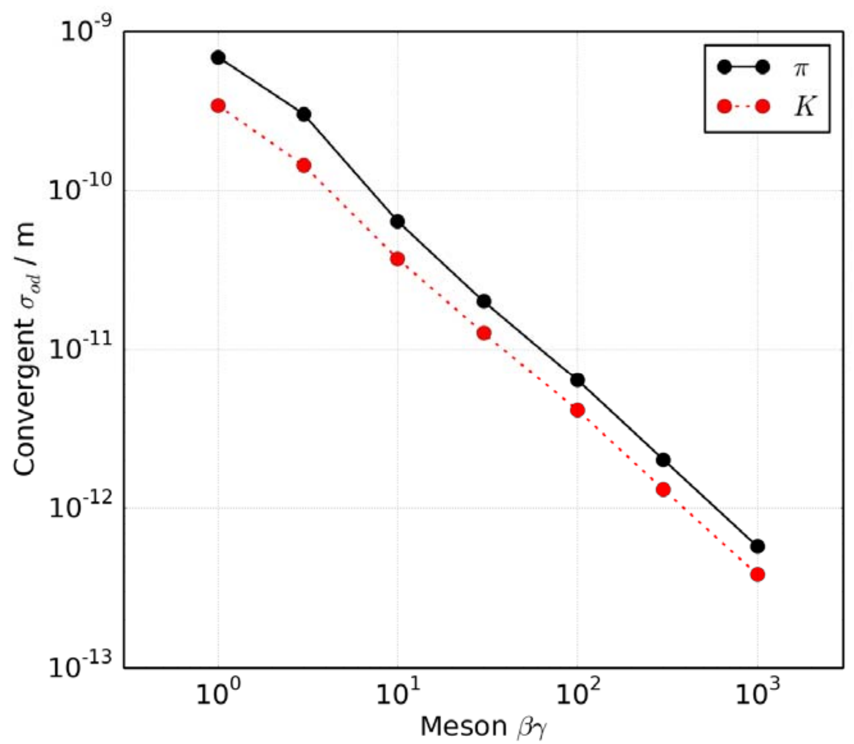
COLLAPSED WIDTHS



Map out collapsed widths as a function of incoming pion momentum

(Dependencies from Lorentz suppression of dispersion, and E-dependence of scattering functions)

COLLAPSED WIDTHS



← We have the collapsed quantum widths as a function of incoming pion momentum

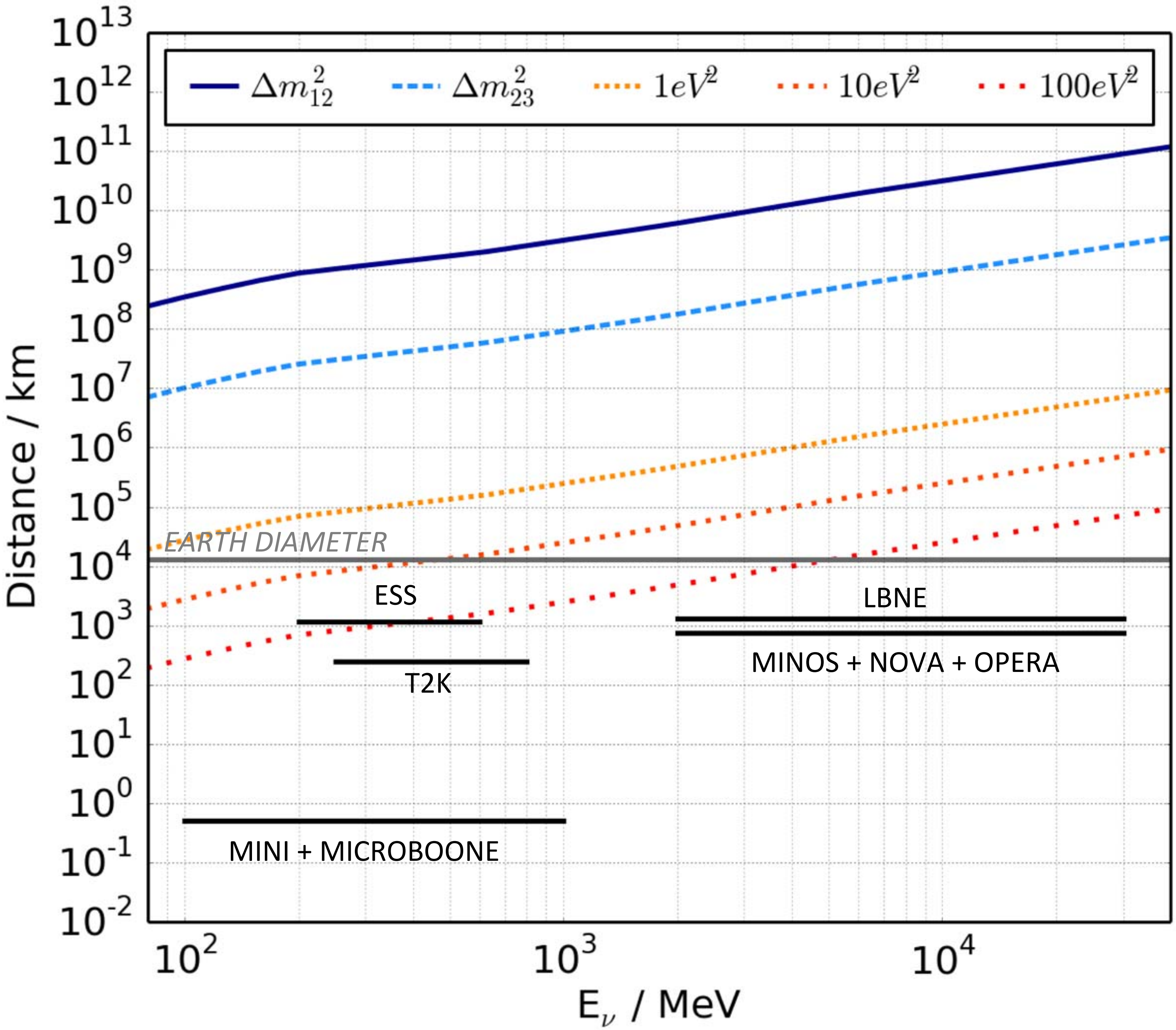
Now substitute into fancy oscillation formula to find coherence loss distance ↓

$$P(\nu_\alpha, L) = N U_{\alpha j} U^\dagger_{\alpha i} U_{\mu i} U^\dagger_{\mu j} \left[e^{-i \frac{m_{ij}^2}{2p_0} \frac{m_\pi^2}{m_\pi^2 - m_\mu^2} L} \right] \left[e^{-\frac{p_0^2 + m_\pi^2}{8\left(m_\pi^2 - m_\mu^2\right)^2} \left(m_{ij}^2\right)^2 \sigma_{diag}^2} \right] \left[e^{-\left(\frac{m_{ij}^2}{2p_0^2} \frac{m_\pi^2}{m_\pi^2 - m_\mu^2}\right)^2 \left(\frac{L^2}{2\sigma_{od}^2}\right)} \right]$$

Standard oscillation

Classical coherence condition

Quantum coherence condition



10^{13}
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
Distance / km
Δm^2_{12}
Δm^2_{23}
$1eV^2$
$10eV^2$
$100eV^2$
EARTH DIAMETER
ESS
T2K
LBNE
MINOS + NOVA + OPERA
MINI + MICROBOONE
10^2
10^3
10^4
E_ν / MeV

THE BOTTOM LINE

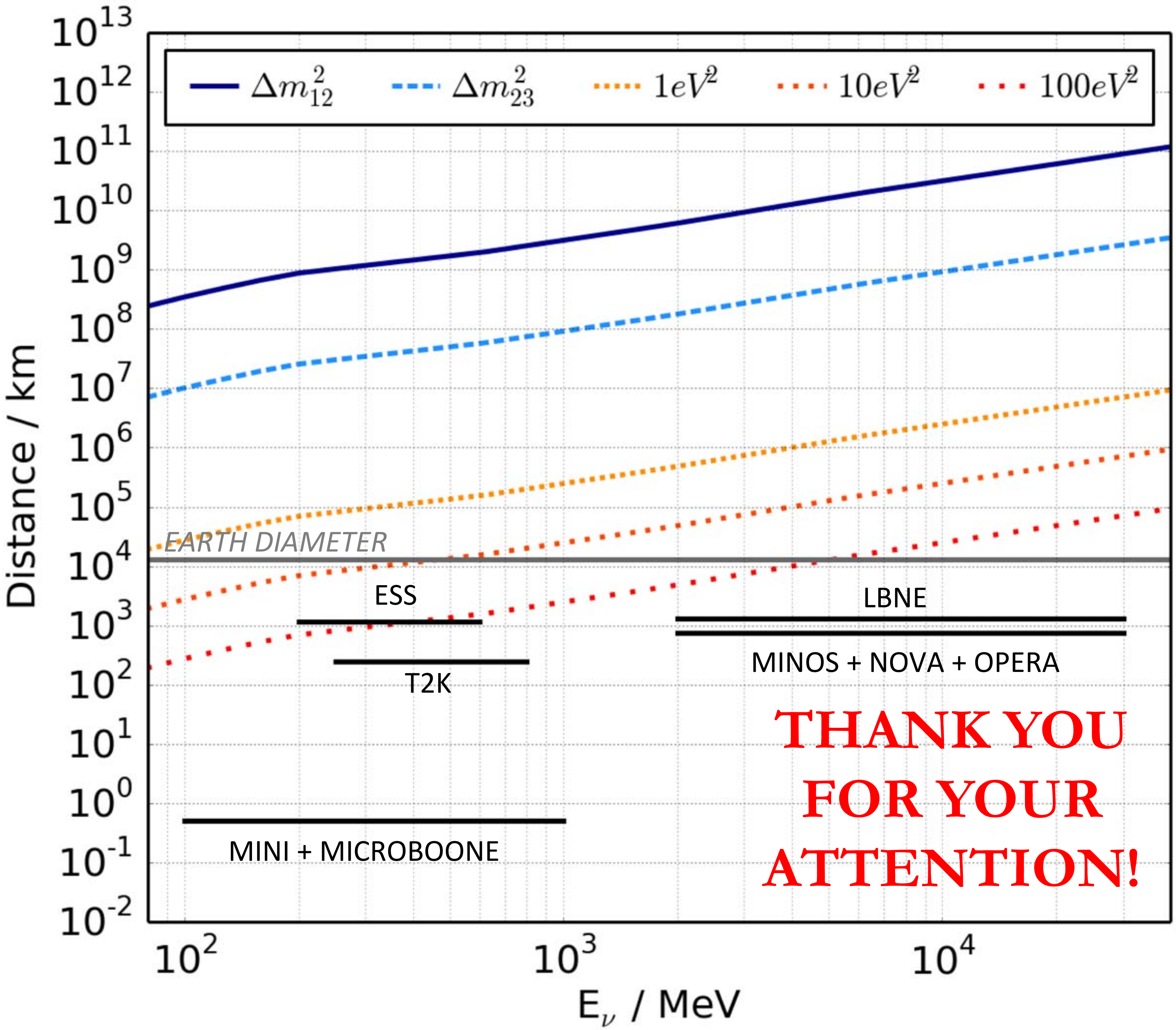
There will be no coherence loss effects for active neutrinos from accelerator neutrino beams anywhere on Earth

There may be coherence loss effects for heavy sterile neutrinos, but we will not see it at SBN experiments, or even existing LBN ones

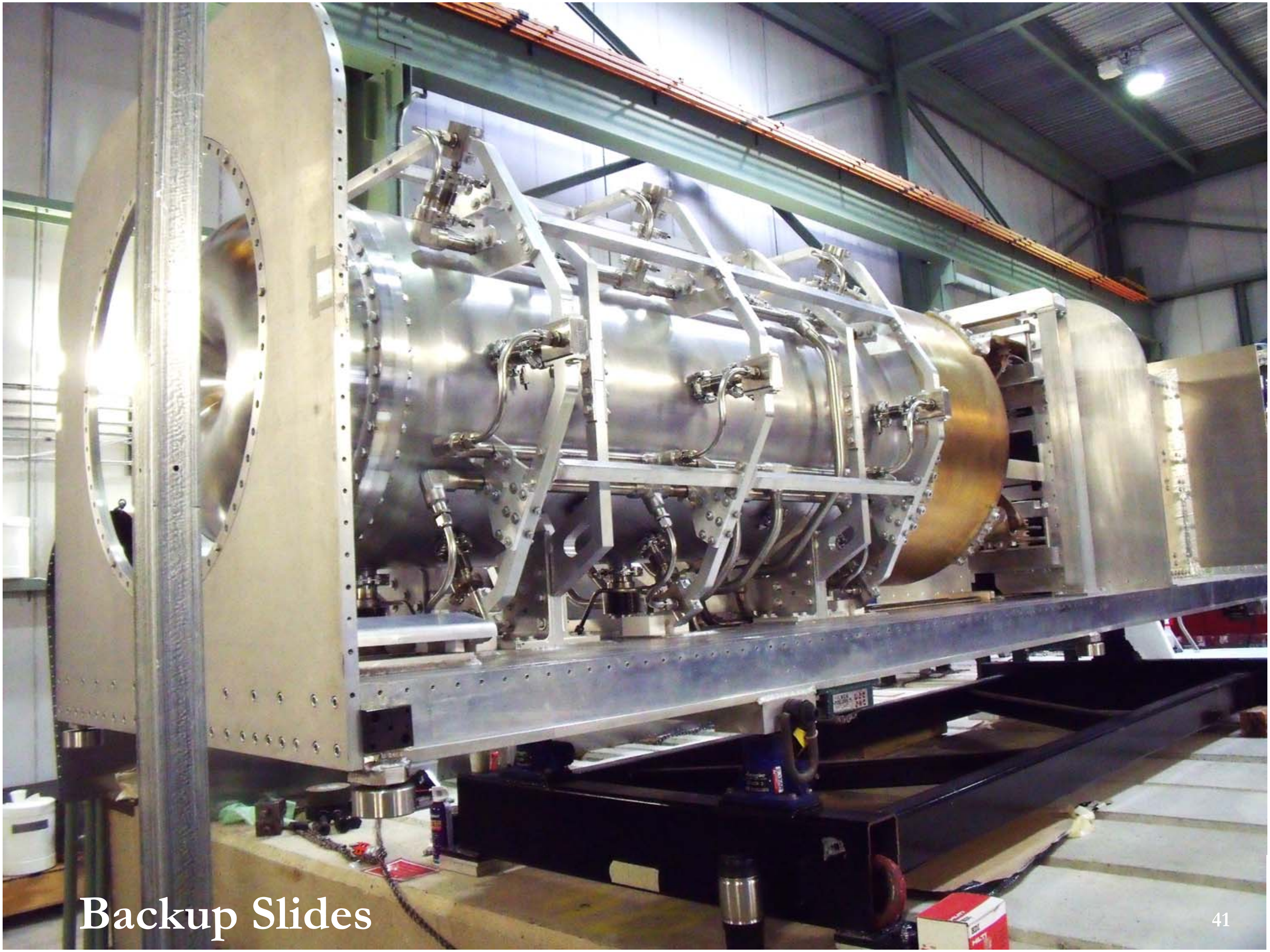
Previous analyses and sensitivities are meaningful

This work demonstrates a rigorous prediction of when neutrinos become incoherent

There are other neutrino sources where the effects are still unknown : reactors, decay-at-rest, etc. Those sound like fun to think about next!



THANK YOU FOR YOUR ATTENTION!



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CHANGING DECAY ENVIRONMENT