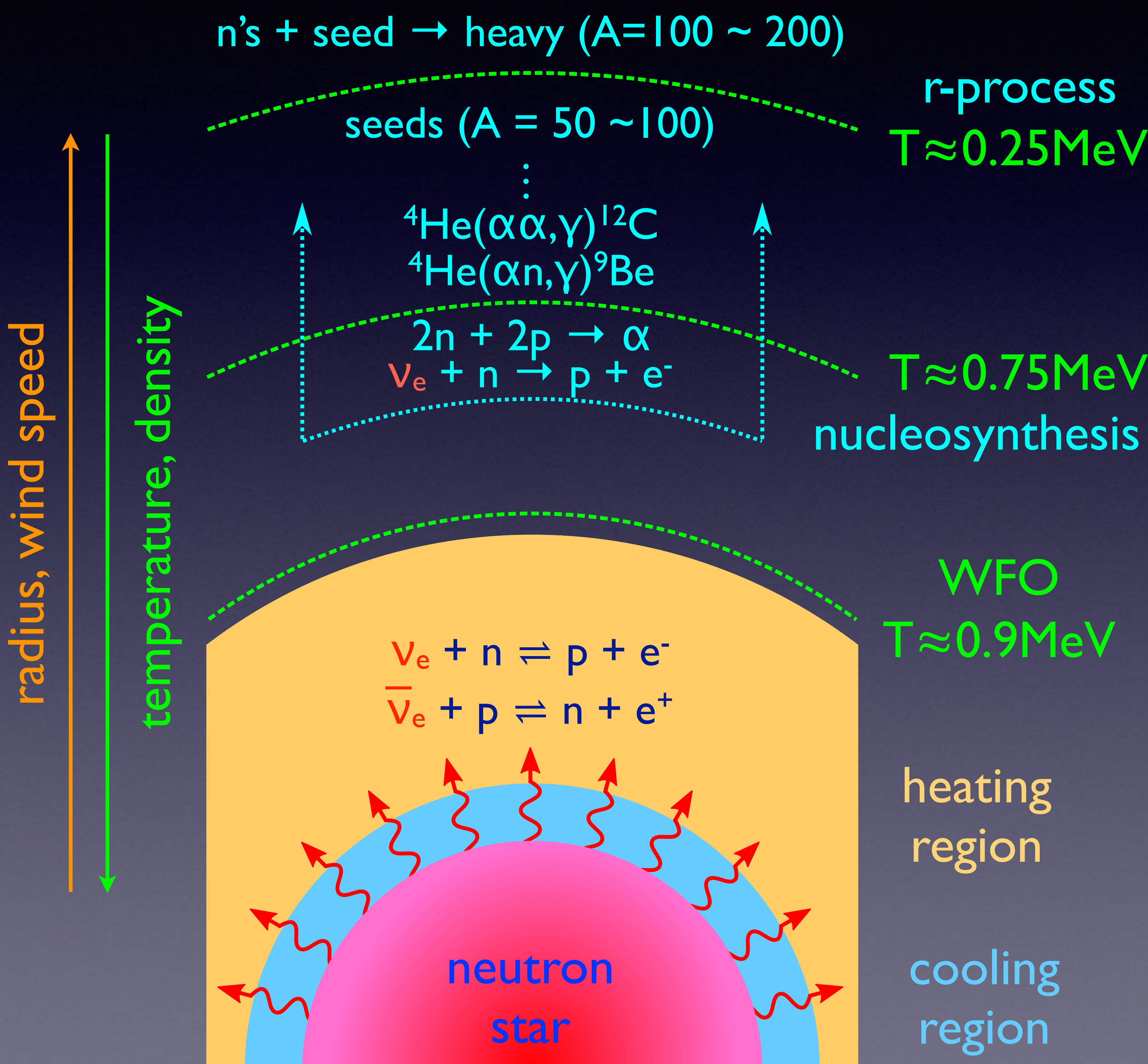
Spontaneous Symmetry Breaking in Collective Neutrino Oscillations

Huaiyu Duan
(U of New Mexico)

Collaborators: Shashank Shalgar
Sajad Abbar

Neutrinos in Supernovae



- ~10^{53} ergs, 10^{58} neutrinos in ~10 seconds
- All neutrino species, 10~30 MeV
- Dominate energetics
- Influence nucleosynthesis
- Probe into SNe

Oscillations in Dense Medium

active neutrinos, coherent forward scattering only

$$\rho(t,\mathbf{x},\mathbf{p}) = \int \mathrm{d}^3\mathbf{x}'\, e^{-\mathrm{i}\mathbf{p}\cdot\mathbf{x}'}\,\psi^\dagger\left(t,\mathbf{x}-\frac{1}{2}\mathbf{x}'\right)\,\psi\left(t,\mathbf{x}+\frac{1}{2}\mathbf{x}'\right)$$

$$(\partial_t + \hat{\mathbf{v}}\cdot\boldsymbol{\nabla})\rho = -\mathrm{i}[\mathsf{H},\ \rho]$$

mass matrix

electron density

$$\mathsf{H} = \frac{\mathsf{M}^2}{2E} + \sqrt{2}G_\mathrm{F}\,\mathrm{diag}[n_e,0,0] + \mathsf{H}_{\nu\nu}$$

neutrino energy

ν–ν forward scattering (self-coupling)

Neutrino Self-Coupling

mass matrix

electron density

$$\mathsf{H} = \frac{\mathsf{M}^2}{2E} + \sqrt{2}G_\mathrm{F}\,\mathrm{diag}[n_e, 0, 0] + \mathsf{H}_{\nu\nu}$$

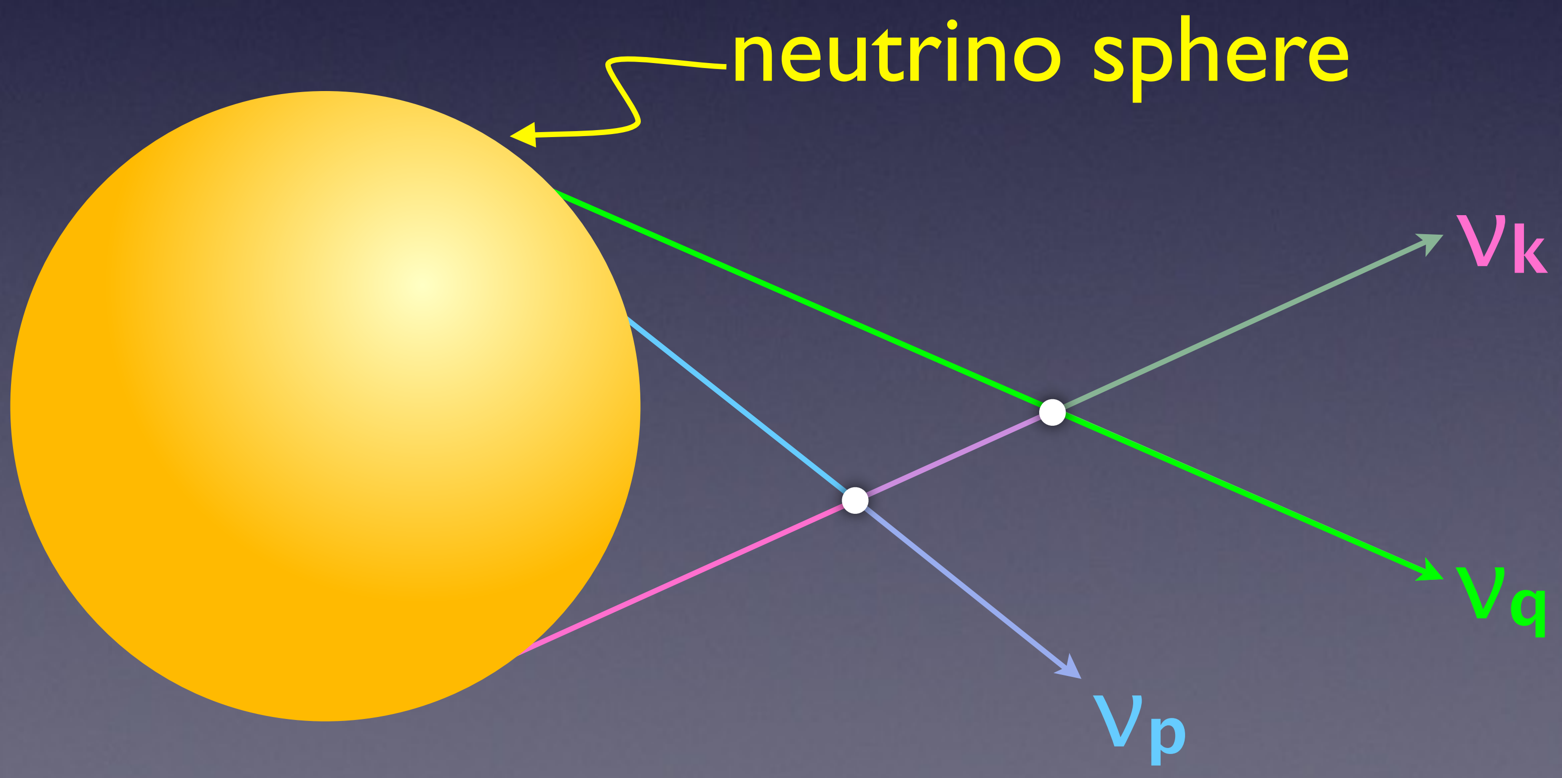
neutrino energy

ν–ν forward scattering
(self–coupling)

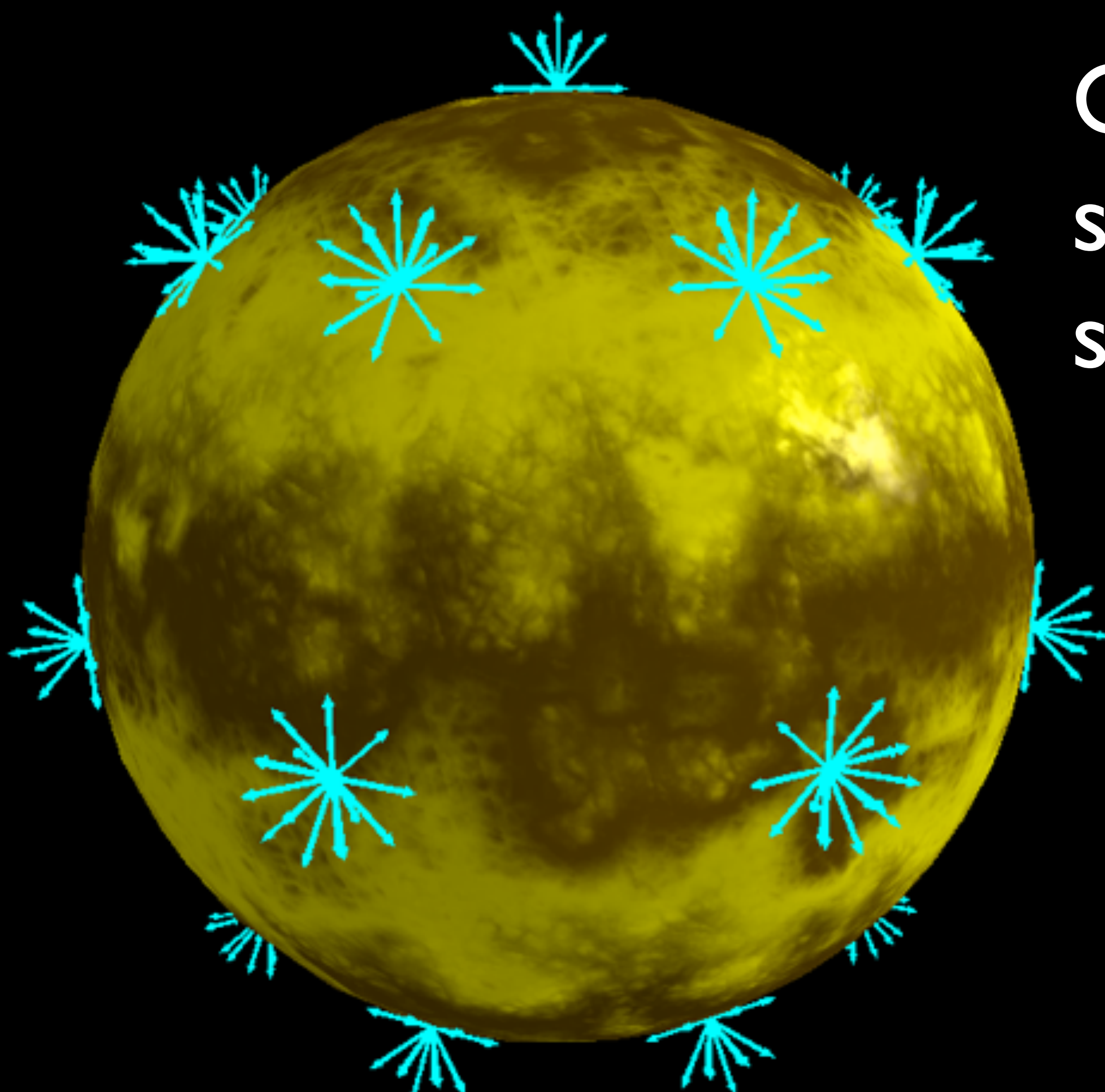
$$\mathsf{H}_{\nu\nu} = \sqrt{2}G_\mathrm{F}\int \mathrm{d}^3\mathbf{p}'(1-\hat{\mathbf{v}}\cdot\hat{\mathbf{v}}')(\rho_{\mathbf{p}'}-\bar{\rho}_{\mathbf{p}'})$$

Oscillations in SN

$$\mathsf{H} = \frac{\mathsf{M}^2}{2E} + \sqrt{2}G_{\mathrm{F}}\,\mathrm{diag}[n_e, 0, 0] + \mathsf{H}_{\nu\nu}$$



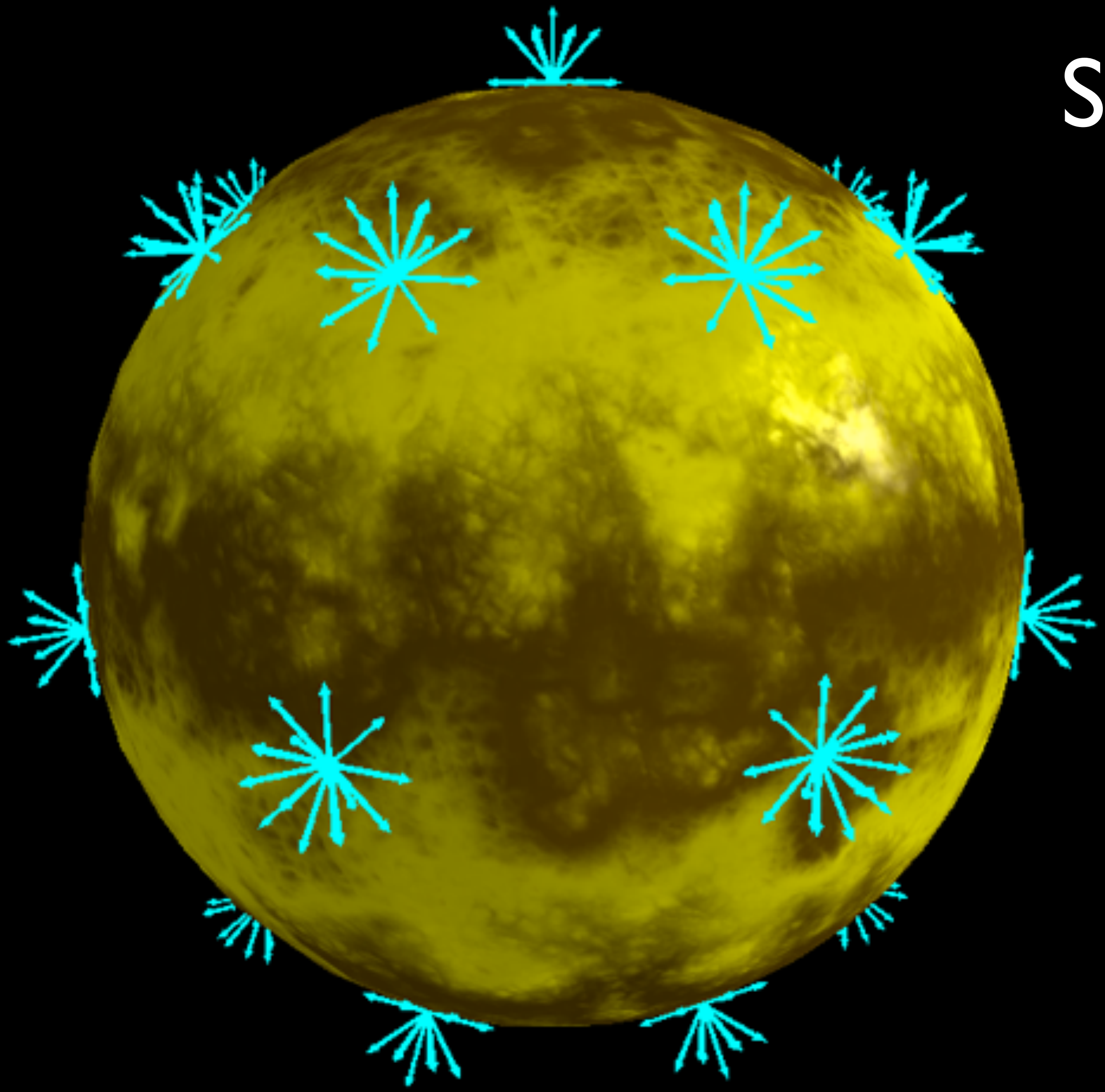
(1+3+3)D



Coherent forward scattering outside neutrino sphere

$$\rho(t; r, \Theta, \Phi; E, \vartheta, \varphi)$$

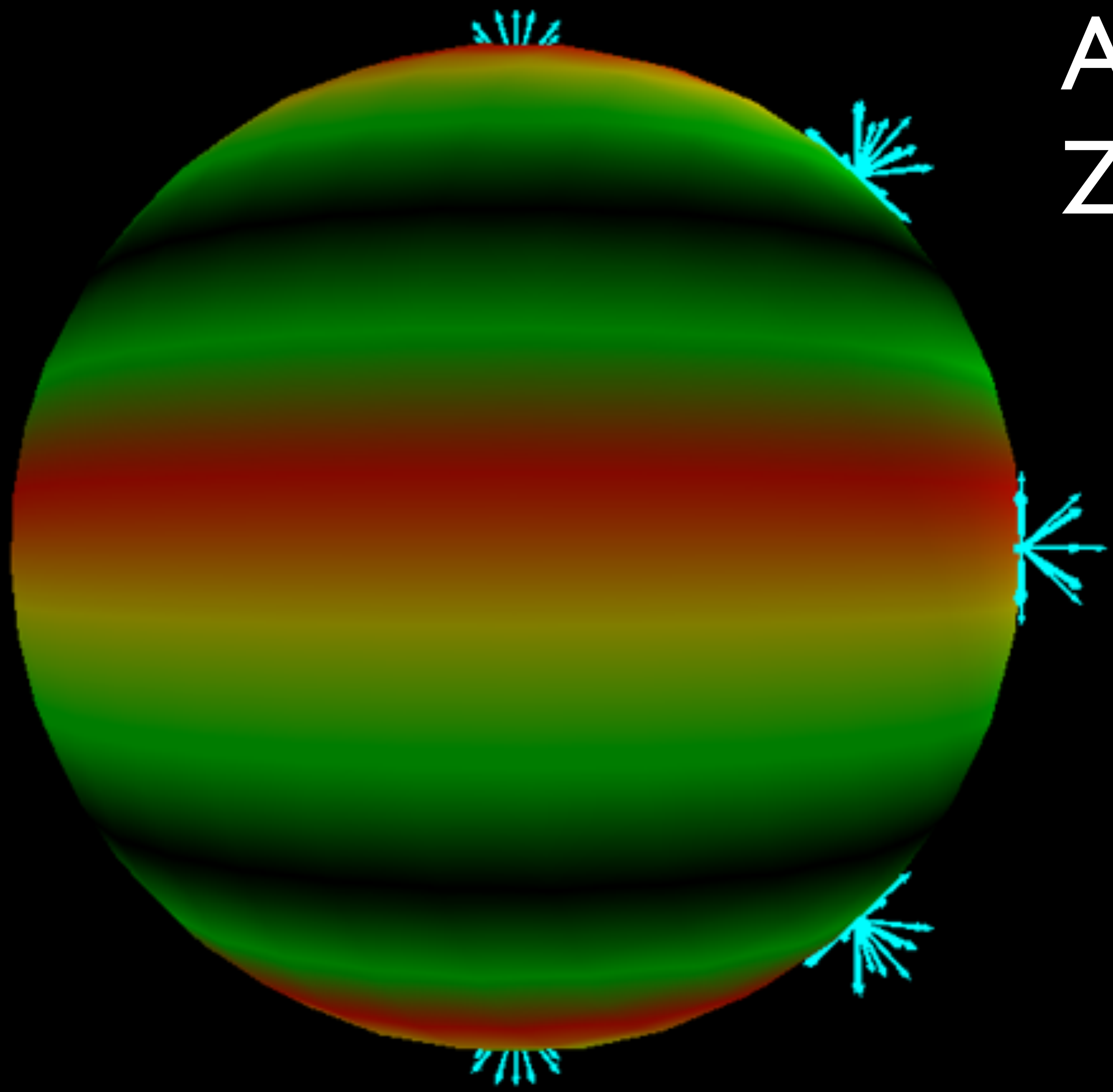
(0+3+3)D



Stationary emission

$$\rho(r, \Theta, \Phi; E, \vartheta, \varphi)$$

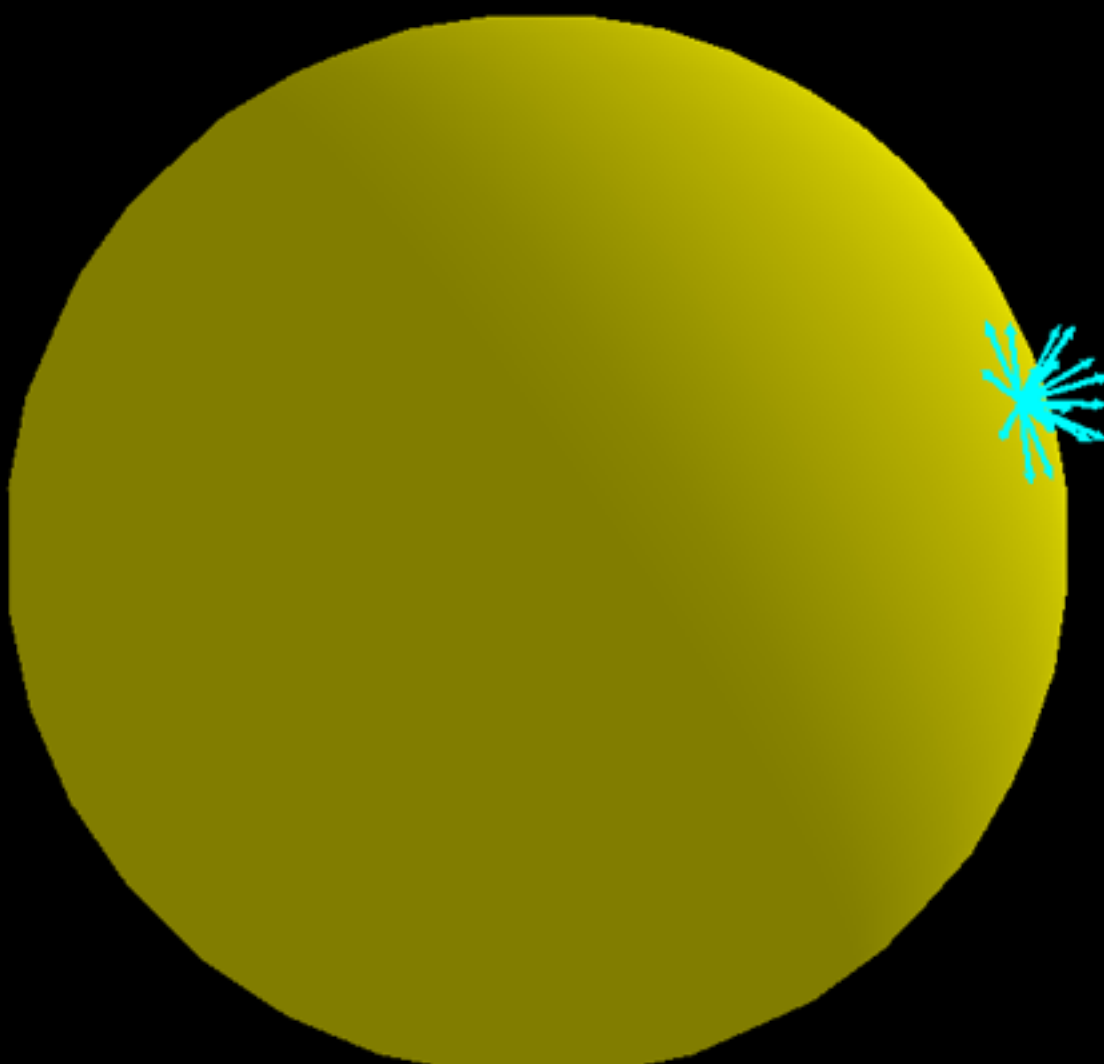
(0+2+3)D



Axial symmetry around the Z axis

$$\rho(r, \Theta; E, \vartheta, \varphi)$$

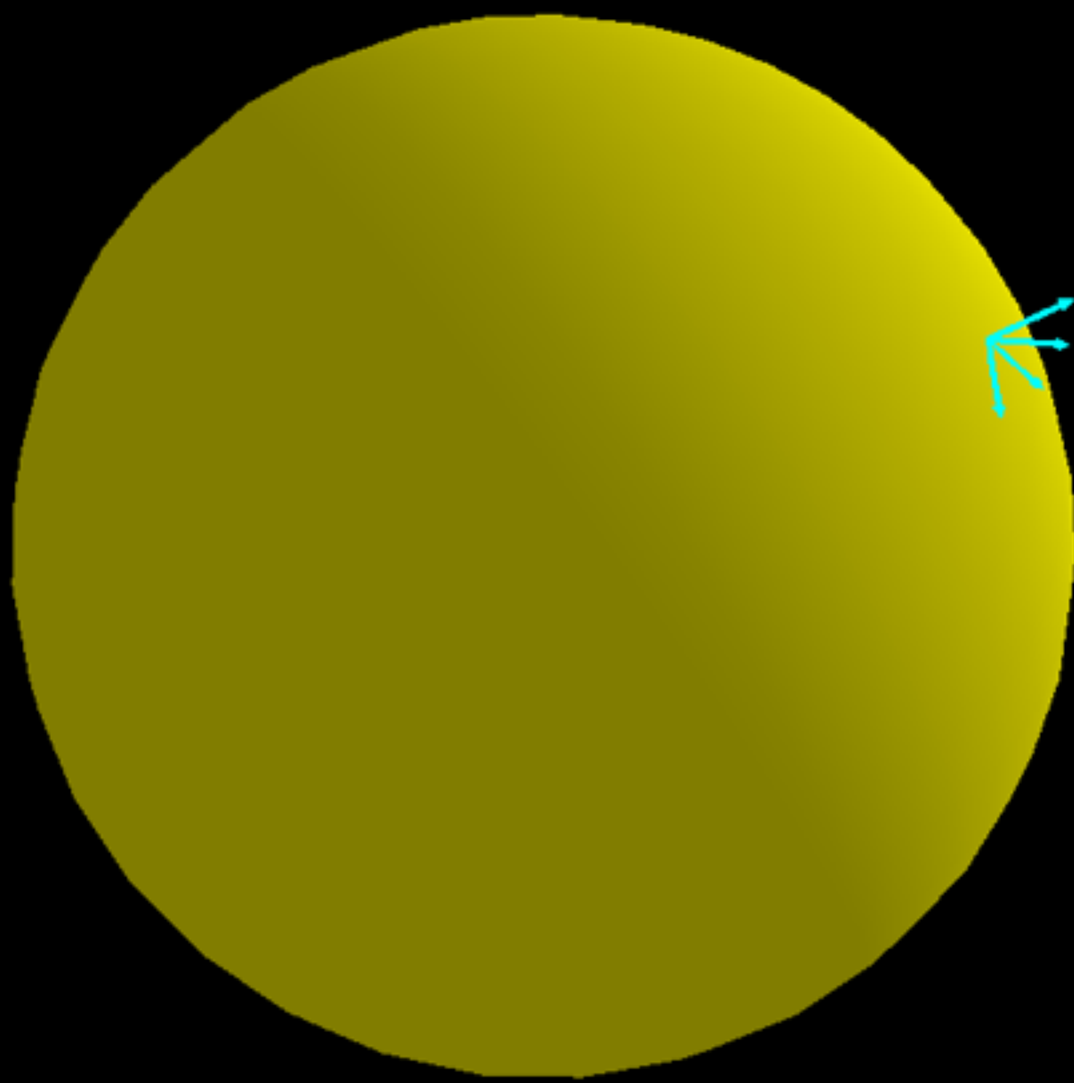
(0+1+3)D



Spherical symmetry about the center (inconsistent?)

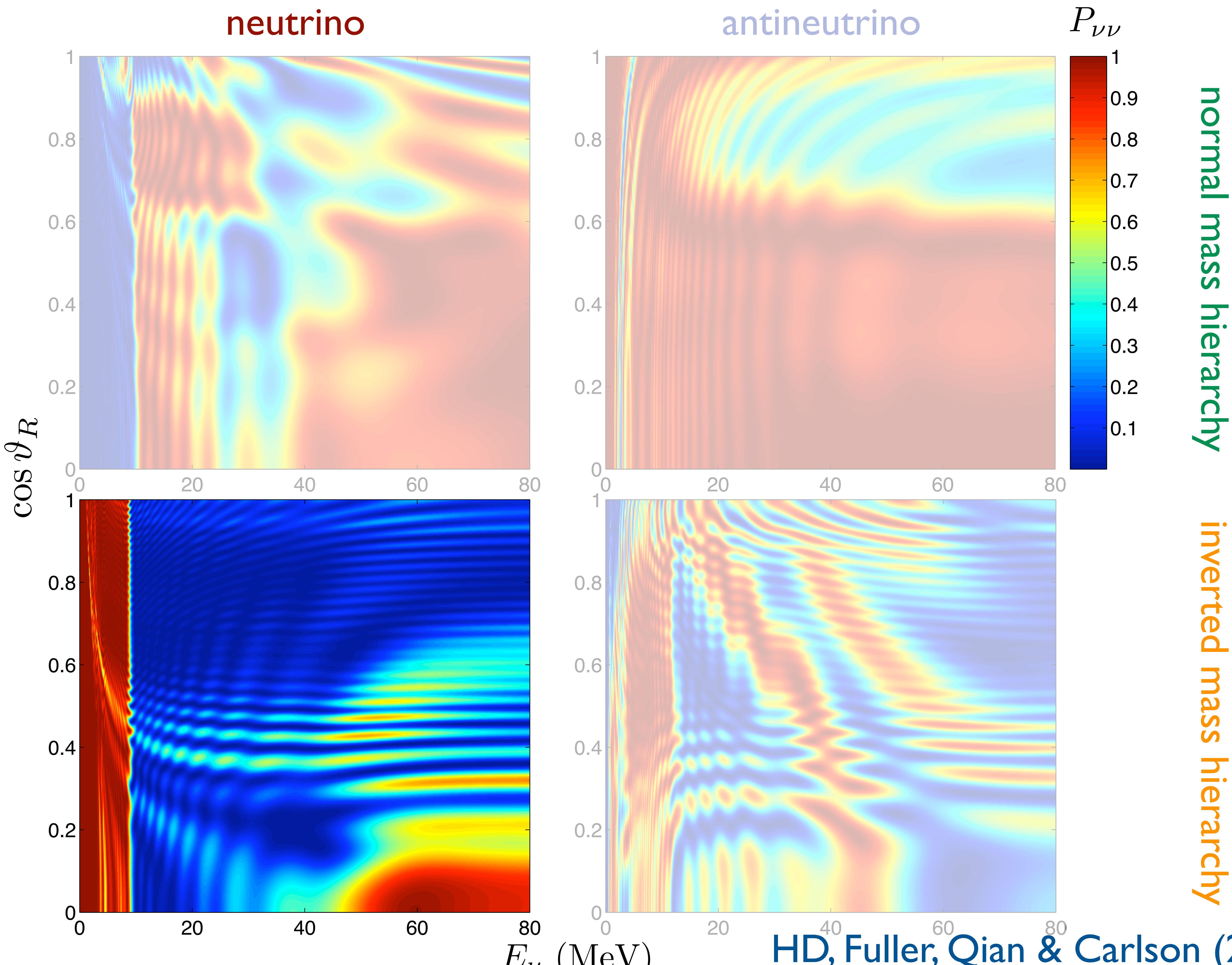
$$\rho(r; E, \vartheta, \varphi)$$

(0+1+2)D
Multi-Angle/Bulb Model



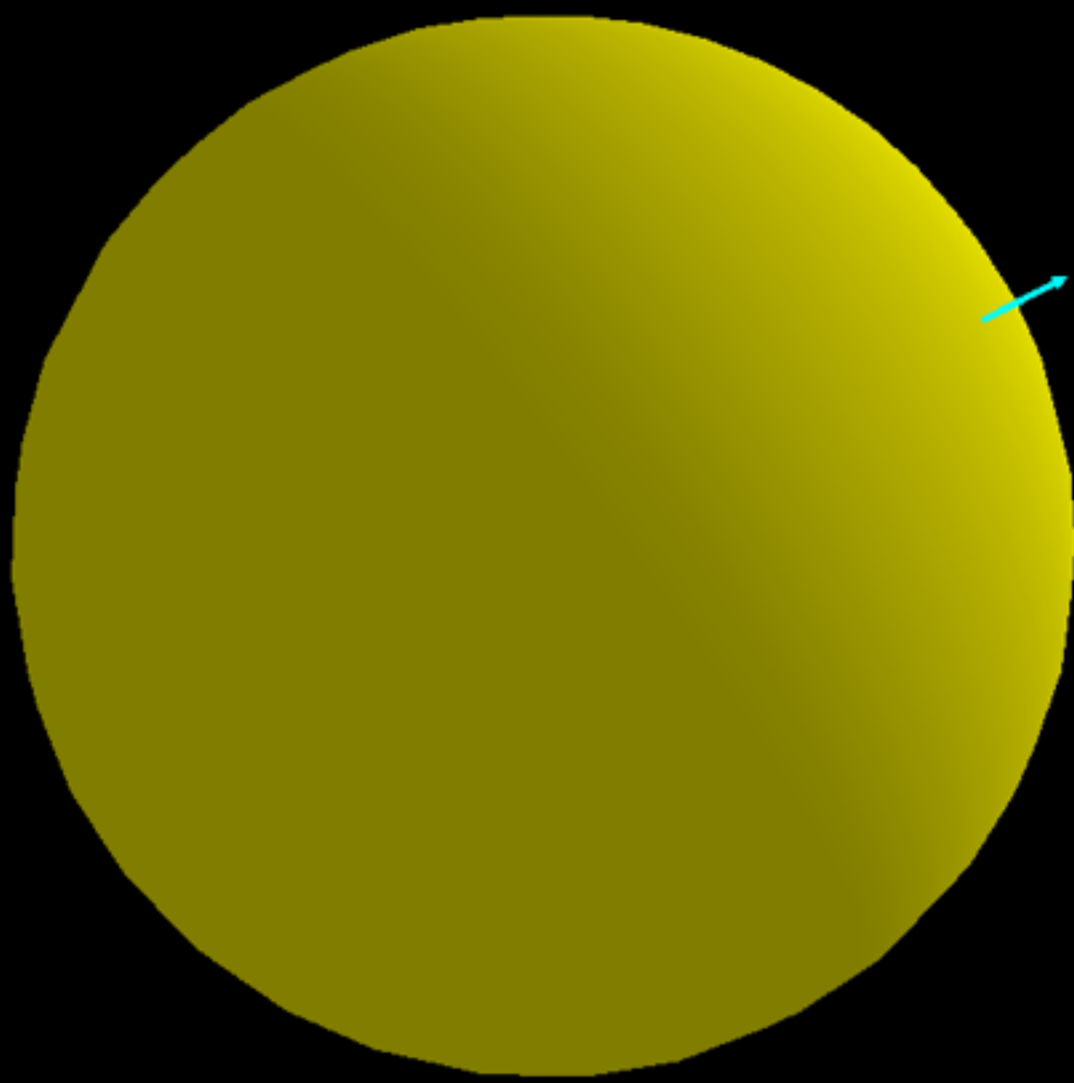
Azimuthal symmetry around any radial direction

$$\rho(r; E, \vartheta)$$



HD, Fuller, Qian & Carlson (2006)

(0+1+1)D
Single-Angle Model



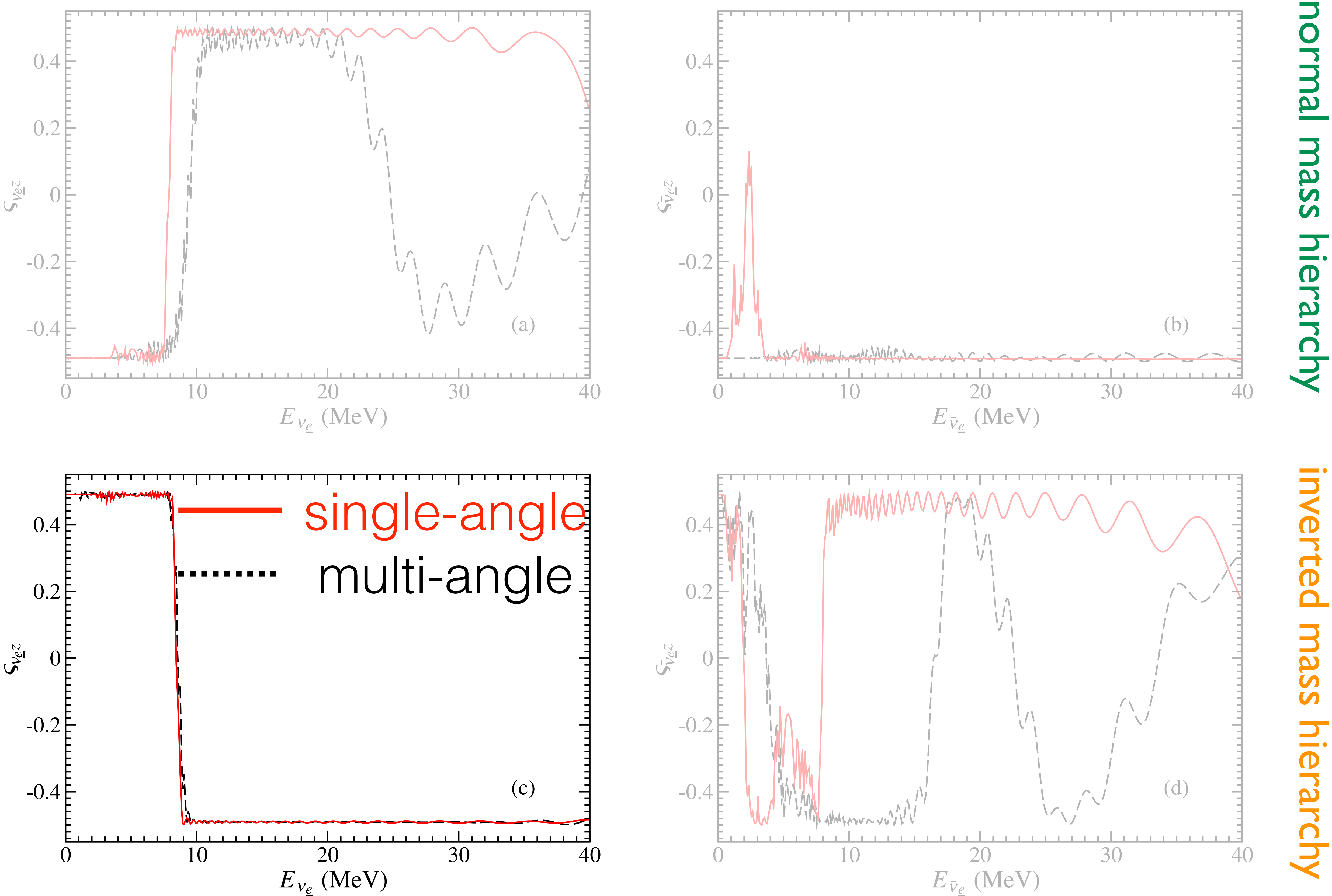
Trajectory independent neutrino flavor evolution

$$\rho(r; E)$$

Equivalent to an homogeneous and isotropic neutrino gas evolving with time.

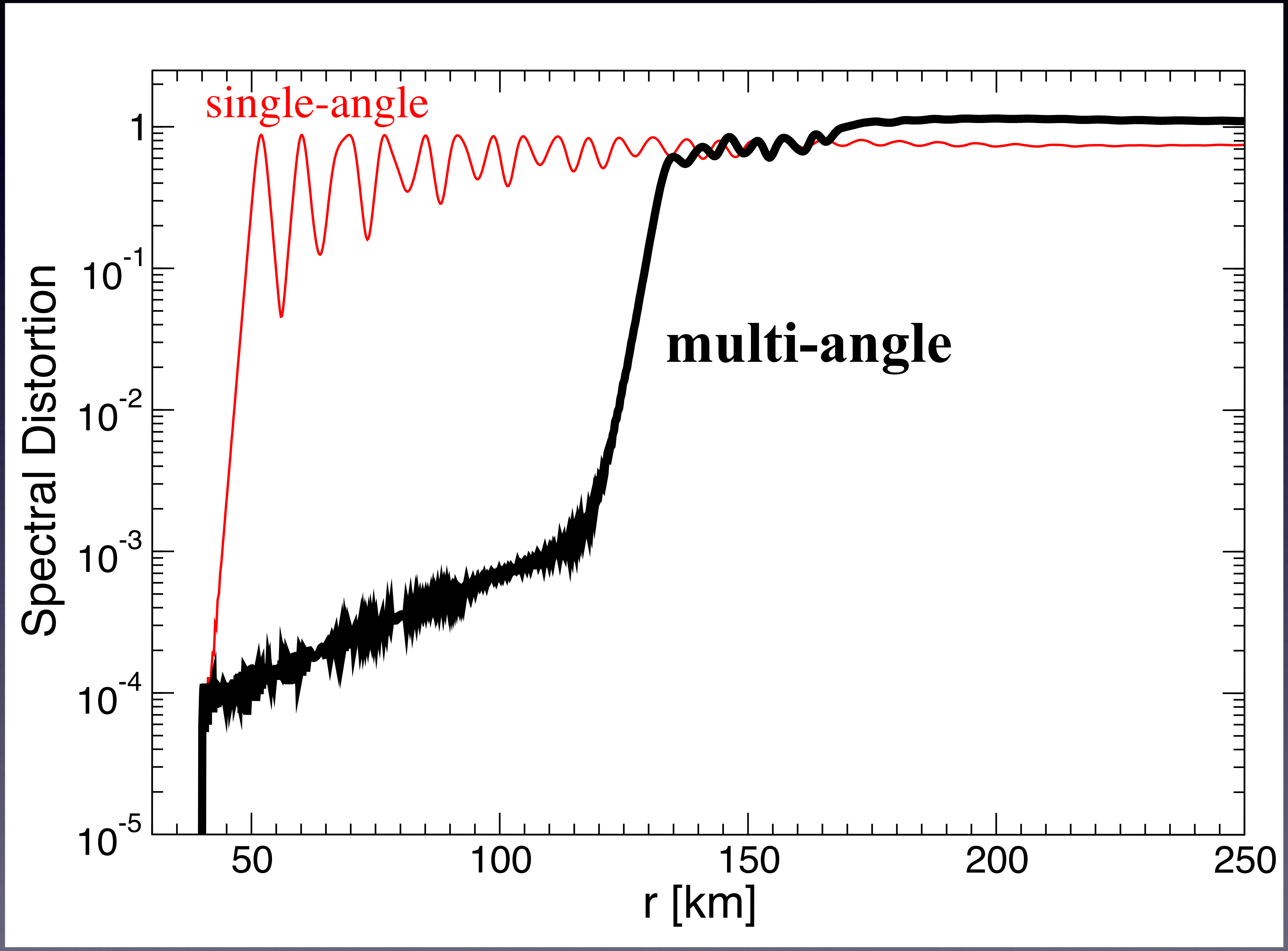
neutrino

antineutrino



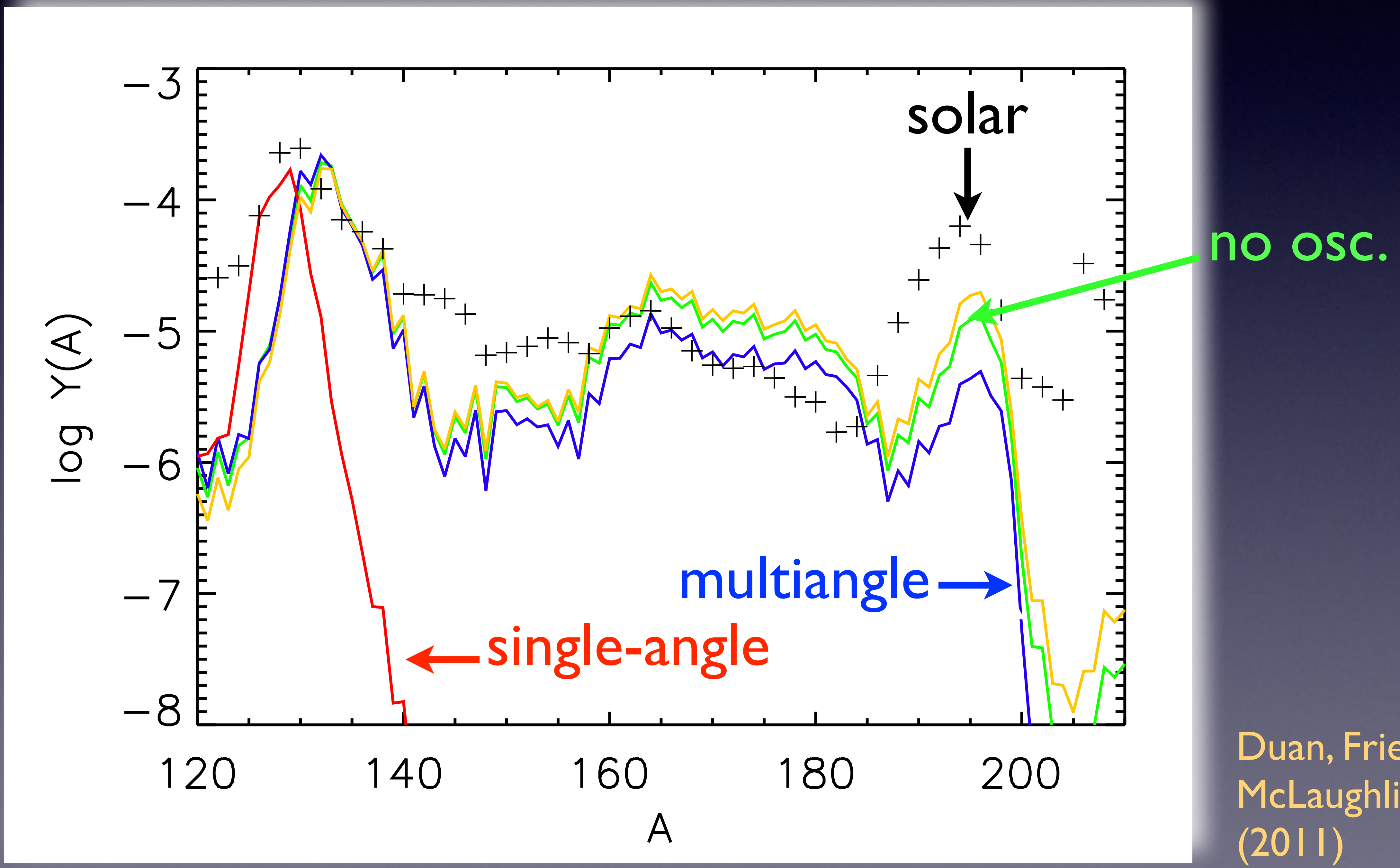
HD, Fuller, Qian & Carlson (2006)

Multiangle Suppression



HD & Friedland (2010)

Nucleosynthesis



Duan, Friedland, McLaughlin & Surman (2011)

Conclusions

- Dimensionality of the supernova model matters in collective neutrino oscillations.

Line Model

Neutrinos emitted in single energy and two beams.

Two active flavors, “no matter & small mixing angle”

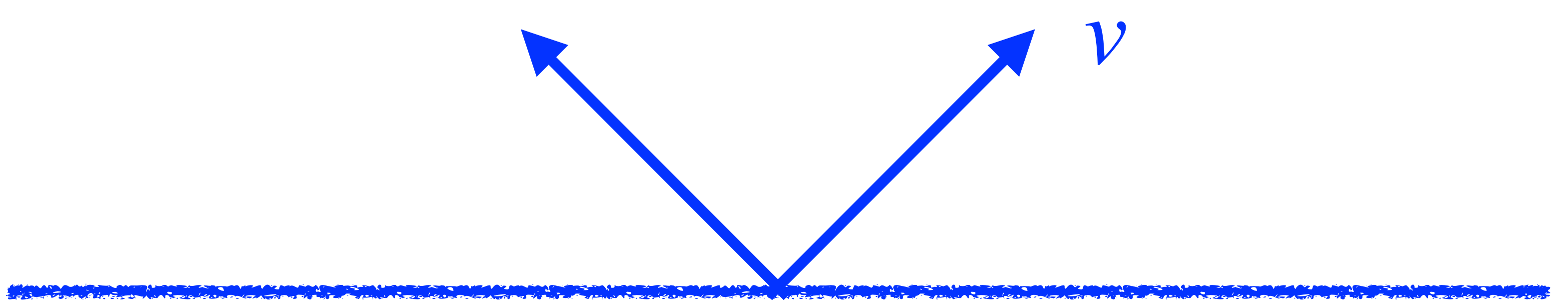
time independent, x translation symmetry, left-right symmetry

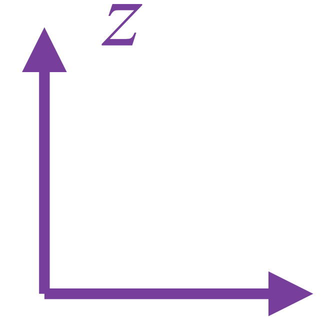
$$\mathrm{i}\hat{\mathbf{v}}\cdot\boldsymbol{\nabla}\rho = [-(\omega/2)\eta\sigma_3 + \mathsf{H}_{\nu\nu},\ \rho]$$

$$\mathrm{i}\hat{\mathbf{v}}\cdot\boldsymbol{\nabla}\bar{\rho} = [-(-\omega/2)\eta\sigma_3 + \mathsf{H}_{\nu\nu},\ \bar{\rho}]$$

$$\omega = \frac{|\Delta m^2|}{2E}$$

$$\eta = \begin{cases} +1 & \text{NH} \\ -1 & \text{IH} \end{cases}$$





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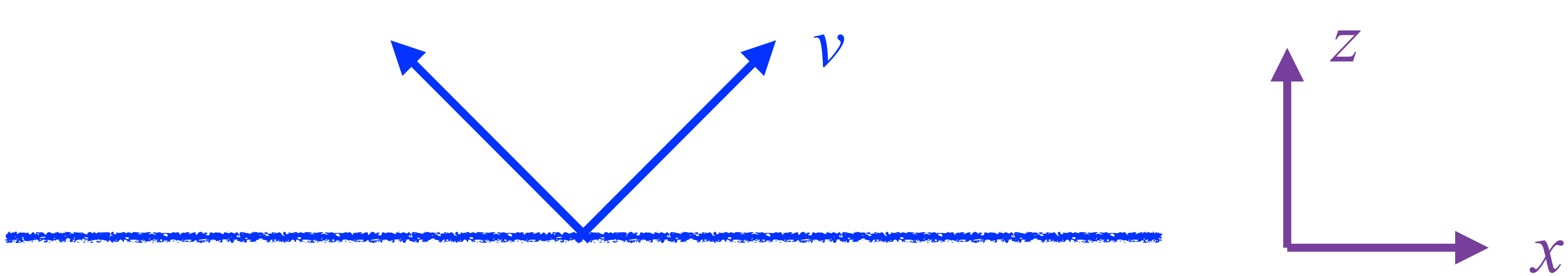
$$\mathrm{i}\hat{\mathbf{v}}\cdot\nabla\bar{\rho} = [-(-\omega/2)\eta\sigma_3 + \cancel{\mathsf{H}_{\nu\nu}},\ \bar{\rho}]$$

$$\rho(z) = e^{\mathrm{i}(\omega/2)(z/v_z)\sigma_3}\rho(0)e^{-\mathrm{i}(\omega/2)(z/v_z)\sigma_3}$$

$$\bar{\rho}(z) = e^{\mathrm{i}(-\omega/2)(z/v_z)\sigma_3}\bar{\rho}(0)e^{-\mathrm{i}(-\omega/2)(z/v_z)\sigma_3}$$



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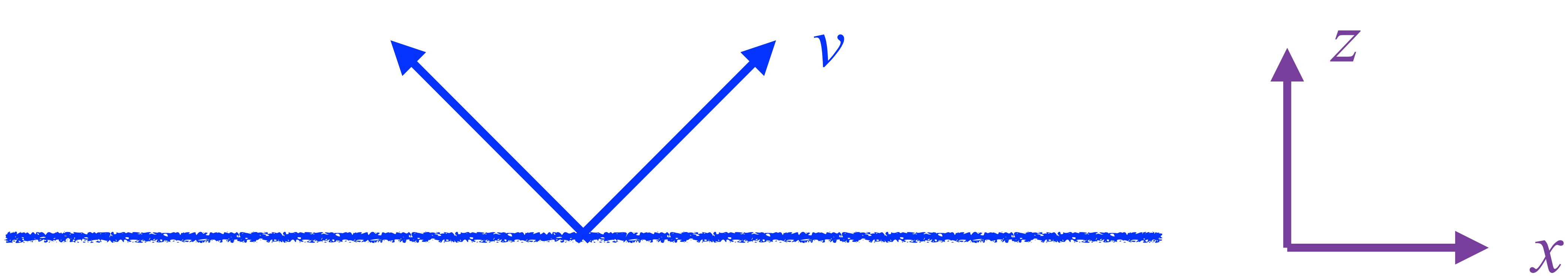


Line Model

Electron flavor neutrinos and antineutrinos

$$\rho \propto \begin{bmatrix} 1 & \epsilon \\ \epsilon^* & 0 \end{bmatrix} \qquad \bar{\rho} \propto \begin{bmatrix} 1 & \bar{\epsilon} \\ \bar{\epsilon}^* & 0 \end{bmatrix}$$

$$\mathrm{i}\partial_z \begin{bmatrix} \epsilon \\ \bar{\epsilon} \end{bmatrix} = v_z^{-1} \begin{bmatrix} -\eta\omega - \alpha\mu & \alpha\mu \\ -\mu & \eta\omega + \mu \end{bmatrix} \begin{bmatrix} \epsilon \\ \bar{\epsilon} \end{bmatrix}$$

$$\alpha = n_{\bar{\nu}}/n_{\nu}$$

$$\mu \propto G_{\mathrm{F}} n_{\nu}$$

Ω can be complex in IH and $\frac{2\omega}{(1+\sqrt{\alpha})^2} < \mu < \frac{2\omega}{(1-\sqrt{\alpha})^2}$

Linear (flavor) stability analysis (Banerjee et al, 2011)

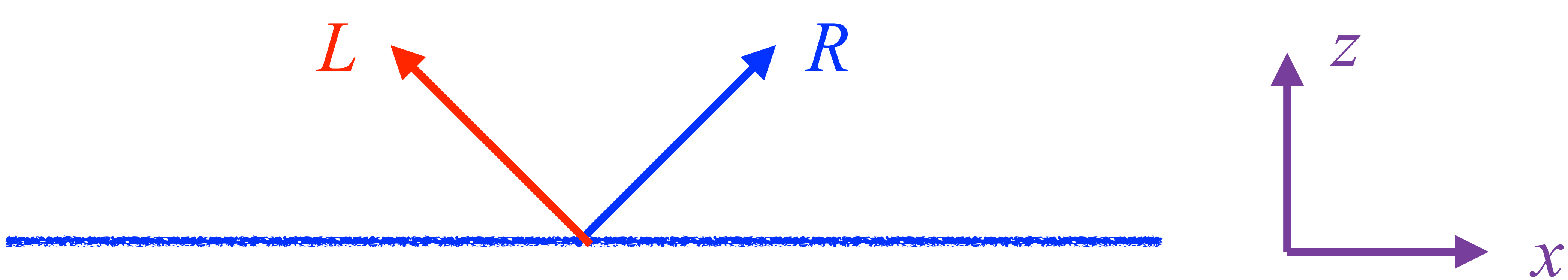
Line Model

time independent, x translation symmetry, ~~left-right symmetry~~

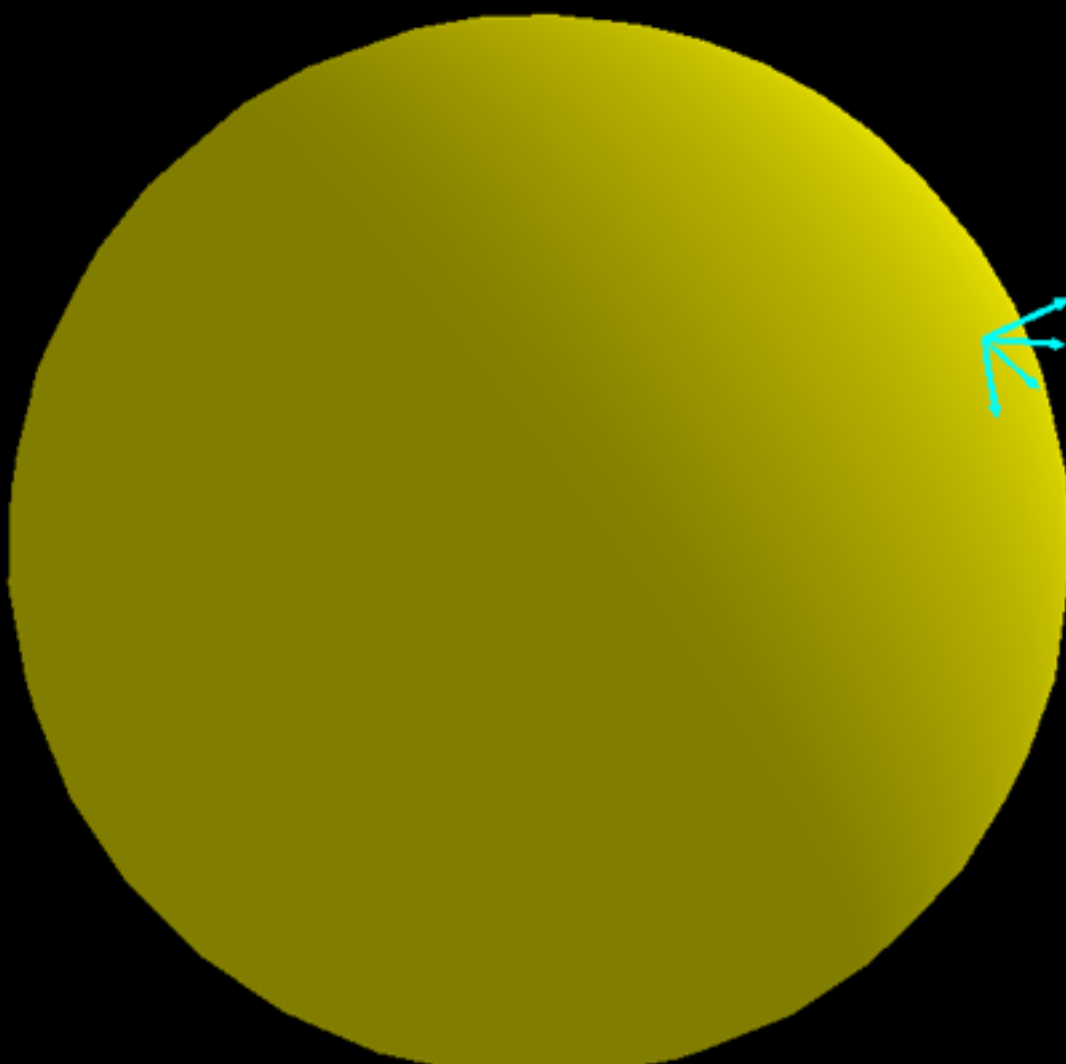
$$i\partial_z \begin{bmatrix} \epsilon_+ \\ \bar{\epsilon}_+ \\ \epsilon_- \\ \bar{\epsilon}_- \end{bmatrix} = \begin{bmatrix} \Lambda_+ & \\ & \Lambda_- \end{bmatrix} \begin{bmatrix} \epsilon_+ \\ \bar{\epsilon}_+ \\ \epsilon_- \\ \bar{\epsilon}_- \end{bmatrix}$$

$$\epsilon_\pm = \frac{\epsilon_L \pm \epsilon_R}{2}$$

$\Omega_{+/-}$ can be complex in IH/NH



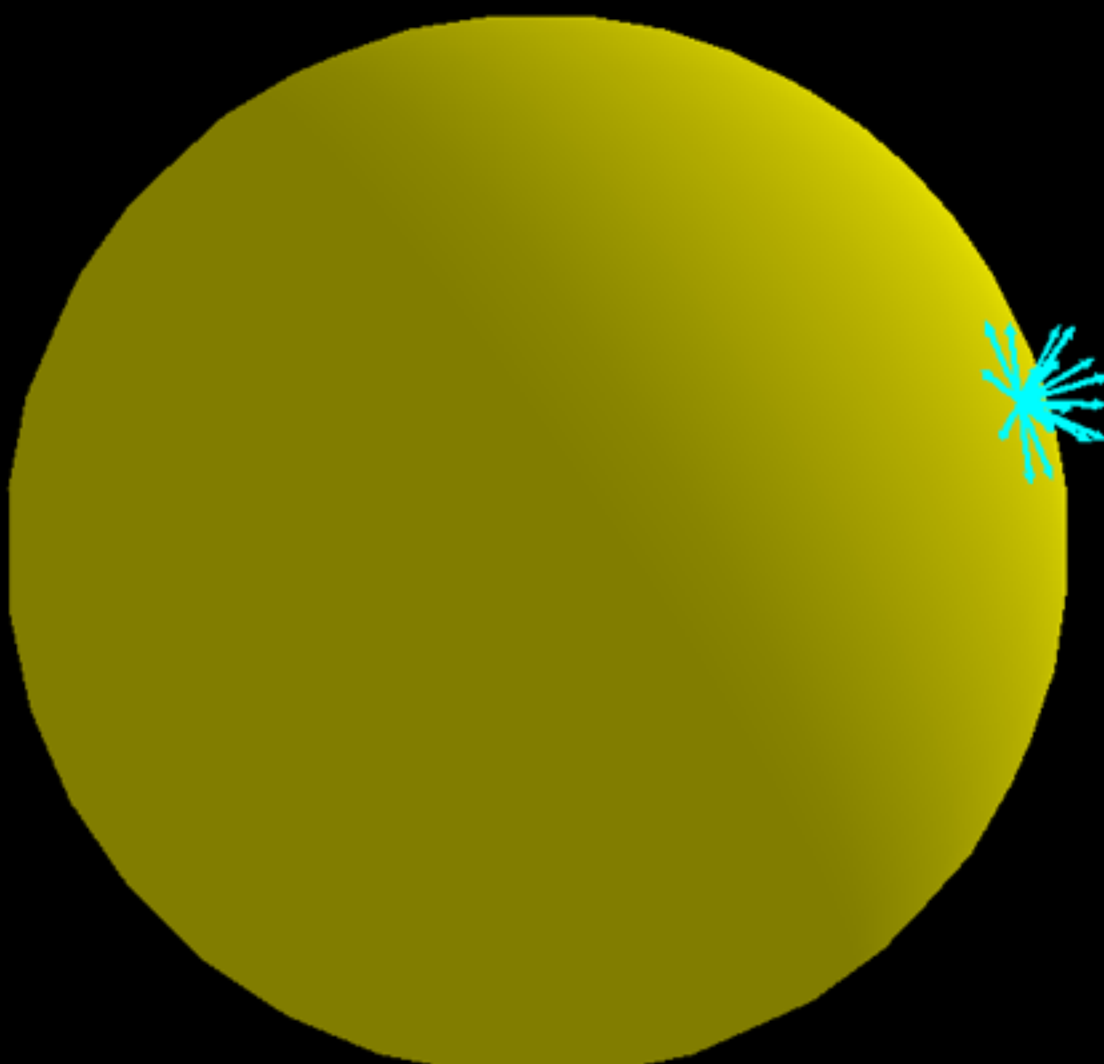
(0+1+2)D
Multi-Angle/Bulb Model



Azimuthal symmetry around any radial direction

$$\rho(r; E, \vartheta)$$

(0+1+3)D



Spherical symmetry about the center

$$\rho(r; E, \vartheta, \varphi)$$

z → r

L, R → φ

Raffelt et al (2013)
Mirizzi (2013)

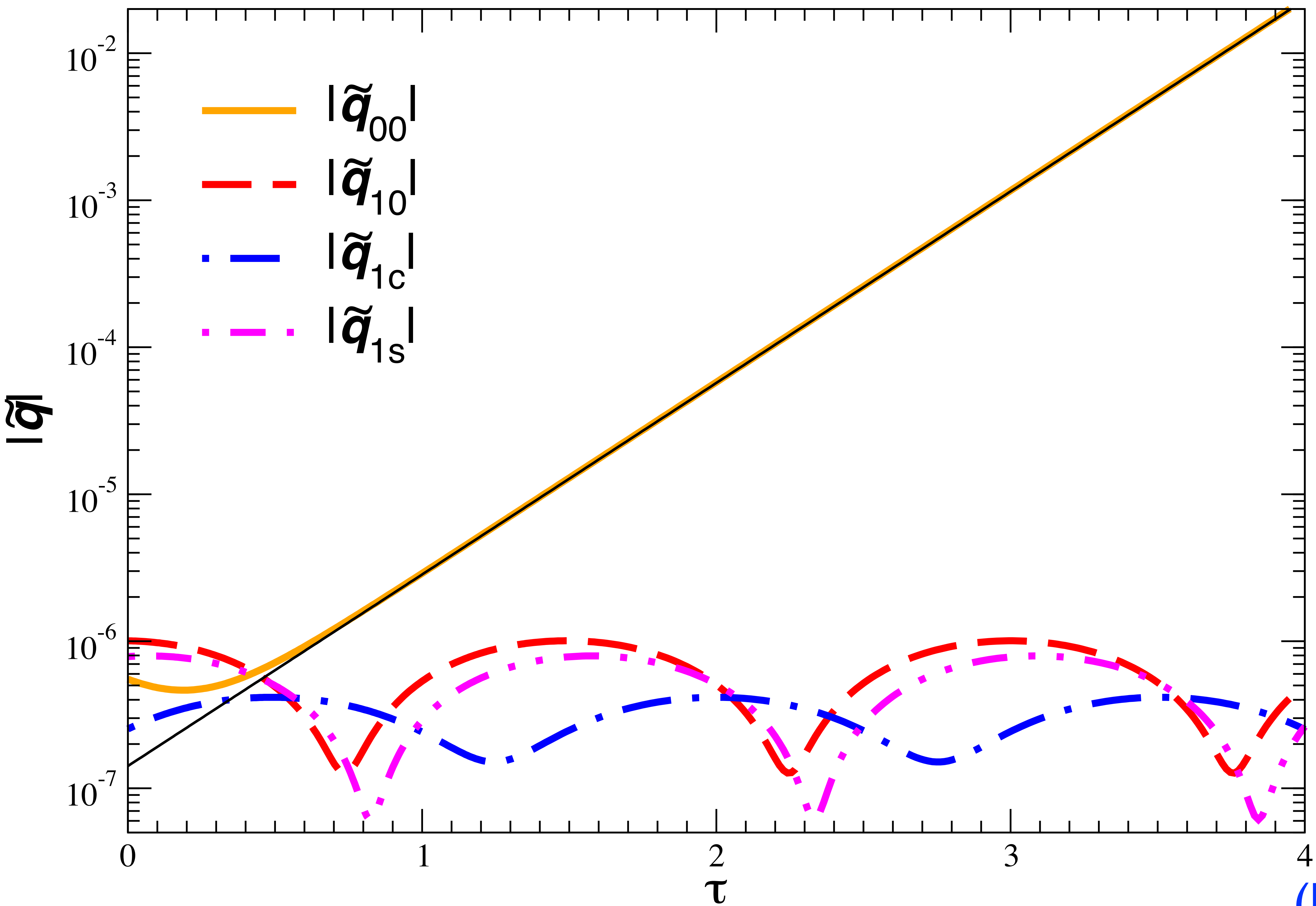
Homogeneous Gas

$$(1 - \hat{\mathbf{v}} \cdot \hat{\mathbf{v}}')\mu = 4\pi\mu \left[Y_{0,0}(\mathbf{p}) Y^*_{0,0}(\mathbf{p}') - \frac{1}{3} \sum_{m=0,\pm 1} Y_{1,m}(\mathbf{p}) Y^*_{1,m}(\mathbf{p}') \right]$$

- Multipole modes are decoupled in the linear regime
- l=0: μ_{eff}= μ, unstable in IH
- l=1: μ_{eff}= -μ/3 unstable in NH (breaking isotropy)

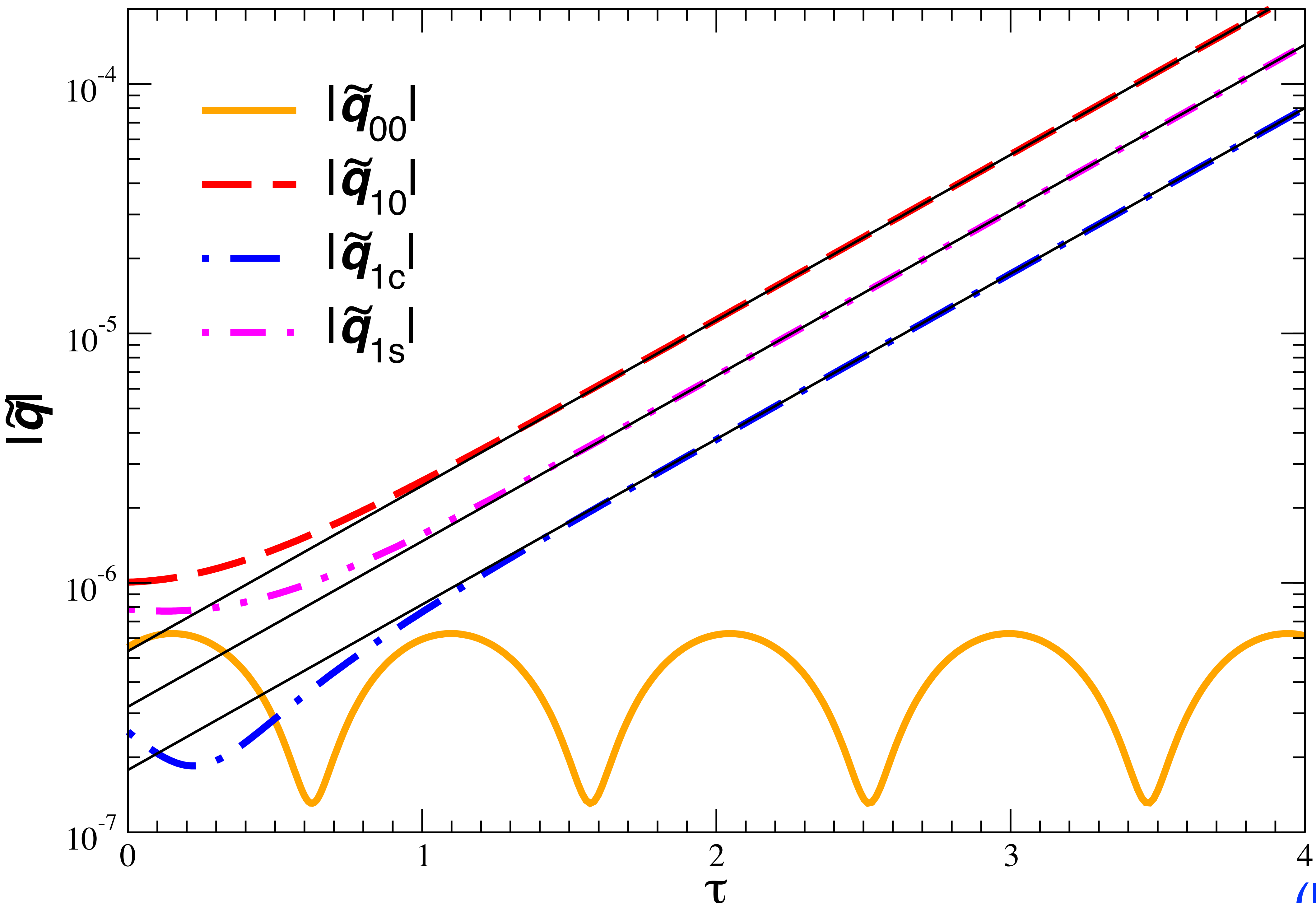
(HD, 2013)

Inverted Hierarchy



(HD, 2013)

Normal Hierarchy



(HD, 2013)

Conclusions

- Dimensionality of the supernova model matters in collective neutrino oscillations.
- Spontaneous breaking of directional symmetry in momentum space $\Longrightarrow$ Collective neutrino oscillations in both mass hierarchies

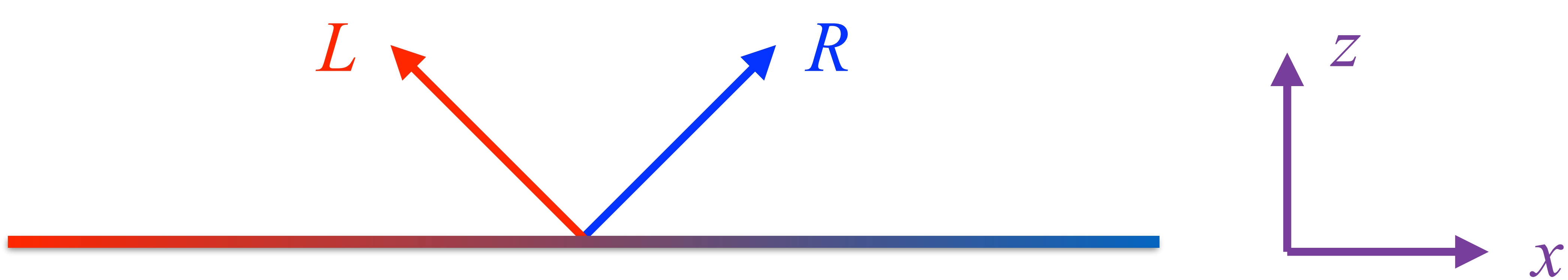
Line Model

time independent, ~~x translation symmetry~~, ~~left-right symmetry~~

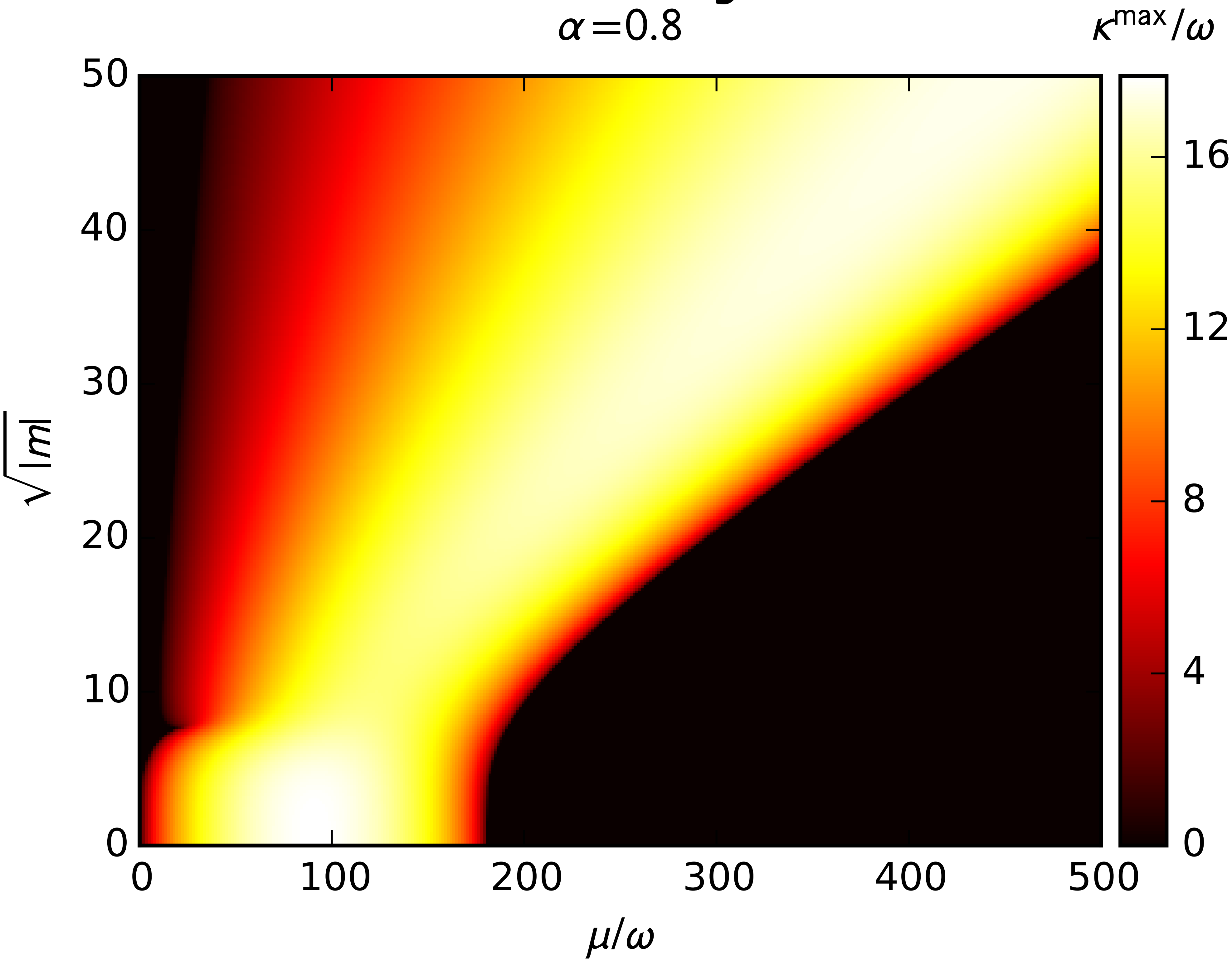
Periodic condition

$$\epsilon_m^{\pm} = \frac{1}{L}\int_0^L \left[\frac{\epsilon_L(x) \pm \epsilon_R(x)}{2}\right] e^{-\mathrm{i}(2m\pi)(x/L)}\,\mathrm{d}x$$

$$\mathrm{i}\partial_z \begin{bmatrix} \epsilon_m^+ \\ \bar{\epsilon}_m^+ \\ \epsilon_m^- \\ \bar{\epsilon}_m^- \end{bmatrix} = \Lambda_m \cdot \begin{bmatrix} \epsilon_m^+ \\ \bar{\epsilon}_m^+ \\ \epsilon_m^- \\ \bar{\epsilon}_m^- \end{bmatrix}$$



A Toy Model

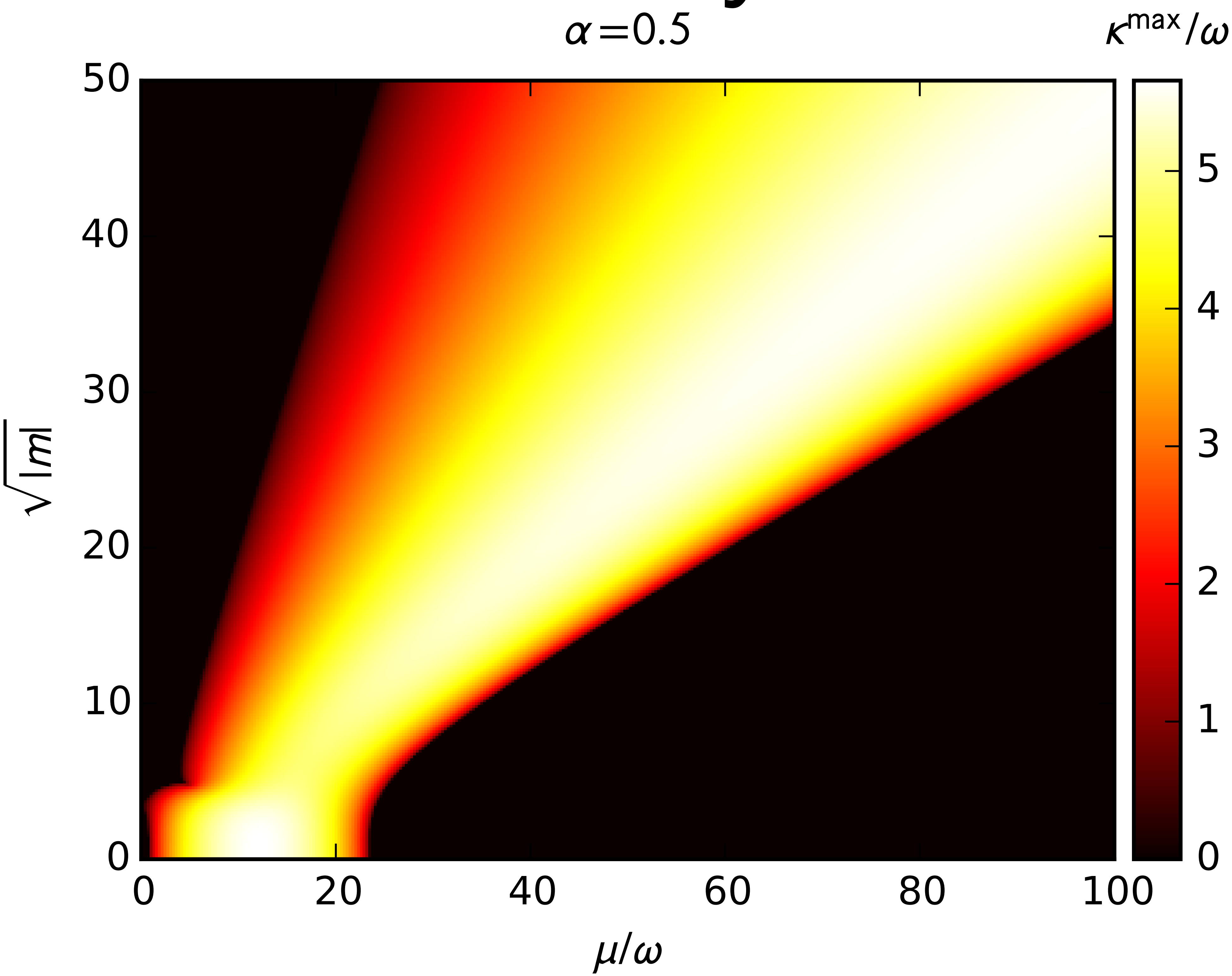


$\alpha = n_{\bar{\nu}}/n_{\nu}$

$\mu \propto G_{\mathrm{F}} n_{\nu}$

HD & Shalgar
1412.7097

A Toy Model

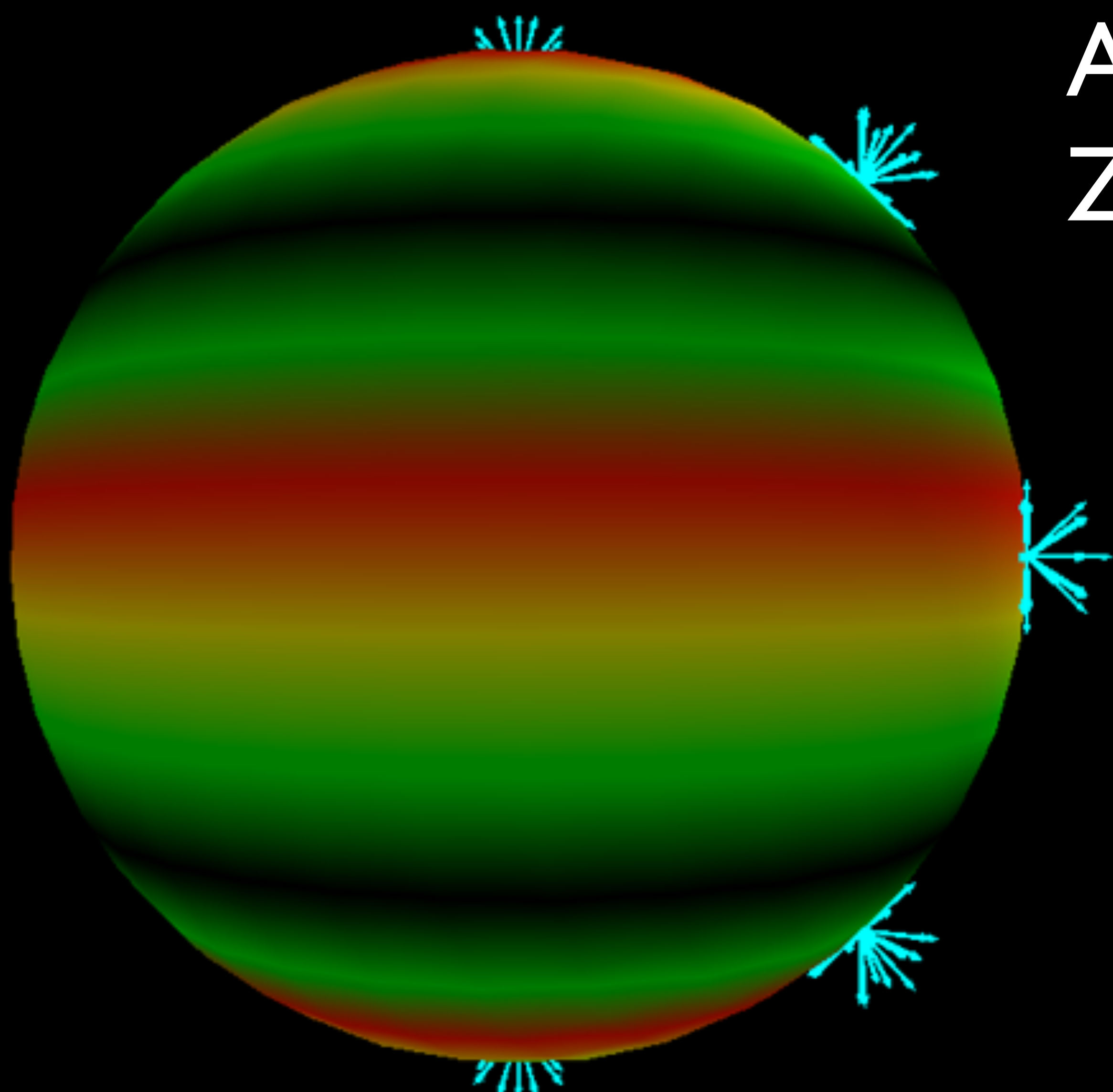


$\alpha = n_{\bar{\nu}}/n_\nu$

$\mu \propto G_F n_\nu$

HD & Shalgar
1412.7097

(0+2+2)D



Axial symmetry around the Z axis

$$\rho(r, \Theta; E, \cancel{\vartheta}, \varphi)$$

Z X L, R

Conclusions

- Dimensionality of the supernova model matters in collective neutrino oscillations.
- Spontaneous breaking of directional symmetry in momentum space $\Longrightarrow$ Collective neutrino oscillations in both mass hierarchies
- Spontaneous breaking of spatial symmetry in position space $\Longrightarrow$ Collective neutrino oscillations at larger density
- Stay tuned