Deriving the Nuclear Shell Model Microscopically

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Arizona's First University.



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Ab Initio Nuclear Structure and Reaction Theory

Tremendous progress has been made in the last 10 to 15 years in calculating the properties of nuclei microscopically due to:

1. Improved NN and NNN interactions based on Chiral EFT

2. New approaches for accurately solving the nuclear many-body problem, e.g., No-Core Shell Model (NCSM), Coupled Cluster method (CC), In-Medium Similarity Renormalization Group technique (IM-SRG), Gorkov-Greens Function theory (GGF), etc.

3. Huge advances in computer technology and computing power.





Towards a unified description of the nucleus The goal of nuclear structure theory:

exact treatment of nuclei based on NN, NNN,... interactions

rightarrow need to build a bridge between:

ab initio few-body & light nuclei calculations: $A \leq 24$ $0\hbar\Omega$ Shell Model calculations: $16 \leq A \leq 60$

Density Functional Theory calculations: $A \ge 60$

OUTLINE

- I. Overview of the No Core Shell Model (NCSM)
- II. Ab Initio Shell Model with a Core Approach
- III. Results: sd-shell
- IV. Summary/Outlook

I. Overview of the No Core Shell Model (NCSM)

No Core Shell Model

"Ab Initio" approach to microscopic nuclear structure calculations, in which <u>all A</u> nucleons are treated as being active.

Want to solve the A-body Schrödinger equation

 $H_{A}\Psi^{A} = E_{A}\Psi^{A}$

R P. Navrátil, J.P. Vary, B.R.B., PRC <u>62</u>, 054311 (2000) BRB, P. Navratil, J.P. Vary, Prog.Part.Nucl.Phys. 69, 131 (2013). P. Navratil, et al., J. Phys. G: Nucl. Part. Phys. 36, 083101 (2009)

From few-body to many-body



Flow chart for a standard NCSM calculation

No-Core Shell-Model Approach

Start with the purely intrinsic Hamiltonian

$$H_{A} = T_{rel} + \mathcal{V} = \frac{1}{A} \sum_{i < j=1}^{A} \frac{(\vec{p}_{i} - \vec{p}_{j})^{2}}{2m} + \sum_{i < j=1}^{A} V_{NN} \left(+ \sum_{i < j < k}^{A} V_{ijk}^{3b} \right)$$

Note: There are <u>no</u> phenomenological s.p. energies!

Can use <u>any</u> NN potentials Coordinate space: Argonne V8', AV18 Nijmegen I, II Momentum space: CD Bonn, EFT Idaho

No-Core Shell-Model Approach

Next, add CM harmonic-oscillator Hamiltonian

$$H_{CM}^{HO} = \frac{\vec{P}^{2}}{2Am} + \frac{1}{2}Am\Omega^{2}\vec{R}^{2}; \quad \vec{R} = \frac{1}{A}\sum_{i=1}^{A}\vec{r}_{i}, \quad \vec{P} = Am\dot{\vec{R}}$$

To H_A, yielding

$$H_{A}^{\Omega} = \sum_{i=1}^{A} \left[\frac{\vec{p}_{i}^{2}}{2m} + \frac{1}{2} m \Omega^{2} \vec{r}_{i}^{2} \right] + \underbrace{\sum_{i< j=1}^{A} \left[V_{NN}(\vec{r}_{i} - \vec{r}_{j}) - \frac{m \Omega^{2}}{2A} (\vec{r}_{i} - \vec{r}_{j})^{2} \right]}_{V_{ij}}$$

V_{ii}

Defines a basis (*i.e.* HO) for evaluating

Effective Interaction

Must truncate to a finite model space



- In general, V_{ij}^{eff} is an *A*-body interaction
- We want to make an *a*-body cluster approximation

$$\mathcal{H} = \mathcal{H}^{(I)} + \mathcal{H}^{(A)} \underset{a < A}{\gtrsim} \mathcal{H}^{(I)} + \mathcal{H}^{(a)}$$

Effective interaction in a projected model space $_{H\Psi_{\alpha}}^{A} = E_{\alpha}\Psi_{\alpha}$ where $H = \sum_{i=1}^{A} t_{i} + \sum_{i\leq j}^{A} v_{ij}$. $\mathcal{H}\Phi_{\beta} = E_{\beta}\Phi_{\beta}$ $\Phi_{\beta} = P\Psi_{\beta}$

P is a projection operator from S into S.

$$\langle \tilde{\Phi}_{\gamma} | \Phi_{\beta} \rangle = \delta_{\gamma\beta}$$

 $\mathcal{H} = \sum_{\beta \in S} | \Phi_{\beta} \rangle E_{\beta} \langle \tilde{\Phi}_{\beta} |$





- NCSM convergence test
 - Comparison to other methods



P. Navratil, INT Seminar, November 13, 2007, online



P. Navrátil and E. Caurier, Phys. Rev. C **69**, 014311 (2004)



II. Ab Initio Shell Model with a Core Approach



PHYSICAL REVIEW C 78, 044302 (2008)

Ab-initio shell model with a core

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We construct effective two- and three-body Hamiltonians for the *p*-shell by performing $12\hbar\Omega ab$ initio no-core shell model (NCSM) calculations for A = 6 and 7 nuclei and explicitly projecting the many-body Hamiltonians onto the $0\hbar\Omega$ space. We then separate these effective Hamiltonians into inert core, one- and two-body contributions (also three-body for A = 7) and analyze the systematic behavior of these different parts as a function of the mass number A and size of the NCSM basis space. The role of effective three- and higher-body interactions for A > 6is investigated and discussed.

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PACS number(s): 21.10.Hw, 21.60.Cs, 23.20.Lv, 27.20.+n

P. Navratil, M. Thoresen and B.R.B., Phys. Rev. C 55, R573 (1997)

FORMALISM

1. Perform a large basis NCSM for a core + 2N system, e.g., 18^AF.

- 2. Use Okubo-Lee-Suzuki transformation to project these results into a single major shell to obtain effective 2-body matrix elements.
- 3. Separate these 2-body matrix elements into a core term, singleparticle energies and residual 2-body interactions, i.e., the standard input for a normal Shell Model calculation.
- 4. Use these values for performing SM calculations in that shell.





Effective Hamiltonian for SSM How to calculate the Shell Model 2-body effective interaction:

Two ways of convergence:

1) For $P \rightarrow 1$ and fixed a: $H^{eff}_{A,a=2} \rightarrow H_A$: previous slide

2) For $a_1 \rightarrow A$ and fixed P_1 : $H_{Aa1}^{eff} \rightarrow H_A$

 $P_1 + Q_1 = P;$ P_1 - small model space; Q_1 - excluded space;

$$\mathcal{H}_{A,a_{1}}^{N_{1,\max},N_{\max}} = \frac{U_{a_{1},P_{1}}^{A,\dagger}}{\sqrt{U_{a_{1},P_{1}}^{A,\dagger}U_{a_{1},P_{1}}^{A}}} E_{A,a_{1},P_{1}}^{N_{\max},\Omega} \frac{U_{a_{1},P_{1}}^{A}}{\sqrt{U_{a_{1},P_{1}}^{A,\dagger}U_{a_{1},P_{1}}^{A}}}$$

Valence Cluster Expansion

 $N_{1,max} = 0$ space (p-space); $a_1 = A_c + a_v$; a_1 - order of cluster; A_c - number of nucleons in core; a_v - order of valence cluster;

$$\mathcal{H}_{A,a_1}^{0,N_{\max}} = \sum_k^{a_v} V_k^{A,A_c+k}$$

III. Results: sd-shell nuclei

Accepted for publication in PRC

Ab initio effective interactions for sd-shell valence nucleons

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We perform *ab initio* no core shell model calculations for A = 18 and 19 nuclei in a $4\hbar\Omega$, or $N_{\rm max} = 4$, model space using the effective JISP16 and chiral N3LO nucleon-nucleon potentials and transform the many-body effective Hamiltonians into the $0\hbar\Omega$ model space to construct the A-body effective Hamiltonians in the *sd*-shell. We separate the A-body effective Hamiltonians with A = 18 and A = 19 into inert core, one- and two-body components. Then, we use these core, one- and two-body components to perform standard shell model calculations for the A = 18 and A = 19 systems with valence nucleons restricted to the *sd*-shell. Finally, we compare the standard shell model results in the $0\hbar\Omega$ model space with the exact no core shell model results in the $4\hbar\Omega$ model space for the A = 18 and A = 19 systems and find good agreement.

ArXiv: Nucl-th 1502.00700

Empirical Single-Particle Energies



 $\mathcal{H}^{sd}(\underline{P}\underline{\Psi})^{sd} = \left\{ \underbrace{\overset{sd}{\underset{i}{\underset{i}{\overset{sd}{\underset{i}{\overset{sd}{\underset{i}{\overset{sd}{\underset{i}{\overset{sd}{\underset{i}{\overset{sd}{\underset{i}{\overset{sd}{\underset{i}{\underset{i}{\underset{i}{\overset{sd}{\underset{i}{\overset{sd}{\underset{i}{\underset{i}{\overset{sd}{\underset{i}{\underset{i}{\underset{i}{\overset{sd}{\underset{i}{\underset{i}{\atopsd}{\underset{i}{\underset{i}{\atopsd}{\atopsd}{\underset{i}{\underset{i}{\atopsd}}}}}}}}}}}}}}}}}}}}}}}}_{i}, i \to \mathcal{V}^{sd}}})^{sd}}}}}}}}}}}}}}}}}}}}}}}}}}}$

Input: The results of N_max = 4 and hw = 14 MeV NCSM calculations

TABLE II: P	roton and n	neutron s	single-particle	energies (ir	ı
MeV) for JISI	P16 effective	interact	ion obtained	for the mass	5
of $A = 18$ and	A = 19.				

	A = 18			A = 19			
	$E_{\rm core} = -115.529$			$E_{\rm core} = -115.319$			
j_i	$\frac{1}{2}$	<u>5</u> 2	3 2	$\frac{1}{2}$	5 2	$\frac{3}{2}$	
$\epsilon_{j_i}^n$	-3.068	-2.270	6.262	-3.044	-2.248	6.289	
$\epsilon^p_{j_i}$	0.603	1.398	9.748	0.627	1.419	9.774	

TABLE III: Proton and neutron single-particle energies (in MeV) for chiral N3LO effective interaction obtained for the mass of A = 18 and A = 19.

	A = 18			A = 19			
	$E_{\rm core} = -118.469$			$E_{\rm core} = -118.306$			
j_i	$\frac{1}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	<u>5</u> 2	$\frac{3}{2}$	
$\epsilon_{j_i}^n$	-3.638	-3.042	3.763	-3.625	-3.031	3.770	
$\epsilon^p_{j_i}$	0.044	0.690	7.299	0.057	0.700	7.307	

A = 18

A = 19

Coupled Cluster, E_core: -130.462 Idaho NN N3LO + 3N N2LO -130.056 from G.R. Jansen et al. PRL 113, 142502 (2014)

IM-SRG, E_core:-130.132-129.637from H. HergertIdaho NN N3LO + 3N N2LOprivate comm.

No-Core Shell-Model Approach

Next, add CM harmonic-oscillator Hamiltonian

$$H_{CM}^{HO} = \frac{\vec{P}^{2}}{2Am} + \frac{1}{2}Am\Omega^{2}\vec{R}^{2}; \quad \vec{R} = \frac{1}{A}\sum_{i=1}^{A}\vec{r}_{i}, \quad \vec{P} = Am\dot{\vec{R}}$$

To H_A, yielding

$$H_{A}^{\Omega} = \sum_{i=1}^{A} \left[\frac{\vec{p}_{i}^{2}}{2m} + \frac{1}{2} m \Omega^{2} \vec{r}_{i}^{2} \right] + \underbrace{\sum_{i< j=1}^{A} \left[V_{NN}(\vec{r}_{i} - \vec{r}_{j}) - \frac{m \Omega^{2}}{2A} (\vec{r}_{i} - \vec{r}_{j})^{2} \right]}_{V_{ij}}$$

V_{ii}

Defines a basis (*i.e.* HO) for evaluating









TABLE III: The NCSM energies (in MeV) of the lowest 28 states J_i^{π} of ¹⁸F calculated in $4\hbar\Omega$ model space using JISP16 and chiral N3LO NN interactions with $\hbar\Omega = 14$ MeV.

J_i^{π}	Т	JISP16	J_i^{π}	Т	N3LO
11	0	-122.742	11	0	-126.964
31+	0	-122.055	31+	0	-126.214
01	1	-121.320	01+	1	-125.510
5_{1}^{+}	0	-120.329	5_{1}^{+}	0	-124.545
2_{1}^{+}	1	-119.505	2_{1}^{+}	1	-123.974
2^{+}_{2}	0	-119.011	2^{+}_{2}	0	-123.890
1_{2}^{+}	0	-118.709	1_{2}^{+}	0	-123.077
0^{+}_{2}	1	-118.410	0^{+}_{2}	1	-122.586
2_{3}^{+}	1	-117.211	2^{+}_{3}	1	-121.588
3^{+}_{2}	1	-117.035	4_{1}^{+}	1	-121.512
41	1	-117.004	3^{+}_{2}	1	-121.450
3_{3}^{+}	0	-116.765	3_{3}^{+}	0	-121.376
1_{3}^{+}	0	-113.565	1_{3}^{+}	0	-119.658
4_{2}^{+}	0	-112.314	4^{+}_{2}	0	-118.656
2_{4}^{+}	0	-111.899	2_{4}^{+}	0	-117.950
1_{4}^{+}	0	-110.357	1_{4}^{+}	0	-116.106
4_{3}^{+}	1	-109.625	4_{3}^{+}	1	-115.785
2_{5}^{+}	1	-109.292	2_{8}^{+}	1	-115.407
1_{5}^{+}	1	-108.752	3_{4}^{+}	0	-115.309
3_{4}^{+}	0	-108.706	1_{5}^{+}	1	-114.870
2_{6}^{+}	0	-108.485	2_{6}^{+}	0	-114.787
1_{6}^{+}	1	-108.055	1_{6}^{+}	1	-114.392
2_{7}^{+}	1	-108.041	3_{8}^{+}	1	-114.258
3_{5}^{+}	1	-107.874	2^{+}_{7}	1	-114.176
3_{6}^{+}	0	-101.528	3_{6}^{+}	0	-109.316
1^{+}_{7}	0	-99.946	1_{7}^{+}	0	-107.798
0_{3}^{+}	1	-99.848	2_{8}^{+}	1	-107.473
2_{8}^{+}	1	-99.607	03+	1	-107.436





Comparison of effective TBMEs in the sd-shell: JISP16 vs USDA by Alex Brown et al.

Preliminary Results

PRELIMINARY RESULTS



Preliminary Results



Summary

Perform a converged NCSM calculation with a NN or NN+NNN interaction for a closed core + 2 valence nucleon system.

An OLS transformation of the results of the above NCSM calculation into a single major shell allows one to obtain core and single-particle energies and two-body residual matrix elements appropriate for shell model calculations in that shell, which have only a weak A-dependence.

The core and single-particle energies and two-body residual matrix elements obtained by this procedure can be used in Standard Shell Model calculations in the sd-shell, yielding results in good agreement with the full space NCSM results. The core and s.p. energies + 2-body effective interactions for A=18 give also good results for A=19 and 20.

Additional calculations are being performed with other NN interactions and for heavier nuclei in the sd-shell.

COLLABORATORS

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Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner, Nogga, Machleidt,...A. Schwenk

Realistic two-body potentials in coordinate and momentum space

Repulsive core makes calculations difficult

Illustration on how the high momentum nodes are integrated out in the Vlowk (a) and in the SRG (b) RG methods

- → Need to decouple high/low momentum modes
- ✓ Achieved by V_{low-k} or Similarity RG approaches (e.g. SRG)

Fig. from S. Bogner et al Prog.Part.Nucl.Phys.65:94-147,2010

- → Observable physics is preserved (e.g. NN phase shifts) AND calculations become easier (work with the relevant degrees of freedom)
- → One has to deal with "induced" many-body forces...

3-body Valence Cluster approximation for A>6

Construct 3-body interaction in terms of 3-body matrix elements: Yes

$$V_3^{A,7} = \mathcal{H}_{A,7}^{0,N_{\max}} - \mathcal{H}_{A,6}^{0,N_{\max}}$$

Ab-initio coupled-cluster effective interactions for the shell model: Application to neutron-rich oxygen and carbon isotopes

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We derive and compute effective valence-space shell-model interactions from ab-initio coupledcluster theory and apply them to open-shell and neutron-rich oxygen and carbon isotopes. Our shell-model interactions are based on nucleon-nucleon and three-nucleon forces from chiral effectivefield theory. We compute the energies of ground and low-lying states, and find good agreement with experiment. In particular our calculations are consistent with the N = 14, 16 shell closures in 22,24 O, while for 20 C the corresponding N = 14 closure is weaker. We find good agreement between our coupled-cluster effective-interaction results with those obtained from standard single-reference coupled-cluster calculations for up to eight valence neutrons.

PACS numbers: 21.30.Fe, 21.60.Cs, 21.60.De, 21.10.-k

arXiv: 1402.2563v1 [nuch-th] 11 Feb. 2014

Core energies: A=18, -130.462; A=19, -130.056 (MeV)