Collective neutrino oscillations

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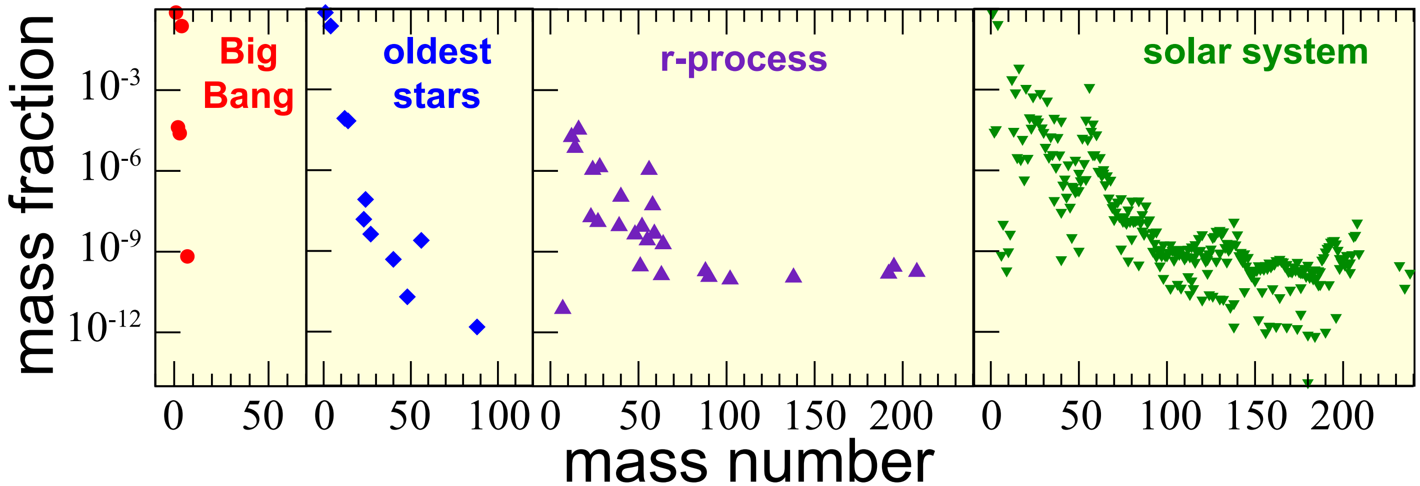
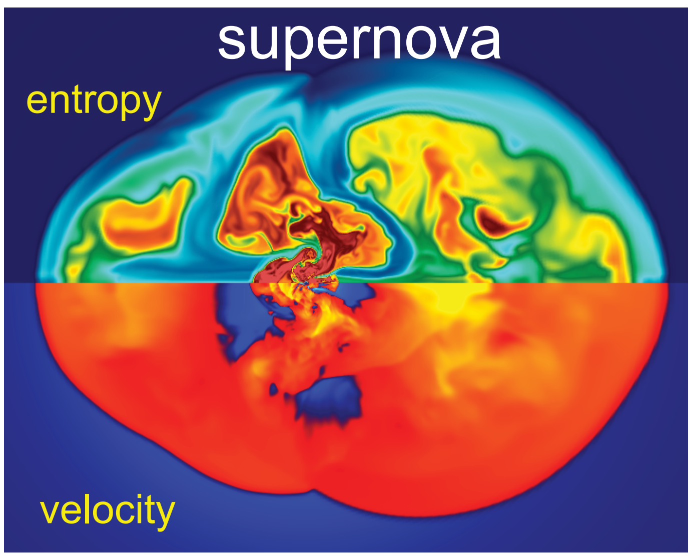
INT-15-2a
Neutrino
Astrophysics
and
Fundamental
Properties
June 2015

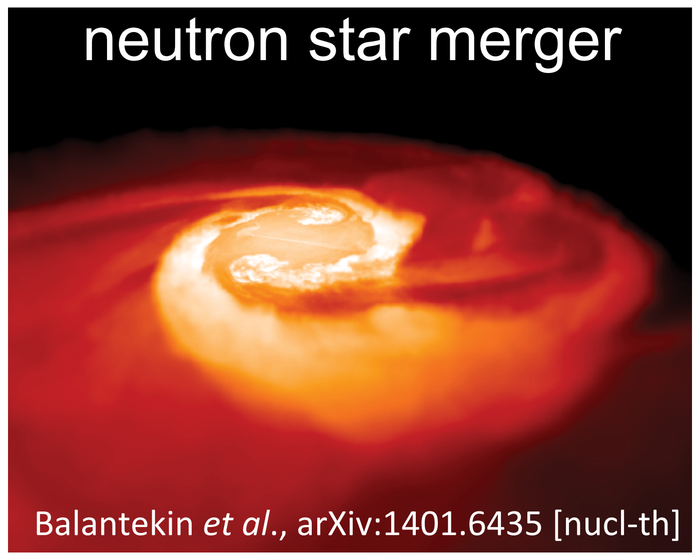
THE UNIVERSITY of WISCONSIN MADISON

NSF

The origin of elements





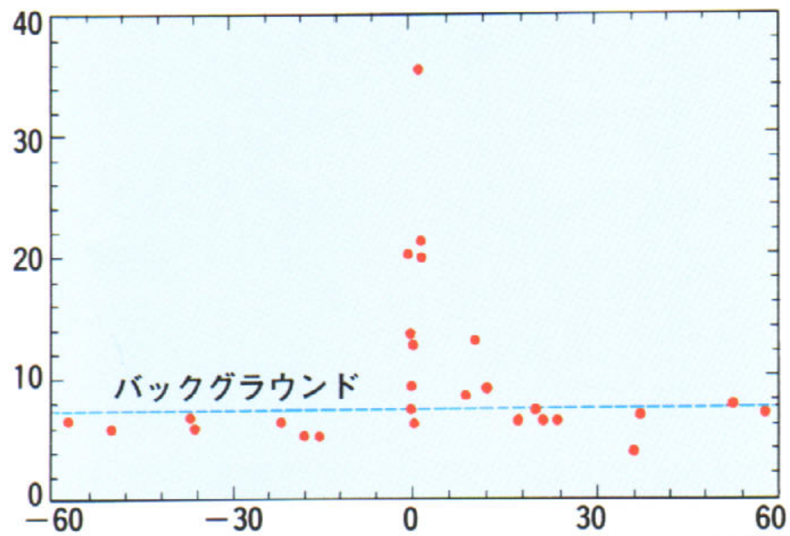


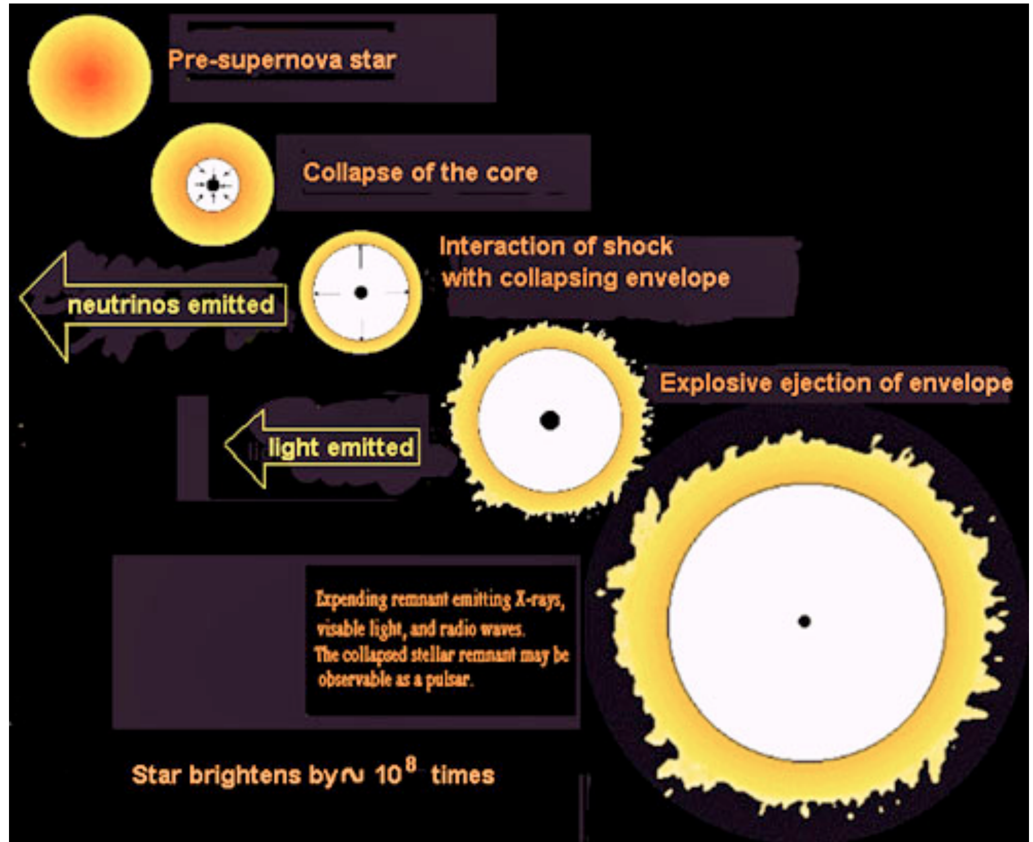
Balantekin *et al.*, arXiv:1401.6435 [nucl-th]

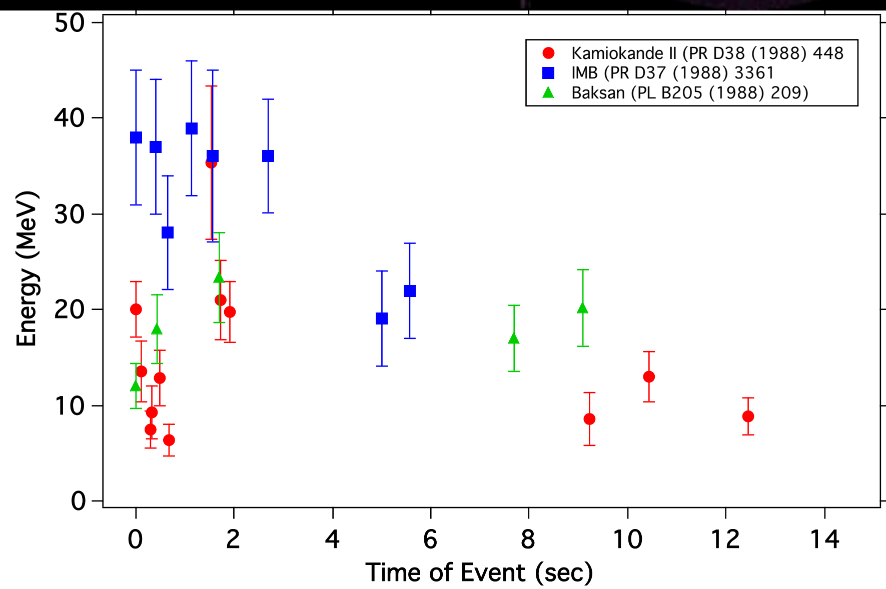
Possible sites for the r-process

Neutrinos not only play a crucial role in the dynamics of these sites, but they also control the value of the electron fraction, the parameter determining the yields of the r-process.

Neutrinos from core-collapse supernovae





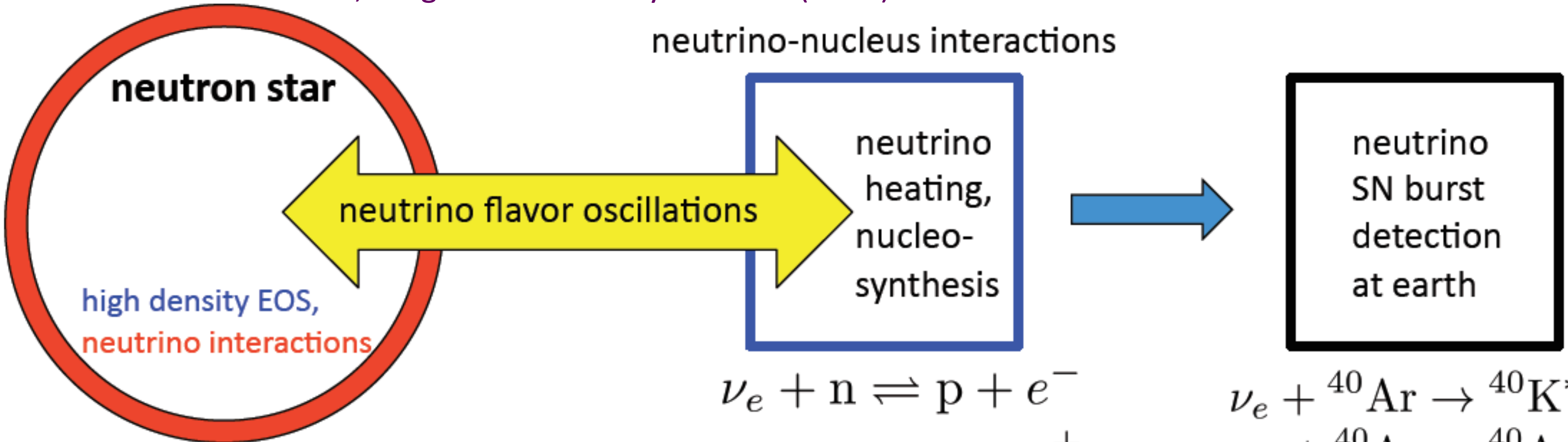


• $M_{prog} \geq 8\ M_{sun} \Rightarrow \Delta E \approx 10^{53}$ ergs $\approx 10^{59}$ MeV

• 99% of the energy is carried away by neutrinos and antineutrinos with $10 \leq E_\nu \leq 30$ MeV $\Rightarrow 10^{58}$ neutrinos

For example understanding a core-collapse supernova requires answers to a variety of questions some of which need to be answered by nuclear physics, both theoretically and experimentally.

Balantekin and Fuller, Prog. Part. Nucl. Phys. **71** 162 (2013)



$$\nu_e + \mathrm{n} \rightleftharpoons \mathrm{p} + e^-$$
$$\bar{\nu}_e + \mathrm{p} \rightleftharpoons \mathrm{n} + e^+$$

$$\nu_e + {}^{40}\mathrm{Ar} \to {}^{40}\mathrm{K}^* + e^-$$
$$\nu_x + {}^{40}\mathrm{Ar} \to {}^{40}\mathrm{Ar}^* + \nu_x$$

If we want to catch a supernova with neutrinos we'd better know what neutrinos do inside a supernova.

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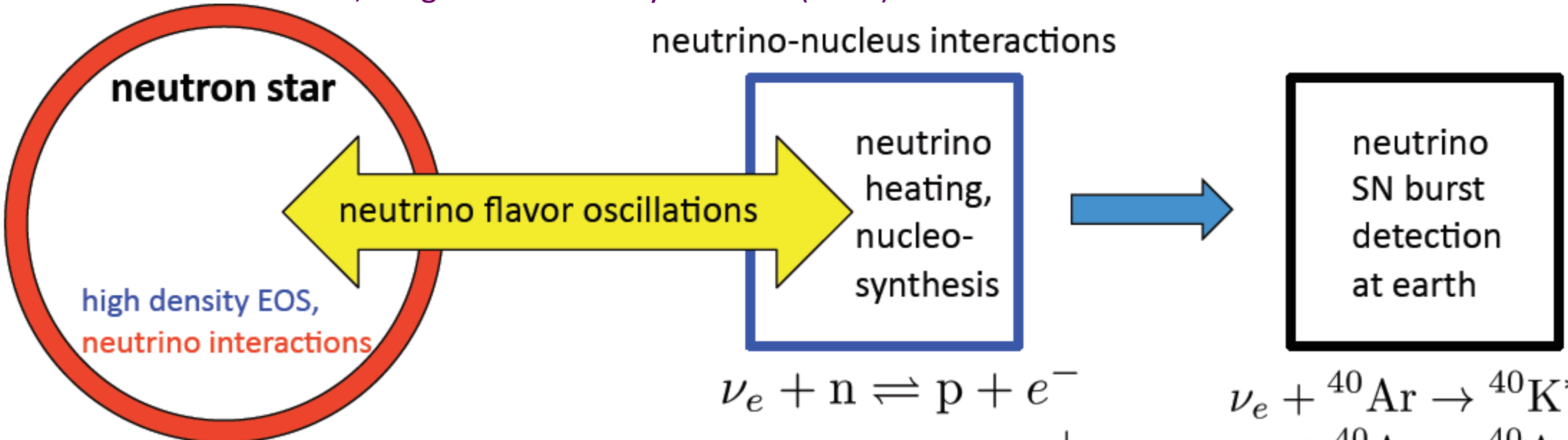
Balantekin and Fuller, Prog. Part. Nucl. Phys. **71** 162 (2013)



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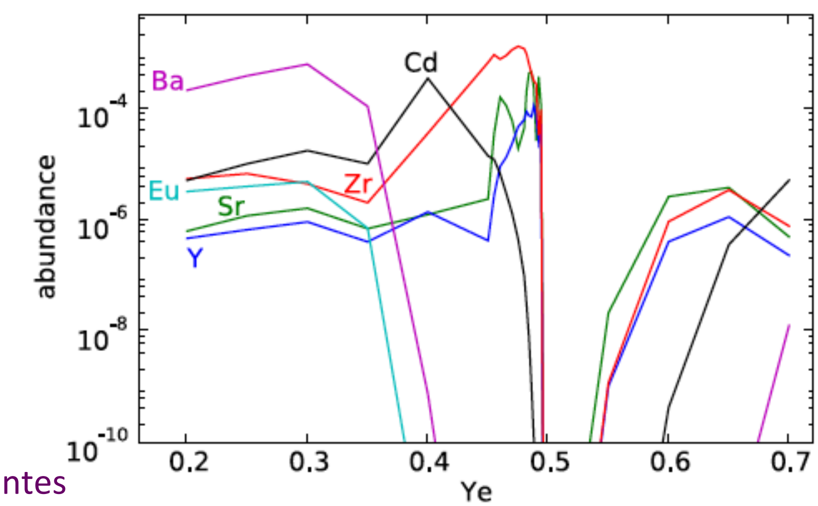
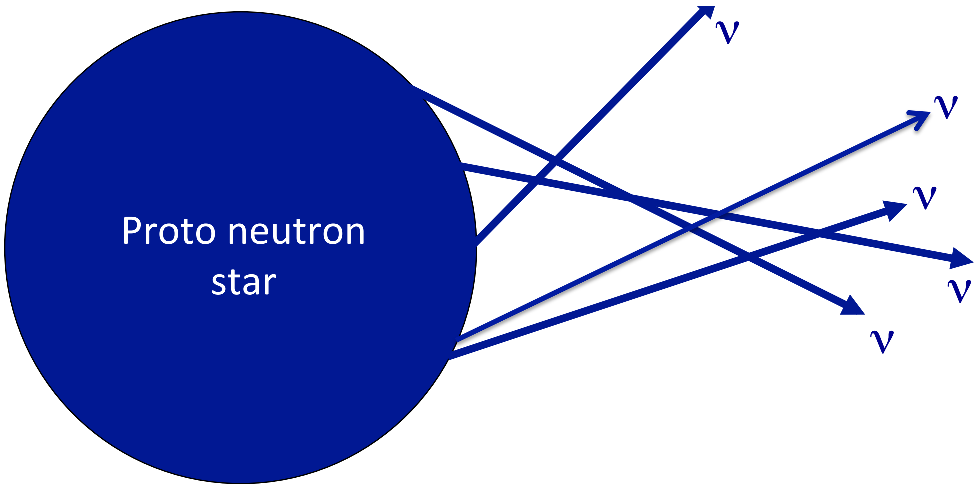
$$Y_e = \frac{N_p}{N_p + N_n} = \frac{1}{1 + \lambda_p / \lambda_n}$$



Arcones and Montes



Energy released in a core-collapse SN: $\Delta E \approx 10^{53}$ ergs $\approx 10^{59}$ MeV
99% of this energy is carried away by neutrinos and antineutrinos!
~ 10^{58} Neutrinos!
This necessitates including the effects of $\nu\nu$ interactions!

$$H = \underbrace{\sum a^\dagger a}_{\text{describes neutrino oscillations interaction with matter (MSW effect)}} + \underbrace{\sum (1-\cos\theta) a^\dagger a^\dagger a a}_{\text{describes neutrino-neutrino interactions}}$$

The second term makes the physics of a neutrino gas in a core-collapse supernova a very interesting many-body problem, driven by weak interactions.

Neutrino-neutrino interactions lead to novel collective and emergent effects, such as conserved quantities and interesting features in the neutrino energy spectra (spectral "swaps" or "splits").

Many neutrino system

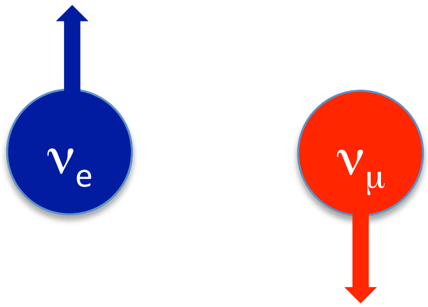
This is the only many-body system driven by the weak interactions:

Table: Many-body systems

Nuclei	Strong	at most $\sim$250 particles
Condensed matter	E&M	at most N_A particles
ν**'s in SN**	Weak	$\sim 10^{58}$particles

Astrophysical extremes allow us to test physics that cannot be tested elsewhere!

Neutrino flavor isospin



$$\hat{J}_+ = a_e^\dagger a_\mu \qquad \hat{J}_- = a_\mu^\dagger a_e$$

$$\hat{J}_0 = \frac{1}{2}\left(a_e^\dagger a_e - a_\mu^\dagger a_\mu\right)$$

These operators can be written in either mass or flavor basis

Free neutrinos (only mixing)

$$\hat{H} = \frac{m_1^2}{2E} a_1^\dagger a_1 + \frac{m_2^2}{2E} a_2^\dagger a_2 + (\cdot\cdot)\hat{1}$$

$$= \frac{\delta m^2}{4E}\cos 2\theta\left(-2\hat{J}_0\right) + \frac{\delta m^2}{4E}\sin 2\theta\left(\hat{J}_+ + \hat{J}_-\right) + (\cdot\cdot)'\hat{1}$$

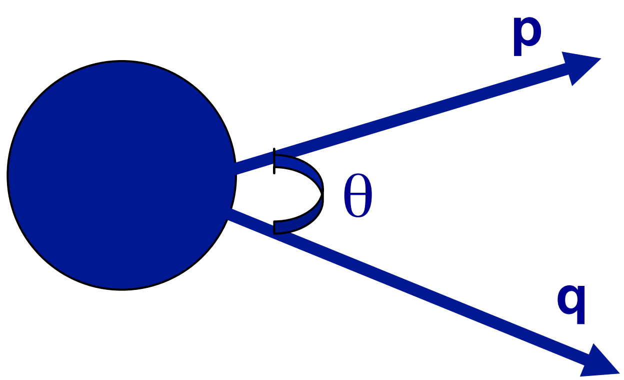
Interacting with background electrons

$$\hat{H} = \left[\frac{\delta m^2}{4E}\cos 2\theta - \frac{1}{\sqrt{2}} G_F N_e\right]\left(-2\hat{J}_0\right) + \frac{\delta m^2}{4E}\sin 2\theta\left(\hat{J}_+ + \hat{J}_-\right) + (\cdot\cdot)''\hat{1}$$

Neutrino-Neutrino Interactions

Smirnov, Fuller and Qian, Pantaleone, McKellar, Friedland, Lunardini, Raffelt, Duan, Balantekin, Kajino, Pehlivan …

$$\hat{H}_{\nu\nu} = \frac{\sqrt{2}G_F}{V}\int dp\,dq\left(1-\cos\theta_{pq}\right)\vec{\mathbf{J}}_p\cdot\vec{\mathbf{J}}_q$$



This term makes the physics of a neutrino gas in a core-collapse supernova a genuine many-body problem

$$\hat{H} = \int dp\left(\frac{\delta m^2}{2E}\vec{\mathbf{B}}\cdot\vec{\mathbf{J}}_p - \sqrt{2}G_F N_e \mathbf{J}_p^0\right) + \frac{\sqrt{2}G_F}{V}\int dp\,dq\left(1-\cos\theta_{pq}\right)\vec{\mathbf{J}}_p\cdot\vec{\mathbf{J}}_q$$

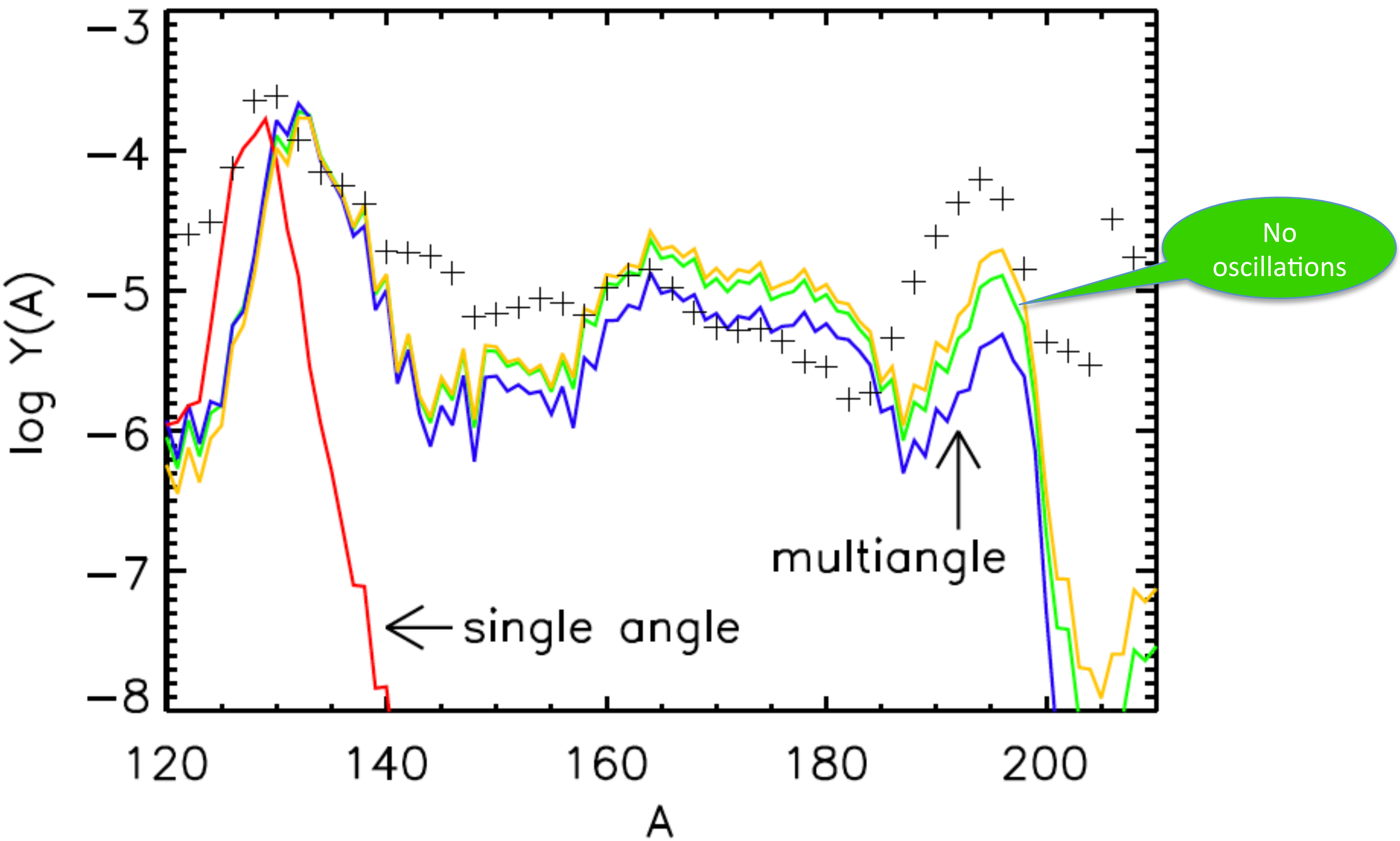
$$\vec{\mathbf{B}} = \left(\sin 2\theta,\ 0, -\cos 2\theta\right)$$

Neutrino-neutrino interactions lead to novel collective and emergent effects, such as conserved quantities and interesting features in the neutrino energy spectra (spectral "swaps" or "splits").

Conserved quantities of the collective motion in the single-angle limit

$$h_p = \vec{\mathbf{B}} \cdot \vec{\mathbf{J}}_p + \frac{4\sqrt{2}G_F}{\delta m^2 V} \sum_{p \neq q} qp \frac{\vec{\mathbf{J}}_p \cdot \vec{\mathbf{J}}_q}{q-p}$$

- There is a second set of conserved quantities for antineutrinos.
- Note the presence of volume. In fact h_p/V are the conserved quantities for the neutrino densities.
- For three flavors a similar expression is written in terms of SU(3) operators.



Duan, Friedland, McLaughlin, Surman

Single-angle approximation Hamiltonian:

$$H = \sum_p \frac{\delta m^2}{2p} J_p^0 + 2\mu \sum_{\substack{p,\,q \\ p \neq q}} \mathbf{J}_p \bullet \mathbf{J}_q$$

Eigenstates:

$$|x_i\rangle = \prod_{i=1}^N \sum_k \frac{J_k^\dagger}{\left(\delta m^2/2k\right) - x_i} |0\rangle$$

$$-\frac{1}{2\mu} - \sum_k \frac{j_k}{\left(\delta m^2/2k\right) - x_i} = \sum_{j \neq i} \frac{1}{x_i - x_j}$$

Bethe ansatz equations

$$\mu = \frac{G_F}{\sqrt{2}V} \langle 1 - \cos\Theta \rangle$$

Invariants:

$$h_p = J_p^0 + 2\mu \sum_{\substack{p,\,q \\ p \neq q}} \frac{\mathbf{J}_p \bullet \mathbf{J}_q}{\delta m^2 \left(\frac{1}{p} - \frac{1}{q}\right)}$$

Pehlivan, ABB, Kajino, & Yoshida
Phys. Rev. D 84, 065008 (2011)

To understand spectral splits consider a simple model with two neutrino energies

$$H = \sum_{p=1,2} \omega_p J_p^0 + 2\mu \sum_{\substack{p,\, q=1,2 \\ p\neq q}} \mathbf{J}_p \bullet \mathbf{J}_q \Rightarrow \begin{cases} \mu \to \infty \quad H = 2\mu \sum\limits_{\substack{p,\, q=1,2 \\ p\neq q}} \mathbf{J}_p \bullet \mathbf{J}_q \\ \mu \to 0 \quad H = \sum\limits_{p=1,2} \omega_p J_p^0 \end{cases}$$

Wavefunctions

$\underline{\mu \to \infty}$	$\underline{\mu \to 0}$
$\frac{1}{\sqrt{2}}\left(J_1^+ + J_2^+\right)\vert 0\rangle$	$J_1^+\vert 0\rangle$
$\frac{1}{\sqrt{2}}\left(J_1^+ - J_2^+\right)\vert 0\rangle$	$J_2^+\vert 0\rangle$

$$J_1^-|0\rangle = 0, \quad J_2^-|0\rangle = 0$$

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$$J_1^-\lvert 0\rangle = 0, \quad J_2^-\lvert 0\rangle = 0$$

To determine the corresponding states use the solutions of Bethe ansatz equations

To understand spectral splits consider a simple model with two neutrino energies

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Wavefunctions

$\mu \to \infty$	$\mu \to 0$
$\frac{1}{\sqrt{2}}\left(J_1^+ + J_2^+\right)\|0\rangle$	$J_1^+\|0\rangle$
$\frac{1}{\sqrt{2}}\left(J_1^+ - J_2^+\right)\|0\rangle$	$J_2^+\|0\rangle$

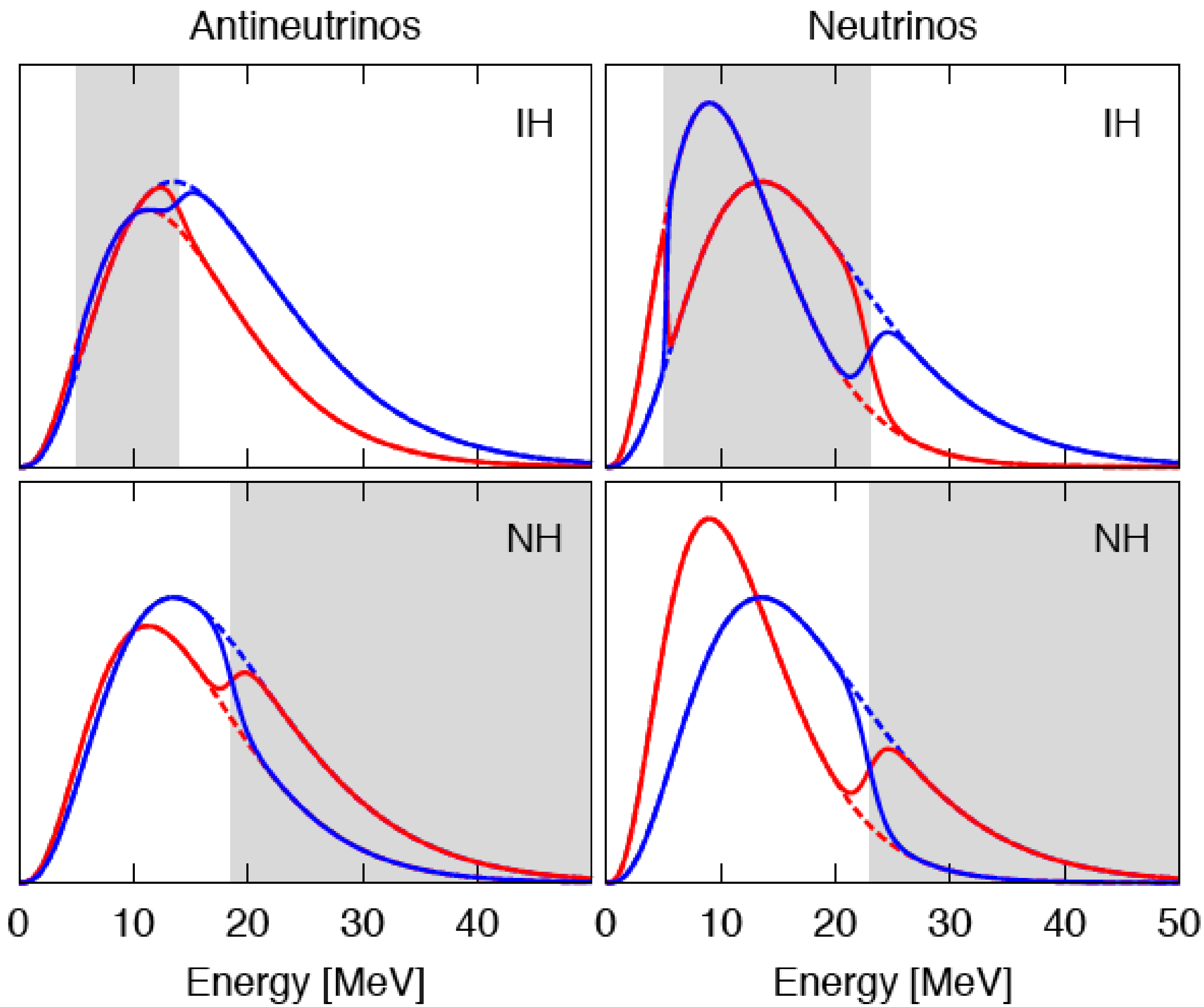
$$J_1^-|0\rangle = 0, \quad J_2^-|0\rangle = 0$$

$$\langle N_e(\omega_1)\rangle_{\text{before}} = \frac{N}{2} \qquad \langle N_e(\omega_1)\rangle_{\text{after}} = 0$$

$$\langle N_e(\omega_2)\rangle_{\text{before}} = \frac{N}{2} \qquad \langle N_e(\omega_2)\rangle_{\text{after}} = N$$

$$\langle N_x(\omega_1)\rangle_{\text{before}} = \frac{N}{2} \qquad \langle N_x(\omega_1)\rangle_{\text{after}} = N$$

$$\langle N_x(\omega_2)\rangle_{\text{before}} = \frac{N}{2} \qquad \langle N_x(\omega_2)\rangle_{\text{after}} = 0$$



Dasgupta *et al.*, 2009

What is the mean-field approximation?

$$\left[\hat{O}_1, \hat{O}_2\right] \cong 0$$

$$\hat{O}_1\hat{O}_2 \approx \hat{O}_1\left\langle\hat{O}_2\right\rangle + \left\langle\hat{O}_1\right\rangle\hat{O}_2 - \left\langle\hat{O}_1\hat{O}_2\right\rangle$$

Expectation values should be calculated with a state $\left|\Psi\right\rangle$ chosen to satisfy:

$$\left\langle\hat{O}_1\hat{O}_2\right\rangle = \left\langle\hat{O}_1\right\rangle\left\langle\hat{O}_2\right\rangle$$

This reduces the two-body problem to a one-body problem:

$$a^\dagger a^\dagger a a \Rightarrow \left\langle a^\dagger a\right\rangle a^\dagger a + \left\langle a^\dagger a^\dagger\right\rangle a a + \text{h.c.}$$

$$\hat{H}_{\nu\nu} = \frac{\sqrt{2}G_F}{V}\int dp\, dq\left(1-\cos\theta_{pq}\right)\vec{\mathbf{J}}_p \cdot \vec{\mathbf{J}}_q \cong \frac{\sqrt{2}G_F}{V}\int dp\, dq\left(1-\cos\theta_{pq}\right)\left\langle\vec{\mathbf{J}}_p\right\rangle \cdot \vec{\mathbf{J}}_q$$

Mean field

Neutrino-neutrino interaction

$$\bar{\Psi}_{\nu L}\gamma^{\mu}\Psi_{\nu L}\bar{\Psi}_{\nu L}\gamma_{\mu}\Psi_{\nu L} \Rightarrow \bar{\Psi}_{\nu L}\gamma^{\mu}\Psi_{\nu L}\left\langle \bar{\Psi}_{\nu L}\gamma_{\mu}\Psi_{\nu L} \right\rangle + \cdots$$

Antineutrino-antineutrino interaction

$$\bar{\Psi}_{\bar{\nu} R}\gamma^{\mu}\Psi_{\bar{\nu} R}\bar{\Psi}_{\bar{\nu} R}\gamma_{\mu}\Psi_{\bar{\nu} R} \Rightarrow \bar{\Psi}_{\bar{\nu} R}\gamma^{\mu}\Psi_{\bar{\nu} R}\left\langle \bar{\Psi}_{\bar{\nu} R}\gamma_{\mu}\Psi_{\bar{\nu} R} \right\rangle + \cdots$$

Neutrino-antineutrino interaction

$$\bar{\Psi}_{\nu L}\gamma^{\mu}\Psi_{\nu L}\bar{\Psi}_{\bar{\nu} R}\gamma_{\mu}\Psi_{\bar{\nu} R} \Rightarrow \bar{\Psi}_{\nu L}\gamma^{\mu}\Psi_{\nu L}\left\langle \bar{\Psi}_{\bar{\nu} R}\gamma_{\mu}\Psi_{\bar{\nu} R} \right\rangle + \cdots$$

Balantekin and Pehlivan, JPG **34**,1783 (2007)

Neutrino-antineutrino can also have an additional mean field

$$\bar{\psi}_{\nu L}\gamma^{\mu}\psi_{\nu L}\bar{\psi}_{\bar{\nu}R}\gamma_{\mu}\psi_{\bar{\nu}R} \Rightarrow \bar{\psi}_{\nu L}\gamma^{\mu}\left\langle \psi_{\nu L}\bar{\psi}_{\bar{\nu}R}\gamma_{\mu}\right\rangle\psi_{\bar{\nu}R} + \cdots$$

However note that

$\left\langle \psi_{\nu L}\bar{\psi}_{\bar{\nu}R}\gamma_{\mu}\right\rangle \propto m_{\nu}$ (negligible if the medium is isotropic)

Fuller *et al.*
Volpe

Polarization vectors

$$\hat{H} \sim \hat{H}^{\text{RPA}} = \sum_{p} \omega_{p} \hat{B} \cdot \vec{J}_{p} + \vec{P} \cdot \vec{J}$$

$$\vec{P}_{\mathbf{p},s} = 2\langle \vec{J}_{\mathbf{p},s} \rangle$$

Eqs. of motion: $$\frac{d}{d\tau}\vec{J}_{p} = -i[\vec{J}_{p}, \hat{H}^{\text{RPA}}] = (\omega_{p}\hat{B} + \vec{P}) \times \vec{J}_{p}$$

RPA Consistency requirement $\Rightarrow \frac{d}{d\tau}\vec{P}_{p} = (\omega_{p}\hat{B} + \vec{P}) \times \vec{P}_{p}$

Invariants $$I_{p} = 2\langle \hat{h}_{p} \rangle = \hat{B} \cdot \vec{P}_{p} + \sum_{q(\neq p)} \frac{\vec{P}_{p} \cdot \vec{P}_{q}}{\omega_{p} - \omega_{q}} \Rightarrow \frac{d}{d\tau} I_{p} = 0$$

Polarization vectors

$$\hat{H} \sim \hat{H}^{\mathrm{RPA}} = \sum_{p} \omega_p \hat{B} \cdot \vec{J}_p + \vec{P} \cdot \vec{J}$$

Calculated using SU(2) coherent states

$$\vec{P}_{\mathbf{p},s} = 2\langle \vec{J}_{\mathbf{p},s} \rangle$$

Eqs. of motion: $$\frac{d}{d\tau}\vec{J}_p = -i[\vec{J}_p, \hat{H}^{\mathrm{RPA}}] = (\omega_p \hat{B} + \vec{P}) \times \vec{J}_p$$

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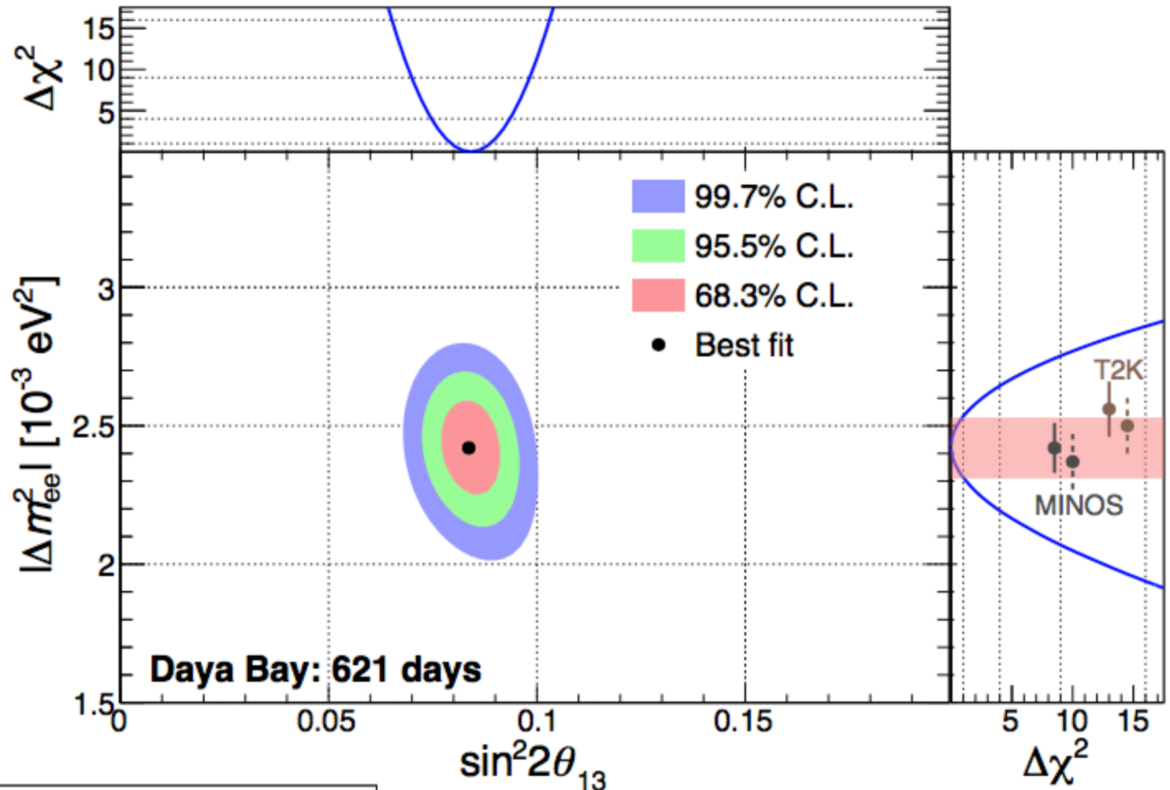
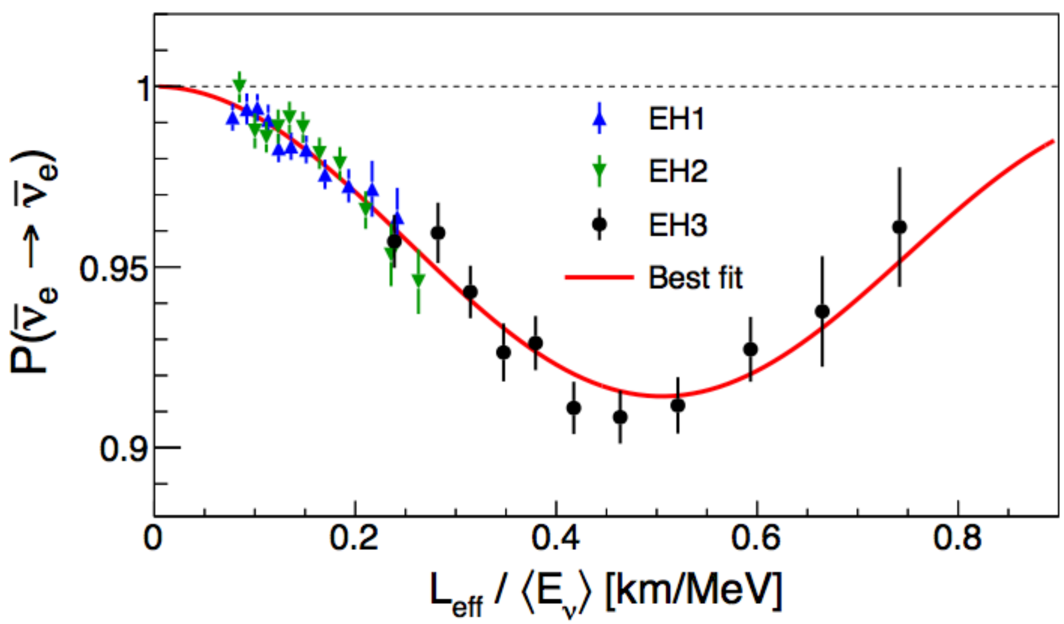
Recent Daya Bay results





$$\sin^2 2\theta_{13} = 0.084 \pm 0.005$$

$$\left|\delta m^2_{ee}\right| = (2.42 \pm 0.11) \times 10^{-3}\,\mathrm{eV}^2$$

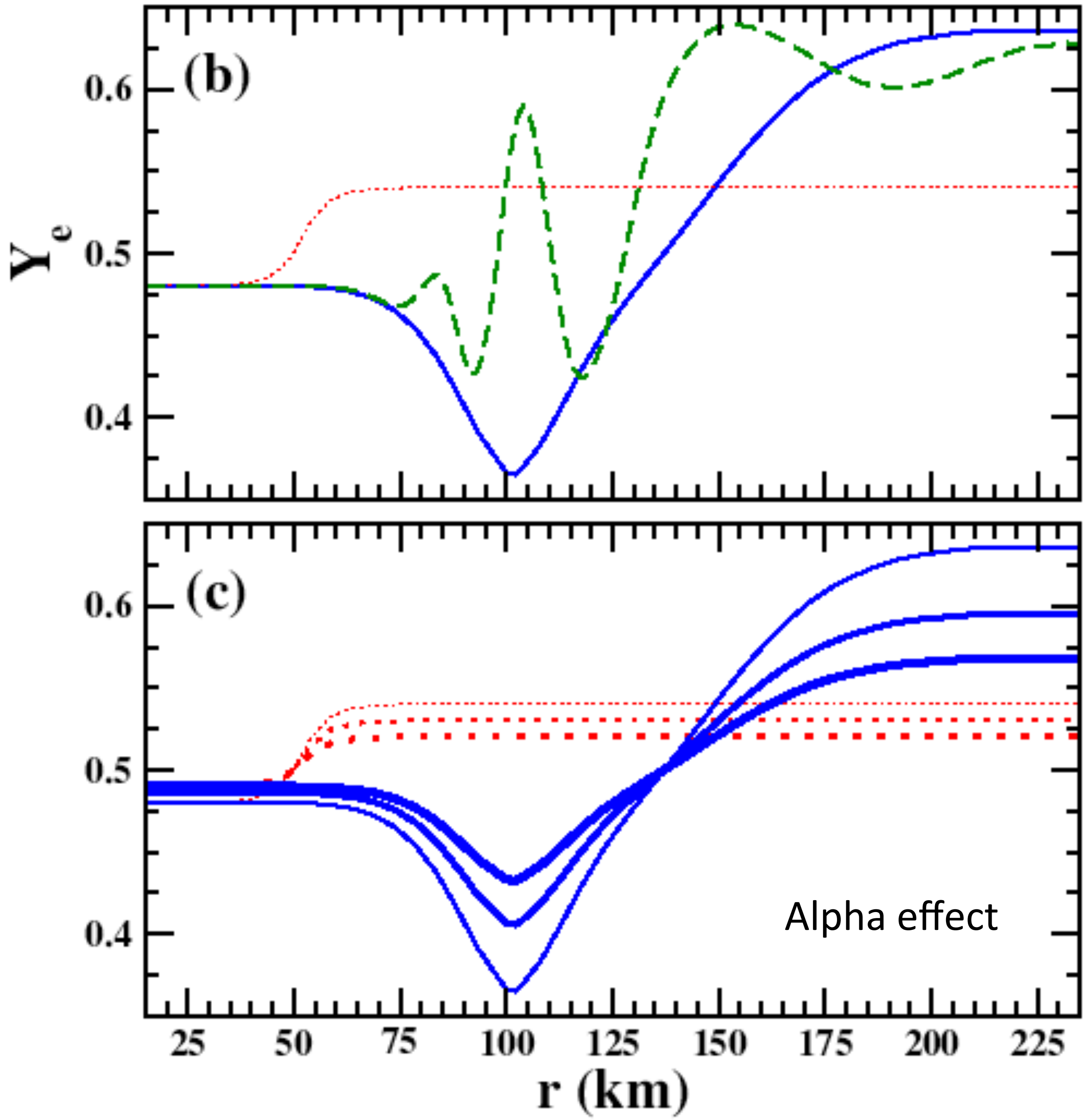
Equilibrium electron fraction with the inclusion of $\nu\nu$ interactions in the mean-field picture

L^{51} = 0.001, 0.1, 50

Balantekin and Yuksel

New J.Phys. 7 (2005) 51

X_α= 0, 0.3, 0.5 (thin, medium, thick lines)



$\sin^2 2\theta_{13}$=0.095

RECALL

Single-angle approximation Hamiltonian:

$$H = \sum_p \frac{\delta m^2}{2p} J_p^0 + 2\mu \sum_{\substack{p,\, q \\ p \neq q}} \mathbf{J}_p \bullet \mathbf{J}_q$$

Eigenstates:

$$|x_i\rangle = \prod_{i=1}^N \sum_k \frac{J_k^\dagger}{\left(\delta m^2/2k\right) - x_i} |0\rangle$$

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Bethe ansatz equations

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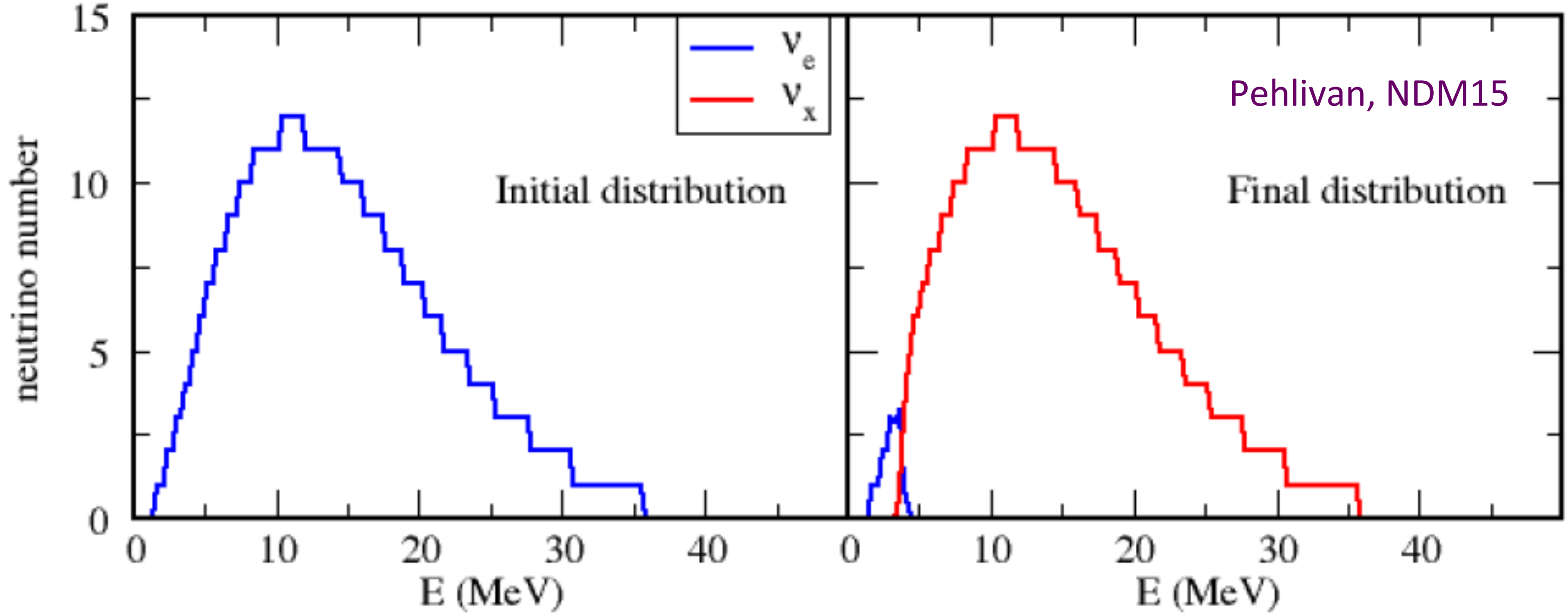
$$h_p = J_p^0 + 2\mu \sum_{\substack{p,\, q \\ p \neq q}} \frac{\mathbf{J}_p \bullet \mathbf{J}_q}{\delta m^2 \left(\frac{1}{p} - \frac{1}{q}\right)}$$

Pehlivan, ABB, Kajino, & Yoshida
Phys. Rev. D 84, 065008 (2011)

Away from the mean-field: First adiabatic solution of the *exact* many-body Hamiltonian



- Solutions of the Bethe ansatz equations for 250 neutrinos. Same behavior as the mean-field.
- Two flavors only
- Inverted hierarchy, no matter effect

Factorization of the CP-violating phase if there are no sterile neutrinos

$$H(\delta) = H_{\nu} + H_{\nu\nu} = \mathbf{S} H(\delta = 0) \mathbf{S}^{\dagger}$$

$$\mathbf{S} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix}$$

Holds if neutrino magnetic moment is ignored.

- MSW Hamiltonian: Balantekin, Gava, Volpe, Phys. Lett B662, 396 (2008).
- Collective Hamiltonian in the mean-field approximation: Gava, Volpe, Phys. Rev. D78, 083007 (2008).
- Exact collective Hamiltonian: Pehlivan, Balantekin, Kajino, Phys. Rev. D90, 065011 (2014).


Thank you!