A rigorous solution to unitary Bose Gases

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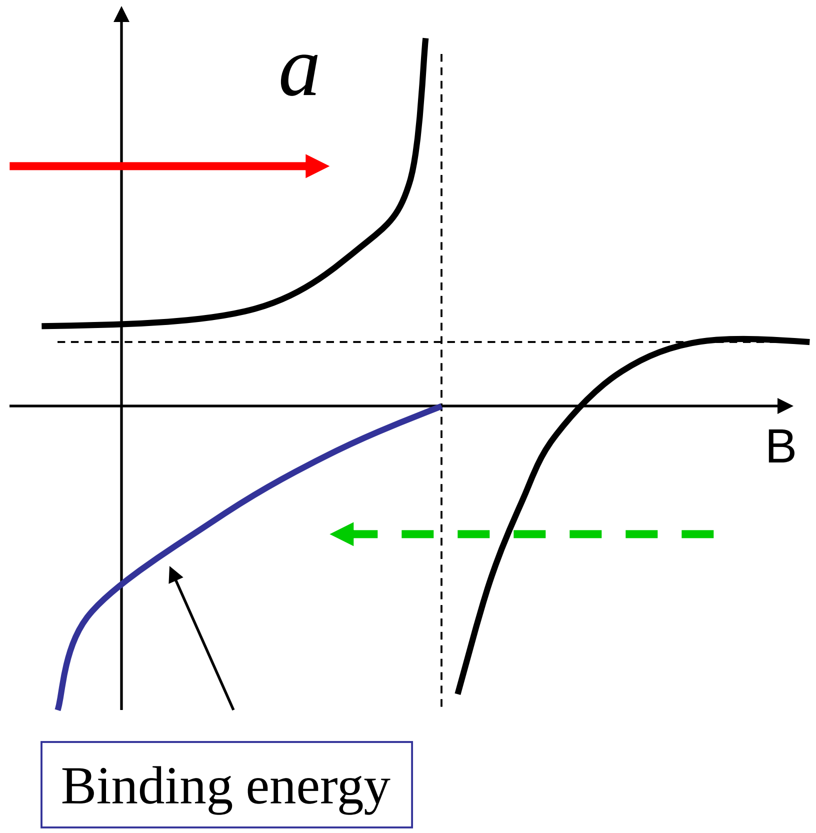
Quantum Bose Gas near Feshbach Resonance (Upper Branch)

Dilute limit



…. Papp + Pino et. al, (Wieman, Jin, Cornell, 2009);
Pollack+ Dries et al., (Hulet, 2009); Navon+Piatecki et al., (Chevy and Salomon, 2011); Wild +Makotyn et al., (Cornell, Jin, 2012); Ha+Hung et al., (Chin, 2012); Makotyn et al., (Cornell, Jin, 2013)…

Dilute Bose Gases

Lee-Yang-Huang (56; 57-58) and Beliaev (58)

And is valid for small scattering lengths.

$$\sqrt{na^3} \propto \frac{a}{\xi} << 1$$

There have been efforts to improve LYH-Beliaev theory by taking into the higher order contributions. (Wu, Sawada, 59……) At infinity a, each term diverges.

$$E = \frac{2\pi\hbar^2 n^2 a}{m}(1 + \frac{128}{15\sqrt{\pi}}\sqrt{na^3}$$

$$+8(4\pi - 3\sqrt{3})/3 \times [\ln(na^3) + ...]na^3 + ...)$$

Approaches to Unitary Bose Gases

1) Quantum Monte Carlo /path integral: of limited utility.

2) Variational approaches (numerical):

Cowell et al. (Pethick group), 2002; Song & Zhou, 2009…
Diederix et al. (Stoof group), 2011, Yin & Radzihovsky, 2013,
Sykes and Corson et al. (JILA +Greene), 2014 ….
2D: Pilati et al., (Giorgini group), 2005.

3) Effective potential via loop expansion (“single shot renormalization method”)

(at UBC, 2011---now)

………

Outline

1) Ideas/Cartoons: scale invariance near resonances and the renormalized running coupling constant.
2) Implementation: Theory frame work for unitary Bose gases
3) Results

Ref:

Jiang, Liu, Semenoff and Zhou, Rigorous solution to strong coupling fixed pt near 4 spatial dimension. Phys. Rev. A 89, 033614 (2014).

Scale invariance versus resonance

$$\psi_{k\to 0}(r) = 1 - \frac{a}{r} \sim \frac{1}{r}$$

Implying the Hamiltonian is scale invariant under the scale transformation.

Scale dependence I: Scale invariance and A cartoon in real Space for square well potential

$$r_0 \to \lambda r_0, V_0 \to V_0 \frac{1}{\lambda^2},$$

$$or \quad g_2 \sim V_0 r_0^3 \to \lambda V_0 r_0^3$$

$$\Lambda = \frac{1}{r_0} \to \frac{\Lambda}{\lambda}, \; g_2(\Lambda) \to g(\frac{\Lambda}{\lambda}) = \lambda g_2(\Lambda)$$

$$\Rightarrow g_2(\Lambda) \sim -\frac{c}{\Lambda} + o(\frac{1}{a\Lambda})$$

$$\Rightarrow \hat{g}_2 = g_2(\Lambda)\Lambda \sim -c + o(\frac{1}{a\Lambda})$$

Renormalization Group equation

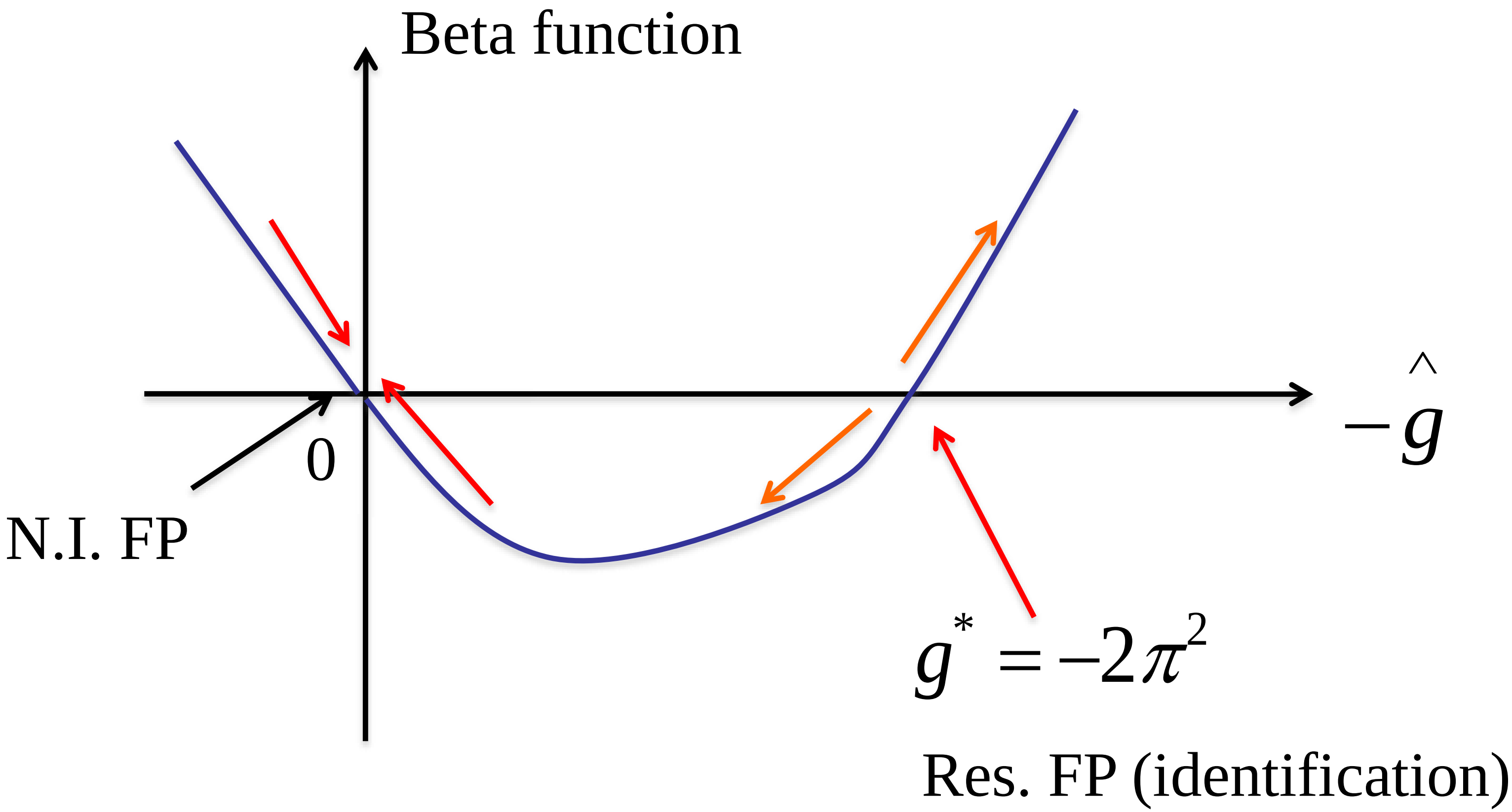
$$\hat{g}(\Lambda) = g_2 * \Lambda,$$

$$\frac{d\hat{g}}{d\log\Lambda} = \beta(\hat{g}), \quad \beta(\hat{g}) = \hat{g} + \frac{1}{2\pi^2}\hat{g}^2$$

… David Kaplan et al., 1998; …..

… Subir Sachdev et al, 2005; Dam Son et al, 2006….

Resonance as a unstable FP (3D)



Running of the coupling constant

$$g_2(\Lambda \sim 0) = 4\pi a(1 + C\Lambda a...), \qquad \Lambda a \Rightarrow \sqrt{2\mu}a \sim \sqrt{na^3}$$

The running in QED and QCD

(Landau et al, Gell-mann-Low, 1954… Wilczek, Politzer and Gross, 1972…)

$$e^2(\Lambda) = \frac{e_R^2}{1 - e_R^2 \beta_1 \ln \Lambda / m_R};$$

$$\alpha(\Lambda) = \frac{1}{\beta_2 \ln \dfrac{\Lambda}{\Lambda_{QCD}}}$$

In QED, Landau Pole is at 10^{227}GeV. Fine structure constant varies from 1/137 to 1/127 when energy increases to 90GeV.

In QCD, the running occurs at distance short than one Fermi !!

Theory Frame Work for near resonance

Condensate

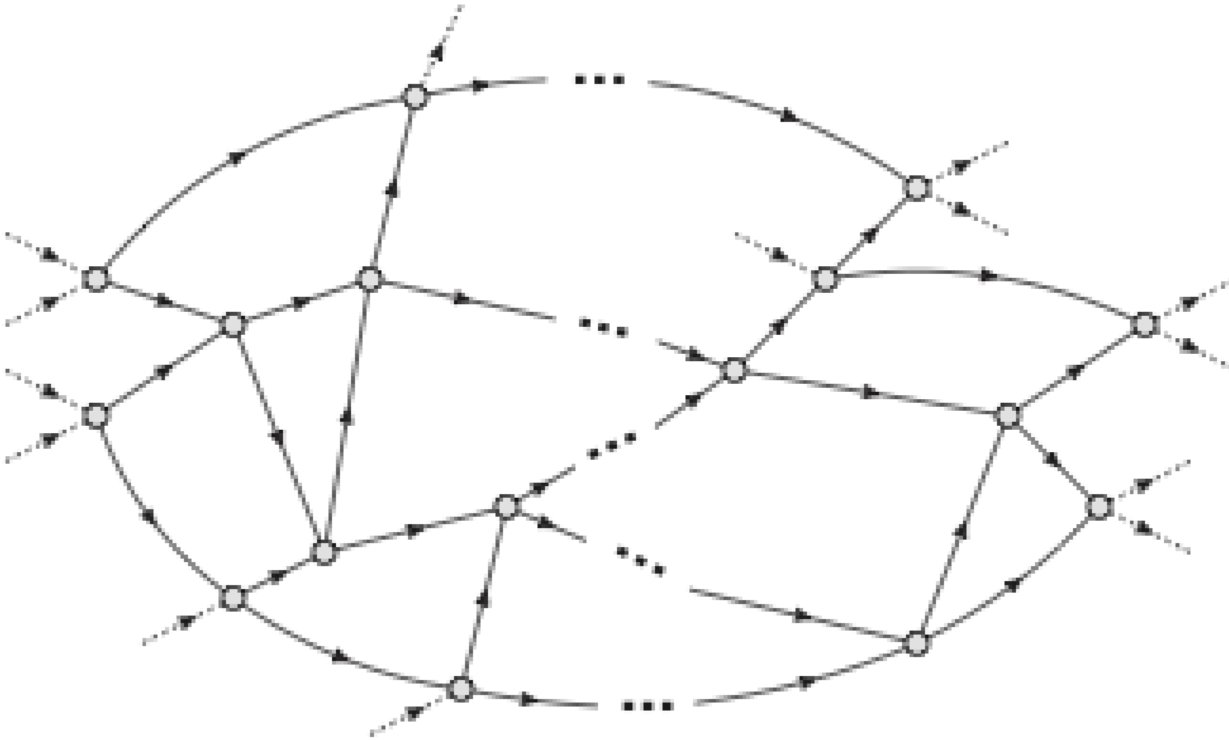
Non-condensed Chem. potential

$$\mu_c(n_0,\mu) = \frac{\partial E(n_0,\mu)}{\partial n_0}, n = n_0 - \frac{\partial E(n_0,\mu)}{\partial \mu},$$
$$\mu = \mu_c(n_0,\mu),$$

Self-consistent Equilibrium Cond.

E is the total interaction energy of condensate at fixed n_0
Is also the effective potential for the quantum bosonic field
(Coleman-Weinberg type but calculated at a finite \mu with attraction).

A typical L-loop diagram for E: Diag (L=10,M=7)



$$G(\varepsilon,k)=\frac{1}{\varepsilon-\frac{k^2}{2}+\mu+i\delta};\quad T(\varepsilon,k)=\frac{4\pi}{\frac{1}{a}-\sqrt{-\varepsilon+\frac{k^2}{4}-2\mu}}$$

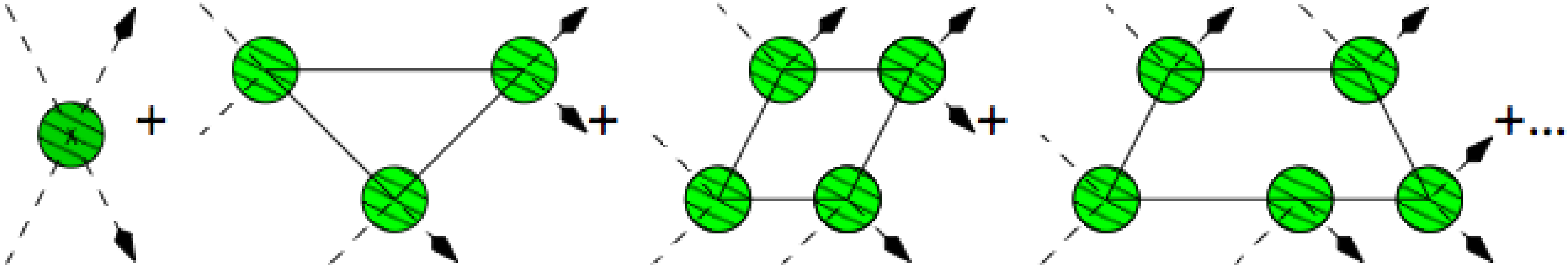
Dashed lines: condensed atoms; solid lines: G, propagators of k-particle ; vertices: T, transition matrix. Both G and T are introduced at prefixed \mu

Summation using the standard method

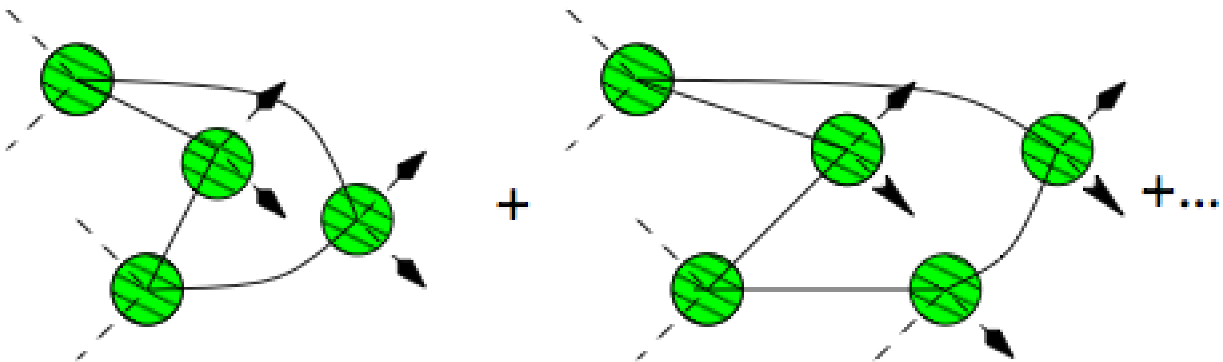
$$E(n_0,\mu) = \sum_{M=2,3,4..} \frac{1}{M!} n_0^M g_M^0(\mu)$$

M-body potentials: all loop diagrams with M incoming/outgoing condensed lines

Diagramatic representation of the LHY: 1-loop



c)



d)

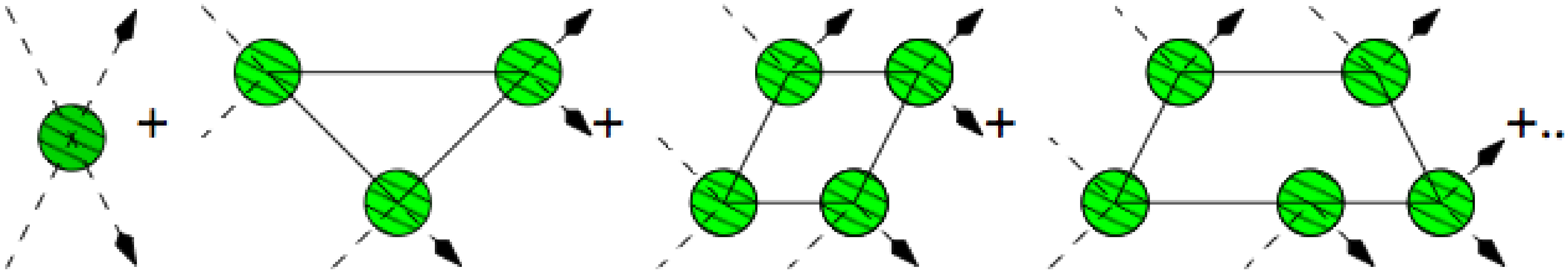
Summation using Irreducible potentials (to go beyond L=1)

$$E(n_0,\mu) = \sum_{M=2,3,4} \frac{1}{M_{irr}!} n_0^{M_{irr}} g_{M_{irr}}^{Irr}(n_0,\mu)$$

$$M_{irr} = M - V_{DS}, \quad g_{M_{irr}}^{Irr}(n_0,\mu) \sim g_M(\Lambda = \sqrt{\mu})$$

M-body irreducible potentials: all loop diagrams with M incoming/outgoing condensed lines but not counting those directly scattered (DS) off by particles with finite k. Irreducible potential induced here is equivalent to the running coupling constant calculated at scale \mu or renormalized coupling constant in the beta function. In 3D, this converges very rapidly.

Diagramatic representation of the LHY



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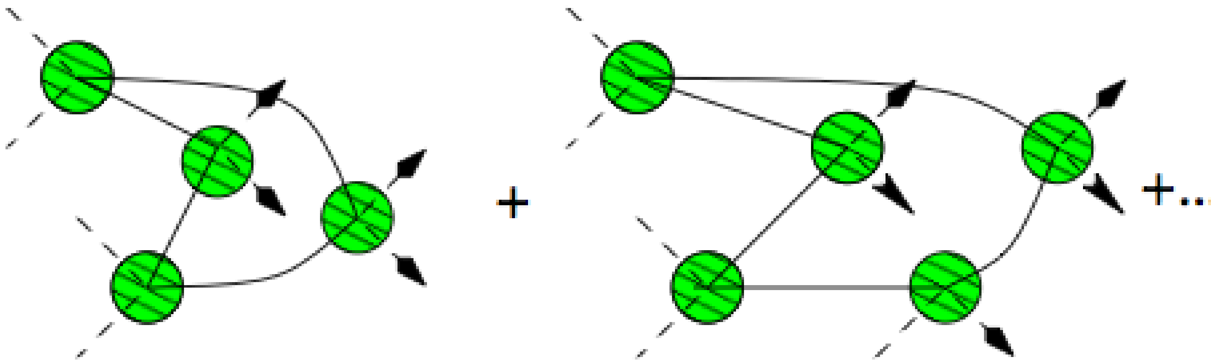


d)

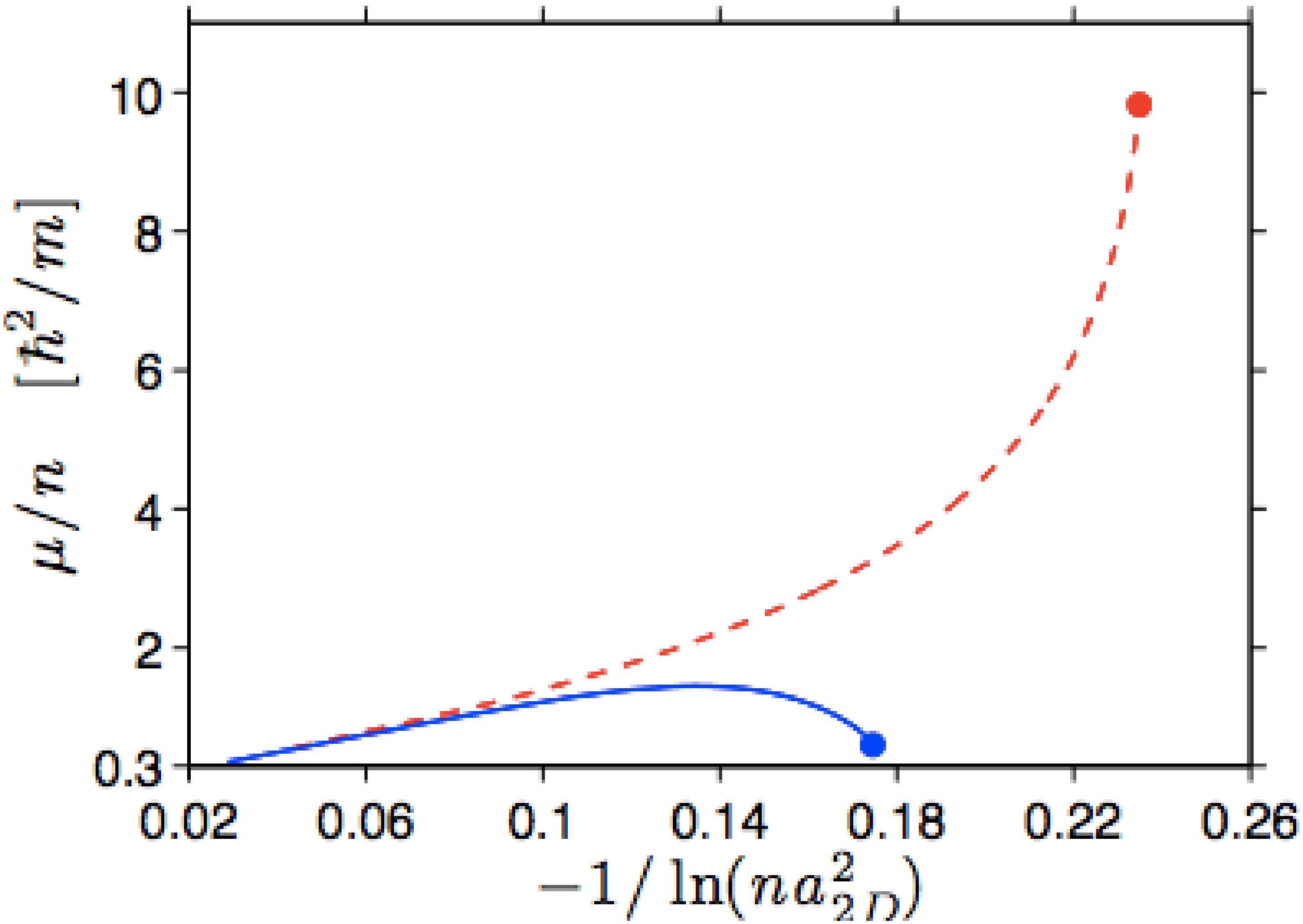
$$\frac{g_2(\Lambda_\mu)-g_2(0)}{LHY}=\frac{(c)}{(c)+(d)}=\frac{9\pi\sqrt{2}}{40}=99.96\%$$

Summation in 2D and 3D

Self-consistent summation was done in 2D and 3D up to M_{irrd}=3 but with L=1,2,3,….

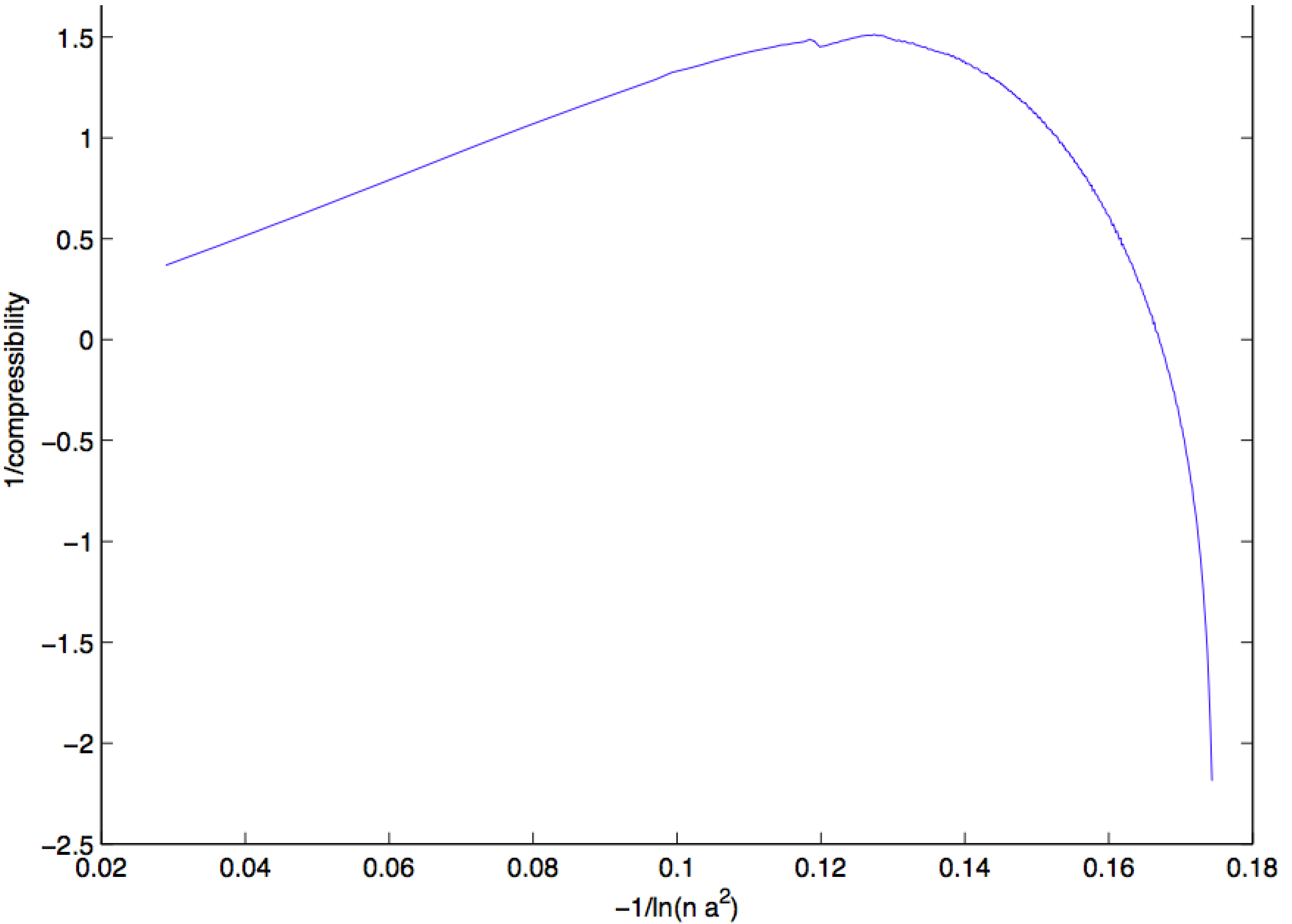
a) Borzov, Mashayekhi, Zhang, Song and FZ, PRA 85, 023620 (2012).
b) FZ and Mashayekhi, 2012, Ann. Phys. 328, 83 (2013).
   (Relation between the loop summation and
   Beta function/running coupling constant.)
c) Mashayekhi, Bernier, Borzov, FZ, PRL 110, 145301 (2013).

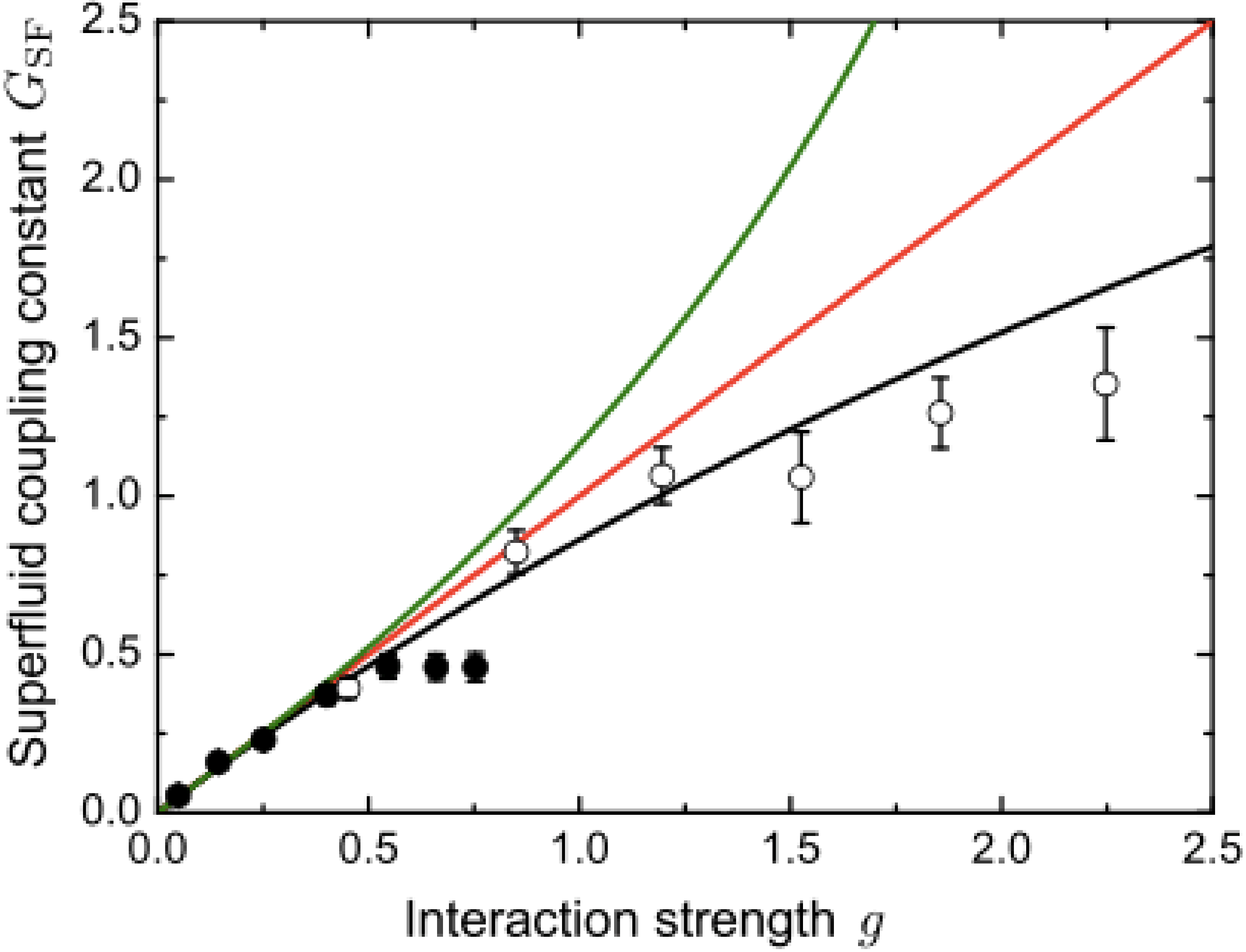
d) Shaojian Jiang, FZ (Finite temperatures physics and exp. Smoking gun) , ArXiv: 1504.03434
   (See his talk next Thursday)



2D, Mashayekhi, Bernier, Brozov and FZ, PRL 2013.

Compressibility Anomalies in 2D (2014)





2D Exp: Ha, Hung, Zhang, Tung, Eismann, Chen, PRL 2013

Special property near 4D

$$\int d^4 r \, |\Psi_2(r)|^2 \sim \Omega_4 \int dr r^3 \times \frac{1}{r^4} \sim \log \frac{\lambda}{r_0}$$

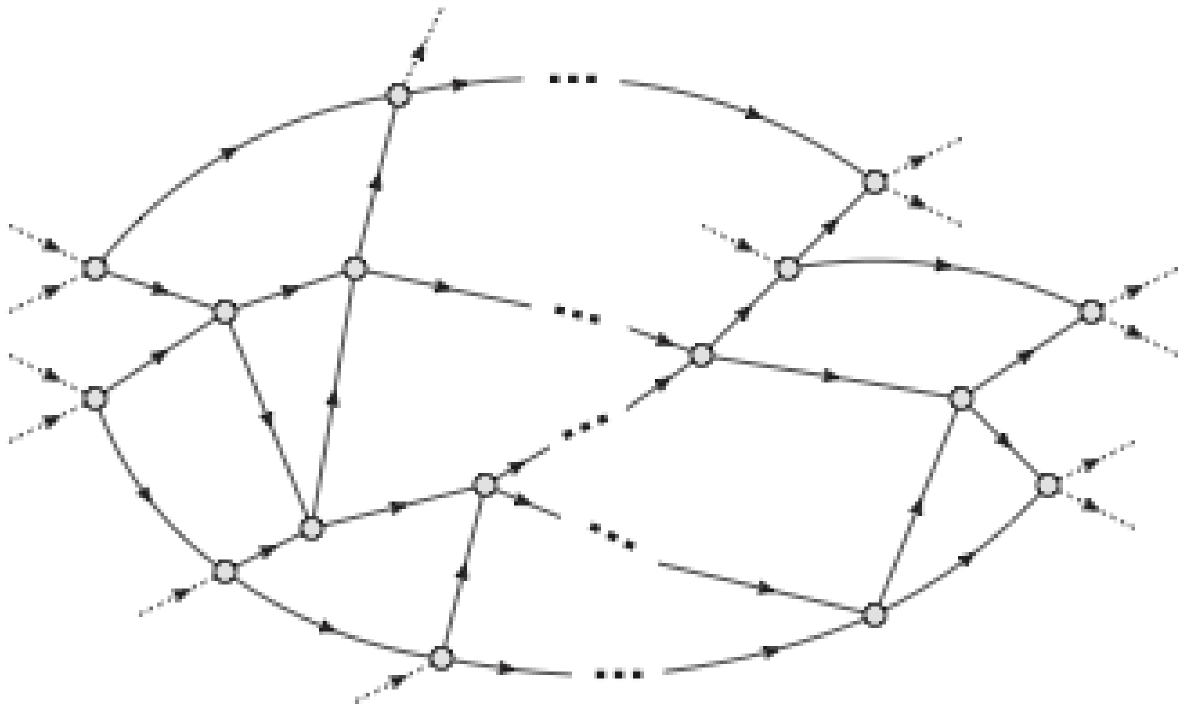
a) Probability is logarithmically concentrated near origin. (Nussinov and Nussinov, 2006; also pseudo potential approach by Yang, 2008; BCS-BEC crossover, Nishida and Son, 2006.)

One can also show

a) Bohn-Oppenheimer potential for 2 heavy 1 light particle is logarithmically small in 4D;

b) Atom-dimmer interactions are logarithmically small;

4D: epsilon expansion---systematic. E(L,M)



$$P = L + 2(M-1), \quad V = L + (M-1)$$

P: Number of propagators; V: number of vertices; L: Number of loops;
M: Number of incoming or outgoing condensed lines

Power counting at resonance: contributions from L-loop M-body diagram

$$E(L,M) \sim {n_0}^M T^V(k) G^P(k) k^{(2+4-\varepsilon)L}$$

$$\Rightarrow \delta\mu(L,M) \sim (\varepsilon n_0)^{\frac{2}{4-\varepsilon}} \varepsilon^L$$

Is the loop summation/ renormalization exact?

Aug. 2011, Prof. C. N. Yang

ANS: A rigorous solution via 4-epsilon expansion

$$\mu = \epsilon^{\frac{2}{4-\epsilon}} \epsilon_F \sqrt{\frac{2}{3}} (1 + 0.474\epsilon - i1.217\epsilon + \cdots),$$

$$n_0 = \frac{2}{3} n (1 + 0.0877\epsilon + \cdots).$$

Jiang, Liu, Semenoff, FZ, PRA, 2014. Life time scales as the inverse of Fermi energy.

Universal life time was also suggested in a JILA's recent experiment. Makotyn et al, Nature physics 2014.

Summary

1) A rigorous solution suggests that near 4D Bose gases are a collection of nearly independent scattering “pair”s, with universal life time and thermodynamic properties etc.

2) Can 4D or extra dimension Physics or equivalent universality be Quantum Simulated in optical lattices ???