

清華大學

Tsinghua University



# Landau-Beliaev Damping in a Bose-Fermi Superfluid Mixture

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**Frontiers in Quantum Simulation with Cold Atoms**

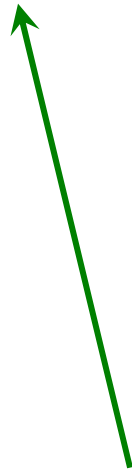
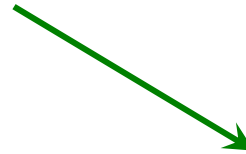
**March 23, 2015 - May 8, 2015**

# Outline

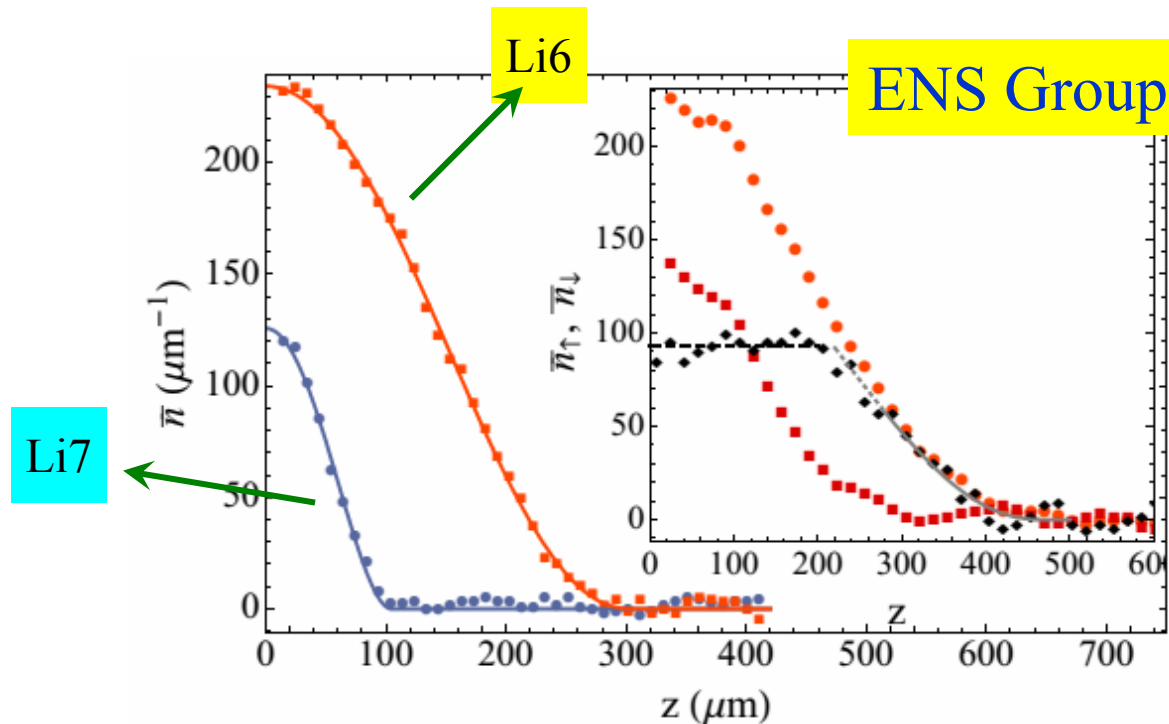
Experiment realization of  
the Bose-Fermi Superfluid Mixture:  
Damping of the dipole mode

Landau-Beliaev damping  
in single component BEC  
High-T and low-T behavior

Landau-Beliaev damping  
in Bose-Fermi Superfluid Mixture  
Damping through the BCS-BEC crossover

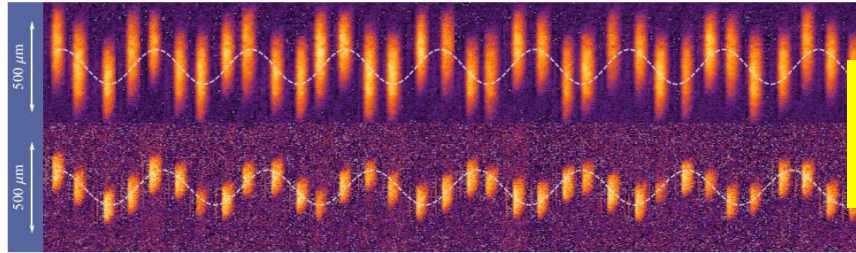


# Experiment of the Bose-Fermi Superfluid Mixture



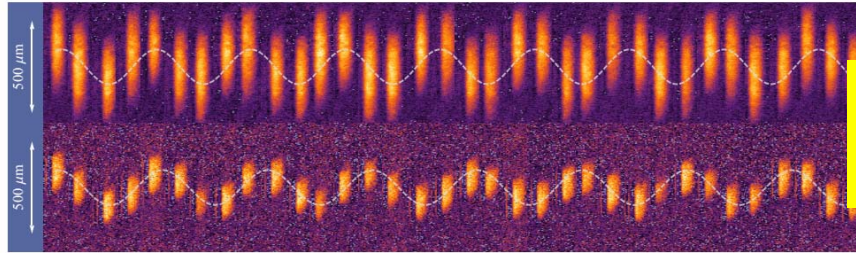
**For the first time, we have  
Bose superfluid mixing with Fermi superfluid**

# Experiment of the Bose-Fermi Superfluid Mixture



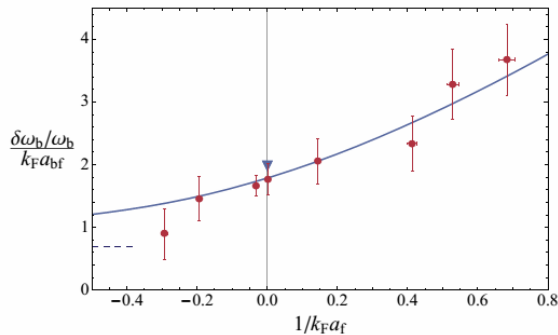
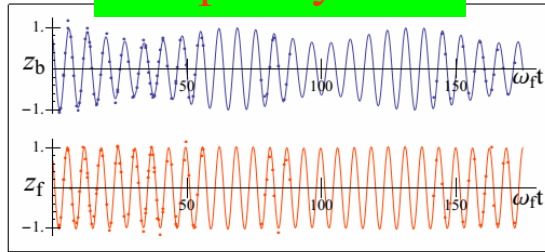
Dynamics:  
Dipole Oscillation

# Experiment of the Bose-Fermi Superfluid Mixture



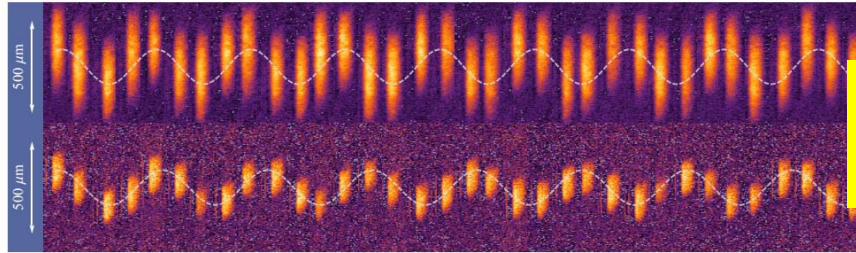
Dynamics:  
Dipole Oscillation

Frequency shift



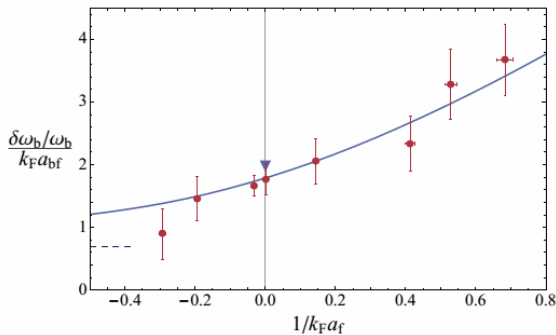
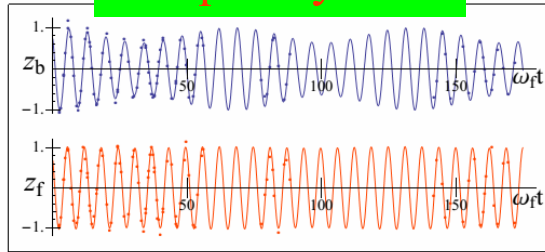
Measure the EOS of Li6

# Experiment of the Bose-Fermi Superfluid Mixture



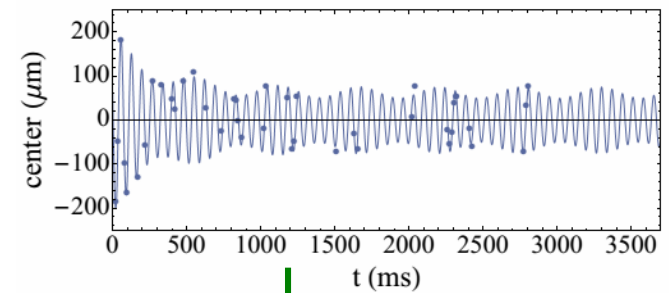
Dynamics:  
Dipole Oscillation

Frequency shift

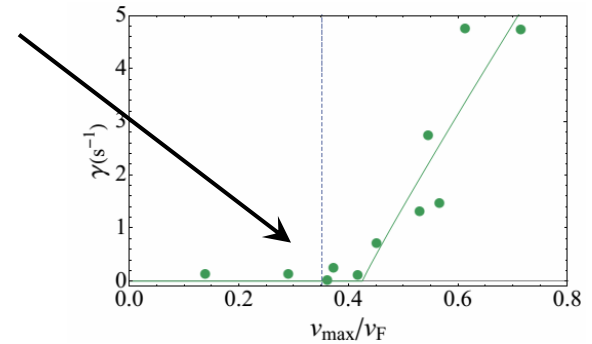


Measure the EOS of Li6

Damping of the oscillation

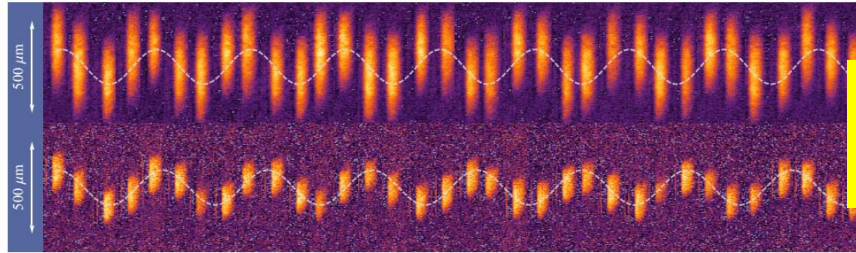


Critical velocity



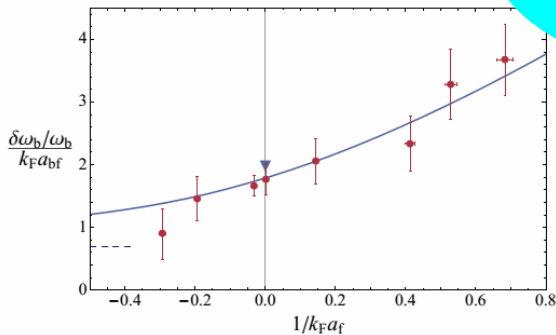
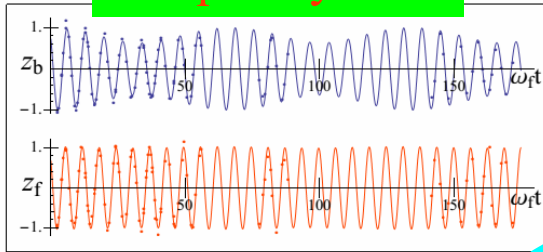
Damping rate

# Experiment of the Bose-Fermi Superfluid Mixture



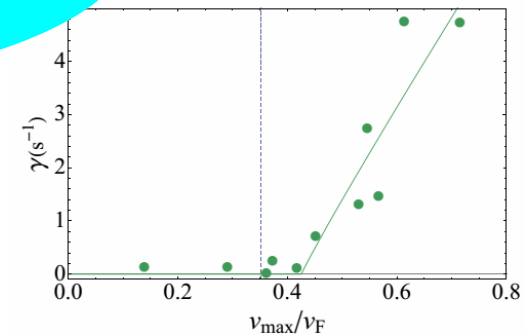
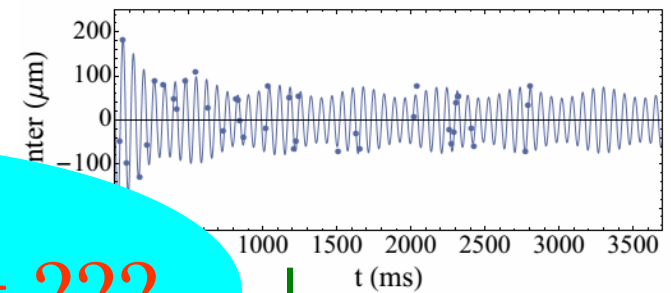
Dynamics:  
Dipole Oscillation

Frequency shift



Measure the EOS of Li6

Damping of the oscillation



Damping rate

Why damping ???

# Landau-Beliaev damping in a single component BEC

$$H = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}} + \frac{1}{2} \frac{g}{V} \sum_{\mathbf{q}, \mathbf{p}, \mathbf{k}} b_{\mathbf{q}+\mathbf{k}}^{\dagger} b_{\mathbf{p}-\mathbf{k}}^{\dagger} b_{\mathbf{q}} b_{\mathbf{p}}$$



# Landau-Beliaev damping in a single component BEC

$$H = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}} + \frac{1}{2} \frac{g}{V} \sum_{\mathbf{q}, \mathbf{p}, \mathbf{k}} b_{\mathbf{q}+\mathbf{k}}^{\dagger} b_{\mathbf{p}-\mathbf{k}}^{\dagger} b_{\mathbf{q}} b_{\mathbf{p}}$$

$$b_0^{\dagger} = b_0 = \sqrt{N_0}$$

$$b_{\mathbf{k}} = u_{\mathbf{k}} \alpha_{\mathbf{k}} - v_{\mathbf{k}} \alpha_{-\mathbf{k}}^{\dagger}$$

Bogoliubov theory

$$H = \sum_{\mathbf{k}} E_{\mathbf{k}} \alpha_{\mathbf{k}}^{\dagger} \alpha_{\mathbf{k}}$$

Quasi-particles

# Landau-Beliaev damping in a single component BEC

$$H = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}} + \frac{1}{2} \frac{g}{V} \sum_{\mathbf{q}\mathbf{p}\mathbf{k}} b_{\mathbf{q}+\mathbf{k}}^{\dagger} b_{\mathbf{p}-\mathbf{k}}^{\dagger} b_{\mathbf{q}} b_{\mathbf{p}}$$

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Bogoliubov theory

$$H = \sum_{\mathbf{k}} E_{\mathbf{k}} \alpha_{\mathbf{k}}^{\dagger} \alpha_{\mathbf{k}}$$

Quasi-particles

$$+ \frac{g\sqrt{N_0}}{V} \sum_{\mathbf{q}\mathbf{k}} M_{\mathbf{q}\mathbf{k}} \alpha_{\mathbf{q}+\mathbf{k}}^{\dagger} \alpha_{\mathbf{q}} \alpha_{\mathbf{k}} + \dots$$

$$+ \frac{g}{V} \sum_{\mathbf{q}\mathbf{p}\mathbf{k}} D_{\mathbf{q}\mathbf{p}\mathbf{k}} \alpha_{\mathbf{q}+\mathbf{k}}^{\dagger} \alpha_{\mathbf{p}-\mathbf{k}}^{\dagger} \alpha_{\mathbf{q}} \alpha_{\mathbf{p}} + \dots$$

# Landau-Beliaev damping in a single component BEC

$$H = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}} + \frac{1}{2} \frac{g}{V} \sum_{\mathbf{q}, \mathbf{p}, \mathbf{k}} b_{\mathbf{q}+\mathbf{k}}^{\dagger} b_{\mathbf{p}-\mathbf{k}}^{\dagger} b_{\mathbf{q}} b_{\mathbf{p}}$$

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$$H = \sum_{\mathbf{k}} E_{\mathbf{k}} \alpha_{\mathbf{k}}^{\dagger} \alpha_{\mathbf{k}}$$

Quasi-particles

$$+ \frac{g\sqrt{N_0}}{V} \sum_{\mathbf{q}, \mathbf{k}} M_{\mathbf{q}, \mathbf{k}} \alpha_{\mathbf{q}+\mathbf{k}}^{\dagger} \alpha_{\mathbf{q}} \alpha_{\mathbf{k}} + \dots$$



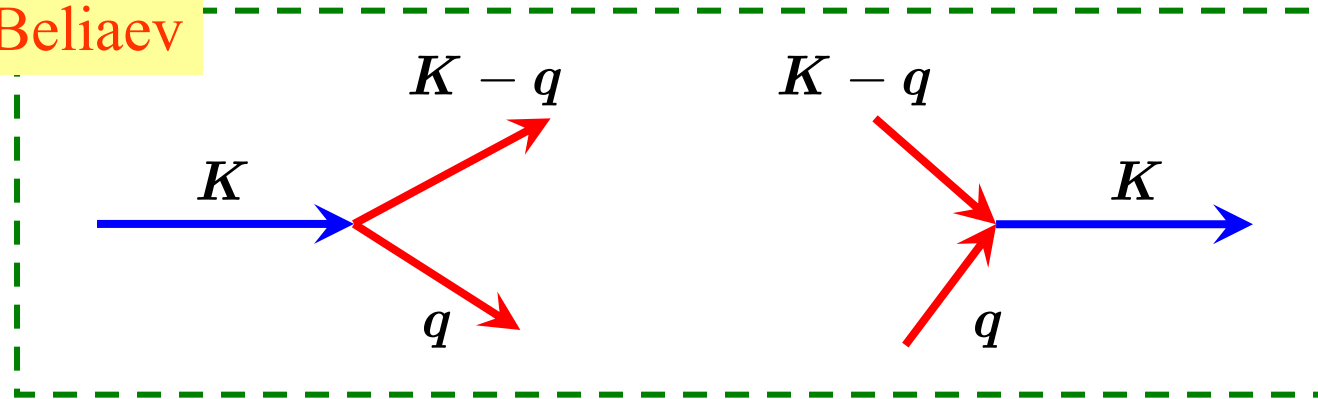
$$+ \frac{g}{V} \sum_{\mathbf{q}, \mathbf{p}, \mathbf{k}} D_{\mathbf{q}, \mathbf{p}, \mathbf{k}} \alpha_{\mathbf{q}+\mathbf{k}}^{\dagger} \alpha_{\mathbf{p}-\mathbf{k}}^{\dagger} \alpha_{\mathbf{q}} \alpha_{\mathbf{p}} + \dots$$

**Interactions**

between the quasi-particles induce the damping

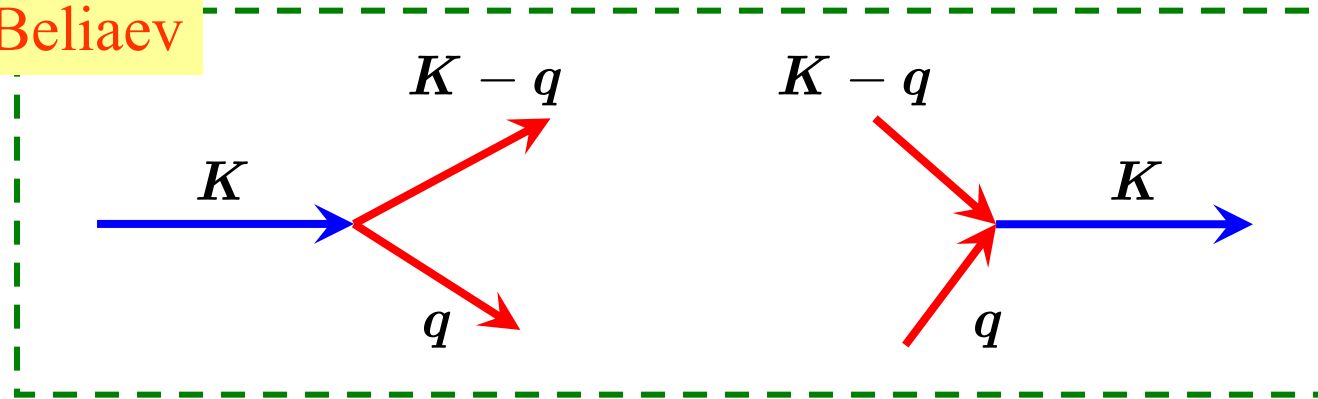
# Beliaev process VS Landau process

Beliaev



# Beliaev process VS Landau process

Beliaev



$$\gamma_B = \frac{\pi}{\hbar} \sum_q |M_{q,K}|^2 (1 + f_q^0 + f_{K-q}^0) \delta(E_q + E_{K-q} - E_K)$$

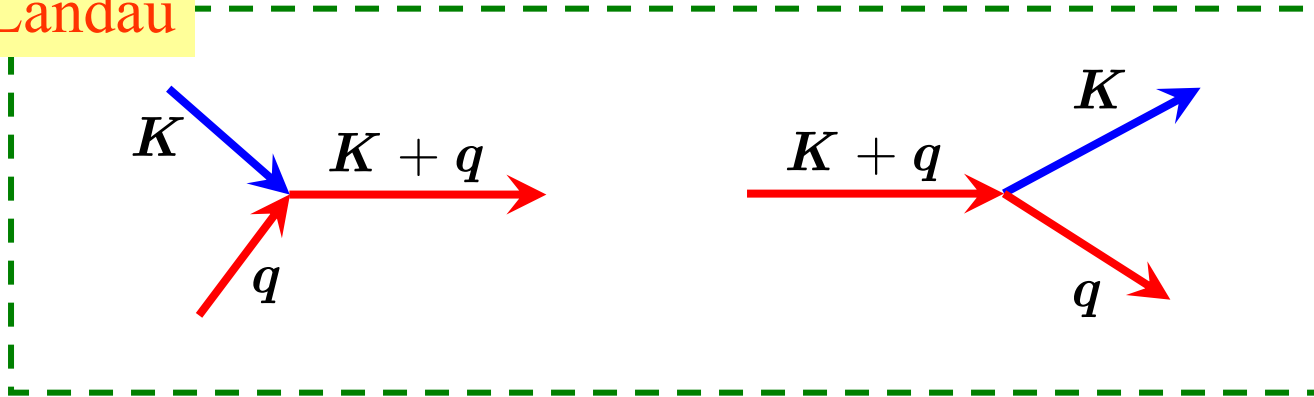
$$T = 0$$

$$f_q^0 = \frac{1}{e^{\beta E_q} - 1}$$

$$\gamma_B = \frac{\pi}{\hbar} \sum_q |M_{q,K}|^2 \delta(E_q + E_{K-q} - E_K)$$

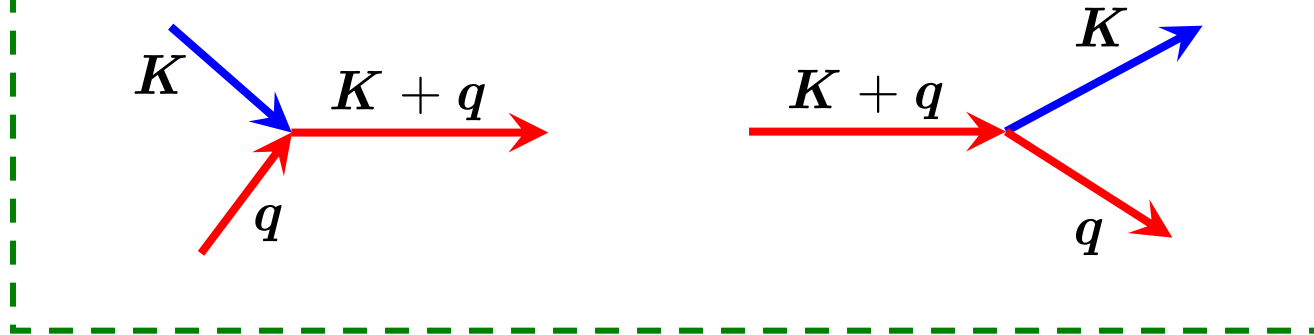
# Beliaev process VS Landau process

Landau



# Beliaev process VS Landau process

Landau



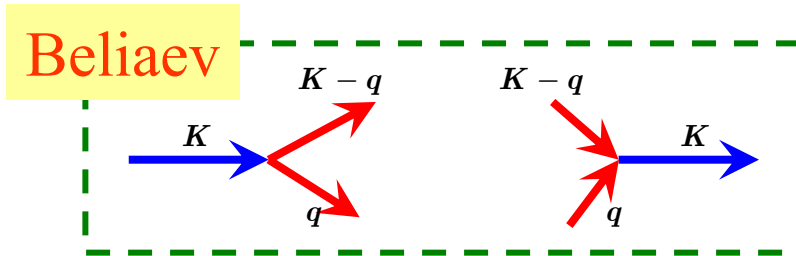
$$\gamma_L = \frac{\pi}{\hbar} \sum_q |M_{q, K+q}|^2 (f_q^0 - f_{K+q}^0) \delta(E_q + E_K - E_{K+q})$$

$$T = 0$$

$$\gamma_L = 0$$

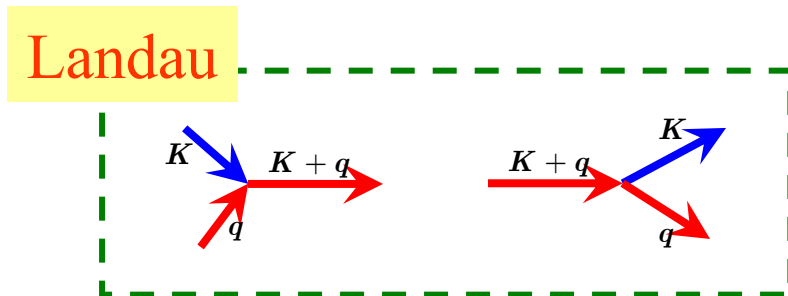
# Landau-Beliaev damping in a single component BEC

Low T, Beliaev process dominate



$$\gamma \approx \frac{3K^5}{640\pi\hbar^3mn}, \quad T = 0$$

High T, Landau process dominate



$$\gamma \approx \frac{3\pi}{8} \frac{k_B T a}{\hbar c} K, \quad k_B T \gg gn$$

- S.T. Beliaev, Soviet Phys. JETP, **34**, 299 (1958);  
P.C. Hohenberg, P.C. Martin, Ann. Phys. (NY), **34**, 291 (1965).  
W. Liu, PRL, **79**, 4056 (1997); S. Giorgini, PRA, **57**, 2949 (1998);  
L.P. Pitaevskii, S. Stringari, Phys. Lett. A **235**, 398 (1997)

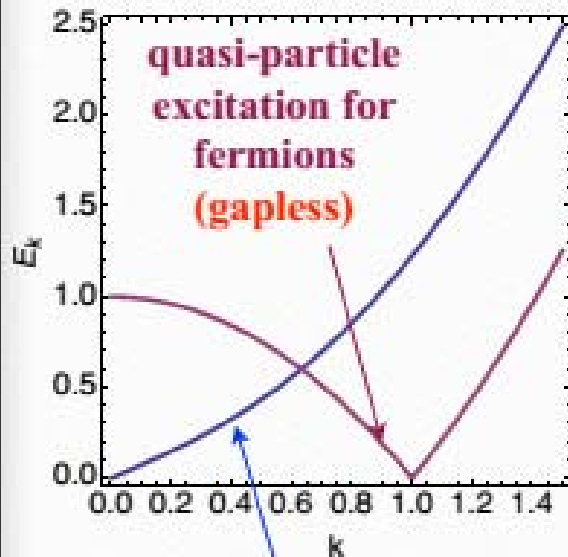


# Quasi-particles in the Bose-Fermi Mixture

**Bose superfluid and  
Fermi gas**

**Two Bose superfluids**

**Bose and Fermi  
superfluids**



**Bogoliubov mode  
for Bose superfluid**

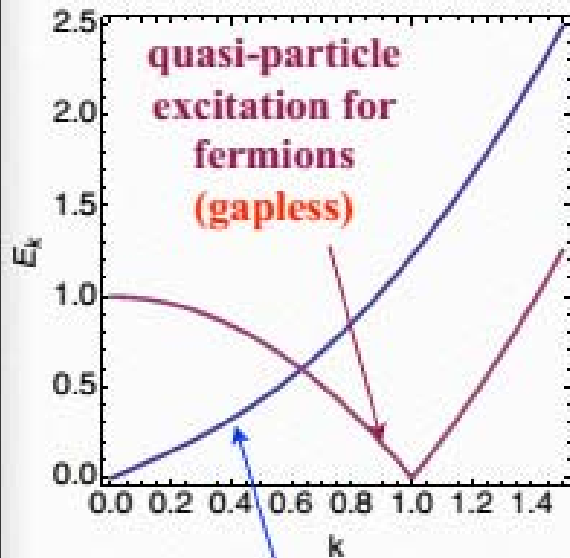
**quasi-particle  
excitation for  
fermions  
(gapless)**

# Quasi-particles in the Bose-Fermi Mixture

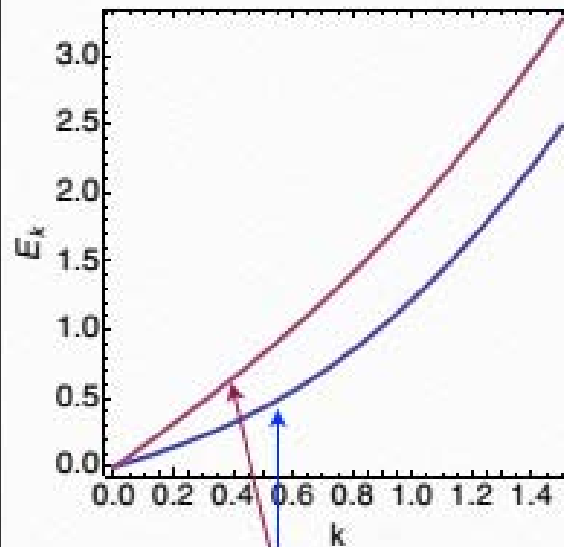
**Bose superfluid and Fermi gas**

**Two Bose superfluids**

**Bose and Fermi superfluids**



**Bogoliubov mode for Bose superfluid**



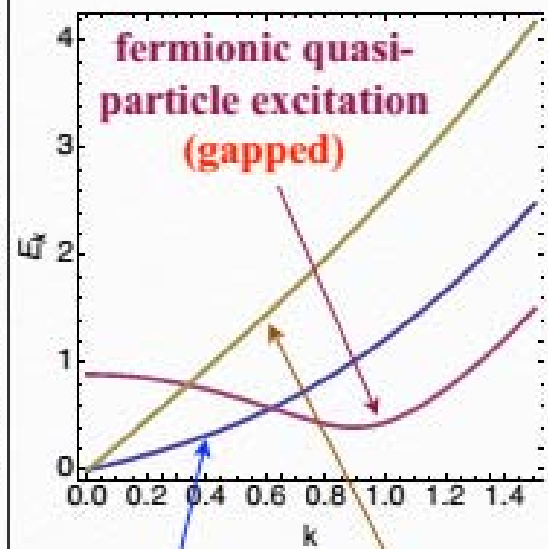
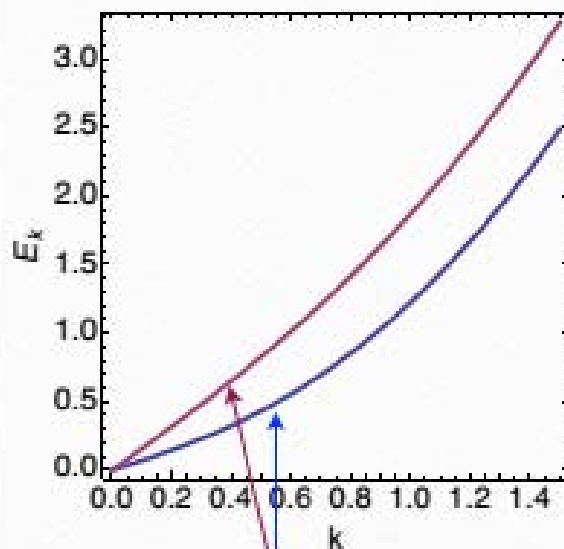
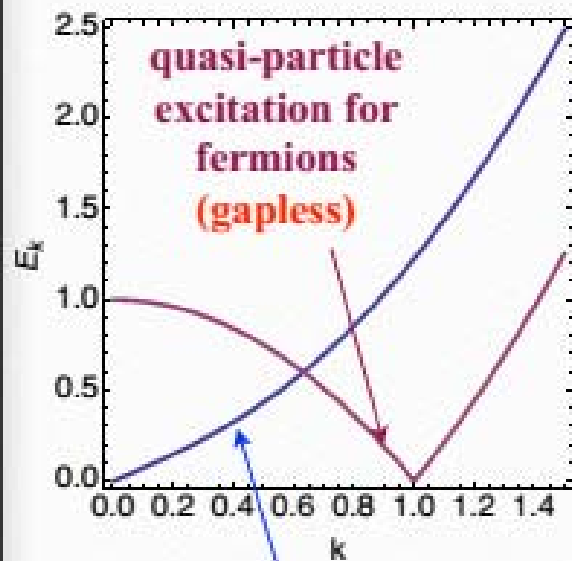
**One Bogoliubov mode for each Bose superfluid**

# Quasi-particles in the Bose-Fermi Mixture

**Bose superfluid and Fermi gas**

**Two Bose superfluids**

**Bose and Fermi superfluids**

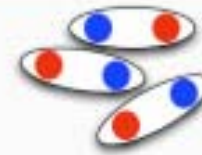


# Quasi-particles in the Bose-Fermi Mixture

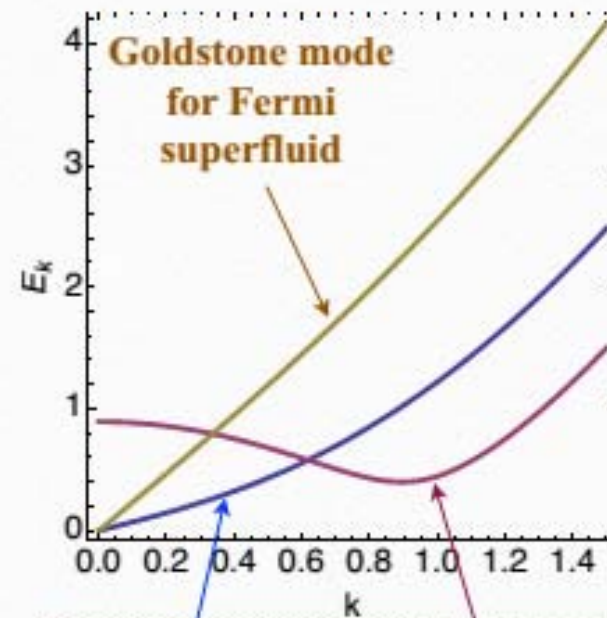
BEC



BCS



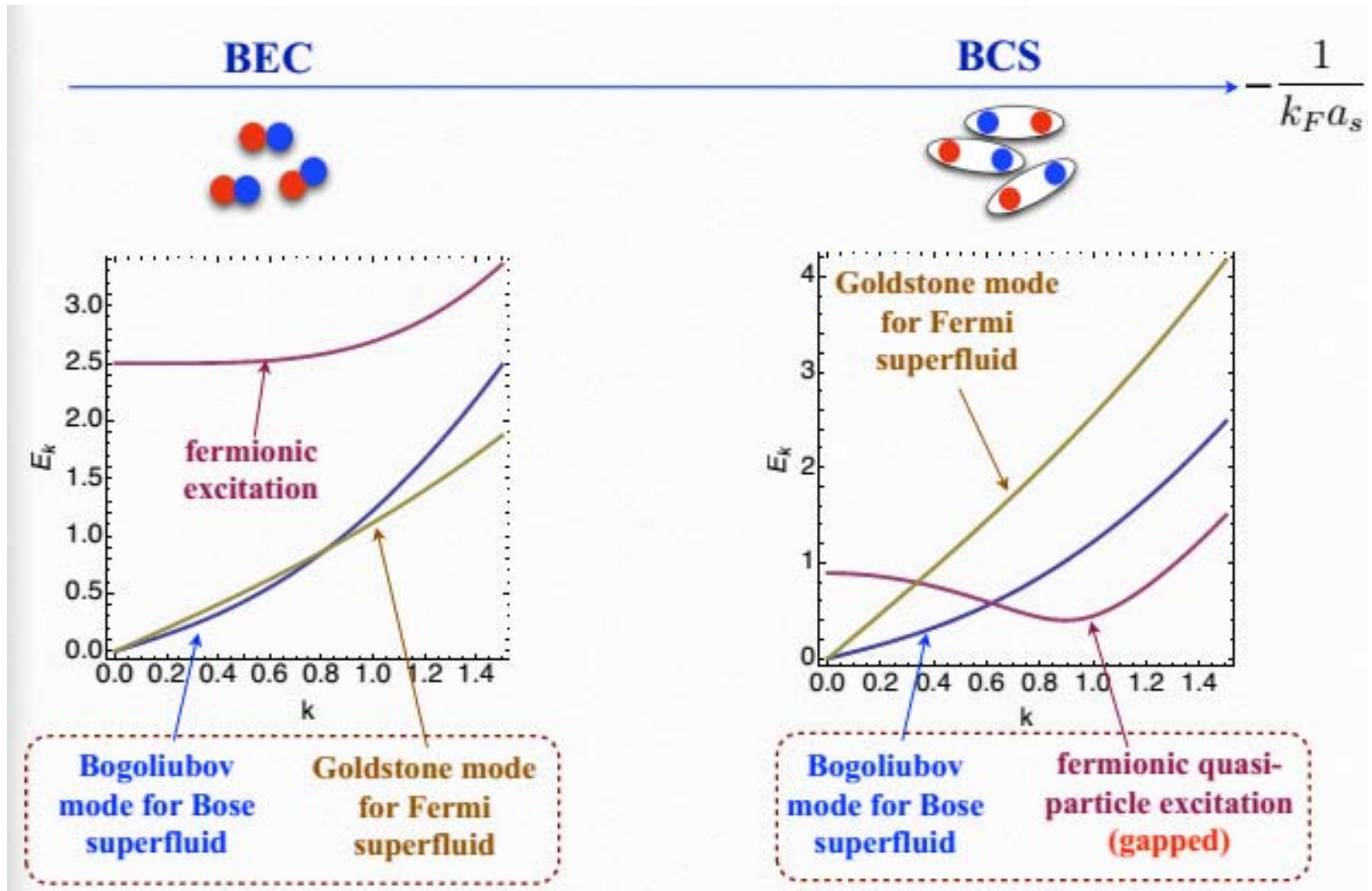
$\frac{1}{k_F a_s}$



Bogoliubov mode for Bose superfluid

fermionic quasi-particle excitation (gapped)

# Quasi-particles in the Bose-Fermi Mixture



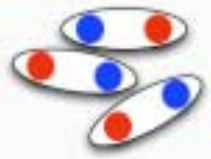
# Quasi-particles in the Bose-Fermi Mixture

**BEC**

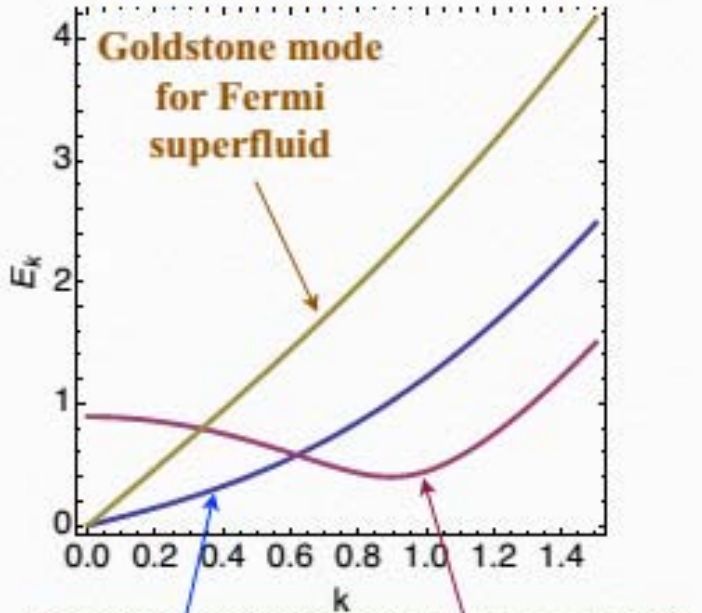
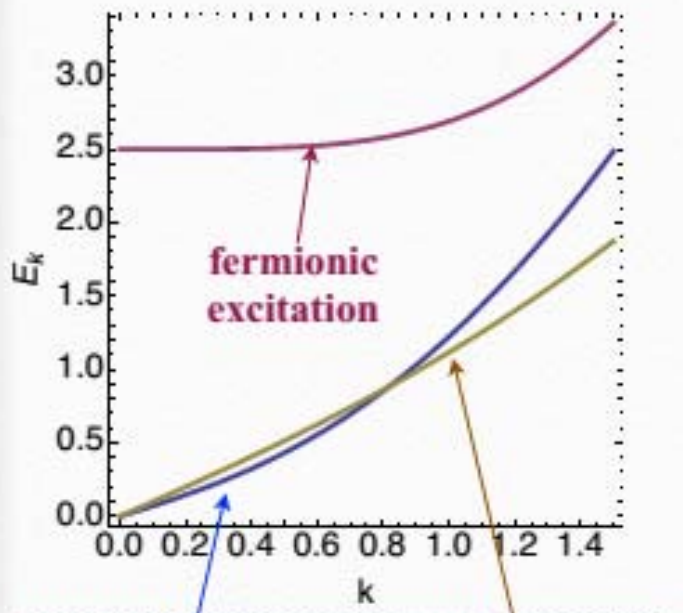


**We focus on the damping of Bogoliubov mode in Bose superfluid**

**BCS**



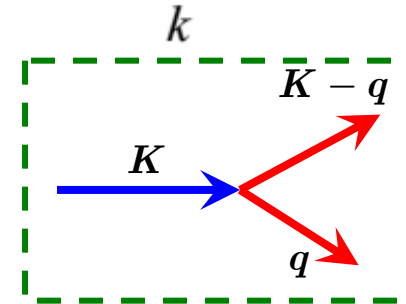
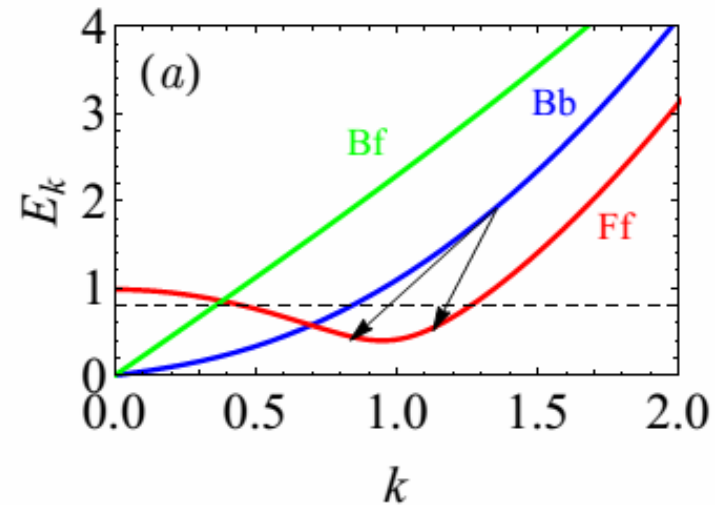
$$\frac{1}{k_F a_s}$$



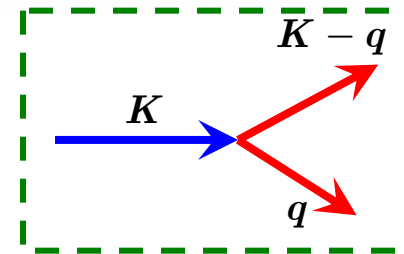
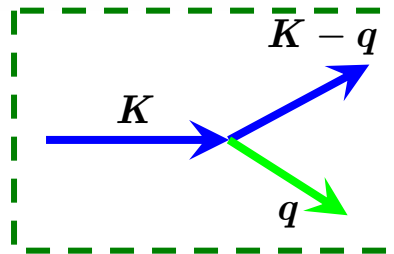
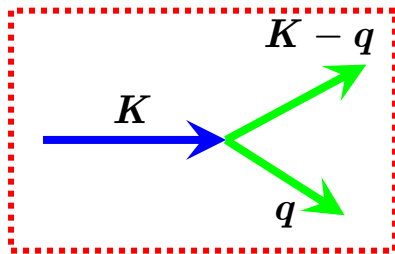
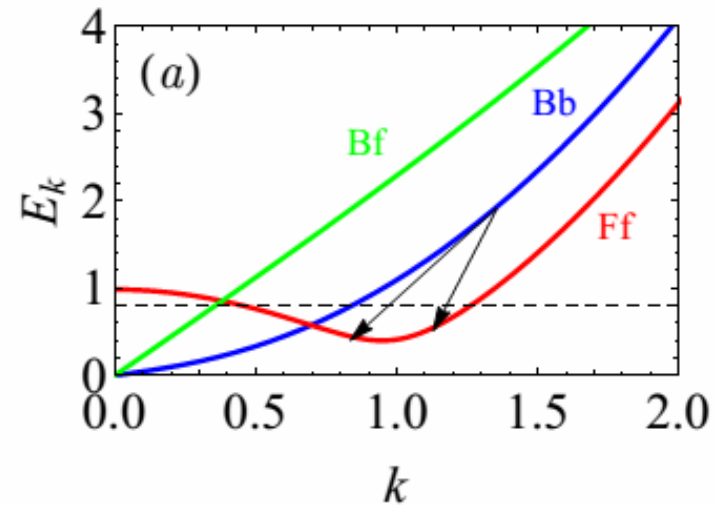
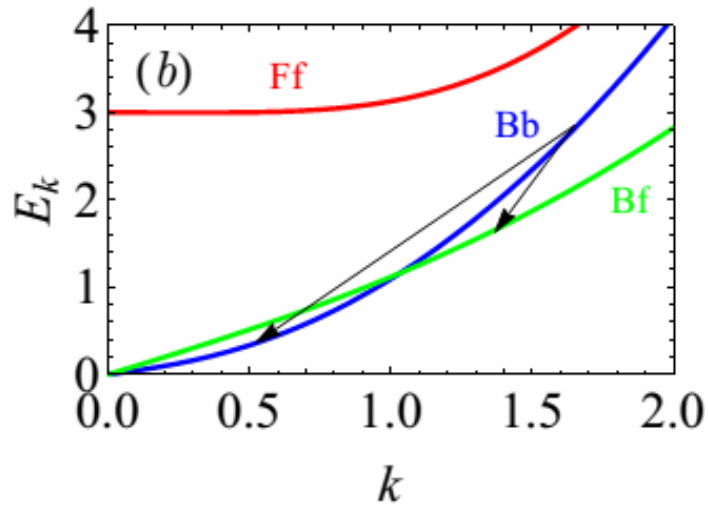
**Bogoliubov mode for Bose superfluid**      **Goldstone mode for Fermi superfluid**

**Bogoliubov mode for Bose superfluid**      **fermionic quasi-particle excitation (gapped)**

# Beliaev damping in the Bose-Fermi superfluid mixture

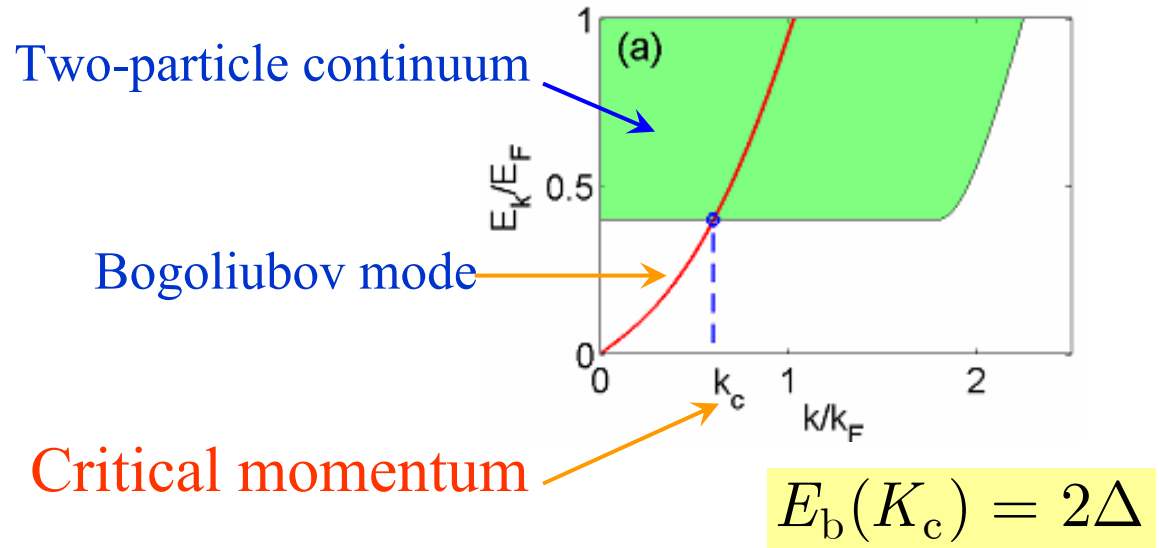


# Beliaev damping in the Bose-Fermi superfluid mixture

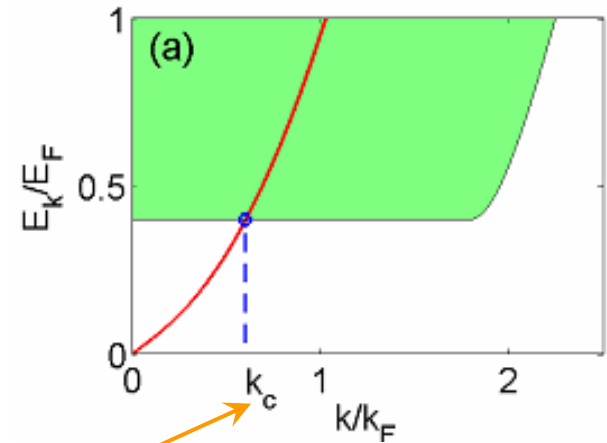
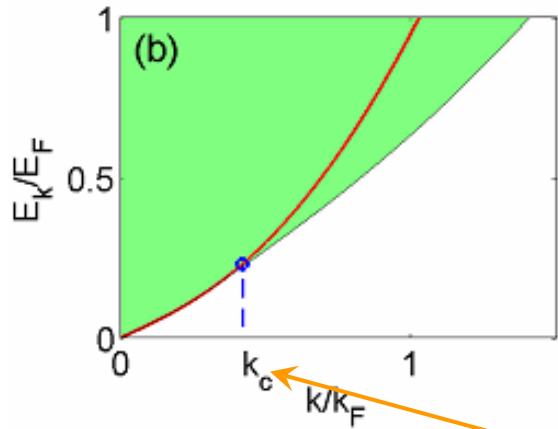




# Beliaev damping in the Bose-Fermi superfluid mixture



# Beliaev damping in the Bose-Fermi superfluid mixture



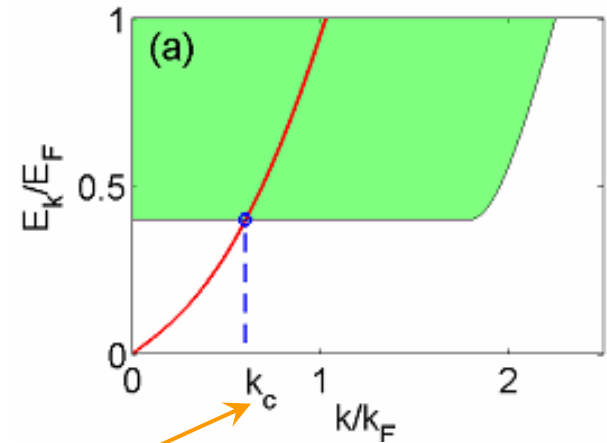
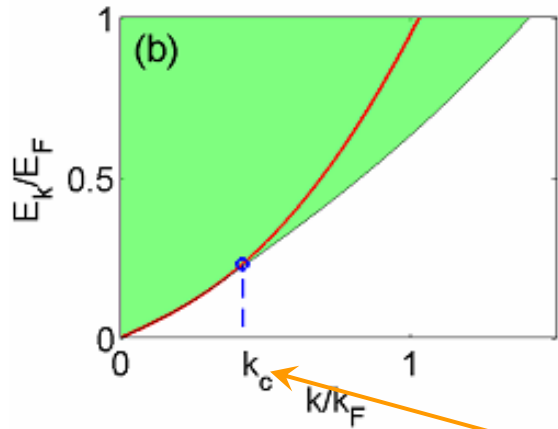
Critical momentum

$$\left. \frac{\partial E_b}{\partial k} \right|_{K_c} = c_m$$

$$E_b(K_c) = 2\Delta$$

Landau criteria for superfluid

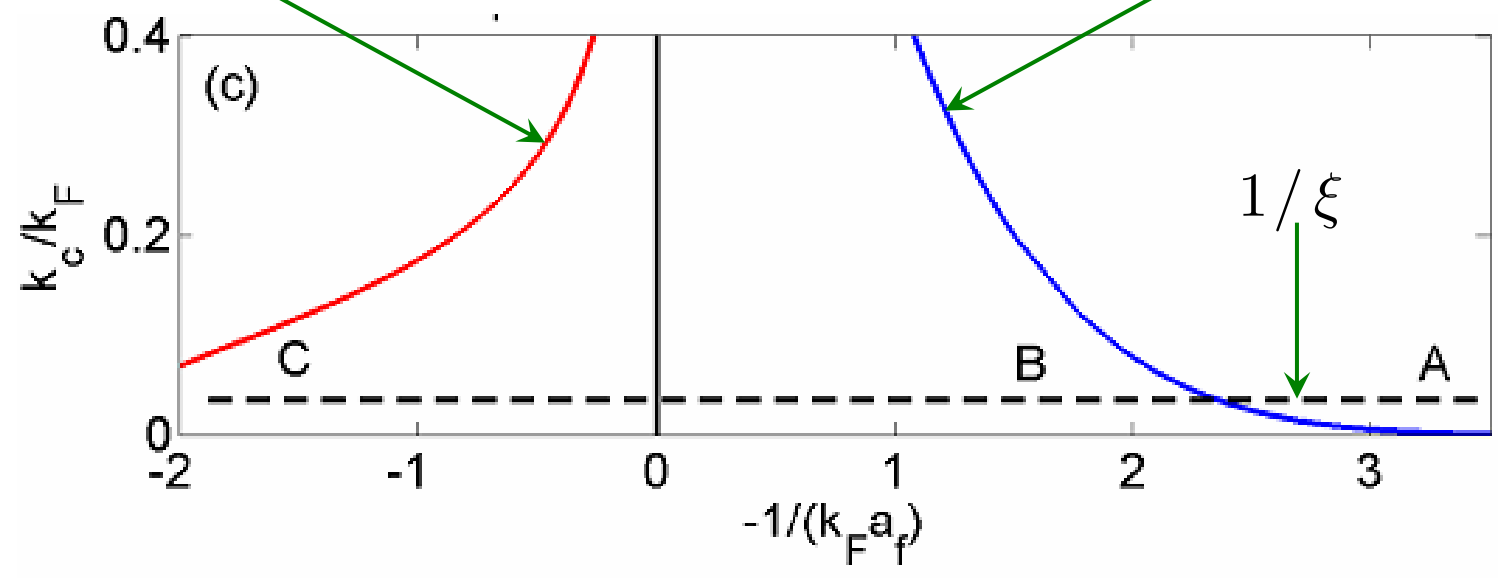
# Beliaev damping in the Bose-Fermi superfluid mixture



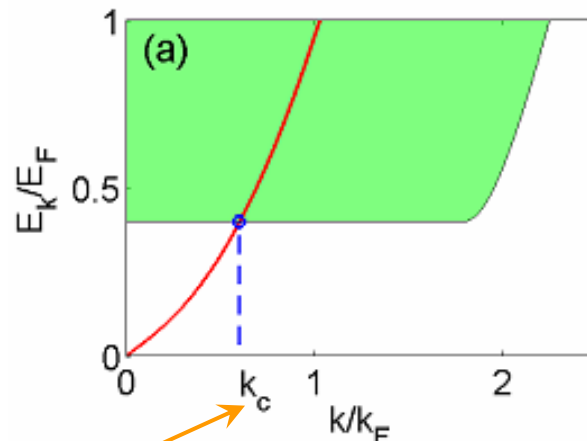
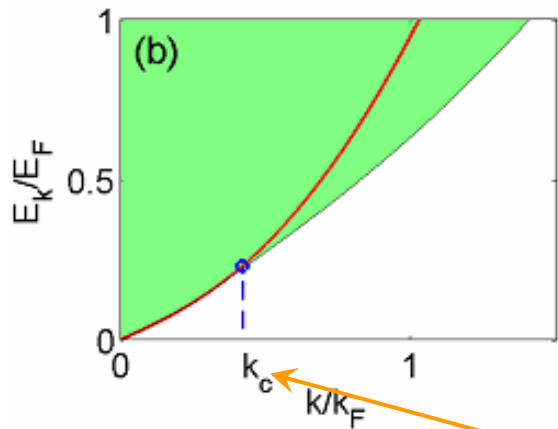
Critical momentum

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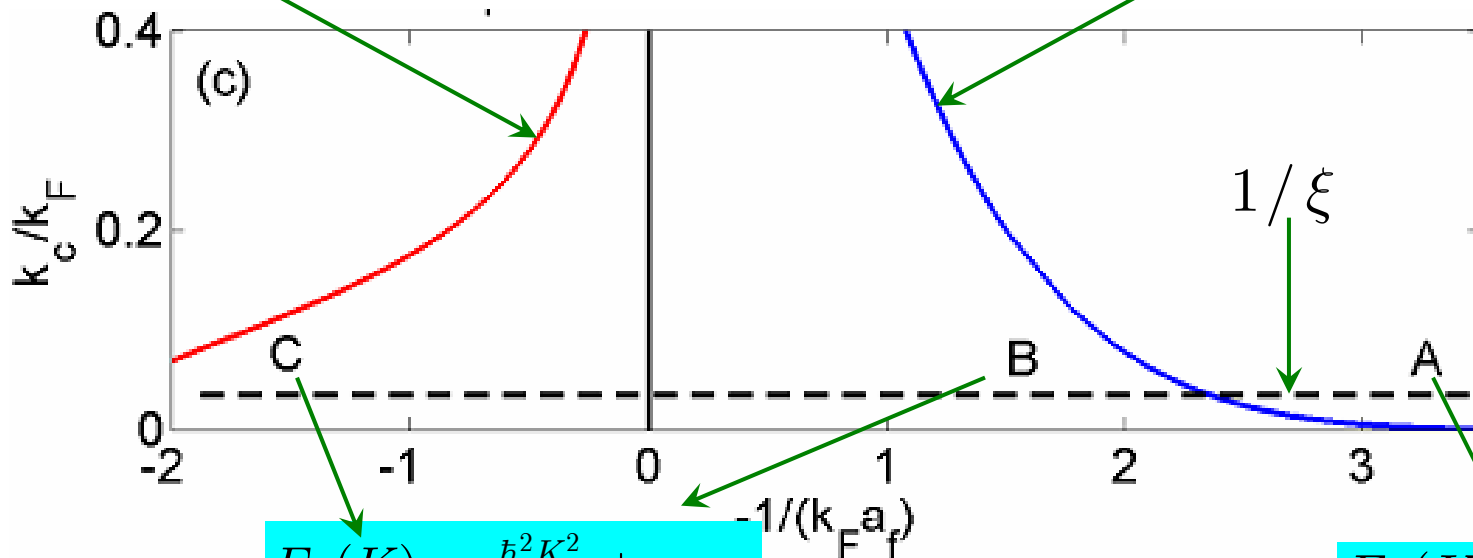
# Beliaev damping in the Bose-Fermi superfluid mixture



Critical momentum

$$\left. \frac{\partial E_b}{\partial k} \right|_{K_c} = c_m$$

$$E_b(K_c) = 2\Delta$$



$$E_b(K) \approx \frac{\hbar^2 K^2}{2m_b} + g_b n_b$$

$$E_b(K) \approx \hbar c_b K$$

# The Hamiltonian of the Superfluid Mixture

$$H_b = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}}^b b_{\mathbf{k}}^+ b_{\mathbf{k}} + \frac{1}{2} \frac{g_b}{V} \sum_{\mathbf{q}, \mathbf{p}, \mathbf{k}} b_{\mathbf{q}+\mathbf{k}}^+ b_{\mathbf{p}-\mathbf{k}}^+ b_{\mathbf{q}} b_{\mathbf{p}}$$

$$H_f = \sum_{\mathbf{k}, \sigma} \varepsilon_{\mathbf{k}}^f c_{\mathbf{k}, \sigma}^+ c_{\mathbf{k}, \sigma} + \frac{g_f}{V} \sum_{\mathbf{q}, \mathbf{p}, \mathbf{k}} c_{\mathbf{q}+\mathbf{k}, \uparrow}^+ c_{\mathbf{p}-\mathbf{k}, \downarrow}^+ c_{\mathbf{q}, \downarrow} c_{\mathbf{p}, \uparrow}$$

$$H_{bf} = \frac{g_{bf}}{V} \sum_{\mathbf{q}, \mathbf{p}, \mathbf{k}, \sigma} c_{\mathbf{q}+\mathbf{k}, \sigma}^+ c_{\mathbf{q}, \sigma} b_{\mathbf{p}-\mathbf{k}}^+ b_{\mathbf{p}}$$

BEC side

BCS side

$$H = \sum_{\mathbf{k}} E_b(\mathbf{k}) \alpha_{\mathbf{k}}^+ \alpha_{\mathbf{k}} + \sum_{\mathbf{k}} E_m(\mathbf{k}) \chi_{\mathbf{k}}^+ \chi_{\mathbf{k}} + H_3^{\text{bm}} + \dots$$

$$H = \sum_{\mathbf{k}} E_b(\mathbf{k}) \alpha_{\mathbf{k}}^+ \alpha_{\mathbf{k}} + \sum_{\mathbf{k}} E_f(\mathbf{k}) (\beta_{\mathbf{k}}^+ \beta_{\mathbf{k}} + \gamma_{\mathbf{k}}^+ \gamma_{\mathbf{k}}) + H_3^{\text{bf}} + \dots$$

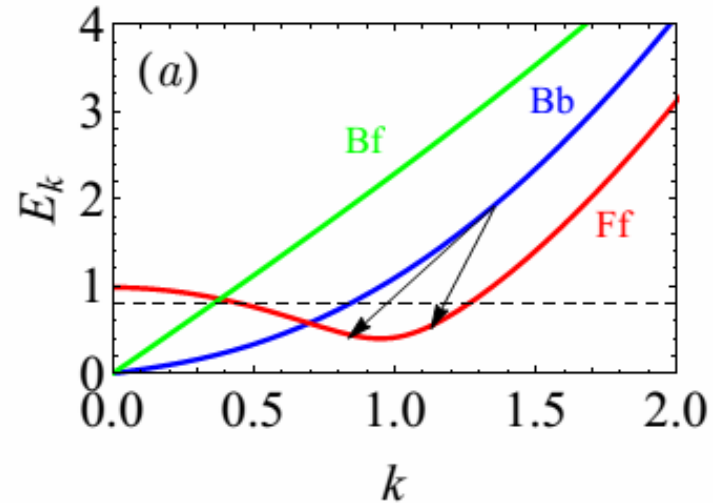
$$E_b(\mathbf{k}) = \sqrt{\varepsilon_{\mathbf{k}}^b (\varepsilon_{\mathbf{k}}^b + 2g_b n_b)}$$

$$E_b(\mathbf{k}) = \sqrt{(\varepsilon_{\mathbf{k}}^f - \mu_f)^2 + \Delta^2}$$

$$E_m(\mathbf{k}) = \sqrt{\varepsilon_{\mathbf{k}}^m (\varepsilon_{\mathbf{k}}^m + 2g_m n_m)}$$

# Beliaev damping in BCS side

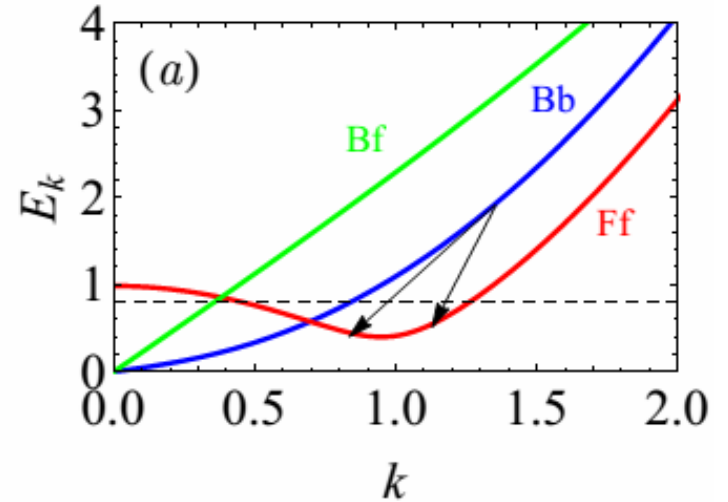
$$\begin{aligned}
 H = & \sum_{\mathbf{k}} E_b(\mathbf{k}) \alpha_{\mathbf{k}}^+ \alpha_{\mathbf{k}} \\
 & + \sum_{\mathbf{k}} E_f(\mathbf{k}) (\beta_{\mathbf{k}}^+ \beta_{\mathbf{k}} + \gamma_{\mathbf{k}}^+ \gamma_{\mathbf{k}}) \\
 & + H_3^{\text{bf}} + \dots
 \end{aligned}$$



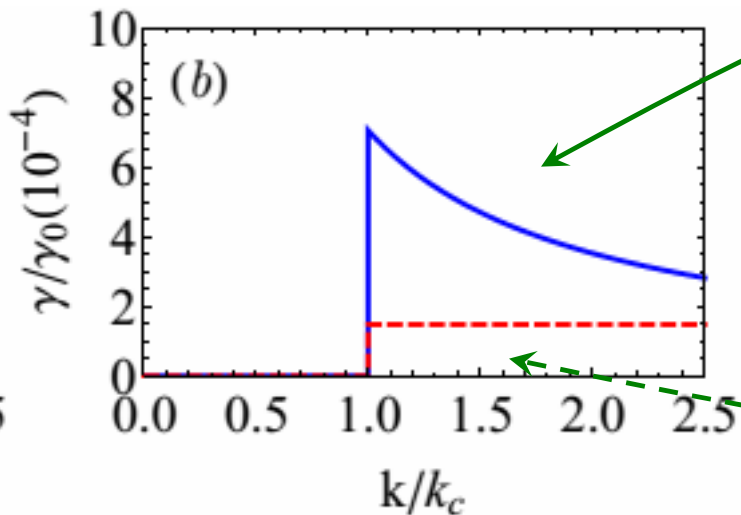
$$H_3^{\text{bf}} = \frac{g_{\text{bf}} \sqrt{N_b}}{V} \sum_{q\mathbf{k}} (u_{\mathbf{k}}^{\text{b}} - v_{\mathbf{k}}^{\text{b}}) (u_{\mathbf{k}-q}^{\text{f}} v_q^{\text{f}} + v_{\mathbf{k}-q}^{\text{f}} u_q^{\text{f}}) \beta_{\mathbf{k}-q}^+ \gamma_q^+ \alpha_{\mathbf{k}} + \text{h.c.}$$

# Beliaev damping in BCS side

$$\begin{aligned}
 H = & \sum_{\mathbf{k}} E_b(\mathbf{k}) \alpha_{\mathbf{k}}^+ \alpha_{\mathbf{k}} \\
 & + \sum_{\mathbf{k}} E_f(\mathbf{k}) (\beta_{\mathbf{k}}^+ \beta_{\mathbf{k}} + \gamma_{\mathbf{k}}^+ \gamma_{\mathbf{k}}) \\
 & + H_3^{\text{bf}} + \dots
 \end{aligned}$$



$$H_3^{\text{bf}} = \frac{g_{\text{bf}} \sqrt{N_b}}{V} \sum_{qk} (u_{\mathbf{k}}^b - v_{\mathbf{k}}^b) (u_{\mathbf{k}-q}^f v_q^f + v_{\mathbf{k}-q}^f u_q^f) \beta_{\mathbf{k}-q}^+ \gamma_q^+ \alpha_{\mathbf{k}} + \text{h.c.}$$



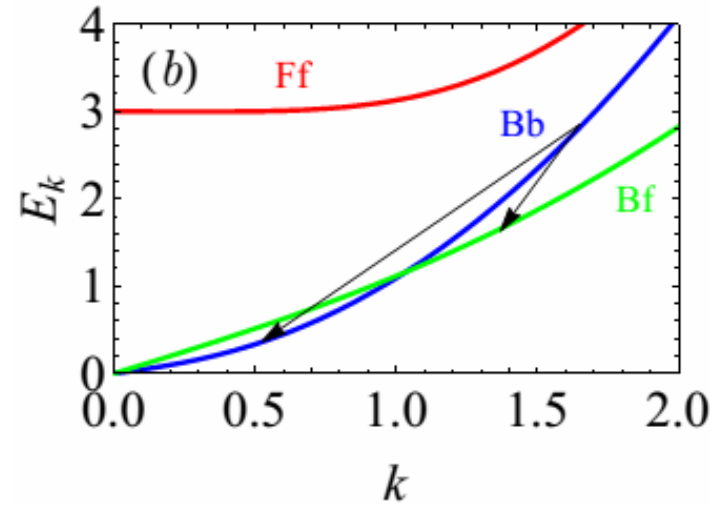
$$\gamma(\mathbf{K}) = \begin{cases} \frac{g_{\text{bf}}^2 n_b \Delta m_f^2}{\hbar^4 K} \Theta(E_{\mathbf{K}}^b - 2\Delta), & \text{regime B} \\ \frac{g_{\text{bf}}^2 c_b \Delta m_f^2}{2\hbar^4 g_{\text{bb}}} \Theta(E_{\mathbf{K}}^b - 2\Delta), & \text{regime A} \end{cases}$$

# Beliaev damping in BEC side

$$\begin{aligned}
 H &= \sum_{\mathbf{k}} E_b(\mathbf{k}) \alpha_{\mathbf{k}}^+ \alpha_{\mathbf{k}} \\
 &+ \sum_{\mathbf{k}} E_m(\mathbf{k}) \chi_{\mathbf{k}}^+ \chi_{\mathbf{k}} \\
 &+ H_3^{\text{bm}} + \dots
 \end{aligned}$$



$$H_3^{\text{bm}} = \frac{g_{\text{mb}} \sqrt{N_m}}{V} \sum_{qk} (u_q^{\text{m}} - v_q^{\text{m}}) (u_{\mathbf{k}-q}^{\text{b}} u_{\mathbf{k}}^{\text{b}} + v_{\mathbf{k}-q}^{\text{b}} v_{\mathbf{k}}^{\text{b}}) \chi_q^+ \alpha_{\mathbf{k}-q}^+ \alpha_{\mathbf{k}} + \text{h.c.}$$



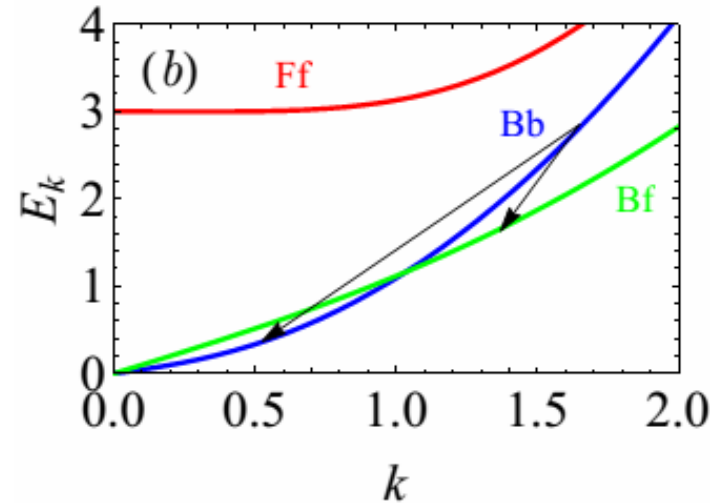
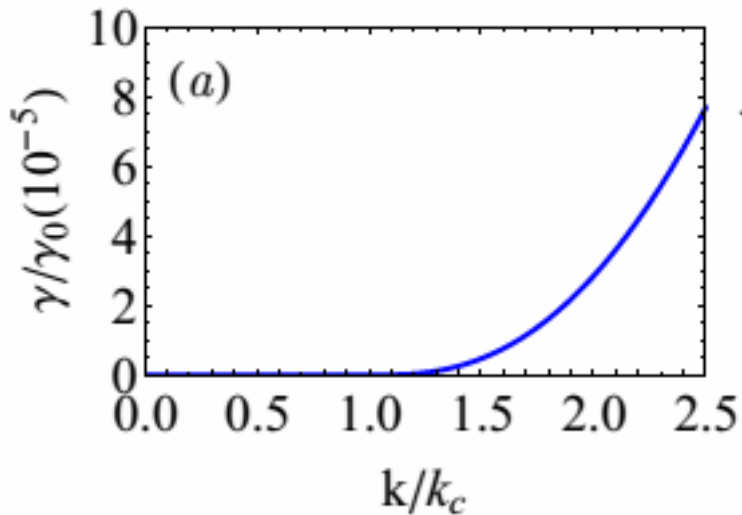


# Beliaev damping in BEC side

$$\begin{aligned}
 H &= \sum_{\mathbf{k}} E_b(\mathbf{k}) \alpha_{\mathbf{k}}^+ \alpha_{\mathbf{k}} \\
 &+ \sum_{\mathbf{k}} E_m(\mathbf{k}) \chi_{\mathbf{k}}^+ \chi_{\mathbf{k}} \\
 &+ H_3^{\text{bm}} + \dots
 \end{aligned}$$

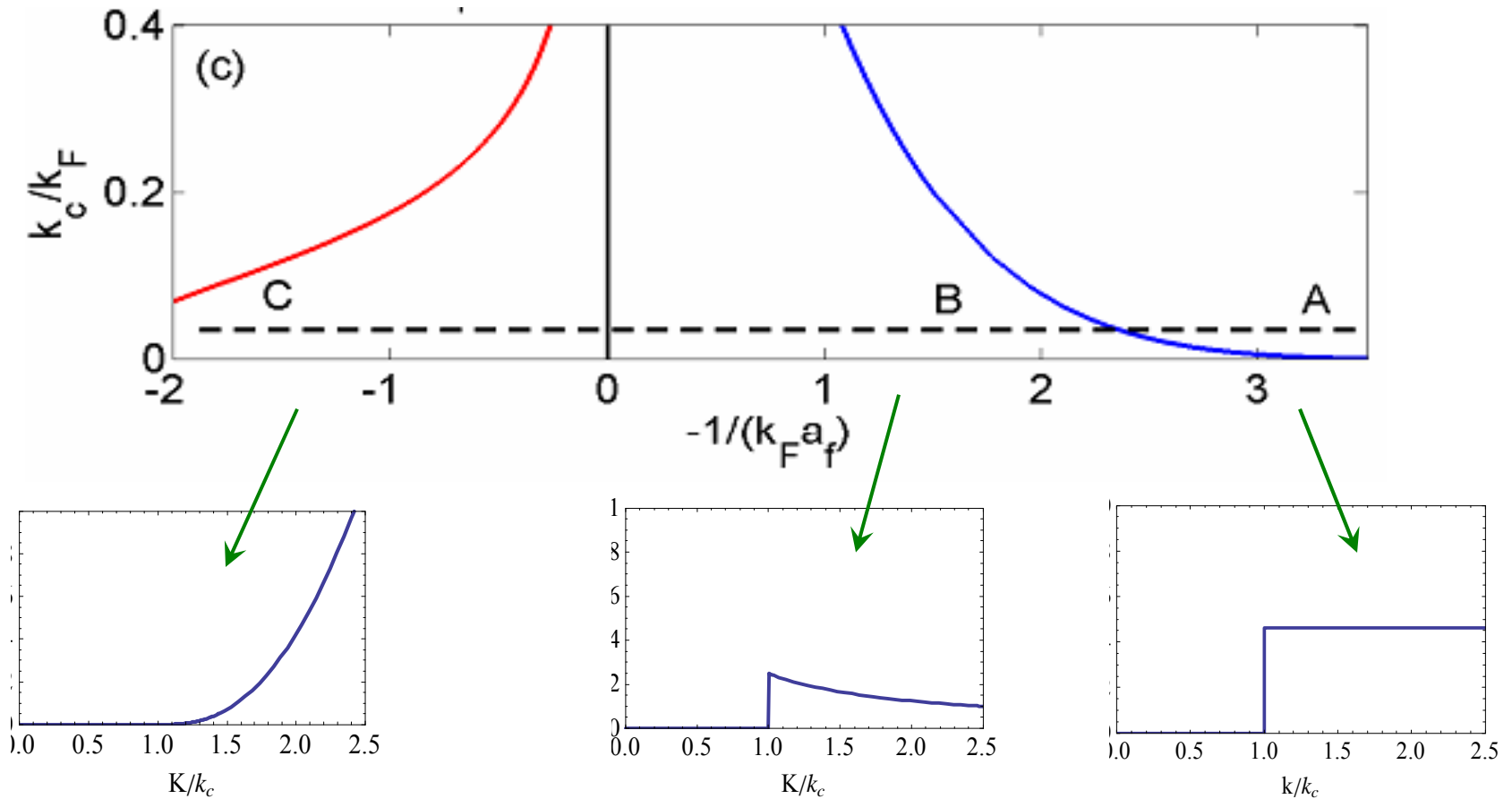


$$H_3^{\text{bm}} = \frac{g_{\text{mb}} \sqrt{N_m}}{V} \sum_{\mathbf{q}, \mathbf{k}} (u_{\mathbf{q}}^{\text{m}} - v_{\mathbf{q}}^{\text{m}}) (u_{\mathbf{k}-\mathbf{q}}^{\text{b}} u_{\mathbf{k}}^{\text{b}} + v_{\mathbf{k}-\mathbf{q}}^{\text{b}} v_{\mathbf{k}}^{\text{b}}) \chi_{\mathbf{q}}^+ \alpha_{\mathbf{k}-\mathbf{q}}^+ \alpha_{\mathbf{k}} + \text{h.c.}$$



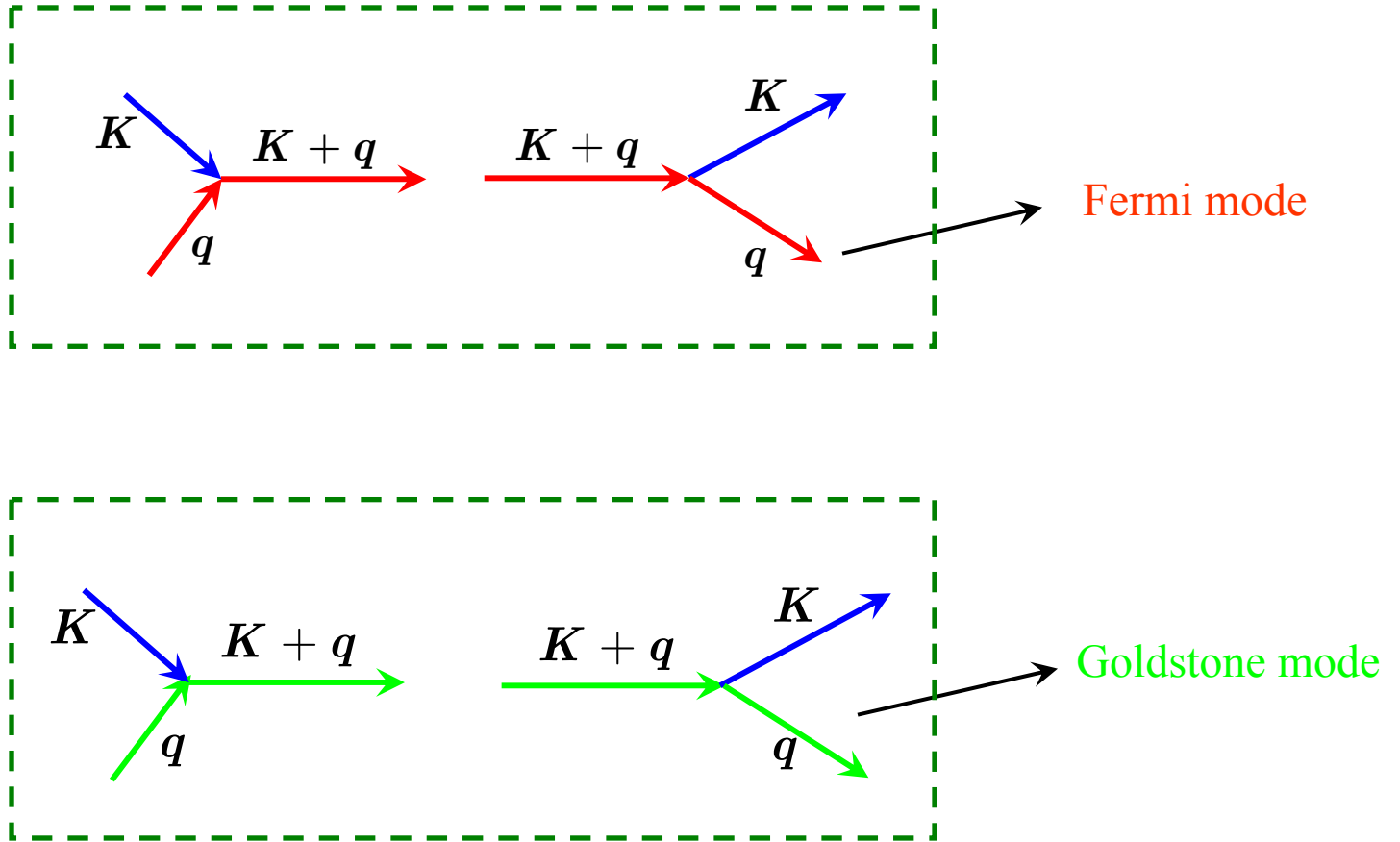
$$\begin{aligned}
 \gamma(\mathbf{K}) &= \frac{2g_{\text{mb}}^2 m_b c_m}{3\pi \hbar^4 g_m K} (K - m_b c_m)^3 \Theta(K - m_b c_m) \\
 &\approx C(\delta K)^3 \Theta(K - K_c), \quad \text{regime C}
 \end{aligned}$$

# Beliaev damping through the crossover



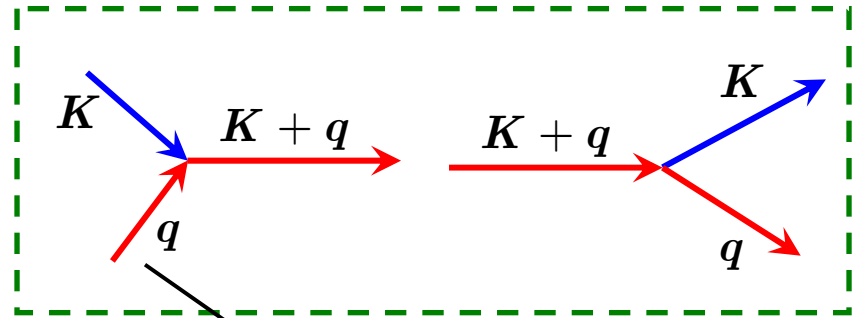
Wei Zheng and Hui Zhai, PRL **113**, 265304 (2014)

# Landau damping in the mixture



**No critical velocity**

# Landau damping in the mixture



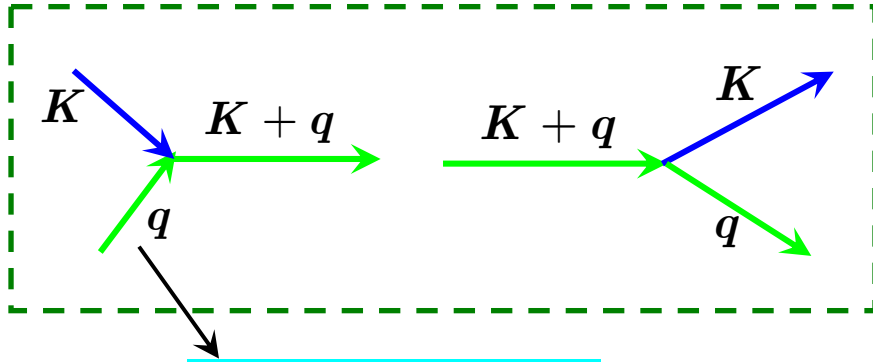
Fermi gap :  $\Delta$

When  $k_B T < \Delta$

damping is suppressed!

$$f_q^f = \frac{1}{e^{\beta E_f(q)} + 1}$$

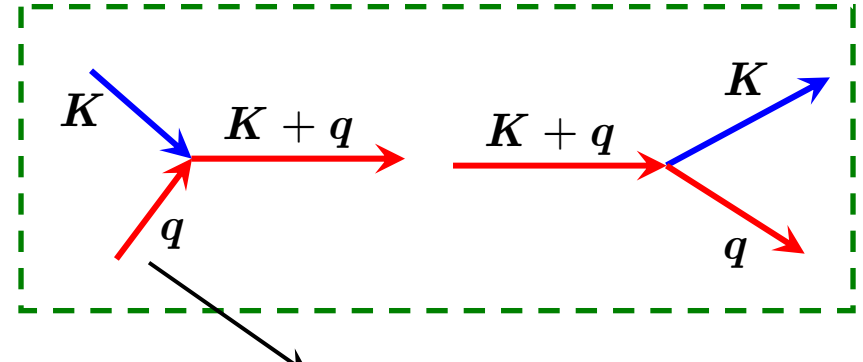
# Landau damping in the mixture



restriction:  $q > q_0$

When  $k_B T < \hbar c_m q_0$   
damping is suppressed!

$$f_q^b = \frac{1}{e^{\beta E_m(q)} - 1}$$

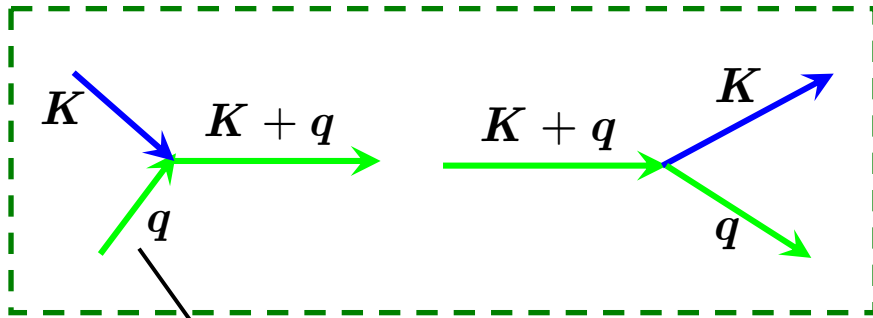


Fermi gap :  $\Delta$

When  $k_B T < \Delta$   
damping is suppressed!

$$f_q^f = \frac{1}{e^{\beta E_f(q)} + 1}$$

# Landau damping in the mixture

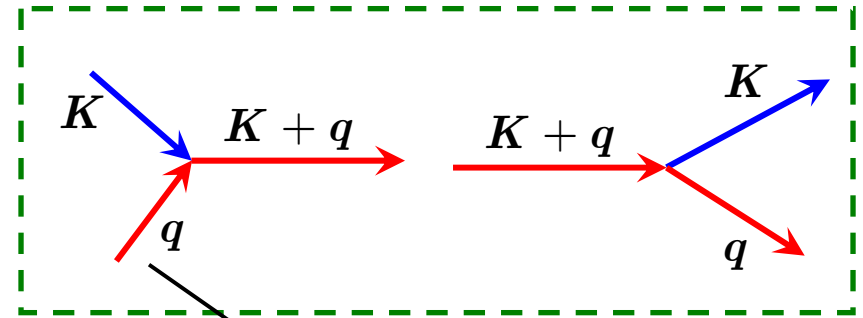


restriction:  $q > q_0$

When  $k_B T < \hbar c_m q_0$   
damping is suppressed!

$$f_q^b = \frac{1}{e^{\beta E_m(q)} - 1}$$

**Dominate in BEC, Suppressed in BCS**



Fermi gap :  $\Delta$

When  $k_B T < \Delta$   
damping is suppressed!

$$f_q^f = \frac{1}{e^{\beta E_f(q)} + 1}$$

**Dominate in BCS, Suppressed in BEC**

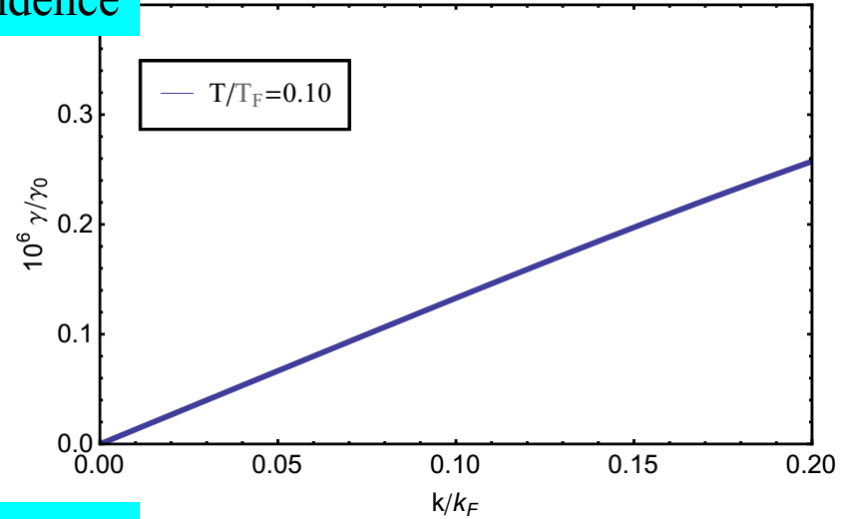


# Landau damping in the mixture

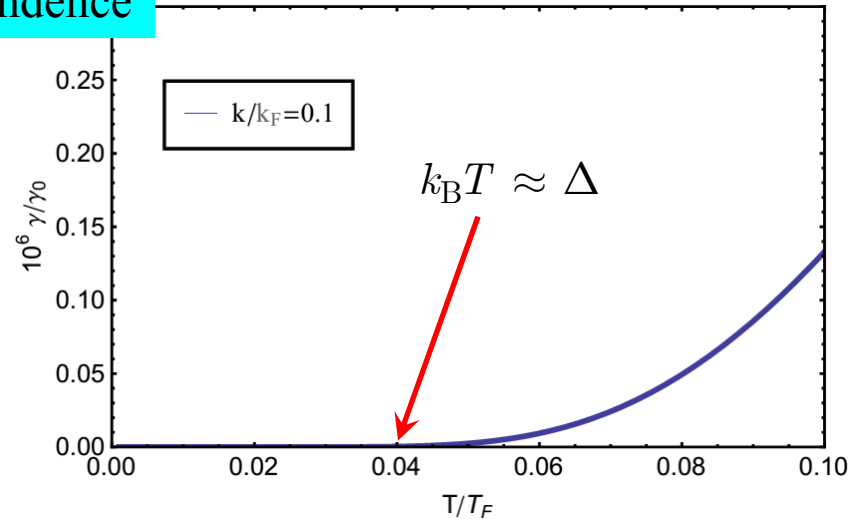
BEC side

BCS side

K-dependence



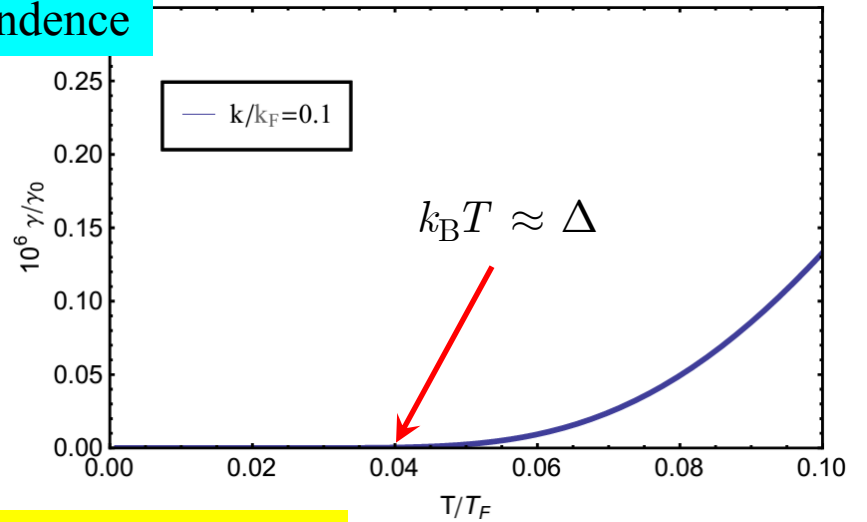
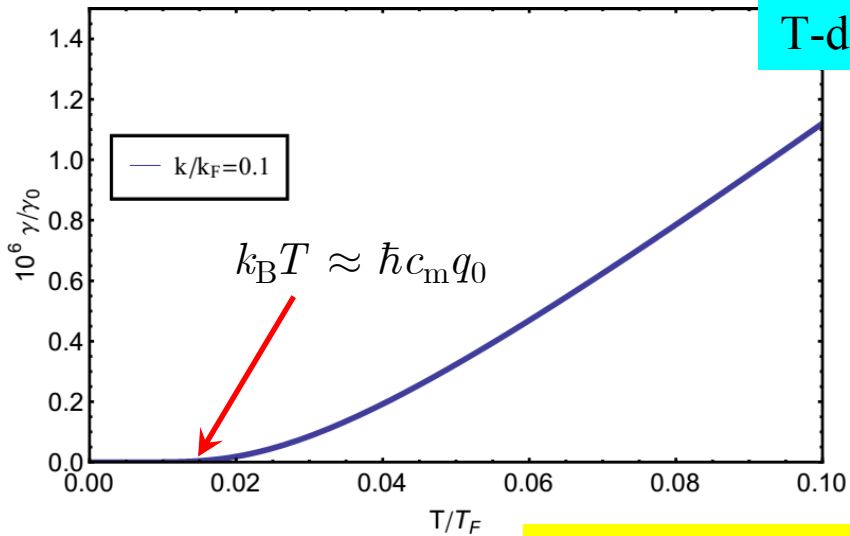
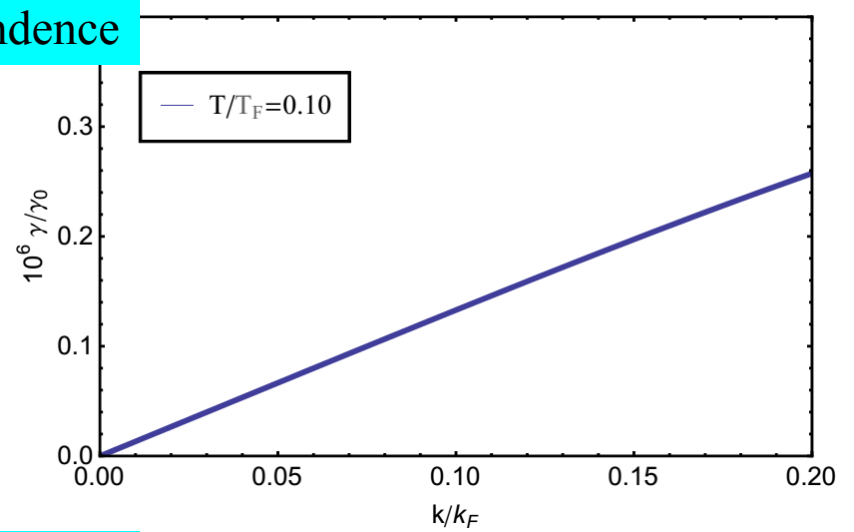
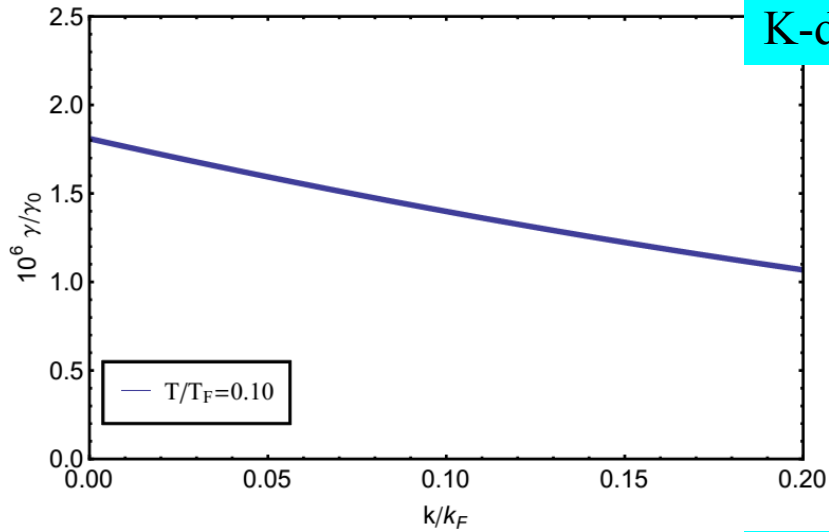
T-dependence



# Landau damping in the mixture

BEC side

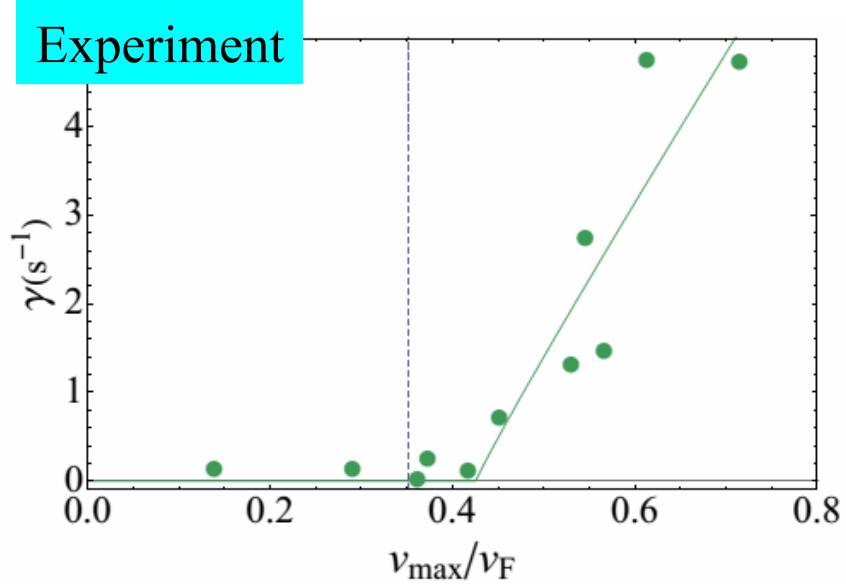
BCS side



**Exponentially suppressed in low-T**

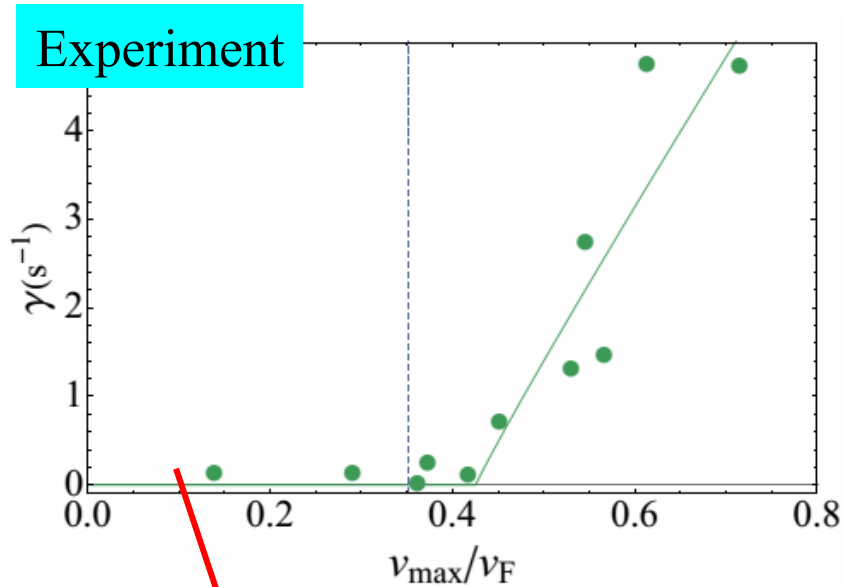


# Compare to the experiment

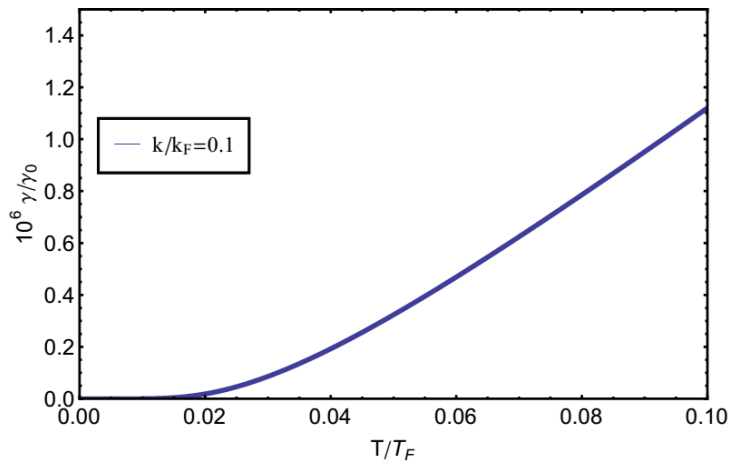


The experiment is in  
the unitary regime,  
Similar to the BEC side

# Compare to the experiment

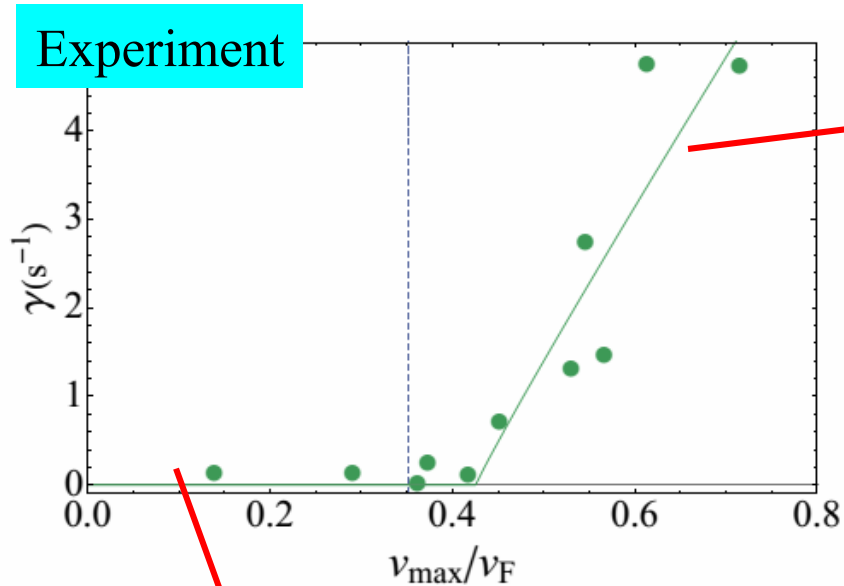


Landau damping is suppressed in low-T

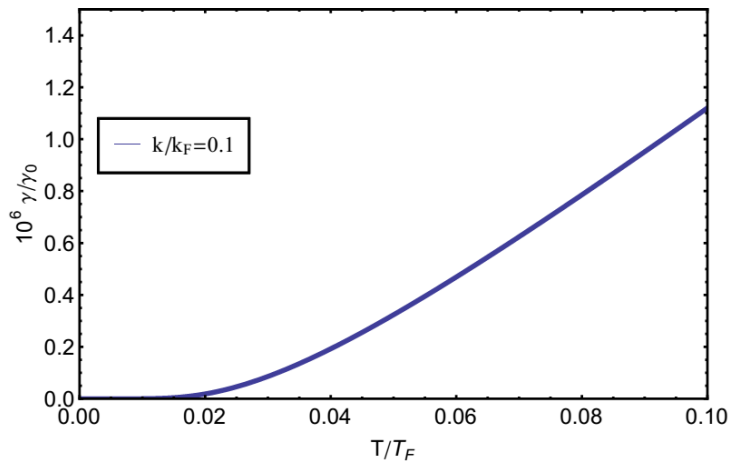


The experiment is in the unitary regime, Similar to the BEC side

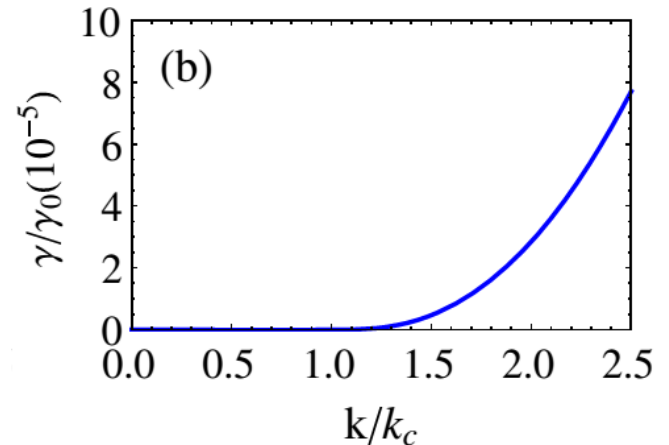
# Compare to the experiment



**Landau damping is suppressed in low-T**



**Beliaev damping dominate**



**critical velocity show up**

The experiment is in the unitary regime, Similar to the BEC side

# Summary

- We try to explain the damping of the dipole oscillation by the Landau-Beliaev damping, which is due to interactions between the quasi-particles in the Bose-Fermi superfluid mixture.
- The experiment was done in a harmonic trap. One need a calculation in the trap.
- The Landau-Beliaev damping gives the quasi-particles a finite lifetime. That will affect many physical properties of the superfluid mixture, such as thermodynamics, transport.

# Thank You !

And thanks to my collaborators:



Huitao Shen



Prof. Hui Zhai