Competing phases in dipolar quantum gas

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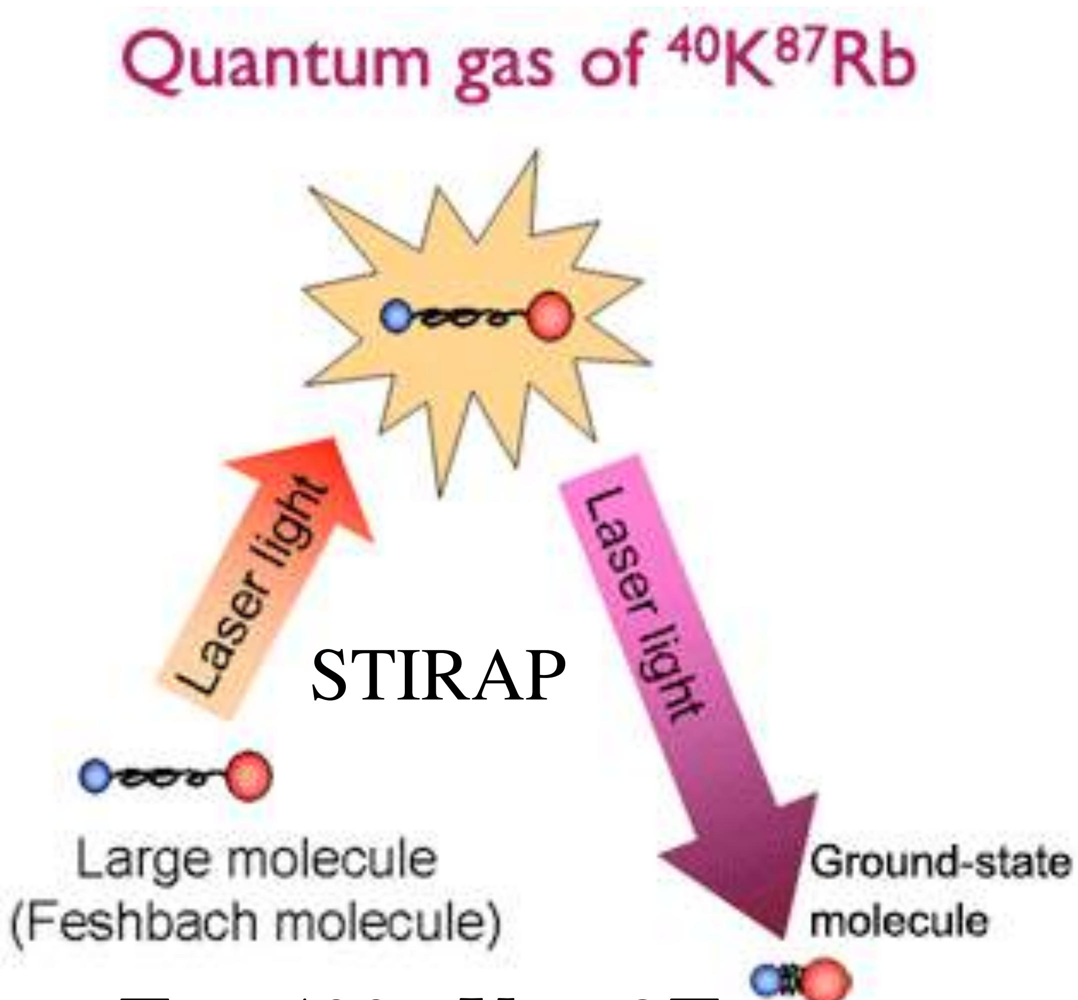
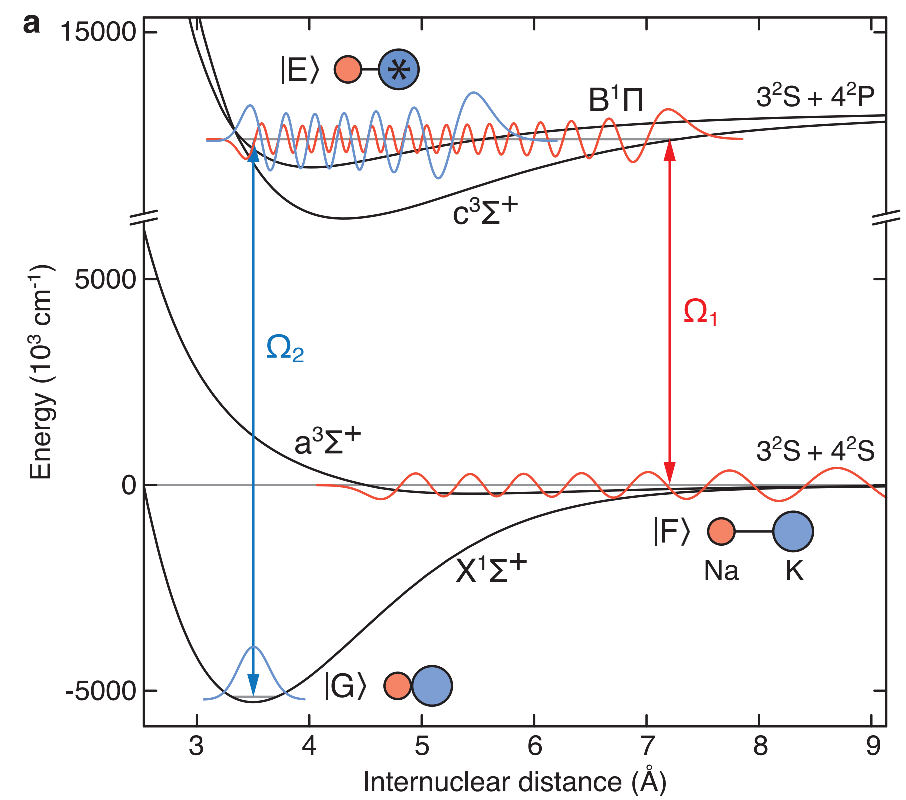
Fermi gas of polar molecules: KRb, NaK



$T \sim 400nK \sim 3T_F$

$d \sim 0.5$ Debye

Ground state $^{23}Na^{40}K$ molecules



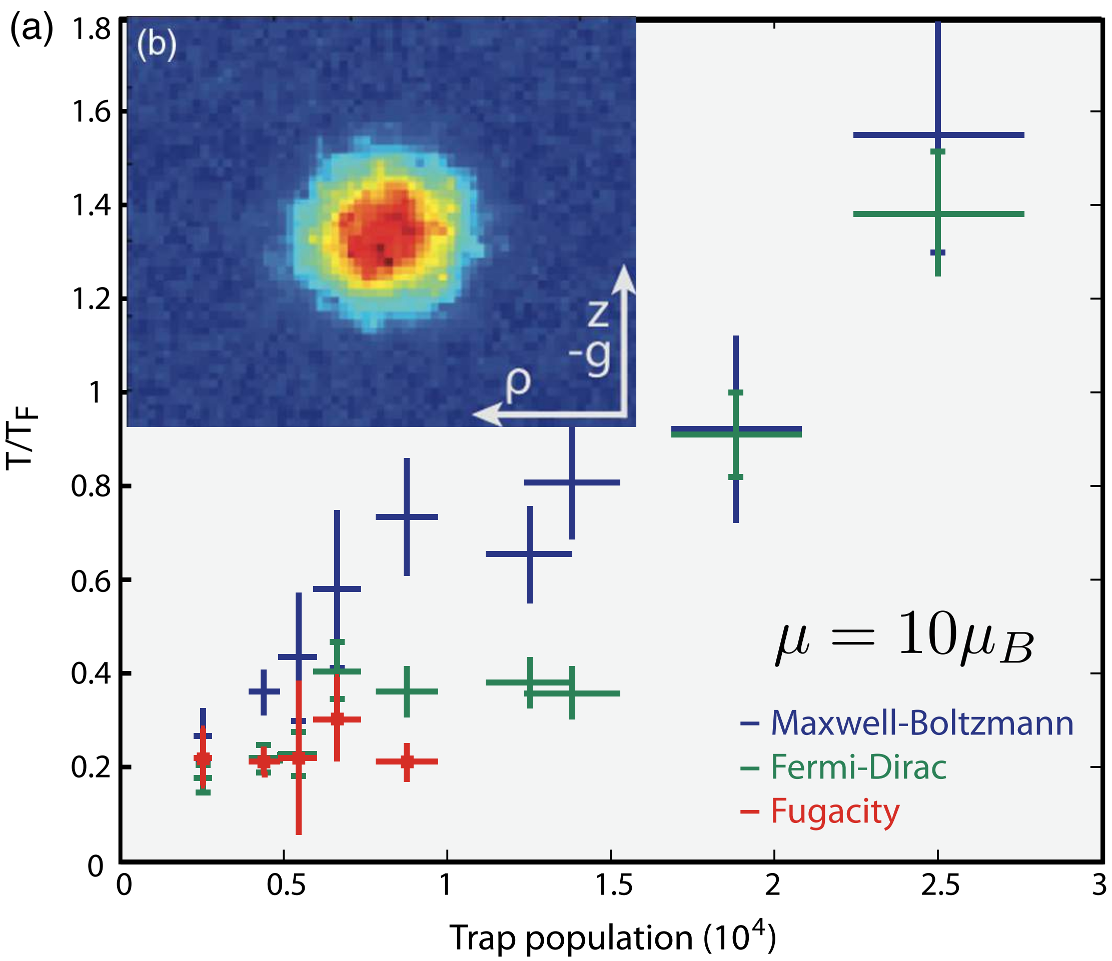
Chemically stable. d~0.8 Debye.
Life time > 2.5s; T ~ 500nK ~ $2T_F$

Ni et al, Science 322, 231-235 (2008)
D. S. Jin and J. Ye, Physics Today 64, 5(2011)
Chotia et al, PRL 108, 080405 (2012)

Wu et al, PRL 109, 085301 (2012)
Park, Will, Zwierlein, arxiv:1505.00473 (2015)

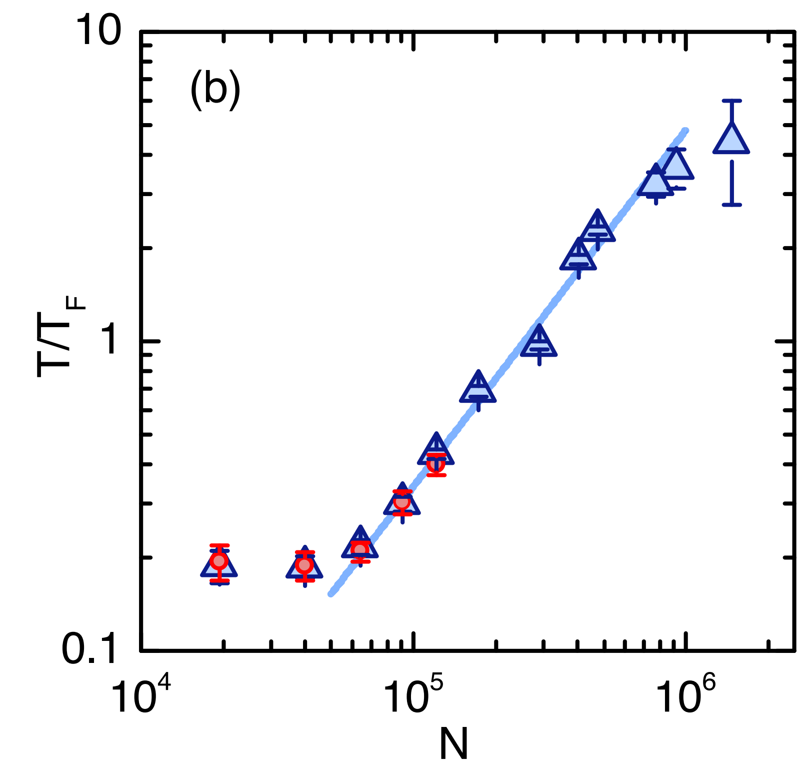
Degenerate Fermi gas of magnetic atoms: ^{161}Dy, ^{167}Er

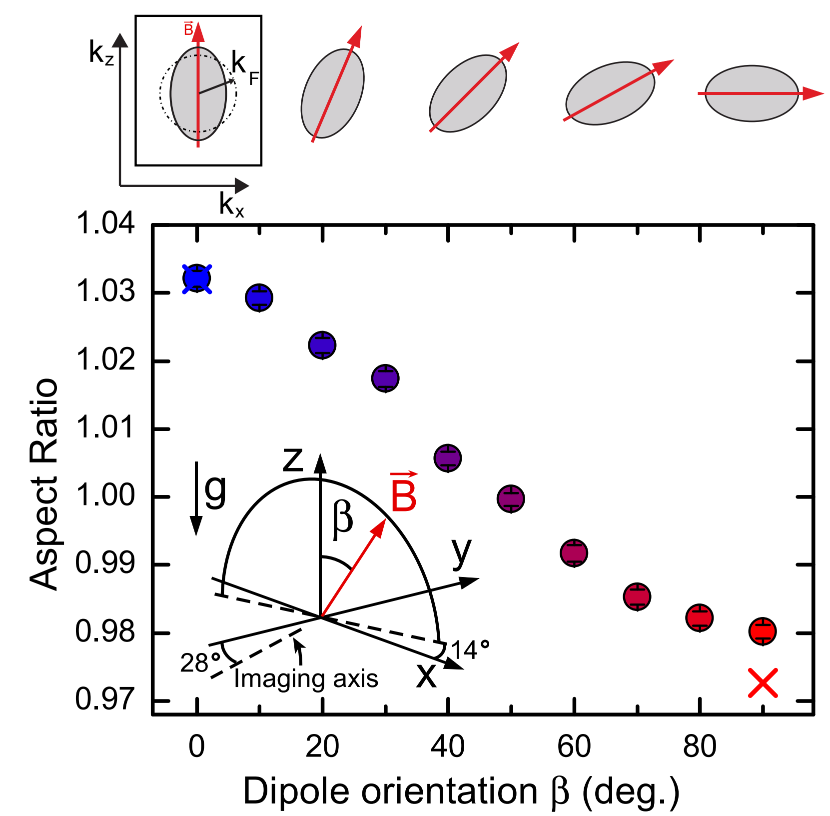
Sympathetic cooling of ^{161}Dy with bosonic ^{162}Dy



$$T/T_F = 0.2 \qquad T_F = 300\ \mathrm{nK}$$

Lu, Burdick, Lev, PRL 108, 215301 (2012)



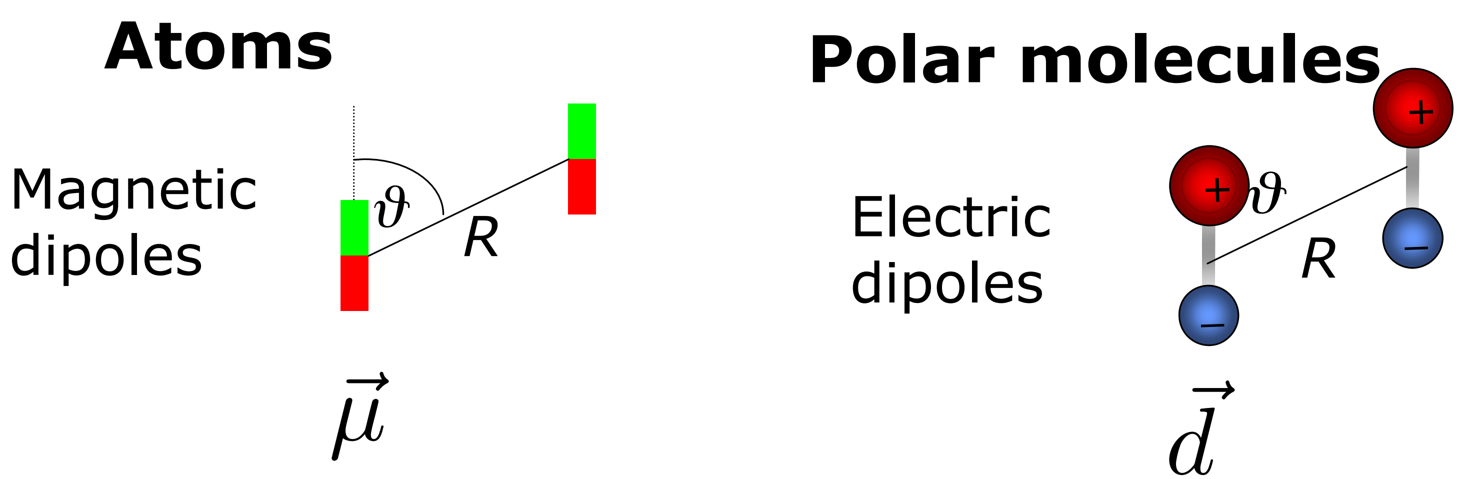


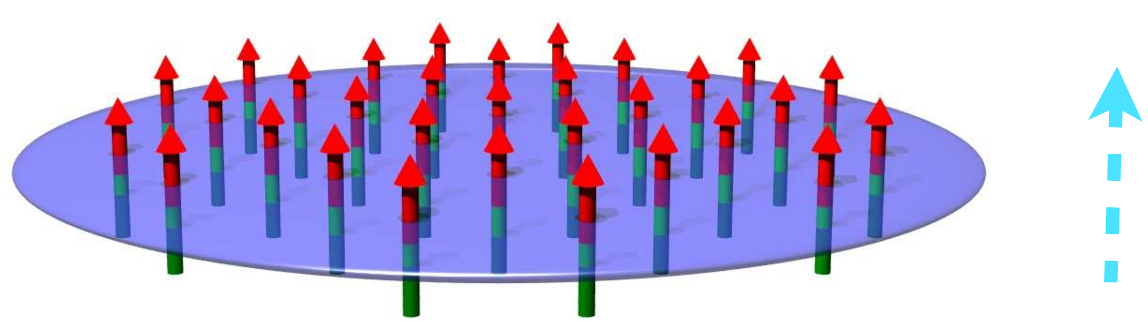
Aikawa et al, PRL 112, 010404 (2014)
Science 345, 1484 (2014)

Quantum phases of dipolar fermi gases

Q: What are the many-body phases of fermions with dipole-dipole interaction?

Are they all "boring," i.e., known and understood in condensed matter physics?





B or ***E*** field

For dipoles pointing in the same direction:

$$V_{dd} = \frac{d^2}{4\pi\epsilon_0}\frac{1-3\cos^2\theta}{r^3}$$

⟶ $P_2(\cos\theta)$ *anisotropic*

⟶ *long-ranged*

Comparing to other Fermi systems

Fermi System	**Interaction**	**Typical Phases**
2D electron gas	Coulomb	Fermi liquid, Wigner crystal
Fermi-Hubbard model	onsite, repulsive	antiferromagnet, *d*-wave superfluid(?)
2-component Fermi gas	contact, attractive	*s*-wave superfluid (BCS-BEC crossover)
dipolar Fermi gas	dipole-dipole	

Candidate phases of dipolar fermions:

- ★ anisotropic Fermi liquid
- ★ charge density waves (CDW)
- ★ p-wave superfluid
- ★ stripes, quantum liquid crystals?
- ★ supersolid? ...

Baranov et al, Chemical Reviews 112, 5012 (2012); Physics Reports, 464, 71 (2008). Lahaye et al, Rep. Prog. Phys. 72, 126401 (2009), etc.

Outline of this talk

1. Dipolar Fermi gas on square lattice @ half filling:
   phase diagram from functional renormalization group

2. Continuum gas of dipolar fermions:
   trying to go beyond Hartree-Fock and RPA

3. Frustrated magnetism of localized (deeply trapped) dipoles:
   hints from exact diagonalization on a small lattice

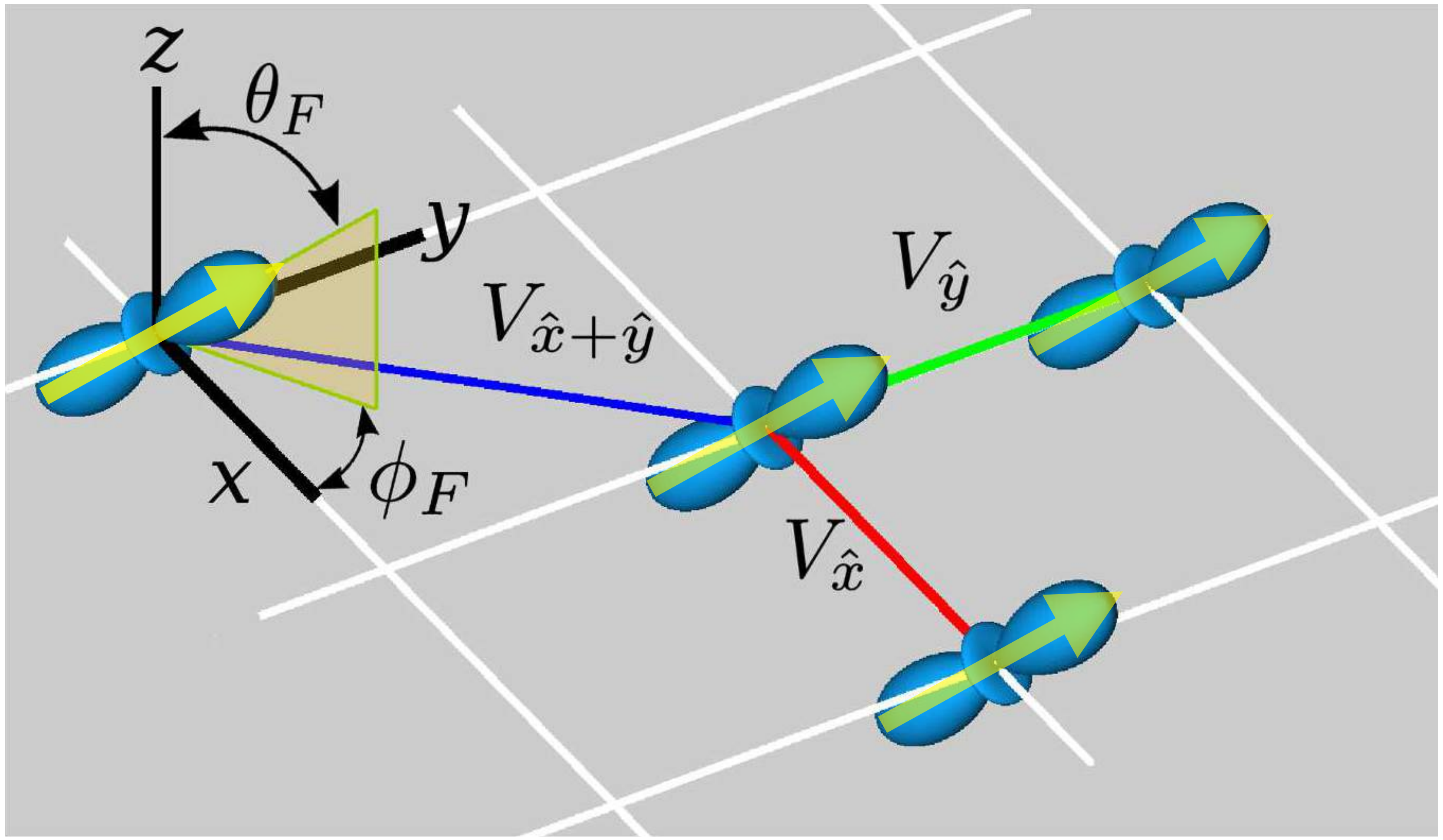
The common theme of the 3 problems is competing order.

Wish: treat (all) orders on the same footing, without a priori bias.

1. Dipolar fermions on lattice

Collaborators:
Satyan Bhongale (GMU)
Ludwig Mathey (Hamburg)
Shan-Wen Tsai (UC Riverside)
Charles Clark (NIST/JQI)

Dipolar fermions on square lattice: model Hamiltonian



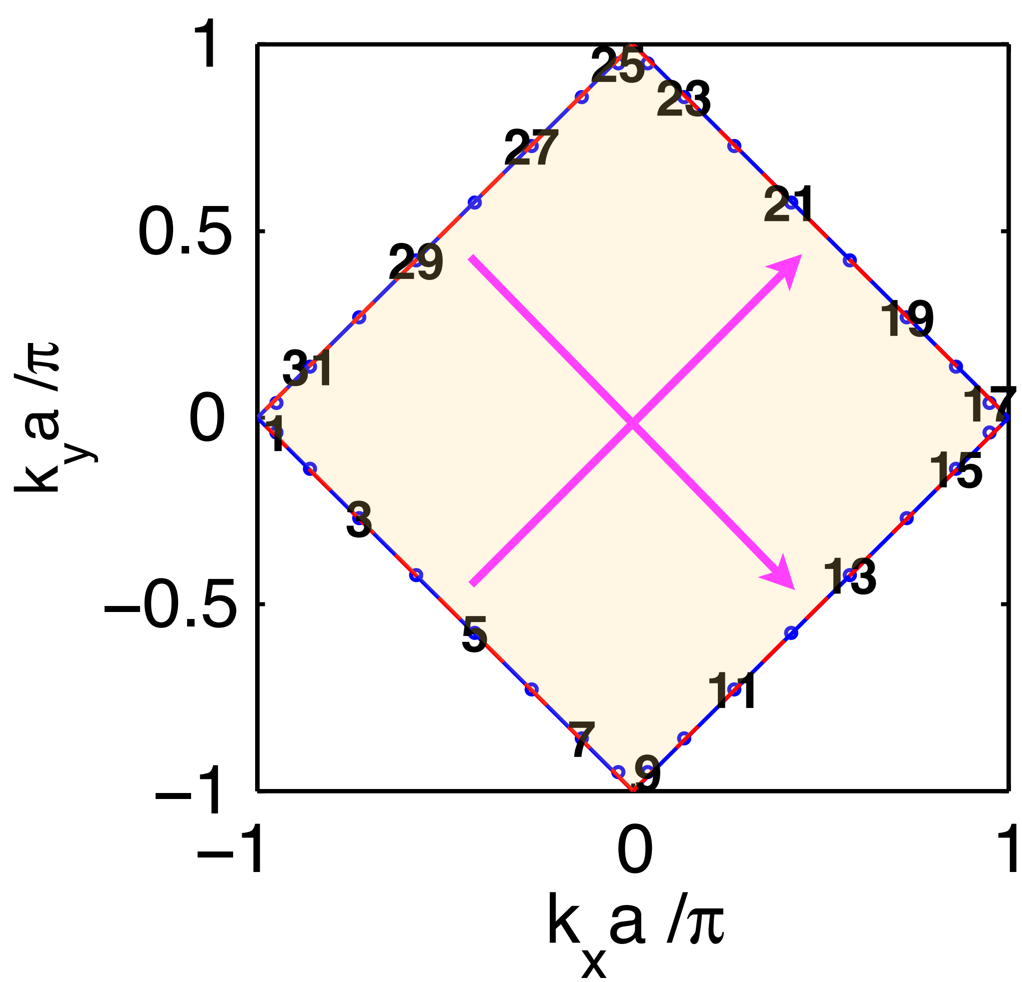
$$H = -t \sum_{\langle ij \rangle} a_i^\dagger a_j + \frac{1}{2} \sum_{i \neq j} V_{dd}(\mathbf{r}_{ij}) n_i n_j,$$

★ Half filling: on average, one fermion every two sites.

★ Zero temperature; Neglect collapse instability.

The Fermi surface is just a square (half filling)

In the absence of dipole-dipole interaction:



$$\mathbf{Q} = (\pi, \pm\pi)$$

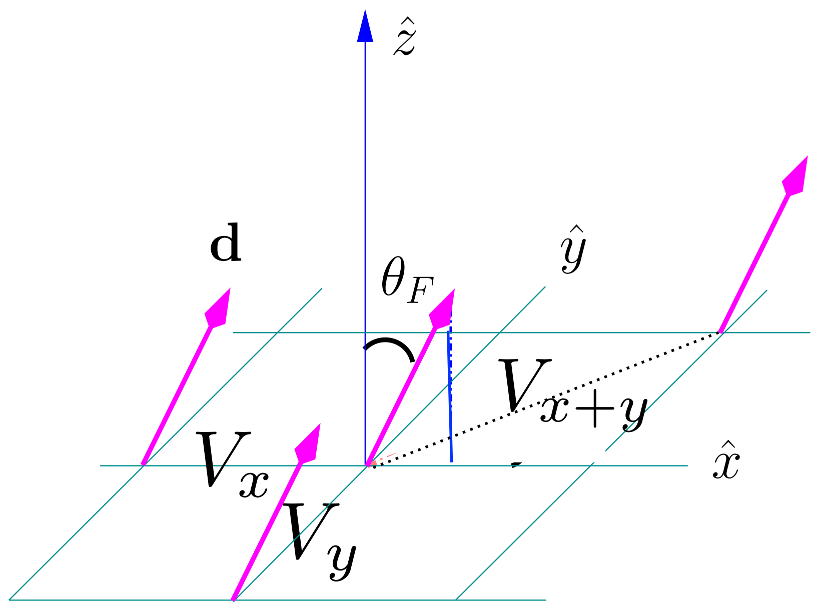
$$(a = 1)$$

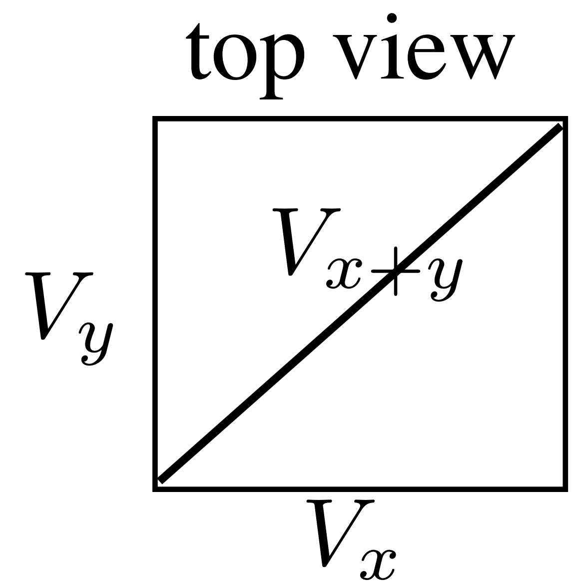
★ Perfect Nesting: **Q** couple **k** points on the opposite sides of the FS.

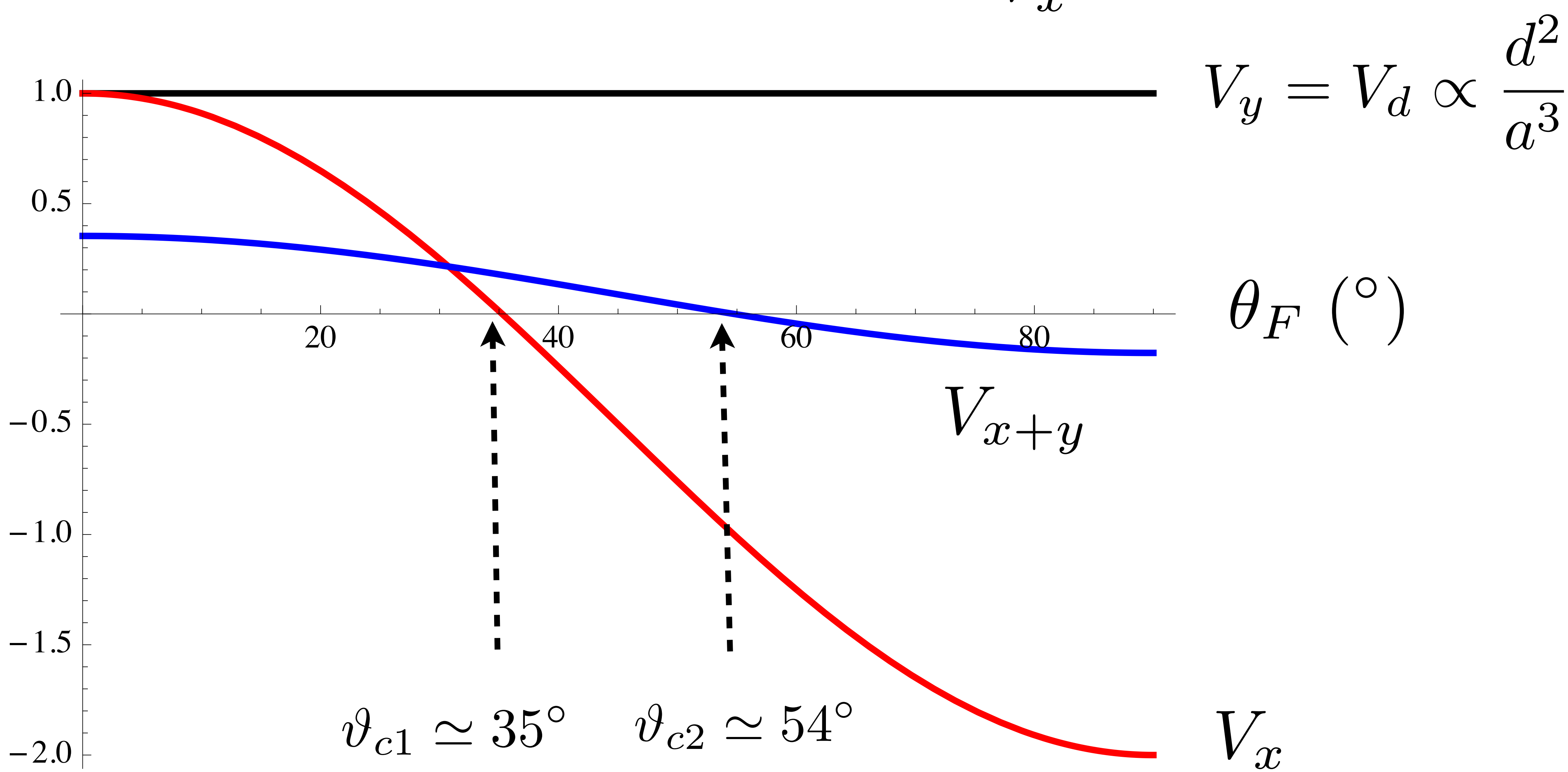
★ We will discretize the Fermi surface into N patches.

★ The Fermi surface may become unstable when V_{dd} is turned on.

Interactions for dipoles tilting in the x direction







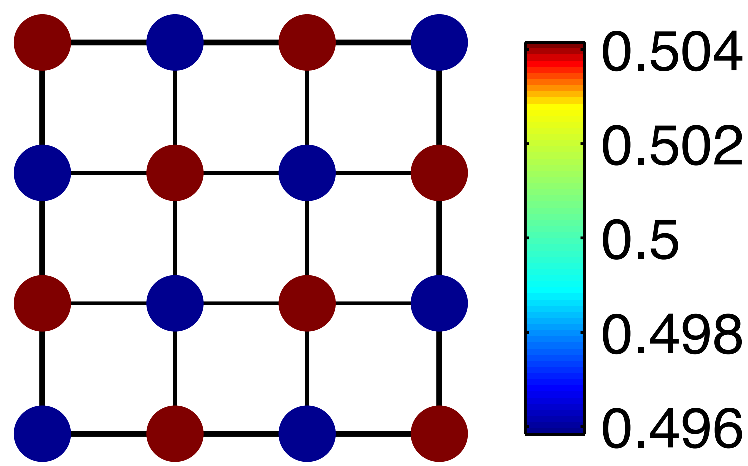
Two limits easy to understand

1. Small tilting angle ($\theta_F < \vartheta_{c1}$): all interactions are repulsive.

Density wave (CDW):
Periodic modulation of on-site density.

$$\langle a_i^\dagger a_i \rangle$$

In **k** space, this is an instability of FS in the particle-hole channel with **Q**.

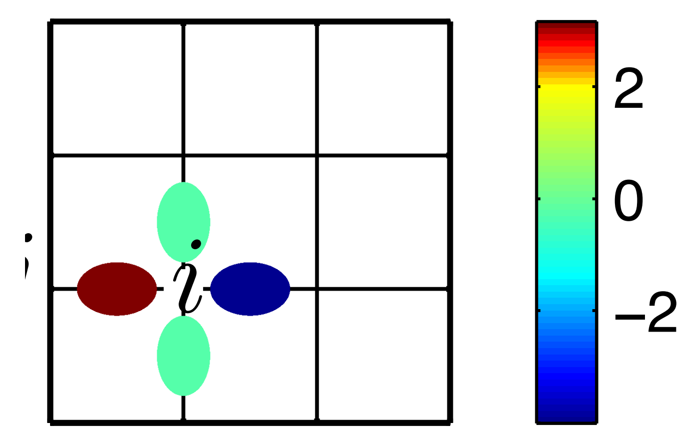


2. Large tilting angle ($\theta_F > \vartheta_{c2}$): V_x and V_{x+y} attractive, but V_y repulsive.

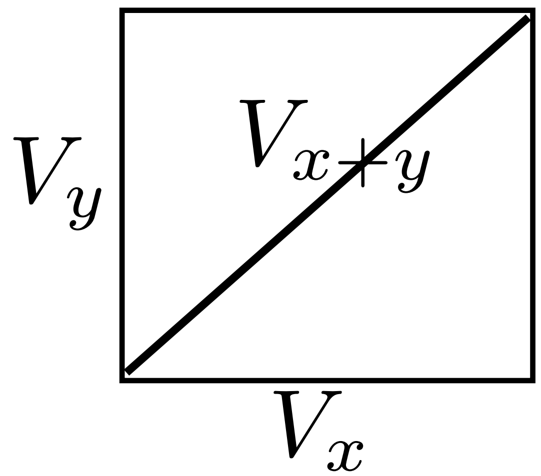
Anisotropic p-wave pairing (BCS):
The pairing order parameter

$$\langle a_i a_{i+\hat{x}} \rangle = -\langle a_i a_{i-\hat{x}} \rangle \quad \langle a_i a_{i\pm y} \rangle = 0$$

In **k** space, this is an instability of FS in the particle-particle channel.

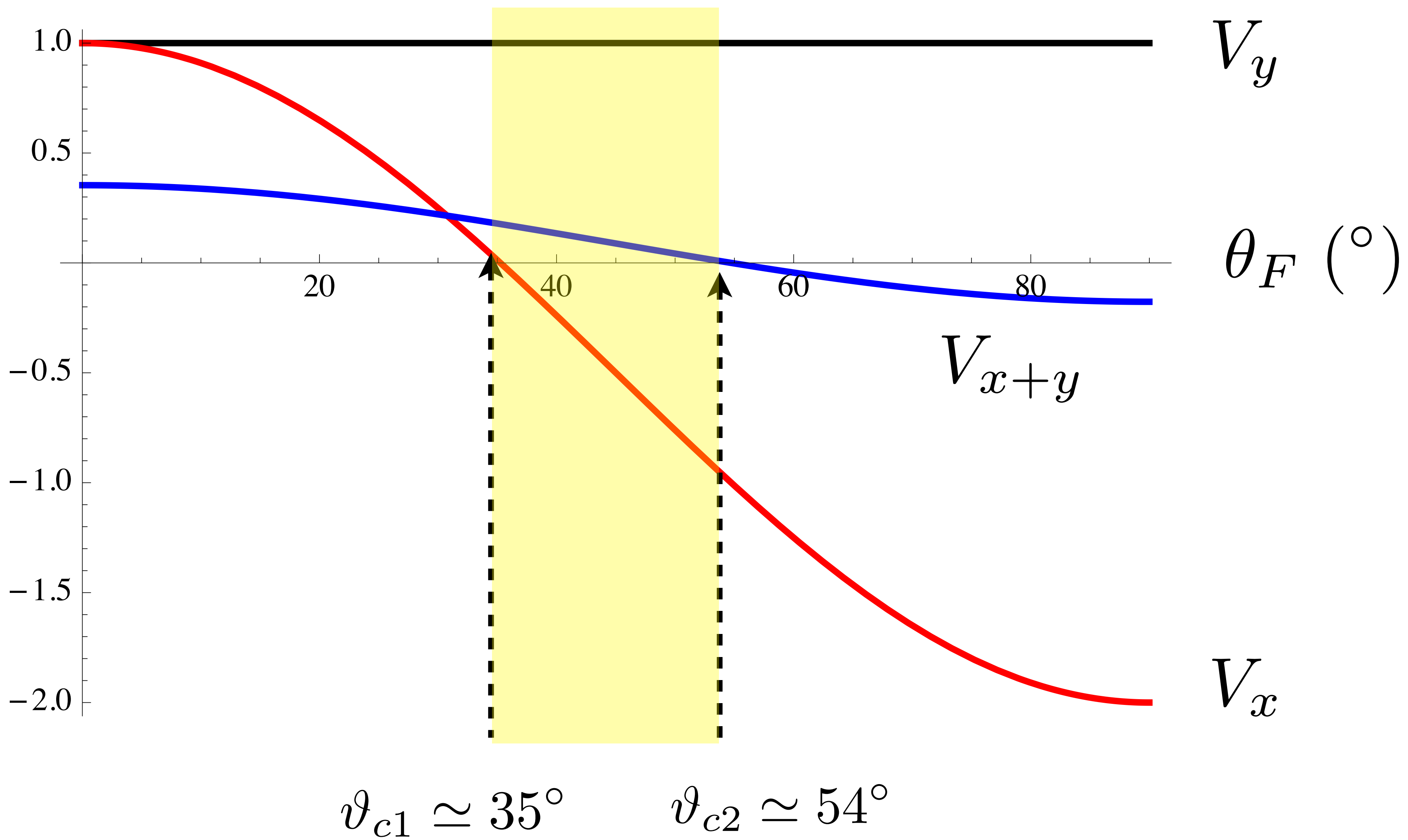


How about the intermediate tilting angle



V_x and V_y opposite in sign and comparable in magnitude. What do the fermions do?

Settle to BCS or CDW? Neither? Both?



Competing orders in interacting dipolar fermions

Three possible scenarios:

★ Direct (1st order) transition from CDW to p-wave BCS superfluid.

★ Coexistence: density modulation + pairing = supersolid.

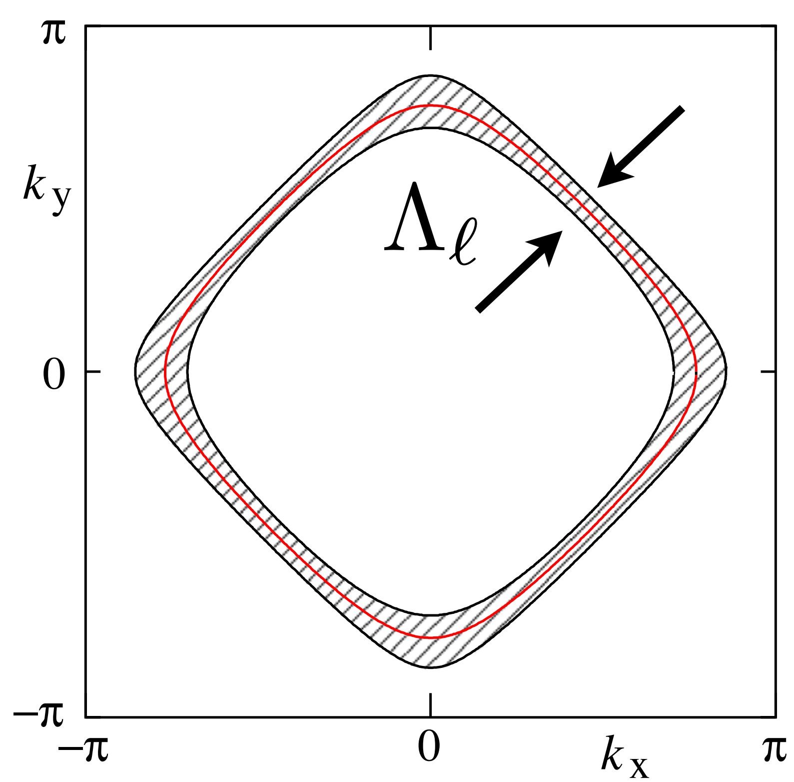
★ Or, some other completely different animal.

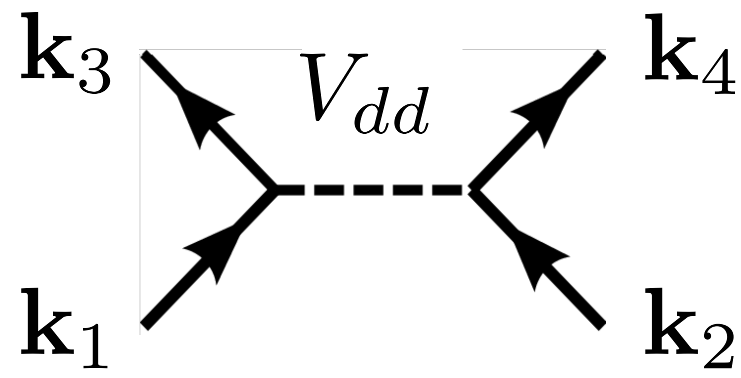
The problem of competing order is at the heart of the many-body physics of dipolar fermions.

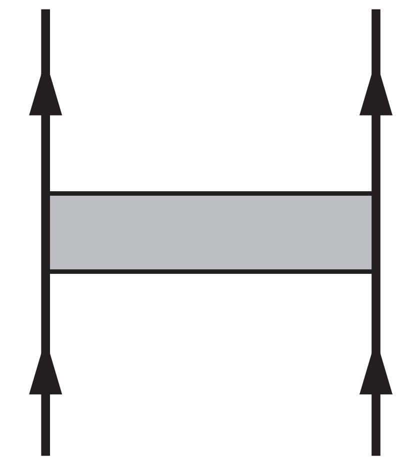
Simple mean field theories or perturbation theories, such as single-channel Renormalization Group or Random Phase Approximation, are insufficient/unreliable to treat competing orders in the regime of intermediate tilting angle.

We need a theory that can treat all ordering instabilities on equal footing, without any a priori assumptions about dominant orders.

Functional Renormalization Group (FRG)







$U_\ell(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$

$$\mathbf{k}_4 = \mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 \qquad \Lambda_\ell = \Lambda_0 e^{-\ell}$$

★ Separate the low-energy modes and high energy modes with scale Λ .

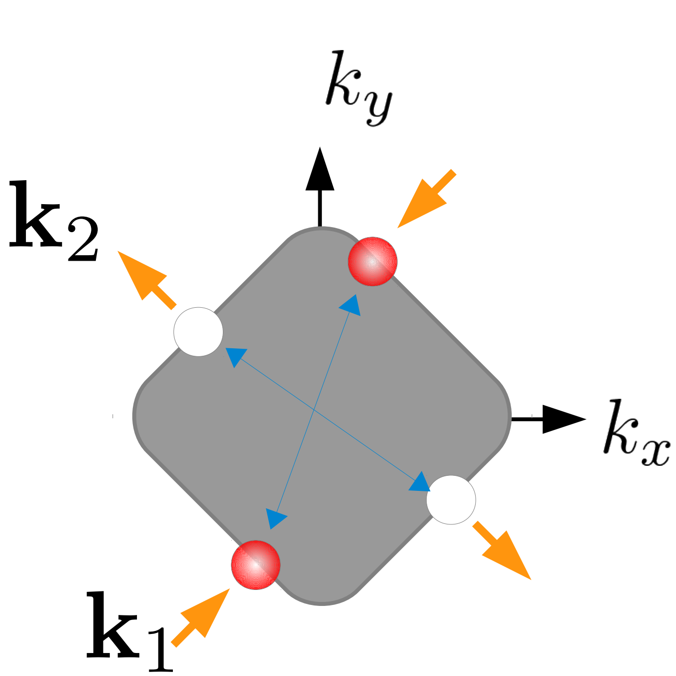
★ At each scale Λ, there is an effective theory description, including the effective interaction (vertex function) *U* between the low energy modes.

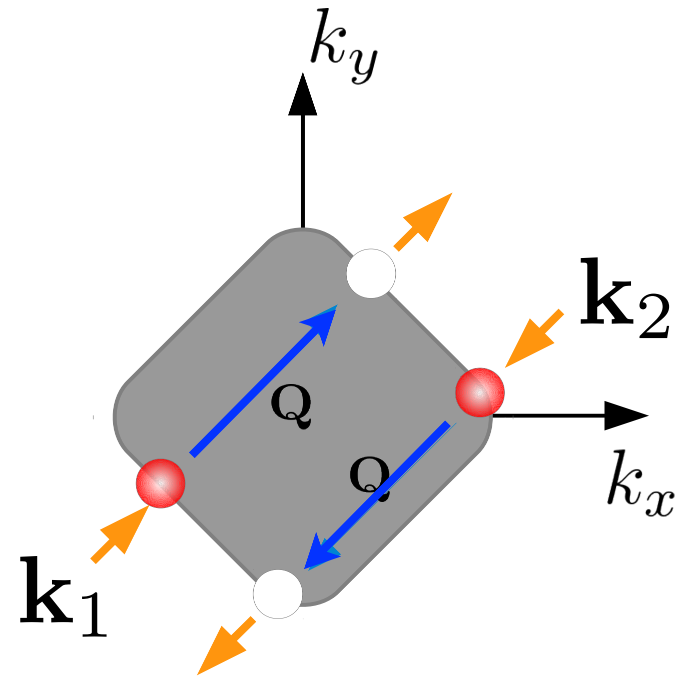
★ As Λ is reduced, the evolution of *U* obeys the exact “flow equation.”

★ For weak coupling, the infinite hierarchy of flow eqns can be truncated and solved numerically by discretizing **k**.

See e.g. Metzner et al, Rev. Mod. Phys. 84, 299–352 (2012); And reference therein.

FRG applied to interacting dipolar fermions





$U_\ell(\mathbf{k}_1, -\mathbf{k}_1, \mathbf{k}_2)$, $U_\ell(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_1 + \mathbf{Q})$,

BCS Channel CDW Channel

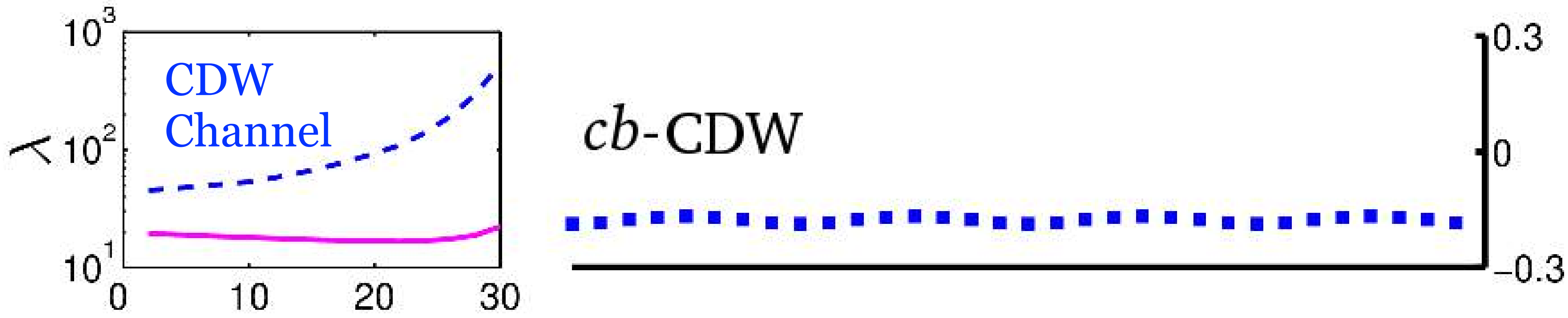
FRG keeps track of all effective interactions as the high energy modes are traced out, including the p-p and p-h channel, as well as their subtle interplay. Especially, we are interested in the BCS and the CDW channel.

The most dominant instability can be inferred from the most diverging eigenvalue of U, which is a matrix of $\mathbf{k}_1$ and $\mathbf{k}_2$. The corresponding eigenvector indicates the symmetry of the incipient order.

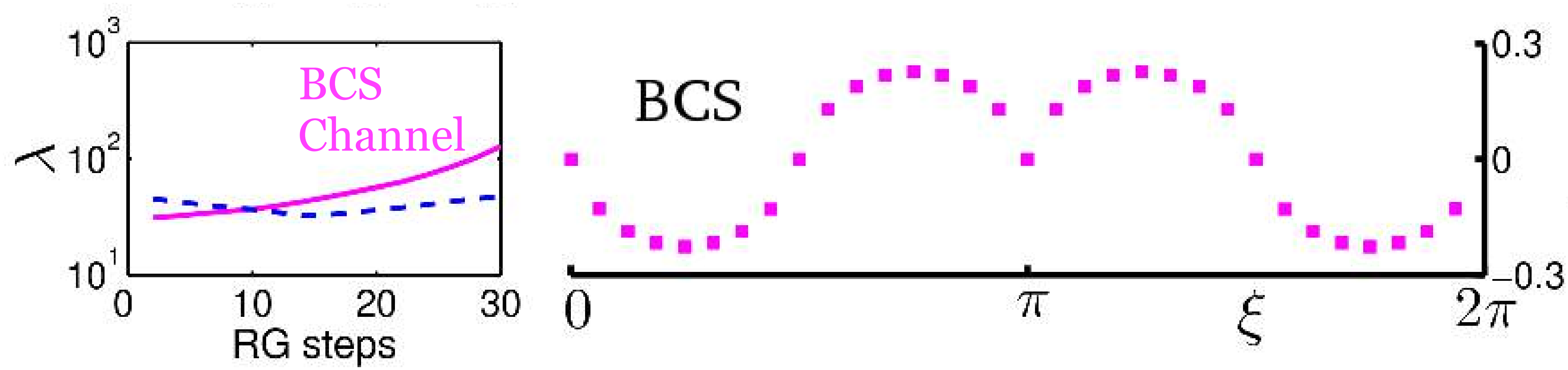
Instability analysis within FRG

Eigenvalue

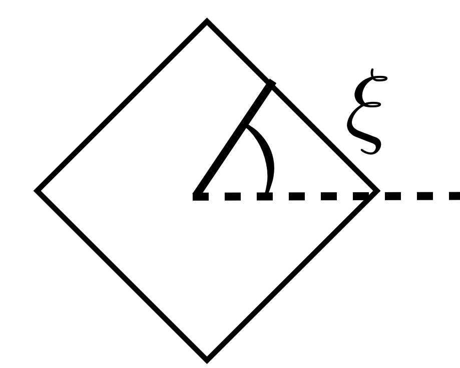
Eigenvector



$\theta_F = 30^\circ$



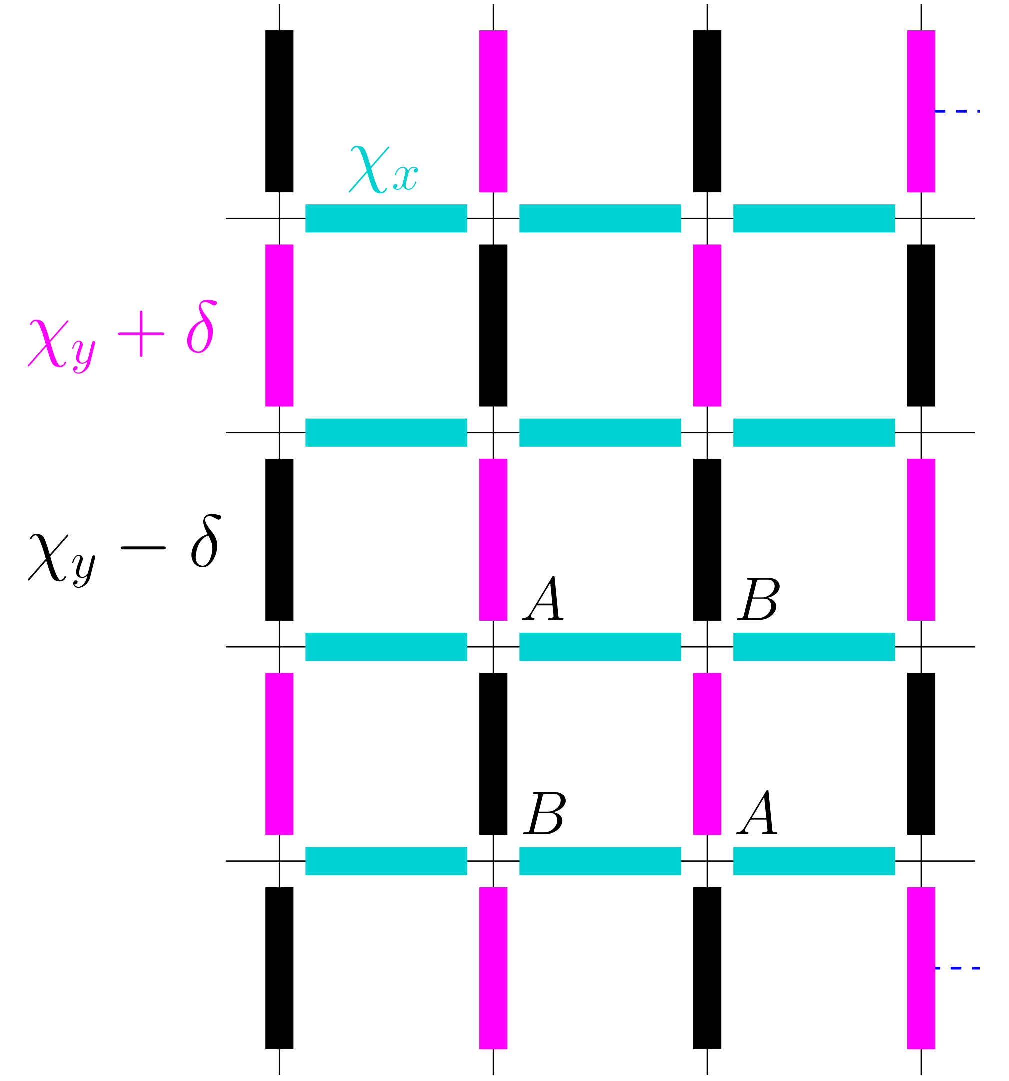
$\theta_F = 70^\circ$



Bond order solid (BOS)

Such p-wave instability in the CDW channel corresponds to a spatial modulation of "bonds", more precisely, the average of hopping

$$\langle a_i^\dagger a_{i+y} \rangle$$



How can such bond order save energy?

A mean field perspective:

$$n_i n_j = -\boxed{a_i^\dagger a_j a_j^\dagger a_i} + n_i$$

$$\hookrightarrow a_i^\dagger a_j \rho_{ji} + \rho_{ij} a_j^\dagger a_i - |\rho_{ij}|^2.$$

with $\rho_{ij} = \langle a_i^\dagger a_j \rangle$

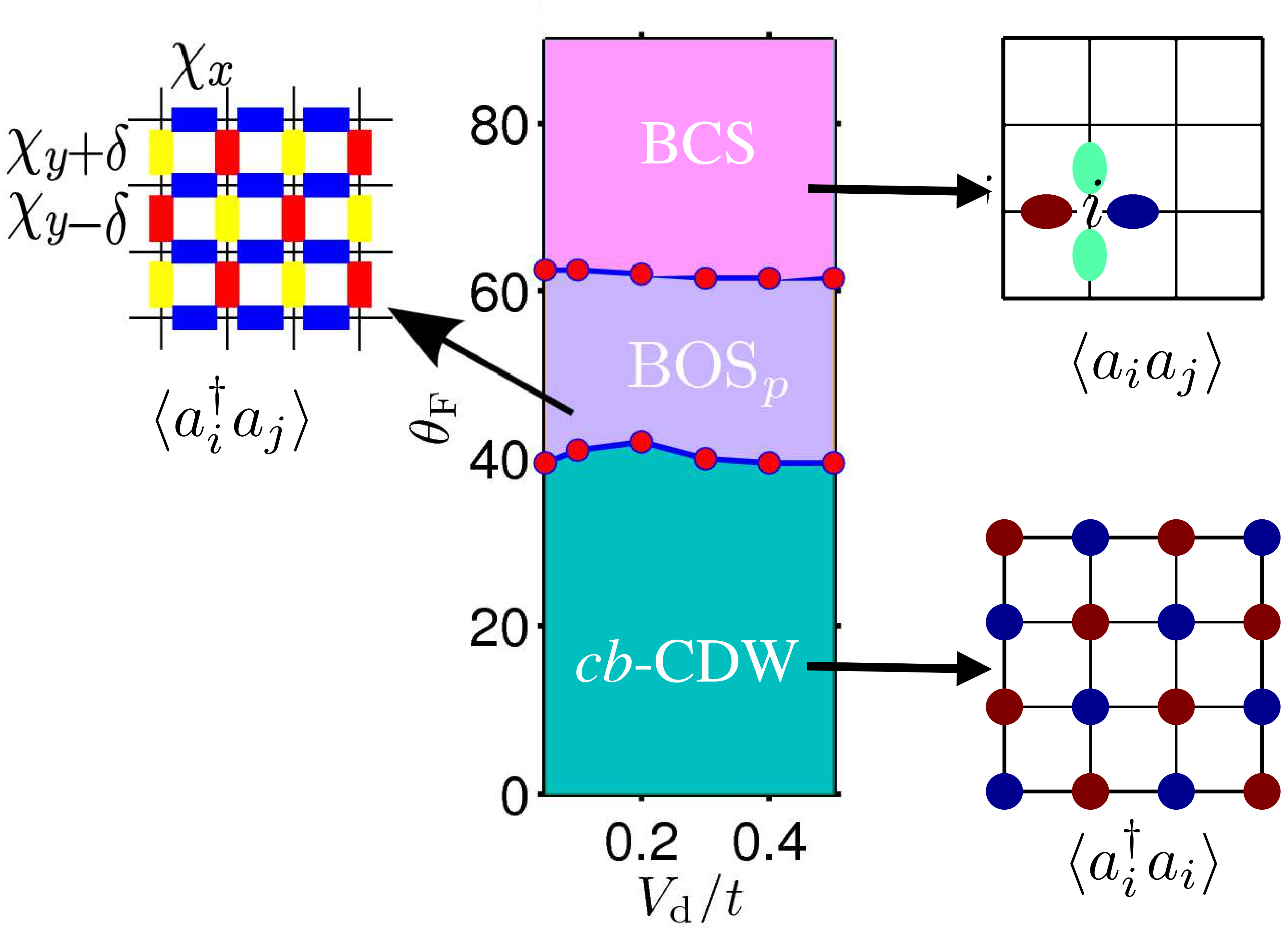
$$\rho_{i,i\pm\hat{x}} = \chi_x, \; \rho_{i,i\pm\hat{y}} = \chi_y \pm \delta$$

★ Opening up a gap at the Fermi surface.

★ Ground state energy: $E_{\rm GS} = -2(\chi_x + \chi_y)(t + V_x + V_y) \boxed{- 2V_y\delta^2}$

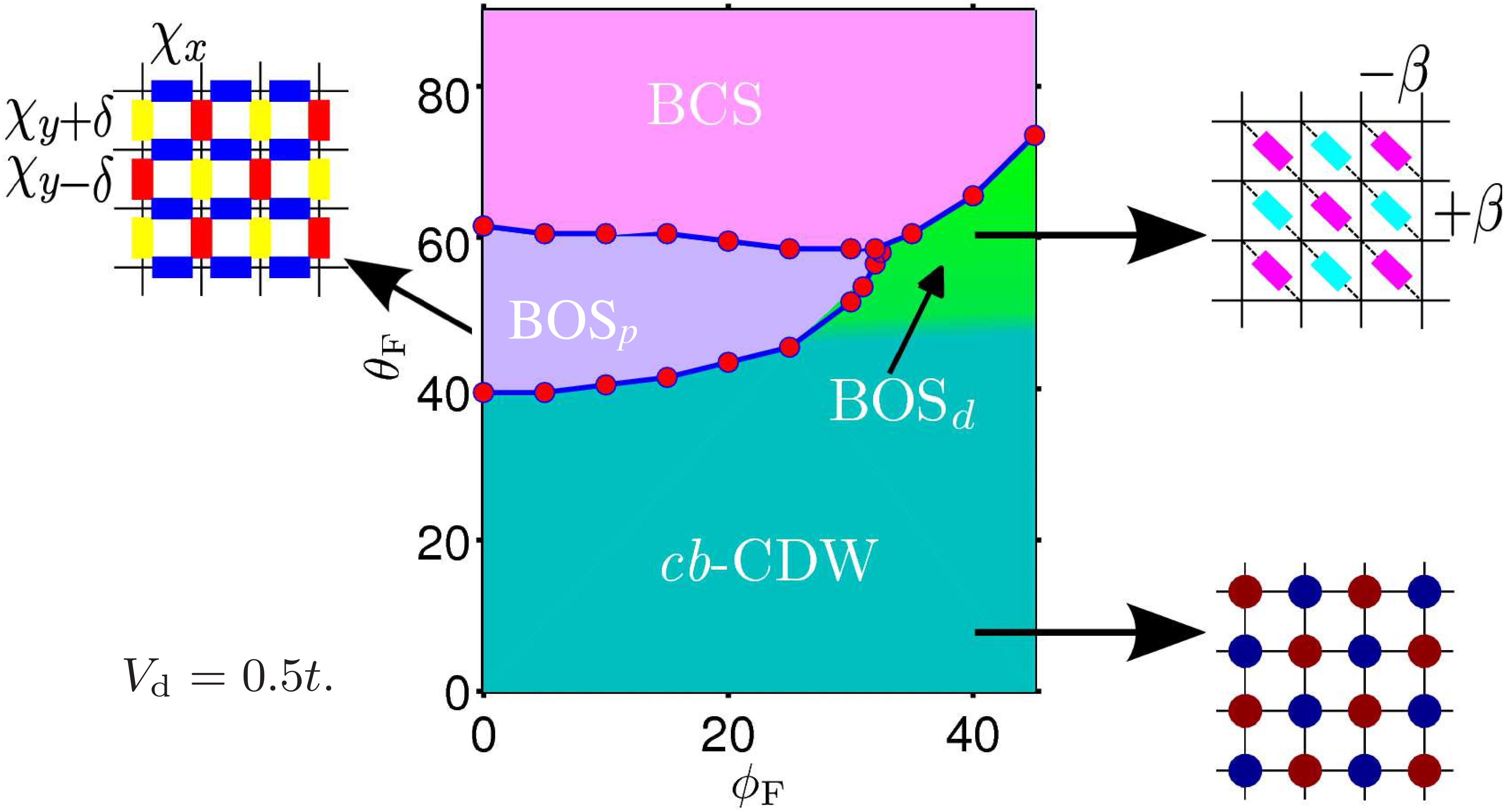
finite bond modulation δ is energetically favored

Phase diagram (T=0, half-filling, ϕ_F=0)



S. G. Bhongale, L. Mathey, S.-W. Tsai, C. W. Clark, EZ, PRL 108, 145301 (2012)

Phase diagram for general dipole tilting



$V_d = 0.5t.$

S. G. Bhongale, L. Mathey, S.-W. Tsai, C. W. Clark, EZ, PRL 108, 145301 (2012)

Classification of density waves

Superconductors (condensate of Cooper pairs):

$$\langle f_\alpha(\mathbf{k}) f_\beta(-\mathbf{k})\rangle = \begin{cases} \Delta(\mathbf{k}) \cdot (i\sigma_y)_{\alpha\beta} & \text{spin singlet, } l=0,2,.. \\ \boldsymbol{\Delta}(\mathbf{k}) \cdot (\boldsymbol{\sigma} i\sigma_y)_{\alpha\beta} & \text{spin triplet, } l=1,3,.. \end{cases}$$

s-wave superconductor, l=0
p-wave superconductors, l=1
d-wave superconductors, l=2
......

Density waves (condensate of particle-hole pairs):

$$\langle f_\alpha^\dagger(\mathbf{k}+\mathbf{Q}) f_\beta(+\mathbf{k})\rangle = \Phi(\mathbf{k})\delta_{\alpha\beta}$$

- s-wave CDW (checkerboard)
- p-wave CDW
- d-wave CDW (DDW) ...

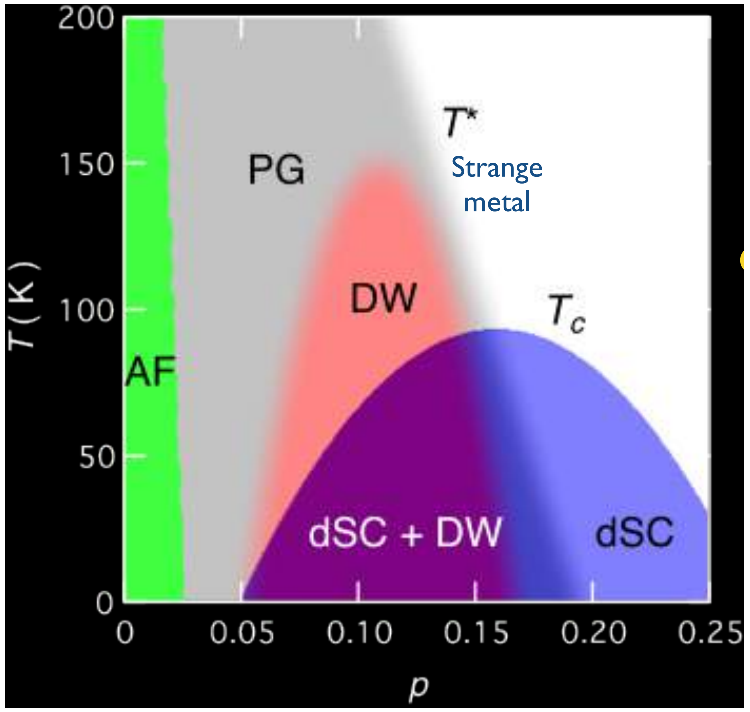
$$\langle f_\alpha^\dagger(\mathbf{k}+\mathbf{Q}) f_\beta(+\mathbf{k})\rangle = \boldsymbol{\Phi}(\mathbf{k}) \cdot \boldsymbol{\sigma}_{\alpha\beta}$$

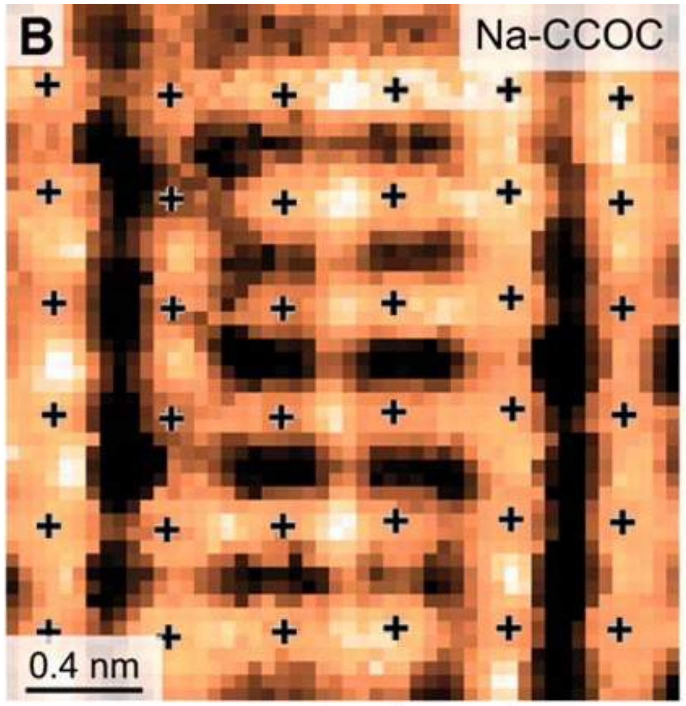
- s-wave SDW (~Neel order)
- p-wave SDW...

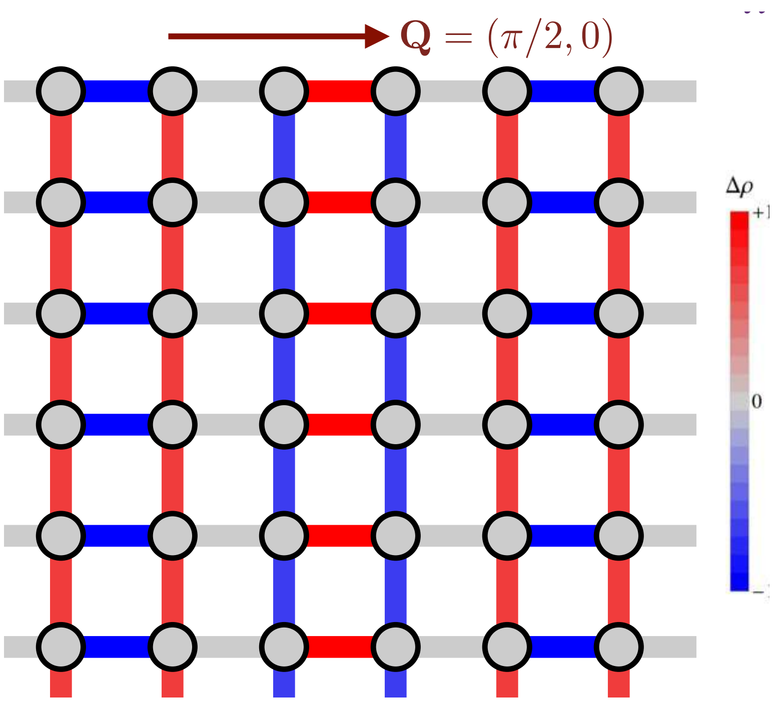
Density-wave states of nonzero angular momentum, Chetan Nayak, Phys. Rev. B 62, 4880 (2000)

They show up in dipolar Fermi gas!

Observation of d-wave density waves?







Observation of d-form factor density waves (in BSCCO and Na-CCOC)

Experiments: Kohsaka et al, Science 315, 1380 (2007). Fujita et al, PNAS 111, E3026 (2014).
Theory: Metlitski & Sachdev, PRB 82, 075128 (2010); PRL 111, 027202 (2013); etc.

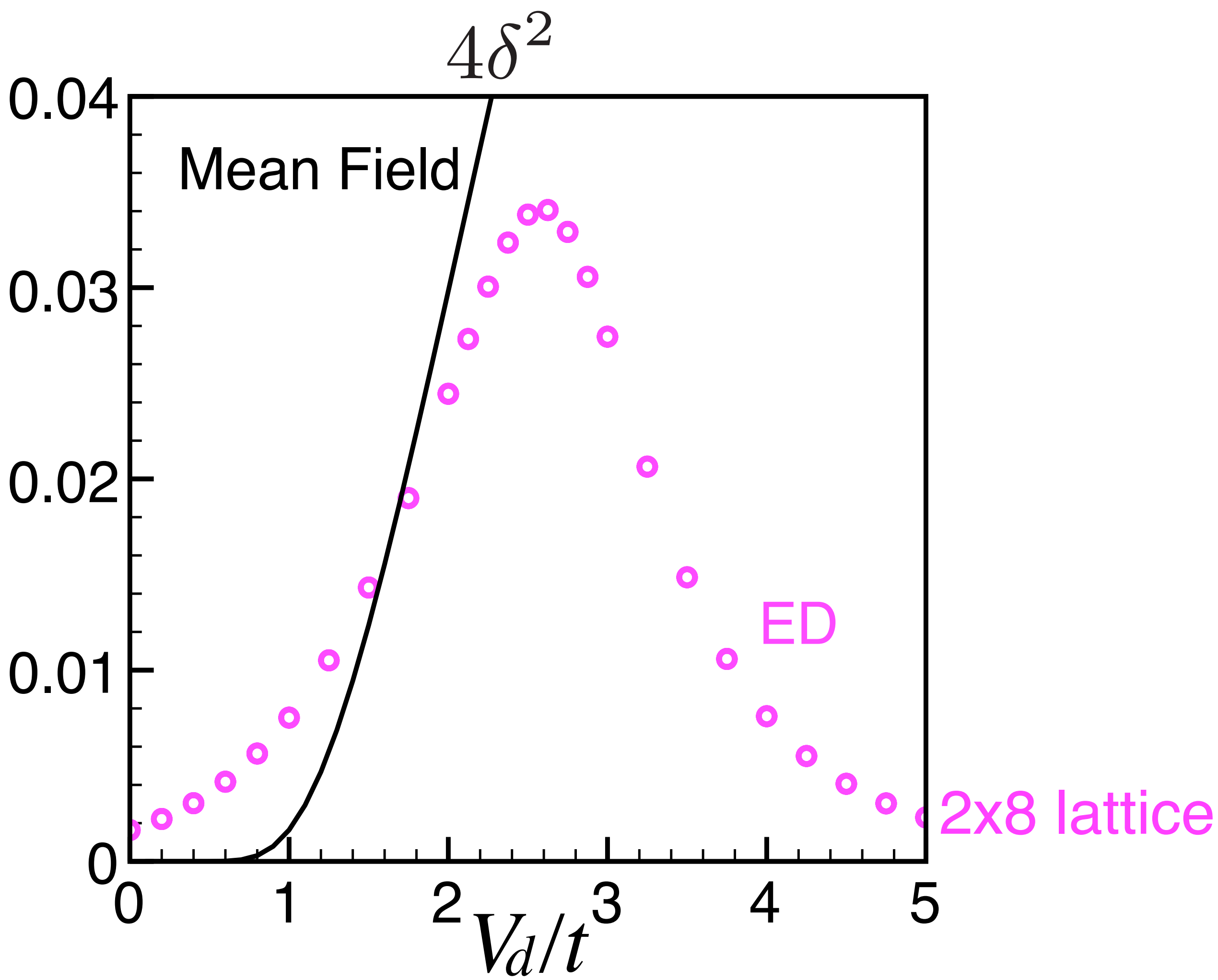
$$P_{ij} = \left\langle c_{i\alpha}^\dagger c_{j\alpha} \right\rangle \text{ for } i = j, \text{ and } i, j \text{ nearest neighbors.}$$

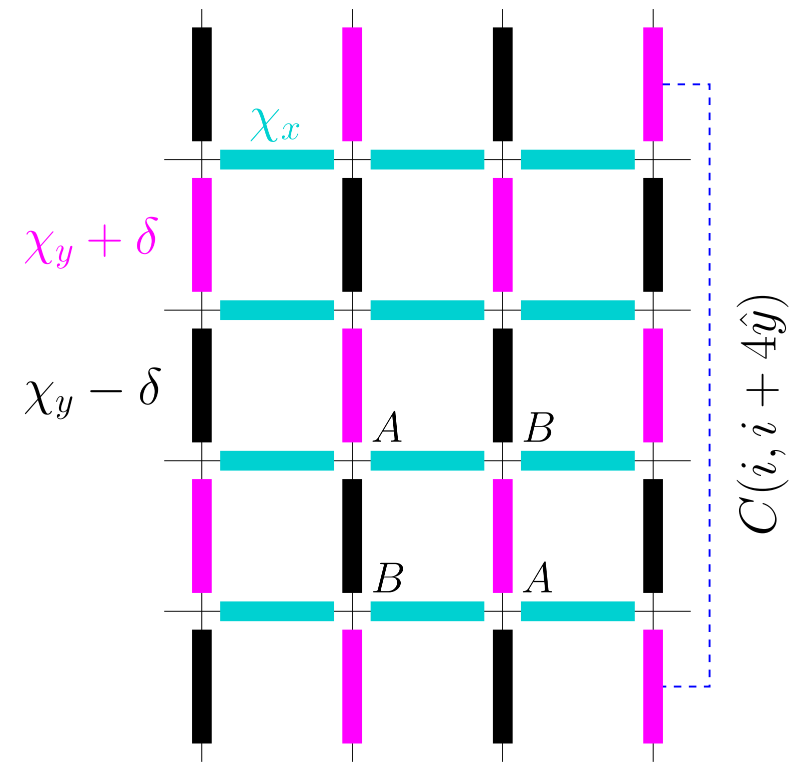
$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} \left[\cos(k_x) - \cos(k_y) \right] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

Beyond weak coupling

Bond order is most robust for intermediate interaction, $V_d \sim 2.5t$, where the mean field gap is $0.23t$, or $0.05\ E_F$.



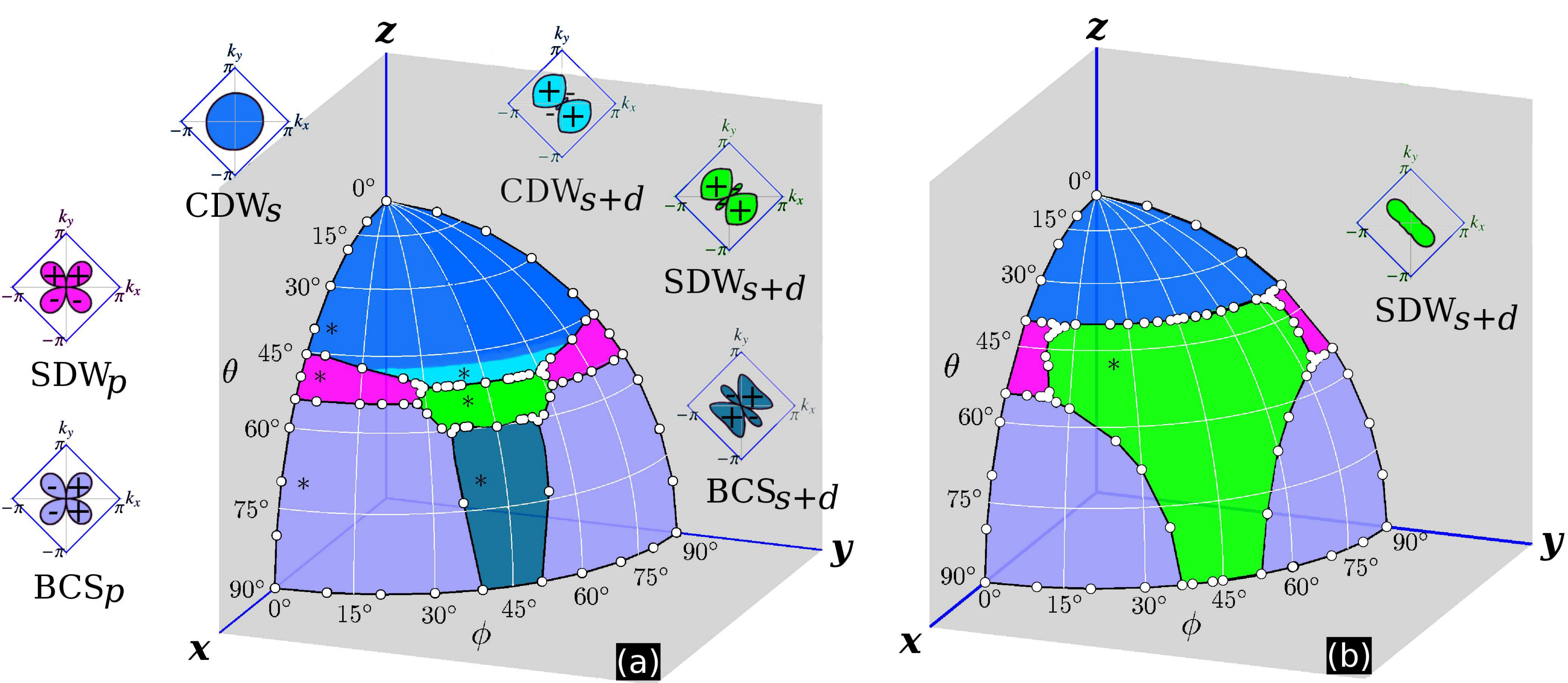


Exact diagonalization (ED) yields the hopping correlation function

$$C(i,j) = \langle K_{i,i+y} K_{j,j+y} \rangle - \langle K_{i,i+y} \rangle \langle K_{j,j+y} \rangle \qquad K_{i,j} \equiv (a_i^\dagger a_j + h.c.)$$

It approaches $4\delta^2$ in the limit of large $|i\text{-}j|$.

Spin half dipolar Fermi gas

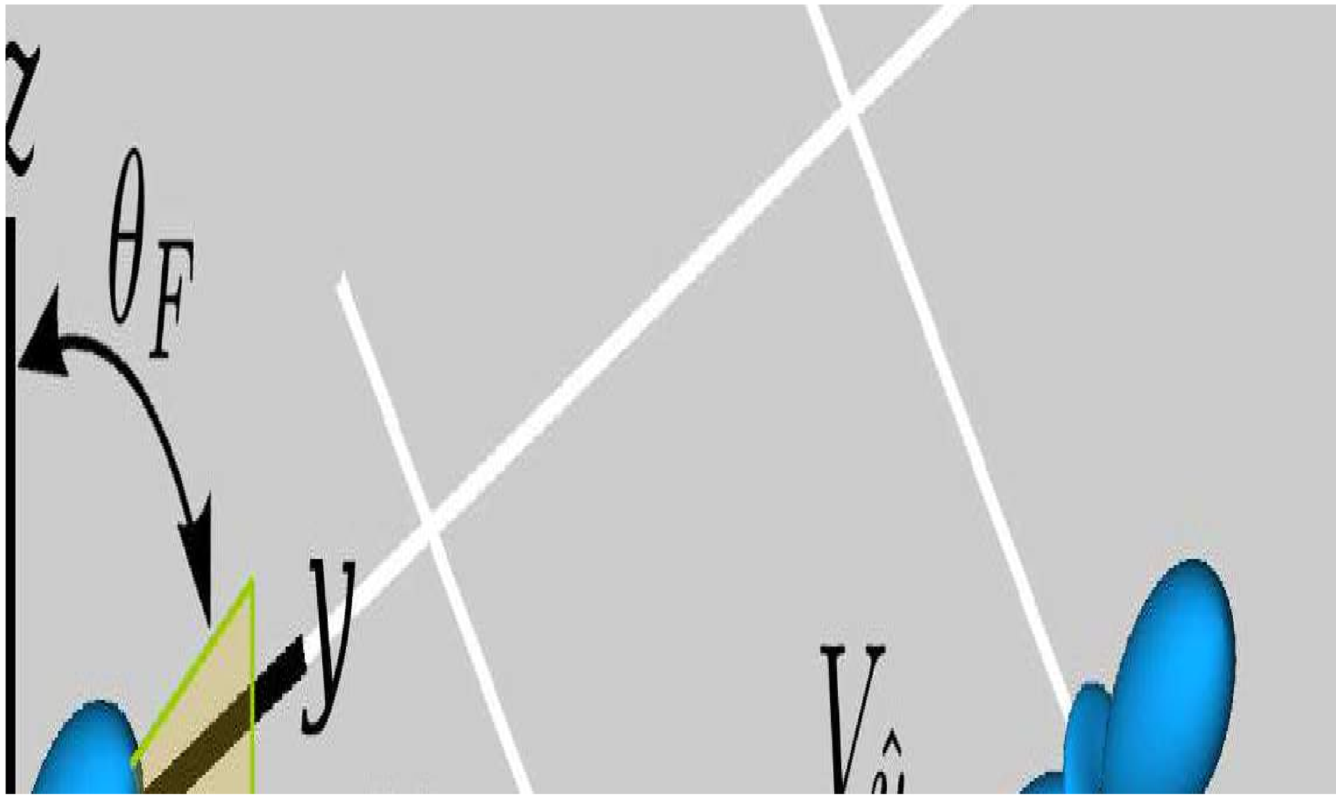


$V_d = 0.5, U = 0.1$ $V_d = 0.5, U = 0.5$

The p-wave spin density wave phase is sandwiched between the CDW and BCS superfluid phases. Its phase boundary depends on U.

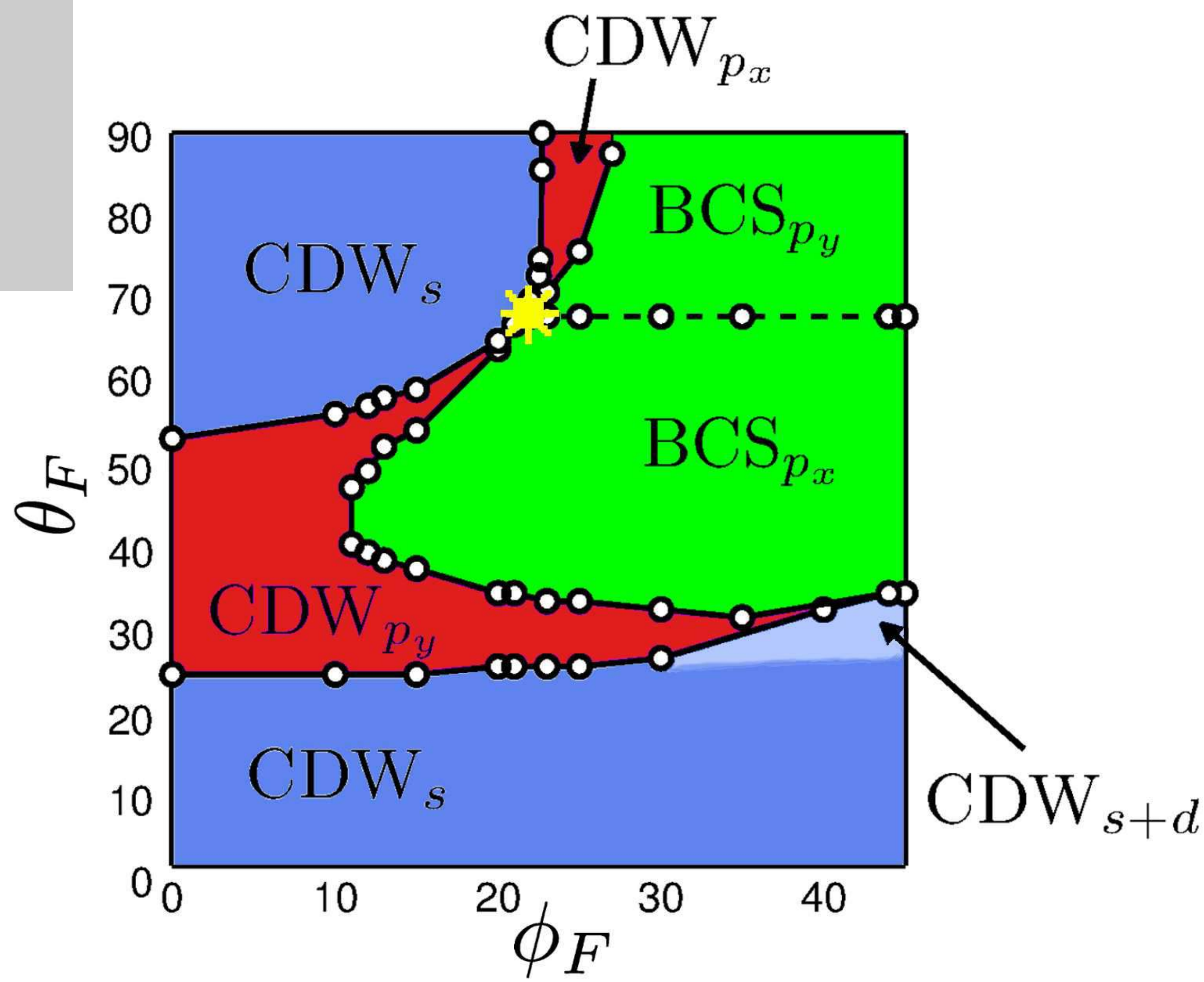
S. G. Bhongale, L. Mathey, S.-W. Tsai, C. W. Clark, EZ, PRA 87, 043604 (2013).

Quadrupolar Fermi gas



$$V^{qq} = V(3 - 30\cos^2\theta + 35\cos^4\theta)/r^5$$

Bond order

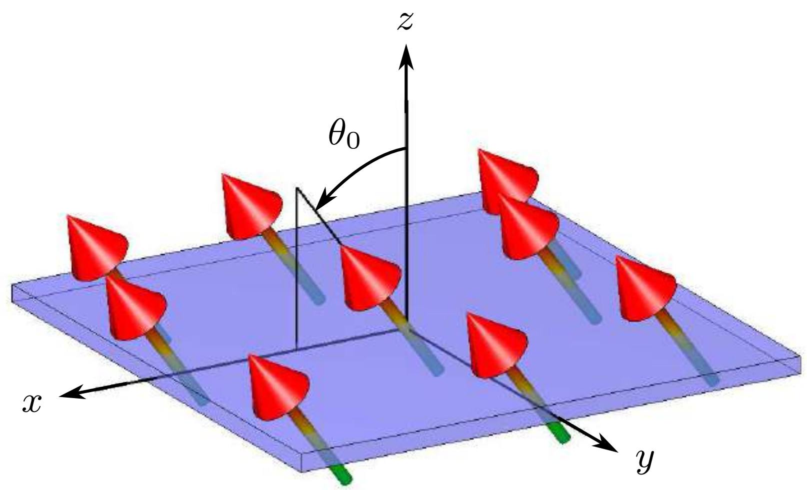


S. G. Bhongale, L. Mathey, EZ, S. F. Yelin, M. Lemeshko, PRL 110, 155301 (2013)

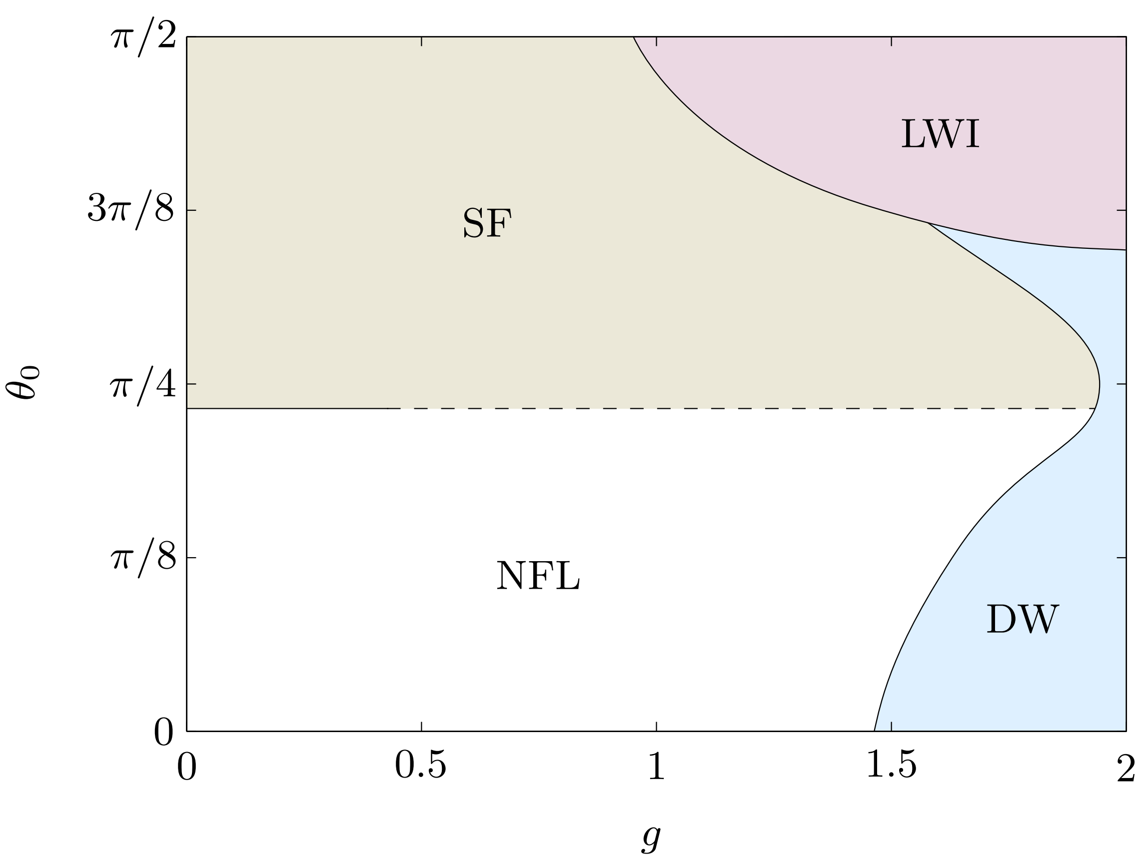
2. Functional renormalization group analysis of continuum dipolar gas in 2D

Collaborator:
Ahmet Keles (Pitt and GMU)

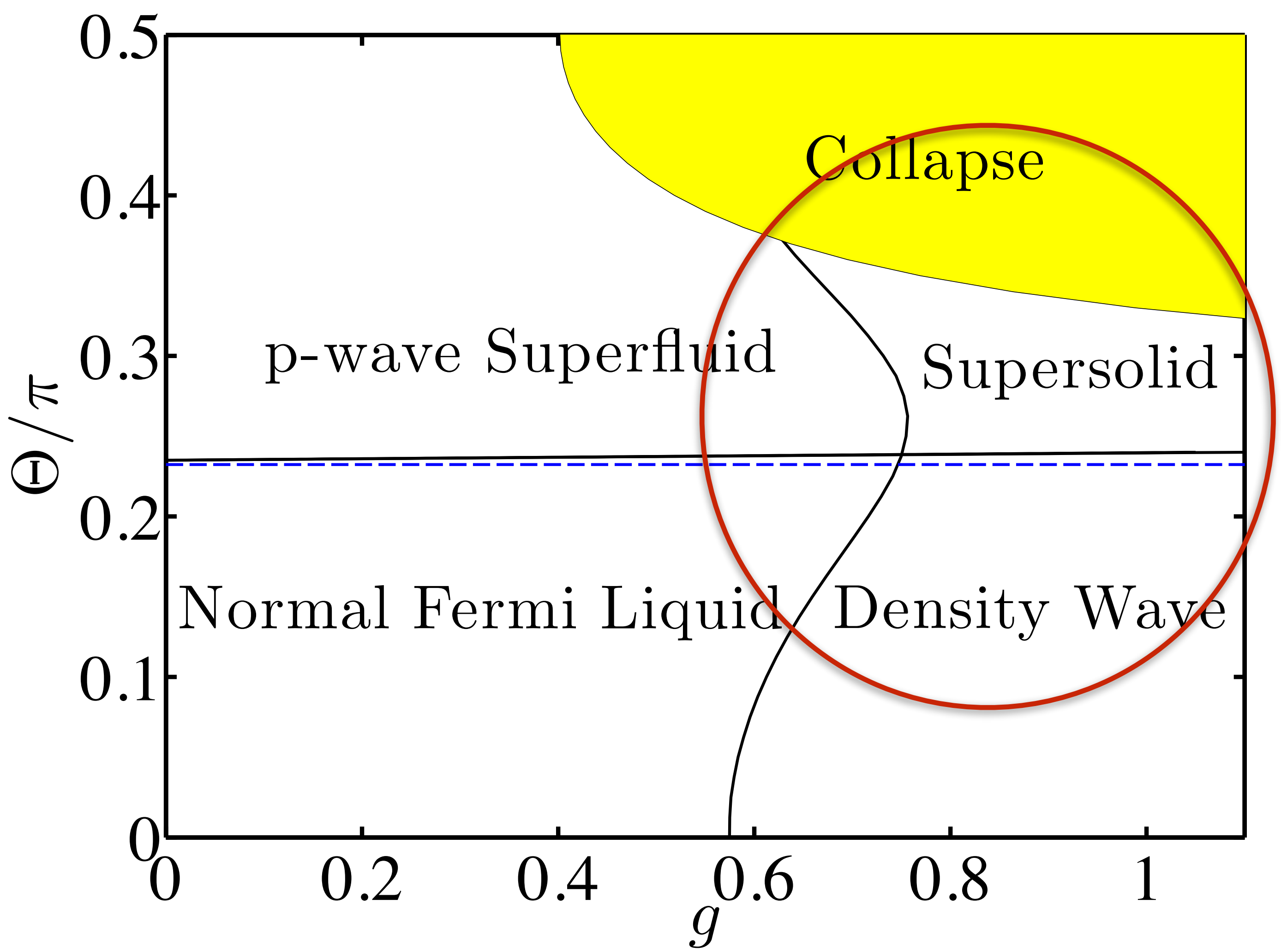
2D dipolar Fermi gas, mean field and RPA predictions



Tightly confined in z direction



Sieberer and Baranov, PRA 84, 063633 (2011)
See also: Babadi & Demler, PRB 2011;
Zhao et al (Pu's group) PRA, 2010;
Bruun and Taylor PRL 2008; and many others.



Wu, Block, Bruun, arXiv:1412.2783 (2014)

Technical slide 1: Flow of effective action

Add infrared regulator R_k to the action S, k being the sliding momentum scale, e.g.,

$$R_{\mathrm{k}}(\mathbf{p}) = \left[\frac{\mathrm{k}^2}{2\mathrm{m}}\mathrm{sgn}(\xi(\boldsymbol{p})) - \xi(\boldsymbol{p})\right]\theta\left(\frac{\mathrm{k}^2}{2\mathrm{m}} - |\xi(\boldsymbol{p})|\right)$$

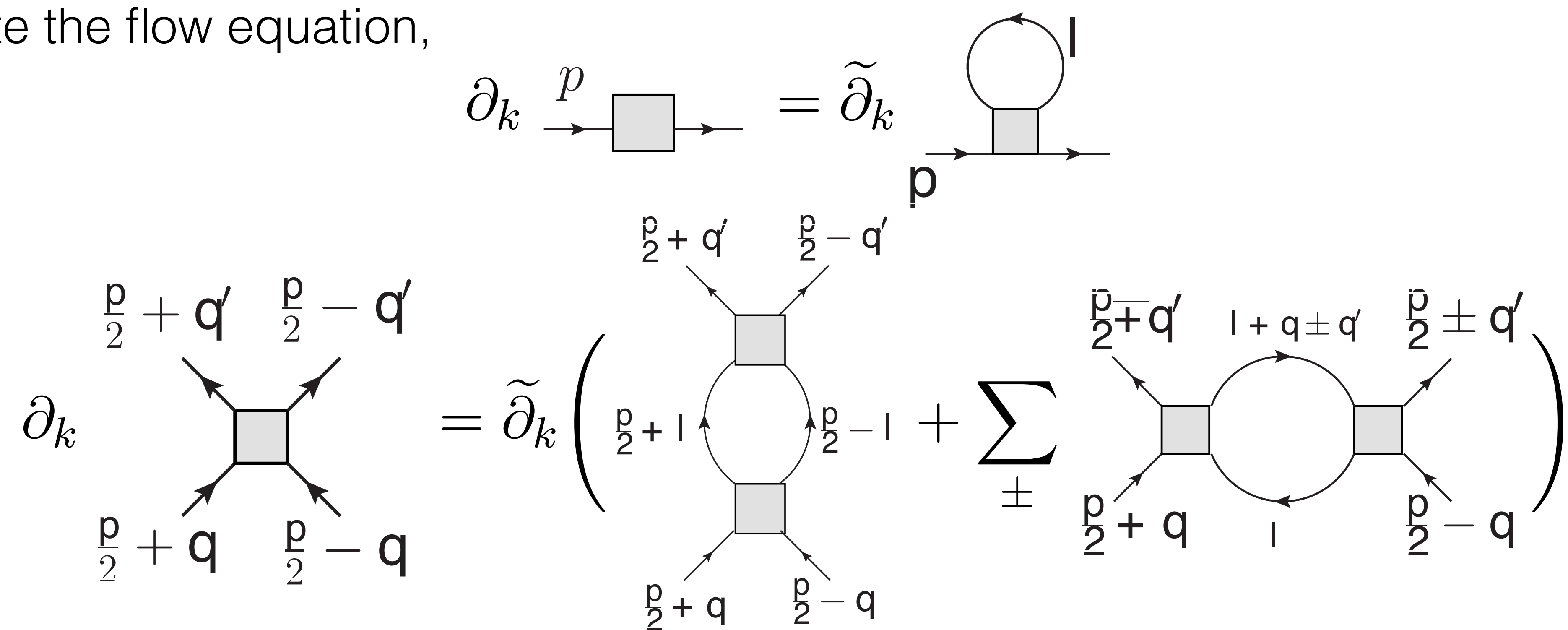
Wetterich's flow equation:

$$\partial_{\mathrm{k}}\Gamma_{\mathrm{k}} = -\frac{1}{2}\tilde{\partial}_{\mathrm{k}}\,\mathrm{Tr}\ln\left[\Gamma^{(2)} + R_{\mathrm{k}}\right]$$

C. Wetterich, Phys. Lett. B, 301(1), 90–94 (1993).
T. R. Morris, Inter. J. of Mod. Phys. A, 9(14), 2411–2450 (1994).

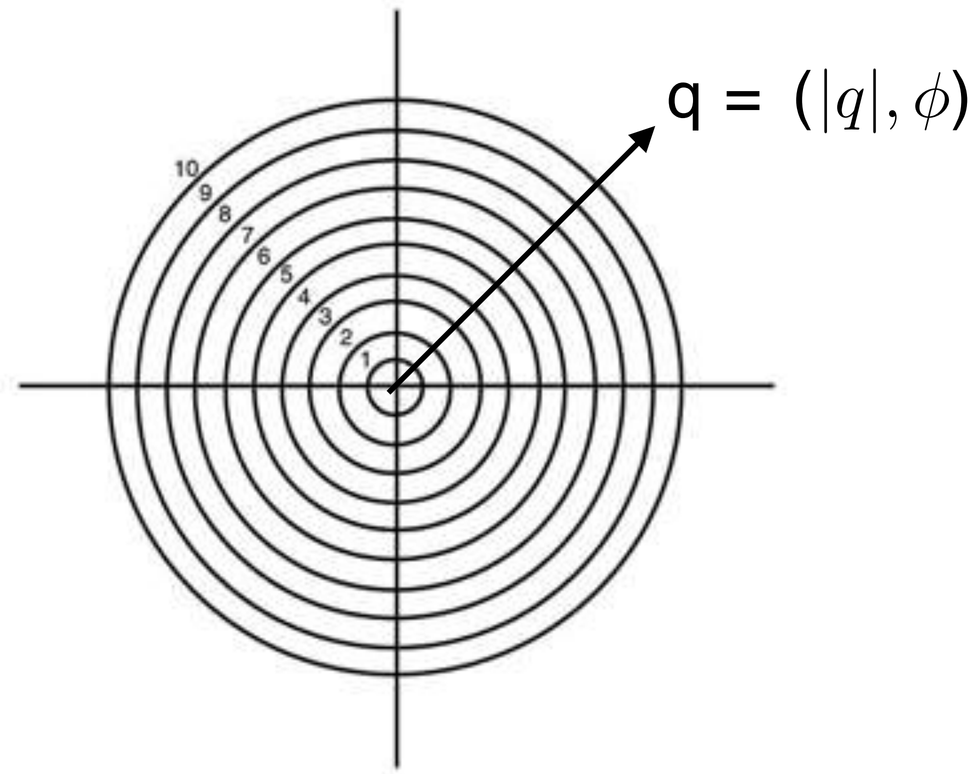
Expand Γ to quartic order, $\Gamma_k = \bar{\psi}_1[G_0^{-1} - \Sigma_k + R_k]\psi_2 + \Gamma^{(4)}\bar{\psi}_1\bar{\psi}_2\psi_3\psi_4 + \ldots$

Truncate the flow equation,



Tanizaki et al, Prog. Theor. Exp. Phys. 043I01, (2014)

Technical slide 2: parametrize the flow



Discretize |q| and decompose Γ into angular momentum channels {*m*}.

$$\Gamma_k(p; q, q') = \sum_m \Gamma_m(p; |q|, |q'|) e^{im(\phi - \phi')}$$

In the limit of large k>>k_F, Γ is the bare interaction.

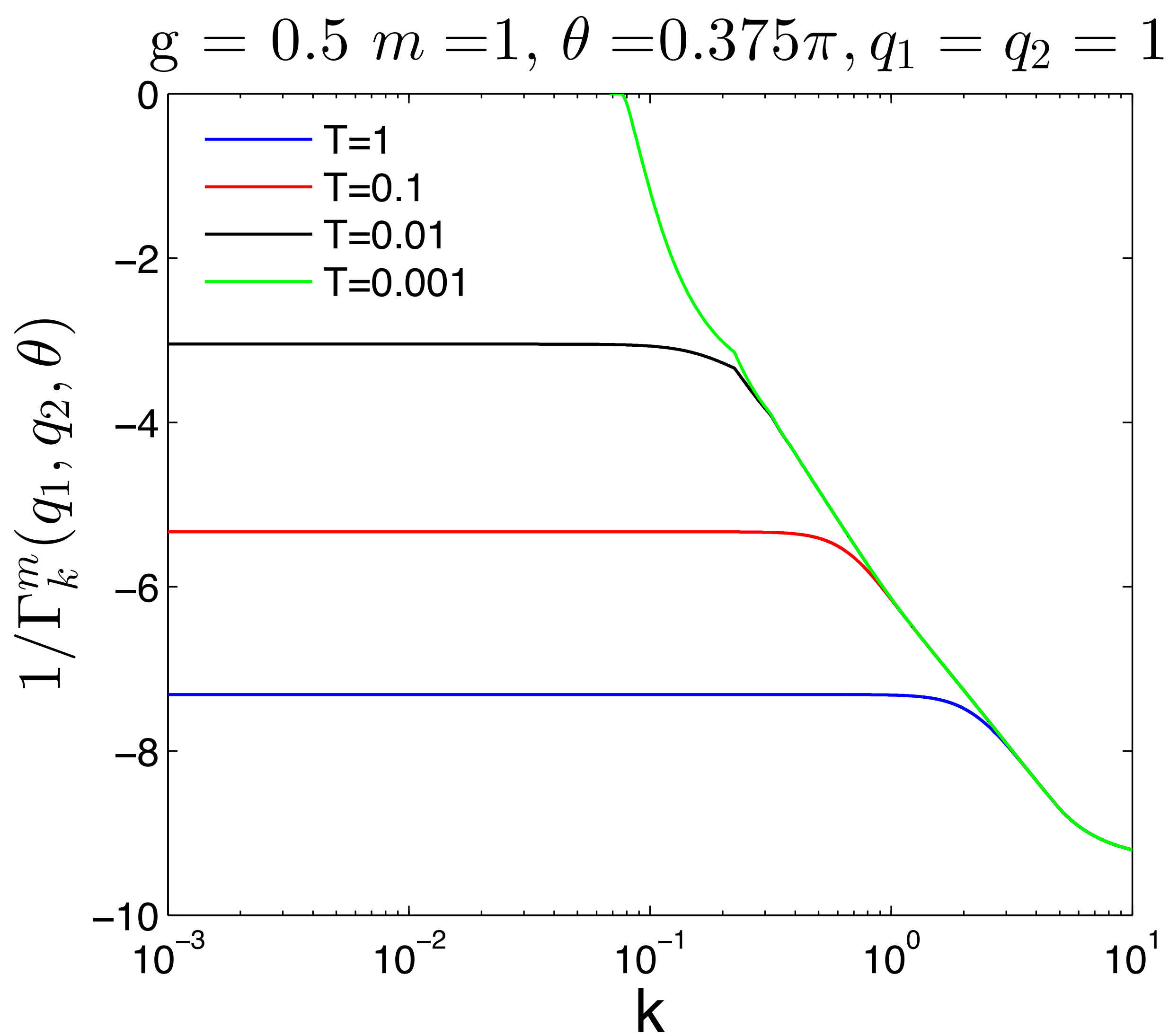
$$\Gamma_{k \to \Lambda}(q, q') = V(\mathbf{q} - \mathbf{q}') \qquad V(\mathbf{p}) = 2\pi p[\cos^2 \phi \sin^2 \theta - \cos^2 \theta] d^2$$

Γ_k at the end of the flow k→0 contains information about the instability and T_c.

Flow in the particle-particle channel: p-wave superfluidity

Divergence of Γ (i.e. zero of $1/\Gamma$) signals the transition to superfluid.

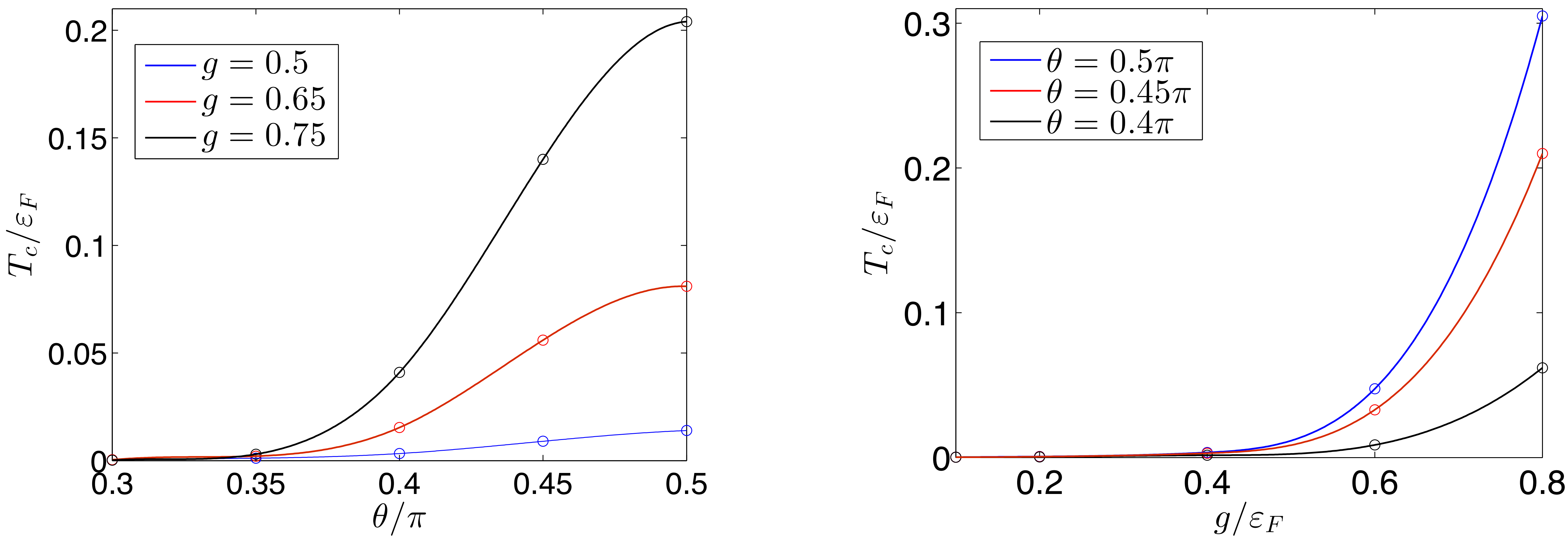
$$\left[\Gamma^{(4)}_{k=0}(p=0)\right]^{-1} = 0 \text{ at } T = T_c.$$



q_1, q_2 and k are in units of p_F

- Neglected the self energy correction;
- Neglected the particle-hole channel.

The superfluid transition temperature



Remaining questions:

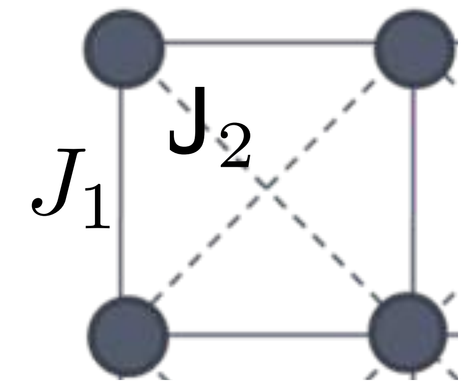
1. How about the particle-hole channel?
2. Self energy corrections (Fermi surface distortion, nematic phase…).
3. Solving the full flow equation numerically.

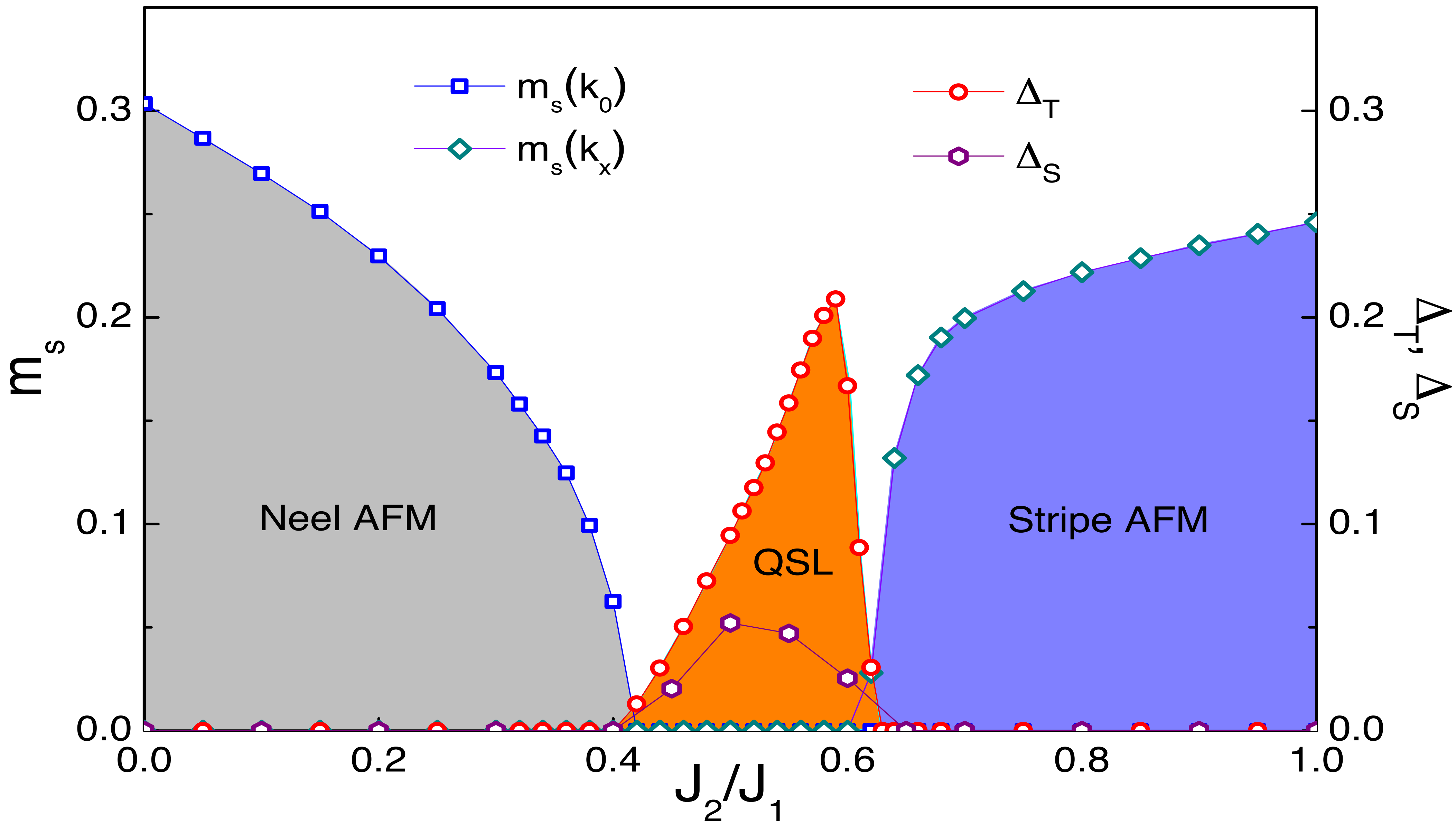
3. Magnetism of confined dipoles on lattice
(preliminary results, very speculative)

Collaborator:
Zhenyu Zhou (Pitt and GMU)

Quantum spin liquid in frustrated spin model: J1-J2 model

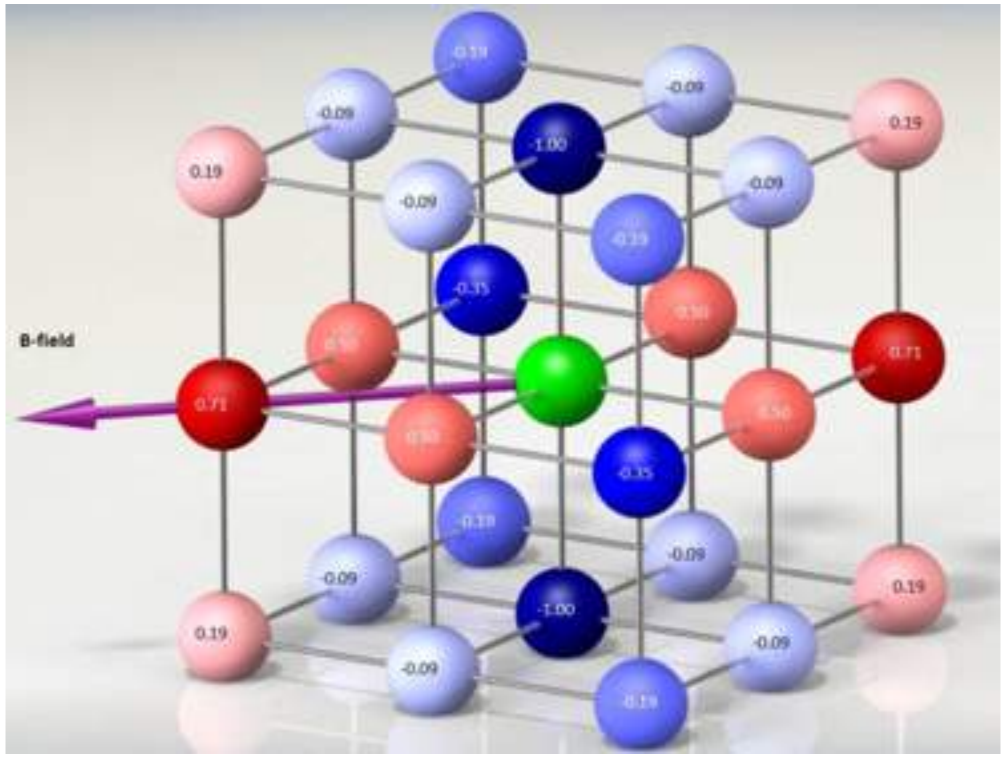
$$H = J_1 \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \vec{S}_i \cdot \vec{S}_j$$





Hong-Chen Jiang, Hong Yao, and Leon Balents, PRB 86, 024424 (2012)

Experiments



Observation of dipolar spin-exchange interactions with lattice-confined polar molecules,

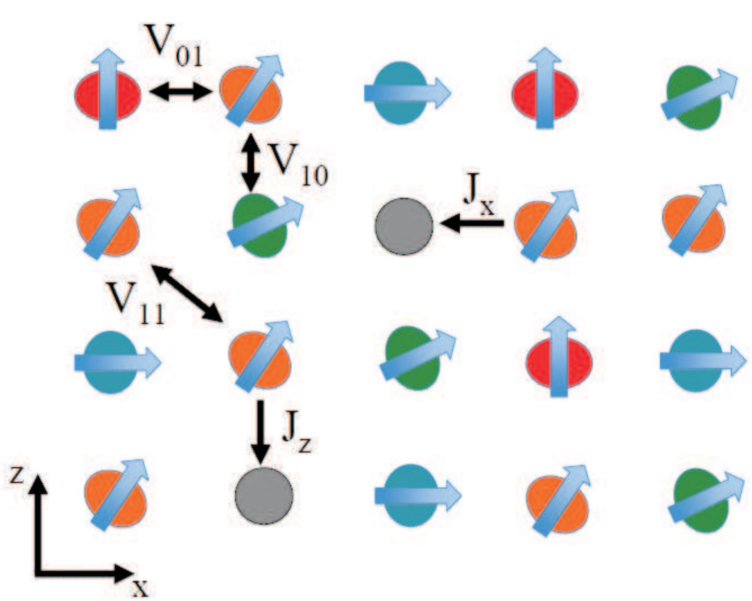
B. Yan, S. A. Moses, B. Gadway, J. P. Covey, K. R. A. Hazzard, A. M. Rey, D. S. Jin & J. Ye,

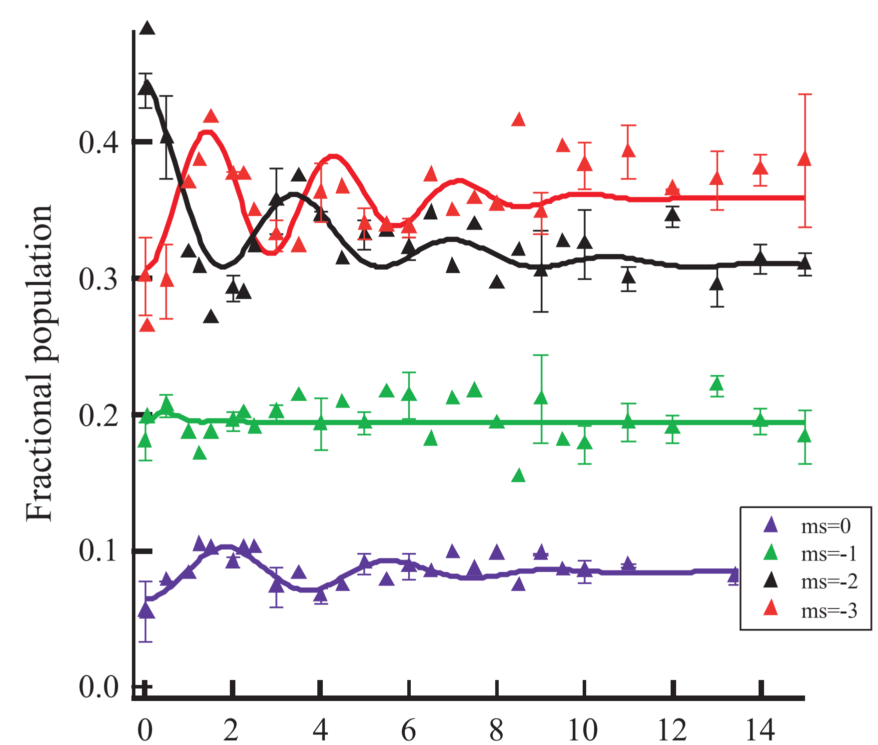
Nature 501, 521–525 (2013).

Nonequilibrium quantum magnetism in a dipolar lattice gas.

A. de Paz, A. Sharma, A. Chotia, E. Maréchal, J. H. Huckans, P. Pedri, L. Santos, O. Gorceix, L. Vernac, and B. Laburthe-Tolra,

Phys. Rev. Lett. 111, 185305 (2013)



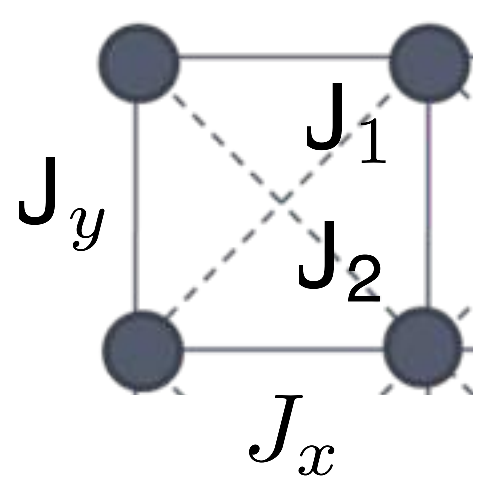


Lattice spin model: 2d, square lattice

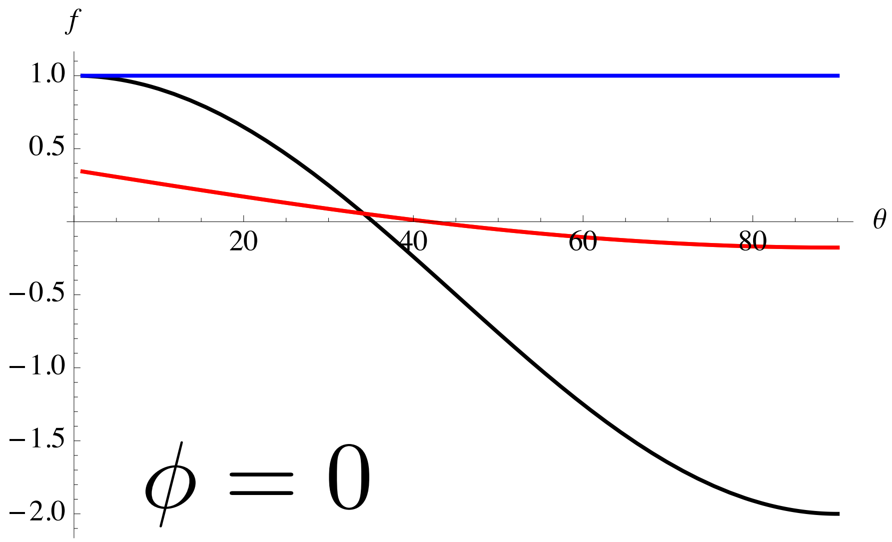
$$H_d = \frac{J}{2}\sum_{i\neq j} f(\mathbf{r}_i - \mathbf{r}_j)(s_i^x s_j^x + s_i^y s_j^y + \eta s_i^z s_j^z).$$

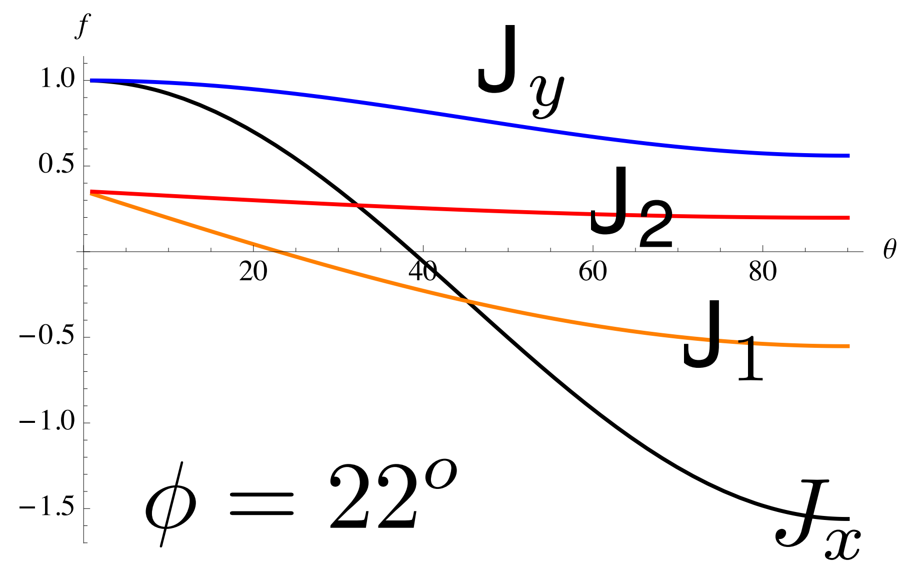
Here $s_i = (s_i^x, s_i^y, s_i^z)$ is the spin at site i, η describes the anisotropy depending on the detailed implementation (e.g., $\eta = 0$ for the KRb experiment), $f(r) = [1 - 3\cos^2(\hat{r}\cdot\hat{d})](a/r)^3$ characterizes the long range, anisotropic spin exchange due to dipolar interaction, and a is the lattice constant.

Consider S=1/2, η=1, and truncate exchange interactions to next nearest neighbor.



Competing exchange interaction

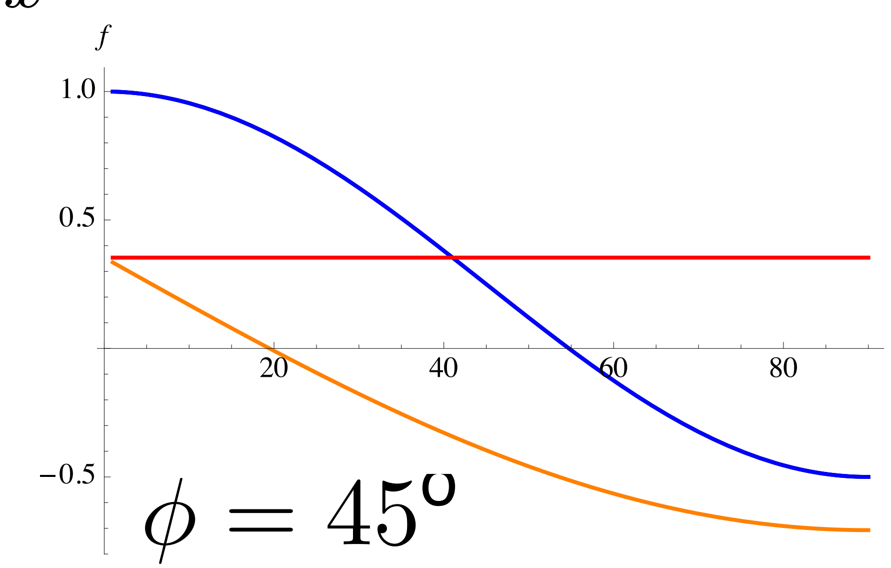


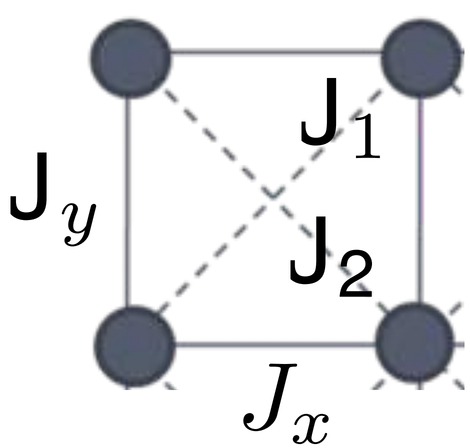


Sweet spots?

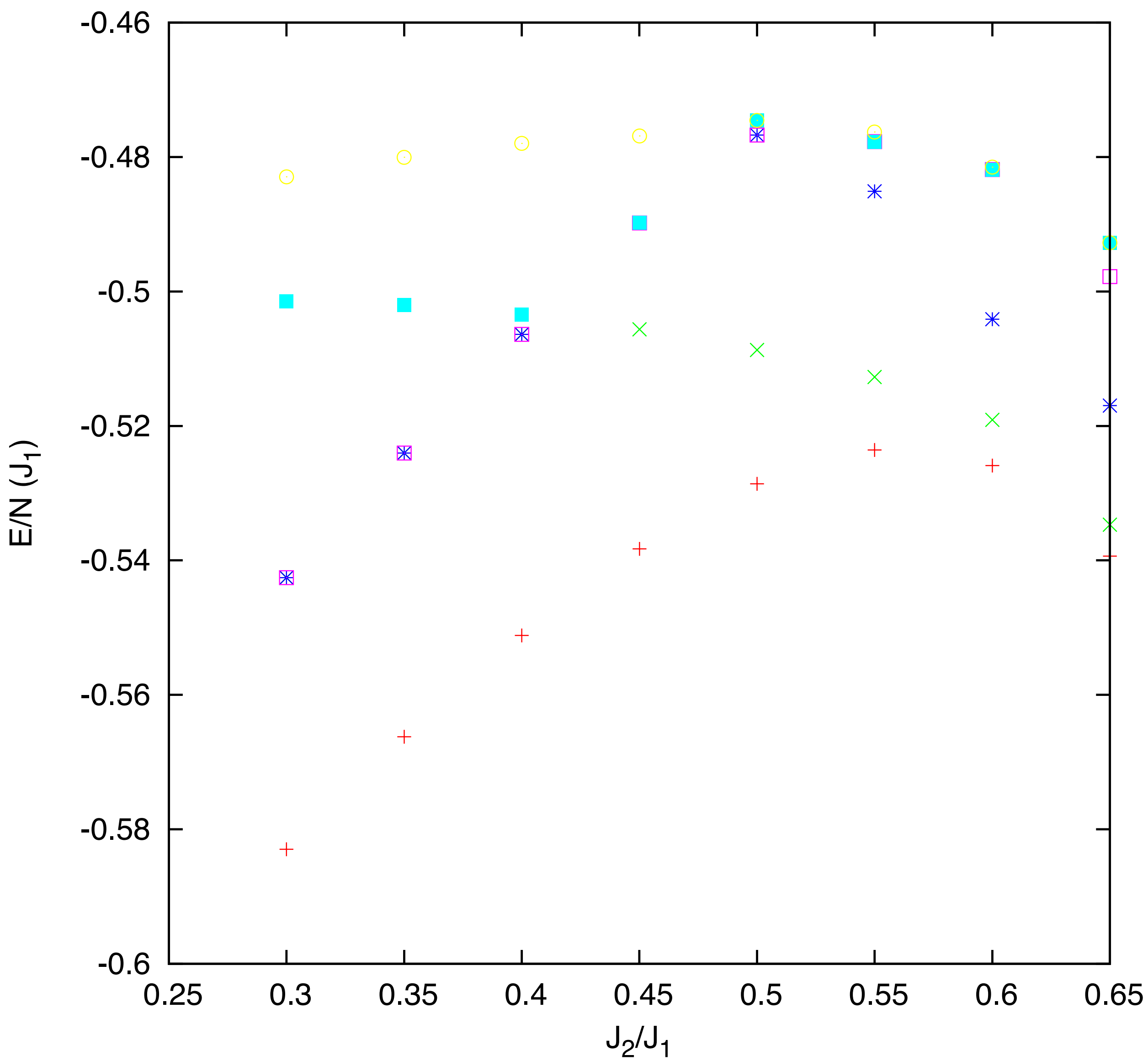
$J_x \sim J_y$

$\sim 2J_1 \sim 2J_2$

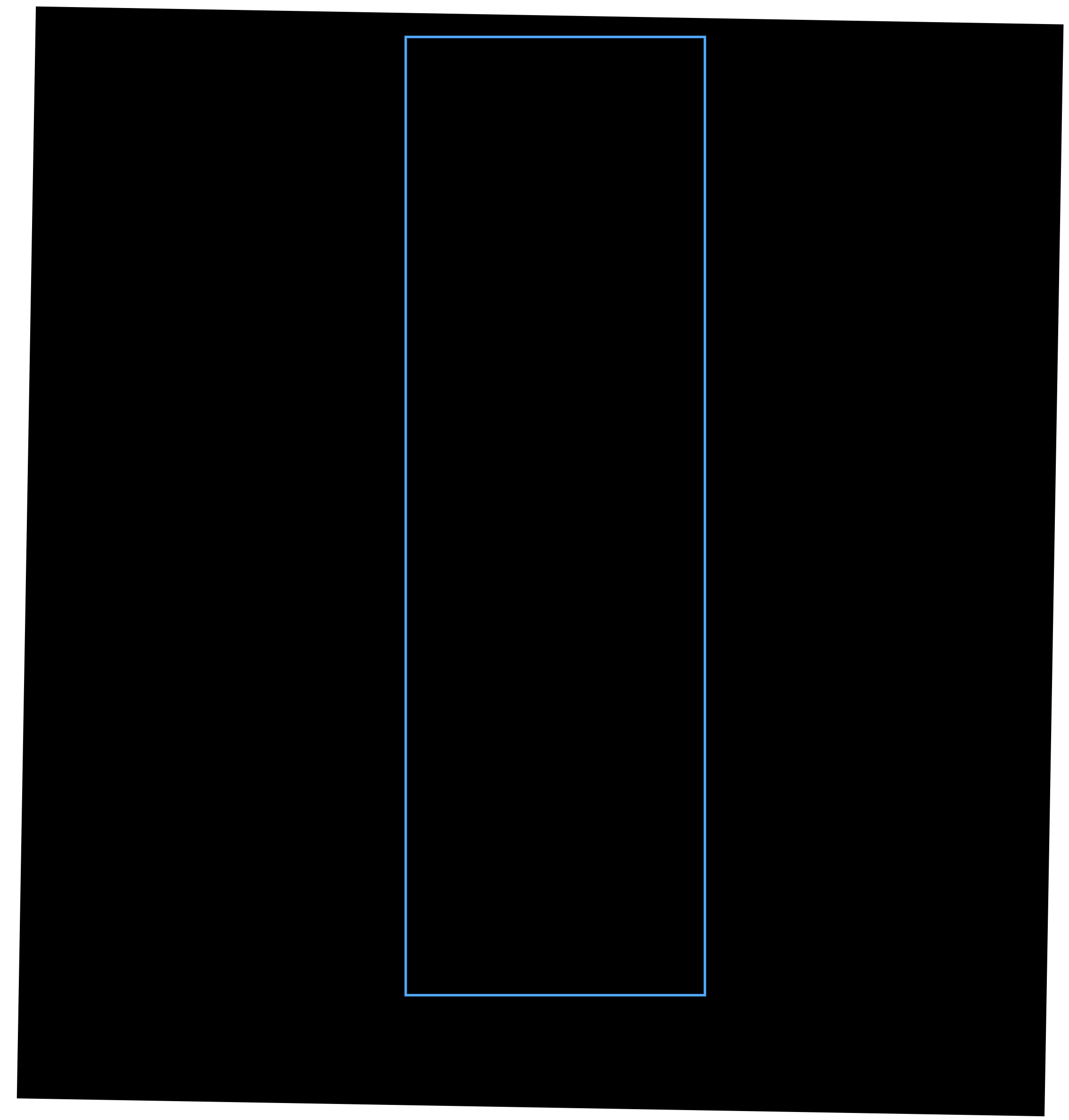




Benchmarking the exact diagonalization: J1-J2 model



16 sites, our calculation.



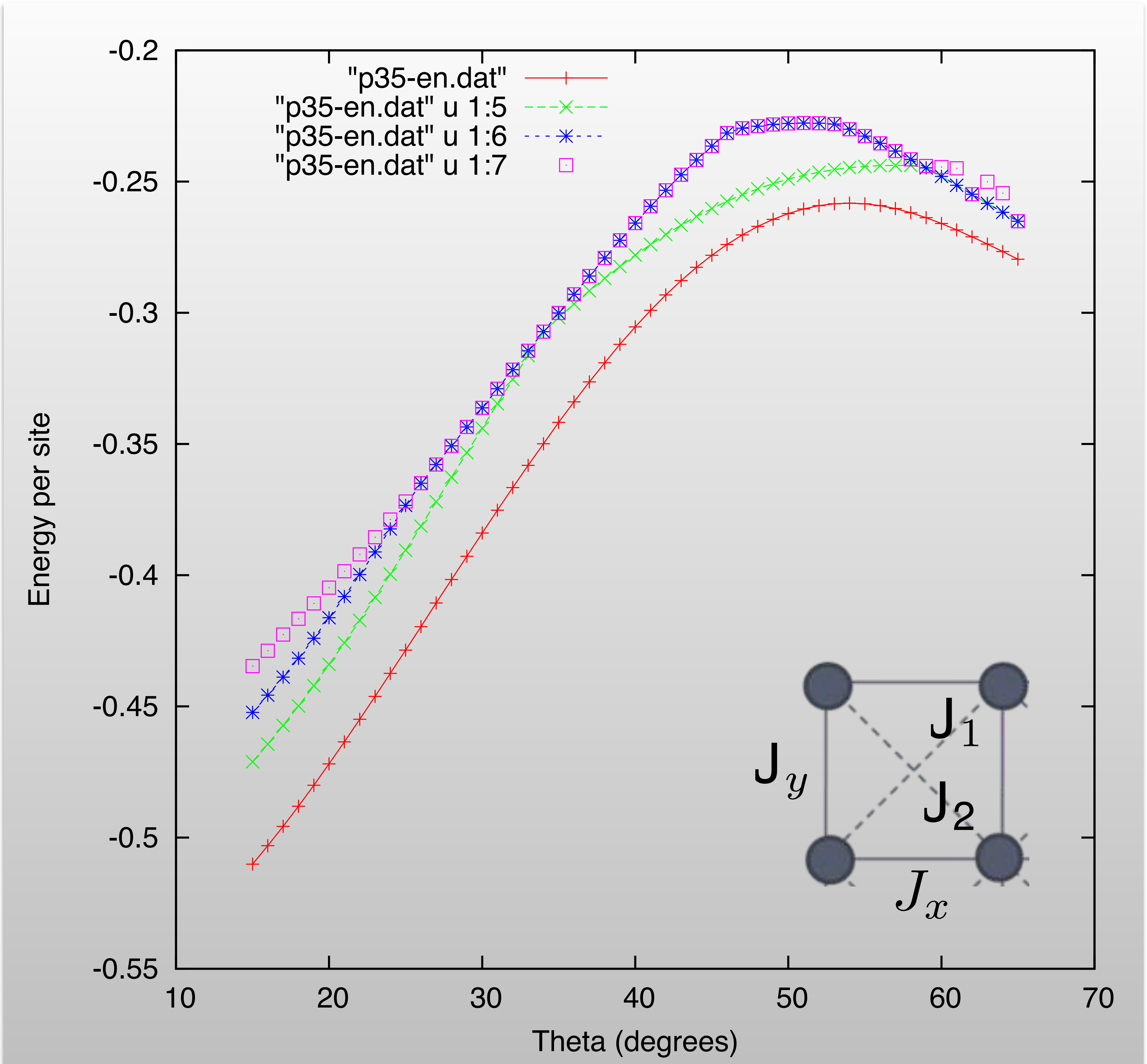
16 sites, with J1=2.

Dagotto and Moreo, PRL 1989

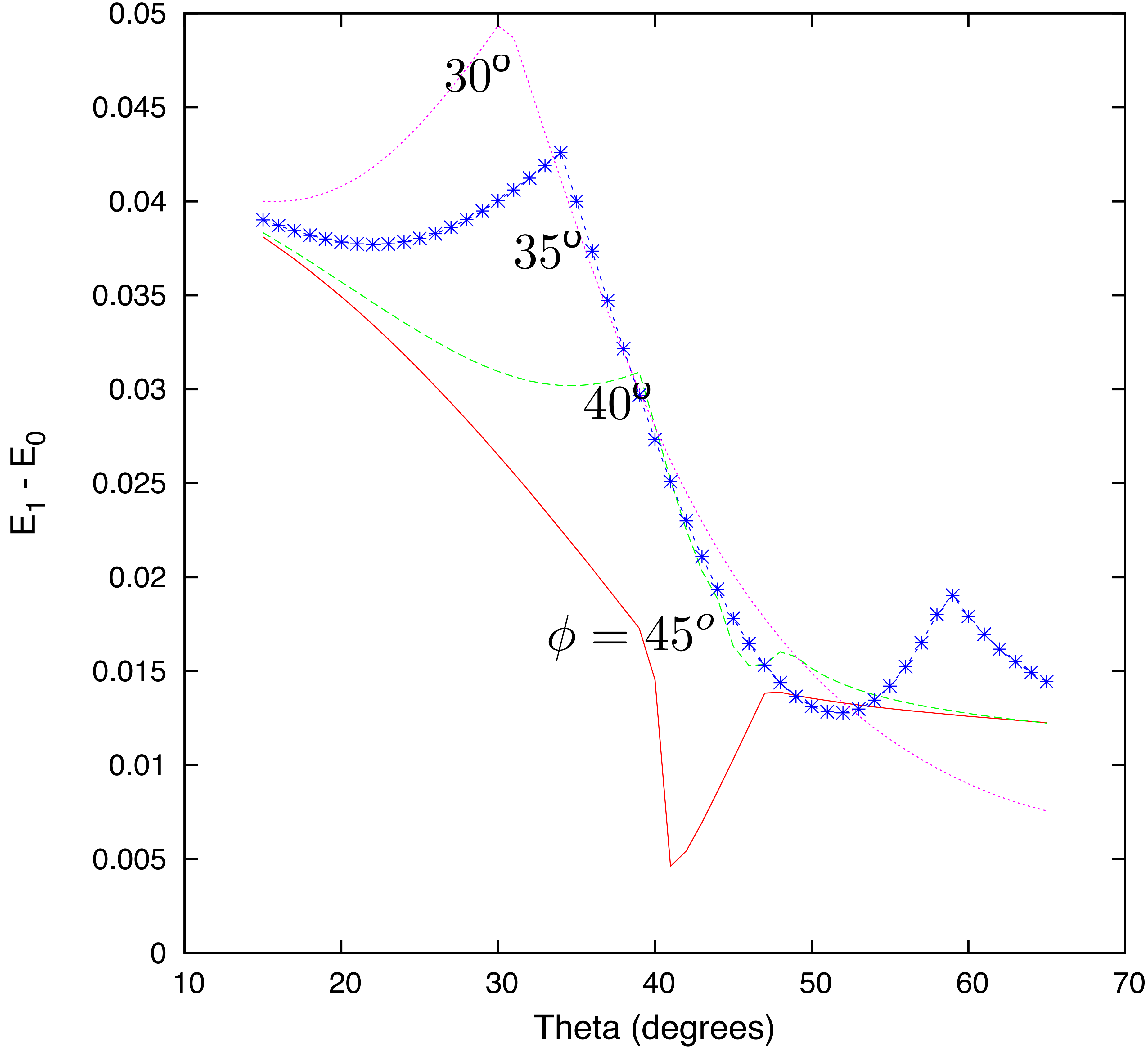
State of the art: 40 sites, # of basis: 430 909 650

J. Richter and J. Schulenburg, Eur. Phys. J. B 73, 117–124 (2010)

An example of the energy spectrum $\phi = 35^o$

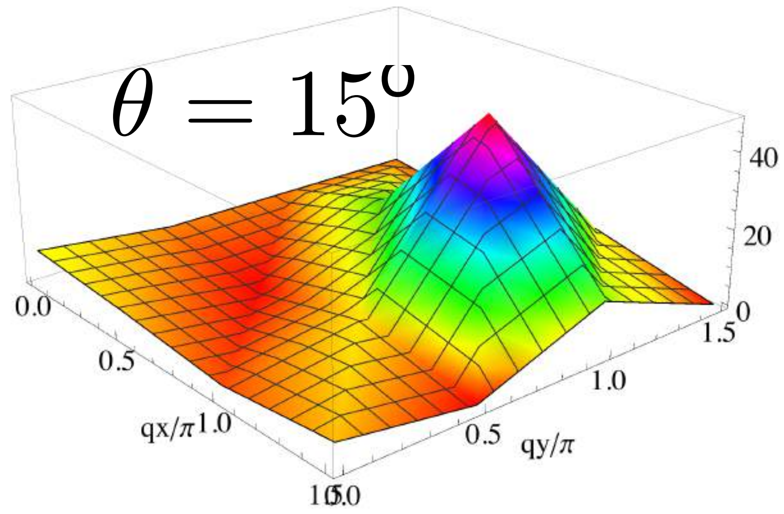


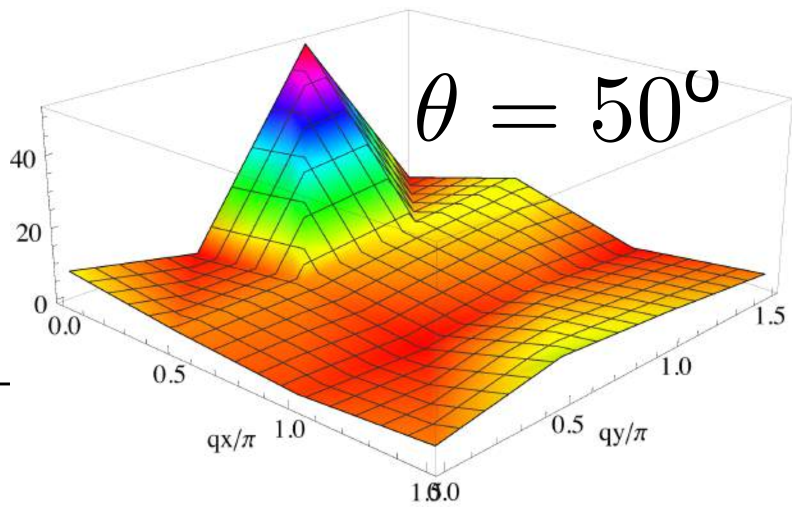
Excitation (spin) gap

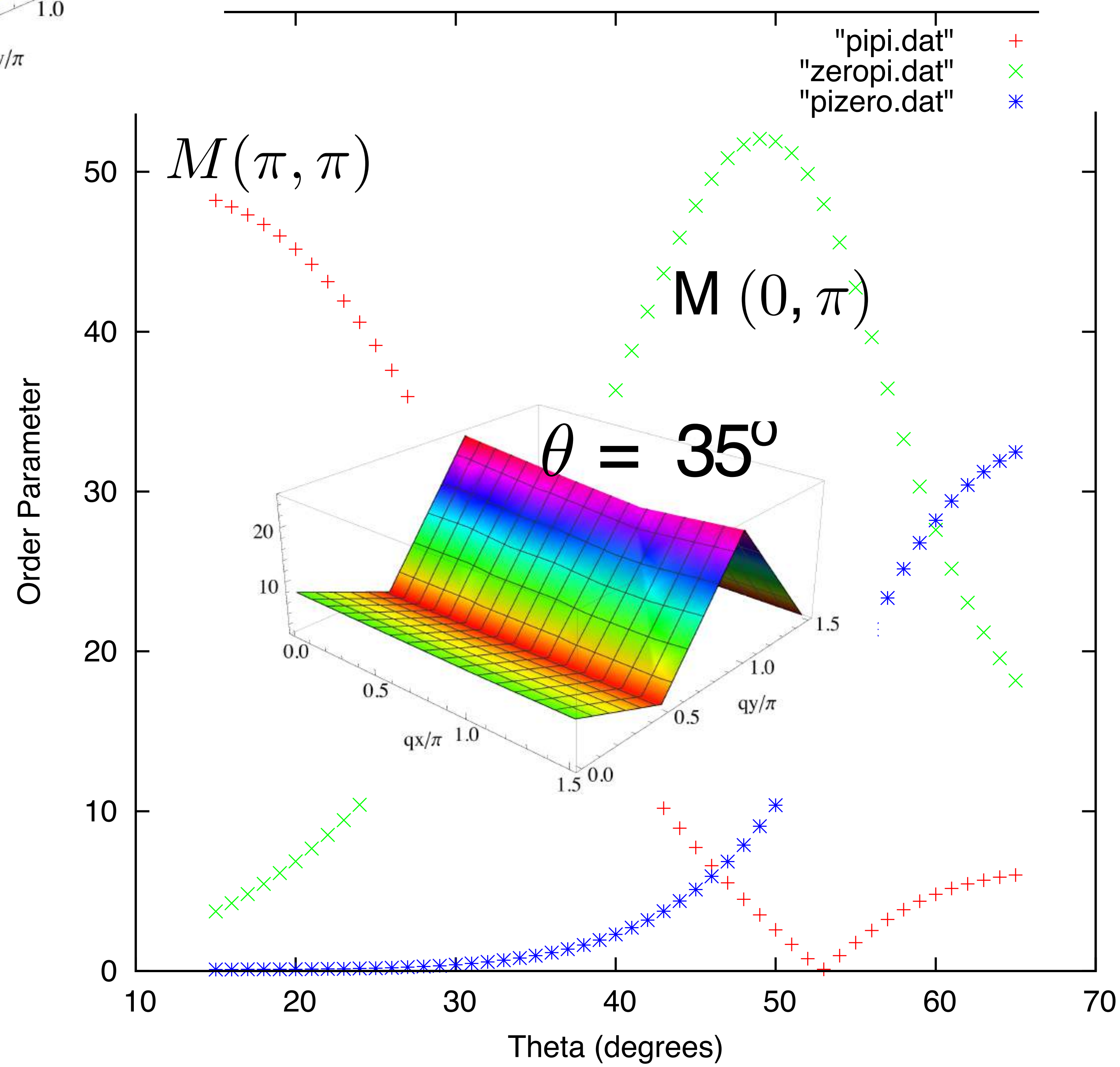


“Order Parameter”

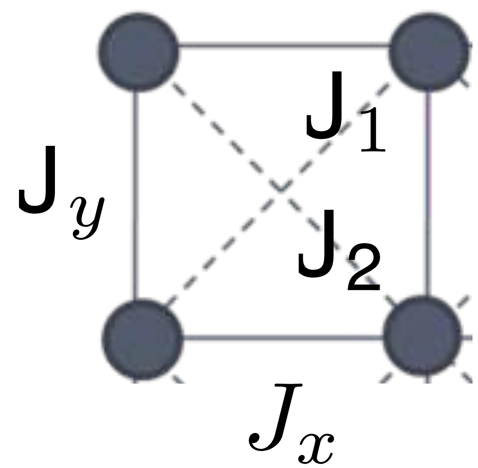
$$M(\mathbf{Q}) = \sum_{i,j} \langle \mathbf{s}_i \cdot \mathbf{s}_j \rangle e^{i\,\mathbf{Q}\cdot(\mathbf{r}_i - \mathbf{r}_j)}$$







Speculations



- This model can be highly, even maximally, frustrated;
- It is closely related to J1-J2 model; but the physics is even richer;
- Our numerical study (for small lattice) suggests a gaped quantum paramagnetic phase between the Neel and collinear ordered phase;
- Numerics on larger size systems is required to resolve the phase diagram.