INT-15-1 Frontiers in Quantum Simulation with Cold Atoms

Competing phases in dipolar quantum gas

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Fermi gas of polar molecules: KRb, NaK

Ni et al, Science 322, 231-235 (2008) D. S. Jin and J. Ye, Physics Today 64, 5(2011) Chotia et al, PRL 108, 080405 (2012)

Ground state 23Na40K molecules

Wu et al, PRL 109, 085301 (2012) Park, Will, Zwierlein, arxiv: 1505.00473 (2015) *v* = 0 1 PRI 109 OS $201 (2012)$ $f(15)$ \mathcal{L} is \mathcal{L}

Degenerate Fermi gas of magnetic atoms: ¹⁶¹Dy, ¹⁶⁷Er the corresponding critical temperature Tc \mathbf{r} Degenerate Fermi gas **Degenerate 1 crim gas** of magnetic stams: $\overline{161}$ \overline{Dv} $\overline{167}$ Degenerate Fermi gas of magnetic atoms: ¹⁶¹Dy, ¹⁶⁷Er Degenerate Fermi gas of magnetic atoms: ¹⁶¹Dy, ¹⁶⁷Er

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Lu, Burdick, Lev, PRL 108, 215301 (2012)

Aikawa et al, PRL 112, 010404 (2014) Lu, Burdick, Lev, PRL 10

1610 yields 1620 yields 1620 yields that Science 345, 1484 (2014) 1610, 1620, 1620, 1620, 1620, 1620, 1620, 1620, 1620, 1620, 1620, 1620, 1620, 1620, 1620, 1620, 1620, 1620, 16
Science 345, 1484 (2014) When TF is reached, we introduce a vertical beam at 1570 nm Aikawa et al, PRL 112, 010404 (2014) FIG. 1: (color online) AR of an expanding dipolar Fermi gas as a tions (solid lines), which include both the FSD and the NBE effects, $\frac{1}{2}$ Science 345, 1484 (2014) function of the angle β . In this measurement, the trap frequencies are β (fx, 1484), fx, fx), socience 345, 1484 Each individual point is obtained from about 39 independent meaand the case of ballistic expansions (dashed lines), $\frac{1}{2}$

Q: What are the many-body phases of fermions with dipole-dipole interaction? Are they all "boring," i.e., known and understood in condensed matter physics?

For dipoles pointing in the same direction:

$$
V_{dd} = \frac{d^2}{4\pi\epsilon_0} \xrightarrow{1 - 3\cos^2{\theta}} \longrightarrow P_2(\cos{\theta}) \text{ anisotropic}
$$

Comparing to other Fermi systems phases at low temperatures. The table below compares dipolar gas to several other archetypical

Candidate phases of dipolar fermions:

tendencies to compete and compete and compete superfluiding superfluiding to charge \sim \triangle shares density waves (CDW) $\sum_{n=1}^{\infty}$ density waves (OD IV) \sim p-wave superinum phases with bond order are in the set of \sim \sim surpes, quantum nquid crystals: ★ anisotropic Fermi liquid ★ charge density waves (CDW) ★ p-wave superfluid ★ stripes, quantum liquid crystals? ★ supersolid? ...

Baranov et al, Chemical Reviews 112, 5012 (2012); Physics Reports, 464, 71
(2008), Lebeve et al, Pen, Preg, Phys. 72, 126401 (2009), etc. $t = 0.00$, the form of the problems of $\frac{1}{2}$ and $\frac{1}{2}$ molecules has a long molecules h (2008). Lahaye et al, Rep. Prog. Phys. 72, 126401 (2009), etc.

Outline of this talk

- 1. Dipolar Fermi gas on square lattice ω half filling: phase diagram from functional renormalization group
- 2. Continuum gas of dipolar fermions: trying to go beyond Hartree-Fock and RPA
- 3. Frustrated magnetism of localized (deeply trapped) dipoles: hints from exact diagonalization on a small lattice

The common theme of the 3 problems is competing order.

Wish: treat (all) orders on the same footing, without a priori bias.

1. Dipolar fermions on lattice

Collaborators: Satyan Bhongale (GMU) Ludwig Mathey (Hamburg) Shan-Wen Tsai (UC Riverside) Charles Clark (NIST/JQI)

Dipolar fermions on square lattice: model Hamiltonian \mathbb{R}^n are known to exhibit a multitude of phases \mathbb{R}^n . In the phases \mathbb{R}^n Dipolar lermions on square lattice. Model Hamiltonian

$$
H = -t \sum_{\langle ij \rangle} a_i^{\dagger} a_j + \frac{1}{2} \sum_{i \neq j} V_{dd}(\mathbf{r}_{ij}) n_i n_j,
$$

 \star Half filling: on average, one fermion every two sites. \star Zero temperature; Neglect collapse instability.

The Fermi surface is just a square (half filling)

In the absence of dipole-dipole interaction: 320 Mars Metzner et al.: Function et al.: $\frac{1}{2}$

18.

15

★ Perfect Nesting: **Q** couple **k** points on the opposite sides of the FS. 1 **25** ★ We will discretize the Fermi surface into N patches. \star The Fermi surface may become unstable when V_{dd} is the med on. $\overline{}$ **1979 2 23** surface into N patches. **29 8** $\sin \sigma$: A couple $\bf k$ points on the oppos $\overline{1}$ V_{dd}

Interactions for dipoles tilting in the x direction

Two limits easy to understand all the corresponding to the corr

the nesting channel has the largest (most divergent) in the largest (most divergent) in the largest (most dive

illustrated in top panel of Fig. 2(a), is almost constant

1. Small tilting angle $(\theta_F<\vartheta_{c1})$: all interactions are repulsive. p_1 \mathcal{L} with \mathcal{L} and \mathcal{L} \mathcal{L} and \mathcal{L} along the Fermi surface. The F σ implies to Γ order with s-wave symmetry, when symmetry, we say show show symmetry, when σ

> Density wave (CDW): Periodic modulation of on-site density. $\langle a_i^{\dagger}a_i \rangle$ $\qquad \qquad$ $\langle a_i^{\dagger} a_i \rangle$

> In **k** space, this is an instability of FS in the particle-hole channel with **Q**. ϵ energy configuration with α $\mathbf x$ particle note enannel with $\mathbf x$.

2. Large tilting angle ($\theta_F > \vartheta_{c2}$): V_x and V_{x+y} attractive, but V_y repulsive. $\overline{2}$.

> Anisotropic p-wave pairing (BCS): The pairing order parameter $\begin{bmatrix} 1 & 1 \end{bmatrix}^2$

$$
\langle a_i a_{i+\hat{x}} \rangle = -\langle a_i a_{i-\hat{x}} \rangle \quad \langle a_i a_{i\pm y} \rangle = 0 \qquad ; \qquad i \qquad \qquad i \qquad \qquad \boxed{\qquad}
$$

In **k** space, this is an instability of FS $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ in the particle-particle channel. responds to the most dominant instability of the Fermi liquid. The corresponding eigenvector ψ defined on the $\frac{1}{\sqrt{2}}$ fluid $\frac{1}{\sqrt{2}}$ is the lattice and $\frac{1}{\sqrt{2}}$ of the particle channel. phase discussed previously for continuum dipolar Fermi

Lattice size is 32 × 32.

How about the intermediate tilting angle

Vx and *Vy* opposite in sign and comparable in magnitude. What do the fermions do?

Settle to BCS or CDW? Neither? Both?

Competing orders in interacting dipolar fermions

Three possible scenarios:

- ★ Direct (1st order) transition from CDW to p-wave BCS superfluid.
- \star Coexistence: density modulation + pairing = supersolid.
- ★ Or, some other completely different animal.

The problem of competing order is at the heart of the many-body physics of dipolar fermions.

Simple mean field theories or perturbation theories, such as single-channel Renormalization Group or Random Phase Approximation, are insufficient/unreliable to treat competing orders in the regime of intermediate tilting angle.

We need a theory that can treat all ordering instabilities on equal footing, without any a priori assumptions about dominant orders.

Functional Renormalization Group (FRG) al in the third order in the two-We now discuss the T \sim 0 phase discussed by the T \sim $f(n)$ u p u r v We now discuss the T $_{\rm eff}$ at half $_{\rm eff}$

- \triangle Conorato the Iour one was modes \star Separate the low-energy modes and high energy modes with scale Λ. $\overline{\mathbf{1}}$ SCd. $\mathbf{J}^{(i)}$ l , Λ . $\frac{1}{\pi}$ ioh energy modes with scale Λ \bigstar Separate the low-energy modes and high energy modes with scale Λ . $\operatorname{de}\Lambda$.
- \triangle At each seale \triangle there is an offer $\overline{}$ \star At each scale Λ , there is an effective theory description, including the effective interaction (vertex function) *U* between the low energy modes. 1 \star \star (\star is equations (See $\begin{array}{c} \text{sep} \ \text{of} \ \text{$ ferrowa
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Notes Ω ergy : Þs — $\boldsymbol{\Omega}$ $\frac{48}{4}$ example $\frac{1}{4}$. rory acsemption, meraanig ene \star At each scale Λ , there is an effective theory description, including the e low energy modes. e ilective interaction (vertex function) σ between the low energy degrees of e ig the modes.
	- ★ As Λ is reduced, the evolution of *U* obeys the exact "flow equation." Þ eys the exact "flow equation." α set of coupled integro-differential equations given by α et "flow ee \bigstar As Λ is reduced, the evolution of U obeys the exact "flow eq in ."
	- \star For weak coupling, the infinite hierarchy of flow eqns can be truncated and solved numerically by discretizing **k**. quatio.
be trui \star For weak coupling, the infinite hierarchy of flow eqns can be truncated and solved numerically by discretizing **K**. incated as a checker

See e.g. Metzner et al, Rev. Mod. Phys. 84, 299–352 (2012); And reference therein. $\frac{1}{100}$ $, '$ $\mathsf{in}.$ erence the ι (και το υποκρατικό ανακτοποιη)
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FRG applied to interacting dipolar fermions $\frac{1}{2}$ is the renormalization group flow parameter that relates the relation $\frac{1}{2}$ FRG applied to interacting dipolar fermions

 EDC keeps track of all effective interactions as the hi FRG keeps track of all effective interactions as the high energy modes are
traced out-including the space of planned as the individual interraced out, including the p-p and p-n channel, as we
Especially, we are interested in the BCS and the CDV \mathbf{r} matrix with the largest divergent eigenvalue \mathbf{r} $B_{\rm{max}}$ divergy modes are
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annel traced out, including the p-p and p-h channel, as well as their subtle interplay. we are interested in the BCS and the CDW channel. $\,$ play. Especially, we are interested in the BCS and the CDW channel.

The most dominant instability can be inferred from the most diverging contractive for a distribution of the supereigenvalue of U, which is a matrix of \mathbf{k}_1 and \mathbf{k}_2 . The corresponding eigenvector indicates the symmetry of the incipient order. $\frac{1}{2}$ matrix with the largest divergent eigenvalue $\frac{1}{2}$ corresponding to $\frac{$ r and the most dominant instability can be interfered to the most diverging r lic, which is a matrix of **N** and N₂. The corresponding eig $\mathcal{F}_{\mathcal{S}}$ minetify of the incipient of a.f. betor ∍ χ . χ The most dominant instability can be inferred from the most diverging eigenvalue of U , which is a matrix of \mathbf{k}_1 and \mathbf{k}_2 . The corresponding eigenvector indicates the symmetry of the incipient order.

Instability analysis within FRG

Bond order solid (BOS) For example, SCMF predicts an additional striped den-

Such p-wave instability in the CDW channel corresponds to a spatial modulation of "bonds", more precisely, the average of hopping ch n-waye instability in the CDW channel corresponds to a spatial ch p wave mstability in the OD iv channel corresponds to a spatial
population of "bonds", more precisely, the average of hopping patial **proprime** who included: The contribution corresponds to a space

mi s The ground state energy per unit cell is the ground by the given by the given by \sim ★ Opening up a gap at the Fermi surface.

Following state chergy $E_{\text{GS}} = 2(\chi x + \chi y)(v + v x + v y)$ and F_{G} $E_{\text{GS}} = -2(\chi_x + \chi_y)(t + V_x + V_y) - 2V_y\delta^2$ annihilation of the separately of the sub-Should state energy. $L'GS = -2(\chi_x + \chi_y)(\iota + \nu_x + \nu_y)$ $\overline{\hspace{1em}}$ $\overline{\hspace{1em}}$ $\overline{\hspace{1em}}$ V_yO^- ★ Ground state energy: $T_{\rm tot}$ and state energy per $F_{\rm esc} = -2(\gamma_0 + \gamma_1)(t + V + V + V)$ EVALUATE CHULES $E - 2(\chi_x + \chi_y)(\iota + \nu_x + \nu_y) - 2\nu_y \sigma_z$

finite bond modulation δ is energetically favored $\frac{1}{2}$ in $\frac{1}{2}$ finite $\frac{1}{2}$ is energetically favored for the production of p energetically favored finite bond modulation δ is energetically favored $\mathcal O$ by performed state energy per unit cell is then given by $\mathcal O$ $\frac{1}{2}$ $\overline{\text{C1C}}$ Phase diagram (T=0, half-filling, $\varphi_F=0$)

Phase diagram for general dipole tilting

 $F = 0$ $F = 0$ online) $F = 0$ online lattice. (a) $F = 0$ $F =$ cal dialectual lattice potential. The induced dipole moment dipole moment dipole moment dipole moment dipole m
Section directions and discussed the directions and discussed the direction of the discussed of the discussed ا K
ا d = cos θFz
= cos θFz $\ddot{}$ + sin $\ddot{}$ sin $\ddot{}$ Fig. 1: Golor online) Dipolarions on son son square lattice. (a) Schematic original fermions on square in square confined to a square of the dipolar fermions continued to a square of the dipolar fermions confined to a squa c. C. Bibliongale, E. Hattic, ˆ S. G. Bhongale, L. Mathey, S.-W. Tsai, C. W. Clark, EZ, PRL 108, 145301 (2012)

Classification of density waves

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Superconductors (condensate of Cooper pairs):

$$
\langle f_{\alpha}(\mathbf{k}) f_{\beta}(-\mathbf{k}) \rangle = \begin{cases} \Delta(\mathbf{k}) \cdot (i\sigma_y)_{\alpha\beta} & \text{spin singlet, } l=0,2,.. \\ \Delta(\mathbf{k}) \cdot (\sigma i \sigma_y)_{\alpha\beta} & \text{spin triplet, } l=1,3,.. \end{cases}
$$

s-wave superconductor, *l*=0 p-wave superconductors, *l*=1 d-wave superconductors, *l*=2

Density waves (condensate of particle-hole pairs):

$$
\langle f^{\dagger}_{\alpha}(\mathbf{k}+\mathbf{Q})f_{\beta}(+\mathbf{k})\rangle = \Phi(\mathbf{k})\delta_{\alpha\beta} \quad \begin{cases} \text{ s-wave CDW (checkerboard)} \\ \text{ p-wave CDW} \\ \text{d-wave CDW (DDW)} \dots \end{cases}
$$

$$
\langle f^{\dagger}_{\alpha}(\mathbf{k}+\mathbf{Q})f_{\beta}(+\mathbf{k})\rangle = \Phi(\mathbf{k})\cdot\boldsymbol{\sigma}_{\alpha\beta}\left\{\begin{array}{l} \text{s-wave SDW (-Neel order)} \\ \text{p-wave SDW...} \end{array}\right.
$$

Density-wave states of nonzero angular momentum, Chetan Nayak, Phys. Rev. B 62, 4880 (2000)

They show up in dipolar Fermi gas!

Observation of d-wave density waves?

d-form factor density wave order

Observation of d-form factor density waves (in BSCCO and Na-CCOC)

Experiments: Kohsaka et al, Science 315, 1380 (2007). Fujita et al, BNAS nd it tar BO26 (2014)111, 027202 (2013). Theory: Metlitski & Sachdev, PRB 82, 075128 (2010); PRL 111, 027202 (2013); etc. M. A. Metlitski and S. Sachdev, Phys. Rev. B 82, 075128 (2010). $U \propto$ decreasing U , and U , and U or U and U (20.10) , $U \cap U$ in $U \cap U$

$$
P_{ij} = \left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle \text{ for } i = j, \text{ and } i, j \text{ nearest neighbors.}
$$

\n
$$
P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}
$$

\n
$$
\mathcal{P}(\mathbf{k}) = e^{i\phi} \left[\cos(k_x) - \cos(k_y) \right] \text{ and } \mathbf{Q} = 2\pi (1/4, 0)
$$

y

x

where the mean fi $\mathbf{D}_{\mathbf{Q}}$ u U
. interaction, $V_d \sim 2.5t$, where the mean field gap is 0.23*t*, or 0.05 E_F . identify each phase, go beyond weak coupling, and study effects of finite temperature and trap potential. Bond order is most robust for intermediate interaction, *V_d* ~ 2.5*t*, \overline{a} Mean field phase diagram obtained for a 32 \overline{a} \overline{a}

Beyond weak coupling

 $\mathcal{L}_{\mathcal{A}}$

Exact diagonalization (ED) yields the hopping correlation function Exact diagonanzation (ED) yiclus the hopping correlation ful rithm and the correlation function correlation μ

It approaches 4*δ*2 in the limit of large |*i*-*j*|. $C(i, j) = \langle K_{i,i+j} K_{i,j+j} \rangle$ It expresses a colorer $4\degree$ is the line it. μ approaches $\pm \sigma$ in the million a) Mean field phase diagram obtained for a 32 \pm 32 \pm 32 \pm 32 \pm 32 \pm $V(i,j) = \langle K_{i,i+j}, K_{i,i+j} \rangle - \langle K_{i,i+j} \rangle \langle K_{i,i+j} \rangle$ $K_{i,j} \equiv (a^{\dagger} a_{i} + b_{i} c_{i})$ ρii = ⟨a† ⁱ ai⟩, the nearest neighbor hopping ρij = ⟨a† ^jai⟩ (with j = i + ˆx or j = i + ˆy), or the pairing gap $\mathfrak a$ ches 4 It approaches $4\delta^2$ in the limit of large $|i-j|$. $C(i,j) = \langle K_{i,i+y}K_{j,j+y} \rangle - \langle K_{i,i+y} \rangle \langle K_{j,j+y} \rangle \qquad K_{i,j} \equiv (a_i^\intercal a_j + h.c.)$ α is the bond modulation along α

Spin half dipolar Fermi gas

the surface of a sphere as a function of the dipole orientation angle θ and φ for fixed interactions (a) V^d = 0.5, U = 0.1; and The p-wave spin density wave phase is sandwiched between the CDW $\frac{1}{2}$ and BCS superfluid phases. Its phase boundary depends on U. and BCS superfluid phases. Its phase boundary depends on U. as the original is increased from U. and BCS superfluid phases. Its phase boundary depends on U.

 A s the on-site interaction is increased from U $=$ 0.1 in (b), the SDWs+d phase expanding and squeezes out the SDWs+d phase expanding and squeezes out the SDWs+d phase expanding and squeezes out the SDWs+d phase expandi S. G. Bhongale, L. Mathey, S.-W. Isai, C. W. Clark, EZ, PRA 87, 043604 (2013). S. G. Bhongale, L. Mathey, S.-W. Tsai, C. W. Clark, EZ, PRA 87, 043604 (2013). \times F7 PRA 87 043604 (2013)

Quadrupolar Fermi gas be controlled by preparing the particles in a particular $\sqrt{3}$ larized photons; the quadrupole-quadrupole interaction is equal in the larger than $\frac{1}{2}$ states.

S. G. Bhongale, L. Mathey, EZ, S. F. Yelin, M. Lemeshko, PRL 110, 155301 (2013) on a square lattice, at half-filling. We assume all particles \mathcal{L}

2. Functional renormalization group analysis of continuum dipolar gas in 2D

Collaborator: Ahmet Keles (Pitt and GMU) 2D dipolar Fermi gas, mean field and RPA predictions

the system and supersolid as understand as understanding the Fourier transformation \mathbb{R} ^V0(p) = [√] Tightly confined in z direction

Sieberer and Baranov, PRA 84, 063633 (2011)
See also: Babadi & Demler, PRB 2011; See also: Babadi & Demler, PRB 2011; angles the system is a normal Fermi liquid (NFL). The tran- \sum riau ut ar (Fuud yruup) i Tri, 2010, 2πab et antitius group) in it. 2010,
Bruun and Taylor PRL 2008; and many others. ∆ + \mathbf{z} sissoro: and Baranor, in this or, occesso Zhao et al (Pu's group) PRA, 2010; Bruun and Taylor PRL 2008; and many others.

Technical slide 1: Flow of effective action Technical slide 1: Flow of effective action

Add infrared regulator R_k to the action S , k being the sliding momentum scale, e.g.,

$$
R_{\mathsf{k}}(\mathbf{p}) = \left[\frac{\mathsf{k}^2}{2\mathsf{m}}\text{sgn}(\xi(\mathbf{p})) - \xi(\mathbf{p})\right]\theta\left(\frac{\mathsf{k}^2}{2\mathsf{m}} - \mathsf{l}\xi(\mathbf{p})\mathsf{l}\right)
$$

 \mathcal{L} Wetterich's flow equation:

$$
\partial_{\mathsf{k}} \Gamma_{\mathsf{k}} = -\frac{1}{2} \tilde{\partial}_{\mathsf{k}} \operatorname{Tr} \ln \left[\Gamma^{(2)} + R_{\mathsf{k}} \right]
$$

 C. Wetterich, Phys. Lett. B, 301(1), 90–94 (1993). T. R. Morris, Inter. J. of Mod. Phys. A, 9(14), 2411–2450 (1994). ka = 0,000 million.
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Expand Γ to quartic order, Expandition quantic order, $\mathbf{1}_k = \psi_1 \mathbf{U}_0$ and $\mathbf{2}_k + \mathbf{1}_k \mathbf{U}_1 \psi_2 + \mathbf{1}_k \psi_1 \psi_2 + \mathbf{1}_k \psi_3$ $\Gamma_k = \bar{\psi}_1[G_0^{-1} - \Sigma_k + R_k]\psi_2 + \Gamma^{(4)}\bar{\psi}_1\bar{\psi}_2\psi_3\psi_4 + ...$

 $F(X) \subseteq X(Y) \cup T(Y|Y) \cup T(Y|Y) \cup T(Y|Y) \cup T(Y|Y) \cup T(Y|Y) \cup T(Y|Y)$ Tanizaki et al, Prog. Theor. Exp. Phys. 043I01, (2014)

Technical slide 2: parametrize the flow

Discretize |q| and decompose Γ into angular momentum channels {*m*}.

$$
\Gamma_k(p;q,q') = \sum_m \Gamma_m(p;|q|,|q'|)e^{im(\phi-\phi')}
$$

In the limit of large k>>k_F, Γ is the bare interaction.

$$
\Gamma_{\mathbf{k}\to\Lambda}(q,q') = V(\mathbf{q}-\mathbf{q}') \qquad V(\mathbf{p}) = 2\pi p[\cos^2\phi\sin^2\theta - \cos^2\theta]d^2
$$

 Γ_k at the end of the flow $k\rightarrow 0$ contains information about the instability and T_c .

Flow in the particle-particle channel: p-wave superfluidity where $=$ **p** m − nergy relative to the energy relative to the energy relative to the Fermi level. For a second-order to the
The Fermi level of a second-order to the Fermi level. For a second-order to the Fermi level. The energy rela

 $\sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{i=1}^{n}$ Divergence of Γ (i.e. zero of 1/Γ) signals the transition to superfluid. by looking at the divergence of the divergence of the divergence of the scattering mat rix at the ϵ production in the

$$
\[\Gamma_{k=0}^{(4)}(p=0)\]^{-1}=0 \text{ at } T=T_c.
$$

 $q_1,\,q_2$ and k are in units of p_F

- Neglected the self energy correction;
- Neglected the particle-hole channel.

The superfluid transition temperature

Remaining questions:

- 1. How about the particle-hole channel?
- 2. Self energy corrections (Fermi surface distortion, nematic phase…).
- 3. Solving the full flow equation numerically.

3. Magnetism of confined dipoles on lattice (preliminary results, very speculative)

Collaborator: Zhenyu Zhou (Pitt and GMU) Quantum spin liquid in frustrated spin model: J1-J2 model

Hong-Chen Jiang, Hong Yao, and Leon Balents, PRB 86, 024424 (2012)

Experiments

Observation of dipolar spin-exchange interactions with lattice-confined polar molecules,

B. Yan, S. A. Moses, B. Gadway, J. P. Covey, K. R. A. Hazzard, A. M. Rey, D. S. Jin & J. Ye, *Nature 501, 521–525 (2013).*

Nonequilibrium quantum magnetism in a dipolar lattice gas. S2.Description of the plaquette simulation for singlons nequilibrium quantum maqne

A. de Paz, A. Sharma, A. Chotia, E. Maréchal, J. H. Huckans, P. Pedri, L. Santos, O. Gorceix, L. Vernac, and B. Laburthe-Tolra, e Paz, A. Sharma, A. Chotia, E. Marechal, J. H. Huckans, P. Pedri, L. Santos, Q $\overline{\mathbf{C}}$ occupied sites $\overline{\mathbf{C}}$

Phys. Rev. Lett. 111, 185305 (2013) v. Lett. 111, 185305 (2013)

0405, Feloczotz. Lattice spin model: 2d, square lattice spin models of dioplar bosons (32) A. Chotia, B. Nevenhuis, S. A. Moses, B. Yan, J. P. Covey, M. Foss-Feig, A. M. Rey, D. S. Jin, and
Park, C. Fize Wish-Like Sandi baggerlies Gird Felooks onder Webling Pan Annapaditioanon. Med Wien (Wier HMay! J $\boldsymbol{\mathcal{U}}$ $\begin{array}{l}\n\text{Hago, } f \left(\mathbf{\hat{p}}_i \text{ Lieqke} \right) \\
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Competing exchange interaction

Benchmarking the exact diagonalization: J1-J2 model VOLUME 63, NUMBER ¹⁹ PHYSICAL REVIEW LETTERS ⁶ NOVEMBER ¹⁹⁸⁹

Dagotto and Moreo, PRL 1989

State of the art: 40 sites, # of basis: 430 909 650 J. Richter and J. Schulenburg, Eur. Phys. J. B 73, 117–124 (2010)

An example of the energy spectrum $\phi = 35^{\circ}$

Excitation (spin) gap

"Order Parameter"

Speculations

- This model can be highly, even maximally, frustrated;
- It is closely related to J1-J2 model; but the physics is even richer;
- Our numerical study (for small lattice) suggests a gaped quantum paramagnetic phase between the Neel and collinear ordered phase;
- Numerics on larger size systems is required to resolve the phase diagram.