# Few-body problems in ultracold alkali-earth atoms and superfluid Boson-Fermion mixture

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# Outline

- Orbital Feshbach Resonance in Alkali-Earth Atoms.

  Ren Zhang, Yanting Cheng, Hui Zhai and PZ, arXiv: 1504.02864

- Calibration of Interaction Energy between Bose and Fermi Superfluids

  Ren Zhang, Wei Zhang, Hui Zhai and PZ, PRA, 90, 063614 (2014).

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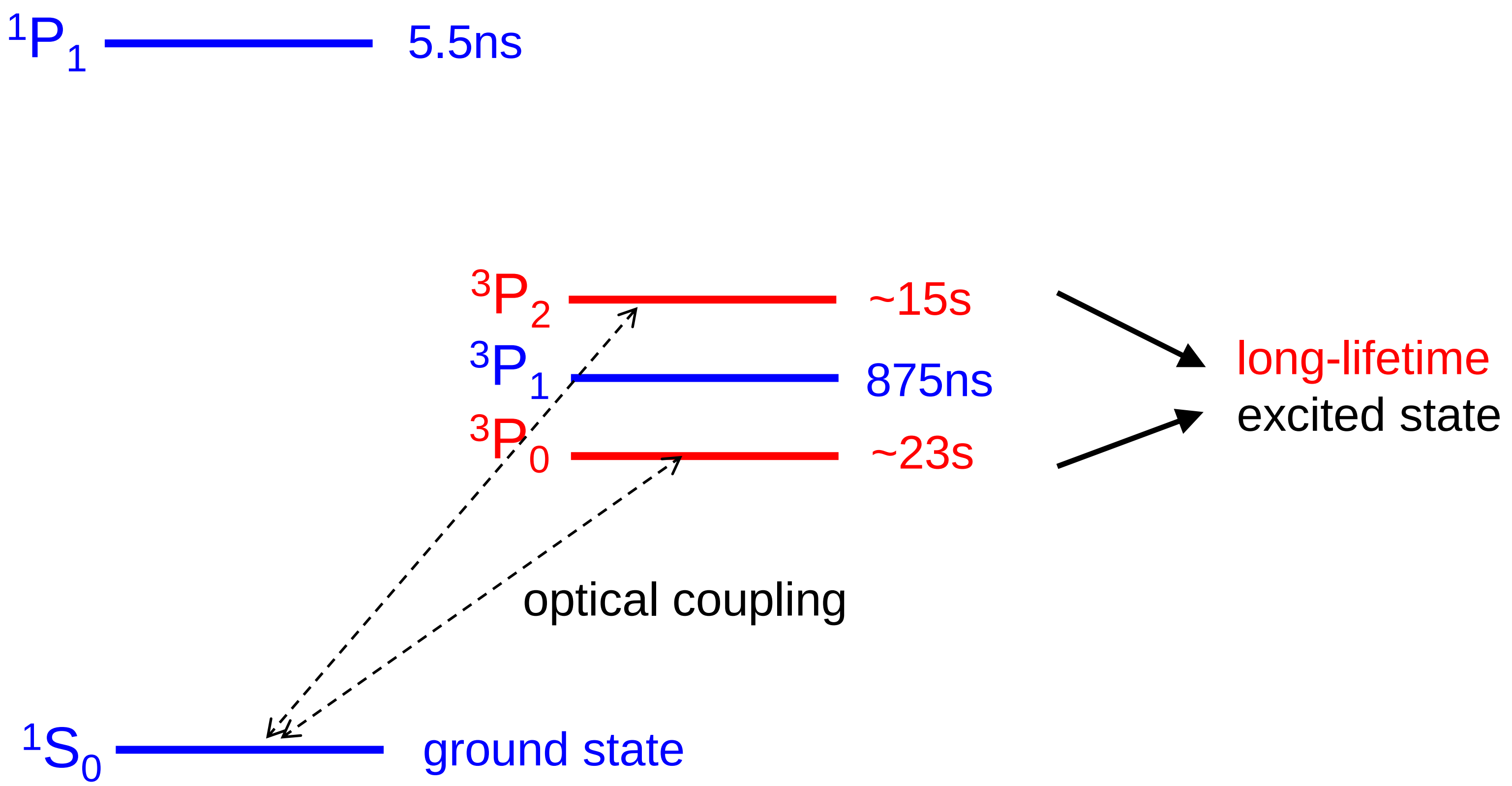
# alkali-earth (like) atoms: long-lifetime excited state

energy levels of Yb atom

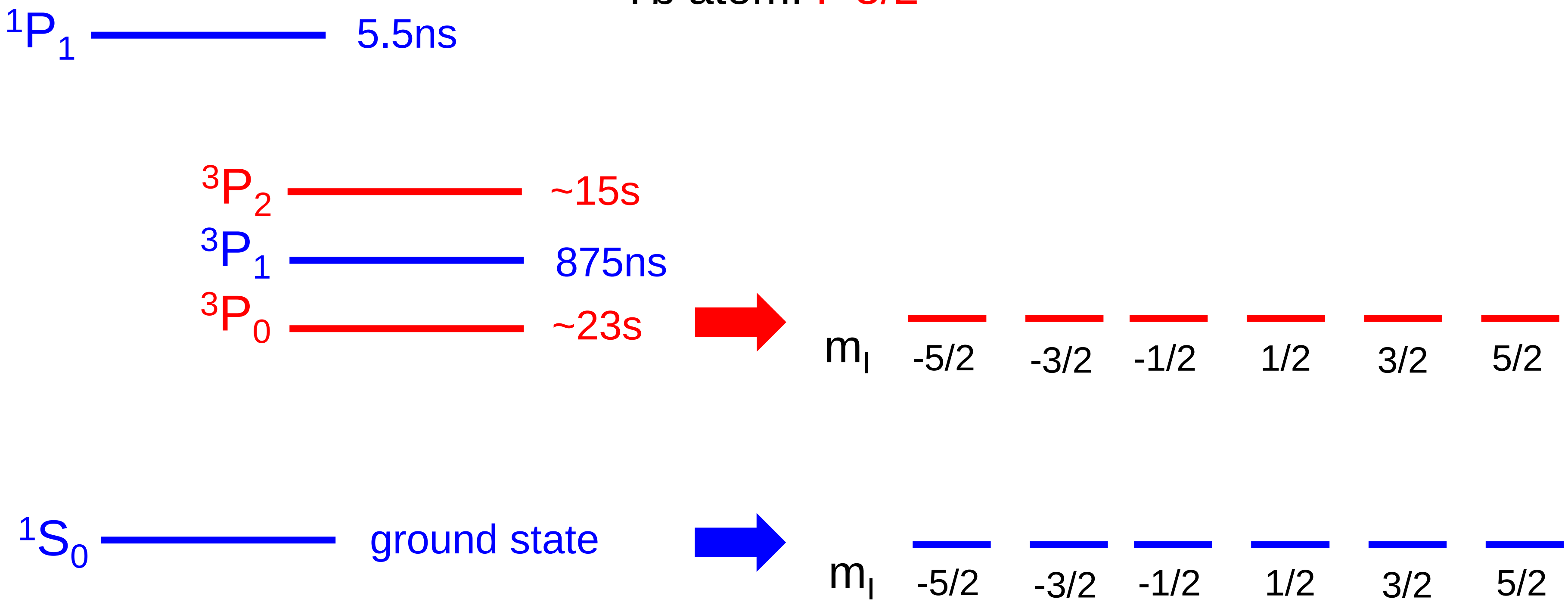



Yoshiro Takahashi's ppt

M. M. Boyd, T. Zelevinsky, A. D. Ludlow, S. Blatt, T. Zanon-Willette, S. M. Foreman, and J. Ye, PRA, **76**, 022510, (2007).

# alkali-earth (like) atoms: SU(N) symmetry of fermions

$^{173}$Yb atom: I=5/2




- interactions between atoms in 1S0 or 3P0 states are independent on $m_I$ : SU(N=6) symmetry ($^{87}$Sr : N=10, $^{171}$Yb: N=2)
- precision of SU(N): $10^{-9}$ for atoms in 1S0 and $10^{-3}$ for atoms in 3P0

X. Zhang, M. Bishof, S. L. Bromley, C. V. Kraus, M. S. Safronova, P. Zoller, A. M. Rey, J. Ye., Science, 345, 1467 (2014). (Sr87, with long supplementary material)

# Quantum degenerate gases of alkali-earth (like) atoms

2003: $^{174}Yb$

2006: $^{170}Yb$, $^{176}Yb$, $^{173}Yb$

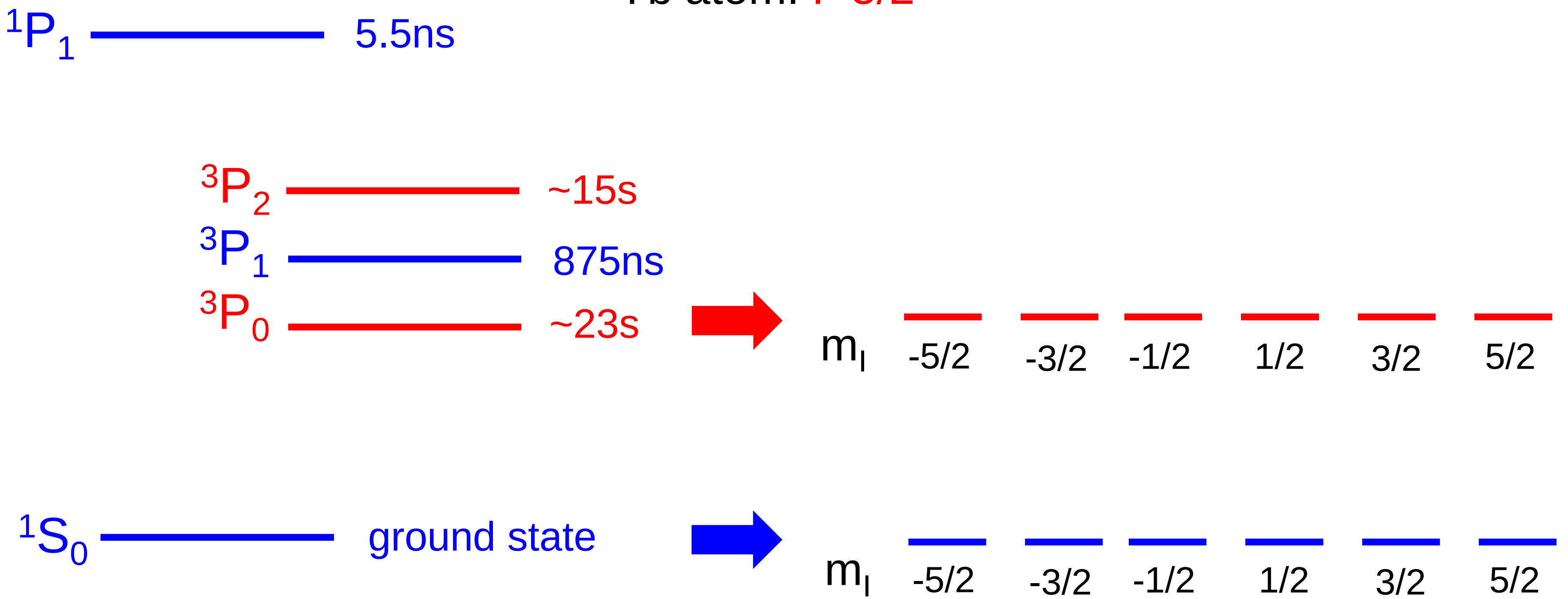
2009: $^{84}Sr$

2010: $^{86}Sr$, $^{88}Sr$, $^{87}Sr$, $^{171}Yb$-$^{173}Yb$ mixture


- Yoshiro Takahashi's ppt,
- S. Stellmer, F. Schreck, T. C. Killian, arXiv: 1307.0601 (in Annual Review of Cold Atoms and Molecules (World Scientific Publishing Co.), volume 2, chapter 1 (2014))
- S. Sugawa, Y. Takasu, K. Enomoto, and Y. Takahashi, Annual Review of Cold Atoms and Molecules (World Scientific Publishing Co., 2013), chap. Ultracold Ytterbium: Generation, Many-Body Physics, and Molecules.

# Our motivation

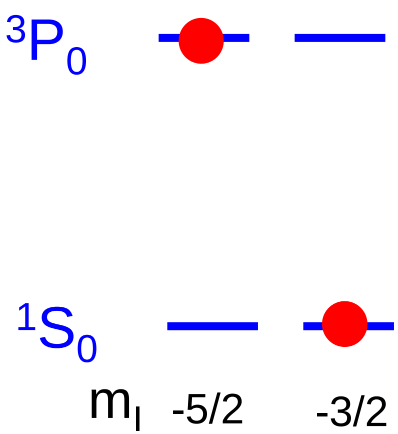
$^{173}$Yb atom: I=5/2



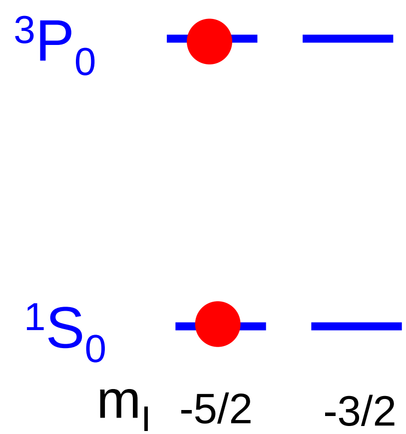

Our problem: how to control interaction between fermonic alkali-earth atoms in 1S0 or 3P0 states?

# Our result

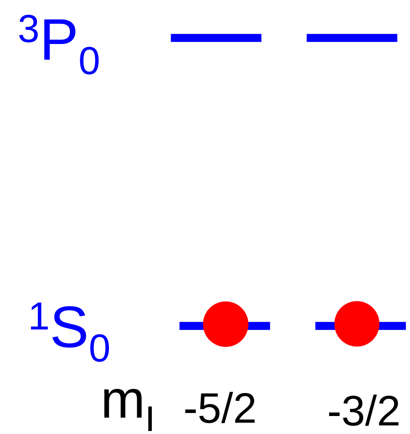
two atoms in different electronic orbital and different nuclear spin states




two atoms in different electronic orbital and same nuclear spin states




two atoms in same electronic orbital and different nuclear spin states



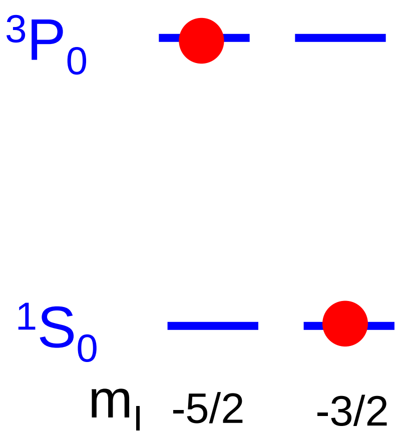

interaction can be controlled by B-field via orbital Feshbach resonance (~60G for Yb173)

so far no good approach

Ren Zhang, Yanting Cheng, Hui Zhai and PZ, arXiv: 1504.02864

# Our result

two atoms in different electronic orbital and different nuclear spin states




interaction can be controlled by B-field via orbital Feshbach resonance (~60G for Yb173)

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# Orbital FR

spin-exchange interaction + Zeeman effect → orbital Feshbach resonance

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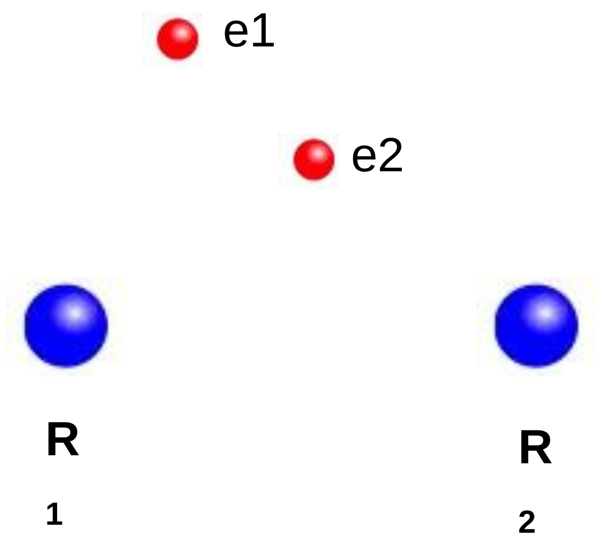
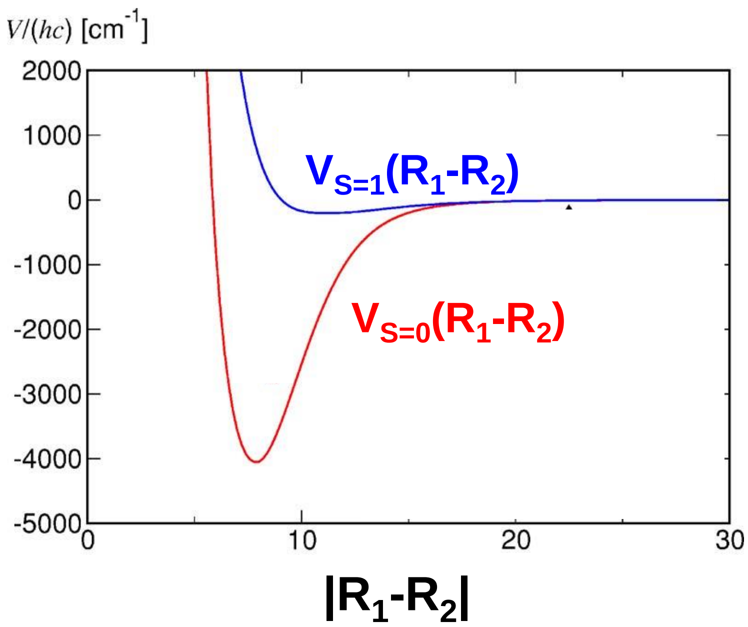
spin-exchange interaction + Zeeman effect ⇒ orbital Feshbach resonance

# Interaction between alkali atoms




**S: total spin of two electrons**



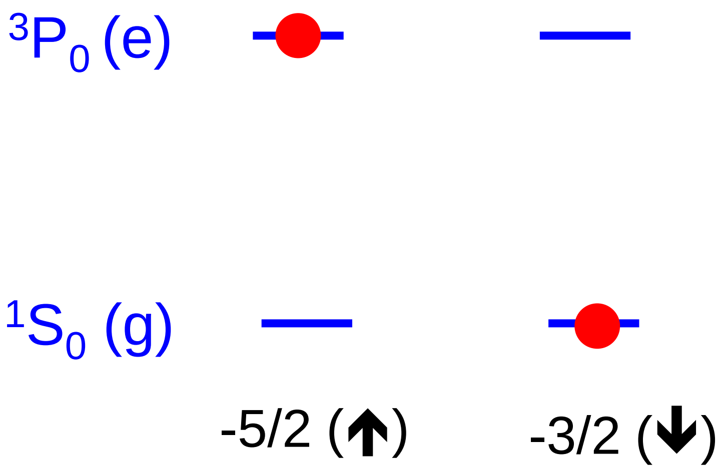

$$V = V_{S=1}(\mathbf{R}_1 - \mathbf{R}_2)\hat{P}_{S=1} + V_{S=0}(\mathbf{R}_1 - \mathbf{R}_2)\hat{P}_{S=0}$$

Two potential curves: necessary condition for FR

T. Köhler, K. Góral and P. S. Julienne, RMP, **78**, 1311 (2006).

# One 1S0 and one 3P0 atom with different nuclear spin

$^{173}$Yb atom: I=5/2

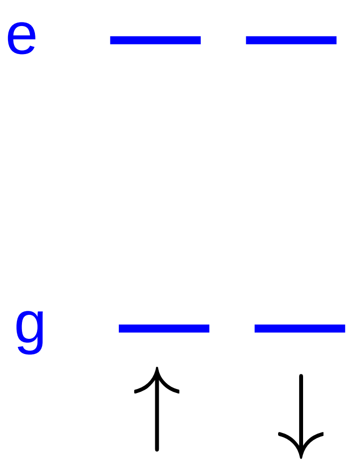



- total electronic spin is one: intuitive speaking, there is only one short-range interaction potential curves

- in fact, NO. There are two interaction potential curves

# consideration I: exchange symmetry in s-wave scattering

two atoms in different electronic orbital and different nuclear spin states




- initial internal state for s-wave scattering of identical fermionic atoms

$$|+\rangle \equiv (|ge\rangle + |eg\rangle)(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$|-\rangle \equiv (|ge\rangle - |eg\rangle)(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

- scattering processes with SU(N) symmetry (nuclear spin is not changed)

$|+\rangle \rightarrow |+\rangle$ scattering length: $a_{eg}^{(+)}$

$|-\rangle \rightarrow |-\rangle$ scattering length: $a_{eg}^{(-)}$

- experiments: $a_{eg}^{(+)} \neq a_{eg}^{(-)}$

$^{173}$Yb: $a_{eg}^{(-)}=3300a_0+i0.78a_0$ ; $a_{eg}^{(+)}=219.5a_0$

$^{87}$Sr: $a_{eg}^{(-)}=169a_0$ ; $a_{eg}^{(+)}=68a_0$

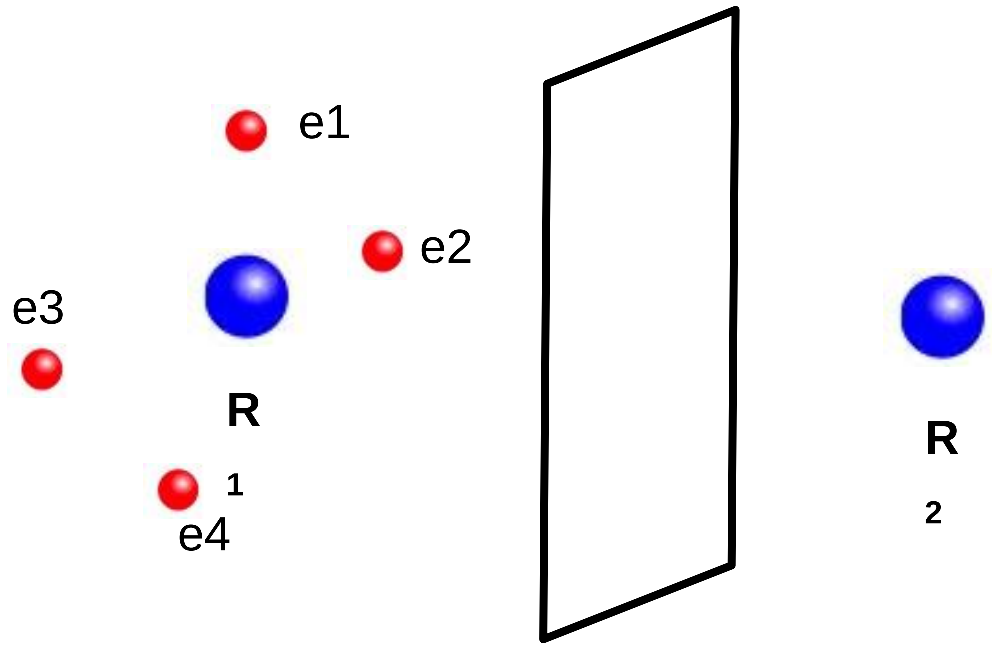
There are two different potential curves for |+> and |->.


- G. Cappellini, M. Mancini, G. Pagano, P. Lombardi, L. Livi, M. S. de Cumis, P. Cancio, M. Pizzocaro, D. Calonico, F. Levi, C. Sias, J. Catani, M. Inguscio, L. Fallani, Phys. Rev. Lett. 113, 120402 (2014) (Yb173).
- F. Scazza, C. Hofrichter, M. Höfer, P. C. De Groot, I. Bloch and S. Fölling, Nature Physics, 10, 779 (2014). (Yb173)
- X. Zhang, M. Bishof, S. L. Bromley, C. V. Kraus, M. S. Safronova, P. Zoller, A. M. Rey, J. Ye., Science, 345, 1467 (2014). (Sr87)

# consideration II: total parity of out-shell electrons

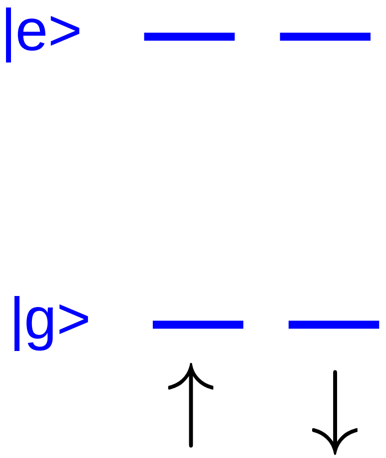
Schroedinger equation of four electrons

$$H_{e1e2e3e4}(\mathbf{R_1}, \mathbf{R_2})|\Psi\rangle_{e1e2e3e4} = V(\mathbf{R_1} - \mathbf{R_2})|\Psi\rangle_{e1e2e3e4}$$

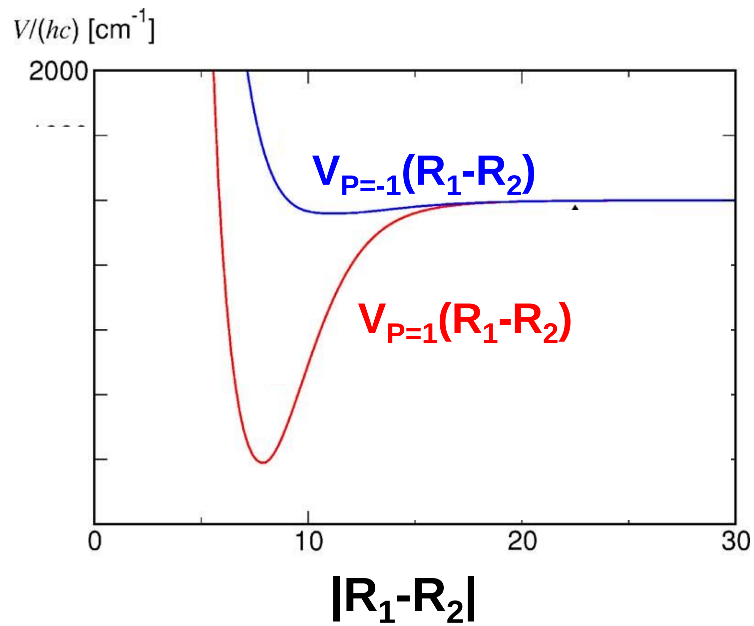



Total parity P of the spatial motion of all four electrons is conserved.

# consideration II: total parity of out-shell electrons




$$H_{e1e2e3e4}(\mathbf{R_1},\mathbf{R_2})|\Psi\rangle_{e1e2e3e4} = V(\mathbf{R_1}-\mathbf{R_2})|\Psi\rangle_{e1e2e3e4}$$




|R1-R2|➔∞ $|\Psi\rangle_{e1e2e3e4} \rightarrow \begin{cases} (|ge\rangle + |eg\rangle)/\sqrt{2} & \text{P=1} \\ (|ge\rangle - |eg\rangle)/\sqrt{2} & \text{P=-1} \end{cases}$

Two different potential curves for $|\pm\rangle \propto (|ge\rangle \pm |eg\rangle)$

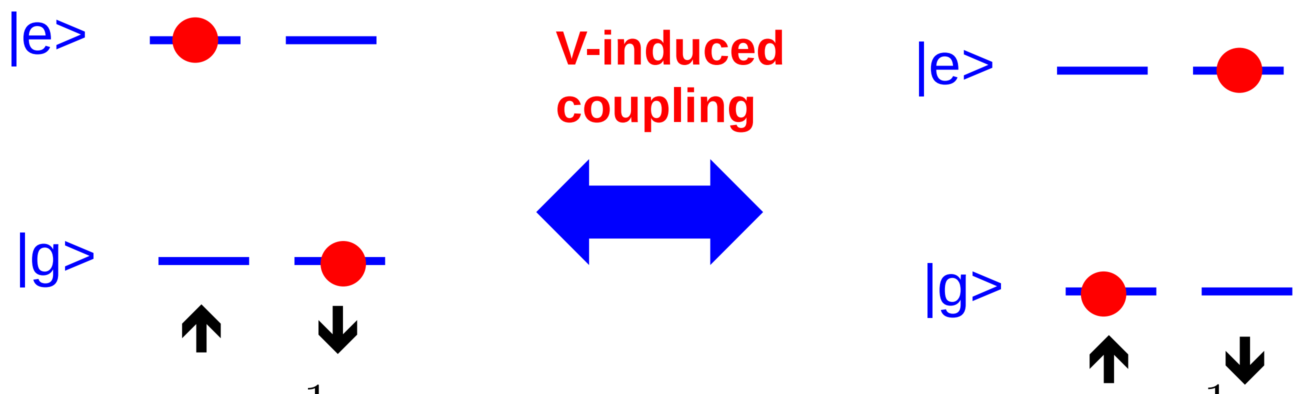
# Spin-exchange interaction

- **ineraction potential**

$$V(R) = V_{P=1}(R)|+\rangle\langle+| + V_{\mathrm{P=0}}(R)|-\rangle\langle-|$$
$$= V_{\mathrm{oo}}(R)|o\rangle\langle o| + V_{\mathrm{cc}}(R)|c\rangle\langle c| + V_{\mathrm{oc}}|o\rangle\langle c| + V_{\mathrm{co}}|c\rangle\langle o|$$

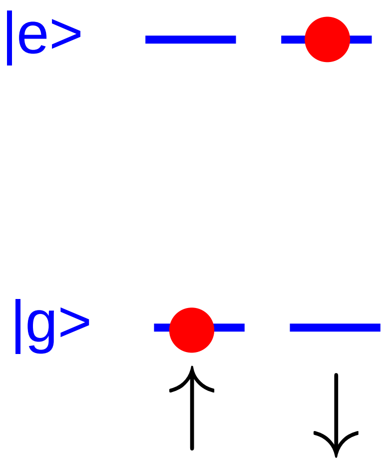
$$V_{oo}(\mathbf{R}) = V_{cc}(\mathbf{R}) = \frac{1}{2}[V_{P=1}(\mathbf{R}) + V_{P=-1}(\mathbf{R})]$$
$$V_{oc}(\mathbf{R}) = V_{co}(\mathbf{R}) = \frac{1}{2}[V_{P=-1}(\mathbf{R}) - V_{P=1}(\mathbf{R})]$$

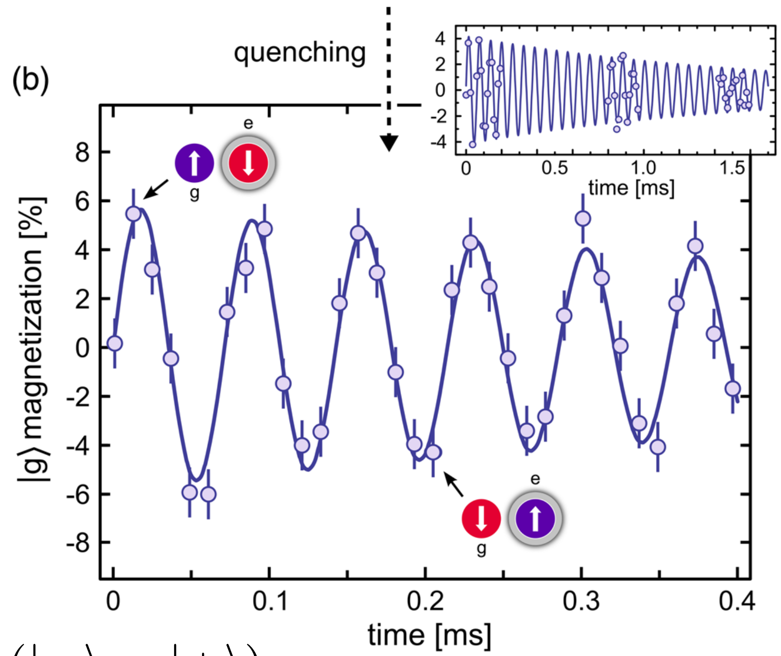
- **convenient basis**

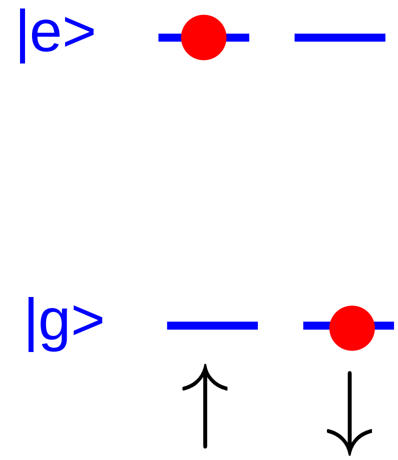



$$|o\rangle = |g\downarrow; e\uparrow\rangle = \frac{1}{\sqrt{2}}(|-\rangle - |+\rangle)$$
$$|c\rangle = |g\uparrow; e\downarrow\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$

# Spin-exchange interaction

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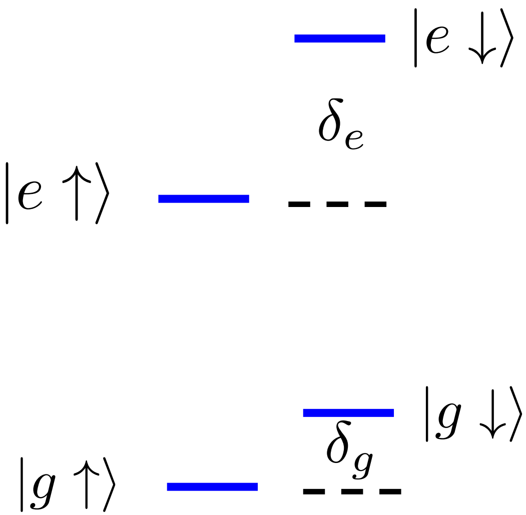
# Orbital FR

spin-exchange interaction + Zeeman effect → orbital Feshbach resonance

- interaction $V(R) = V_{P=1}(R)|+\rangle\langle+| + V_{\mathrm{P}=0}(R)|-\rangle\langle-|$

# Zeeman effect

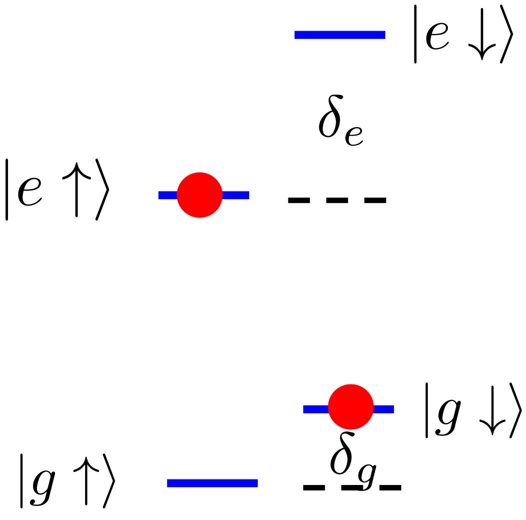


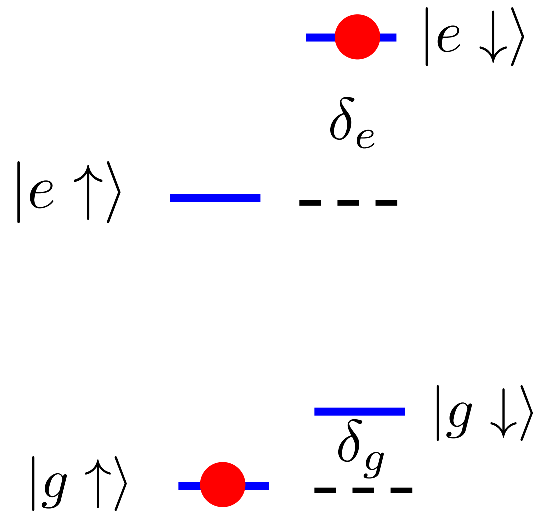

$$\delta = \delta_e - \delta_g = \Delta\mu B$$

$^{173}$Yb: $\Delta\mu \sim (2\pi)112\mathrm{Hz/Gauss}$

M. M. Boyd, T. Zelevinsky, A. D. Ludlow, S. Blatt, T. Zanon-Willette, S. M. Foreman, and J. Ye, PRA, **76**, 022510, (2007).

# Free Hamiltonian for two-atom relative motion




$$|o\rangle = |g\downarrow; e\uparrow\rangle = \frac{1}{\sqrt{2}}(|-\rangle - |+\rangle)$$

$$|c\rangle = |g\uparrow; e\downarrow\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$

## Free Hamiltonian

$$H_0 = -\nabla^2 + \delta|c\rangle\langle c|$$

$$\delta = \delta_e - \delta_g = \Delta\mu B$$

# Orbital FR

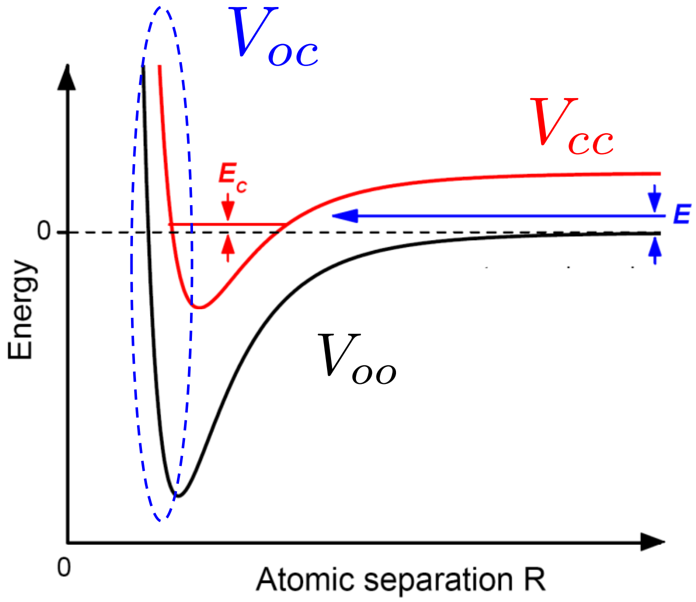
spin-exchange interaction + Zeeman effect ⇒ orbital Feshbach resonance

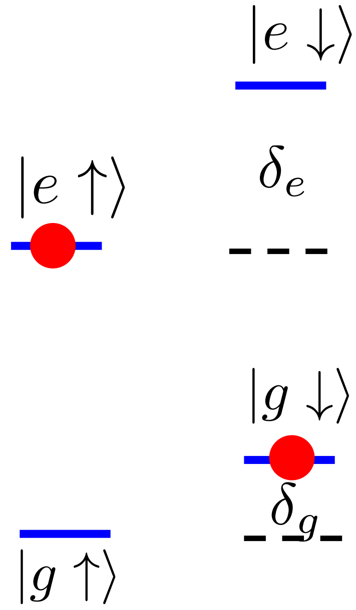
- interaction $V(R) = V_{P=1}(R)|+\rangle\langle+| + V_{\mathrm{P}=0}(R)|-\rangle\langle-|$
- free Hamitltonian $H_0 = -\nabla^2 + \delta|c\rangle\langle c|$

# Orbital Feshbach resonance

Total two-atom Hamiltonian

$$H = H_0 + V = -\nabla^2 + \delta|c\rangle\langle c| + \sum_{i,j=o,c} V_{ij}(\mathbf{R})|i\rangle\langle j|$$



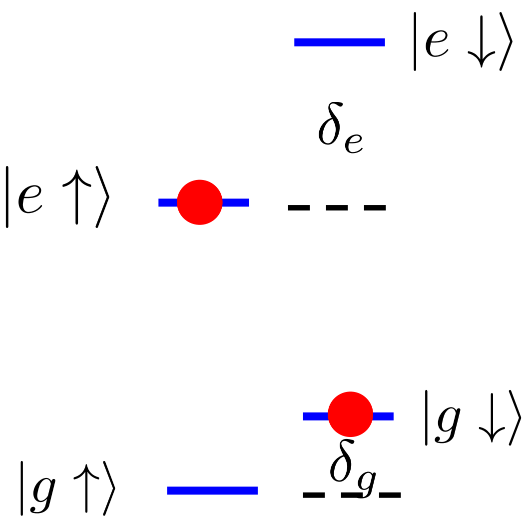




Incident channel: |o>

Orbital Feshbach resonance: scattering length of channel |o> can be controlled by B

# Physical realization

**Yb173 atoms**




Incident channel: |o>

$$\delta = \delta_e - \delta_g$$

- **Free Hamiltonian**

$$\hat{H}_0 = \left(-\frac{\hbar^2\nabla^2}{m} + \delta\right)|c\rangle\langle c| - \frac{\hbar^2\nabla^2}{m}|o\rangle\langle o|$$

$$\delta = \Delta\mu B \;\; \Delta\mu \sim (2\pi)112\text{Hz}/\text{Gauss}$$

- **Interaction (|+>, |-> basis)**

$$\hat{V} = \frac{4\pi\hbar^2}{m}\left[a_s^+|+\rangle\langle+| + a_s^-|-\rangle\langle-|\right]\delta(\mathbf{r})\frac{\partial}{\partial r}(r\cdot)$$

$$a_s^- = 3300a_0 + i0.78a_0$$

$$a_s^+ = 219.5a_0$$

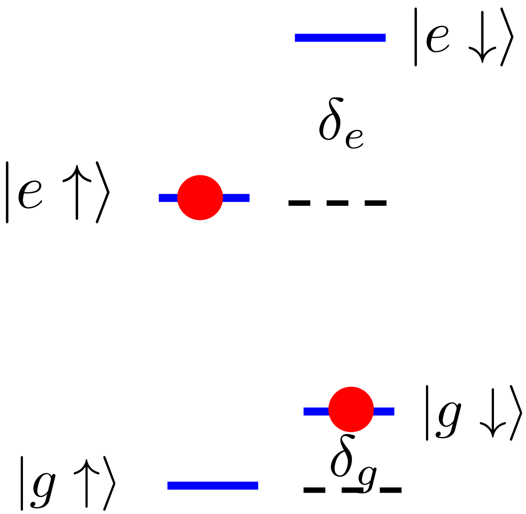
two-atom loss

Scattering length: F. Scazza, C. Hofrichter, M. Höfer, P. C. De Groot, I. Bloch and S. Fölling, Nature Physics, 10, 779 (2014). (Yb173)

$\delta \ll E_{vdW}$ ($\sim 10^6$Hz): Huang-Yang potential is applicable.

# Physical realization

**Yb173 atoms**




Incident channel: |o>

- **Free Hamiltonian**

$$\hat{H}_0 = \left(-\frac{\hbar^2\nabla^2}{m} + \delta\right)|c\rangle\langle c| - \frac{\hbar^2\nabla^2}{m}|o\rangle\langle o|$$

$$\delta = \Delta\mu B \ \ \Delta\mu \sim (2\pi)112\mathrm{Hz}/\mathrm{Gauss}$$

- **Interaction (|c>, |o> basis)**

$$\hat{V} = \frac{4\pi\hbar^2}{m} a_{\mathrm{s}0}\delta(\mathbf{r})\frac{\partial}{\partial r}(r\cdot)\left[|o\rangle\langle o| + |c\rangle\langle c|\right] + \frac{4\pi\hbar^2}{m} a_{\mathrm{s}1}\delta(\mathbf{r})\frac{\partial}{\partial r}(r\cdot)\left[|o\rangle\langle c| + |c\rangle\langle o|\right]$$

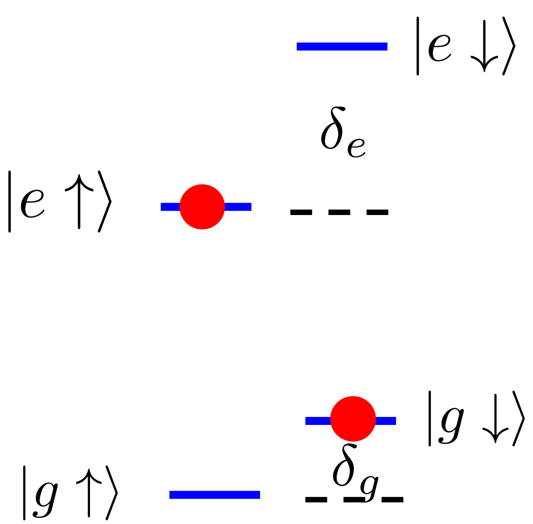
$$a_{\mathrm{s}0,1} = (a_{\mathrm{s}}^{-} \pm a_{\mathrm{s}}^{+})/2$$

- **Binding energy of bound-state in |c>:**

$$\hbar^2/(m a_{\mathrm{s}0}^2) \sim 10^4\mathrm{Hz}$$

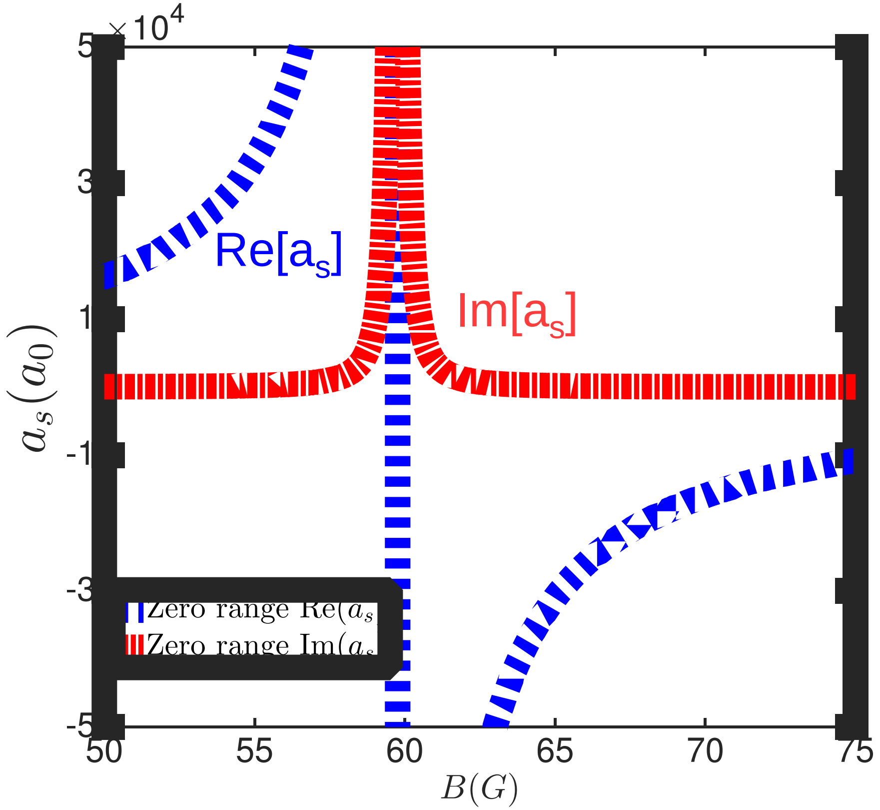
# Physical realization

**Yb173 atoms**




Incident channel: |o>

$$\delta = \delta_e - \delta_g$$



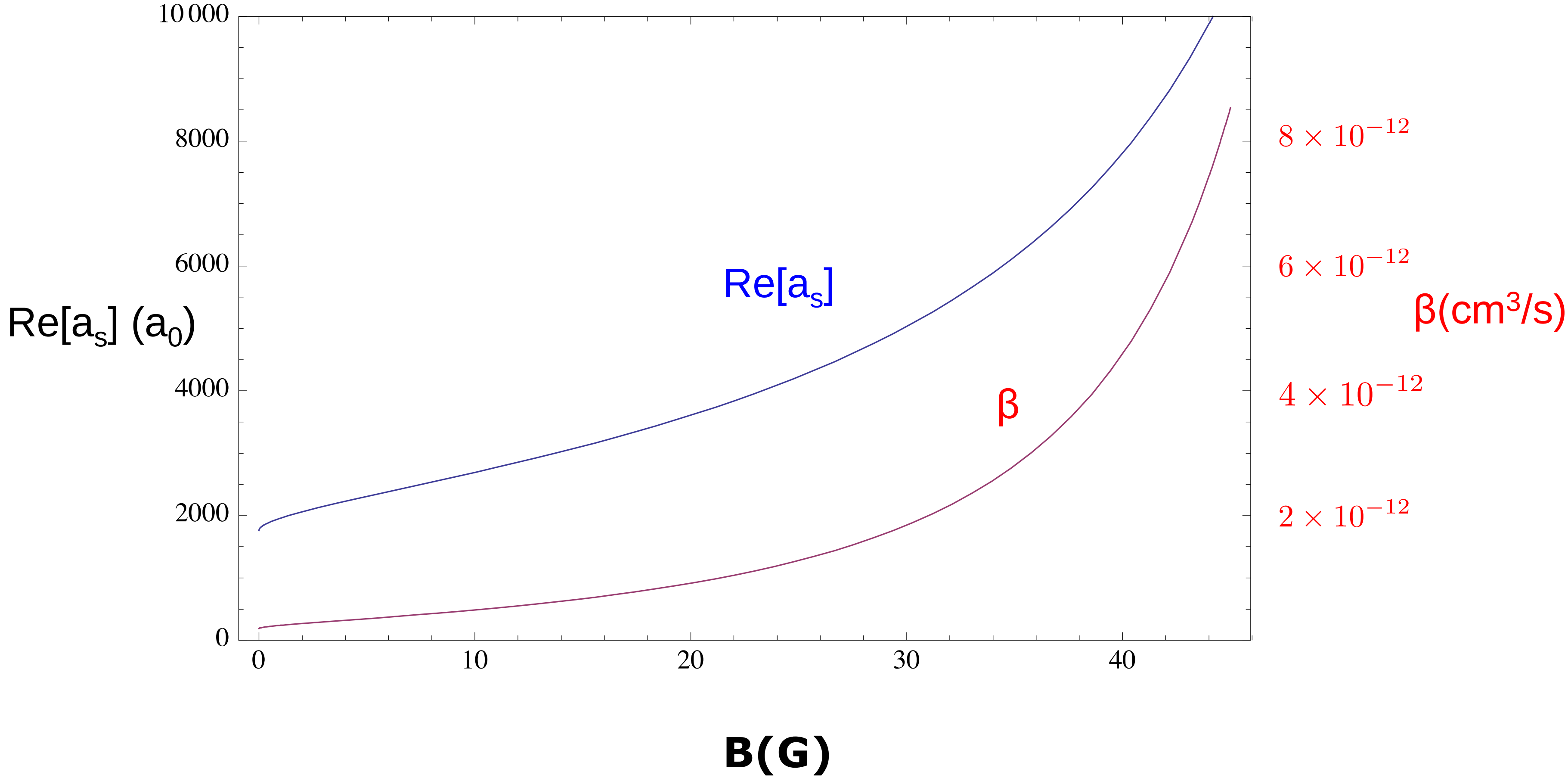

**2-body loss:**

**$dn/dt=-\beta n^2$; $\beta=8\pi \mathrm{Im}[a_s]$**

Orbital Feshbach resonance: $\delta \sim 10^4$Hz<<$E_{vdW}$

# Physical realization

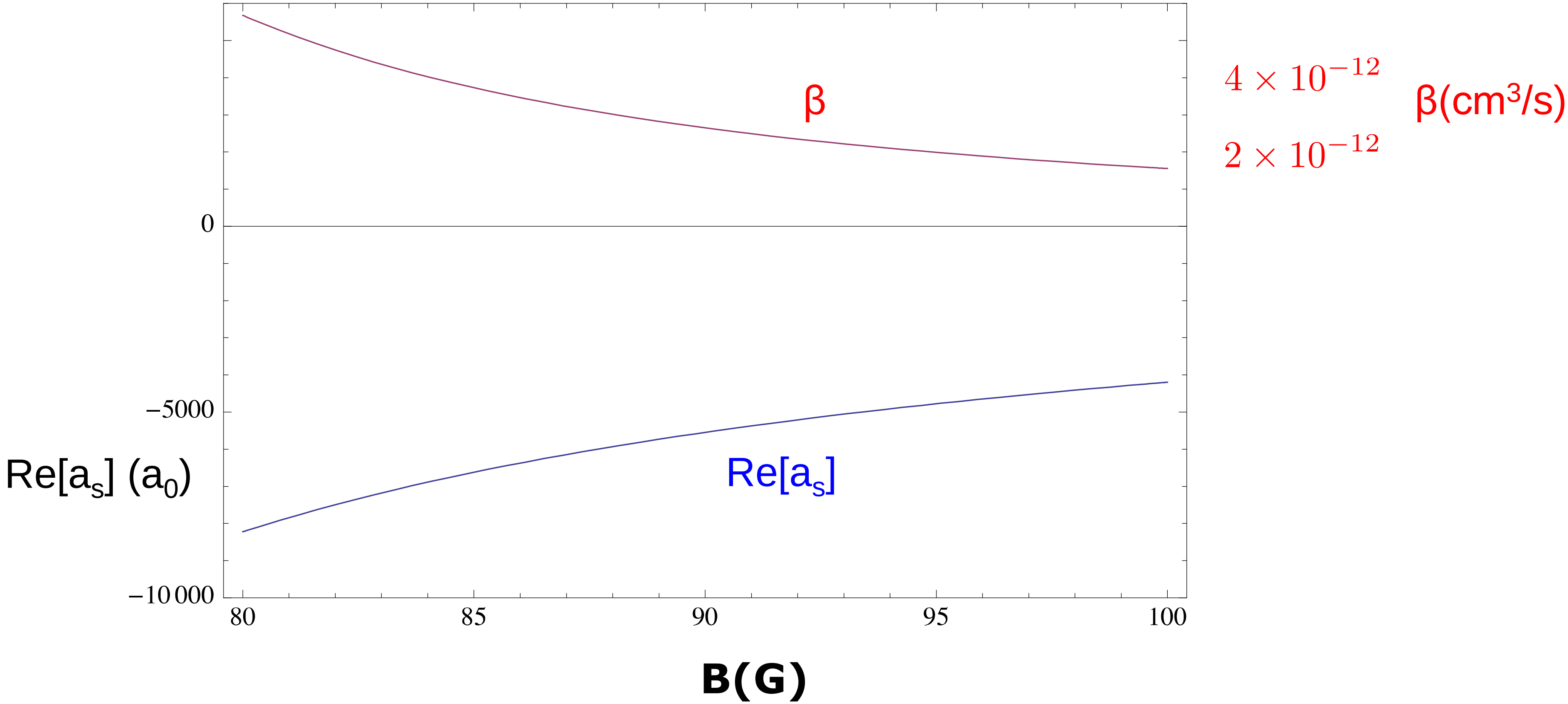
## $Re[a_s]$ and two-body loss rate β of Yb173 atoms




Lifetime~1ms-10ms for density $10^{14}/cm^3$

# Physical realization

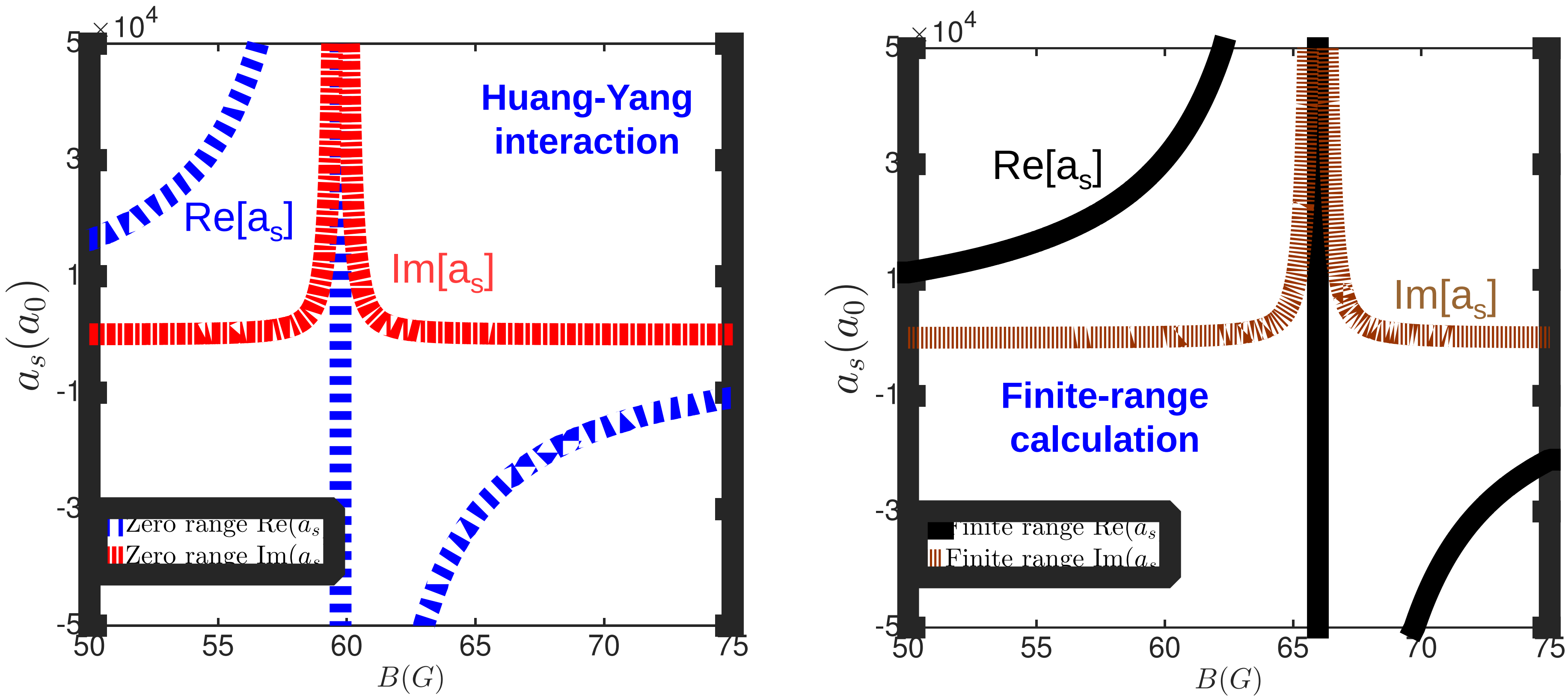
## $Re[a_s]$ and two-body loss rate $\beta$ of Yb173 atoms




Lifetime~1ms-10ms for density $10^{14}/cm^3$

# Zero-range V.S. finite-range

## Yb173 atoms



# Summary

- Orbital Feshbach resonance can occur between one 1S0 and one 3P0 alkali-earth (like) atoms with different nuclear spin.

  - **Two alkali atoms:**

    $$V = V_{S=1}(\mathbf{R}_1 - \mathbf{R}_2)\hat{P}_{S=1} + V_{S=0}(\mathbf{R}_1 - \mathbf{R}_2)\hat{P}_{S=0}$$

  - **Two alkali-earth (like) atoms:**

    $$V(\mathbf{R}_1 - \mathbf{R}_2) = V_{P=1}(\mathbf{R}_1 - \mathbf{R}_2)|+\rangle\langle+| + V_{P=-1}(\mathbf{R}_1 - \mathbf{R}_2)|-\rangle\langle-|$$

    In orbital FR, the electronic orbital degree of freedom plays the same role as the electronic spin in magnetic FR.

- May occur for Yb173 when B~60G.

# Outline

- Orbital Feshbach Resonance in Alkali-Earth Atoms.

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- Calibration of Interaction Energy between Bose and Fermi Superfluids

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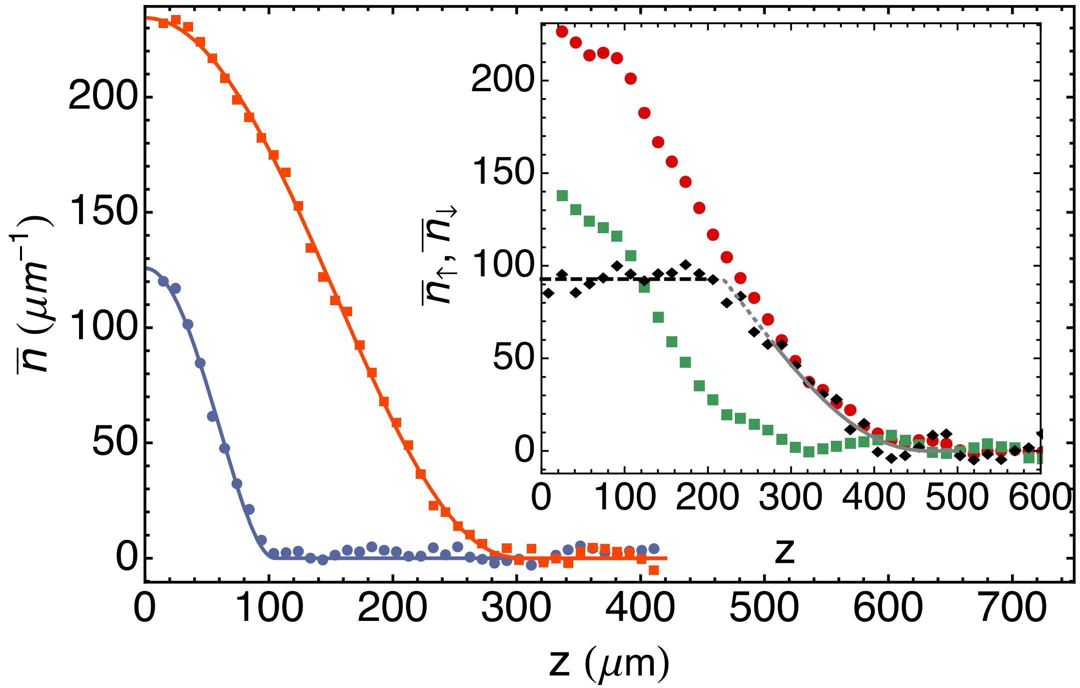
# Mixture of Bose and Fermi Superfluids

Li6 (2-component )+ Li7 (1-component )

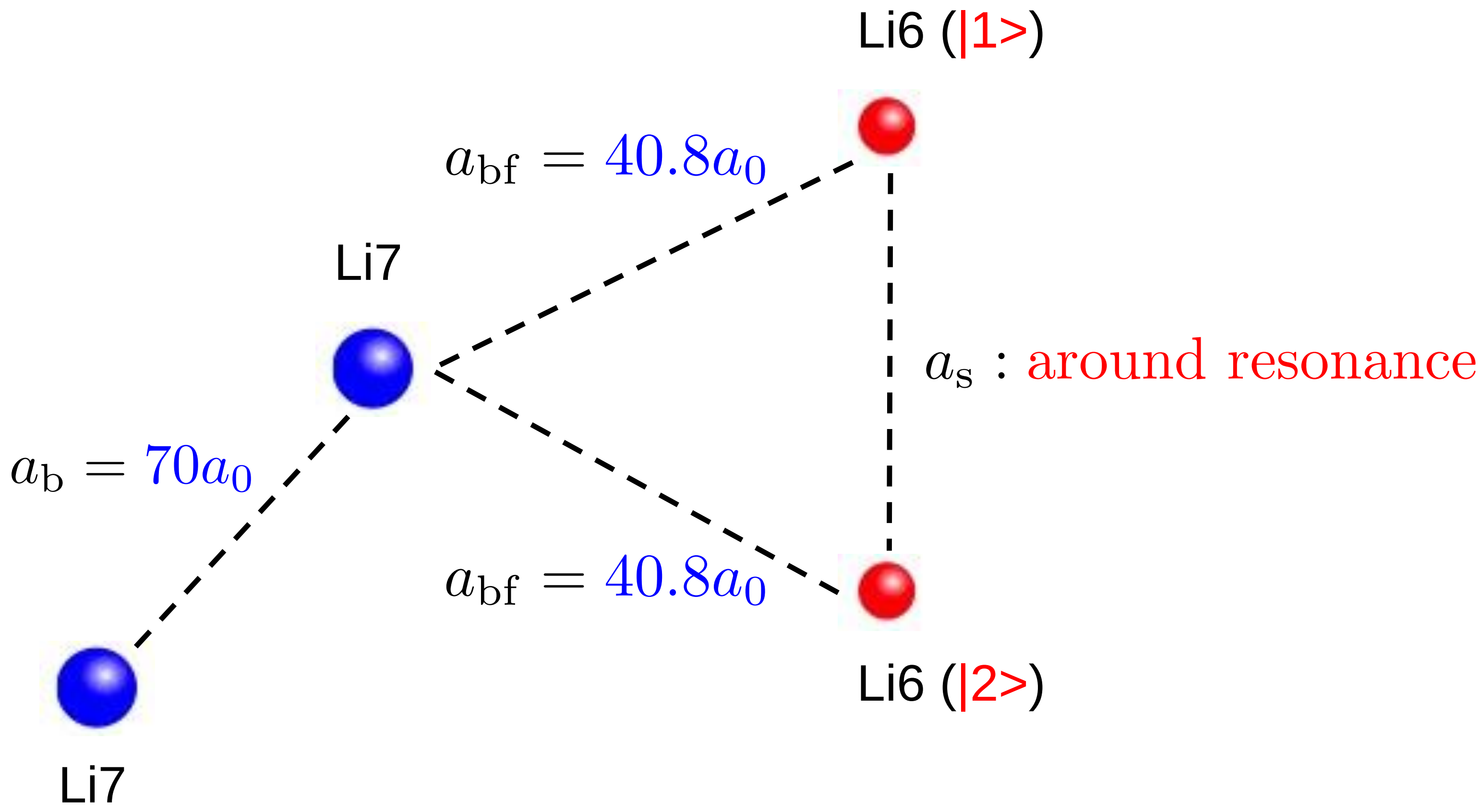



atom number

- Li6: $3.5 \times 10^5$
- Li7: $4 \times 10^4$

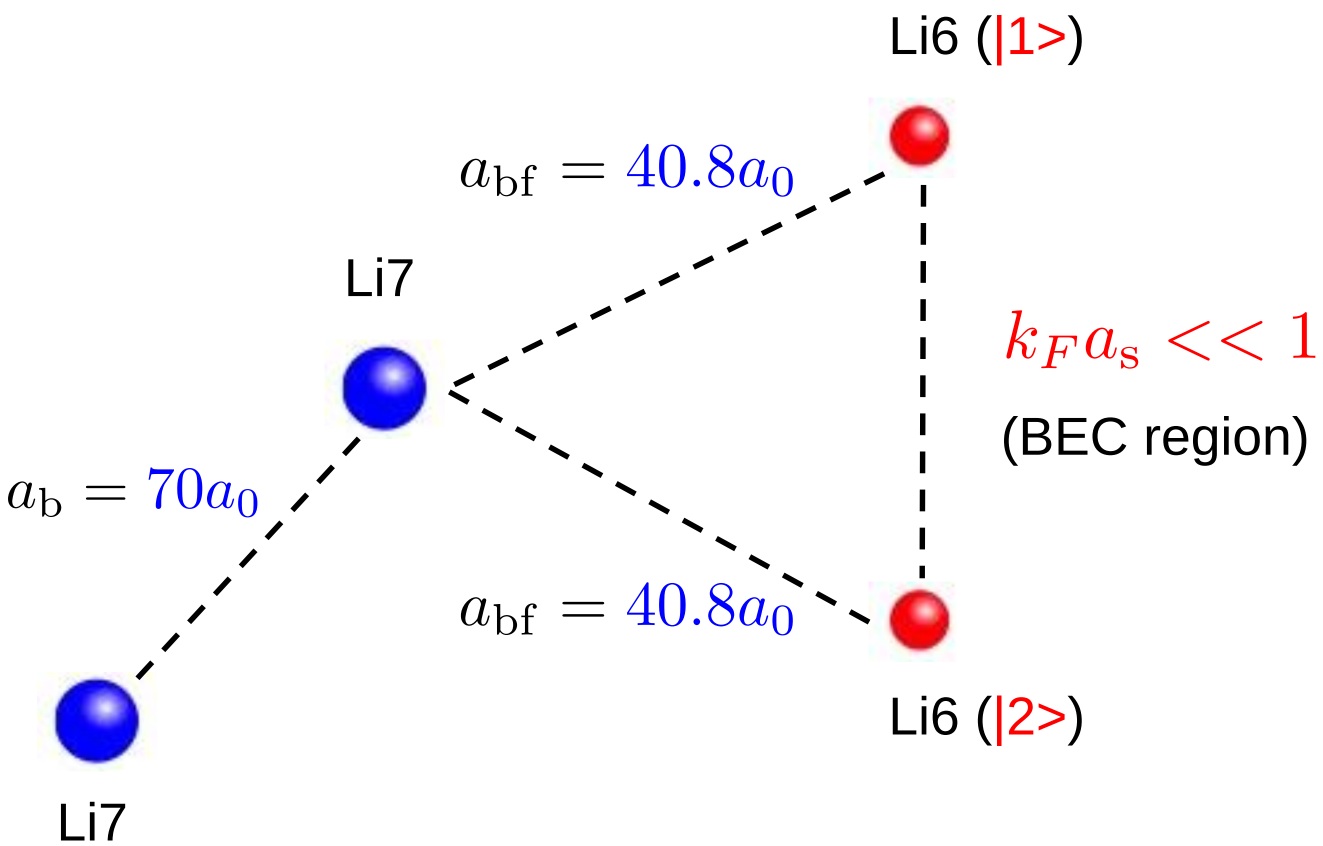

I. Ferrier-Barbut, M. Delehaye, S. Laurent, A. T. Grier, M. Pierce, B. S. Rem, F. Chevy, C. Salomon, Science, **345**, 1035 (2014).

# Mixture of Bose and Fermi Superfluids




I. Ferrier-Barbut, M. Delehaye, S. Laurent, A. T. Grier, M. Pierce, B. S. Rem, F. Chevy, C. Salomon, Science, **345**, 1035 (2014).

# BEC region of Fermi Superfluid




How large is Boson-Fermion interaction energy?

# Calculation I: simple mean-field (MF) theory

- interaction energy

$$V = \frac{2\pi\hbar^2 a_{\rm bf}}{m_{\rm bf}} \sum_{\sigma=1,2} \int d\mathbf{r} \hat{b}^\dagger(\mathbf{r})\hat{b}(\mathbf{r})\hat{c}^\dagger_\sigma(\mathbf{r})\hat{c}_\sigma(\mathbf{r})$$

field operators of bosonic atoms

field operators of fermonic atoms (with internal state |σ>)

- mean-field approximation

$$\sum_\sigma \langle \hat{b}^\dagger(\mathbf{r})\hat{b}(\mathbf{r})\hat{c}^\dagger_\sigma(\mathbf{r})\hat{c}_\sigma(\mathbf{r})\rangle \approx \sum_\sigma \langle \hat{b}^\dagger(\mathbf{r})\hat{b}(\mathbf{r})\rangle\langle\hat{c}^\dagger_\sigma(\mathbf{r})\hat{c}_\sigma(\mathbf{r})\rangle = n_{\rm b} n_{\rm f}$$

- Interaction energy density given by simple MF

$$E_{\rm MF} = \frac{4\pi\hbar^2 a_{\rm bf}}{m_{\rm bf}} n_{\rm b} n_{\rm f}$$

# Calculation II: weakly-interacting Bose gas

- interaction energy density given by weakly-interacting Bose gas theory

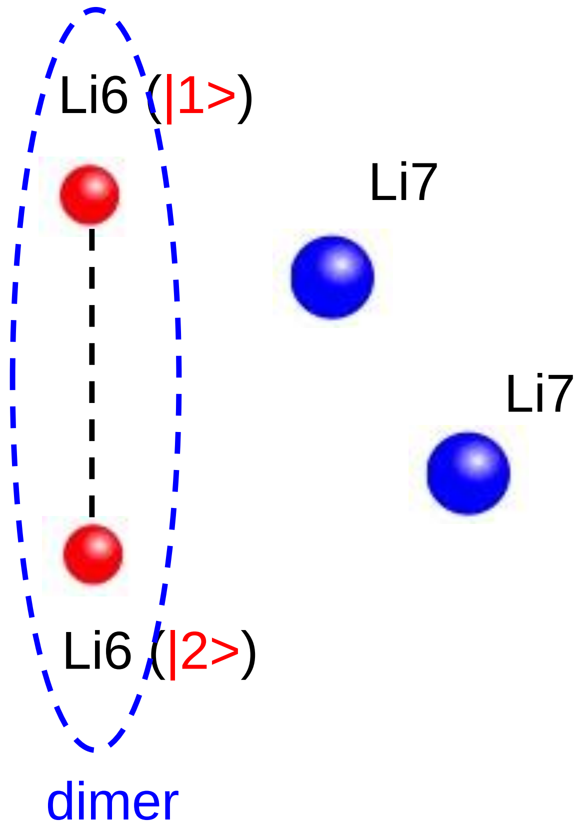
$$E_{\rm IB} = \frac{2\pi\hbar^2 a_{\rm ad}}{m_{\rm ad}} n_{\rm b} n_{\rm d}$$

$a_{\rm ad}$ : atom-dimer scattering length

$n_{\rm d} = 2n_{\rm f}$

$m_{\rm ad}$ : atom-dimer reduced mass

$k_F a_{\rm s} << 1$



# Comparison

- Interaction energy density given by simple MF

$$E_{\mathrm{MF}} = \frac{4\pi\hbar^2 a_{\mathrm{bf}}}{m_{\mathrm{bf}}} n_{\mathrm{b}} n_{\mathrm{f}}$$

- interaction energy density given by weakly-interacting Bose gas theory

$$E_{\mathrm{IB}} = \frac{2\pi\hbar^2 a_{\mathrm{ad}}}{m_{\mathrm{ad}}} n_{\mathrm{b}} n_{\mathrm{d}} \qquad n_{\mathrm{d}} = 2n_{\mathrm{f}}$$

- $E_{MF}$=$E_{IB}$ only when (Li6/Li7 system)

$$a_{\mathrm{ad}} = \frac{2m_{\mathrm{ad}}}{m_{\mathrm{bf}}} a_{\mathrm{bf}} \approx 2.74 a_{\mathrm{bf}}$$

atom-dimer scattering length given by mean-field theory

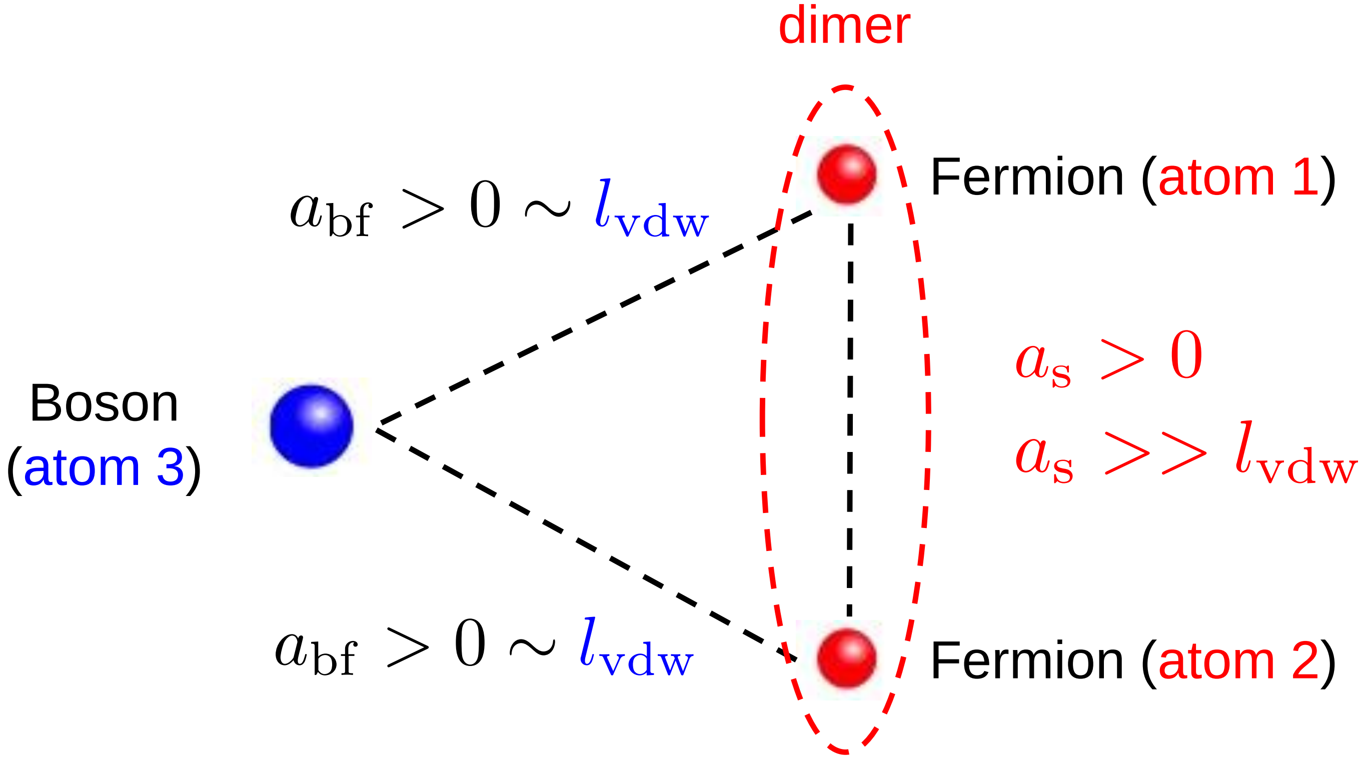
# $a_{ad}$ and MF

- Error of simple mean-field approximation is determined by

$$a_{\mathrm{ad}} - 2.74 a_{\mathrm{bf}}$$

- We calculate the exact value of $a_{ad}$
    - the accurate boson-fermion interaction energy
    - the validity of the simple MF treatment

# System




- 2-body interaction (Huang-Yang potential)

$$V_{ij} = \frac{2\pi\hbar^2 a_{ij}}{m_{ij}} \delta(\mathbf{r}_{ij}) \frac{\partial}{\partial r_{ij}} (r_{ij} \cdot)$$

$$a_{12} = a_{\rm s} \quad a_{23} = a_{31} = a_{\rm bf}$$

$a_{bf}$ <0 or $a_{bf}$ >>$l_{vdw}$ : Xiaoling Cui, arXiv: 1406.1242

# STM equation for our system

$$\frac{\frac{M+2}{M}a_{ad}(K)}{\frac{1}{a_s}+\sqrt{\frac{M+2}{4M}K^2+1/a_s^2-i\varepsilon}}+\frac{2}{\pi^2}\int^{\Lambda}\frac{K'dK'}{K}\log\left[\frac{\frac{M+1}{2M}K^2+K'^2+KK'+1/a_s^2-i\varepsilon}{\frac{M+1}{2M}K^2+K'^2-KK'+1/a_s^2-i\varepsilon}\right]\zeta(K')=0$$

$$\left[\frac{1}{a_{bf}}-\frac{\sqrt{2M(M+1)}}{M+1}\sqrt{\frac{M+2}{2(M+1)}K^2+1/a_s^2-i\varepsilon}\right]\zeta(K)$$

$$-\frac{2(M+1)}{M}\int^{\Lambda}\frac{K'dK'}{K(K'^2-\frac{4M}{M+2}i\varepsilon)}\log\left[\frac{K^2+\frac{M+1}{2M}K'^2+KK'+1/a_s^2-i\varepsilon}{K^2+\frac{M+1}{2M}K'^2-KK'+1/a_s^2-i\varepsilon}\right]a_{ad}(K')$$

$$+\frac{M+1}{2\pi}\int^{\Lambda}\frac{K'dK'}{K}\log\left[\frac{\frac{M+1}{2M}(K^2+K'^2)+\frac{1}{M}KK'+1/a_s^2-i\varepsilon}{\frac{M+1}{2M}(K^2+K'^2)-\frac{1}{M}KK'+1/a_s^2-i\varepsilon}\right]\zeta(K')$$

$$=\frac{2\pi(M+1)}{M}\left[\frac{i\varepsilon}{(K^2+1/a_s^2-i\varepsilon)(K^2+1/a_s^2)}-\frac{1}{K^2+1/a_s^2-i\varepsilon}\right]$$

$\varepsilon\to 0^+$

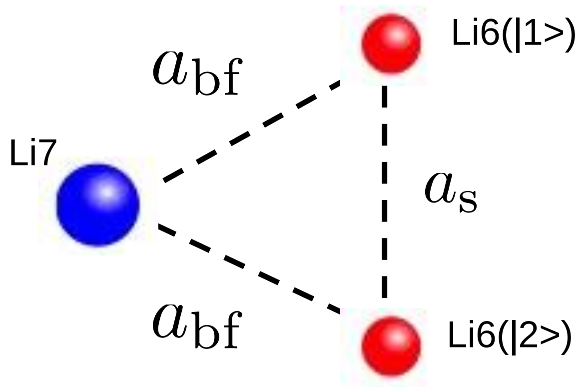
$M=m_b/m_f$

$\Lambda$: momentum cutoff, or the boundary condition for region where all 3 atoms are close. $|\Lambda|\in(\frac{2}{a_{\text{bf}}},\frac{8}{a_{\text{bf}}})\quad \arg\Lambda\in(0,0.08)$

G.V. Skorniakov, K.A. Ter-Martirosian, Sov. Phys. JETP 4, 648 (1957) .

P. Naidon, and M. Ueda, Comptes Rendus Physique, 12, 13 (2011).

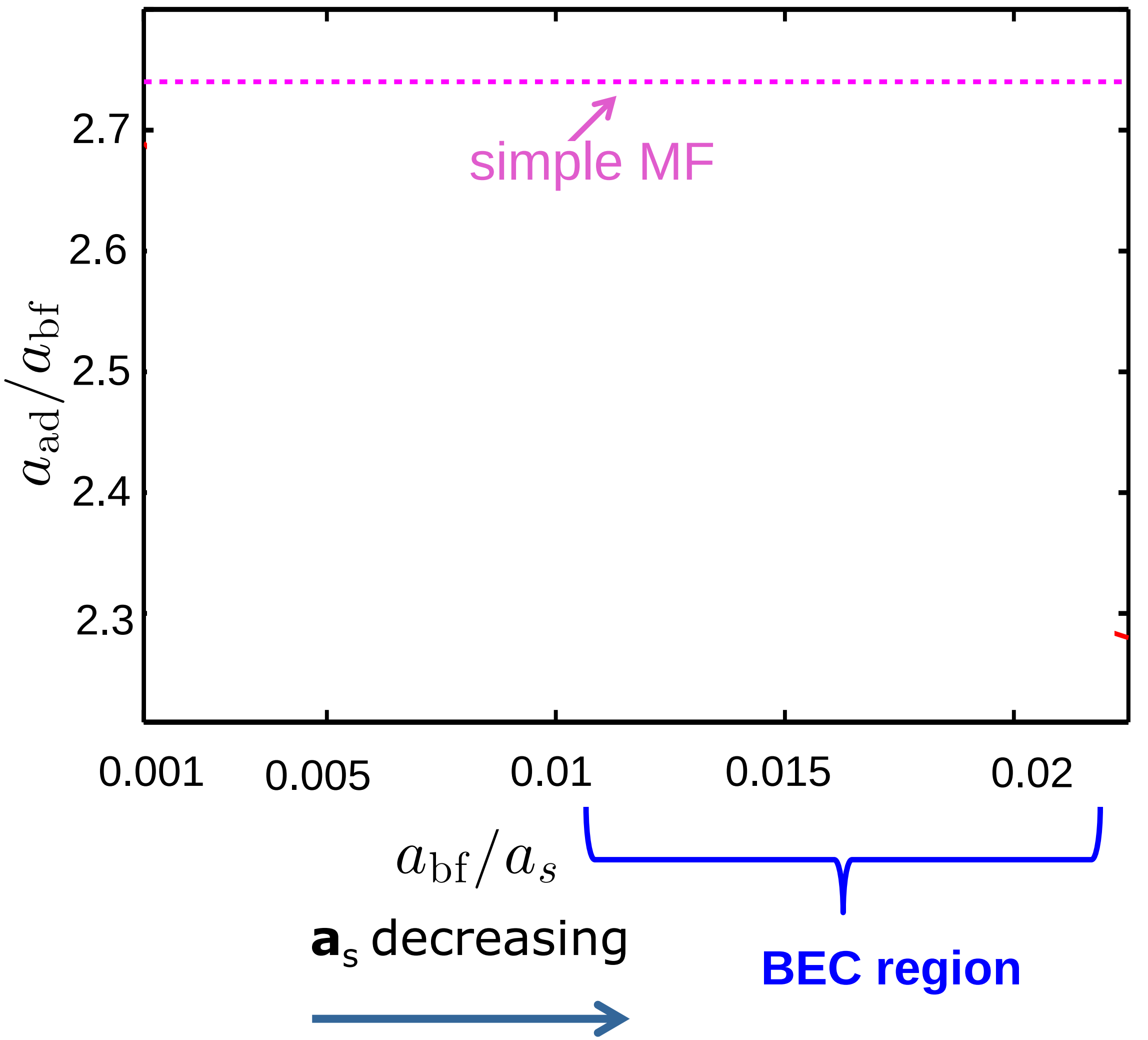
# $a_{ad}$ for Li6/Li7



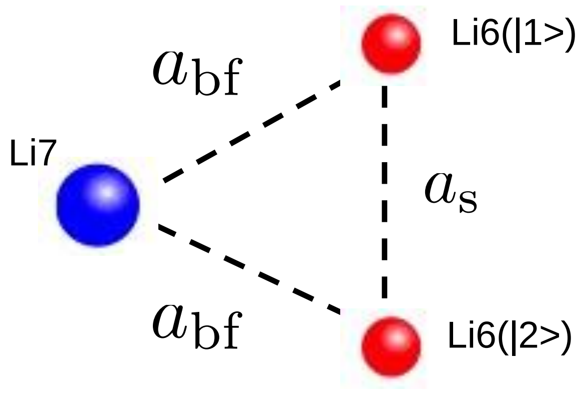

- simple MF:

$$a_{\rm ad} \approx 2.74 a_{\rm bf}$$

- Fix Λ, $a_{bf}$ , change $a_s$



# $a_{ad}$ for Li6/Li7



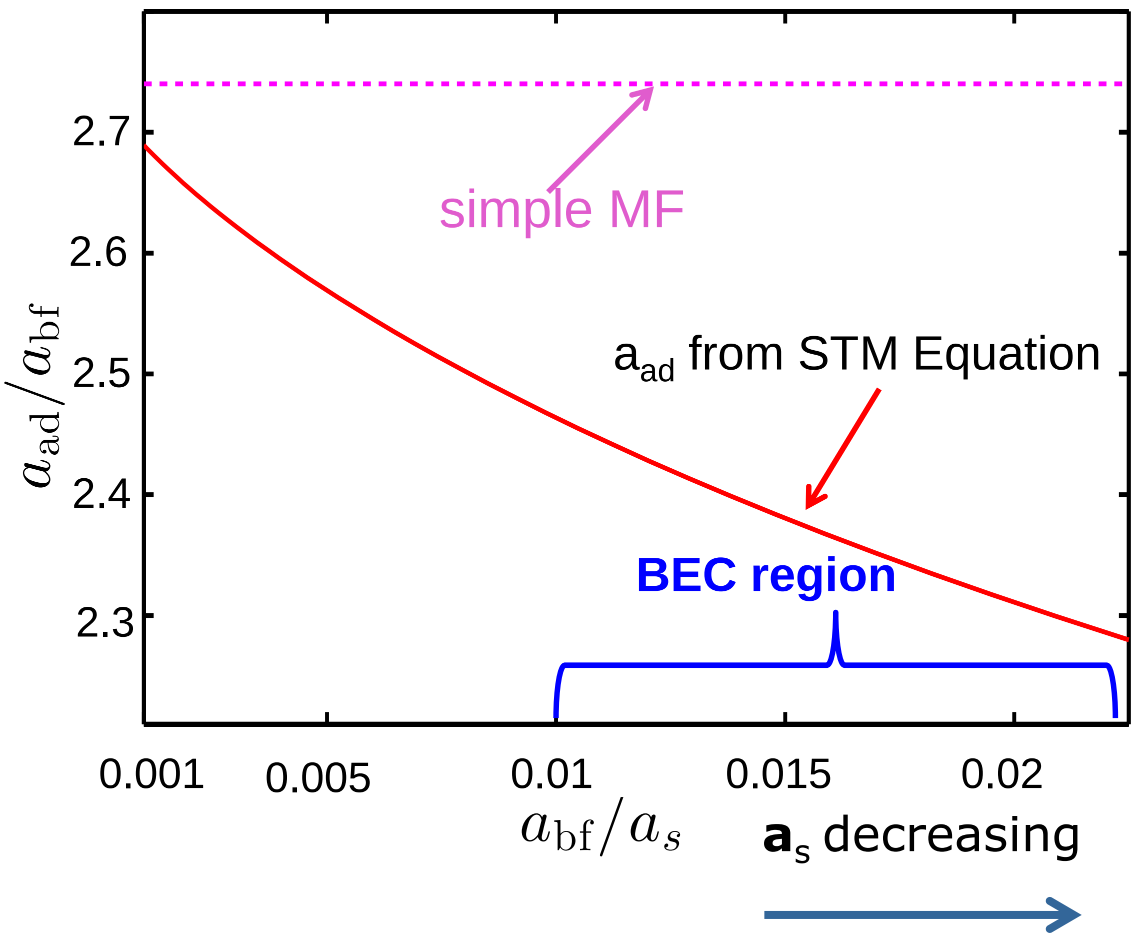

- simple MF:

$$a_{\rm ad} \approx 2.74 a_{\rm bf}$$

- $a_{ad}$ is almost Λ-independent when

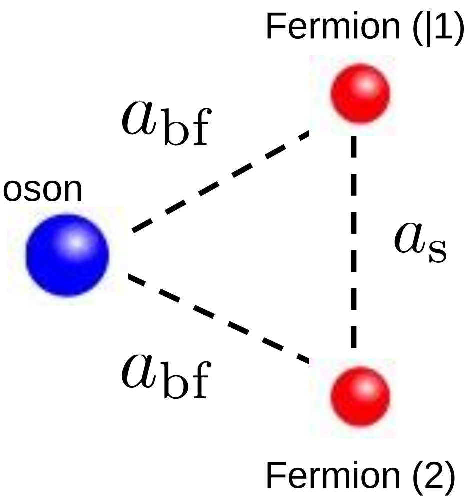
$$|\Lambda| \in (\frac{2}{a_{\rm bf}}, \frac{8}{a_{\rm bf}})$$

$$\arg\Lambda \in (0, 0.08)$$




- ➢ $a_{ad}=2.74a_{bf}$ when $a_s=\infty$;
- ➢ $(2.74a_{bf} - a_{ad})/a_{ad}$ ~10% when $a_{bf}/a_s$ ~1%

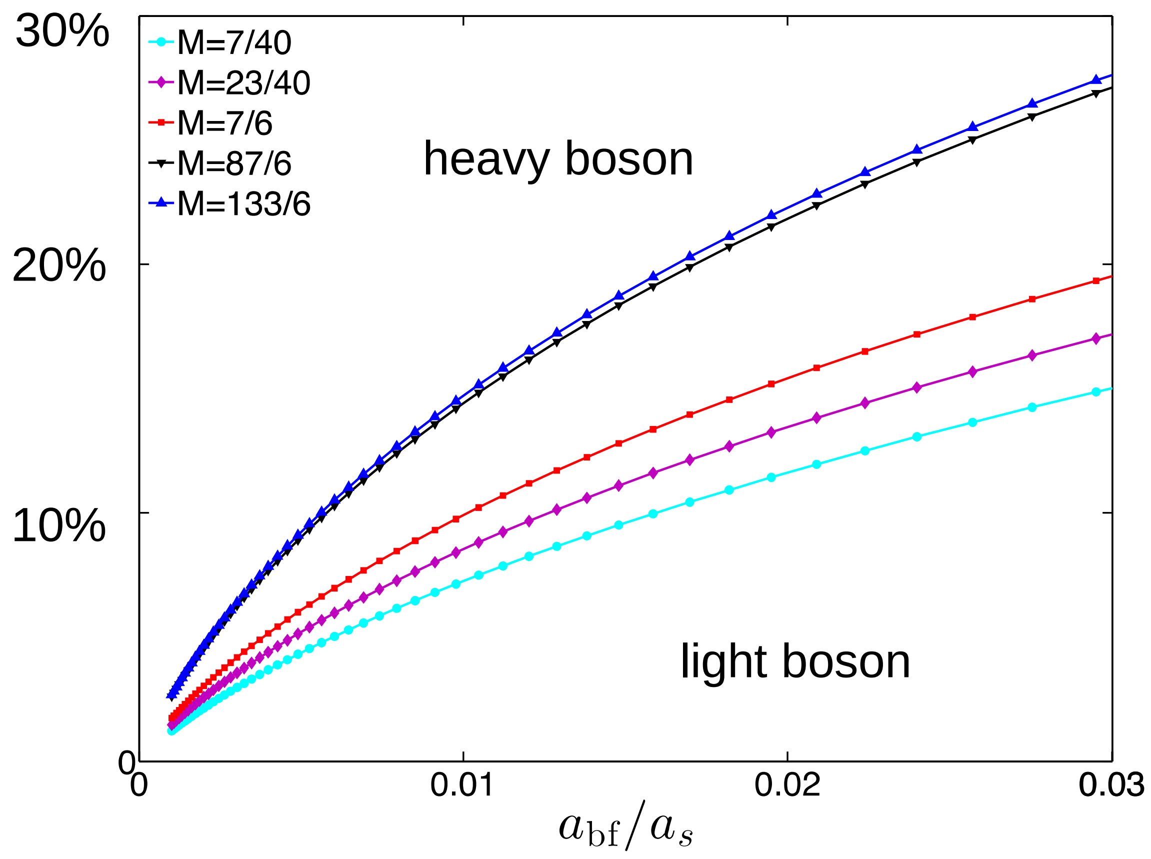
# Influence of mass ratio




- Mass ratio

  M=$m_{boson}$/ $m_{fermion}$

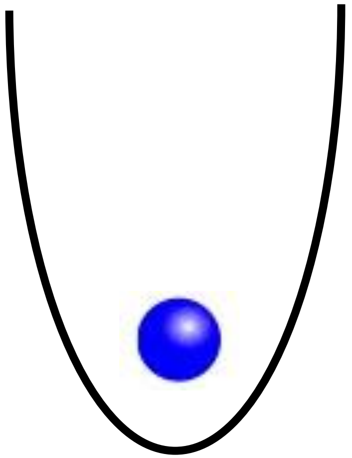
$(a_{ad}^0 - a_{ad})/a_{ad}^0$ (relative error of simple MF)




Error of MF is large when Boson is heavy.

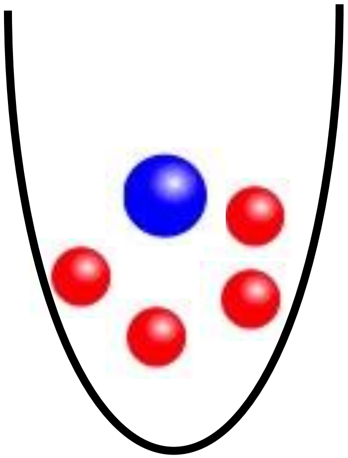
# Frequency of effective potential for Bosons

- without fermions



$$V(\mathbf{r}) = \frac{m\omega_0^2}{2} r^2$$

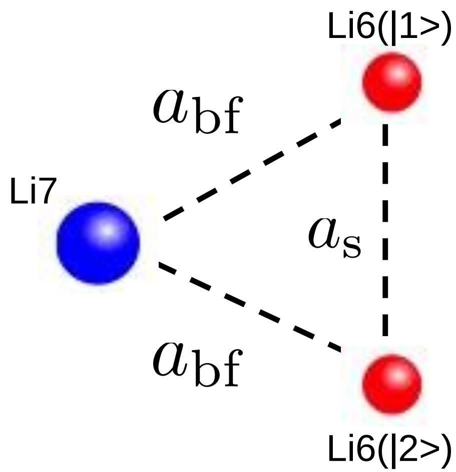
- with fermions

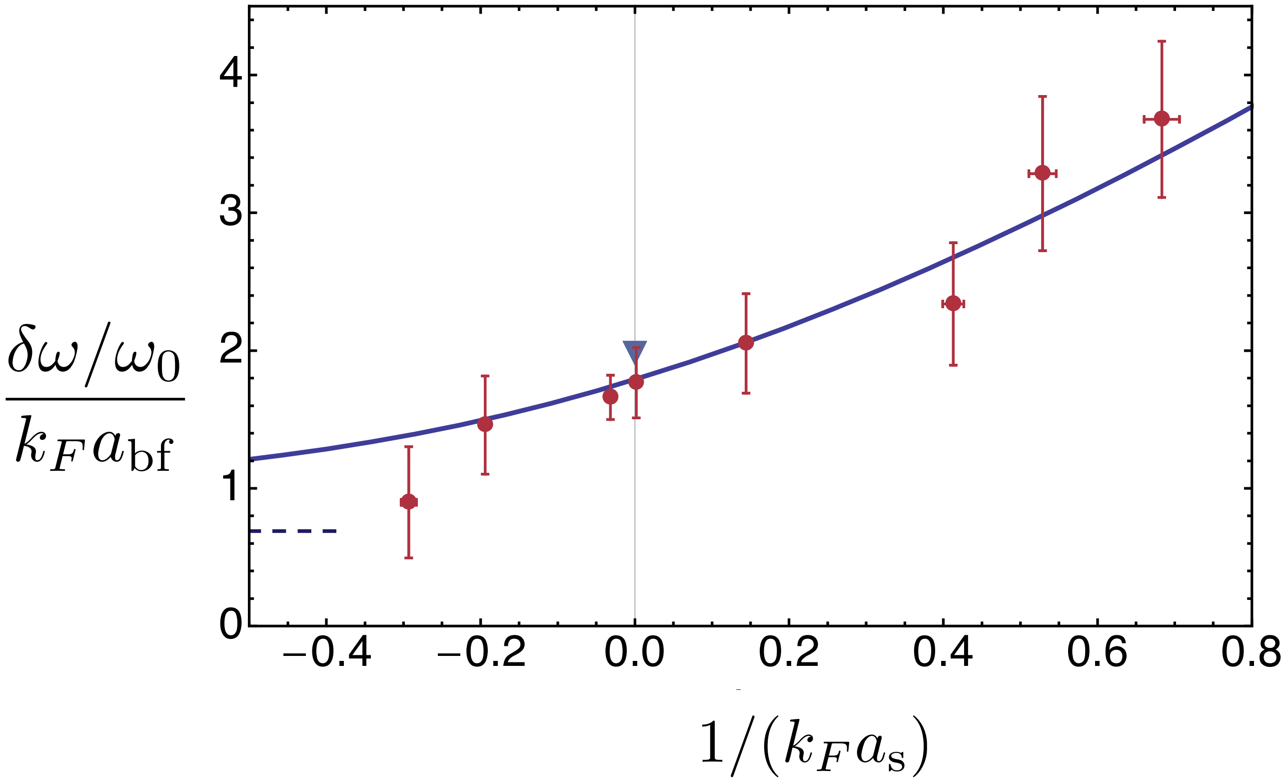


$$V_{\text{eff}}(\mathbf{r}) = \frac{m\omega_0^2}{2} r^2 + \frac{2\pi a_{\text{ad}}}{m_{\text{ad}}} n_{\text{d}}(\mathbf{r})$$

$$= \frac{m(\omega_0 + \delta\omega)^2}{2} r^2$$

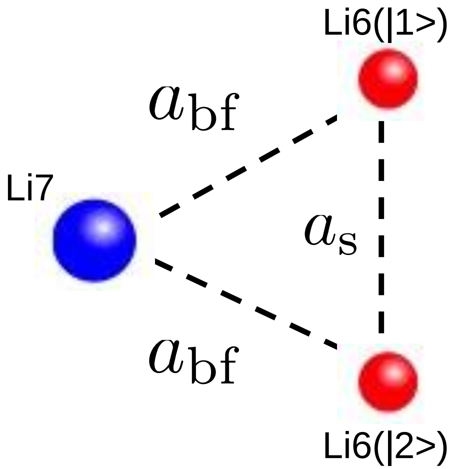
# Experimental observation



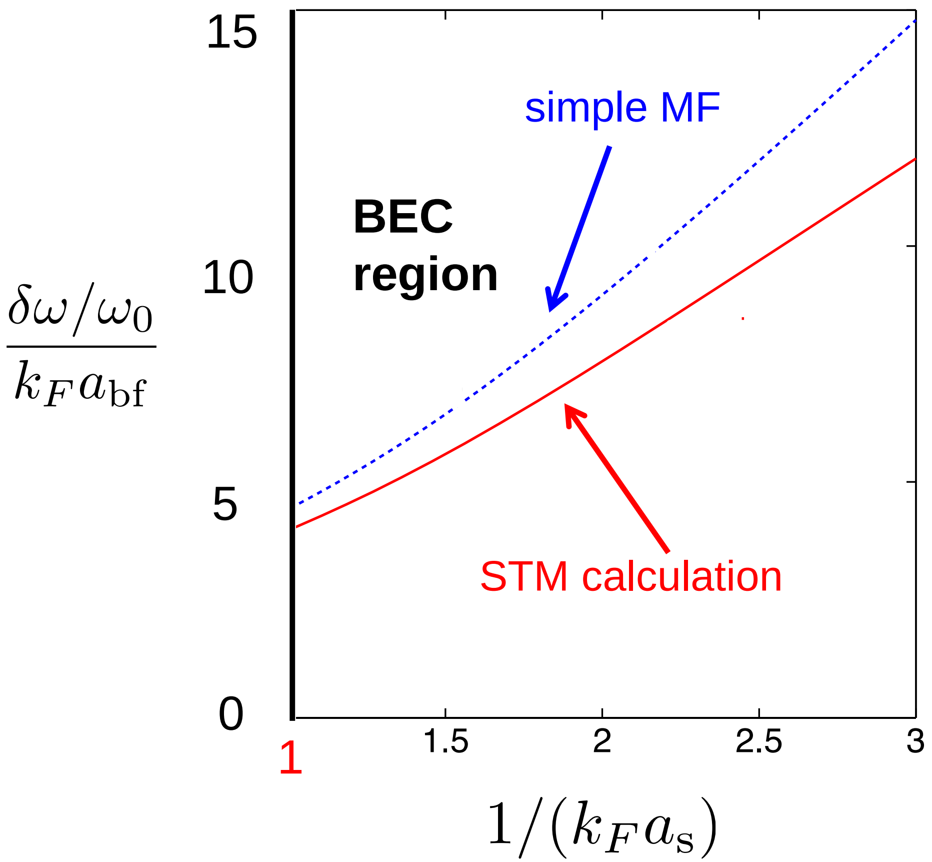




I. Ferrier-Barbut, M. Delehaye, S. Laurent, A. T. Grier, M. Pierce, B. S. Rem, F. Chevy, C. Salomon, arXiv: 1404.2548

# Frequency shift: BEC region




$$k_F = 4.6 \times 10^{-6} m^{-1}$$

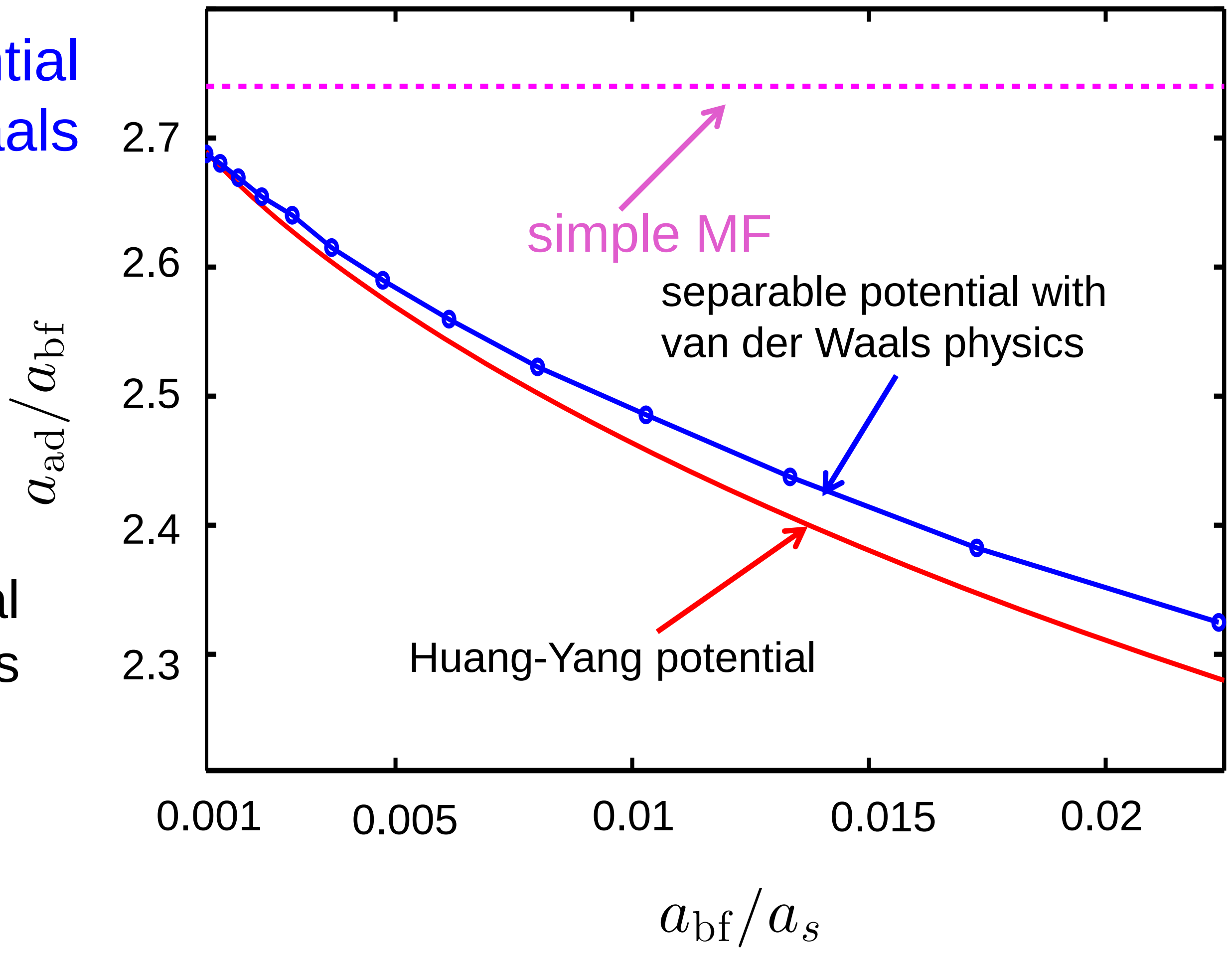


# Van der Waals physics

- separable potential with van der Waals physics

V=ξ|χ><χ|

zero-range potential can give good results (relative error~2%)

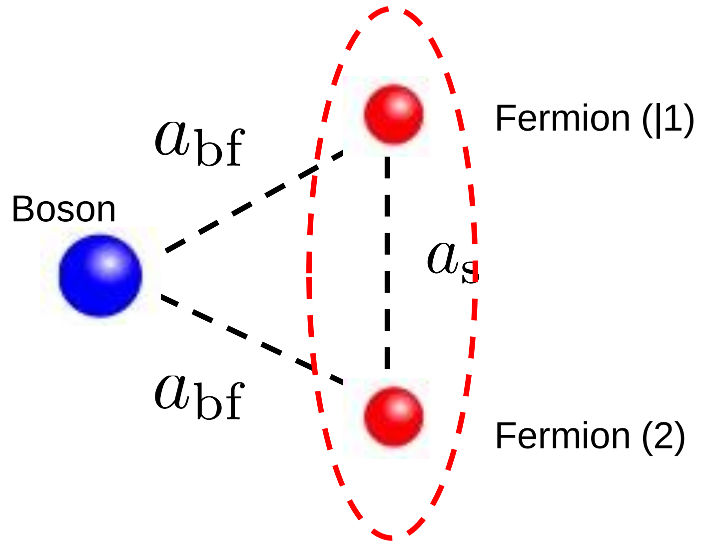


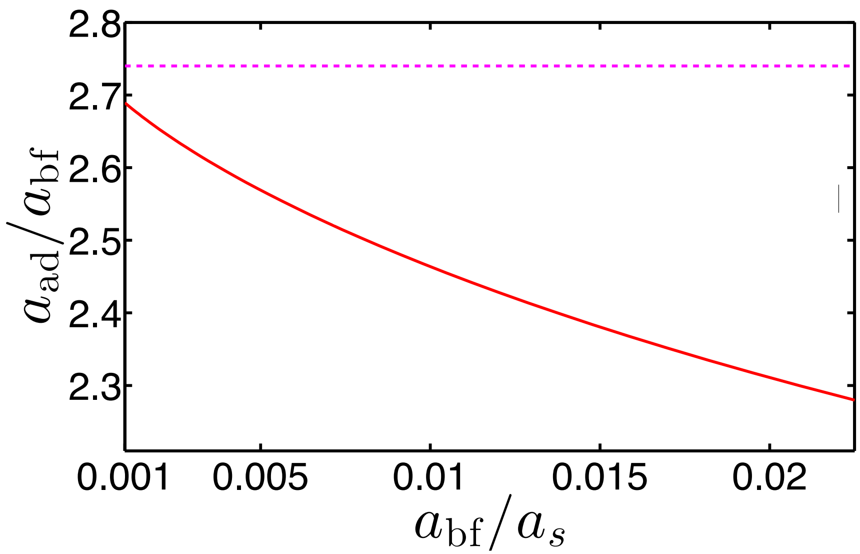

P. Naidon, S. Endo, and M. Ueda, Phys. Rev. Lett. 112, 105301 (2014).

# Summary:

- Error of simple MF approximation rapidly increases with $a_{bf}/a_s$ .

- Effects beyond simple MF can be experimentally observed.

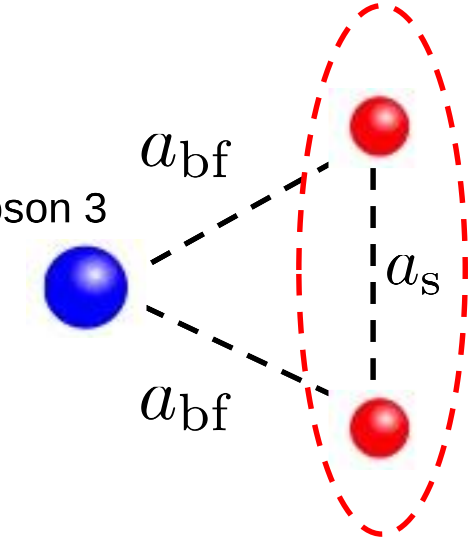


Thank you!

Thank you!

# Formal Expansion with $a_{bf}$




- scattering length

$$a_{\rm ad} = 4\pi^2 m_{\rm ad} \langle \Psi_{\rm in} | (V_{23} + V_{31}) | \Psi + \rangle$$

- scattering state

$$|\Psi+\rangle = \lim_{\varepsilon \to 0^+} \frac{i\varepsilon}{-\frac{\hbar^2}{2m_{12}a_s^2} + i\varepsilon - H} |\Psi_{\rm in}\rangle$$

- Formal expansion with $a_{bf}$

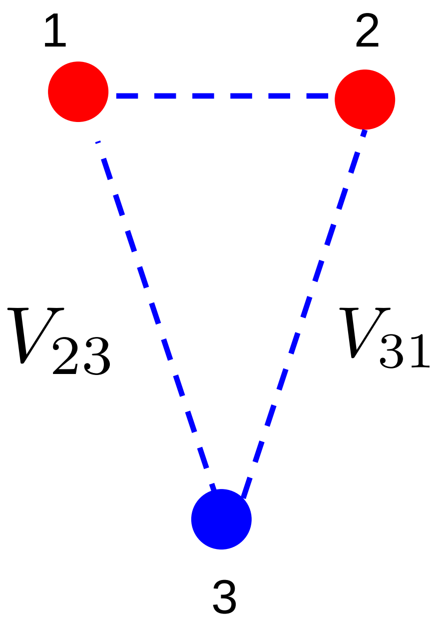
$$a_{\rm ad} = 4\pi^2 m_{\rm ad} \langle \Psi_{\rm in} | (V_{23} + V_{31}) | \Psi_{\rm in} \rangle$$
$$+ 4\pi^2 m_{\rm ad} \langle \Psi_{\rm in} | (V_{23} + V_{31}) G_3 (V_{23} + V_{31}) | \Psi_{\rm in} \rangle + \ldots.$$

G3: Green's function for free boson and interacting fermions

# Born approximation

$$a_{\rm ad} \approx 4\pi^2 m_{\rm ad} \langle \Psi_{\rm in} | (V_{23} + V_{31}) | \Psi_{\rm in} \rangle$$

$$= \frac{2m_{\rm ad}}{m_{\rm bf}} a_{\rm bf} \approx 2.74 a_{\rm bf}$$

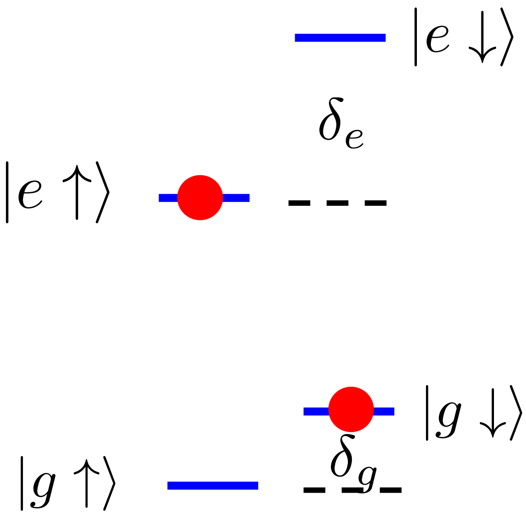



$a_{ad}$ from Born approximation is nothing but the one from simple MF

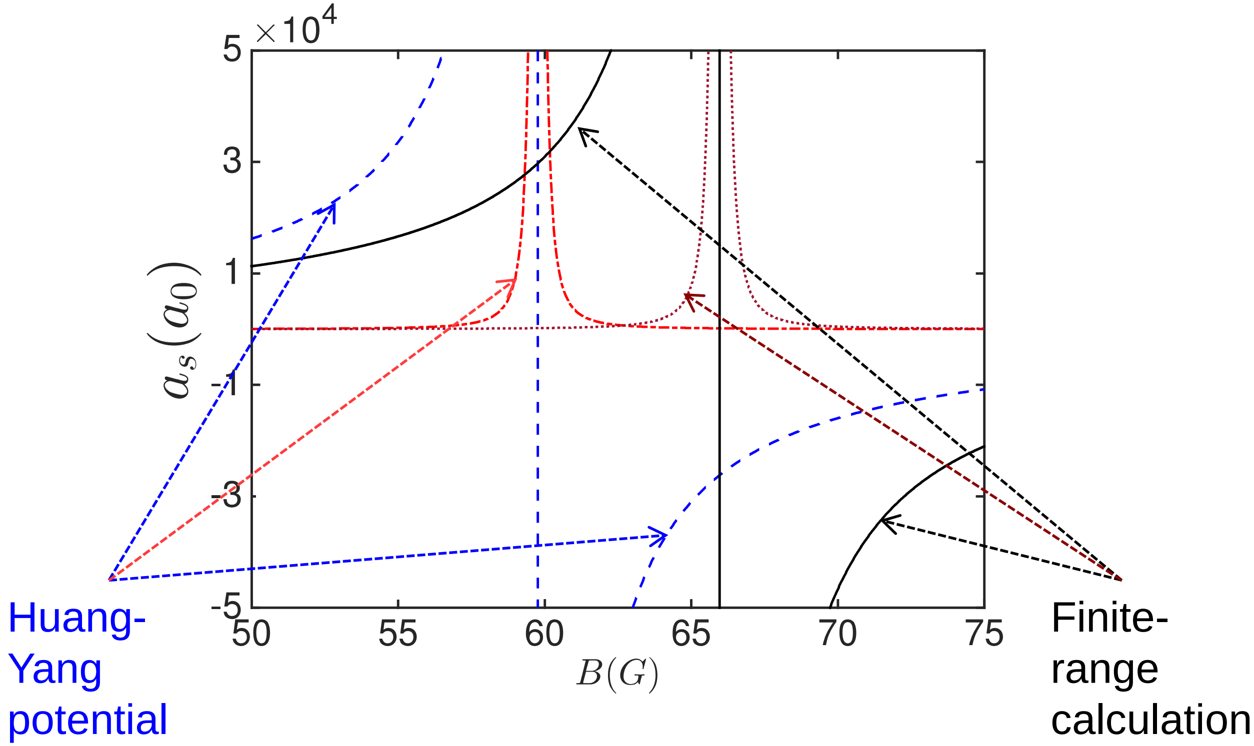
Beyond simple MF: high-order processes should be included

# Physical realization

**Yb173 atoms**




$$\delta = \delta_e - \delta_g$$




Orbital Feshbach resonance: $\delta \sim 10^4$Hz<<$E_{vdW}$