# Interaction Effects on Topological Models with Cold Atoms

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Frontiers in Quantum Simulation with Cold Atoms

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Mar 2015

# Experimental Progresses on Topological Model with Cold Atoms

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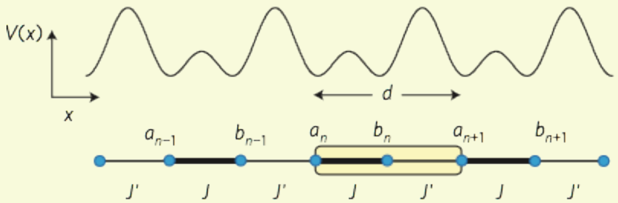
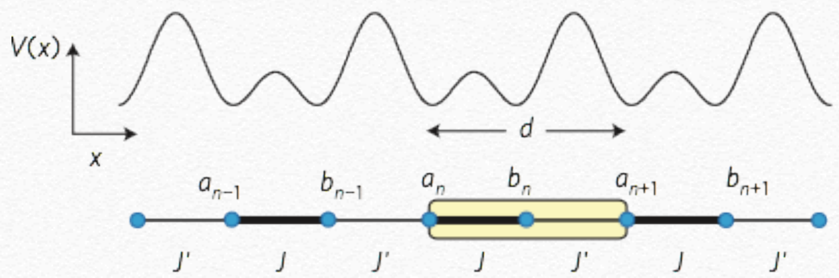
## Su-Schrieffer-Heeger Model

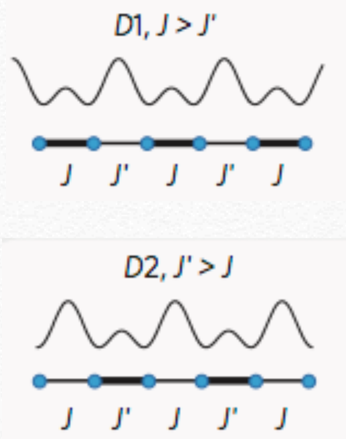


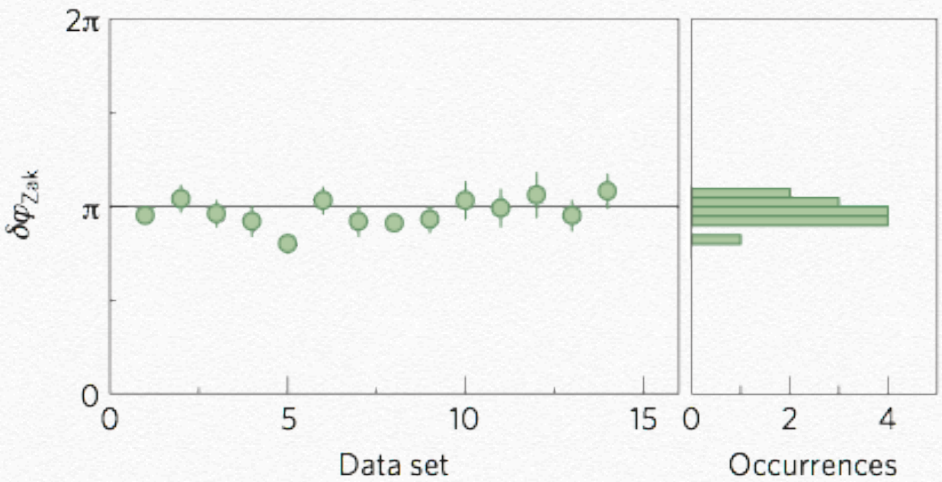
# Experimental Progresses on Topological Model with Cold Atoms

## I. Su-Schrieffer-Heeger Model




Zak phase: $\varphi_{\mathrm{Zak}} = i\int_{-G/2}^{G/2} \langle u_k|\partial_k|u_k\rangle \,\mathrm{d}k$

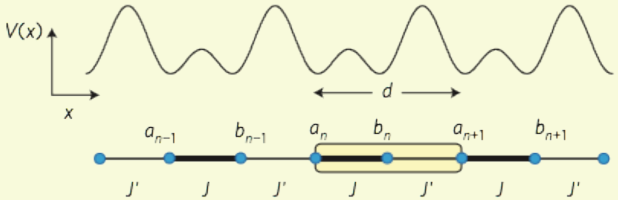


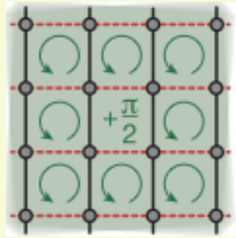

M. Atala et.al (Munich), Nat. Phys. 9, 795 (2013)

# Experimental Progresses on Topological Model with Cold Atoms

## Su-Schrieffer-Heeger Model




## Hofstadter Model



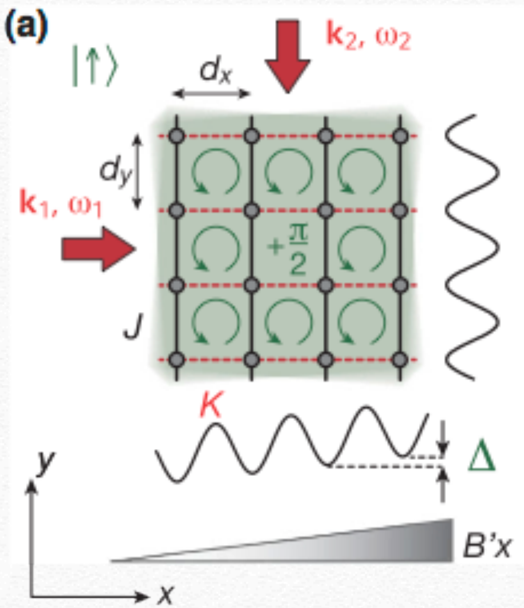
# Experimental Progresses on Topological Model with Cold Atoms

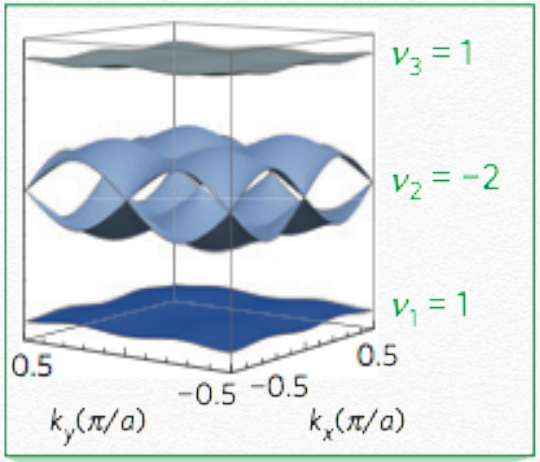
## II. Hofstadter Model

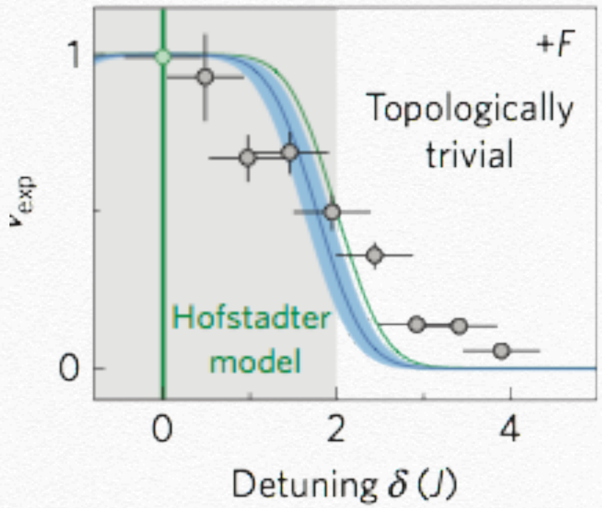
**Chern number**

$$\Omega_\mu = i\left(\langle \partial_{k_x} u_\mu | \partial_{k_y} u_\mu \rangle - \langle \partial_{k_y} u_\mu | \partial_{k_x} u_\mu \rangle\right)$$

$$\nu_\mu = \int_{\mathrm{FBZ}} \Omega_\mu \mathrm{d}^2 k / (2\pi)$$



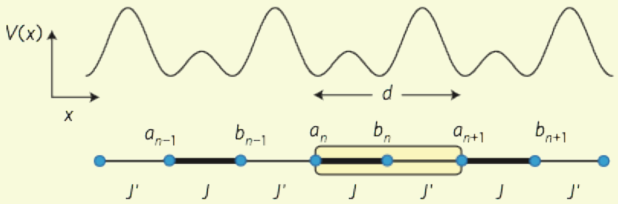

M. Aidelsburger et.al (Munich)
PRL, 111, 185301 (2013)

H. Miyake, et.al. (MIT)
PRL, 111, 185302 (2013)

M. Aidelsburger et.al. (Munich)
Nat. Phys. 11, 162 (2015)

# Experimental Progresses on Topological Model with Cold Atoms

## Su-Schrieffer-Heeger Model

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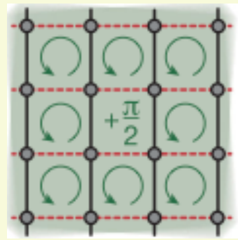
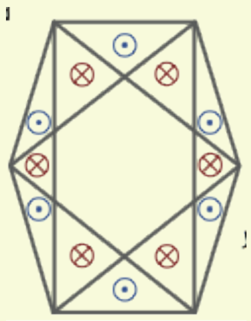
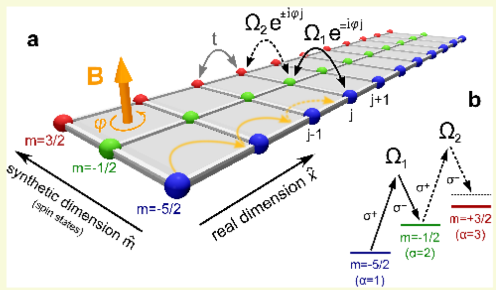



## Haldane Model

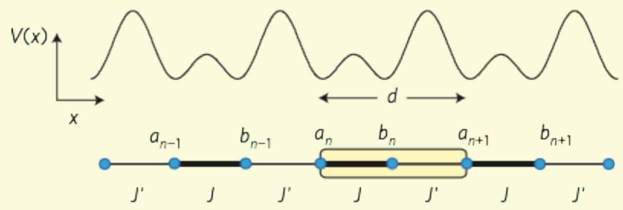


## Synthetic Dimension



# Theoretical Interests:
# Interaction Effects

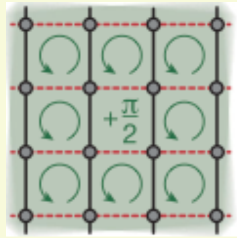
## Su-Schrieffer-Heeger Model



Many studies since early 80s

Fradkin, Hirsch, Kivelson ...

## Hofstadter Model



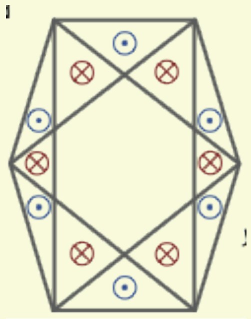
Many studies recently

J. Zhang, C.-M. Jian, F. Ye and HZ, PRL, (2010)

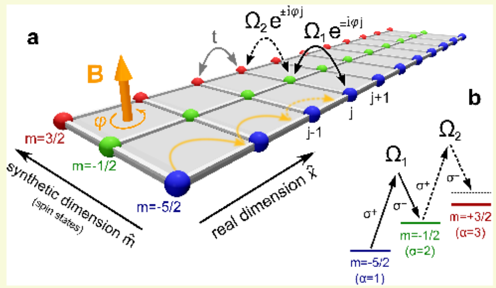
HZ, R. O. Umucalılar and M. O. Oktel, PRL, (2010)

......

## Haldane Model

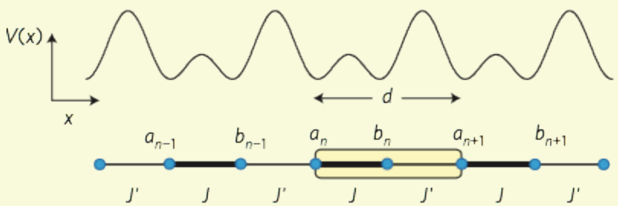


## Synthetic Dimension



# Theoretical Interests: Interaction Effects

## Su-Schrieffer-Heeger Model

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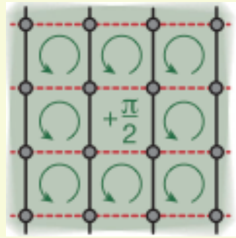
## Hofstadter Model

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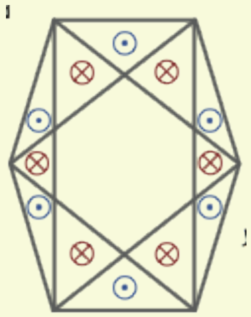
## Haldane Model



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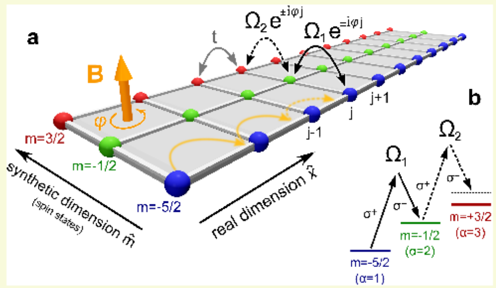
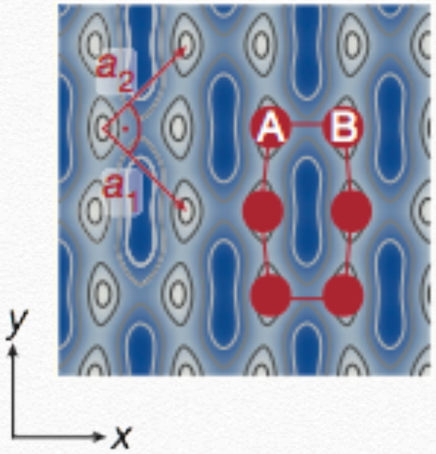



Focus !

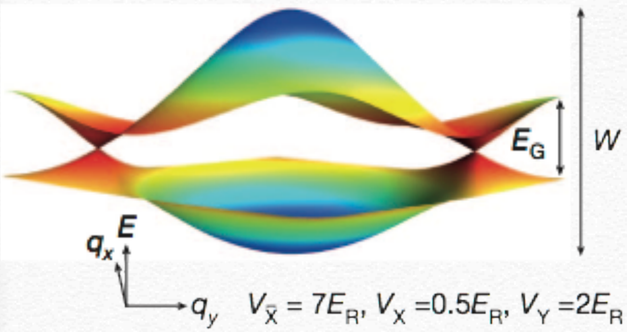
# Part A

# Haldane-Hubbard Model

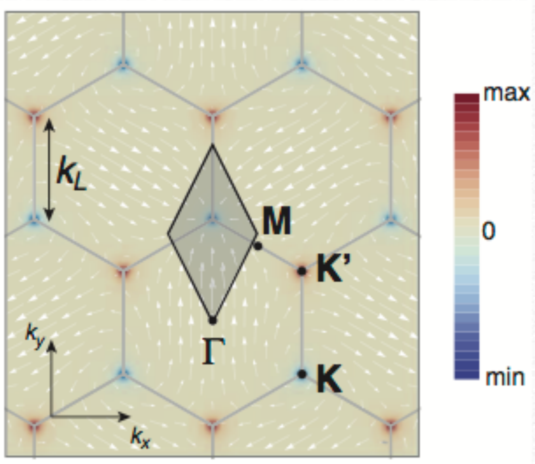
# The Haldane Model



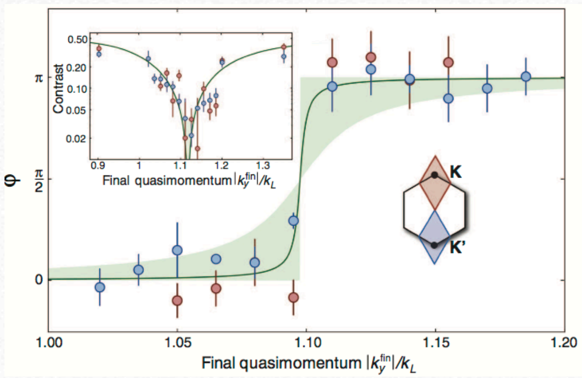

$$\psi_{\mathbf{k}} = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}$$




Dirac point




Berry phase around each Dirac point



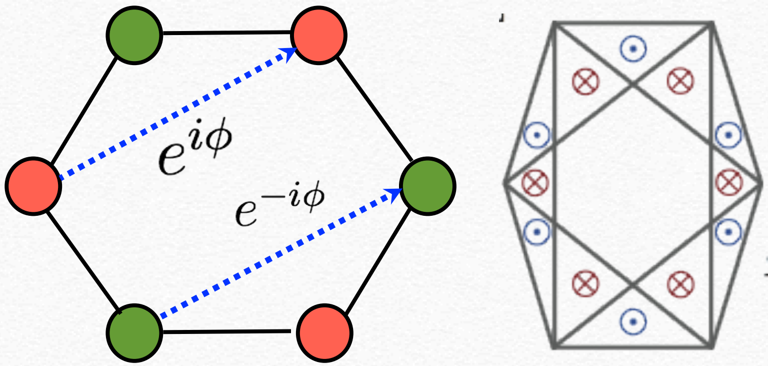

L. Duca, et.al (Munich group), Science, 347, 288 (2015)

L. Tarruell et. al. (ETH) Nature. 483, 302 (2012)

# The Haldane Model

$$\hat{H}_{\mathrm{H}} = -t_1 \sum_{\langle ij\rangle,s} \left(\hat{c}^{\dagger}_{i,s}\hat{c}_{j,s} + \text{h.c.}\right)$$

$$- t_2 \sum_{\langle\langle ij\rangle\rangle,s} \left(e^{i\phi_{ij}}\hat{c}^{\dagger}_{i,s}\hat{c}_{j,s} + \text{ h.c.}\right) - M\sum_{i,s}\epsilon_i\hat{c}^{\dagger}_{i,s}\hat{c}_{i,s}$$




AB sublattice imbalance

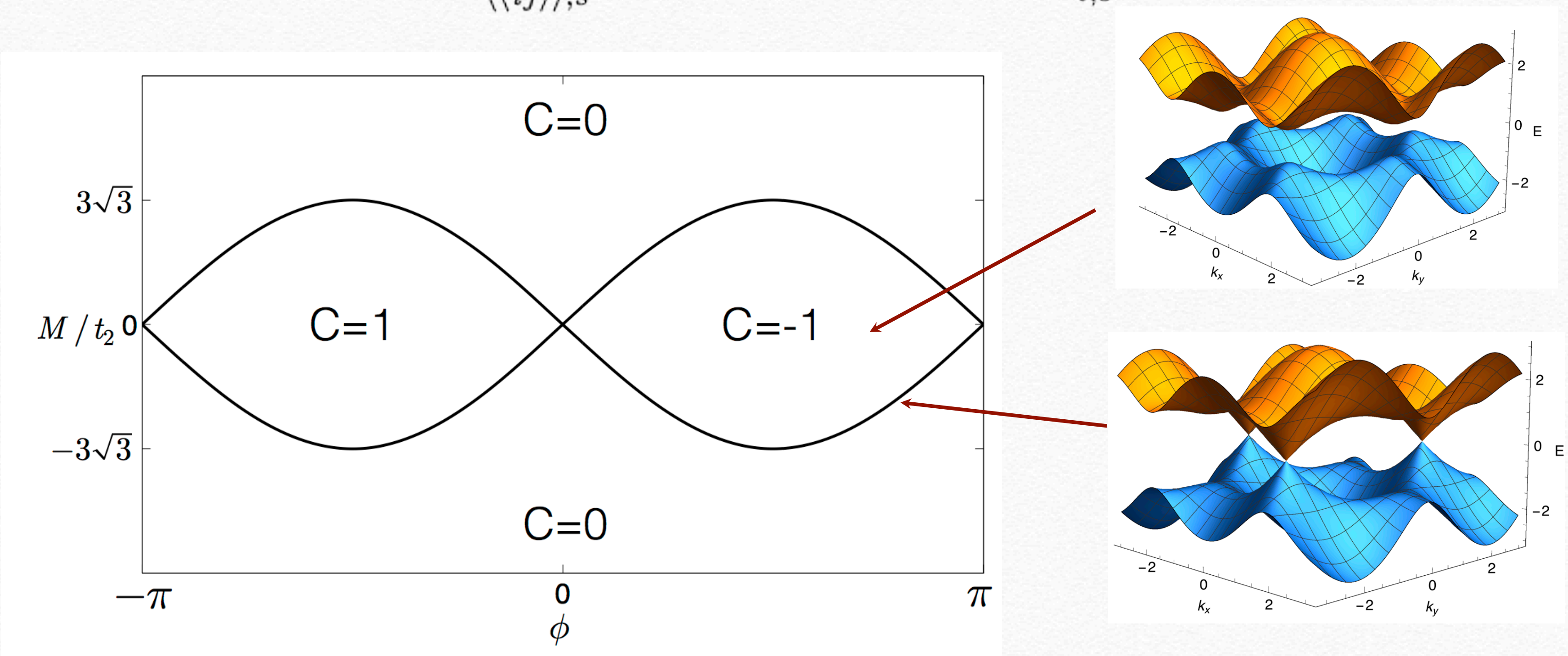
Break Inversion Symmetry

Next nearest hopping with staggered flux

Break Time-Reversal Symmetry

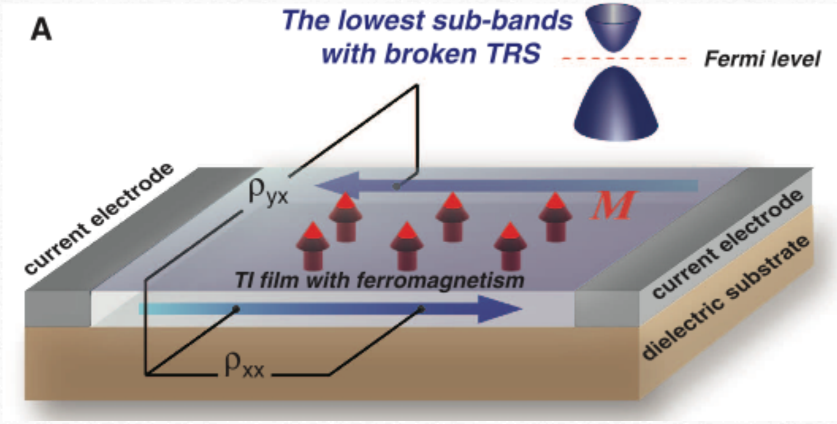
# The Haldane Model

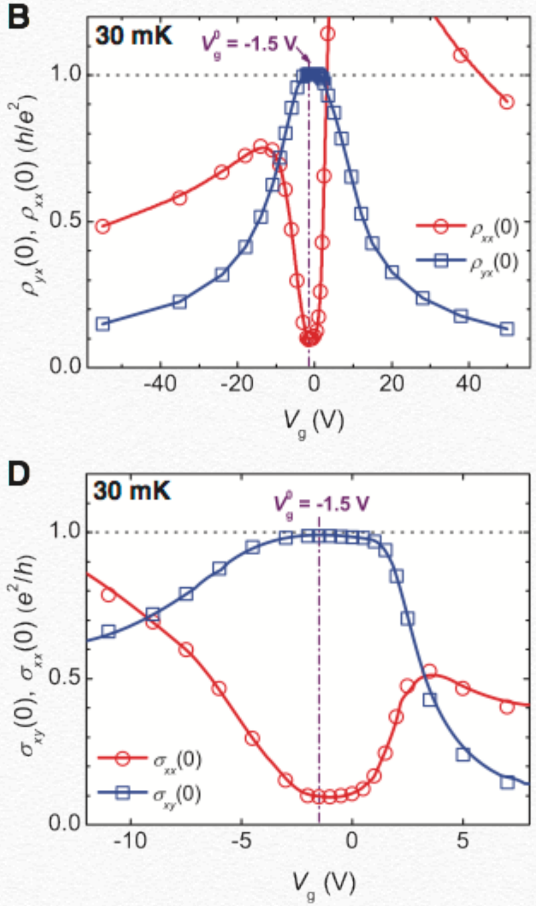
$$\hat{H}_{\mathrm{H}} = -t_1 \sum_{\langle ij \rangle, s} \left( \hat{c}^{\dagger}_{i,s} \hat{c}_{j,s} + \mathrm{h.c.} \right) - t_2 \sum_{\langle\langle ij \rangle\rangle, s} \left( e^{i\phi_{ij}} \hat{c}^{\dagger}_{i,s} \hat{c}_{j,s} + \mathrm{h.c.} \right) - M \sum_{i,s} \epsilon_i \hat{c}^{\dagger}_{i,s} \hat{c}_{i,s}$$




Quantum Anomalous Hall Effects: Quantized Hall conductance without external magnetic field

# Quantum Anomalous Hall Effect




chromium-doped $(Bi,Sb)_2Te_3$,

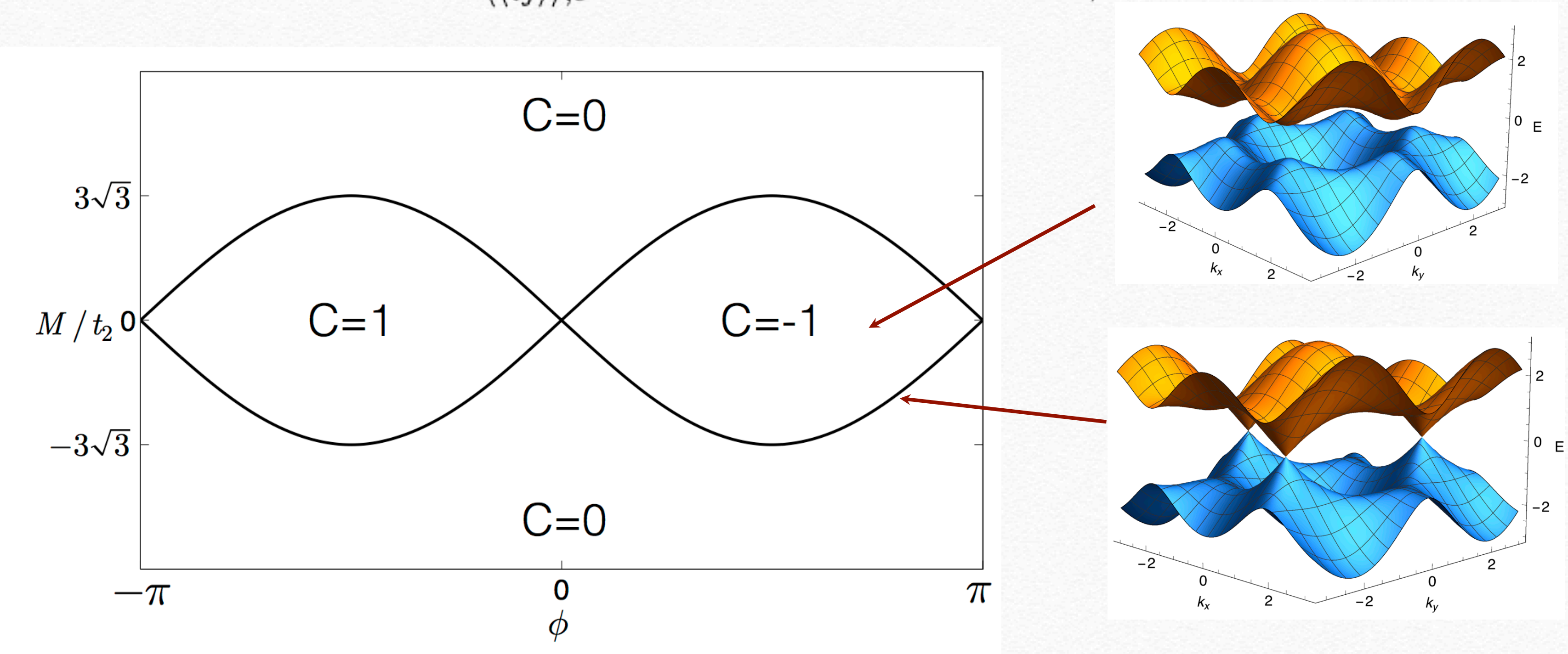
Magnetic topological insulator

1. Model is much complicated than Haldane model;
2. Growing this material is very challenging;
3. It lacks of flexibility of tuning parameters (e.g. interactions).

C. Z. Chang, et.al. (Tsinghua and IoP), Science, (2013)

# The Haldane Model

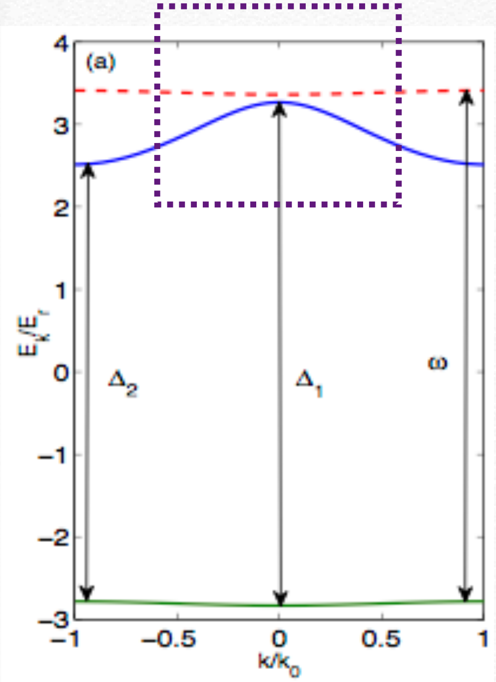
$$\hat{H}_{\mathrm{H}} = -t_1 \sum_{\langle ij \rangle, s} \left( \hat{c}^{\dagger}_{i,s} \hat{c}_{j,s} + \mathrm{h.c.} \right) - t_2 \sum_{\langle\langle ij \rangle\rangle, s} \left( e^{i\phi_{ij}} \hat{c}^{\dagger}_{i,s} \hat{c}_{j,s} + \mathrm{h.c.} \right) - M \sum_{i,s} \epsilon_i \hat{c}^{\dagger}_{i,s} \hat{c}_{i,s}$$

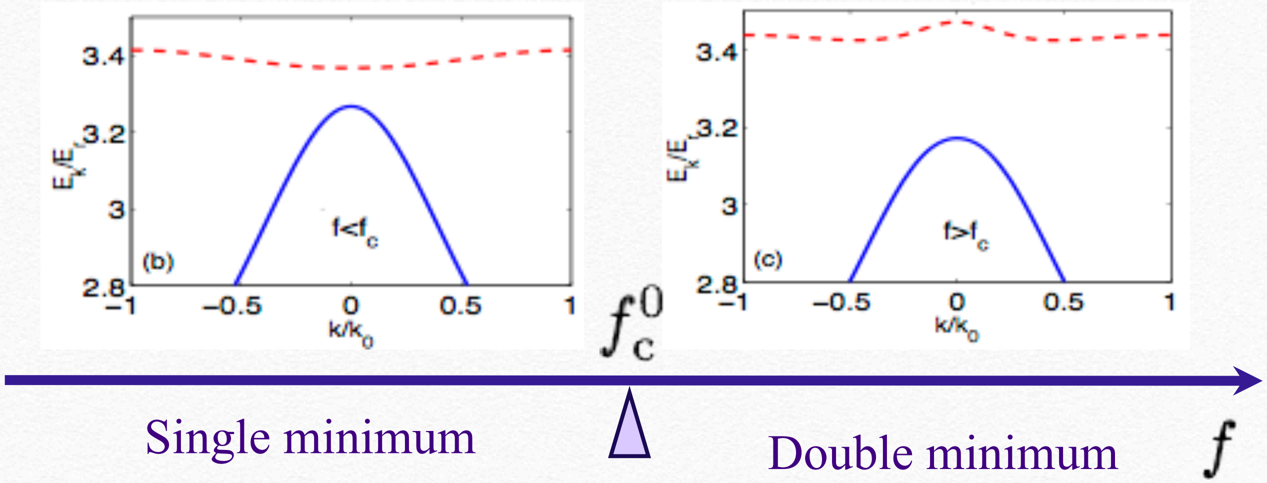



Quantum Simulation of the Haldane Model

How to implement this next nearest hopping in cold atom system ?

# Shaking Lattice Scheme

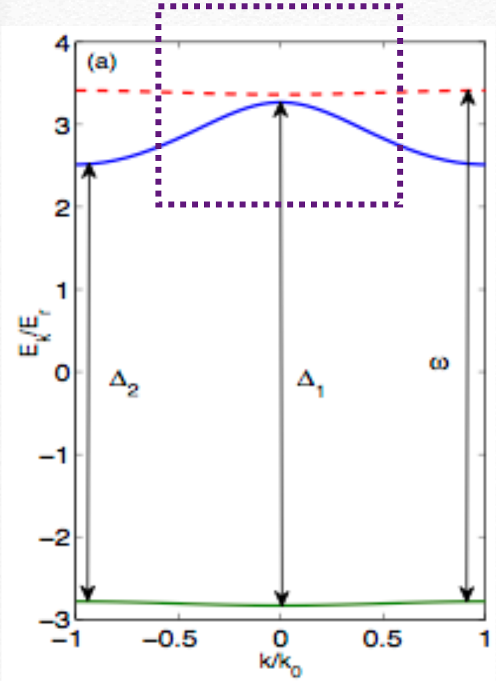


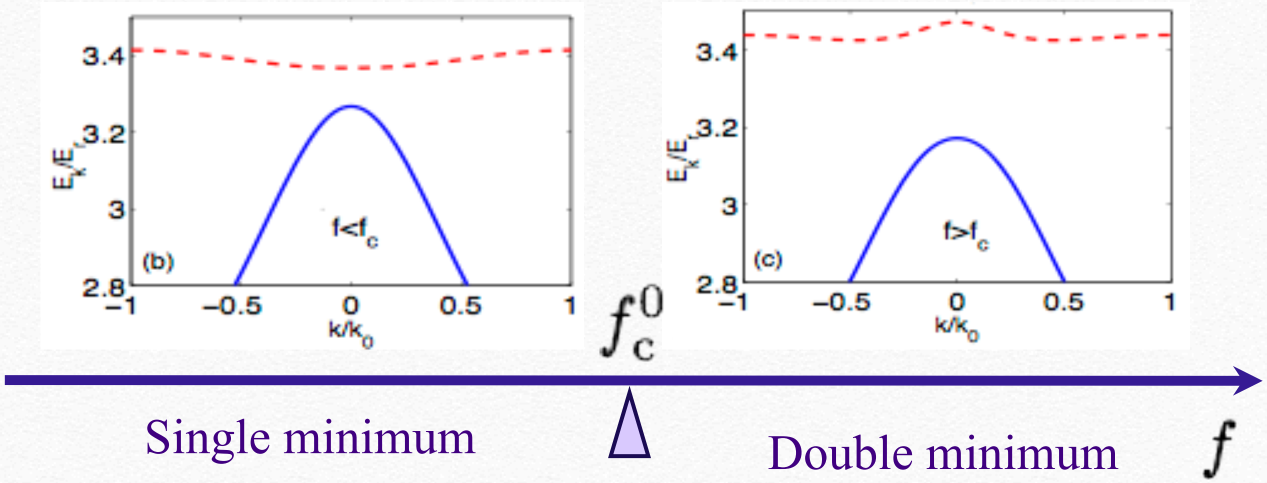




With resonant shaking, small shaking amplitude can make a significant change in dispersion

C. V. Parker, et.al. (Chicago), Nat. Phys. 9, 769 (2014)

# Shaking Lattice Scheme

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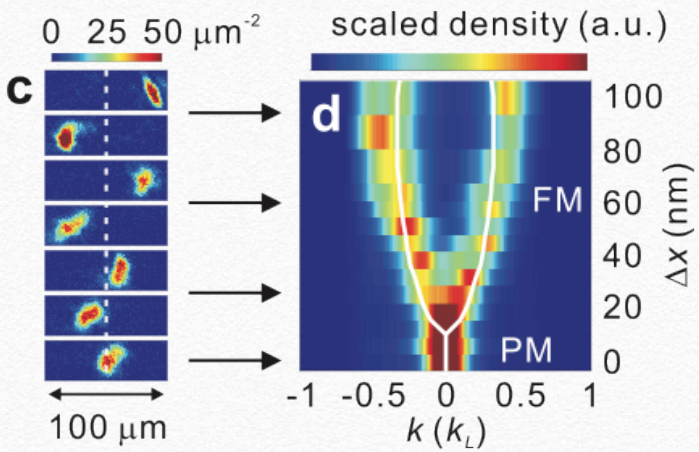
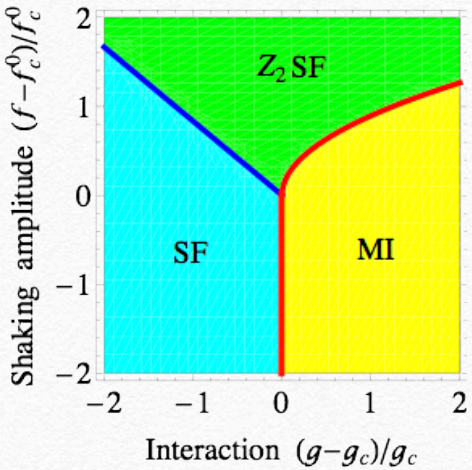



C. V. Parker, et.al. (Chicago), Nat. Phys. 9, 769 (2014)

W. Zheng, B. Liu, J. Miao, C. Chin and HZ, PRL, 113, 155303 (2014)

# Shaking Lattice Scheme

Method I:

Floquet operator:

$$\hat{F} = \hat{U}\left(T_i + T, T_i\right) = \hat{T} \exp\left\{-i \int_{T_i}^{T_i+T} dt \hat{H}\left(t\right)\right\}$$

$$\hat{F}\left|\varphi_n\right\rangle = e^{-i\varepsilon_n T}\left|\varphi_n\right\rangle$$

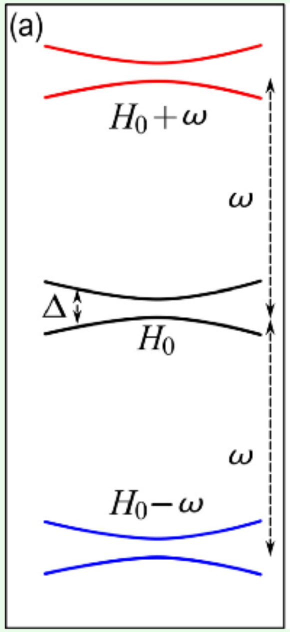
Quasi-energy $\mathcal{E}_n$

Method II:

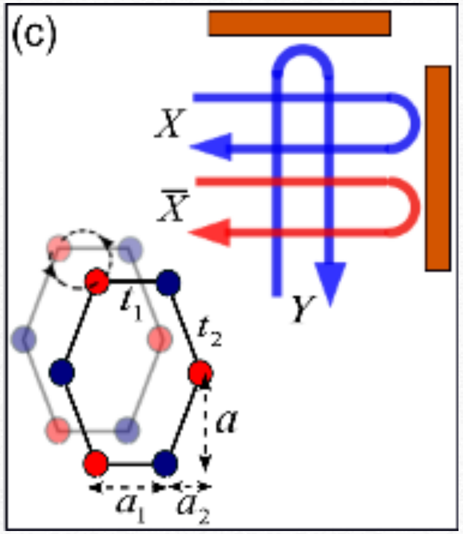
$$\hat{H}(t) = \sum_{n=-\infty}^{\infty} \hat{H}_n(t) e^{in\omega t}$$

Effective Hamiltonian $\hat{F} = e^{-i\hat{H}_{\text{eff}}T}$

$$\hat{H}_{\text{eff}} = \hat{H}_0 + \sum_{n=1}^{\infty} \left\{ \frac{\left[\hat{H}_n, \hat{H}_{-n}\right]}{n\omega} - \frac{\left[\hat{H}_n, \hat{H}_0\right]}{e^{-2\pi n i \alpha} n\omega} + \frac{\left[\hat{H}_{-n}, \hat{H}_0\right]}{e^{2\pi n i \alpha} n\omega} \right\}$$

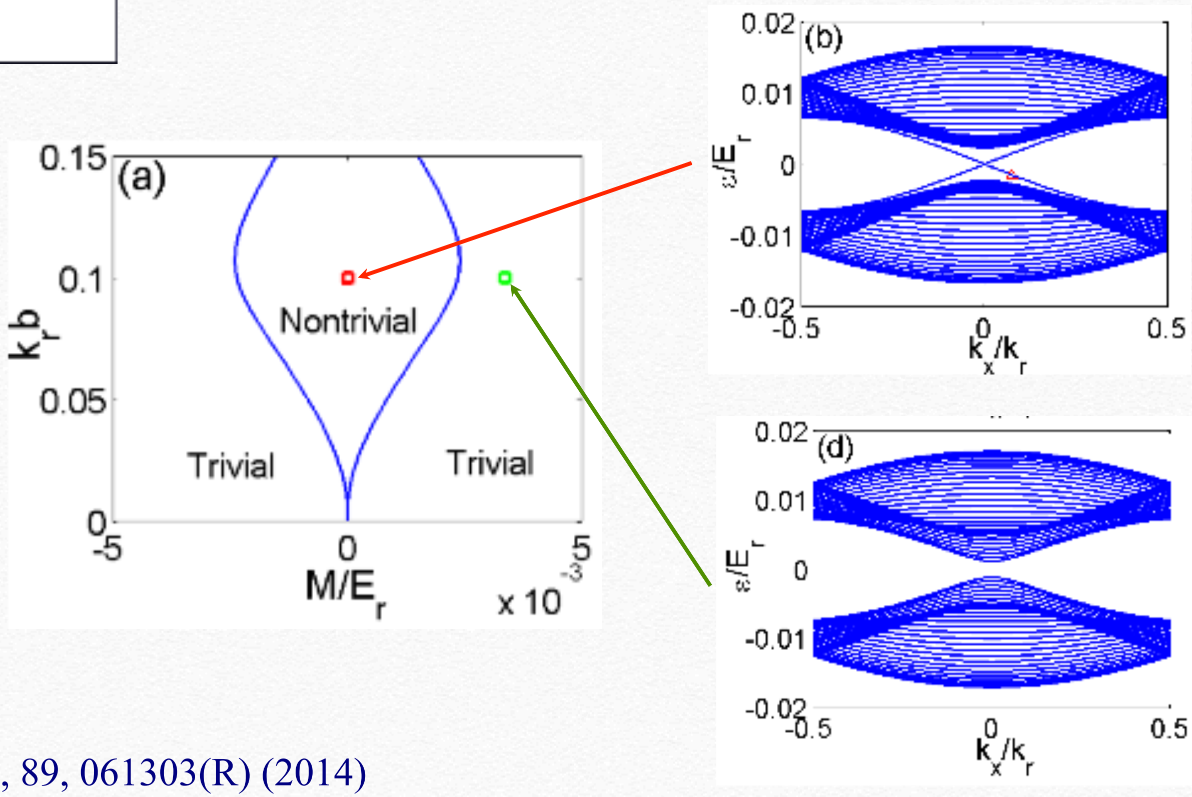


# Shaking Lattice Scheme



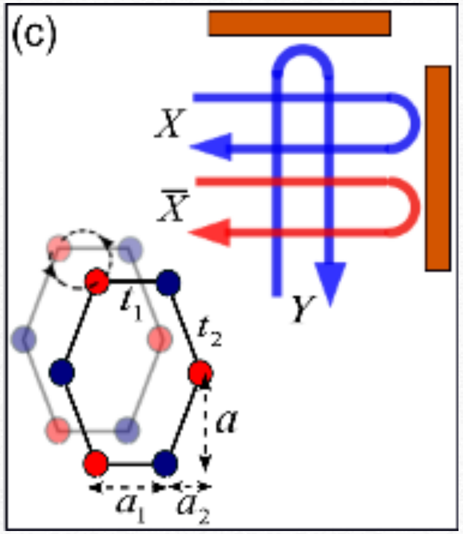

$$\hat{H} = \frac{\mathbf{p}^2}{2m} + V(x + b\cos(\omega t), y + b\sin(\omega t))$$

Method I:

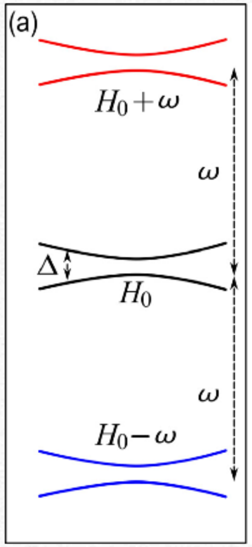


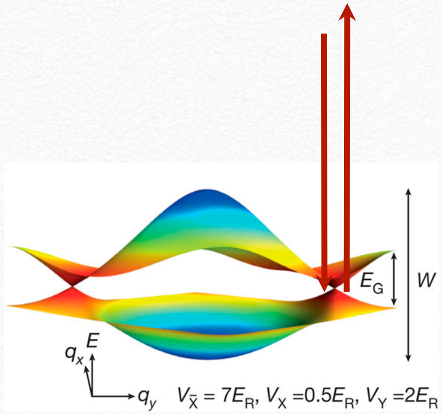

W. Zheng and HZ, PRA, 89, 061303(R) (2014)

# Shaking Lattice Scheme




$$\hat{H} = \frac{\mathbf{p}^2}{2m} + V(x + b\cos(\omega t), y + b\sin(\omega t))$$

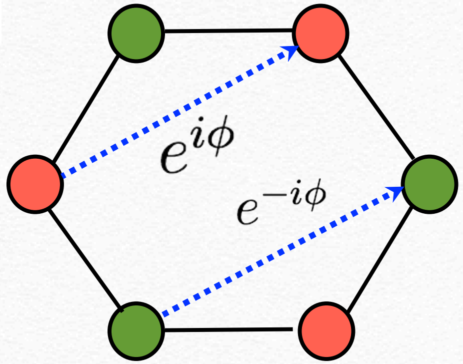



Method II: $\hat{H}_{eff} = \hat{H}_0 + \frac{1}{\omega}[\hat{H}_1, \hat{H}_{-1}] + \dots$

$$\hat{H}_{\rm H} = -t_1 \sum_{\langle ij \rangle, s} \left( \hat{c}^\dagger_{i,s} \hat{c}_{j,s} + \text{h.c.} \right)$$

$$- t_2 \sum_{\langle\langle ij \rangle\rangle, s} \left( e^{i\phi_{ij}} \hat{c}^\dagger_{i,s} \hat{c}_{j,s} + \text{ h.c.} \right) - M \sum_{i,s} \epsilon_i \hat{c}^\dagger_{i,s} \hat{c}_{i,s}$$




W. Zheng and HZ, PRA, 89, 061303(R) (2014)

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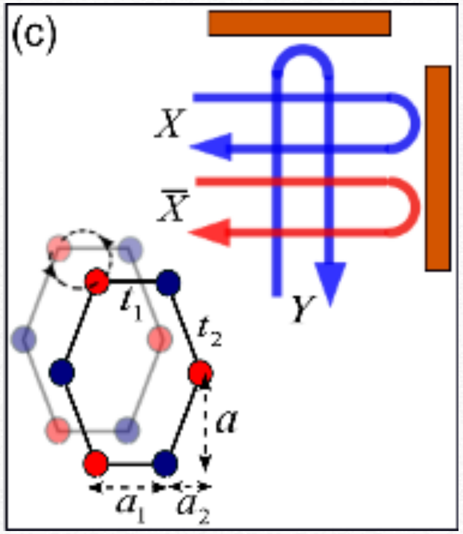


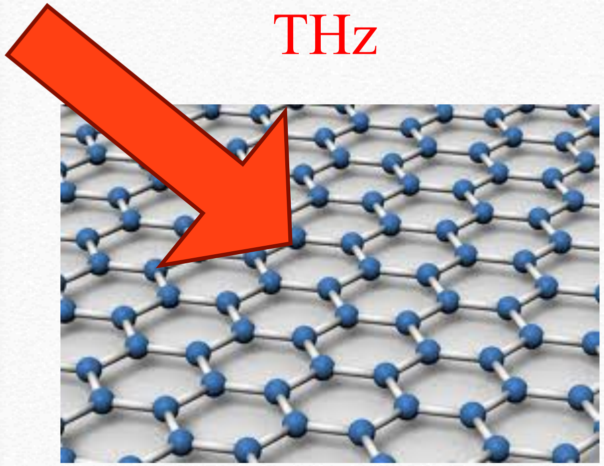

$$\hat{H} = \frac{\mathbf{p}^2}{2m} + V(x + b\cos(\omega t), y + b\sin(\omega t))$$

$$x \to x + b\cos(\omega t) \quad y \to y + b\sin(\omega t)$$

$$\hat{H} = \frac{(\mathbf{p} - \mathbf{A})^2}{2m} + V(x, y)$$

$$A_x = -b\sin(\omega t) \quad A_y = b\cos(\omega t)$$

Soft X-ray !
frequency about 3500 THz




T. Oka and H. Aoki, PRB, 79, 081406 (2009)
T. Kitagawa, E. Berg, M. Rudner and E. Demler, PRB, 82, 235114 (2010)
N. H. Linder, G. Refael and V. Galitski, Nat. Phys. 7, 490, (2011)
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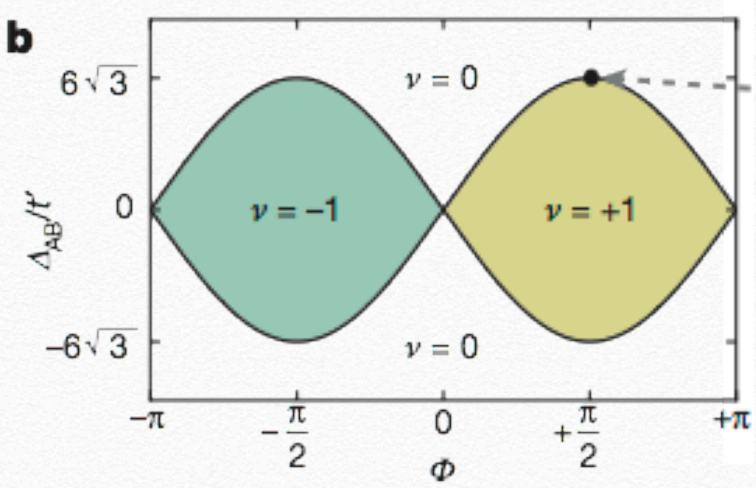
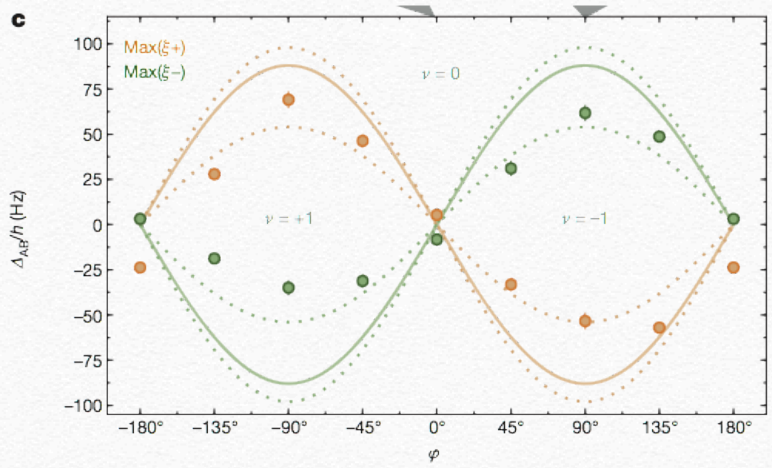
# Experimental Progresses on the Haldane model



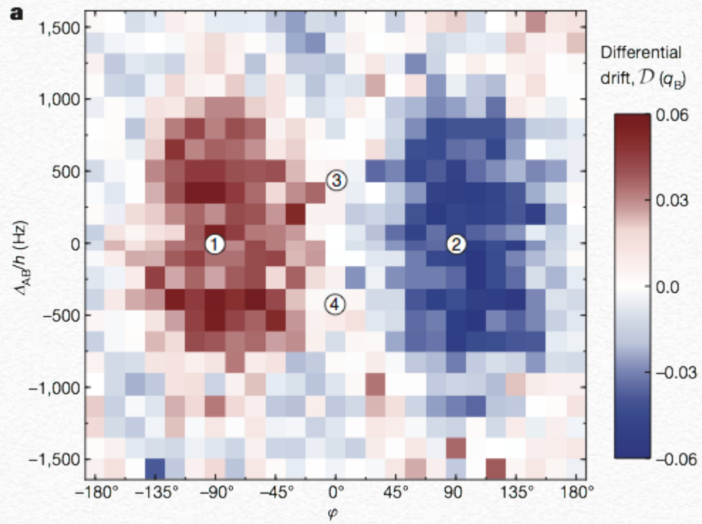




Bloch oscillation

Extending our work to interacting systems requires sufficiently low heating. We investigate this with a repulsively interacting spin mixture in the honeycomb lattice previously used for studying the fermionic Mott insulator[27]. We measure the entropy in the Mott insulating regime by loading atoms into the lattice and reversing the loading procedure (see Methods and Extended Data Fig. 3). The entropy increase is only 25% larger than without modulation. This opens up the possibility of studying topological models with interactions[28] in a controlled and tunable

" Little is known "

----- Tilman / Yesterday




Drift measurement

G. Jotzu, et.al. (ETH group), Nature, 515, 237 (2014)

# The Haldane-Hubbard Model

## Spin-1/2 fermions

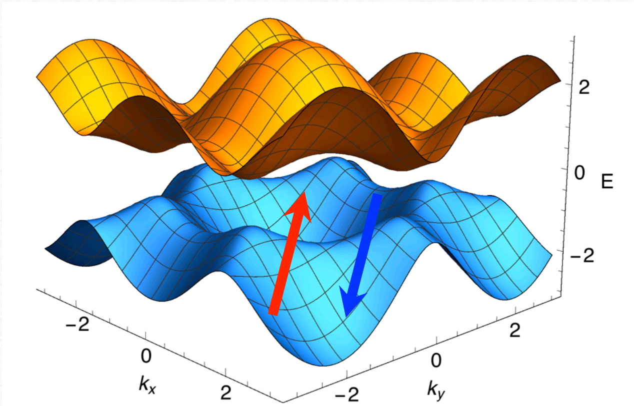
$$\hat{H}_{\mathrm{HH}} = \hat{H}_{\mathrm{H}} + U \sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$

$$n = 1/2 + 1/2 = 1$$

$$\hat{H}_{\mathrm{H}} = -t_1 \sum_{\langle ij \rangle, s} \left( \hat{c}^\dagger_{i,s}\hat{c}_{j,s} + \mathrm{h.c.} \right)$$

$$- t_2 \sum_{\langle\langle ij \rangle\rangle, s} \left( e^{i\phi_{ij}} \hat{c}^\dagger_{i,s}\hat{c}_{j,s} + \mathrm{h.c.} \right) - M \sum_{i,s} \epsilon_i \hat{c}^\dagger_{i,s}\hat{c}_{i,s}$$

$$C = 1 + 1 = 2$$




$U$

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$$\hat{H}_{\mathrm{HH}} = \hat{H}_{\mathrm{H}} + U\sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$

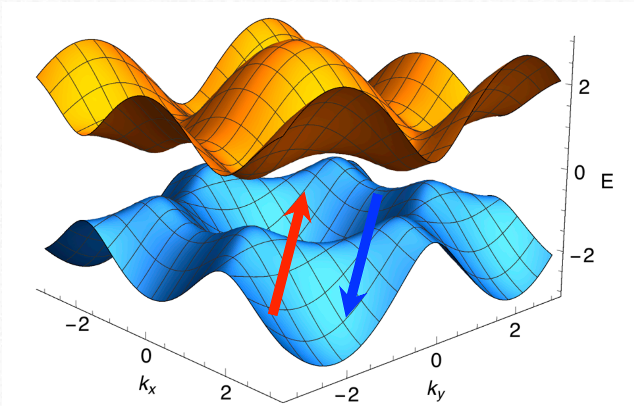
$$n = 1/2 + 1/2 = 1$$

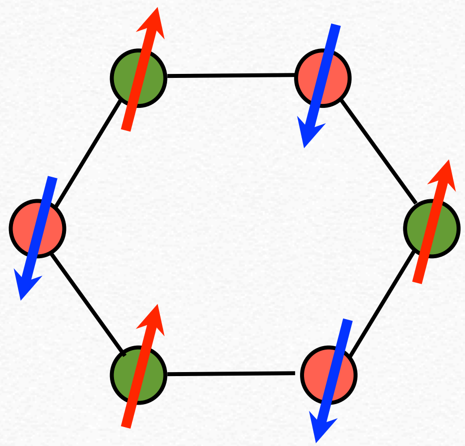
$$\hat{H}_{\mathrm{H}} = -t_1 \sum_{\langle ij\rangle,s}\left(\hat{c}^\dagger_{i,s}\hat{c}_{j,s} + \mathrm{h.c.}\right) - t_2 \sum_{\langle\langle ij\rangle\rangle,s}\left(e^{i\phi_{ij}}\hat{c}^\dagger_{i,s}\hat{c}_{j,s} + \mathrm{h.c.}\right) - M\sum_{i,s}\epsilon_i \hat{c}^\dagger_{i,s}\hat{c}_{i,s}$$

$$U\sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$

$$C = 1 + 1 = 2$$

$$C = 0$$



$U$

# The Haldane-Hubbard Model

Spin-1/2 fermions

$$\hat{H}_{\mathrm{HH}} = \hat{H}_{\mathrm{H}} + U \sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$

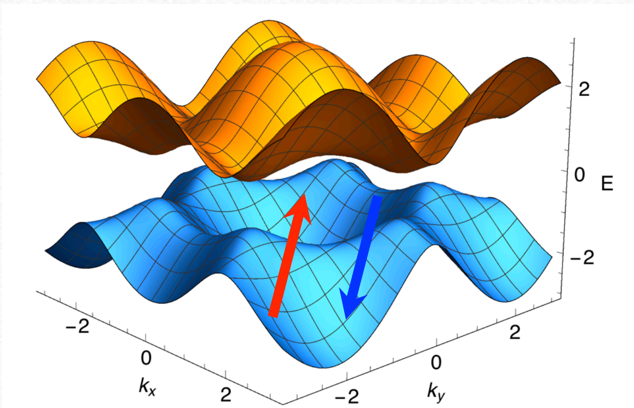
$$n = 1/2 + 1/2 = 1$$

$$\hat{H}_{\mathrm{H}} = -t_1 \sum_{\langle ij \rangle, s} \left( \hat{c}^\dagger_{i,s}\hat{c}_{j,s} + \mathrm{h.c.} \right) - t_2 \sum_{\langle\langle ij \rangle\rangle, s} \left( e^{i\phi_{ij}} \hat{c}^\dagger_{i,s}\hat{c}_{j,s} + \mathrm{h.c.} \right) - M \sum_{i,s} \epsilon_i \hat{c}^\dagger_{i,s}\hat{c}_{i,s}$$

$$U \sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}$$

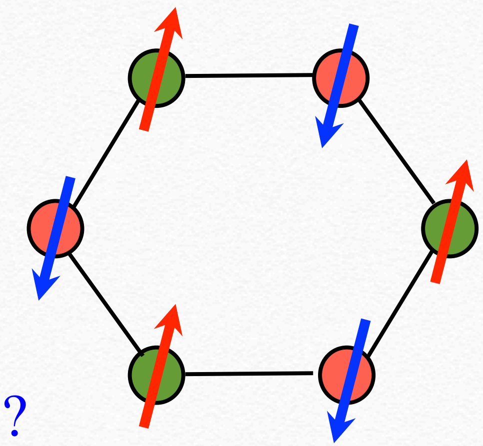
$$C = 1 + 1 = 2$$

$$C = 0$$




Gap close ?

First-order transition ?



$U$

Topological Phase Transition

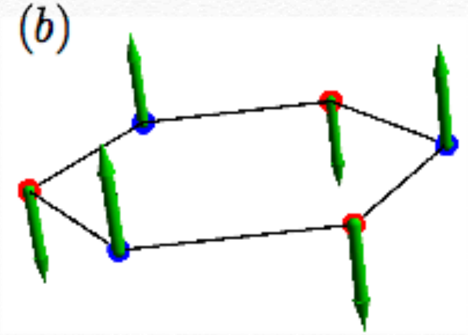
# Relation between magnetic order and topology

$$U\sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow} = \frac{1}{2}U\hat{N} - \frac{2}{3}U\sum_i \mathbf{S}_i^2$$
$$\approx \frac{1}{2}U\hat{N} + \sum_i \left(-\mathbf{m}_i\cdot\mathbf{S}_i + \frac{3\mathbf{m}_i^2}{8U}\right)$$

Mean-field Hamiltonian

$$\hat{H}_{\rm MF} = \hat{H}_{\rm H} - \sum_i \mathbf{m}_i\cdot\mathbf{S}_i$$

(b)



$$\mathbf{m} = m\hat{z}.$$

Free Hamiltonian

$$\hat{H}_{\rm H} = -t_1 \sum_{\langle ij\rangle,s}\left(\hat{c}^\dagger_{i,s}\hat{c}_{j,s} + \text{h.c.}\right)$$
$$- t_2 \sum_{\langle\langle ij\rangle\rangle,s}\left(e^{i\phi_{ij}}\hat{c}^\dagger_{i,s}\hat{c}_{j,s} + \text{ h.c.}\right) - M\sum_{i,s}\epsilon_i\hat{c}^\dagger_{i,s}\hat{c}_{i,s}$$

For Neel AF, MF Hamiltonian = Free Hamiltonian with modified

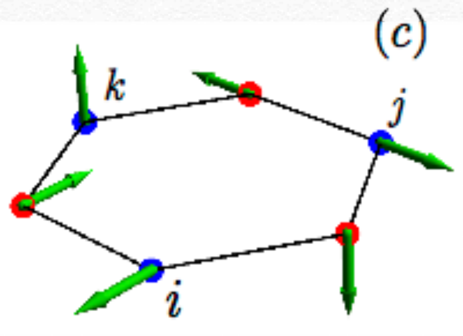
$$M \rightarrow M + sm$$

# Relation between magnetic order and topology

$$U\sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow} = \frac{1}{2}U\hat{N} - \frac{2}{3}U\sum_i \mathbf{S}_i^2$$
$$\approx \frac{1}{2}U\hat{N} + \sum_i\left(-\mathbf{m}_i\cdot\mathbf{S}_i + \frac{3\mathbf{m}_i^2}{8U}\right)$$

## Mean-field Hamiltonian

$$\hat{H}_{\mathrm{MF}} = \hat{H}_{\mathrm{H}} - \sum_i \mathbf{m}_i\cdot\mathbf{S}_i$$

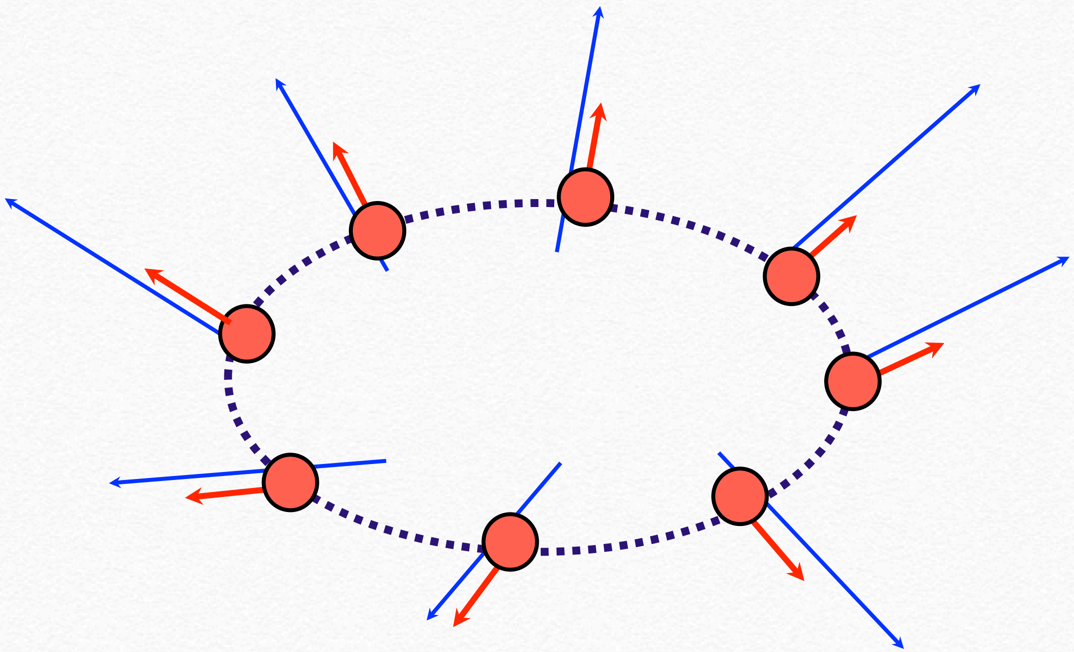



$$\mathcal{S} = \langle\hat{\mathbf{S}}_i\rangle\cdot(\langle\hat{\mathbf{S}}_j\rangle\times\langle\hat{\mathbf{S}}_k\rangle)$$

## Free Hamiltonian

$$\hat{H}_{\mathrm{H}} = -t_1\sum_{\langle ij\rangle,s}\left(\hat{c}^\dagger_{i,s}\hat{c}_{j,s} + \mathrm{h.c.}\right)$$
$$- t_2\sum_{\langle\langle ij\rangle\rangle,s}\left(e^{i\phi_{ij}}\hat{c}^\dagger_{i,s}\hat{c}_{j,s} + \mathrm{h.c.}\right) - M\sum_{i,s}\epsilon_i\hat{c}^\dagger_{i,s}\hat{c}_{i,s}$$

# Magnetic field texture and gauge field



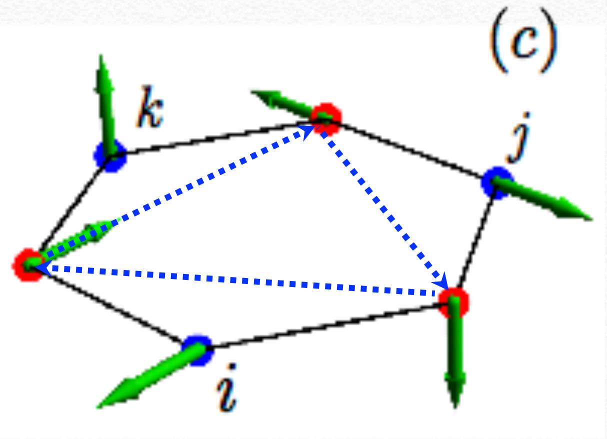
Berry phase == solid angle expanded by spin vector

# Relation between magnetic order and topology

$$U\sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow} = \frac{1}{2}U\hat{N} - \frac{2}{3}U\sum_i \mathbf{S}_i^2$$
$$\approx \frac{1}{2}U\hat{N} + \sum_i\left(-\mathbf{m}_i\cdot\mathbf{S}_i + \frac{3\mathbf{m}_i^2}{8U}\right)$$

## Mean-field Hamiltonian

$$\hat{H}_{\mathrm{MF}} = \hat{H}_{\mathrm{H}} - \sum_i \mathbf{m}_i\cdot\mathbf{S}_i$$



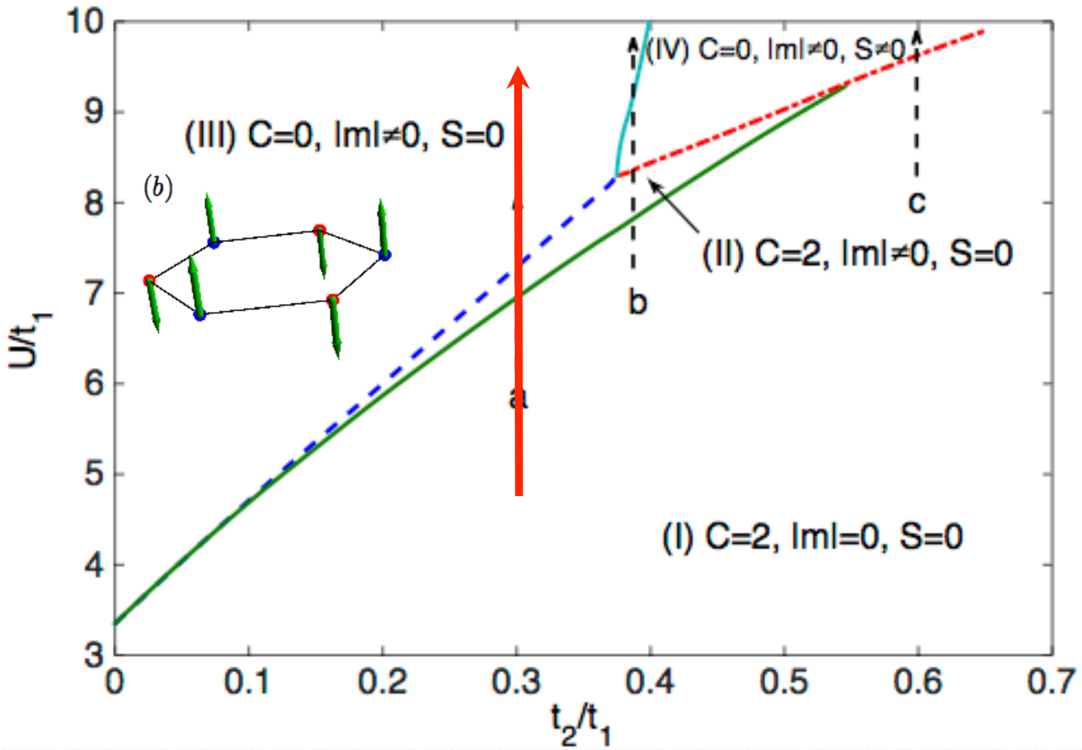

## Free Hamiltonian

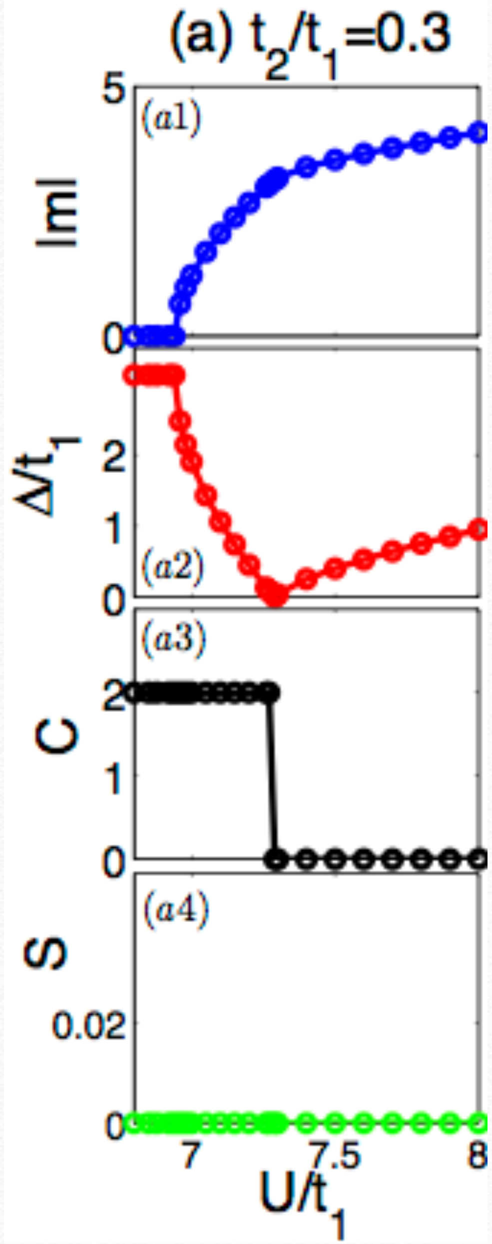
$$\hat{H}_{\mathrm{H}} = -t_1\sum_{\langle ij\rangle,s}\left(\hat{c}^\dagger_{i,s}\hat{c}_{j,s} + \mathrm{h.c.}\right)$$
$$-t_2\sum_{\langle\langle ij\rangle\rangle,s}\left(e^{i\phi_{ij}}\hat{c}^\dagger_{i,s}\hat{c}_{j,s} + \mathrm{h.c.}\right) - M\sum_{i,s}\epsilon_i\hat{c}^\dagger_{i,s}\hat{c}_{i,s}$$

For Canted AF, MF Hamiltonian = free Hamiltonian with modified

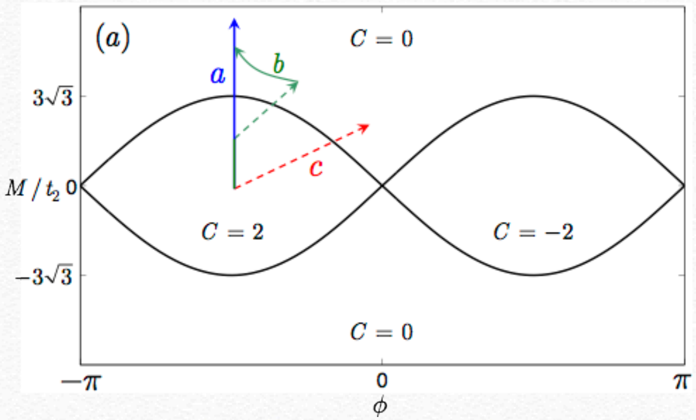
$$\phi_{\mathrm{eff}} = \phi + \tilde{\phi}.$$

# How magnetic order driven topological transition

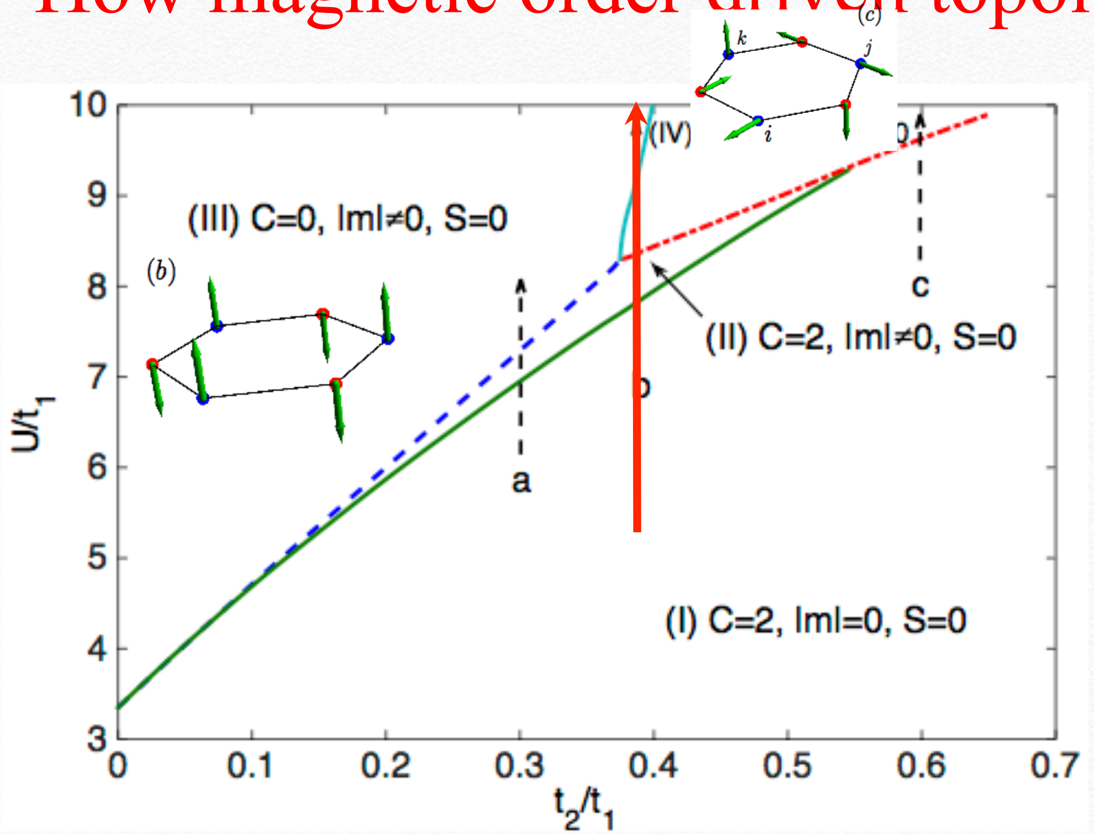


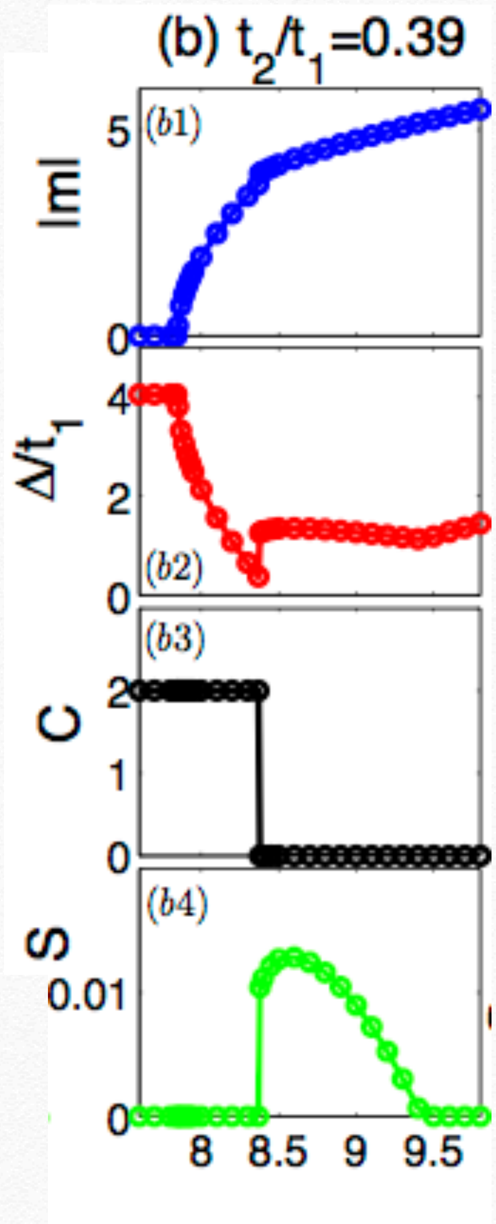




$$M \rightarrow M + sm$$



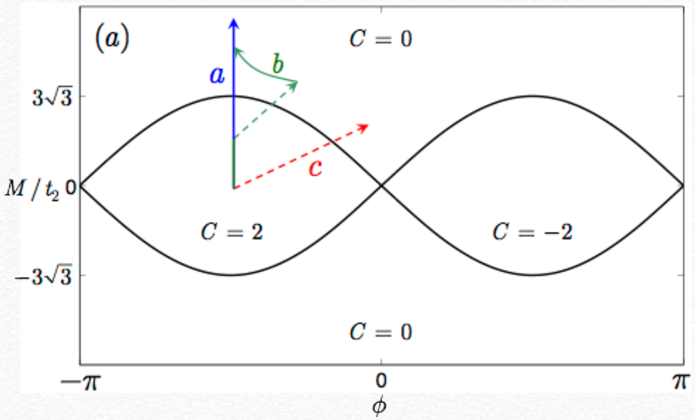
# How magnetic order driven topological transition



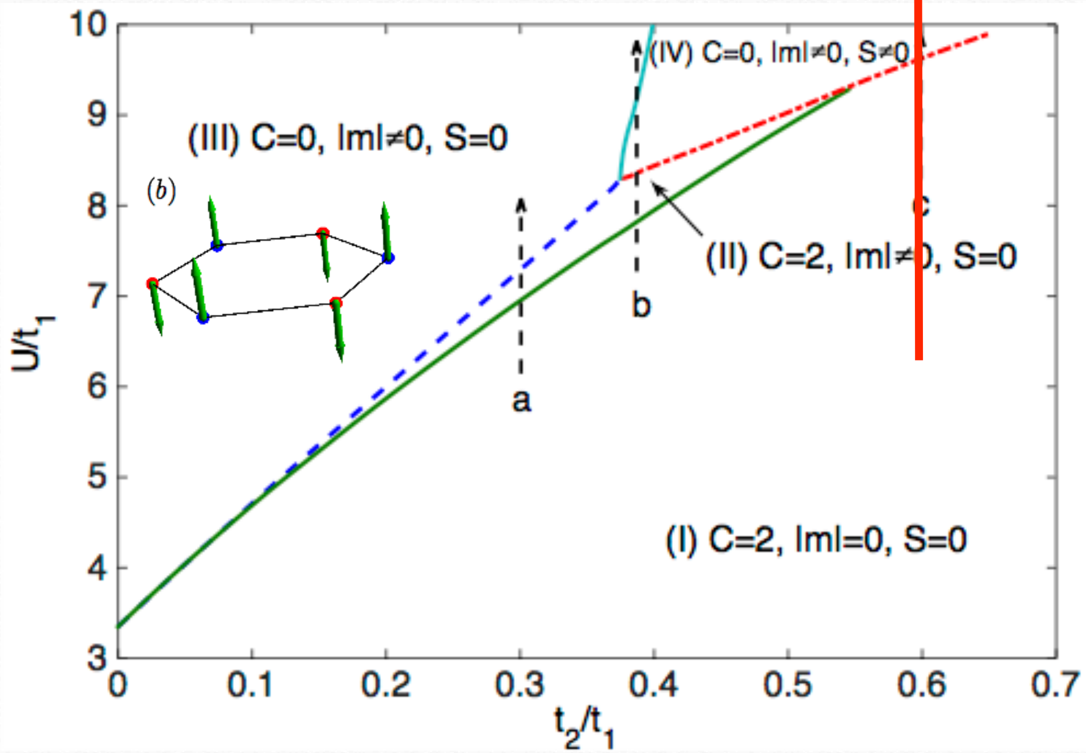



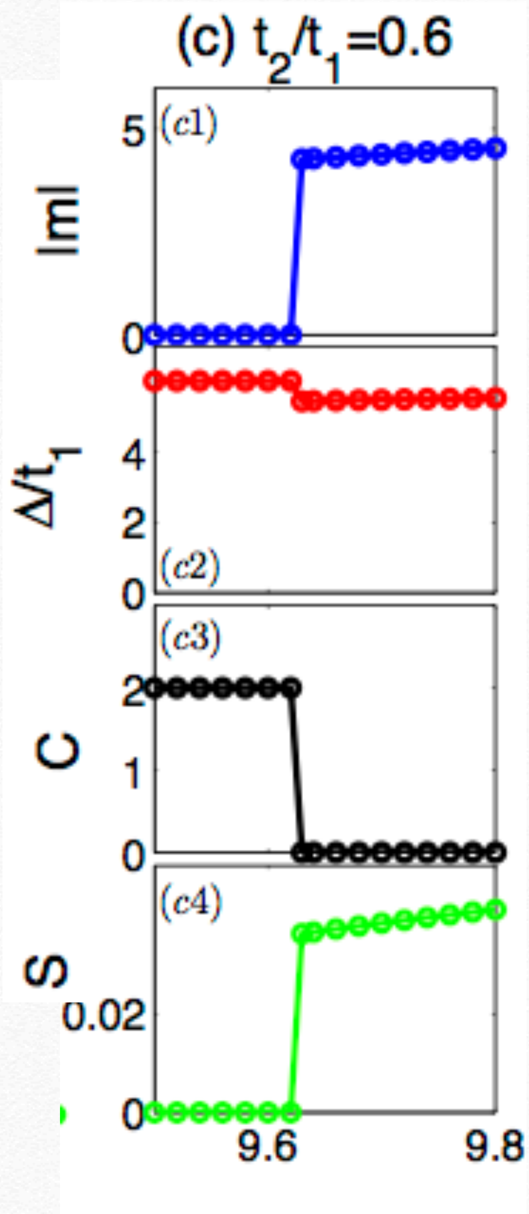

$$M \rightarrow M + sm$$

$$\phi_{\text{eff}} = \phi + \tilde{\phi}.$$

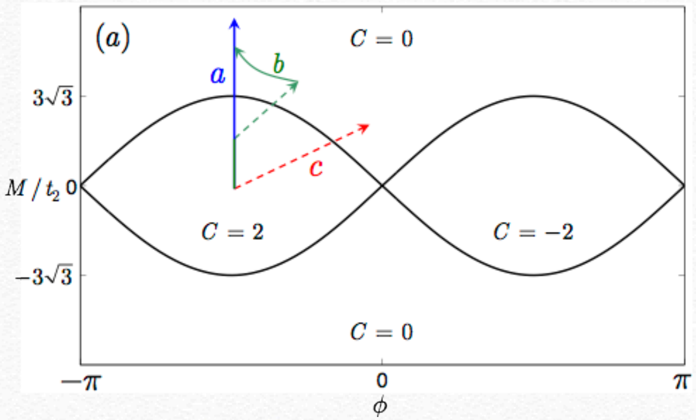


# How magnetic order driven topological transition

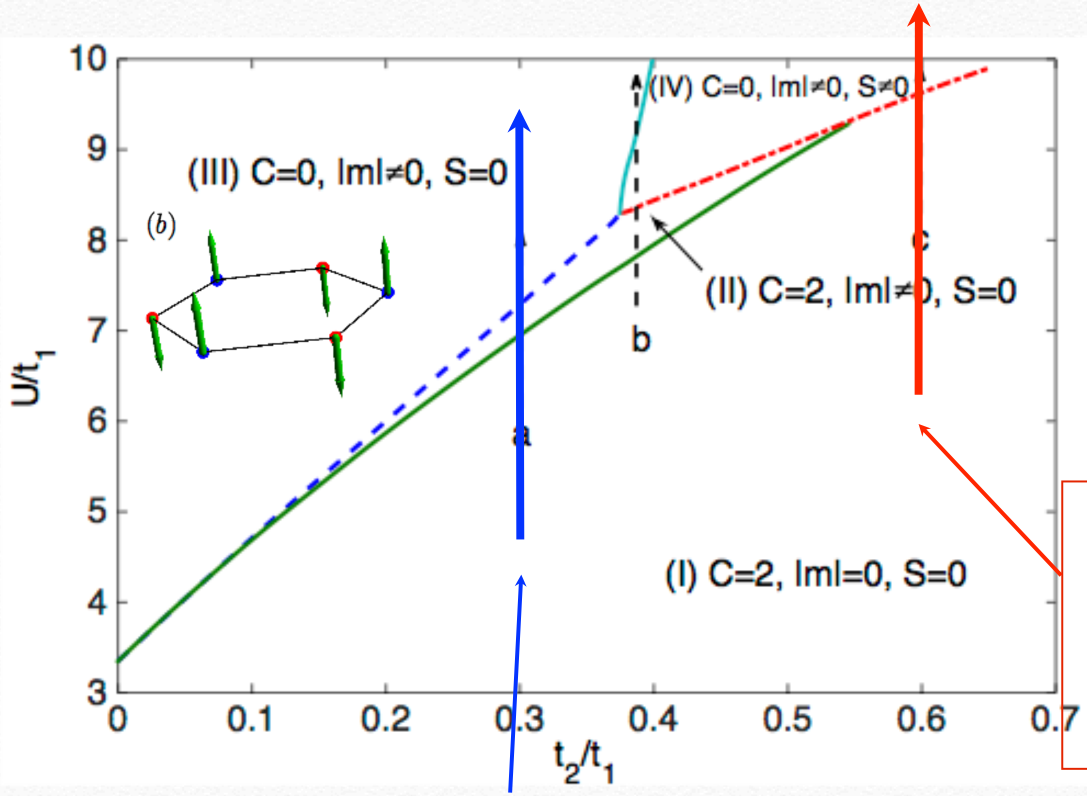



$$\phi_{\rm eff} = \phi + \tilde{\phi}.$$



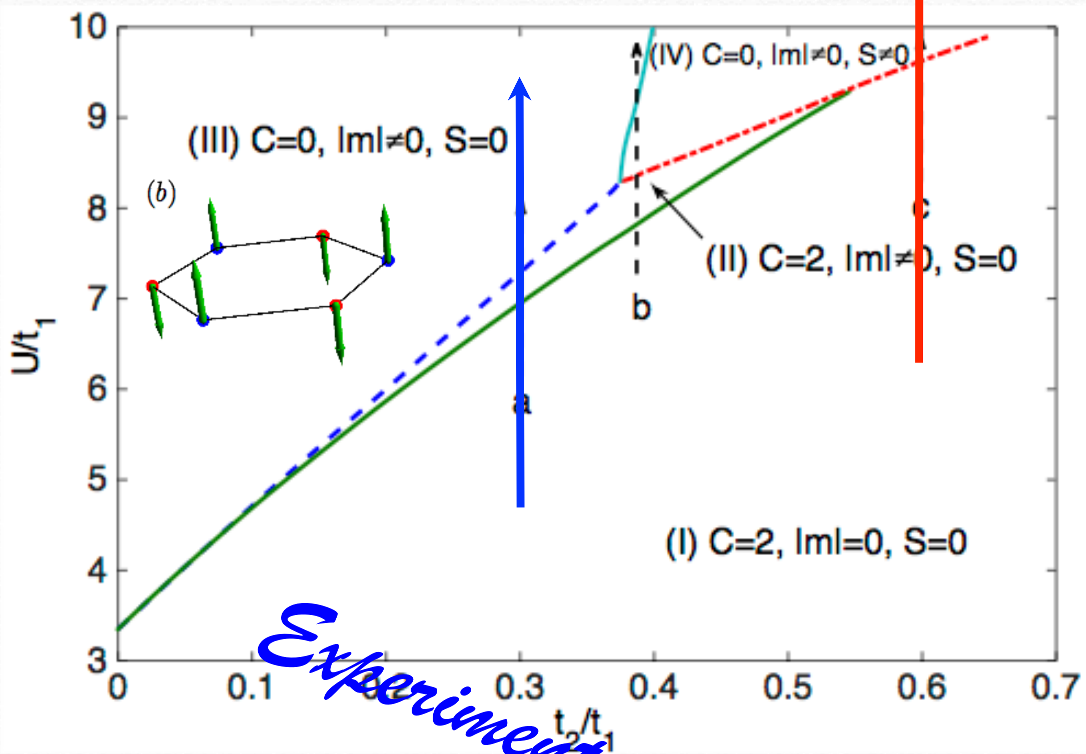
# Magnetic order driven topological transition



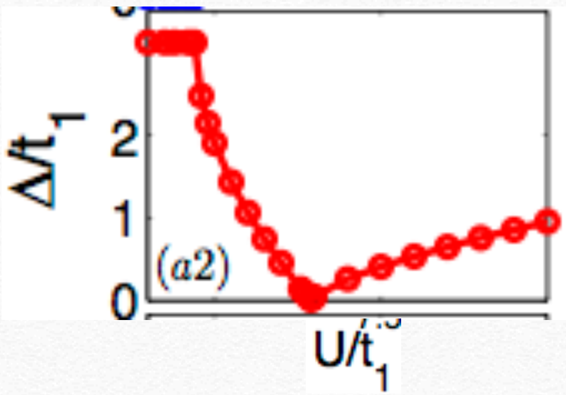

First-order scenario:
Magnetic order increases
discontinuously
Gap does not close

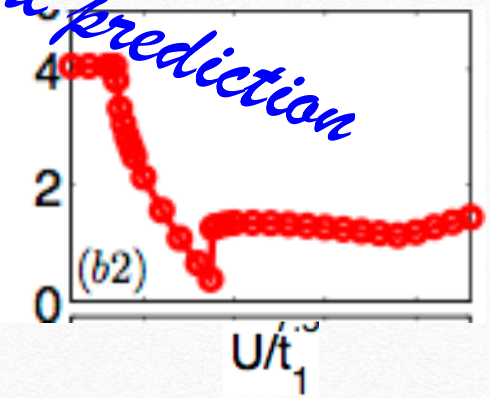
Second-order scenario:
Magnetic order increases continuously
Gap closes

# Magnetic order driven topological transition



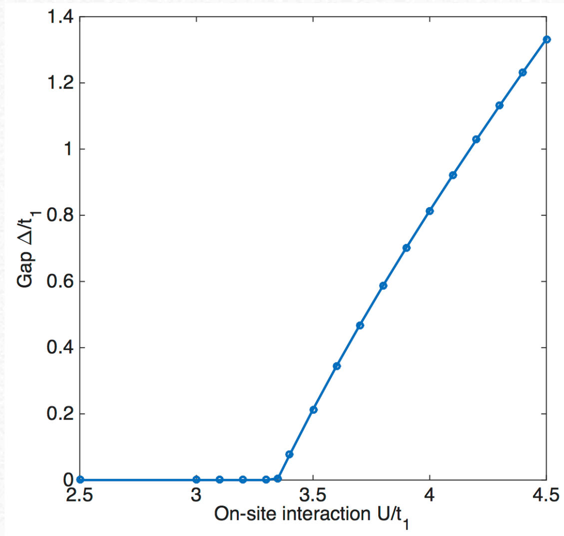

Experimental prediction




In contrast with non-topological case

$t_2 = 0$




How about finite temperature ?

# Field Theory Approach

$$S = \int dt d^2\mathbf{r} \left(\mathcal{L}_{\mathrm{n}} + \mathcal{L}_{\mathrm{f}} + \mathcal{L}_{\mathrm{I}}\right)$$

$$\mathcal{L}_{\mathrm{n}} = \frac{1}{2g}\left[(\partial_t \mathbf{n})^2 - c^2(\nabla \mathbf{n})^2\right]$$

Nonlinear Sigma model:
Neel order/Fluctuation of Neel order

$$\mathcal{L}_{\mathrm{f}} = \Psi^\dagger\left[i\partial_t + iv_{\mathrm{F}}\tau_z\sigma_x\partial_x + iv_{\mathrm{F}}\sigma_y\partial_y - m\tau_z\sigma_z\right]\Psi$$

Low-energy fermions

$\sigma$ sublattices

$\tau$ valley

$$\mathcal{L}_{\mathrm{I}} = -\lambda\Psi^\dagger[\sigma_z \otimes (\mathbf{n}\cdot\mathbf{s})]\Psi$$

Interaction between
Neel order and fermions

# Field Theory Approach

$$S = \int dt d^2\mathbf{r} \left(\mathcal{L}_\mathrm{n} + \mathcal{L}_\mathrm{f} + \mathcal{L}_\mathrm{I}\right)$$

$$\mathcal{L}_\mathrm{n} = \frac{1}{2g}\left[(\partial_t \mathbf{n})^2 - c^2(\nabla \mathbf{n})^2\right]$$

$$\mathcal{L}_\mathrm{f} = \Psi^\dagger\left[i\partial_t + iv_\mathrm{F}\tau_z\sigma_x\partial_x + iv_\mathrm{F}\sigma_y\partial_y - m\tau_z\sigma_z\right]\Psi$$

$$-\Psi^\dagger\sigma_z \otimes (m\tau_z \otimes I + \lambda I \otimes s_z)\Psi$$

$$\mathcal{L}_\mathrm{I} = -\lambda\Psi^\dagger[\sigma_z \otimes (\mathbf{n}\cdot\mathbf{s})]\Psi \quad \Rightarrow \quad -\lambda\Psi^\dagger\sigma_z s_z\Psi$$

Magnetic ordered phase $\langle \mathbf{n} \rangle = 1\hat{z}$

Topological phase transition at $\lambda = m$

# Field Theory Approach

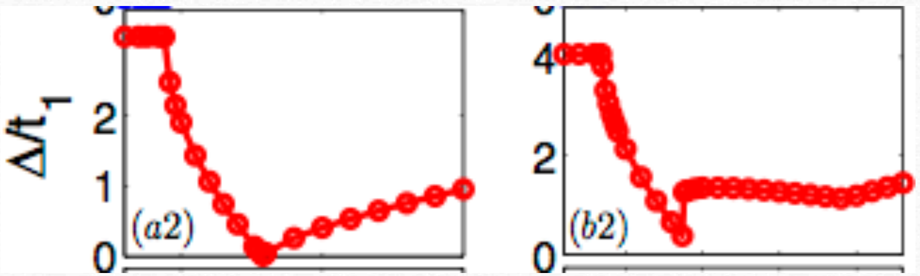
$$S = \int dt d^2\mathbf{r} \left(\mathcal{L}_{\rm n} + \mathcal{L}_{\rm f} + \mathcal{L}_{\rm I}\right)$$

$$\mathcal{L}_{\rm n} = \frac{1}{2g}\left[(\partial_t \mathbf{n})^2 - c^2(\nabla \mathbf{n})^2\right]$$

$$\mathcal{L}_{\rm f} = \Psi^\dagger\left[i\partial_t + iv_{\rm F}\tau_z\sigma_x\partial_x + iv_{\rm F}\sigma_y\partial_y - m\tau_z\sigma_z\right]\Psi$$

$$\mathcal{L}_{\rm I} = -\lambda\Psi^\dagger[\sigma_z \otimes (\mathbf{n}\cdot\mathbf{s})]\Psi$$

Suitable for studying fermion gap about Neel temperature with AF fluctuations

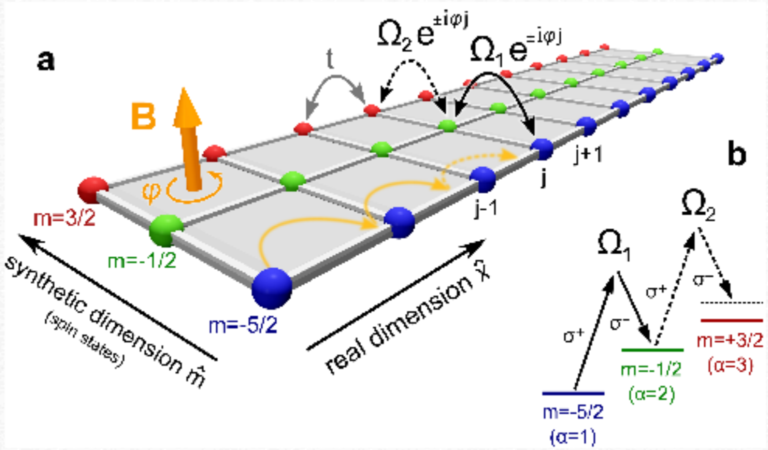


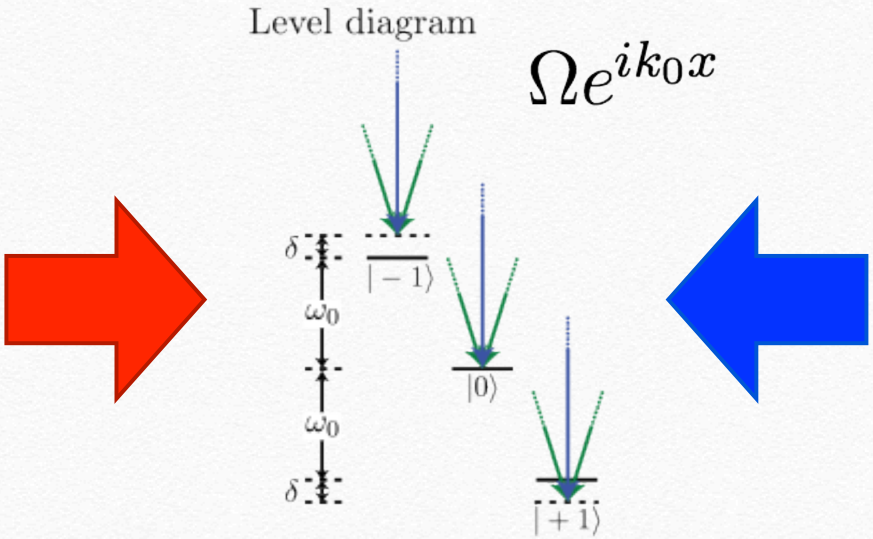

Work in progress .......

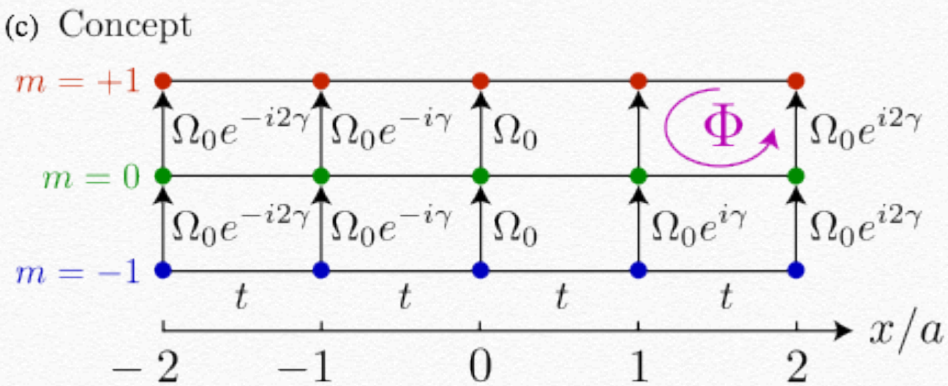
# Part B

# Synthetic Dimension

# Concept of Synthetic Dimension



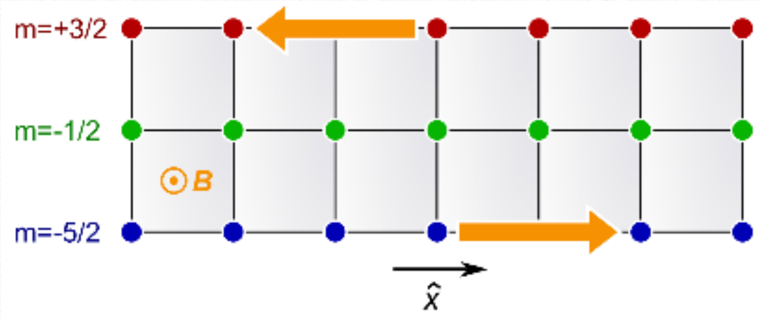

well defined edge

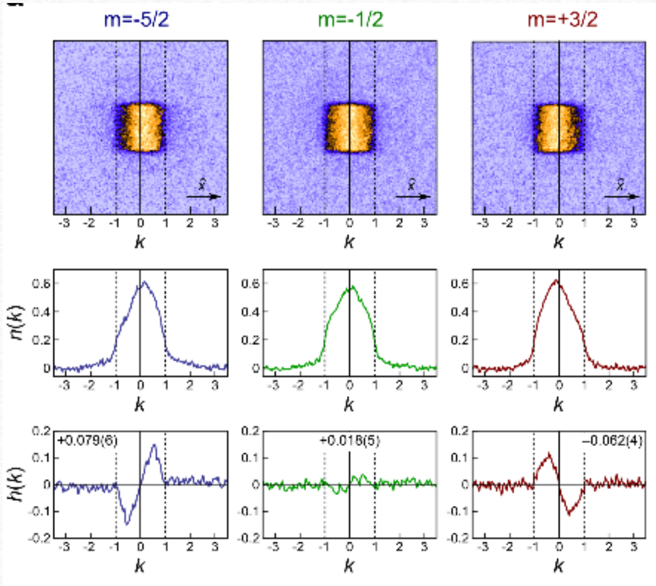
A. Celi et.al. PRL, 112, 043001 (2014)

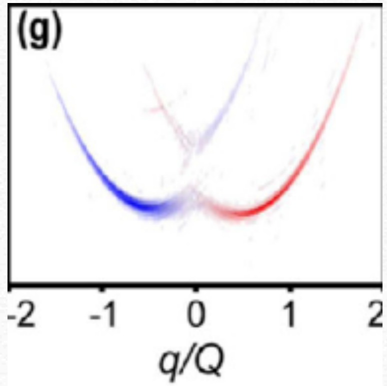
# Experimental Progresses on Synthetic Dimension

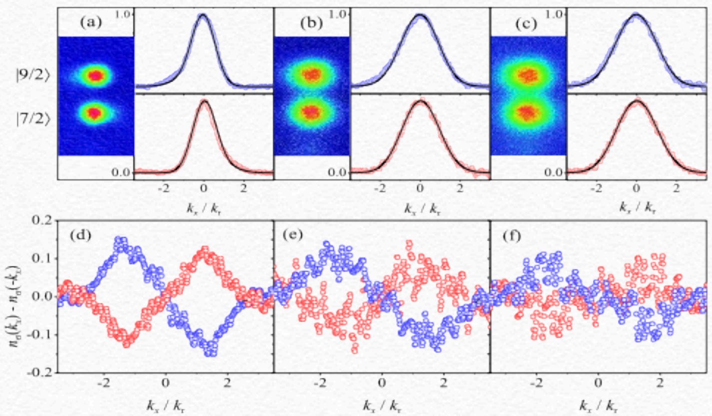
Magnetic flux lattice in 2d:
Chiral edge state

Spin-orbit coupling in 1d:
Spin-momentum locking



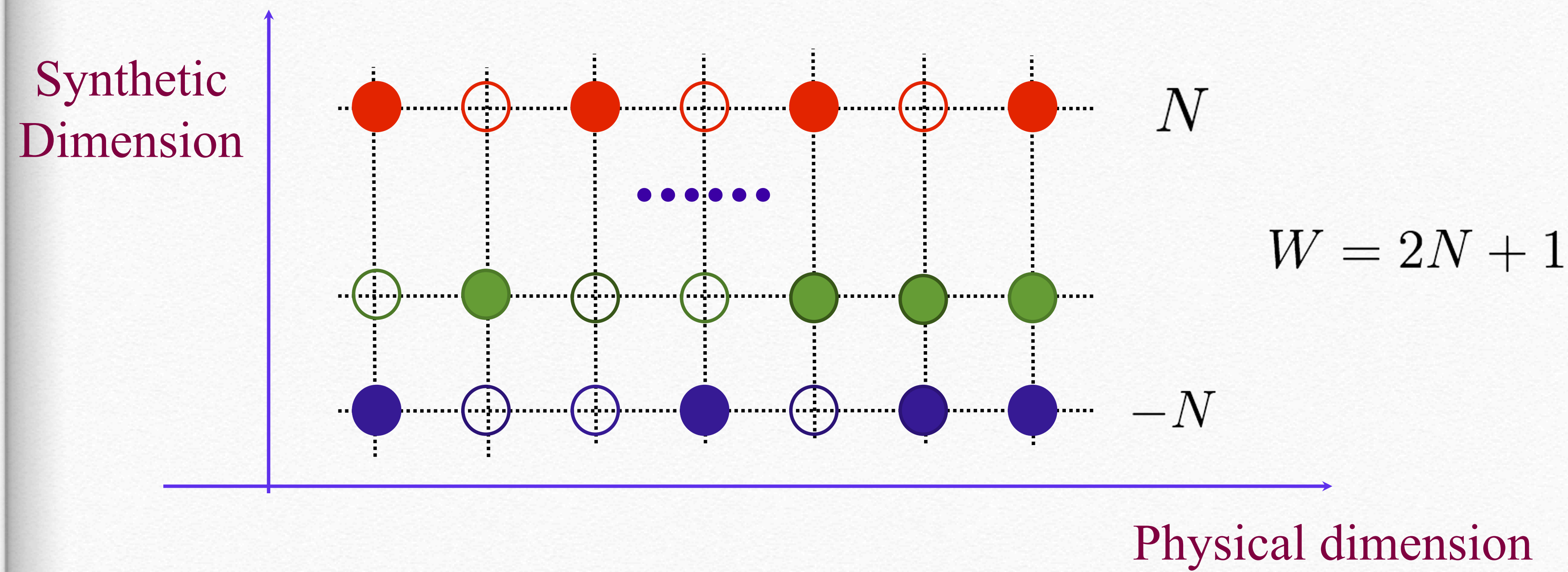




M. Mancini, et.al. (Florence), arXiv: 1502.02495
B. K. Stuhl, et.al. (NIST), arXiv: 1502.02496

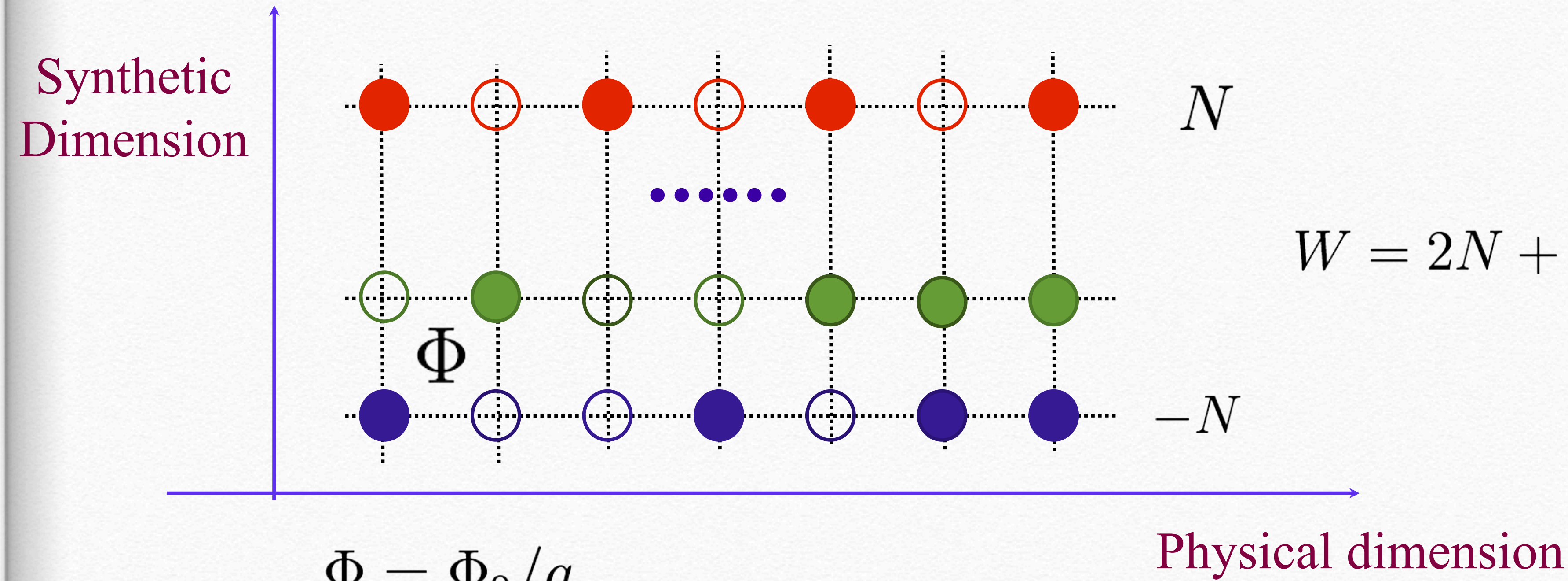
P. Wang et.al. (Shanxi and Tsinghua)
PRL. 109, 095301 (2012)
L. W. Cheuk et.al. (MIT), PRL, 109, 095302 (2012)

# Interaction Effects on Synthetic Dimension




1. Interaction is short-ranged in physical dimension, but long-ranged in synthetic dimension.

# Interaction Effects on Synthetic Dimension




$$W = 2N + 1$$

$$\Phi = \Phi_0/q$$

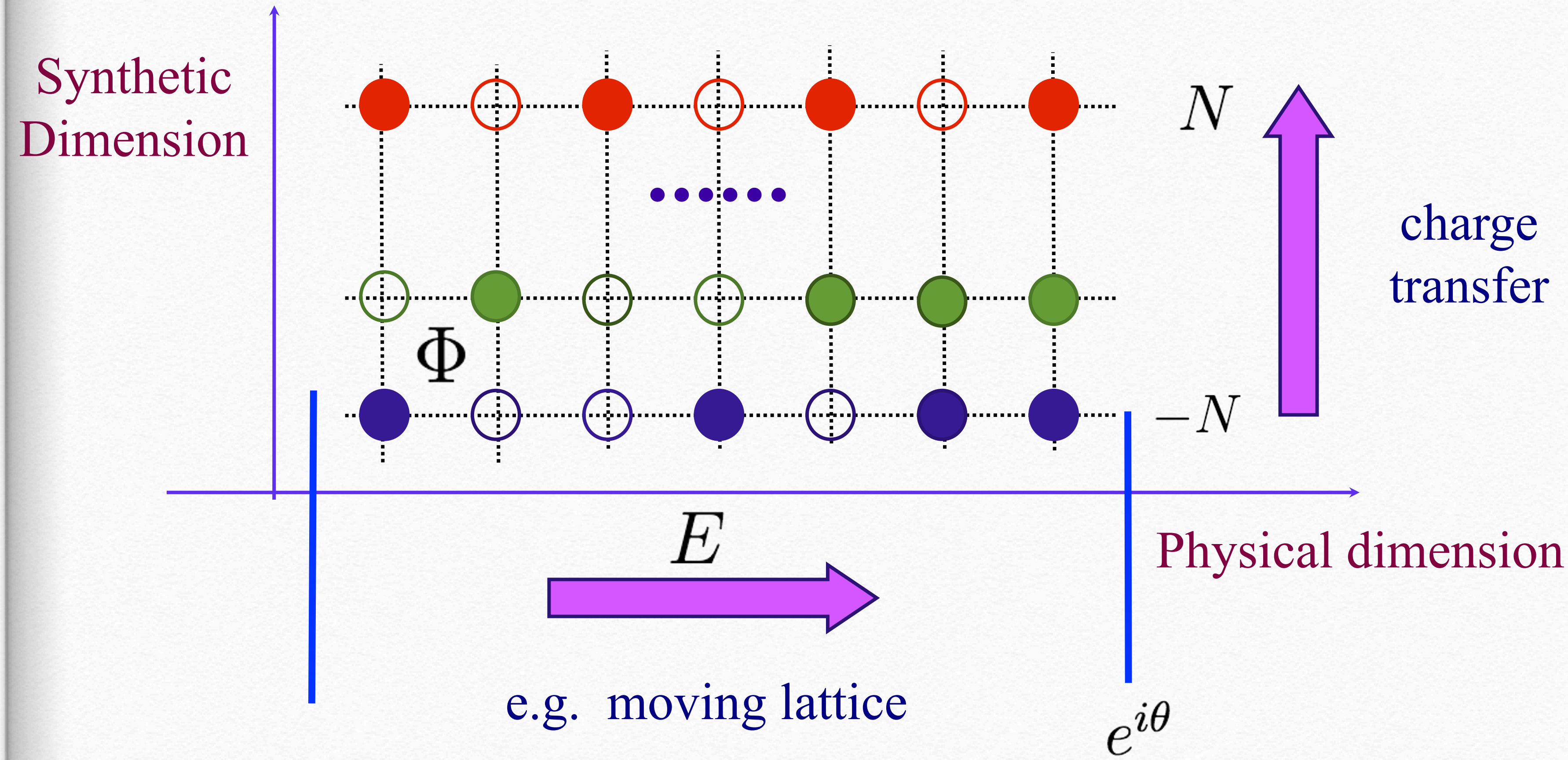
$$\nu = \frac{N}{N_{flux}} = \frac{N}{WL/q} = \frac{Nq}{LW} \qquad \nu_{1d} = \frac{N}{L}$$

$\nu = 1/3$ Fractional QH ?

$\nu_{1d} = 1$ Mott insulator ?

2. $\nu$ is not the only relevant parameter.

# Interaction Effects on Synthetic Dimension




Charge pumping:

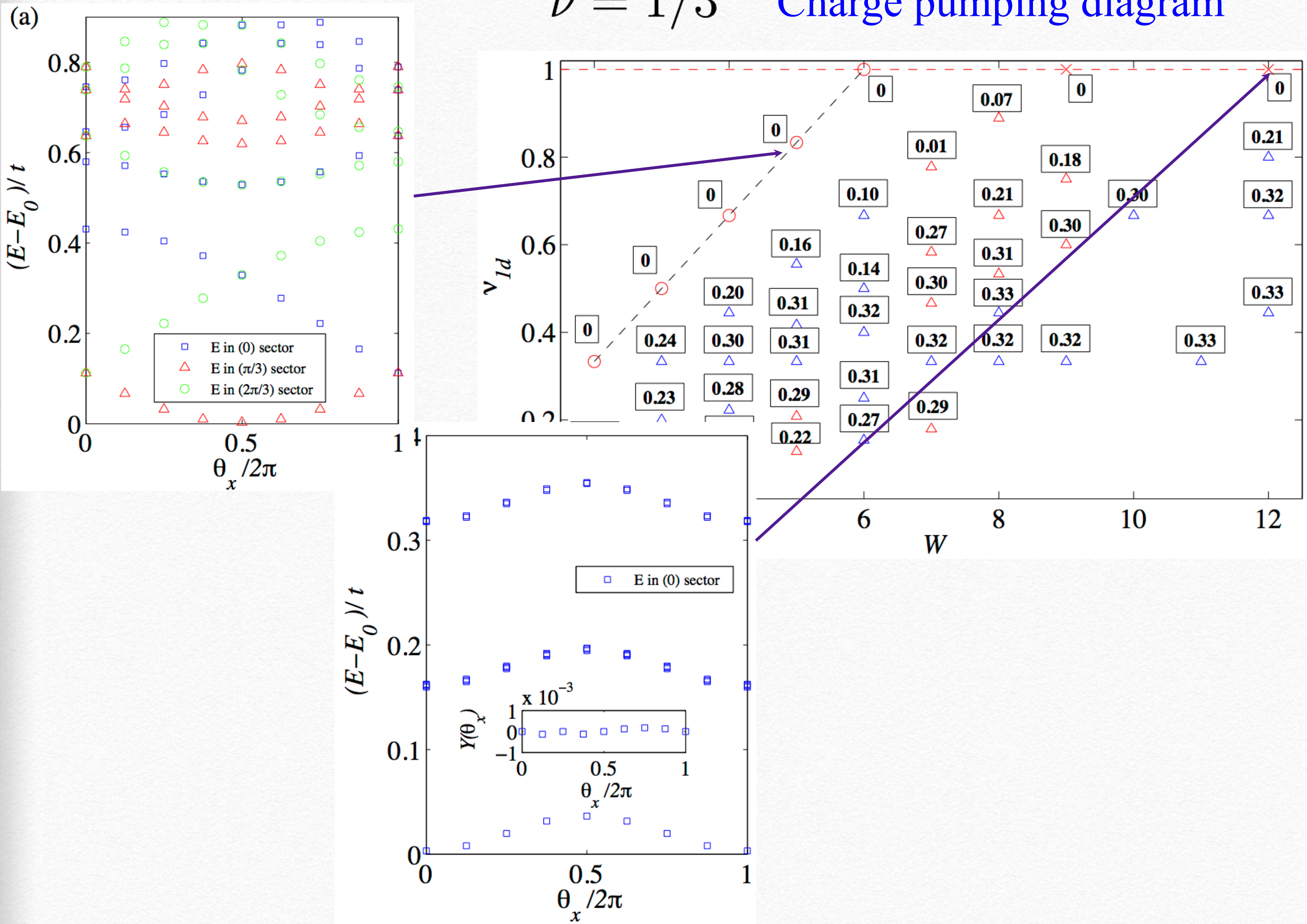
Twisted boundary condition along physical dimension

Detecting change of spin population

$$P = \frac{1}{W}[\langle M_z\rangle(\theta = 2\pi) - \langle M_z\rangle(\theta = 0)]$$

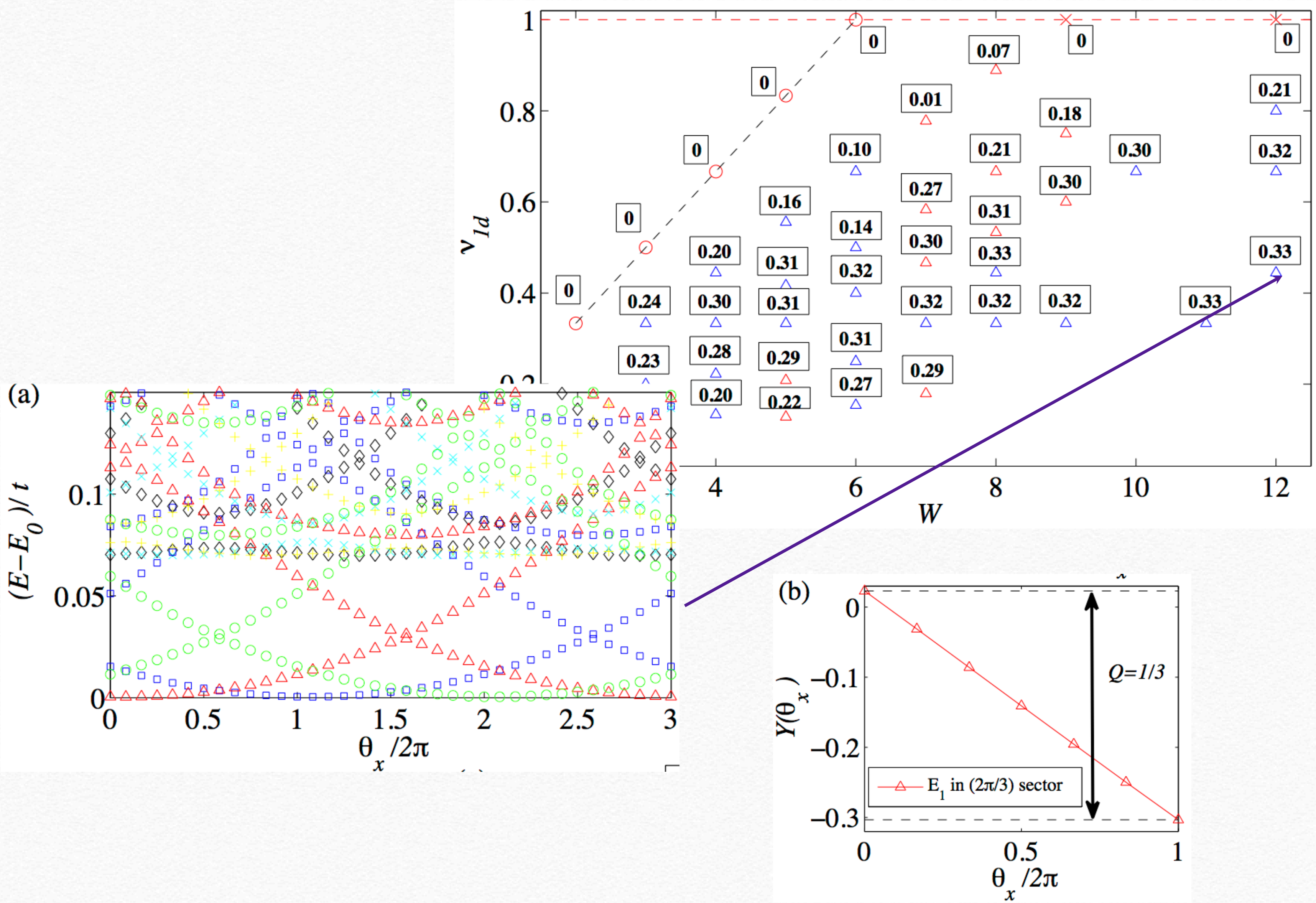
# Interaction Effects on Synthetic Dimension

$\nu = 1/3$ Charge pumping diagram



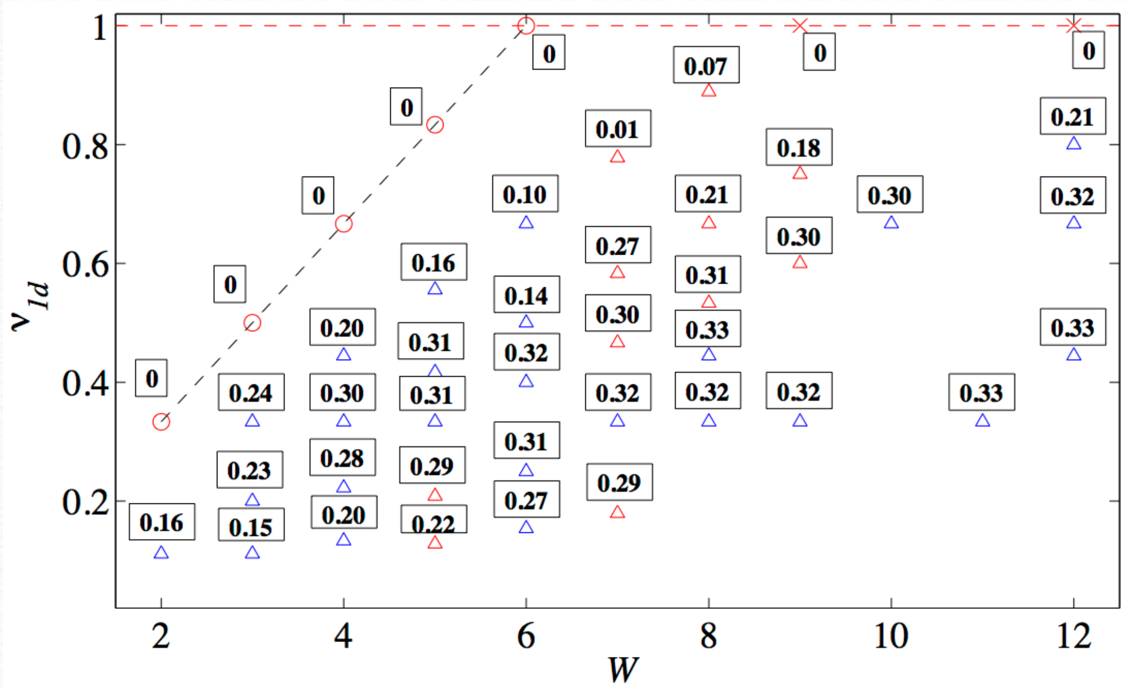
# Interaction Effects on Synthetic Dimension

$\nu = 1/3$ Charge pumping diagram



# Interaction Effects on Synthetic Dimension

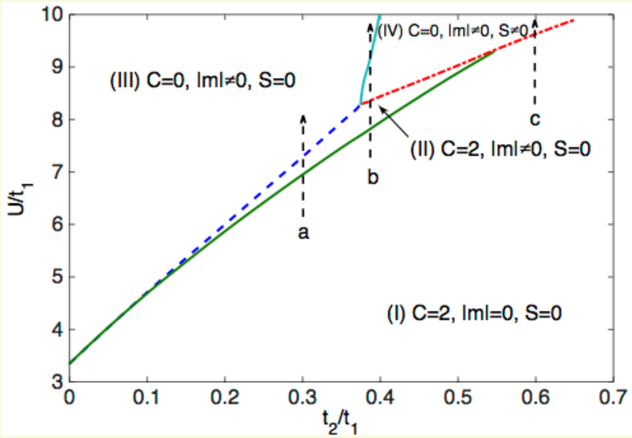
$\nu = 1/3$ Charge pumping diagram

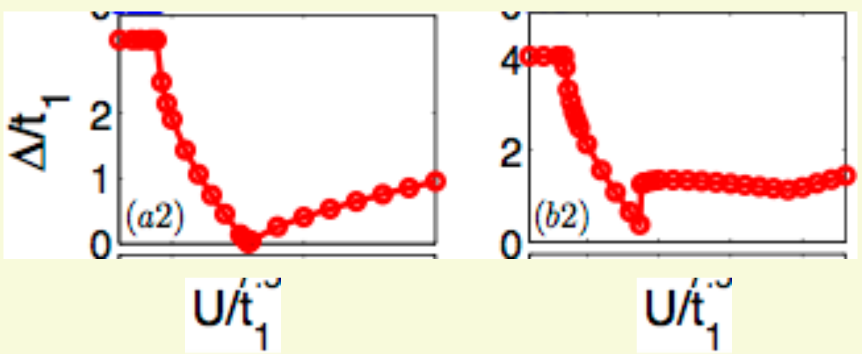



The smaller $\nu_{1d}$ the larger $W$ Charge pumping ==> $1/3$

# Conclusion

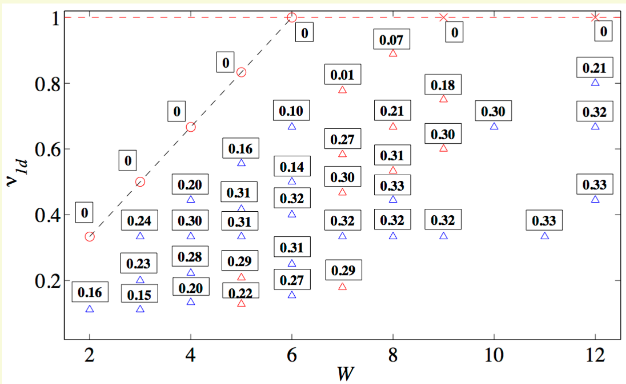
## Haldane-Hubbard Model

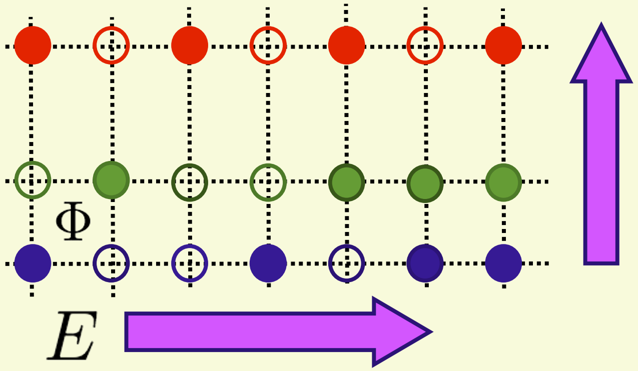



W. Zheng, H. T. Shen. Z. Wang and HZ, arXiv: 1501.024t5

## Fractional Charge Pumping in Synthetic Dimension




T. S. Zeng, C. Wang and HZ , to appear

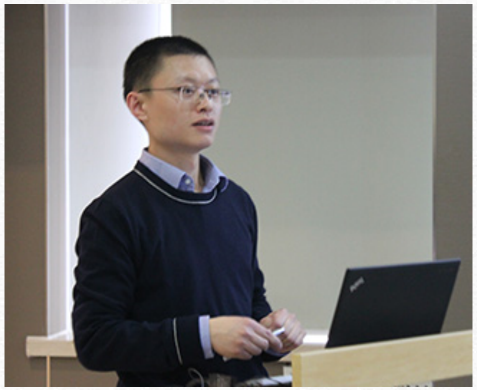
# Collaborators


Wei Zheng

Hui-Tao Shen

Tian-Sheng Zeng


Ce Wang



Zhong Wang

Thank you very much for your attention !