Superradiance of Degenerate Fermi Gases in a Cavity

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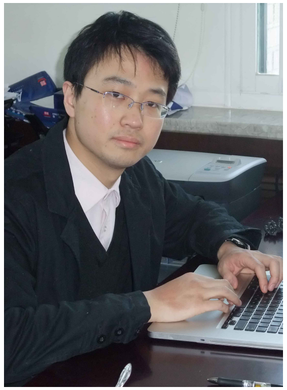
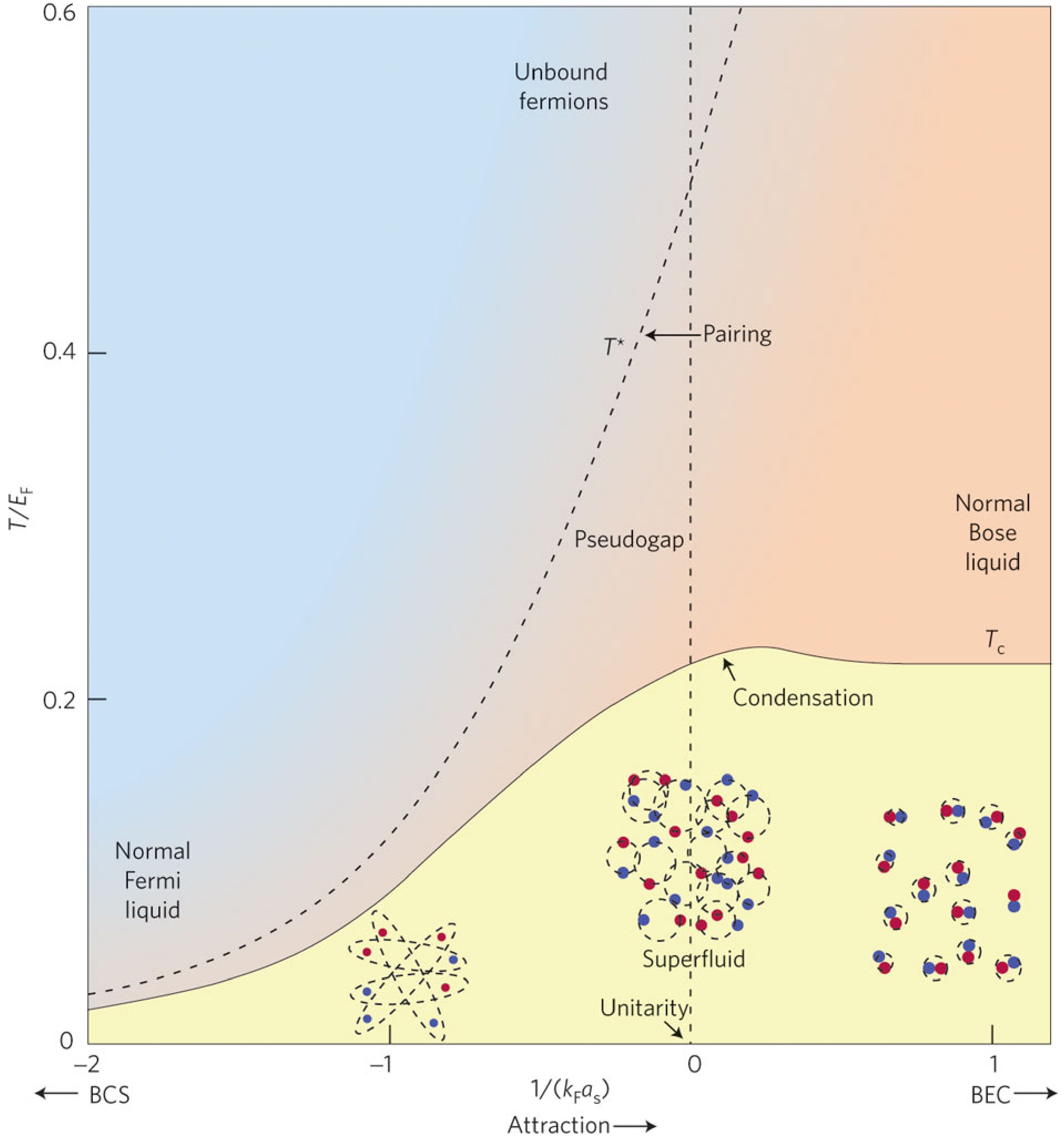
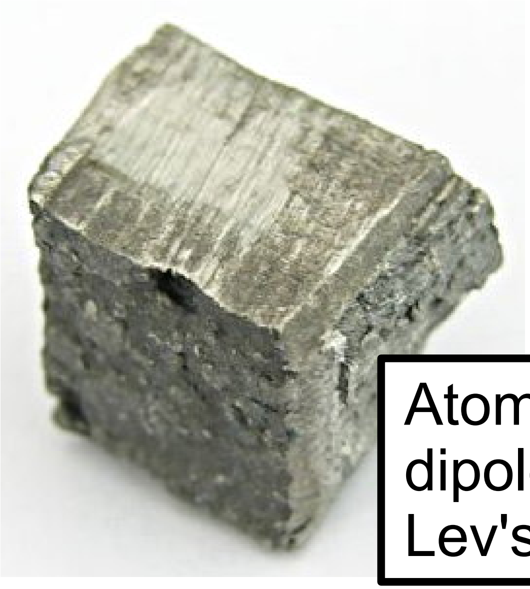


INT, April 10, 2015

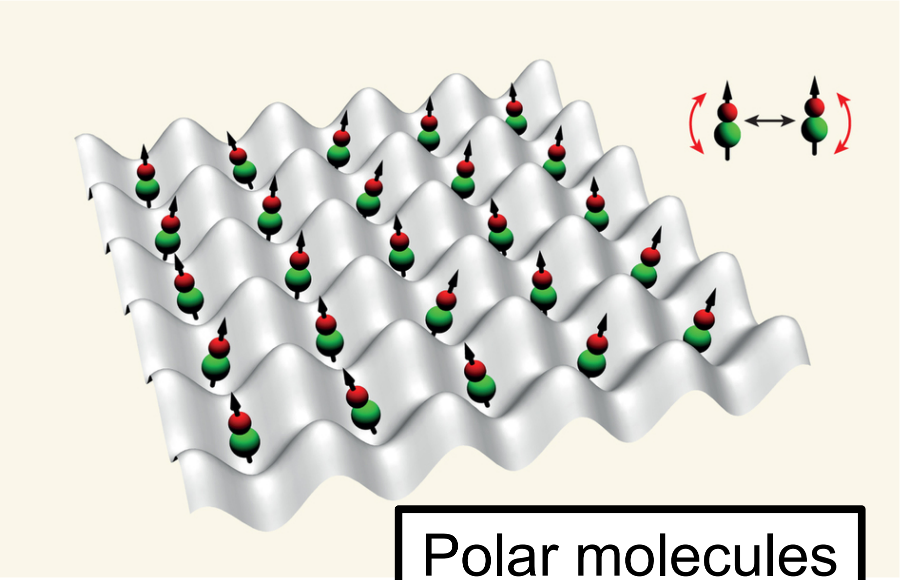
BEC-BCS Crossover



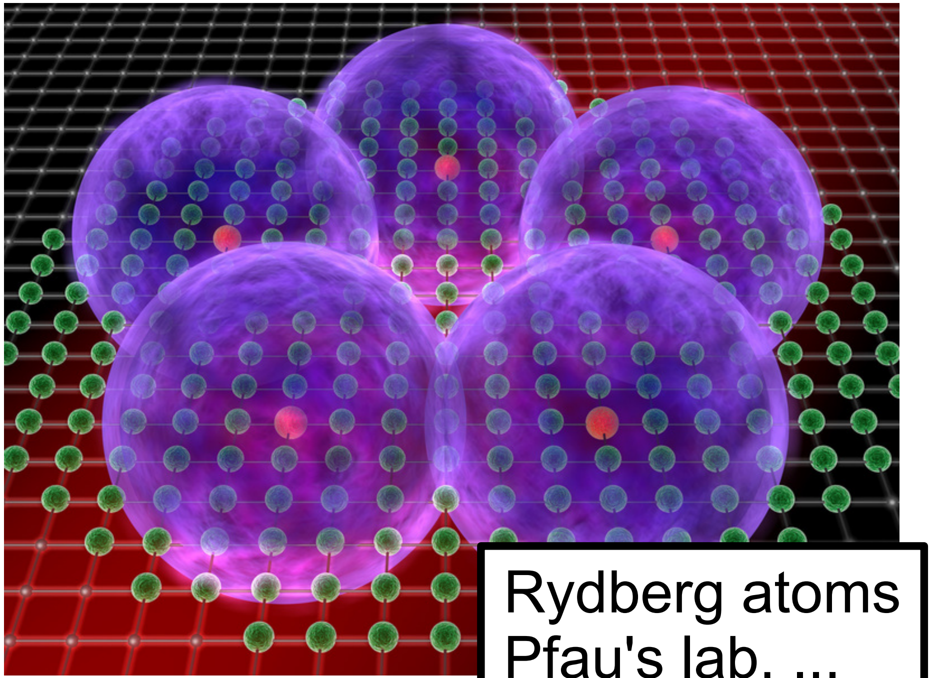
Realizing Long Range Interactions



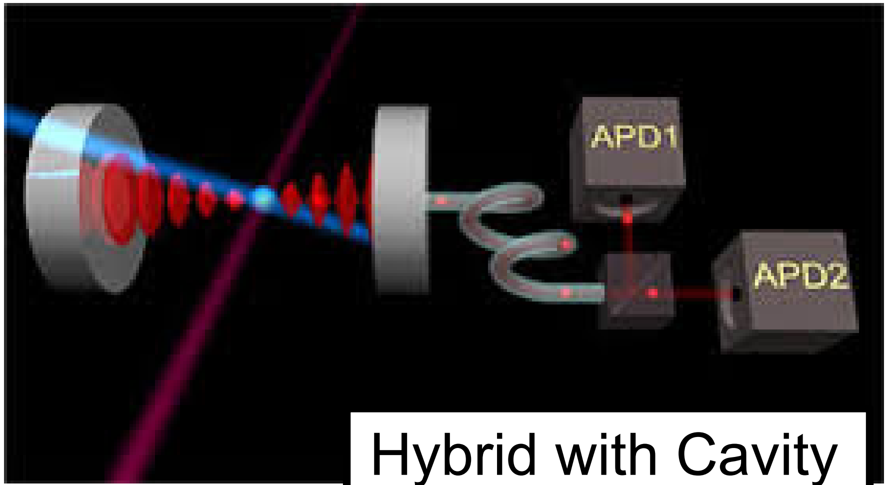
Atoms with magnetic dipole moments (Dy)
Lev's lab, ...



Polar molecules
JILA, ...



Rydberg atoms
Pfau's lab, ...



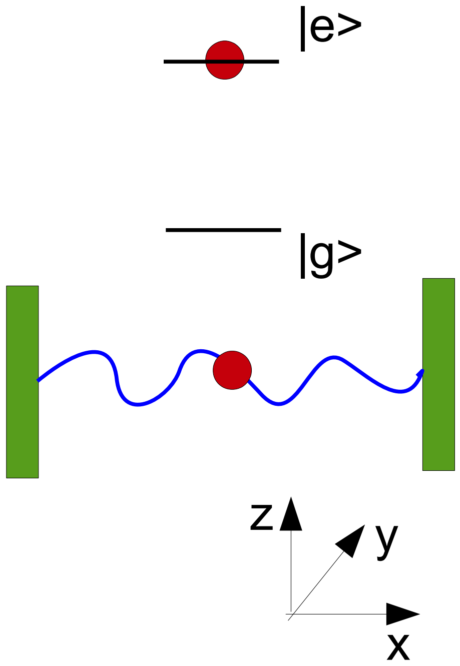
Hybrid with Cavity
Esslinger's Lab, ...

Cavity QED

*Discrete spectrum -> one or a few relevant cavity modes
*Possible strong coupling at ***SINGLE*** photon level

Electronic dipole coupling between single atoms and light field

$$h_{dip} = \boldsymbol{E} \cdot \boldsymbol{d}$$



With canonical quantization

$$h_{dip} = \sum_{\boldsymbol{k},\epsilon} g_{k,\epsilon} \left(i\, a_{\boldsymbol{k},\epsilon} e^{i \boldsymbol{k} \cdot \boldsymbol{r}} + h.c. \right)$$

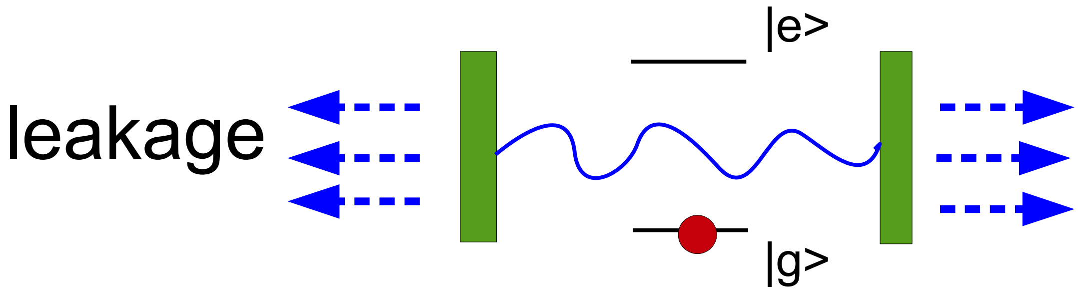
$$g_{k,\epsilon} = \sqrt{\frac{\omega_k}{2\Omega}} \boldsymbol{\epsilon} \cdot \boldsymbol{d} \sim \sqrt{\omega_k \frac{e^2}{a_0}} \sqrt{\frac{a_0^3}{\Omega}} \sim 10\, MHz$$

$e\, a_0$ $\quad 10^{14}\, Hz$ $\quad 1\, \mu m^3$

Strong Coupling Regime

Decay rate

$$\kappa \sim 1\,MHz$$

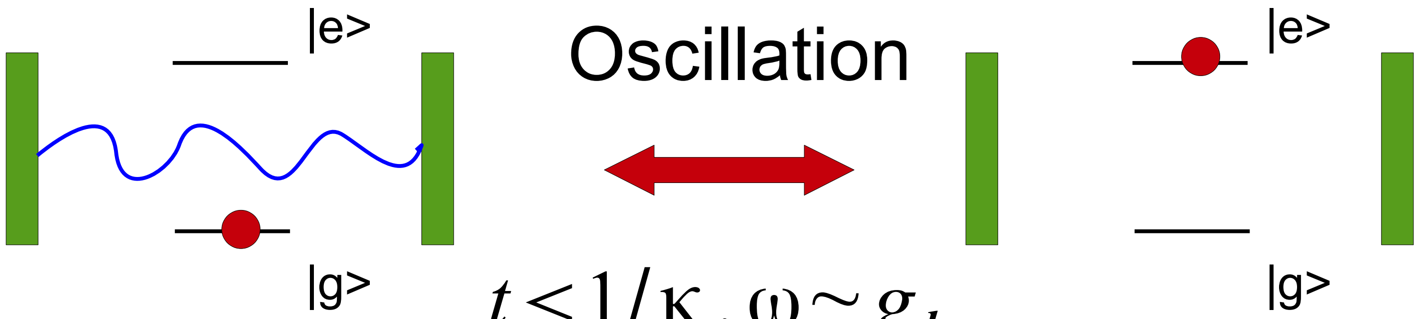


For a resonant cavity mode whose frequency is equal to the electronic excitation energy of the atom,

$$i\frac{\partial a_{\boldsymbol{k},\epsilon}}{\partial t} = -i\kappa a_{\boldsymbol{k},\epsilon} - i g_{\boldsymbol{k},\epsilon}\sigma_{-} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}$$

$$\sigma_{-} = |\mathrm{g}\rangle\langle\mathrm{e}|$$

$$i\frac{\partial}{\partial t}\sigma_{-} = i\,g_{\boldsymbol{k},\epsilon} a_{\boldsymbol{k},\epsilon} e^{i\boldsymbol{k}\cdot\boldsymbol{r}},$$



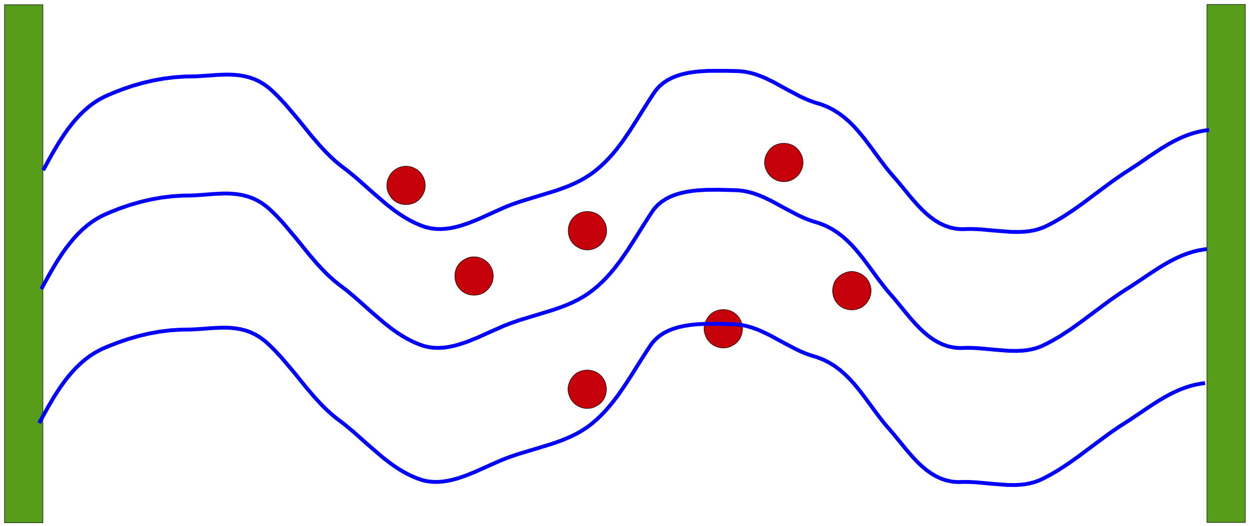
$$t < 1/\kappa,\ \omega \sim g_{\boldsymbol{k},\epsilon}$$

Mediated Interactions

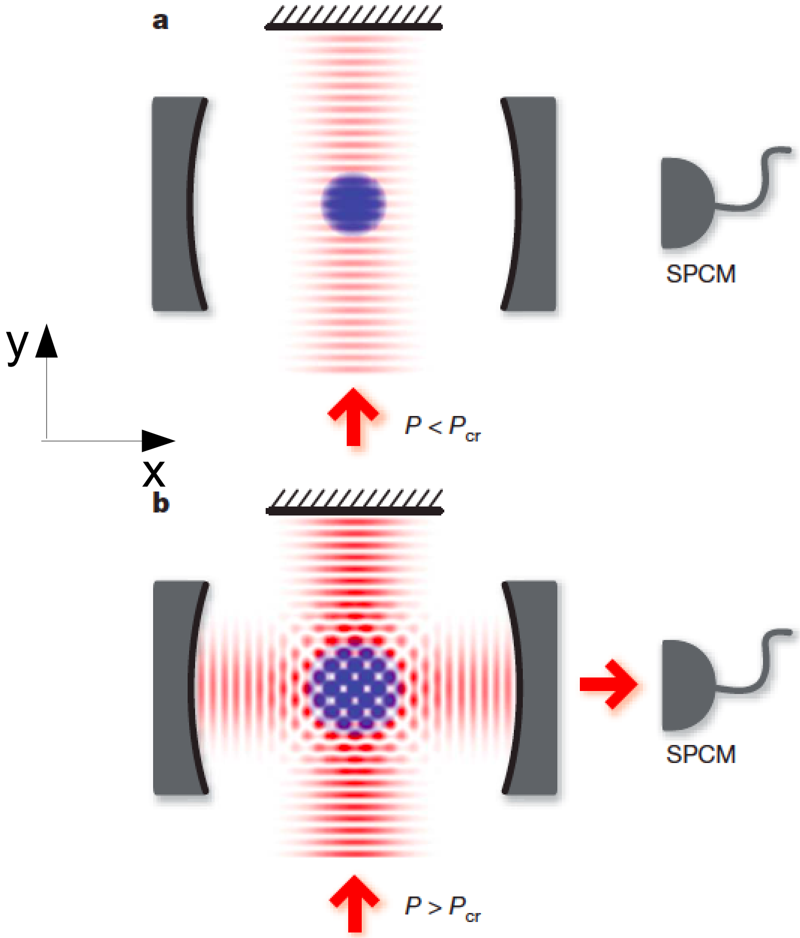
Atoms coupled to the same cavity field

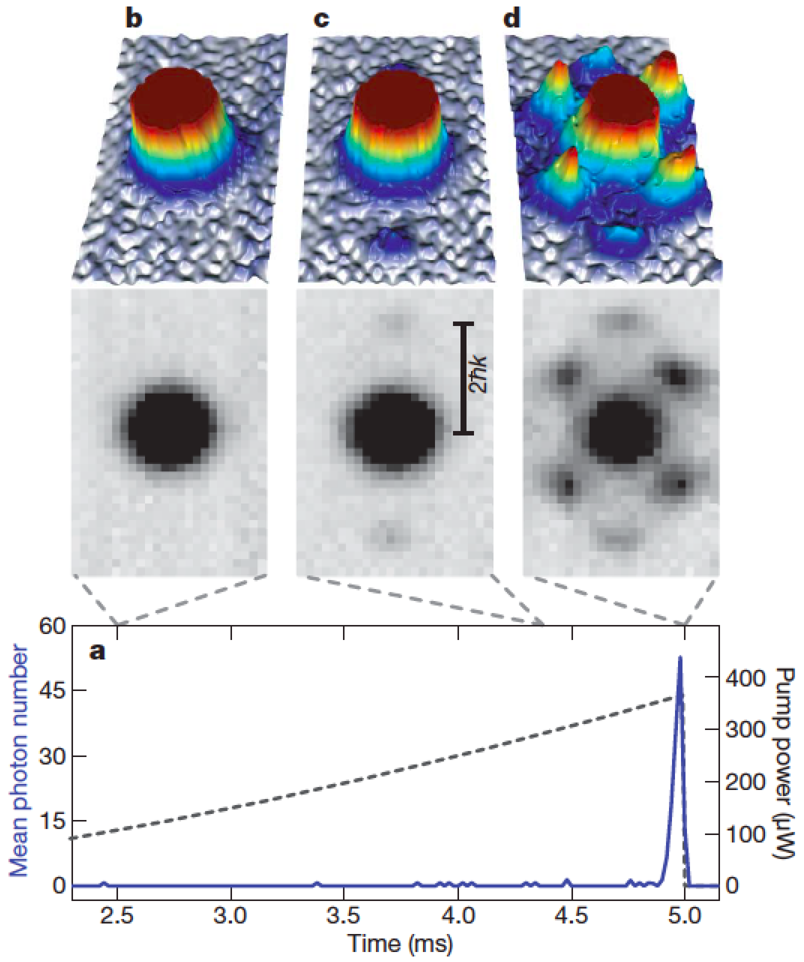
↓

Interatomic interactions via the cavity field



Superradiance of BEC





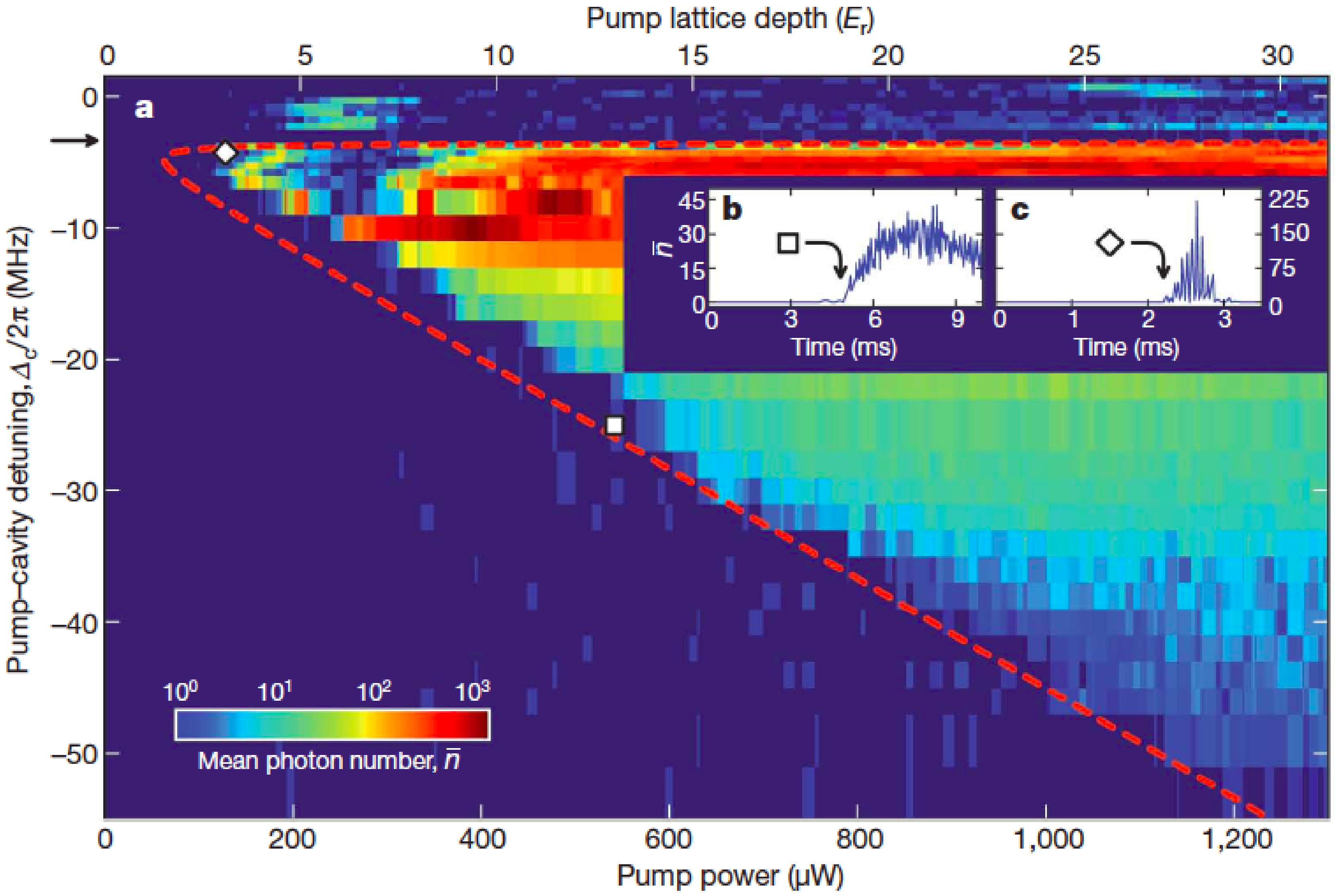
Red far-detuned pumping lasers

$$\Delta_a = \omega_p - \omega_a < 0$$

Cavity detuning $\Delta_c = \omega_p - \omega_c < 0$

K. Baumann *& el at*, Nature 464, 1301 (2010); PRL 107, 140402 (2011);
R. Mottl *& el at*, Science 336, 1570 (2012)

Phase Diagram



Explained by linear stability analysis of the Gross-Pitaevskii eqaution, K. Baumann & *el at*, Nature 464, 1301 (2010)

Transition related to the density correlations of the atomic gases

Superradiance of Degenerate Fermi Gases

Consider spinless fermions, no direct interatomic interactions

$$H=\int d\boldsymbol{r}[\psi^{+}(\boldsymbol{r})h_0\psi(\boldsymbol{r})]-\Delta_c a^{+}a$$

$$h_0=h_{at}+\eta(\boldsymbol{r})(a^{+}+a)+U(\boldsymbol{r})a^{+}a,$$

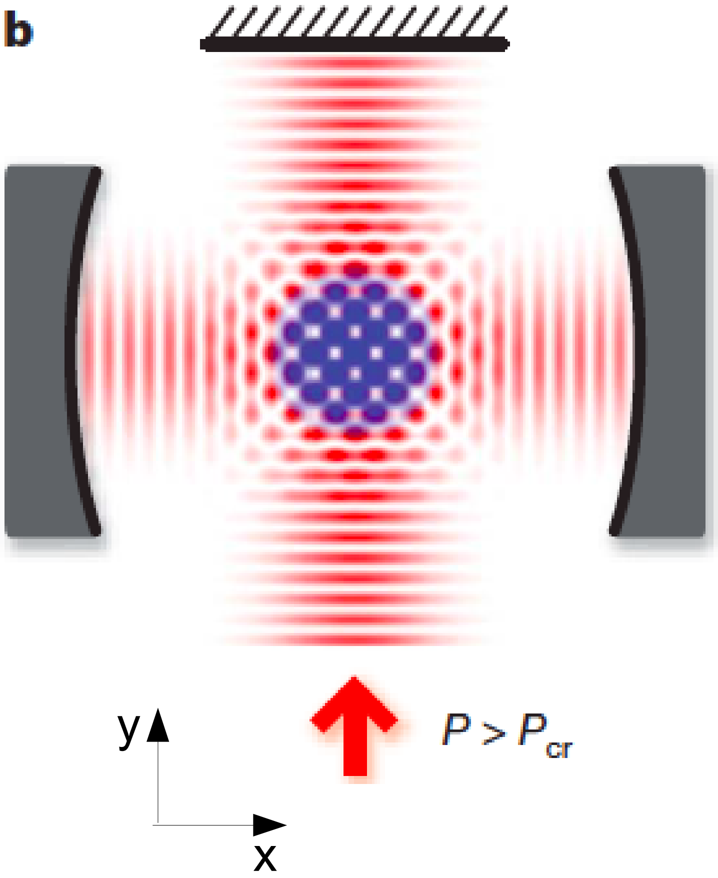
$$h_{at}=\frac{\boldsymbol{P}^2}{2\mathrm{m}}+\frac{\Omega_p^2}{\Delta_a}\cos^2(k_0 y)$$

$$\eta(\boldsymbol{r})=\eta_0\cos(k_0 x)\cos(k_0 y),$$

$$U(\boldsymbol{r})=\frac{g^2}{\Delta_a}\cos^2(k_0 x)$$

Cavity mode

Pumping laser mode



$$\eta_0=\frac{g\,\Omega_p}{\Delta_a}$$

Pumping laser
Rabi frequency

Cavity mode coupling

Yu Chen, ZY and Hui Zhai, PRL **112**, 143004 (2014);
J. Keeling, M. J. Bhaseen, B. D. Simons,
PRL **112**, 143002 (2014);
F. Piazza, P. Strack, PRL **112**, 143003 (2014)

Nature of Superradiance

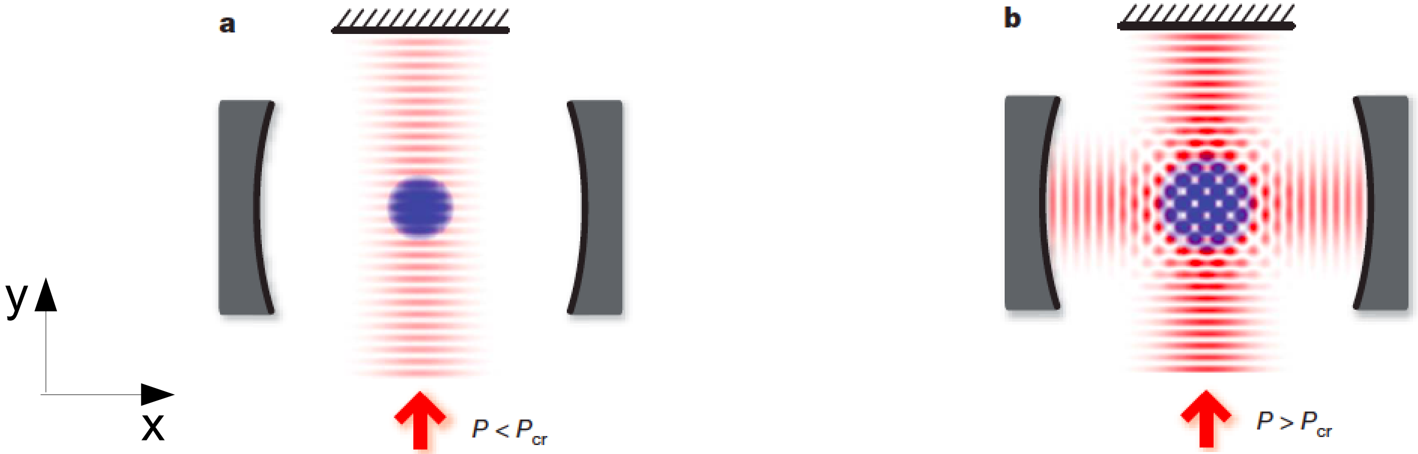
$$H=\int d\boldsymbol{r}[\psi^{+}(\boldsymbol{r})h_{0}\psi(\boldsymbol{r})]-\Delta_{c}a^{+}a$$

$$h_{0}=h_{at}+\underline{\eta(\boldsymbol{r})(a^{+}+a)+U(\boldsymbol{r})a^{+}a},\ \eta(\boldsymbol{r})=\eta_{0}\cos(k_{0}x)\cos(k_{0}y)$$

Equation of motion: $i\frac{\partial a}{\partial t}=-(\tilde{\Delta}_{c}+i\kappa)a+\eta_{0}\Theta$

$$\boxed{\text{Density order: } \Theta=\int d\boldsymbol{r}\, n(\boldsymbol{r})\eta(\boldsymbol{r})/\eta_{0}}$$

Effective cavity detuning $\tilde{\Delta}_{c}=\Delta_{c}-\int d\boldsymbol{r}\, n(\boldsymbol{r})U(\boldsymbol{r})<0$



Mean Field Theory

Order parameter: $\Theta = \int d\boldsymbol{r} \langle n(\boldsymbol{r}) \rangle \cos(k_0 x) \cos(k_0 y)$

Steady solution: $0 = i \frac{\partial \langle a \rangle}{\partial t} = -(\tilde{\Delta}_c + i\kappa) \langle a \rangle + \eta_0 \Theta$

$$\langle a \rangle = \frac{\eta_0 \Theta}{\tilde{\Delta}_c + i\kappa} \qquad \kappa \gg k_0^2 / 2\mathrm{m} \sim 10\, KHz$$

Free energy:

$$F = -\frac{1}{\beta} \ln Tr\, e^{-\beta H} = -\left[\frac{\tilde{\Delta}_c}{\tilde{\Delta}_c^2 + \kappa^2} + \eta_0^2 \chi \frac{4 \tilde{\Delta}_c^2}{(\tilde{\Delta}_c^2 + \kappa^2)^2} \right] (\eta_0 \Theta)^2$$

$$\chi = -\frac{1}{2\beta \eta_0^2} Tr[\langle T\, \hat{n}(\boldsymbol{r}, t) \hat{n}(\boldsymbol{r}', t') \rangle \eta(\boldsymbol{r}) \eta(\boldsymbol{r}')] > 0$$

Density susceptibility to modulation $\eta(\boldsymbol{r}) / \eta_0$

Transition Condition

$$\eta_0^{cr} = \frac{1}{2}\sqrt{\frac{\tilde{\Delta}_c^2+\kappa^2}{(-\tilde{\Delta}_c)\chi}} \qquad \eta_0 = \frac{g\,\Omega_p}{\Delta_a}$$

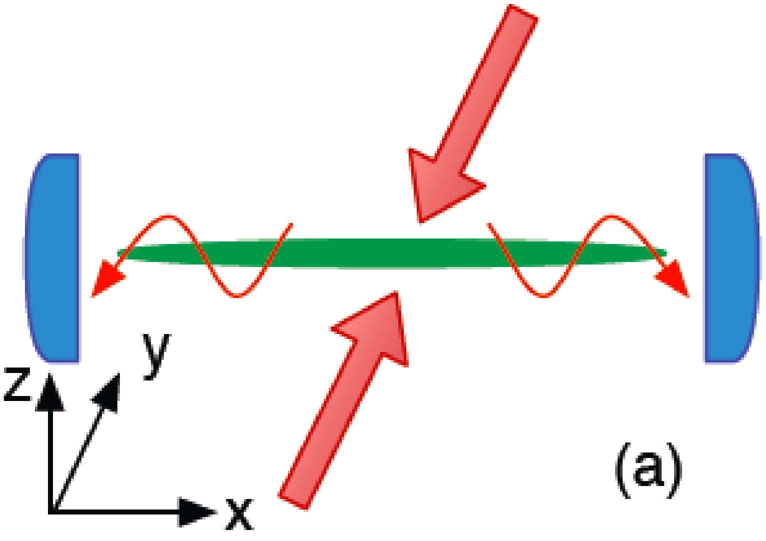
In terms of the single particle states $\phi_{\boldsymbol{k}}$

Single particle distribution function

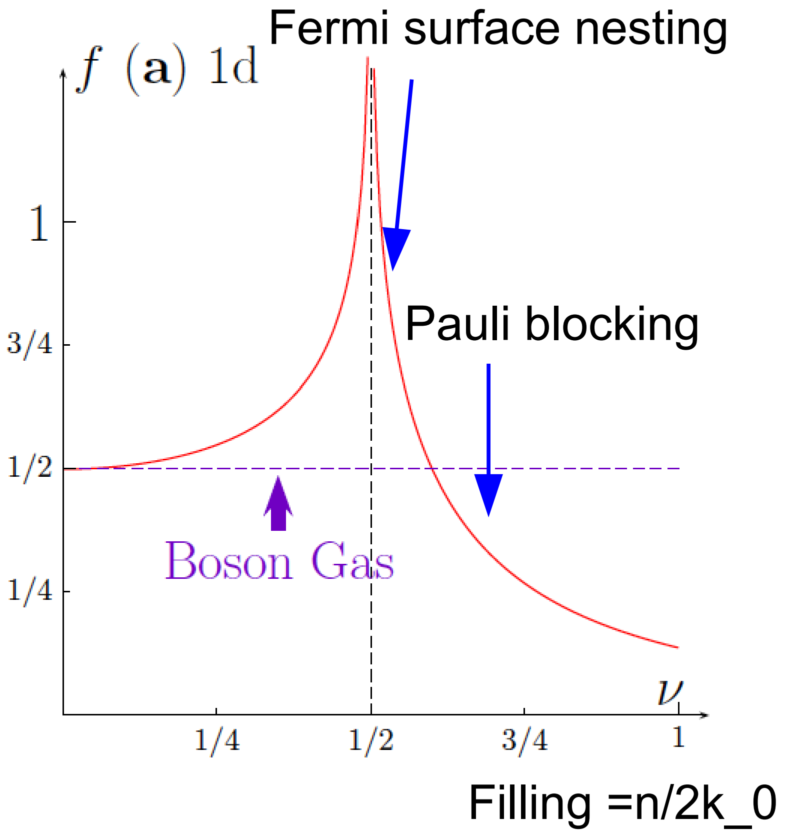
$$\chi = \frac{1}{2\,\eta_0^2}\sum_{\boldsymbol{k},\boldsymbol{k}'}\left|\int d\boldsymbol{r}\,\phi_{\boldsymbol{k}}^*(\boldsymbol{r})\,\phi_{\boldsymbol{k}'}(\boldsymbol{r})\,\eta(\boldsymbol{r})\right|^2 \frac{n(\epsilon_{\boldsymbol{k}})-n(\epsilon_{\boldsymbol{k}'})}{\epsilon_{\boldsymbol{k}}-\epsilon_{\boldsymbol{k}'}}$$

Also applies to BEC

1d @ T=0



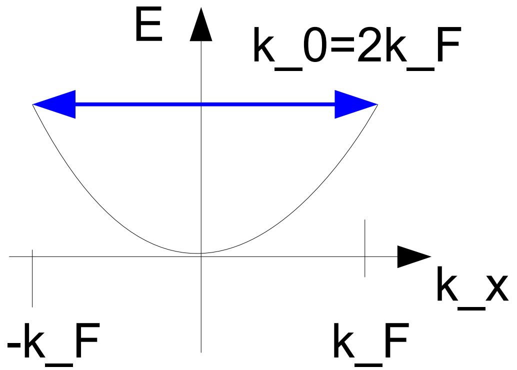
$$\eta(\boldsymbol{r}) \sim \cos(k_0 x)$$

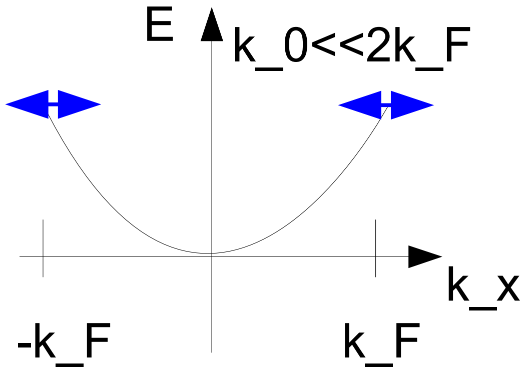


Normalized and dimensionless susceptibility

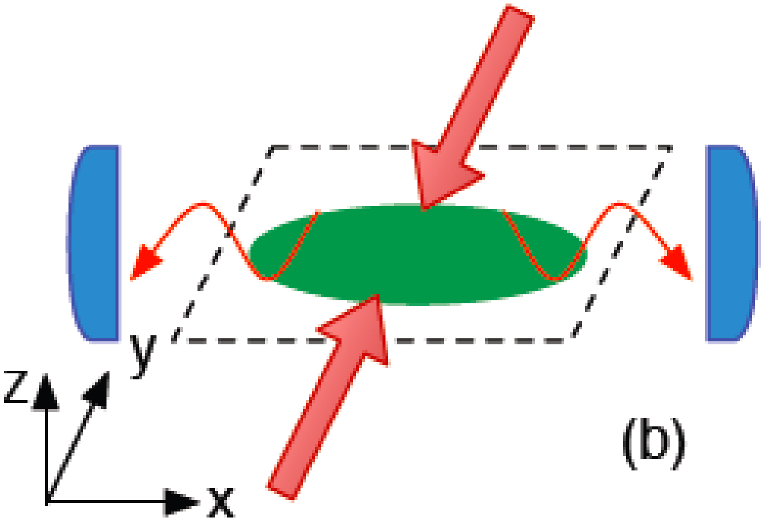
$$f = \chi E_r / N_{at} \qquad E_r = k_0^2 / 2m$$

$$f = \frac{k_0}{8k_F} \ln \left| \frac{k_0 + 2k_F}{k_0 - 2k_F} \right|$$

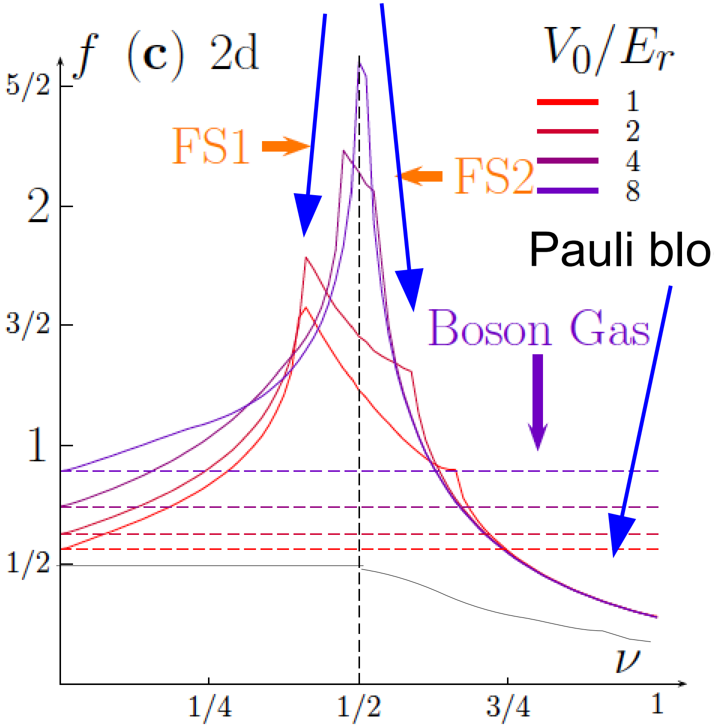




2d @ T=0



Fermi surface nesting



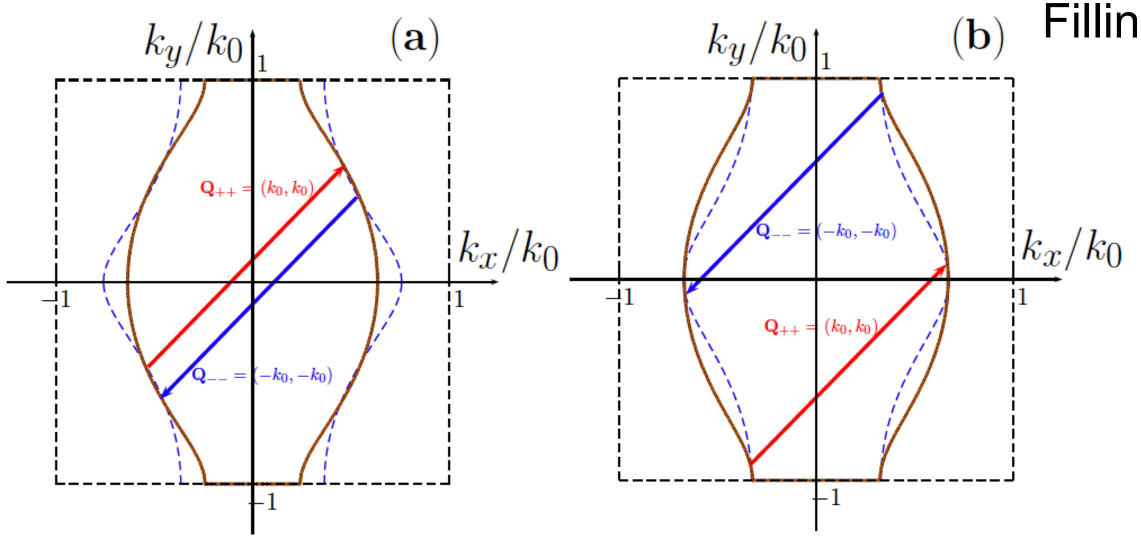
Pauli blocking

$$\eta(\boldsymbol{r}) \sim \cos(k_0 x)\cos(k_0 y)$$

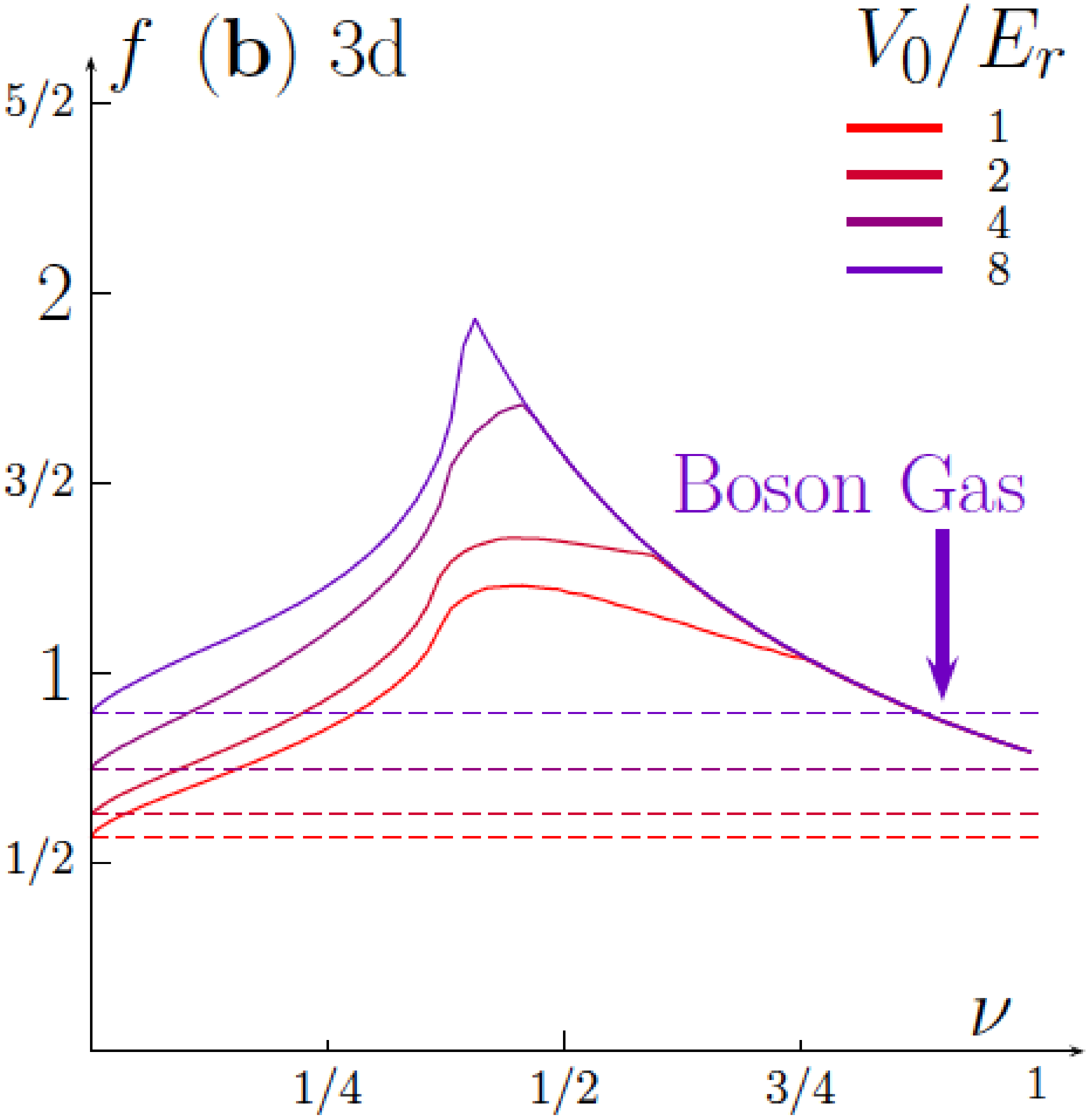
$$h_{at} = \frac{\boldsymbol{P}^2}{2\mathrm{m}} + \frac{\Omega_p^2}{\Delta_a}\cos^2(k_0 y)$$

$$V_0 = \frac{\Omega_p^2}{\Delta_a}$$

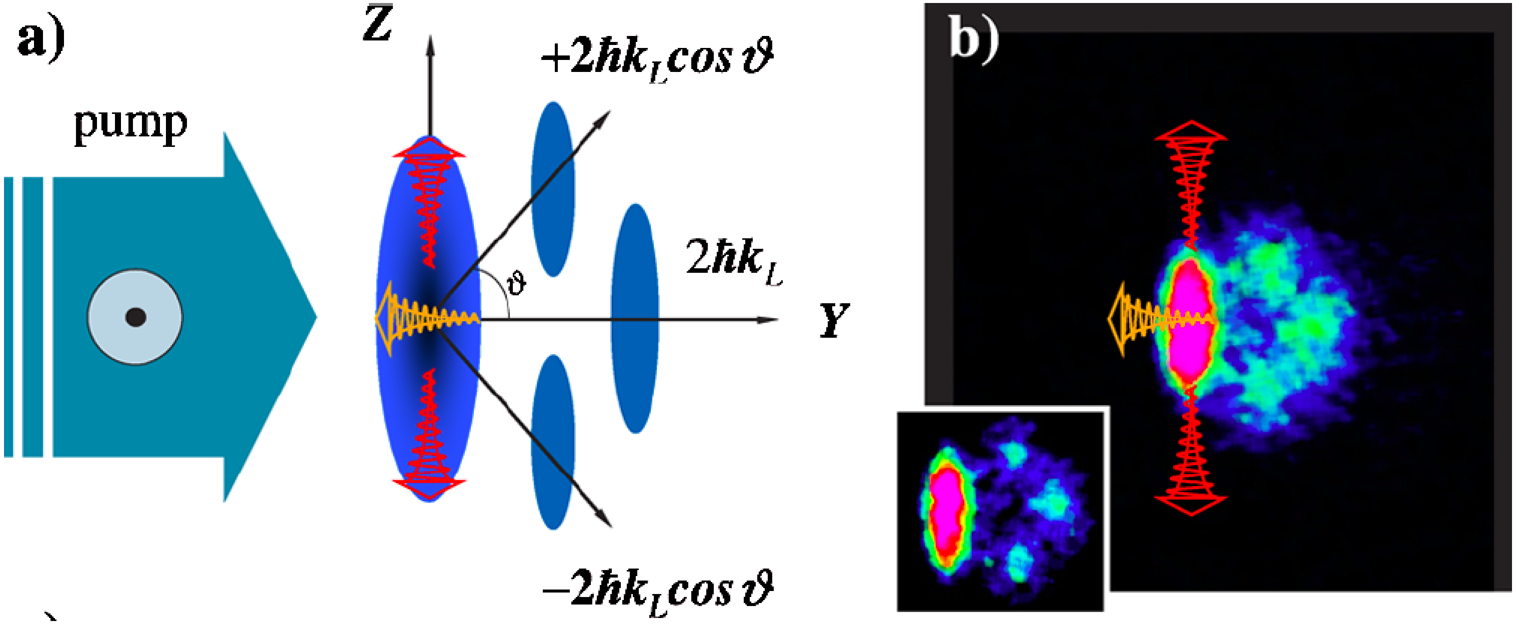
Filling =n/4k_0^2



3d @ T=0



Superradiance in free space



Bosons: Ketterle's group
Science 285, 571 (1999)

Fermions: Zhang Jing's group
PRL 106, 210401 (2011)

Phase Diagram for 3d

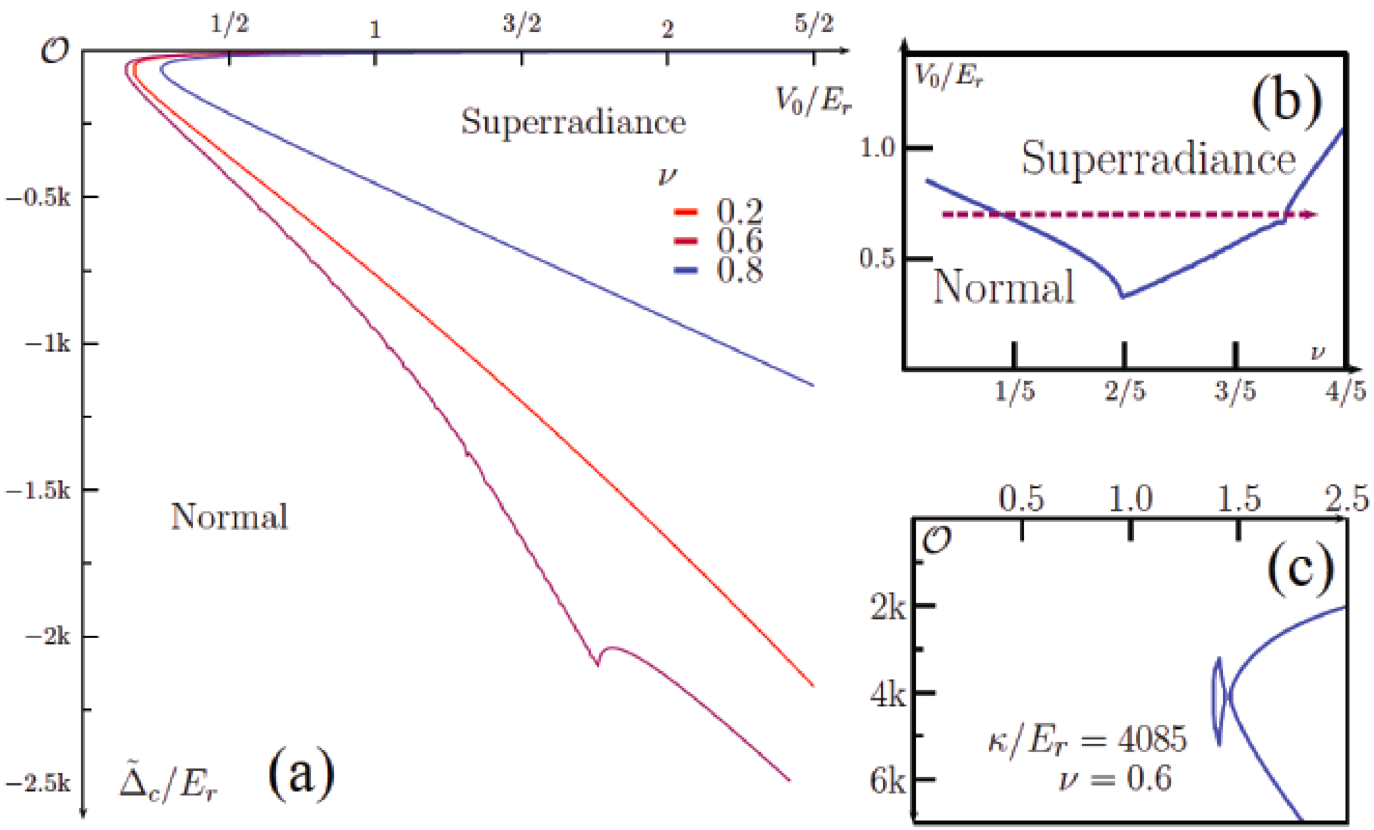
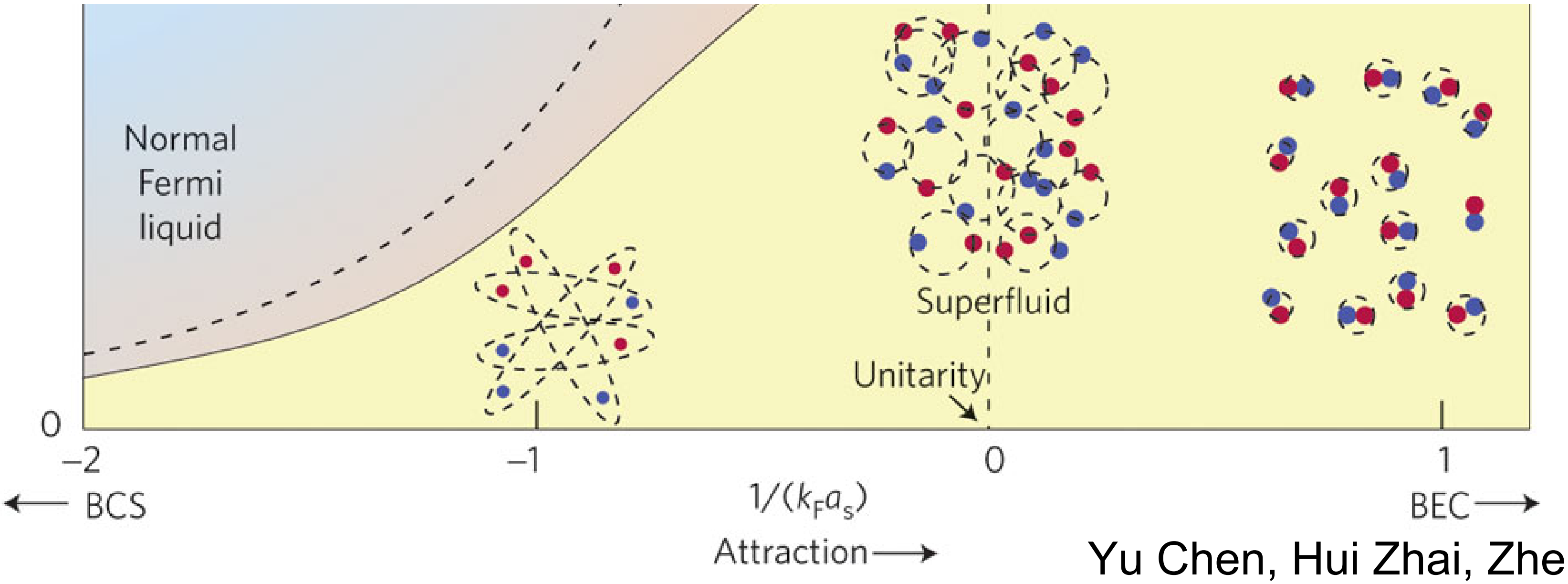


FIG. 4: (a) and (c): The phase diagram for two-dimension case, in terms of effective detuning $\tilde{\Delta}_c/E_r$ and pumping lattice depth V_0/E_r. Different lines in (a) represent phase boundary with different fillings. (b) Critical V_0/E_r as a function of filling ν for $\tilde{\Delta}/E_r$ fixed at 2×10^3. $\kappa/E_r = 250$ for (a) and (b); $\kappa/E_r = 4085$ for (c) and $U_0 N_{at}/E_r = 1 \times 10^3$ for (a-c).

Interplay with Interatomic Interactions

Consider a two component Fermi gas across a Feshbach resonance

$$H_i = \bar{g} \int d\boldsymbol{r}\, \psi_\uparrow^+(\boldsymbol{r}) \psi_\downarrow^+(\boldsymbol{r}) \psi_\downarrow(\boldsymbol{r}) \psi_\uparrow(\boldsymbol{r})$$



Yu Chen, Hui Zhai, Zhenhua Yu,
PRA 91, 021602 (2015)

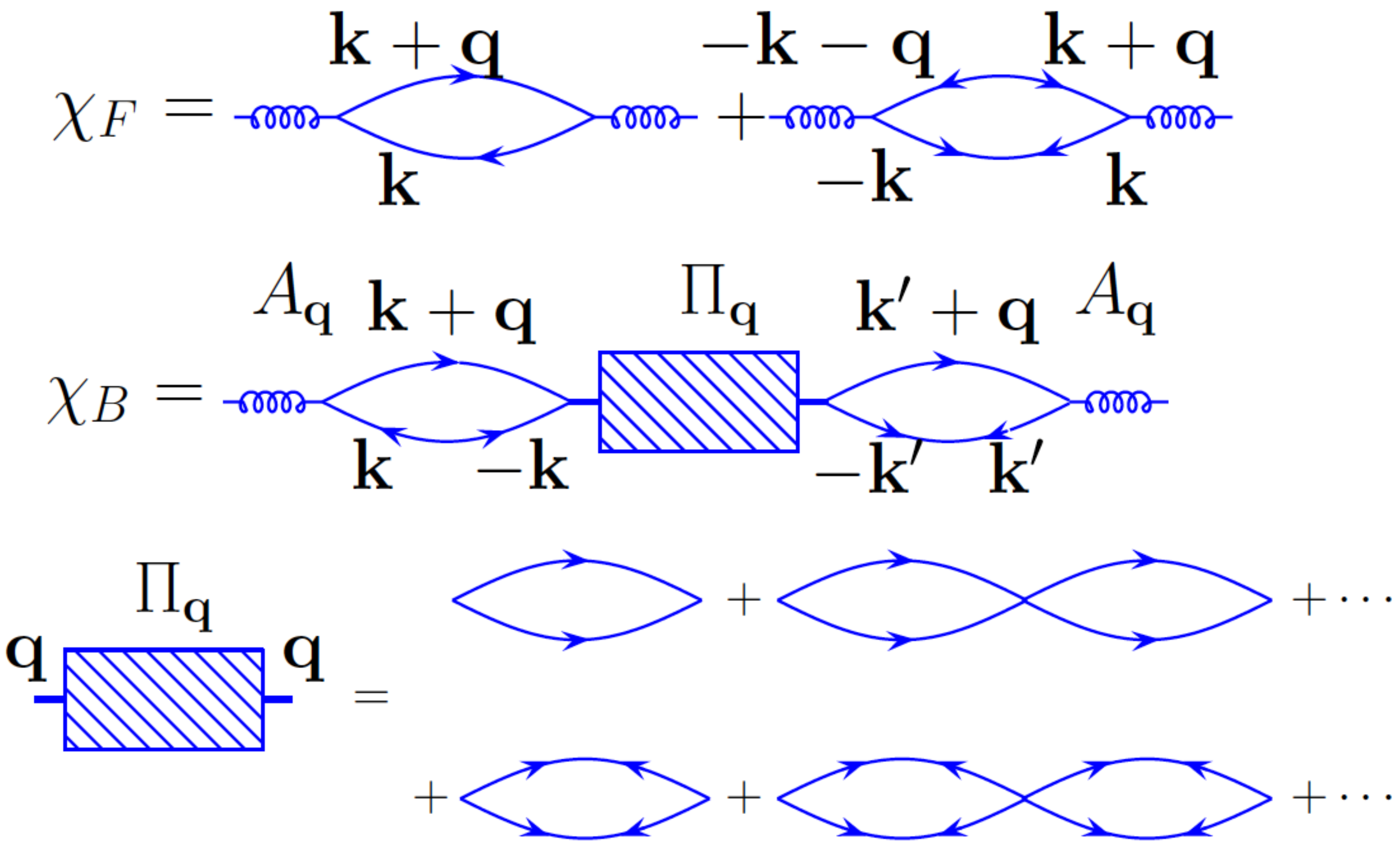
Free energy:

$$F = -\left[\frac{\tilde{\Delta}_c}{\tilde{\Delta}_c^2 + \kappa^2} + \eta_0^2 \chi \frac{4\tilde{\Delta}_c^2}{(\tilde{\Delta}_c^2 + \kappa^2)^2} \right] (\eta_0 \Theta)^2$$

$$\chi = -\frac{1}{2\beta\eta_0^2} Tr[\langle T \hat{n}(\boldsymbol{r}, t) \hat{n}(\boldsymbol{r}', t') \rangle \eta(\boldsymbol{r}) \eta(\boldsymbol{r}')]$$

Superradiance through the BEC-BCS Crossover

$$\chi = \chi_F + \chi_B$$

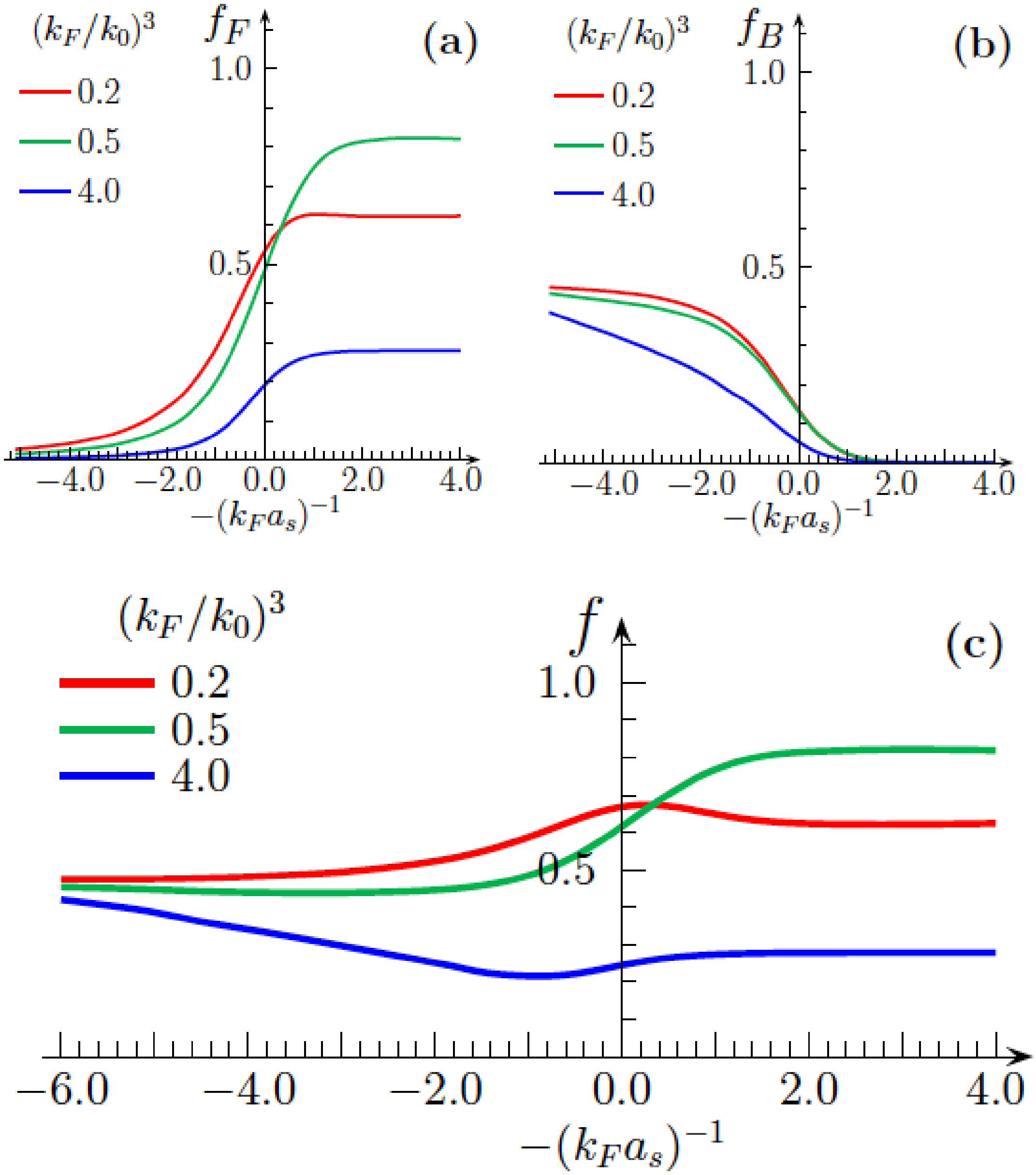


Susceptibility through the BEC-BCS Crossover

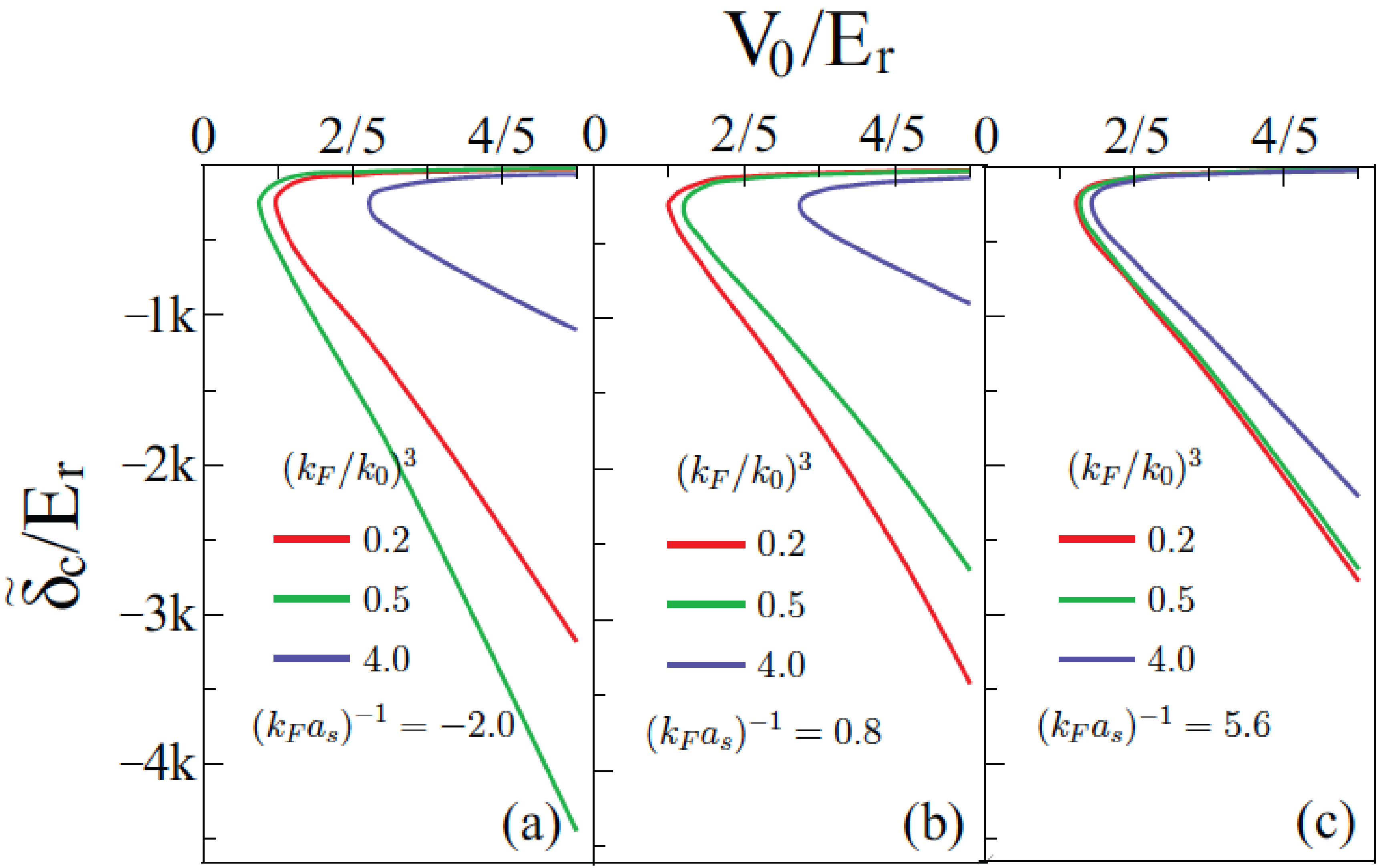
$f = \chi E_r / N_{at}$

The BCS limit

The BEC limit



Phase Diagram



Summary

- Landau mean field theory for superradiant transition of quantum gases in a cavity
- Transition point depends on the density susceptibility
- Fermi surface nesting and Pauling blocking for fermions vs bosons
- Interatomic interaction effects

Thank you