Novel Sp(2N)/SU(2N) quantum magnetism and Mott physics – large spins are different

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Current work:

1. Z. C. Zhou, Z. Cai, C. Wu, Y. Wang, Phys. Rev. B, Phys. Rev. B 90, 235139 (2014) .
2. D. Wang, Y. Li, Z. Cai, Z. Zhou, Y. Wang, C. Wu, Phys. Rev. Lett. 112, 156403 (2014).
3. Z. Cai, H. Hung, L. Wang, D. Zheng, C. Wu, Phys. Rev. Lett. 110, 220401 (2013) .
4. C. Wu, Nature Physics 8, 784 (2012) (News and Views).

Earlier work:

1. C. Wu, J. P. Hu, and S. C. Zhang, Phys. Rev. Lett. 91, 186402 (2003).
2. C. Wu, Phys. Rev. Lett. 95, 266404 (2005),
3. C. Wu, Mod. Phys. Lett. B 20, 1707 (2006) (brief review).

March 24, 2015, INT, University of Washington, Seattle

Current collaborators

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Da Wang	(UCSD→ Nanjing Univ.)
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Hsiang-hsuan Hung	(UCSD→UIUC→ UT Austin)
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Outline

- **Introduction: what is large?**

  Large symmetry (large N) rather than large spin magnitude (large S).
  Quantum spin fluctuations are enhanced rather than suppressed.

- Generic Sp(4) symmetry in spin-3/2 systems – unification of AFM, SC and CDW.

  http://online.kitp.ucsb.edu/online/coldatoms07/wu2/

- Novel quantum phase transitions: Slater v.s. Mott – interplay between charge and spin degrees of freedom (QMC).

- Thermodynamics: enhancement of Pomeranchuk cooling – QMC.

- Interaction effects v.s. N (1D - QMC)

The simplest interacting model of lattice fermions

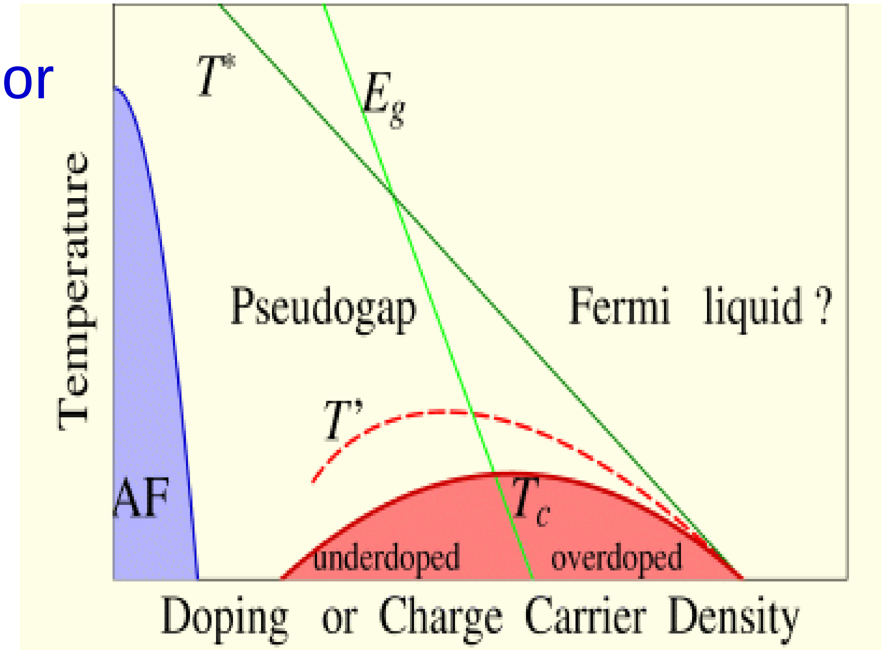
$$H = -\sum_{\langle i,j \rangle,\sigma} t\{c^{+}_{i,\sigma} c_{j,\sigma} + h.c.\} - \mu \sum_{i,\sigma} c^{+}_{i,\sigma} c_{i,\sigma} + U \sum_{i} n_{i,\uparrow} n_{i,\downarrow}$$

- Hubbard 1963: itinerant ferromagnetsim (FM), not successful.

- But useful for metal-Mott insulator transitions.

- Can the single band Hubbard describe high Tc cuprates?

  --- Still in debates.



What do we know for sure?

- **1D Mott physics**: half-filled (U>0).

1) Charge gap opens at infinitesimal U: Umklapp process is relevant.

2) Spin channel remains critical exhibiting power-law AFM correlation.

C. N. Yang, PRL 19, 1312 (1967); Lieb and F. Y. Wu, PRL 20, 1445, (1968).

Field theoretical methods, DMRG simulations

- **2D AFM long-range-order:** the square lattice (half-filled).

Determinant quantum Monte-Carlo (DQMC):
Sign-problem free at half filling -- non-peturbative method, asymptotically exact

Blackenbecler, Scalapinio, Sugar, PRD (1981); J. Hirsch, PRB 31, 4403 (1985).

Hidden pseudo-spin symmetry

- C. N. Yang's η pairing → generators of charge SUc(2).

$$\eta^- = \sum_i (-)^i c_{i\downarrow} c_{i\uparrow}, \quad \eta^+ = \sum_i (-)^i c_{i\uparrow}^+ c_{i\downarrow}^+, \quad [\eta^-, \eta^+] = 2N$$

C. N. Yang and S. C. Zhang, Mod Phys. Lett. 4, 759 (1990).

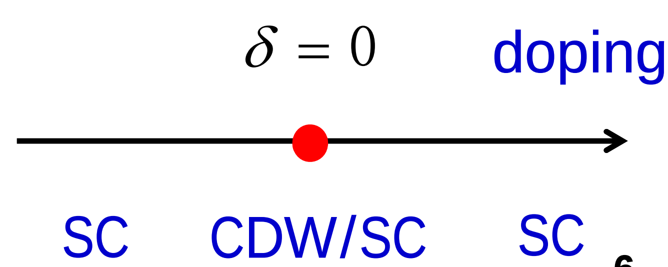
- U<0: degeneracy between CDW and superconductivity at half-filling.

$$O_{cdw} = \sum_i (-)^i n_i, \; \Delta = \sum_i c_{i\uparrow} c_{i\downarrow}, \; \Delta^+ = \sum_i c_{i\downarrow}^+ c_{i\uparrow}^+$$

- Pseudo-Goldstone η mode (eigen-mode)

$$[\eta^+, \Delta] = O_{CDW} \qquad [H, \eta^\pm] = \mp(\mu - \mu_0)\eta^\pm$$

$$H(\eta^+ | G_{SC}\rangle) = (\mu - \mu_0)\,(\eta^+ | G_{SC}\rangle), \; (\mu \geq \mu_0)$$



Theory progress with large-spin fermions

- Novel physics **inaccessible** in usual solid state systems.

- Early work by Ho and Yip (PRA and PRL 1999):

Richer Fermi liquid properties and Cooper pairing structures than those in spin-1/2 electron systems.

- **A new view point: high symmetries, Sp(2N) / SU(2N).**

Sp(4)/SO(5) /SU(4): C. Wu, S. C. Zhang, S. Chen, Y. P. Wang, A. Tsvelik, G. M. Zhang, Lu Yu, X. W. Guan, Azaria, Lecheminant, et al. (2003 ---).

SU(2N): V. Guriare, M. Hermele, A. Rey, E. Demler, M. Lukin, P. Zoller, et al. (2010 ---).

Experiment breakthrough of large-spin fermions

Selected for a Viewpoint in *Physics*

PRL 105, 190401 (2010)

90401 (2010) PHYSICAL REVIEW LETTERS wee 5 NOVE

Realization of a SU(2) × SU(6) System of Fermions in a Cold Atomic Gas

Shintaro Taie,[1,*] Yosuke Takasu,[1] Seiji Sugawa,[1] Rekishu Yamazaki,[1,2] Takuya Tsujimoto,[1] Ryo Murakami,[1] and Yoshiro Takahashi[1,2]

)2 (2010) PHYSICAL REVIEW LETTERS

PRL 105, 030402 (2010)

Degenerate Fermi Gas of ^{87}Sr

B. J. DeSalvo, M. Yan, P. G. Mickelson, Y. N. Martinez de Escobar, and T. C. Killian

Viewpoint

Physics 3, 92(2010)

Exotic many-body physics with large-spin Fermi gases

Congjun Wu
Department of Physics, University of California, San Diego, CA 92093, USA
Published November 1, 2010

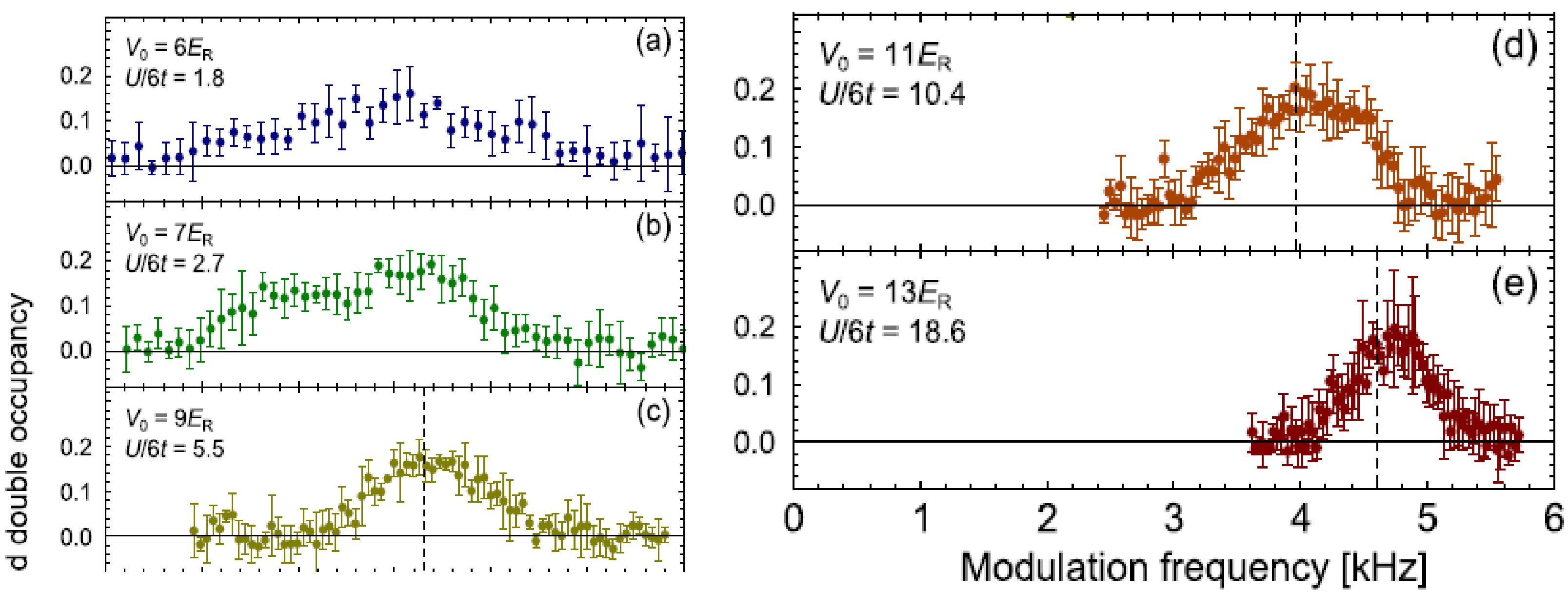
The experimental realization of quantum degenerate cold Fermi gases with large hyperfine spins opens up a new opportunity for exotic many-body physics.

nature physics
ARTICLES
PUBLISHED ONLINE: 23 SEPTEMBER 2012 | DOI: 10.1038/NPHYS2430

An SU(6) Mott insulator of an atomic Fermi gas realized by large-spin Pomeranchuk cooling

S. Taie,et.al, Nature phys. 8, 825(2012).

Shintaro Taie[1]*, Rekishu Yamazaki[1,2], Seiji Sugawa[1] and Yoshiro Takahashi[1,2]



- Many recent progresses: Fallani et al; Jun Ye et al; K. Sengstock et al; Foelling/Bloch et al, …….

What is large?

- High symmetry (large N, SU(2N), Sp(2N)) rather than large spin magnitude (large S).

• High symmetries do not occur frequently in nature, since every new symmetry brings with itself a possibility of new physics, they deserve attention.

--- comment from D. Controzzi and A. M. Tsvelik, cond-mat/0510505

- Quantum spin fluctuations are enhanced NOT suppressed.

• SU(2N) and Sp(2N) were introduced to condensed matter physics as a math tool, say, 1/N-expansion.

• Now they have become realistic in lab.

Transition metal oxides **(large S → classical)**

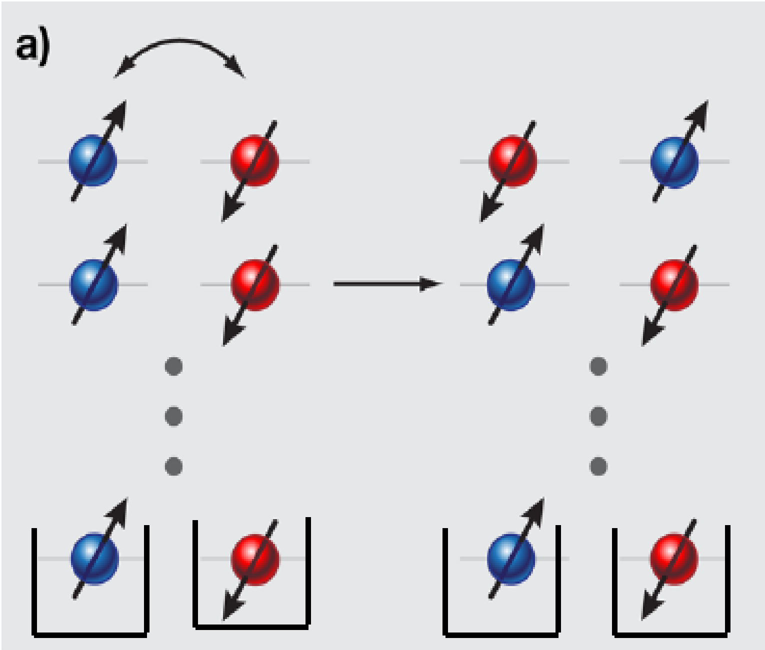
- **Large spin magnitude** from Hund's coupling.

- Inter-site coupling: exchange **a single pair** of electrons.

- **1/S-fluctuations**: $\Delta S_z = \pm 1$

- Bilinear exchange dominates

$$\frac{t^2}{U}\vec{S}_i \cdot \vec{S}_j + \frac{t^4}{U^3}(\vec{S}_i \cdot \vec{S}_j)^2 + \ldots$$

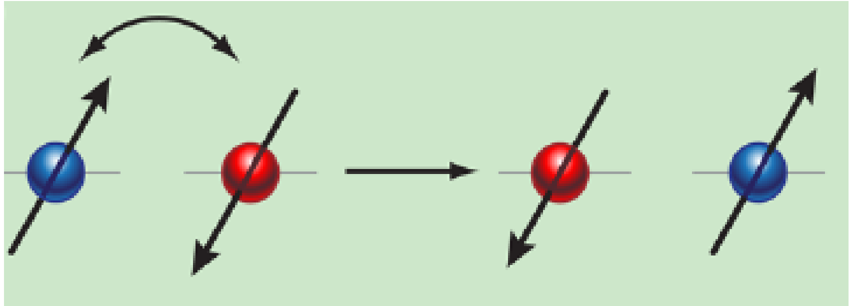


C. Wu, Physics 3, 92 (2010).

C. Wu, Nature Physics 8, 784 (2012) (News and Views).

Cold fermions: **large N→ enhanced fluctuations!**

- Large-hyperfine-spin as a whole object (no ionization).



$$\Delta S_z = \pm 1, \pm 2, \ldots \pm S$$

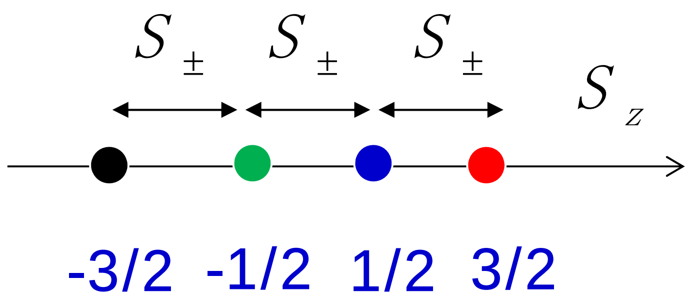
- One step of super-exchange can completely overturn spin config.

- Bilinear, bi-qudratic, bi-cubic terms, etc., are all at equal importance.

$$\vec{S}_i \cdot \vec{S}_j, (\vec{S}_i \cdot \vec{S}_j)^2, (\vec{S}_i \cdot \vec{S}_j)^3$$

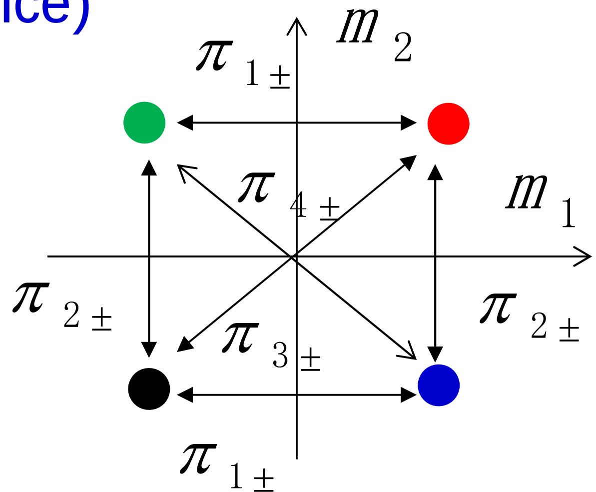
Two views of the spin quartet (weight diagrams (lattice) of Lie algebra): c.f. synthetic lattice

Solid: SU(2) (1D lattice)



- A high rank spinor Rep. of a small group.
- Off-diagonal operator: (fluctuation) $S_\pm$

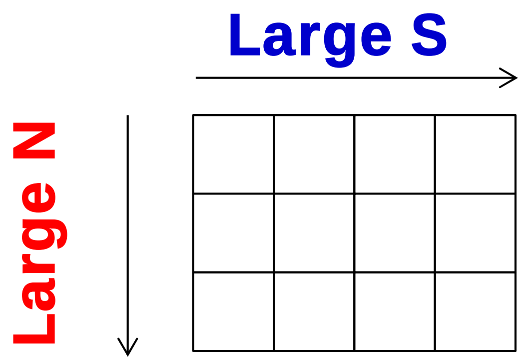
Cold fermions Sp(4)/SO(5) (2D lattice)



- The fundamental spinor Rep of a large group.
- Much more off-diagonal operators.

$$\pi_{1\pm},\ \pi_{2\pm},\ \pi_{3\pm},\ \pi_{4\pm}$$

SU(2N), Sp(2N) (2N=2S+1)



- Alkaline-earth fermions: fully filled electronics shells; interactions are insensitive to nuclear moments; 2N components are equivalent.

- Alkali fermions: spin-dependent interactions; SU(2N) symmetry is not generic.

- The next high symmetry: symplectic symmetry

$$SU(2N) \rightarrow Sp(2N)$$

Good properties under time-reversal transformation.

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The simplest case spin-3/2: **Hidden symmetry**!

- Spin 3/2 atoms: ^{132}Cs, ^{9}Be, ^{135}Ba, ^{137}Ba, ^{201}Hg.

- **Sp(4) (SO(5))** symmetry without fine tuning regardless of dimensionality, particle density, and lattice geometry!

Sp(4) in spin 3/2 systems ←→ SU(2) in spin ½ systems

- SU(4) symmetry is realized iff the interaction is spin-independent.

- Importance of high symmetries: unification of competing orders, description of strong spin fluctuations, etc.

C. Wu et al. Phys. Rev. Lett. 91, 186402 (2003).

Spin-3/2 Hubbard model in optical lattices

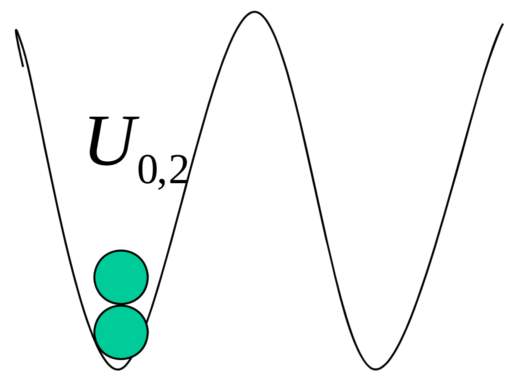
$$H = \sum_{\langle ij\rangle,\alpha} -t\{c^{+}_{i,\alpha} c_{j,\alpha} + h.c.\} - \mu \sum_{i} c^{+}_{i,\alpha} c_{i,\alpha}$$
$$+ U_0 \sum_{i} \eta^{+}(i)\eta(i) + U_2 \sum_{a=1\sim 5} \chi^{+}_{a}(i)\chi_{a}(i)$$

$$\Uparrow \left|\tfrac{3}{2}\right\rangle \quad \uparrow \left|\tfrac{1}{2}\right\rangle \quad \downarrow \left|-\tfrac{1}{2}\right\rangle \quad \Downarrow \left|-\tfrac{3}{2}\right\rangle$$

- Fermi statistics: only F_{tot}=0, 2 are allowed; F_{tot}=1, 3 are forbidden.

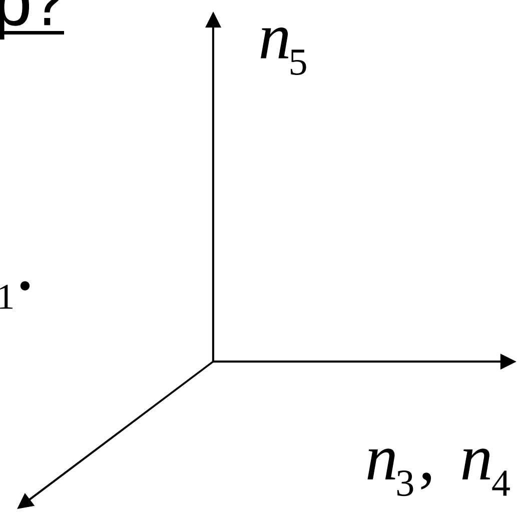
singlet: $$\eta^{+}(i) = \sum_{\alpha\beta} \langle 00 \,|\, \tfrac{3}{2}\tfrac{3}{2}; \alpha\beta \rangle c^{+}_{\alpha}(i) c^{+}_{\beta}(i)$$

quintet: $$\chi^{+}_{a}(i) = \sum_{\alpha\beta} \langle 2a \,|\, \tfrac{3}{2}\tfrac{3}{2}; \alpha\beta \rangle c^{+}_{\alpha}(i) c^{+}_{\beta}(i)$$



- For arbitrary values of t, μ, U_0, U_2 and lattice geometry, there is an **exact** Sp(4), or SO(5) symmetry.

What is Sp(4)(SO(5)) group?



- SU(2) (SO(3)) group.

  3-vector: x, y, z; 3-generator: $L_{12}, L_{23}, L_{31}.$

  2-spinor: $|\uparrow\rangle,\quad |\downarrow\rangle$

- Sp(4)(SO(5)) group.

  5-vector: n_1, n_2, n_3, n_4, n_5

  10-generator: $L_{ab}\ (1 \le a < b \le 5)$

  4-spinor: ⬆ $\left|\frac{3}{2}\right\rangle$ ↑ $\left|\frac{1}{2}\right\rangle$ ↓ $\left|-\frac{1}{2}\right\rangle$ ⬇ $\left|-\frac{3}{2}\right\rangle$

- We will see what quantities correspond to these 5-vector and 10-generator.

spin-3/2 algebra $\psi^{+}_{\alpha} M_{\alpha\beta} \psi_{\beta}$

- Total degrees of freedom: $4^2=16=1+3+5+7$.

1 density operator and 3 spin operators are far from complete.

	rank:	
	0	$1,$
	1	F_x, F_y, F_z
$M_{\alpha\beta}$	2	$\xi^a_{ij} F_i F_j \ (a=1\sim 5):$ ⟷ $F_x^2 - F_y^2, F_z^2 - \frac{5}{4},$ $\{F_x, F_y\}, \{F_y, F_z\}, \{F_z, F_x\}$
	3	$\xi^a_{ijk} F_i F_j F_k \ (a=1\sim 7)$

- **Spin-quadrupole matrices** (rank-2 tensors) form five-Γ matrices (SO(5) vector) --- the same Γ-matrices in Dirac equation.

$$\Gamma^a = \xi^a_{ij} F_i F_j \ , \quad \{\Gamma^a, \Gamma^b\} = 2\delta_{ab}, \quad (1 \le a, b \le 5)$$

Hidden conserved quantities: **spin-octupoles**

- Both $F_{x,y,z}$ and $\xi^a_{ijk}F_iF_jF_k$ commute with Hamiltonian. 10 SO(5) generators: 10=3+7.

- **7 spin-octupole operators** are the hidden conserved quantities.

$$\Gamma^{ab} = \tfrac{i}{2}[\Gamma^a, \Gamma^b] \ (1 \le a < b \le 5)$$

- **SO(5): 1 scalar + 5 vectors + 10 generators = 16**

		Time Reversal
1 density:	$n = \psi^+ \psi;$	even
5 spin-quadrupole:	$n_a = \frac{1}{2}\psi^+ \Gamma^a \psi;$	even
3 spins + 7 spin-octupole:	$L_{ab} = \frac{1}{2}\psi^+ \Gamma^{ab} \psi;$	odd

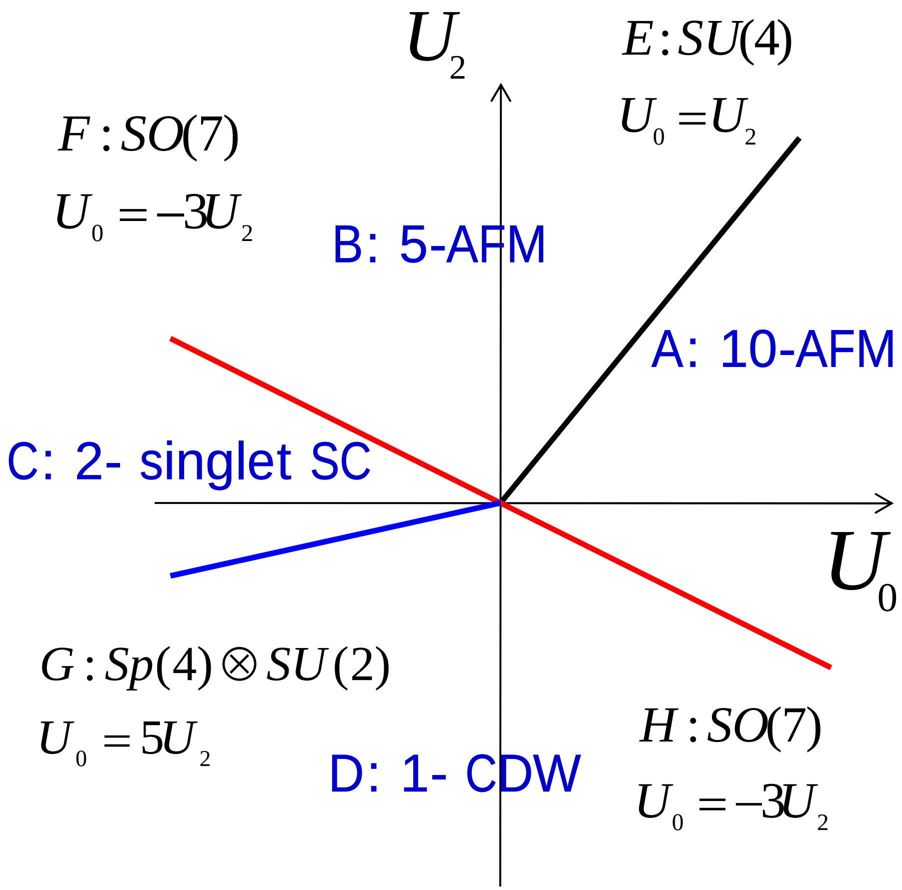
Unify AF, SC, CDW with **exact** symmetries

- Even higher symm. appear in bipartite lattice at half-filling.

- AF (5-spin quadrupole) + SC (singlet) by SO(7) symmetry.

- CDW + SC (singlet) by pseudo-spin SU(2) symmetry. Generalization of C. N. Yang's eta-pairing.

- AF(10-spin+spin octupole) +SC (10-quintet)+ CDW by the adjoint rep. of SO(7).

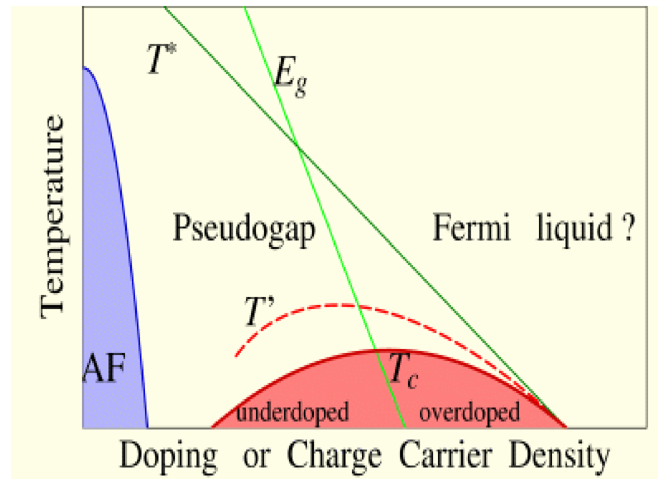


“Grand-unifications” – elegancy and power of the group theory

- Pseudo-spin SO(3=2+1), or, SU(2) symm. unifies SC (singlet) +CDW – C. N. Yang, S. C. Zhang.

- Approx. SO(5=2+3) symm. unifies SC (d-wave singlet) + AFM – S. C. Zhang, E. Demler, et al.

41mev neutron resonance mode in the high Tc SC state: pseudo-Goldstone mode (▲ -mode)



- Exact SO(7=2+5) symm. Unifies SC + AFM (5-spin quadrupole).

$$[\chi_a^+, \Delta] = AF_{a,qd} \qquad [H, \chi_a^\pm] = \mp(\mu - \mu_0)\chi_a^\pm$$

$$H(\chi_a^+ | G_{SC}\rangle) = [E_G + (\mu - \mu_0)]\,(\chi_a^+ | G_{SC}\rangle)$$

5-⇨ models: rotate SC←→AF.

Analogy to the ▲ modes in high Tc.

Sign-problem free QMC algorithm away from half-filling

- An equivalent formulation:

$$H = \sum_{\langle ij \rangle,\sigma} -t\{c^{+}_{i,\alpha} c_{j,\alpha} + h.c.\} - \mu \sum_i c^{+}_{i,\alpha} c_{i,\alpha} - \sum_{i,1\le a\le 5} \{V(n(i)-2)^2 + W n_a^2(i)\}$$

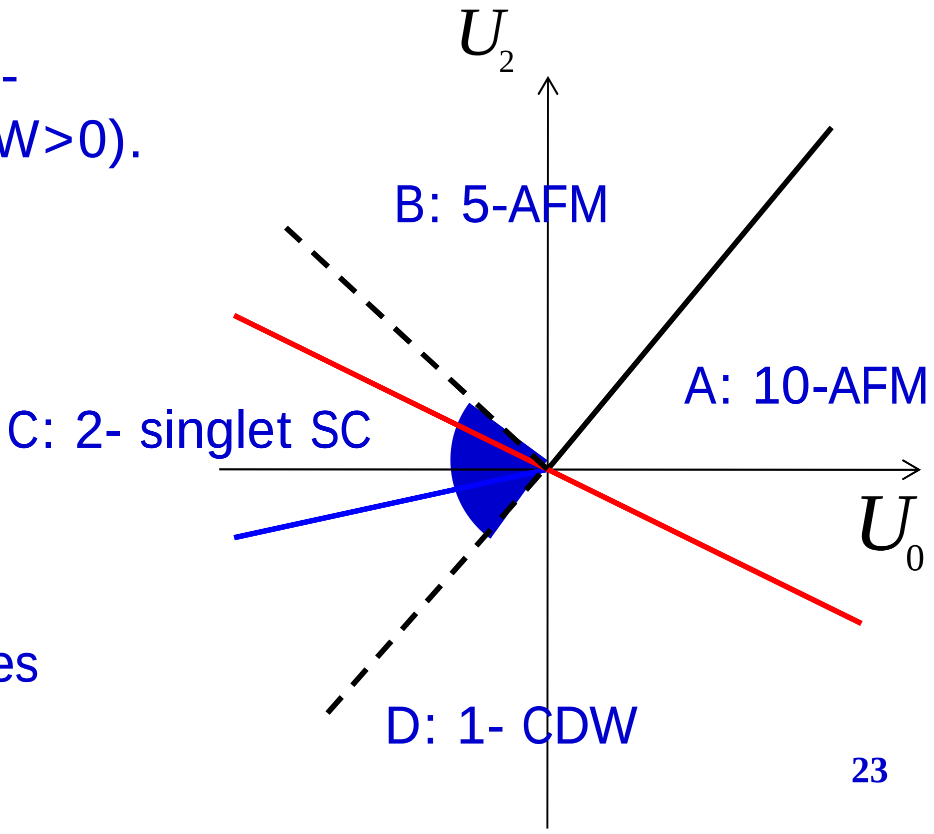
$$V = -\frac{3U_0 + 5U_2}{16}, \quad W = \frac{U_2 - U_0}{4}$$

- Time-reversal invariant Hubbard-Stratonovich decomposition at (V,W>0).

- Fermion determinant remains positive-definite at any filling.

$$U_0 < U_2 < -\frac{3}{5} U_0$$

- Sign problem free region includes Superconductivity, CDW, AFM.

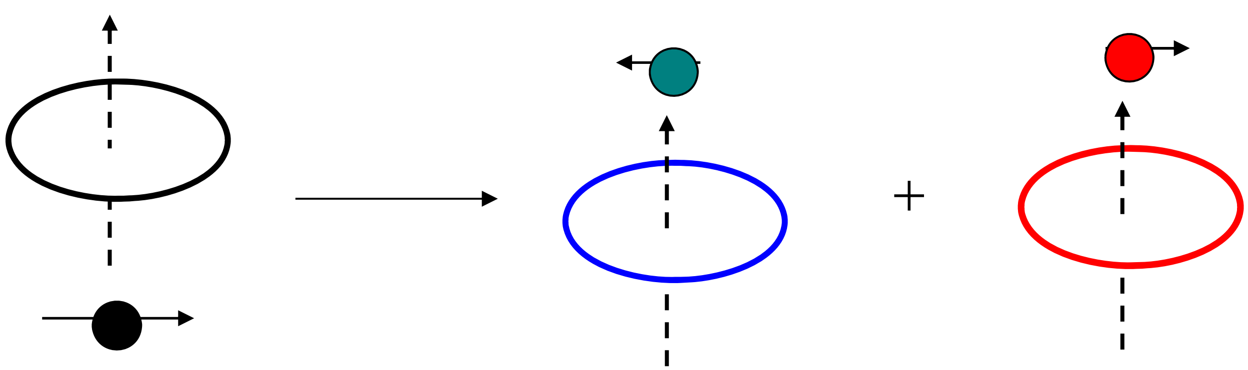


Non-abelian statistics – Alice vortex loop/particle (SO(4) Cheshire charge)

- Quintet pairing (S=2) → half-quantum vortex loop carrying spin quantum number.

$$|init\rangle = \left|\tfrac{3}{2}\right\rangle_p \otimes |\text{zero charge}\rangle_{vort} \longrightarrow$$

$$|final\rangle = \left|\tfrac{1}{2}\right\rangle_p \otimes |S_z = 1\rangle_{vort} - \left|\tfrac{-1}{2}\right\rangle_p \otimes |S_z = 2\rangle_{vort}$$



$$|00;00\rangle_{vt} \otimes \left|\tfrac{1}{2}\tfrac{1}{2};00\right\rangle_{qp}, \qquad \left|\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{-1}{2}\right\rangle_{vt} \otimes \left|00;\frac{1}{2}\frac{1}{2}\right\rangle_{qp} - \left|\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}\right\rangle_{vt} \otimes \left|00;\frac{1}{2}\frac{-1}{2}\right\rangle_{qp}.$$

C. Wu, J. P. Hu, and S. C. Zhang, cond-mat/0512602.

More details

Brief Review

Modern Physics Letters B, Vol. 20, No. 27 (2006) 1707–1738

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HIDDEN SYMMETRY AND QUANTUM PHASES IN SPIN-3/2 COLD ATOMIC SYSTEMS

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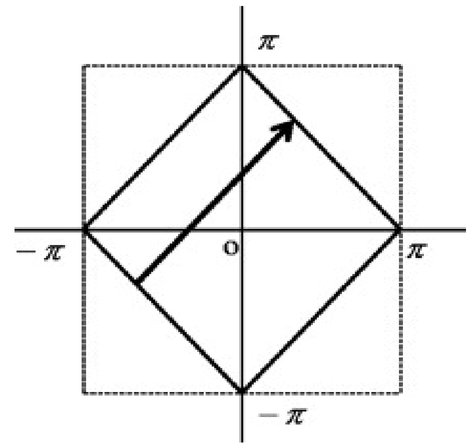
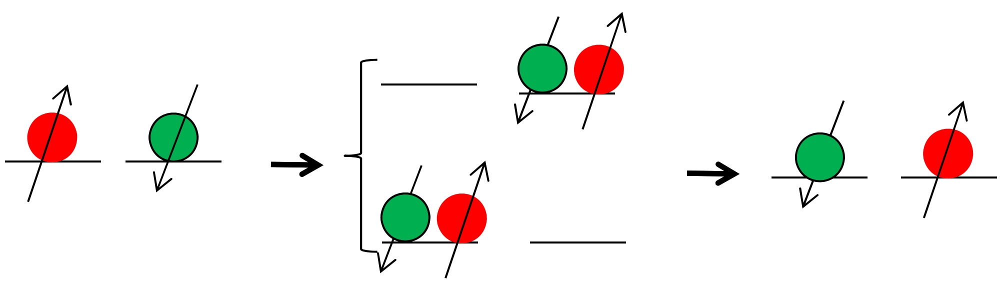
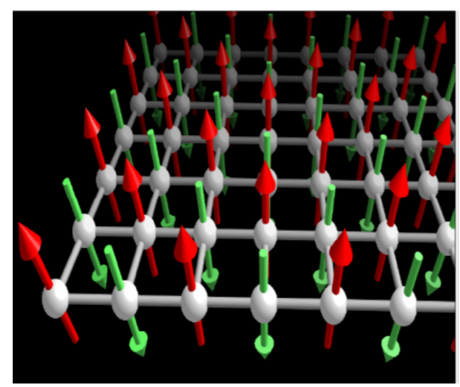
Slater V. S. Mott (half-filling)



- Small U/t (Fermi surface nesting): divergence of AFM susceptibility; charge fluctuation cannot be neglected!

- Large U/t (local moment): charge fluctuation suppressed; AFM super-exchange.





$$H = J\sum_{i}(\vec{S}_i \cdot \vec{S}_j - \frac{1}{4}) \qquad J = \frac{4t^2}{U}$$

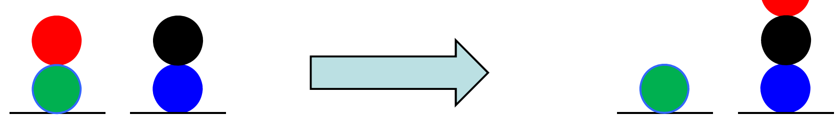
SU(2N) Hubbard model at half-filling

$$H = -t \sum_{\langle ij \rangle, \sigma=1}^{2N} \{ c_{i,\sigma}^{+} c_{j,\sigma} + h.\ c. \} + \frac{U}{2} \sum_i (n_i - N)^2 \qquad n_i = \sum_{\sigma=1}^{2N} n_{i,\sigma}$$

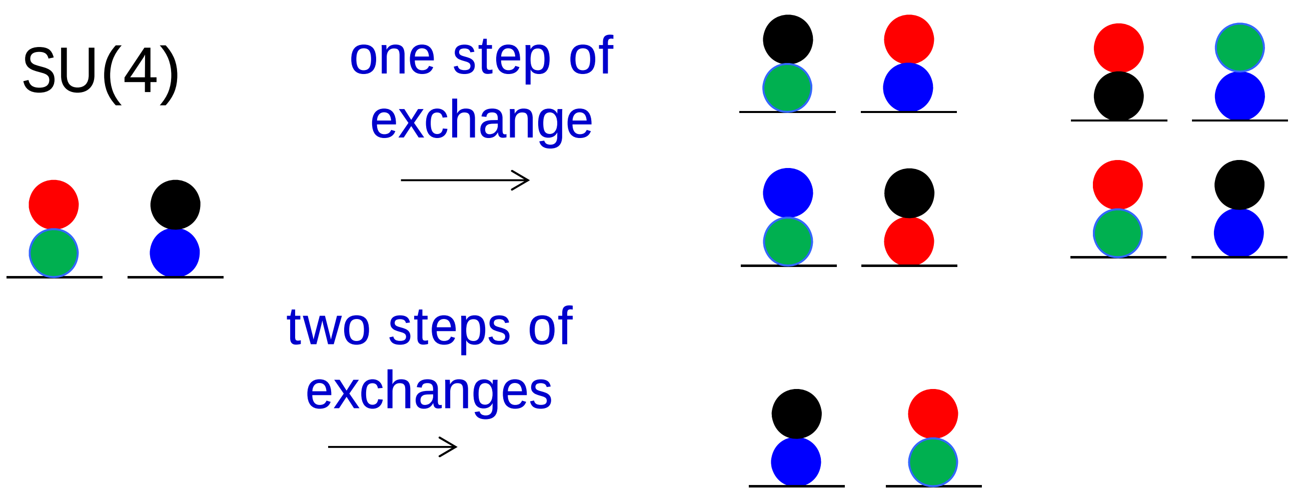
- SU(4) as an example.

In the atomic limit, t=0.

$$\Delta E = U$$



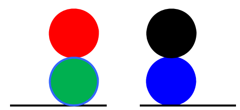
- Turning on t, number of super-exchange processes scales as N^2.



Enhancement of quantum spin fluctuations

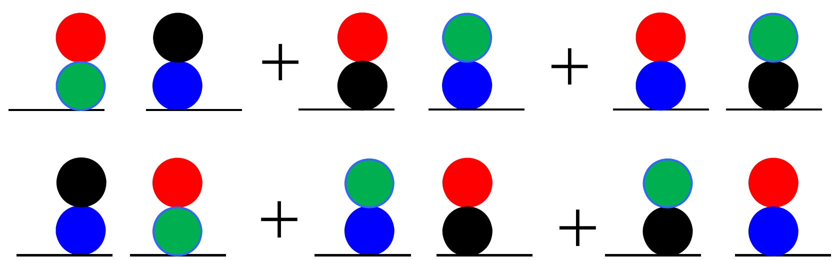
- As increasing 2N, the Neel states become unfavorable.

$$\Delta E = -2N\frac{t^2}{U}$$



classic-Neel

$$\Delta E = -2N(N+1)\frac{t^2}{U}$$



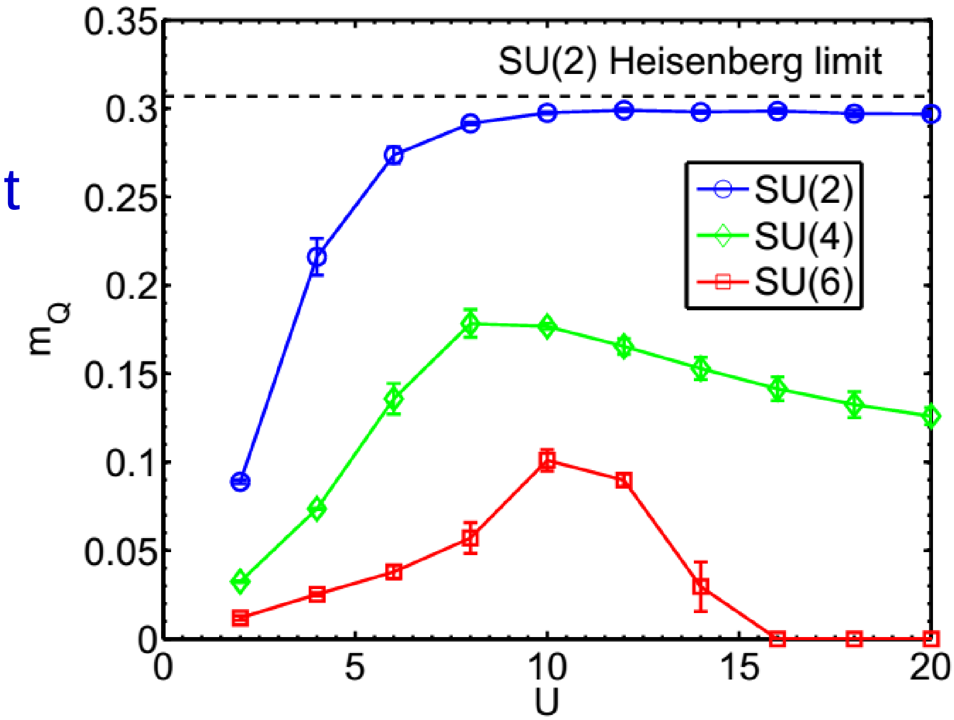
bond SU(2N) singlet

- Bond dimer state consists of $\binom{2N}{N}$ resonating Neel configurations.

- As N > coordination number, valence bond dimering is favored (Sachdev + Read).

A new phase transition inside the Mott phase (zero T)

Neel moment



Projector determinant QMC + pinning field.

D. Wang, Y. Li, Z. Cai, Z. Zhou, Y. Wang, C. Wu, Phys. Rev. Lett. 112, 156403 (2014).

- SU(2): smooth cross-over (J. Hirsch)
- SU(4) and SU(6): non-monotonic behavior of Neel moment.
- Complete suppression of AFM for SU(6).

T=0 projector determinant QMC algorithm (sign problem free at half-filling)

- Projection to the ground state.

$$|\Psi_G\rangle = \lim_{\beta\to\infty} \mathrm{e}^{-\beta H/2}|\Psi_T\rangle.$$

S. R. White et al., PRB (1989);
F. F. Assaad and H. G. Evertz, computational many-particle physics (2008)

- Trotter-Suzuki decomposition.

$$\mathrm{e}^{-\Delta\tau(K+V)} = \mathrm{e}^{-\Delta\tau K/2}\mathrm{e}^{-\Delta\tau V}\mathrm{e}^{-\Delta\tau K/2} + o[(\Delta\tau)^3],$$

- Exact Hubbard-Stratonovich (HS) decoupling for multi-component fermions:

$$e^{\lambda X^2} = \frac{1}{4}\sum_{l=\pm1,\pm2}\gamma_i(l)e^{\eta_i(l)X},$$

|eig(X)|=0,1,2,3

where $a = e^{\lambda}$, $d = \sqrt{8+a^2(3+a^2)^2}$.

$$\gamma(\pm1) = \frac{-a(3+a^2)+d}{d}, \qquad \gamma(\pm2) = \frac{a(3+a^2)+d}{d},$$

$$\eta(\pm1) = \pm\cosh^{-1}\left\{\frac{a+2a^3+a^5+(a^2-1)d}{4}\right\}$$

$$\eta(\pm2) = \pm\cosh^{-1}\left\{\frac{a+2a^3+a^5-(a^2-1)d}{4}\right\},$$

Da Wang et al., PRL (2014)

Projector QMC with the pinning field

- Usual methods to identify long-range-order in simulations:

1) 2-point correlation function: $\lim_{r\to\infty} < S(r)S(0) > \neq 0$

2) Structure factor: $\frac{1}{L^2}\sum < S(r)S(0) > e^{iQr} \neq 0$

Square of order parameter

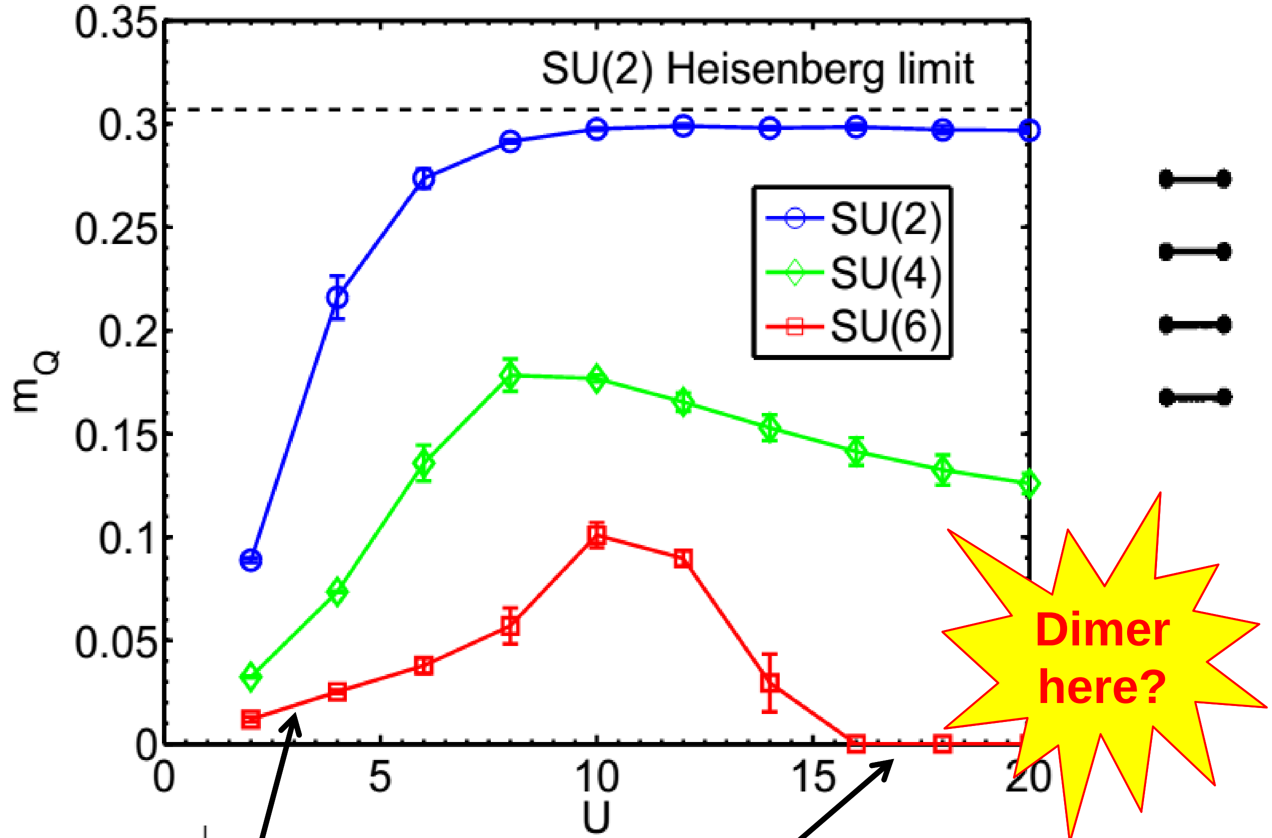
- The pinning field method (sensitive to weak ordering):

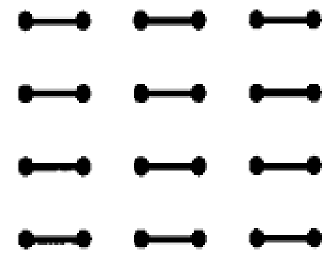
Add an external field at the center, and measure the spatial decay of induced magnetic moment.

$\lim_{r\to\infty} < S(r) >_h \neq 0$ → Order parameter

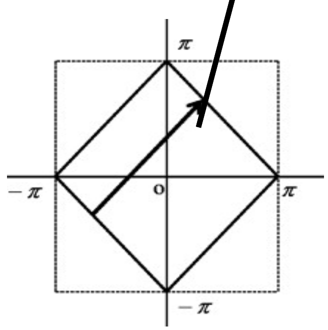
S. R. White and A. L. Chernyshev, PRL (2007); F. F. Assaad and I. F. Herbut, PRX (2013)

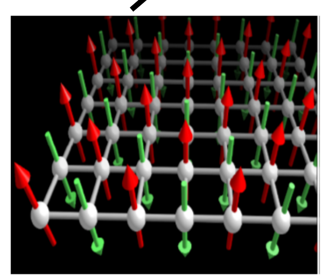
Competition between FS nesting and local moment!





Itinerancy:
FS nesting
Q(pi, pi)



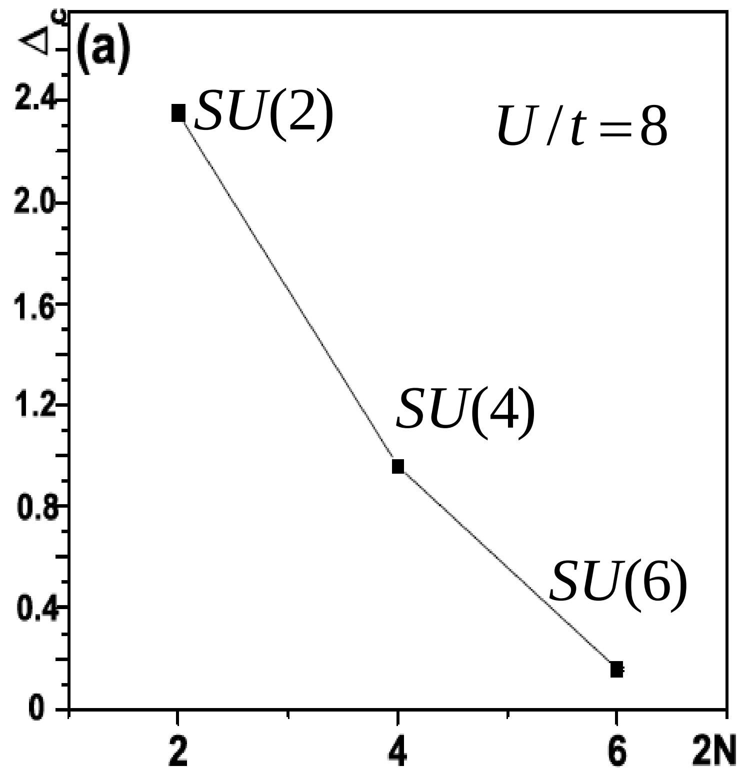


Local moment
Q(pi,0)

Mott gap: short-range charge fluctuations

- Single particle gap extracted from Green's function.

$$G(i,i,\tau) = \left\langle G \mid c_\alpha^+(i,\tau) c_\alpha(i,0) \mid G \right\rangle$$
$$\rightarrow e^{-\Delta_c \tau}$$



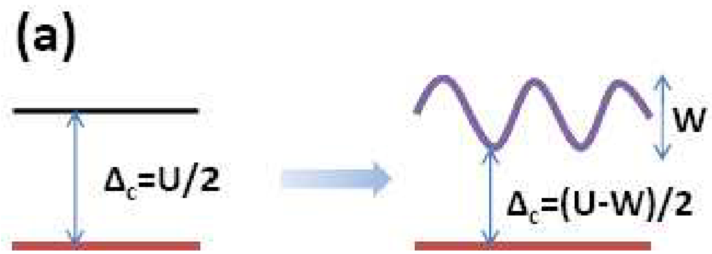
- Mott insulating states do not mean that charge does not move! Charge localization length.

$$\xi_c / a_0 \approx t / \Delta_c$$

- Enhancing charge fluctuations as N increases! It is NOT legitimate to neglect charge degree of freedom.

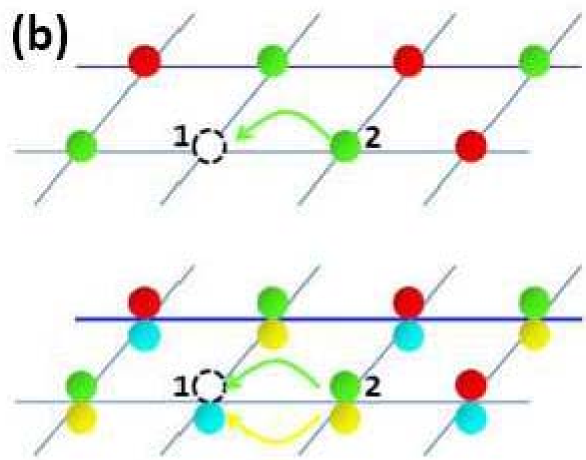
Estimation of single particle gap v.s N (large U)

- Charge gap decreases due to the enhanced number of hopping processes of charge excitations.



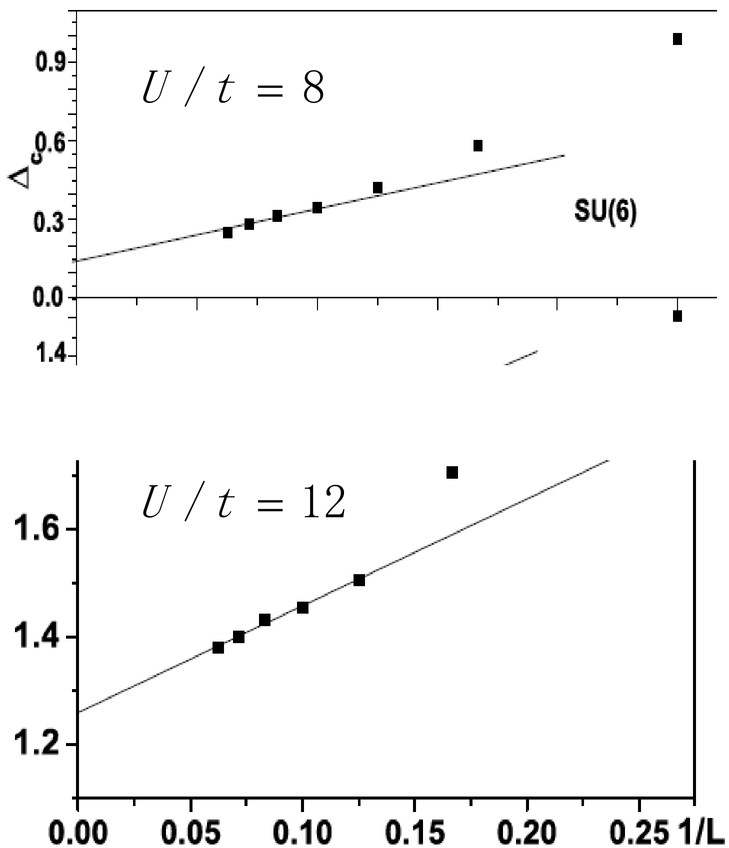
$SU(2)$

$SU(4)$

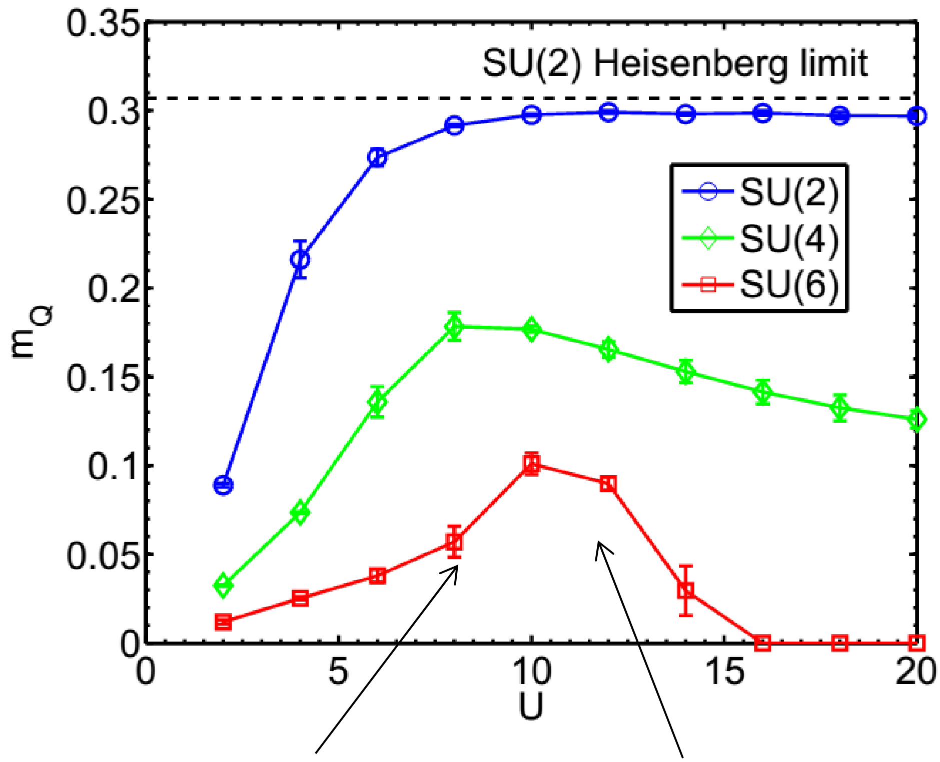


$$W \propto Nt$$

Rapid increase of Mott gap around U~10 (SU(6))



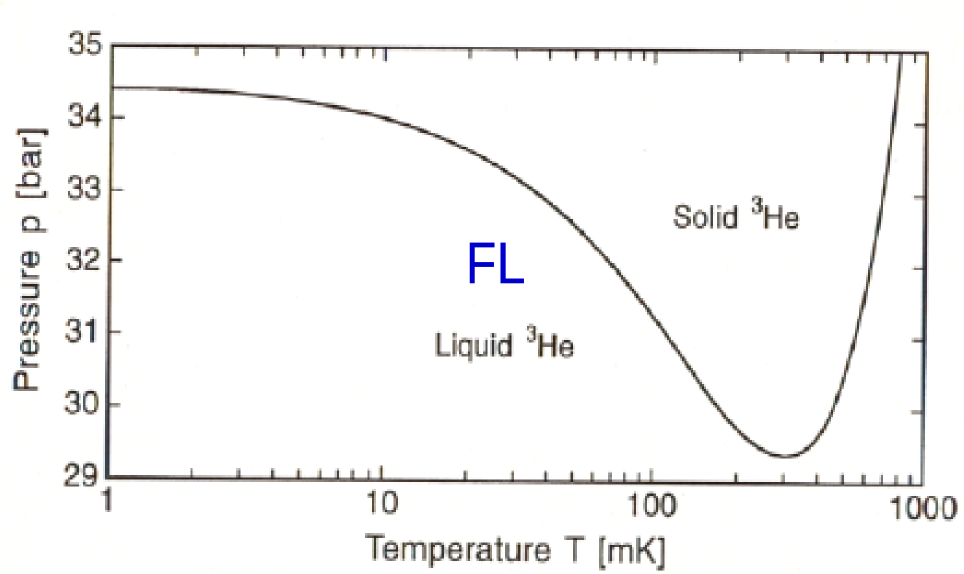
Signature of Mottness



$\Delta_c / t \approx 0.2$ $\Delta_c / t \approx 1.26$

Thermodynamics: Pomeranchuk effect

- In Mott-insulators, all the sites contribute to entropy through spin configurations, while in Fermi liquids, only fermions close to Fermi surfaces contribute.



- Pomeranchuk effect is more efficient in large spin systems due to the enhanced entropy capability.

S. Taie, arXiv 1208.4883; K. R. Hazzard, et al PRA 2012, Z. Cai et al, PRL, 2013.

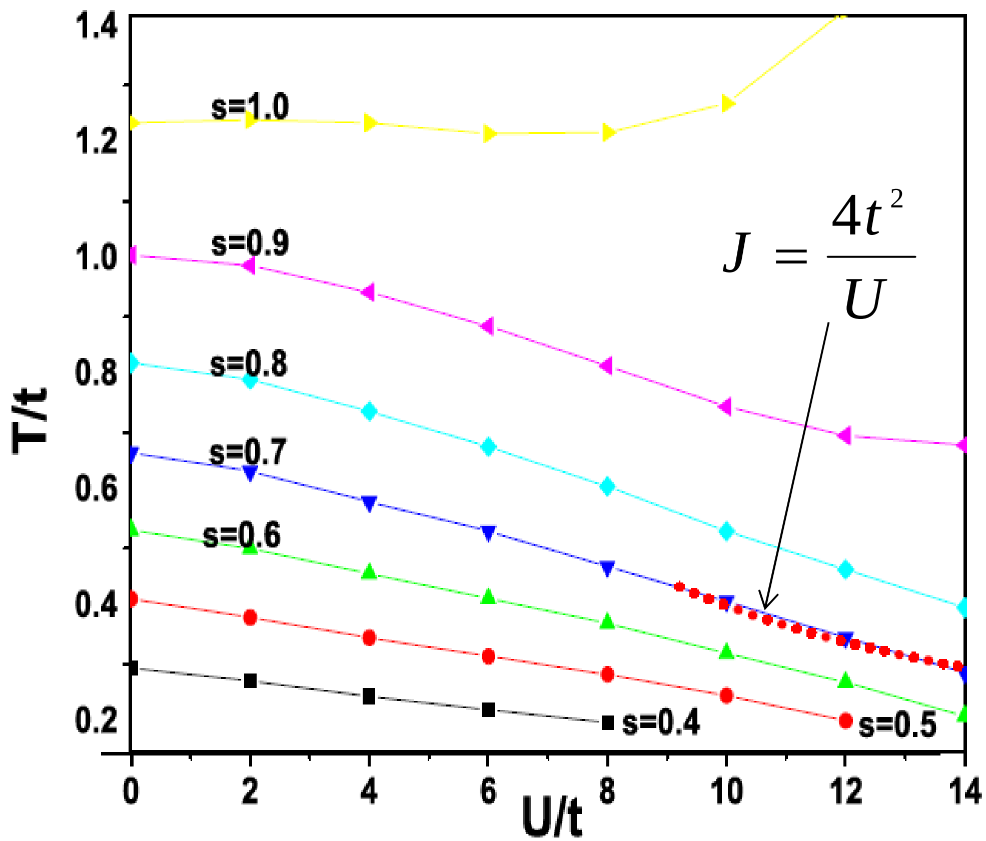
Pomeranchuk effect (SU(6), half-filling)

- Iso-entropy curve (three-particle per site).

$$S_{su(2N)} = S/(NL^2)$$

$$\frac{S_{su(2N)}(T)}{k_B} = \ln 4 + \frac{E(T)}{T} - \int_T^{\infty} dT' \frac{E(T')}{T'^2},$$

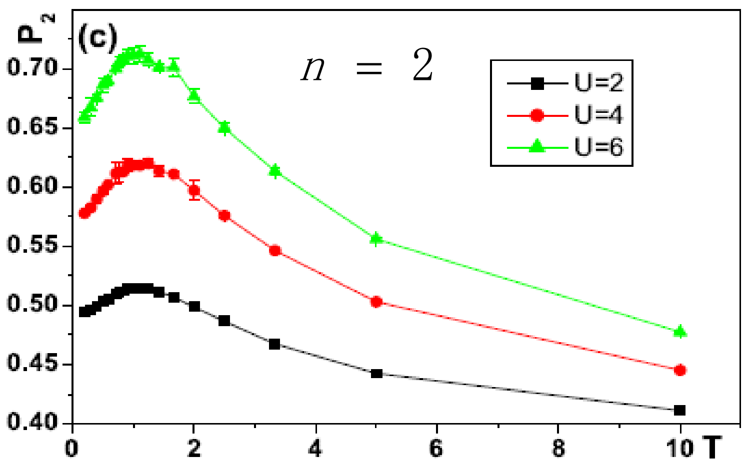
- As entropy per particle s<0.7, increasing U can cool the system below the anti-ferro energy scale J.



Sample size 10×10

Z. Cai, H. H. Hung, L. Wang, D. Zheng, and C. Wu, PRL 2013.

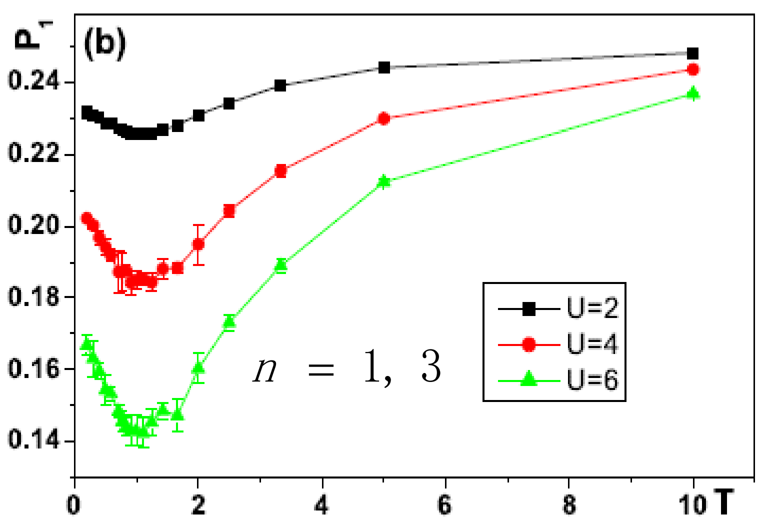
Probability of onsite occupation (SU(4))

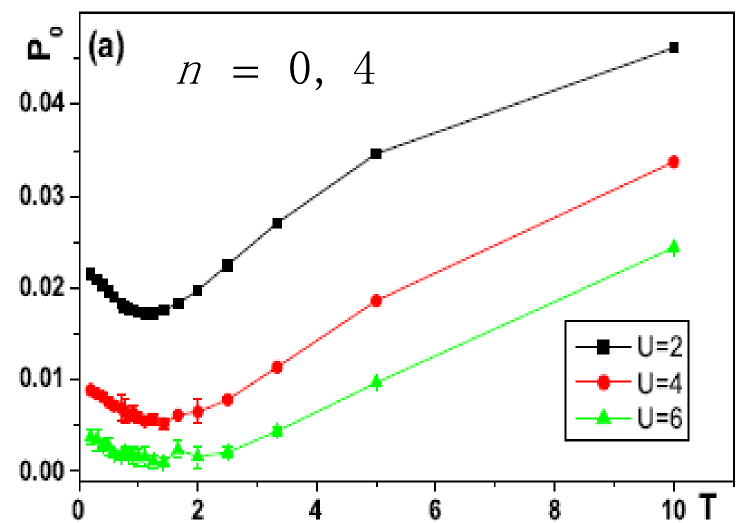


$$P(0) = \prod_{\alpha=1}^{4}(1-n_i^{\alpha});$$

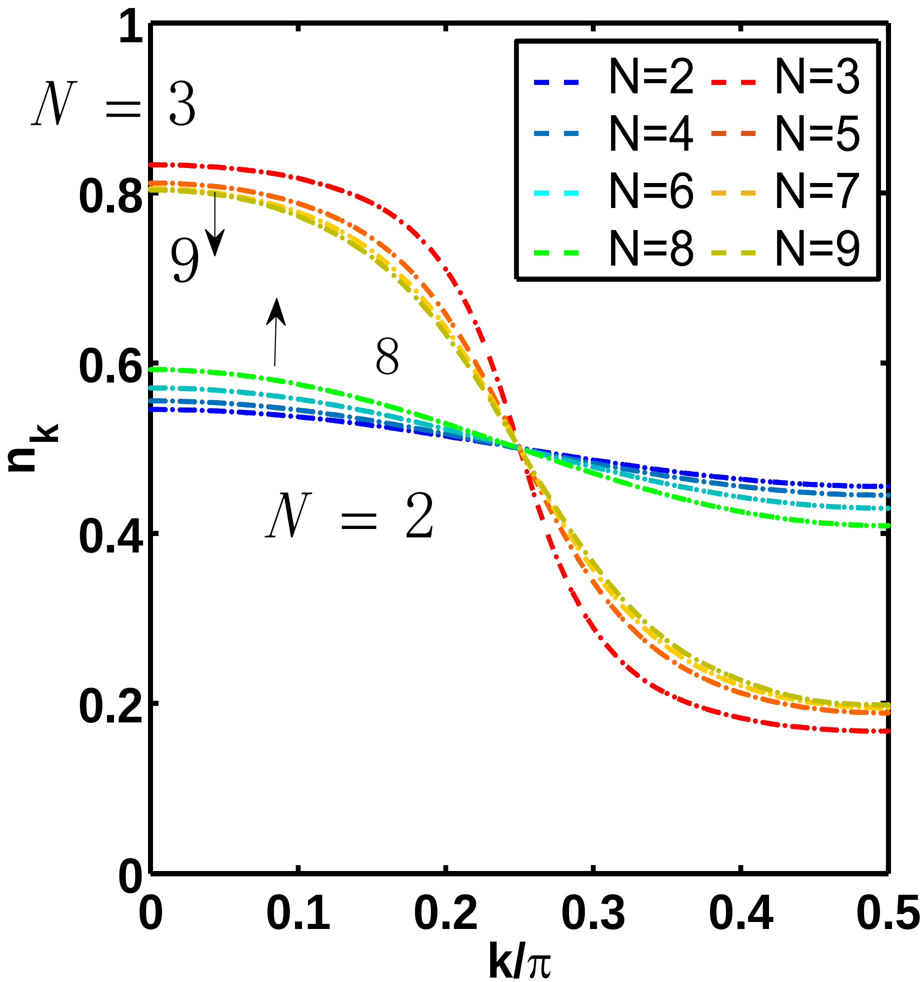
$$P(1) = \sum_{\alpha=1}^{4} n_i^{\alpha} \prod_{\beta\neq\alpha}(1-n_i^{\beta});$$

$$P(2) = \sum_{\alpha\neq\beta} n_i^{\alpha} n_i^{\beta} \prod_{\gamma\neq\alpha\,\beta}(1-n_i^{\gamma}).$$





1D SU(N): interaction effects v.s. N



- Fermi distribution n(k) at strong coupling at half-filling.

$$U = 40; \beta = 10$$

- Even N: interaction effect is weakened as increasing N.

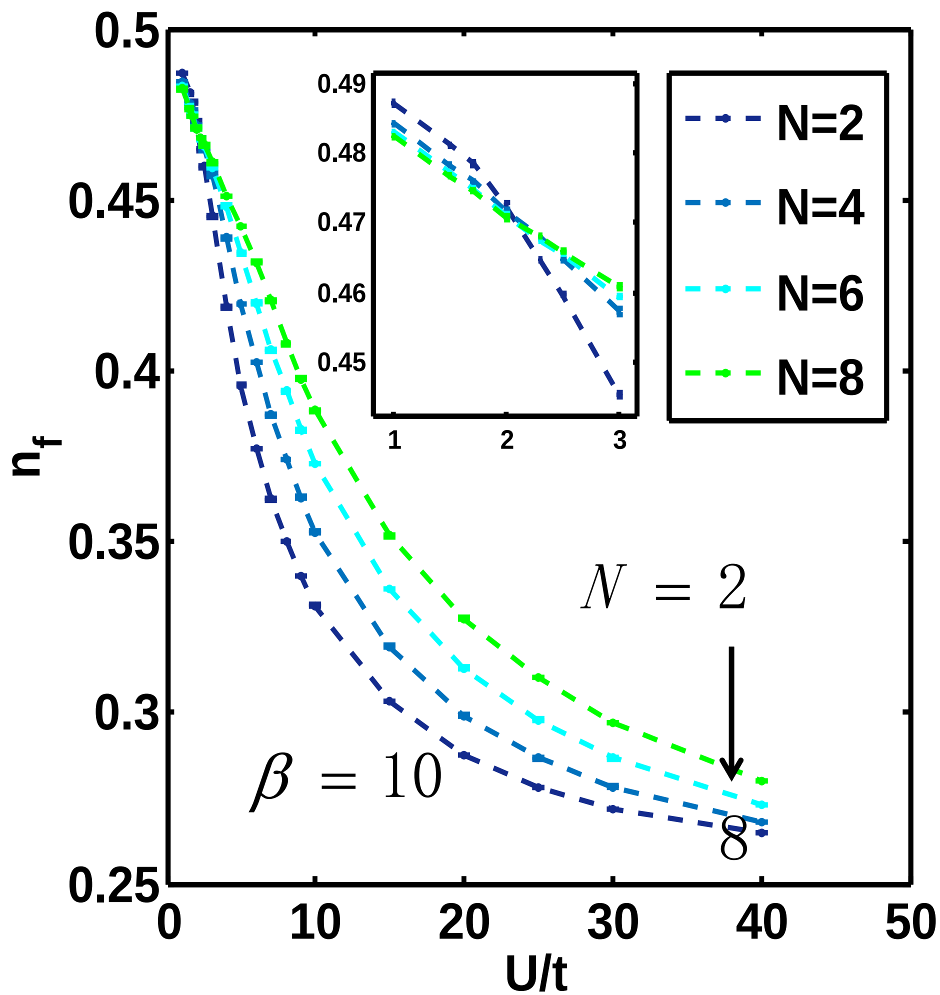
- odd N: interaction effect is enhanced as increasing N.

Universal (?) crossing (weak to strong coupling)

- Density of particles within kf as a probe of effect of U and N

$$n_f = \frac{1}{2\pi}\int_{-k_f}^{k_f} n_k$$

- All curves cross over from weak coupling to strong coupling at same point



Digression: itinerant FM from the Hubbard model

- Absence of FM in 1D Hubbard model – correlation effect. (Stoner mechanism overestimated exchange effect).

Lieb, Mattis, PR, 125, 164 (1962)

- Nagaoka FM (single hole, infinite U), and flat-band FM.

Nagaoka PR 147, 392 (1966), Mielke J. Phys. A (1991), Tasaki PRL (1992).

- A large stable phase of itinerant FM in 2D square/3D cubic lattice (quasi-1D band) by multi-orbital Hund's rule coupling
- Relevant to cold atom p-orbital systems and $SrTiO_3/LaAlO_3$ interface.

Y. Li, E. Lieb, C. Wu, PRL 112, 217201 (2014). S. Xu, Y. Li, C. Wu, arxiv:1411:0340.

Conclusion

- **Large-spin cold fermions are quantum-like NOT classical**!

- Elegancy of unification (group theory based on Sp(4)):

  AFM, SC and CDW phases/ Non-abelian Alice/Cheshire physics

- SU(6) Mott-ness: competition between Fermi surface (Slater) and local moments (Mott).

  Quantum phase transitions in the Mott regime.

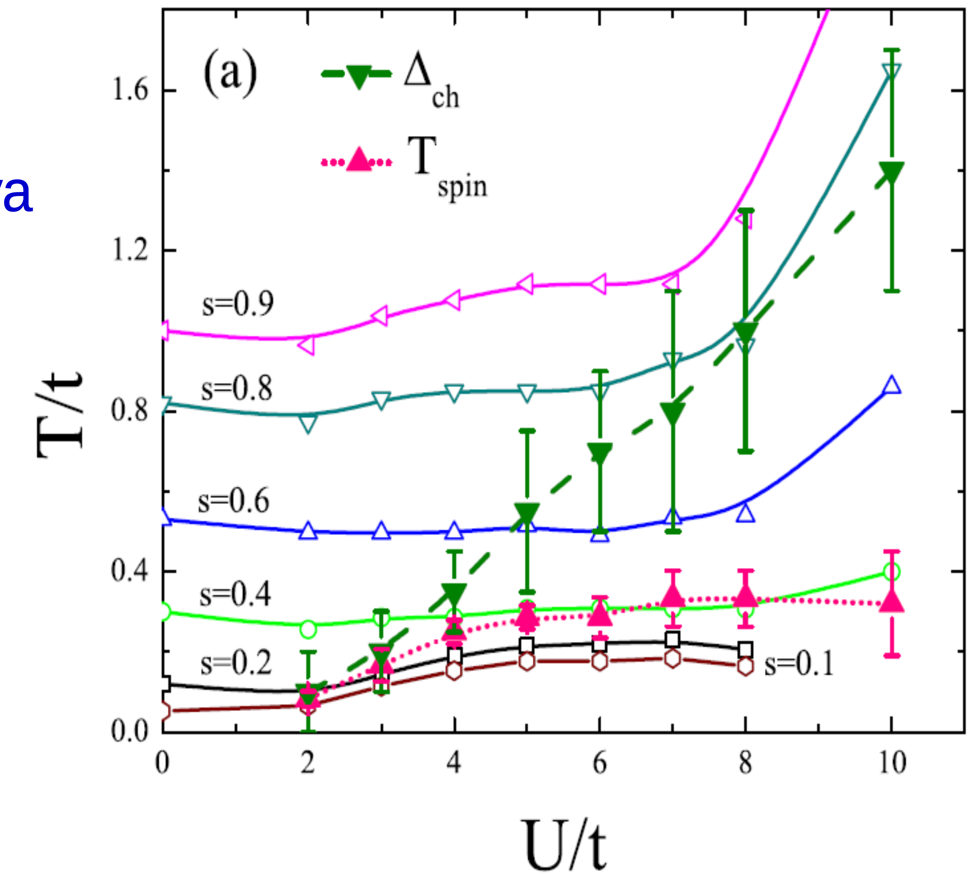
- Pomeranchuk cooling of 2D SU(6) Hubbard model.

- 1D SU(N) Hubbard model: interaction effects v.s. N.

Inefficiency of Pomeranchuk cooling of SU(2) fermions

- The iso-entropy curve for spin-1/2 Hubbard model at half-filling – QMC by T. Paiva et al, PRL 2010.

- The ordering tendency of the SU(2) AFM suppresses the spin entropy.



T. Paiva, et al, PRL 104, 066406 (2010).

Entropy capability per particle for half-filled SU(2N) Hubbard model

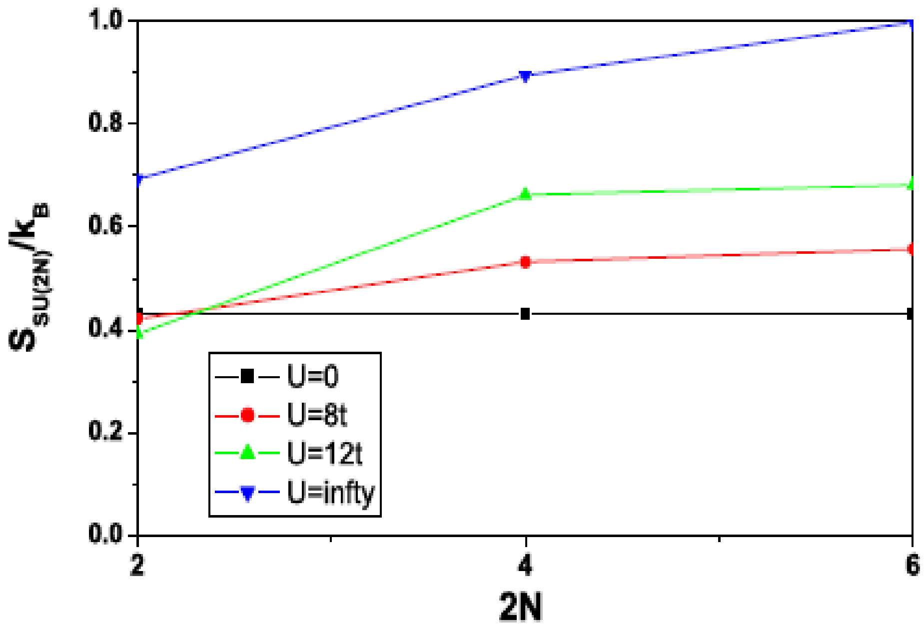


FIG. 1: Entropy per particle $S_{su(2N)}$ for the SU($2N$) Hubbard model at half-filling v.s. $2N$ in a 10×10 square lattice. The temperature is fixed at $T/t = \frac{1}{3}$. The line of $U/t = \infty$ is from the results of Eq. 3.

- Entropy per particle at U→ infinity and N→ infinity.

$$\frac{S_{Su(2N)}}{k_B} = \frac{1}{N} \ln \frac{(2N)!}{N!N!} \underset{N\to\infty}{\longrightarrow} \ln 4$$

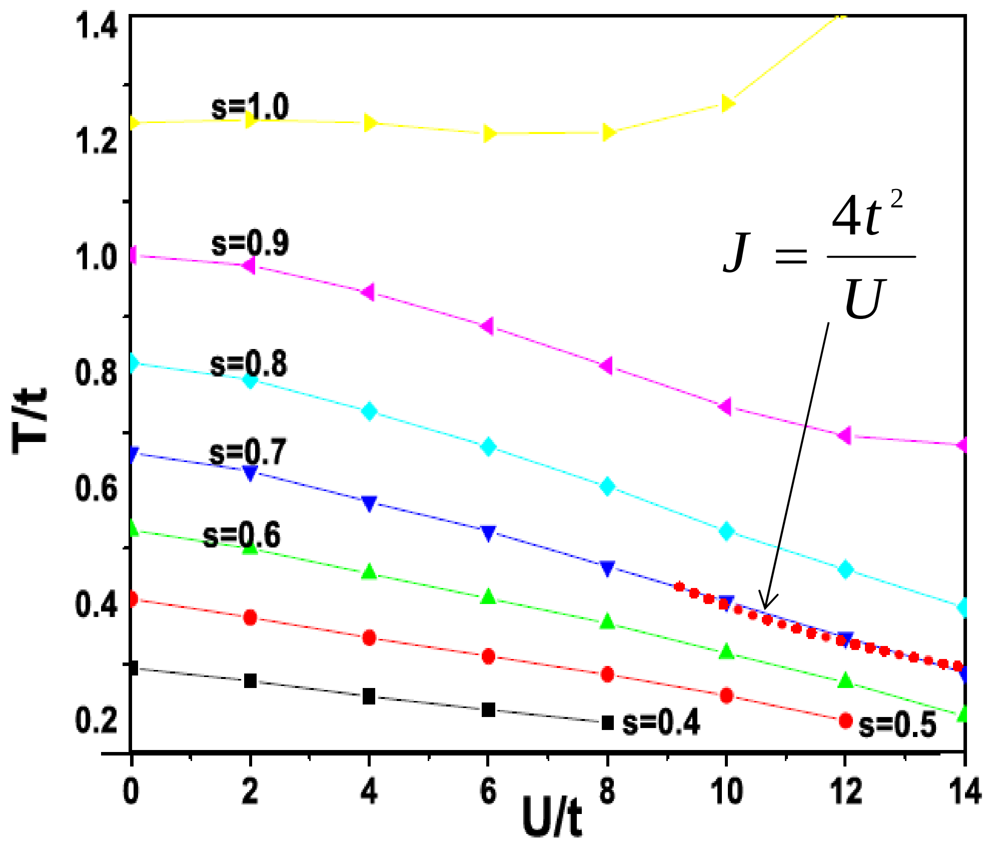
Pomeranchuk cooling for SU(6) fermions at half-filling

- Iso-entropy curve at half-filling (three-particle per site).

$$S_{su(2N)} = S/(NL^2)$$

$$\frac{S_{su(2N)}(T)}{k_B} = \ln 4 + \frac{E(T)}{T} - \int_T^\infty dT' \frac{E(T')}{T'^2},$$

- As entropy per particle s<0.7, increasing U can cool the system below the anti-ferro energy scale J.



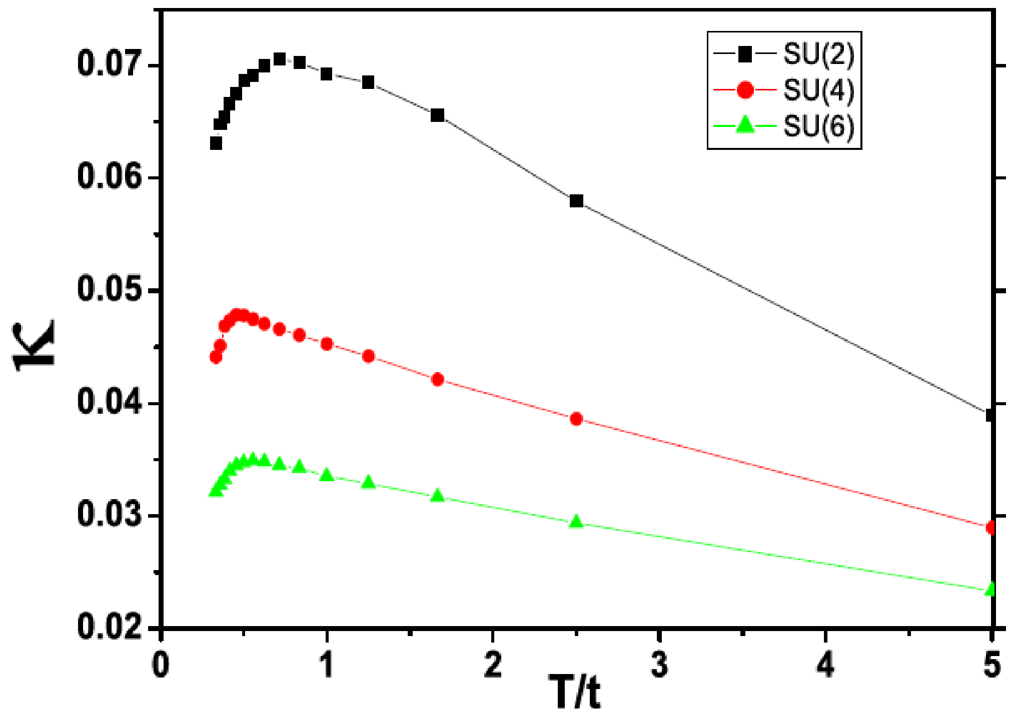
Sample size 10×10

Z. Cai, H. H. Hung, L. Wang, D. Zheng, and C. Wu, arxiv1202.6323.

Compressibility

- Charge fluctuation energy scale.

$$\kappa_{SU(2N)} = \frac{1}{L^2}\frac{\partial N_f}{\partial \mu} = \frac{1}{TL^2}(\langle \hat{N}_f^2 \rangle - \langle \hat{N}_f \rangle^2)$$



The normalized compressibility $\kappa_{su(2N)}/(2N)$ v.s. T

Sample size 10×10

$U/t = 4$

Z. Cai, H. H. Hung, L. Wang, D. Zheng, and C. Wu, arxiv1202.6323.

Magnetic susceptibility v.s. T

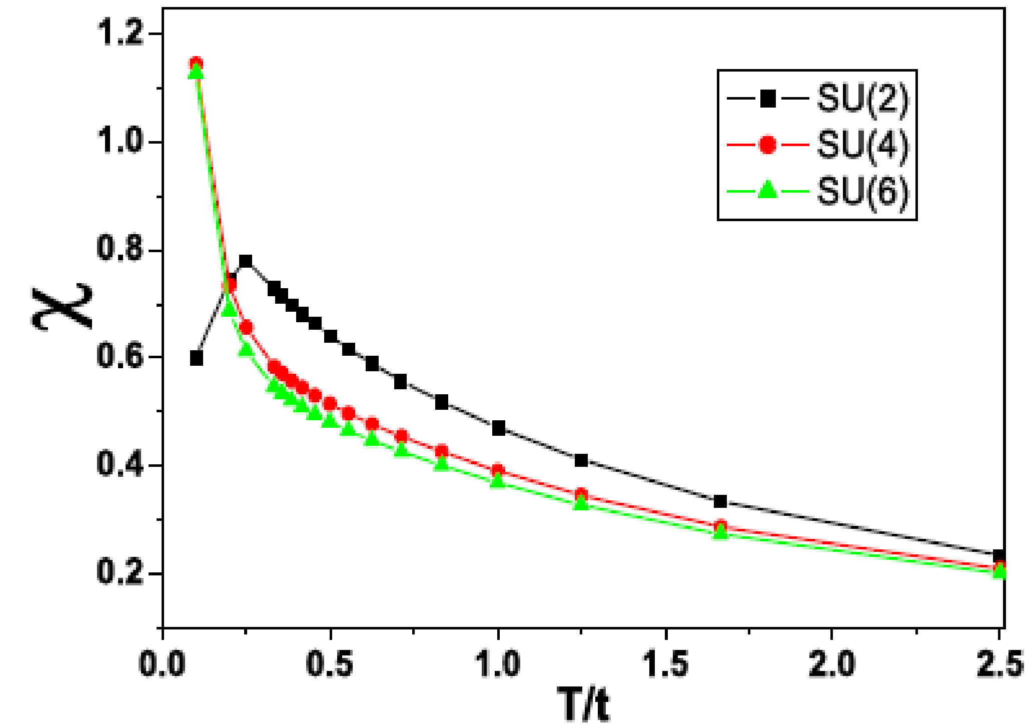


FIG. 4: The normalized SU(2N) susceptibilities $\chi_{su(2N)}$ v.s. T with fixed $U/t = 4$ for $2N = 2, 4, 6$

Sample size 10×10

Z. Cai, H. H. Hung, L. Wang, D. Zheng, and C. Wu, arxiv1202.6323.

1D systems: strongly correlated but understandable

- Bethe ansatz results for 1D SU(2N) model:

2N particles form an SU(2N) singlet; Cooper pairing is not possible because 2 particles can not form an SU(2N) singlet.

P. Schlottmann, J. Phys. Cond. Matt 6, 1359(1994).

- Competing orders in 1D spin 3/2 systems with Sp(4) symmetry.

Both quartetting and singlet Cooper pairing are allowed.

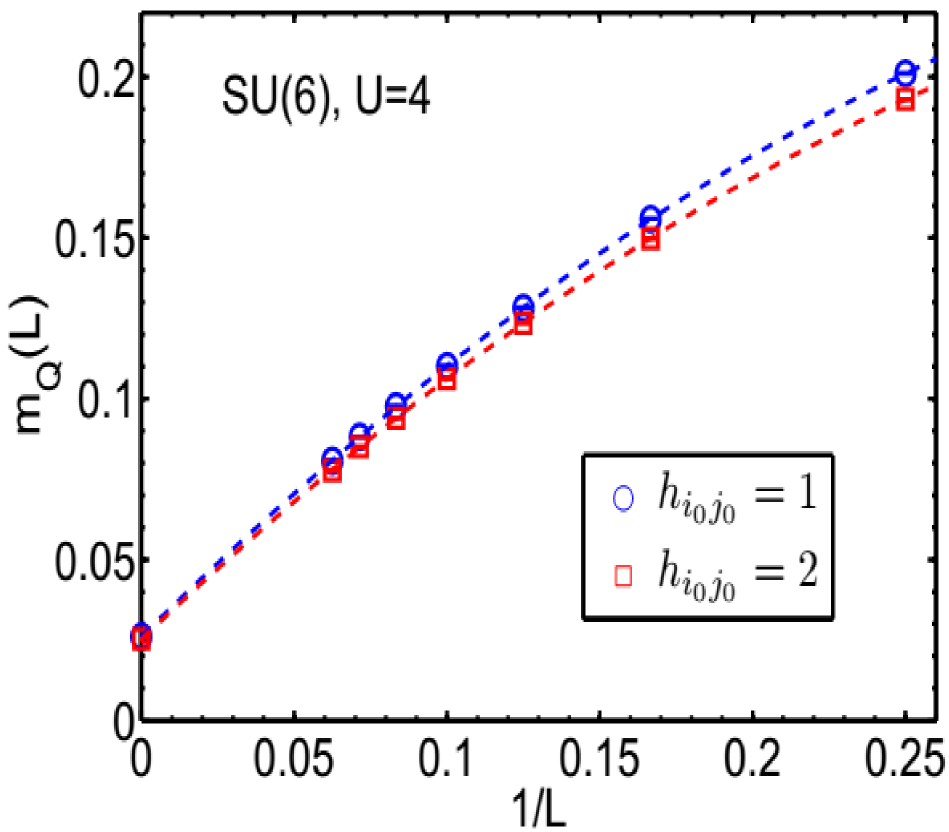
Transition between quartetting and Cooper pairing.

C. Wu, Phys. Rev. Lett. 95, 266404(2005).

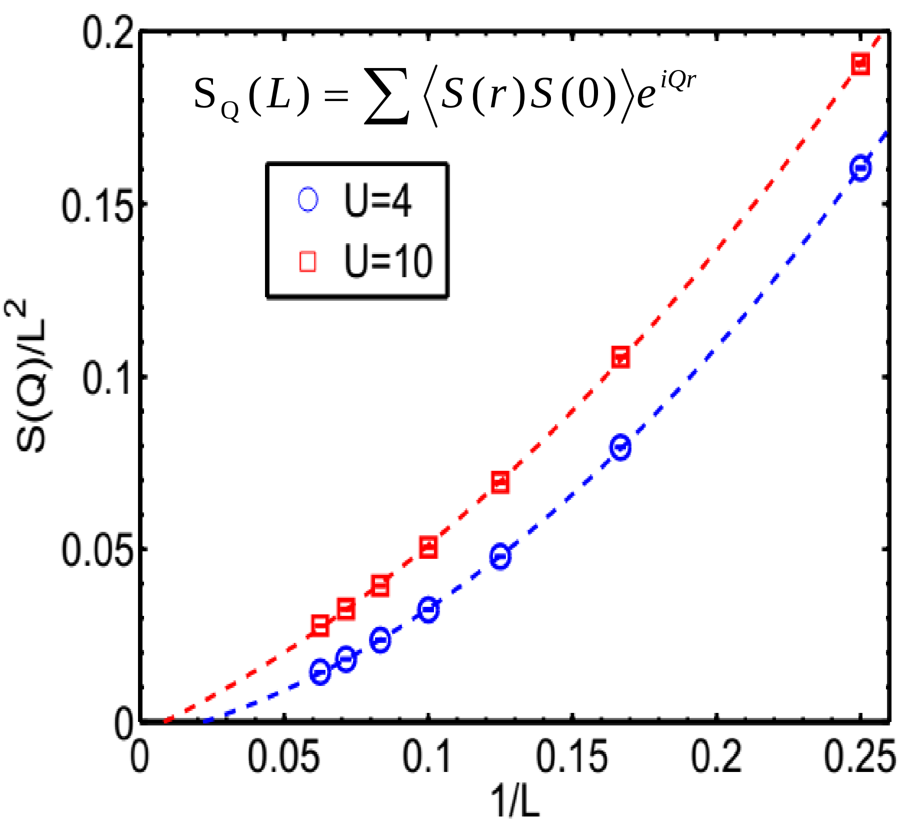
QMC with pinning field: sensitive to Neel order

- Local pinning field for Neel order: $H_{pin,n} = h\{m_{i_0} - m_{j_0}\}$
- Long range order $m_Q = \lim_{L\to\infty} m_Q(L)$

$$m_Q(L) = \frac{1}{L^d}\sum \langle S(r)\rangle e^{iQr}$$



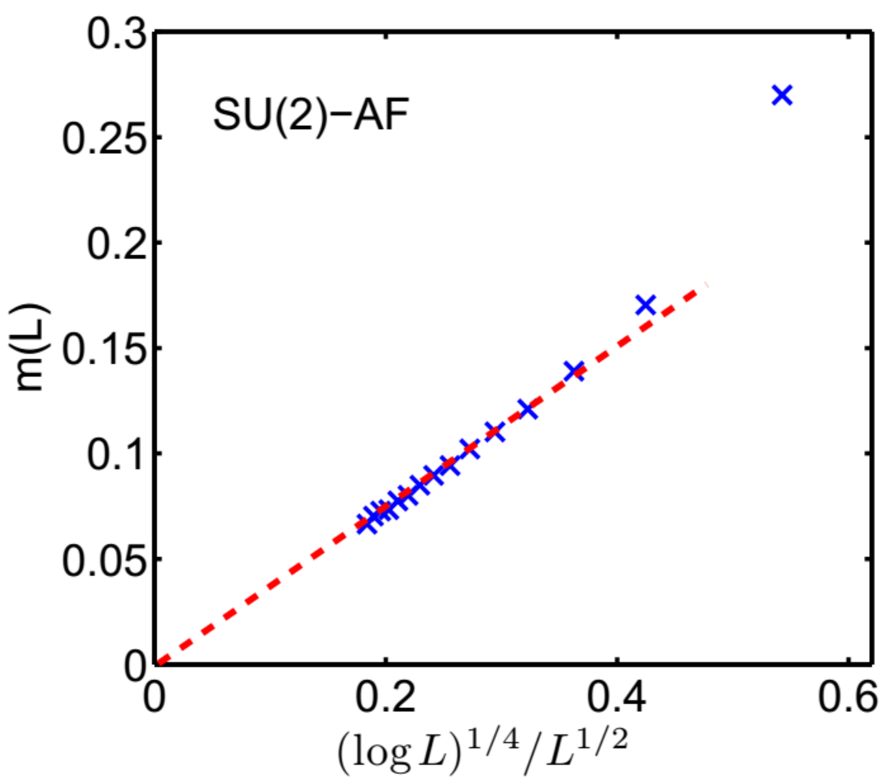
Comparison: structure factor



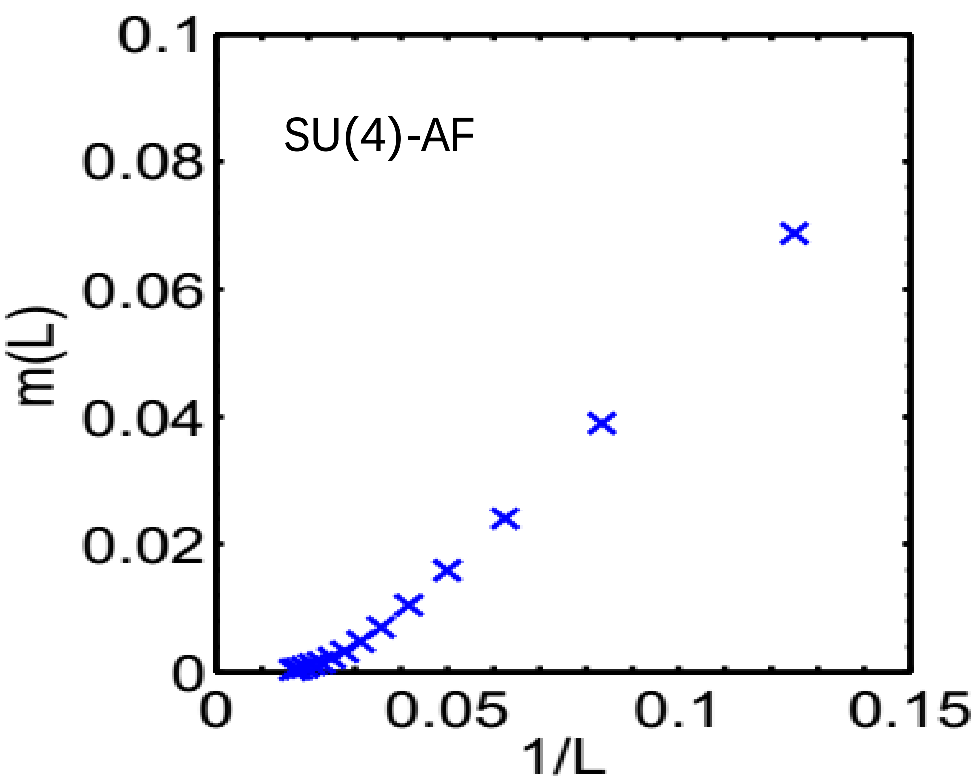
QMC with pinning field: NOT over-sensitive to Neel order

- 1D Hubbard model:

SU(2): critical behavior

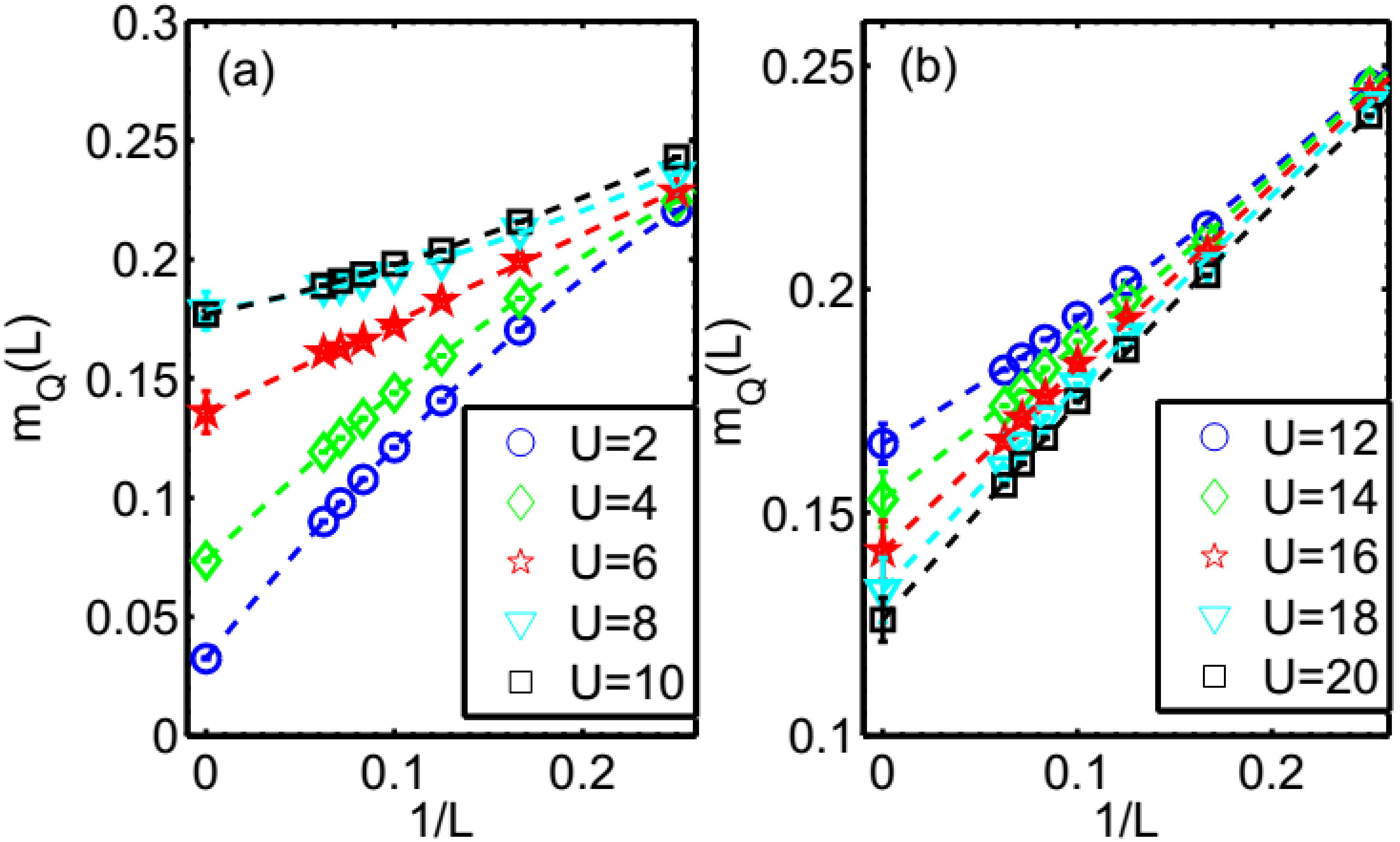


SU(4): no Neel order



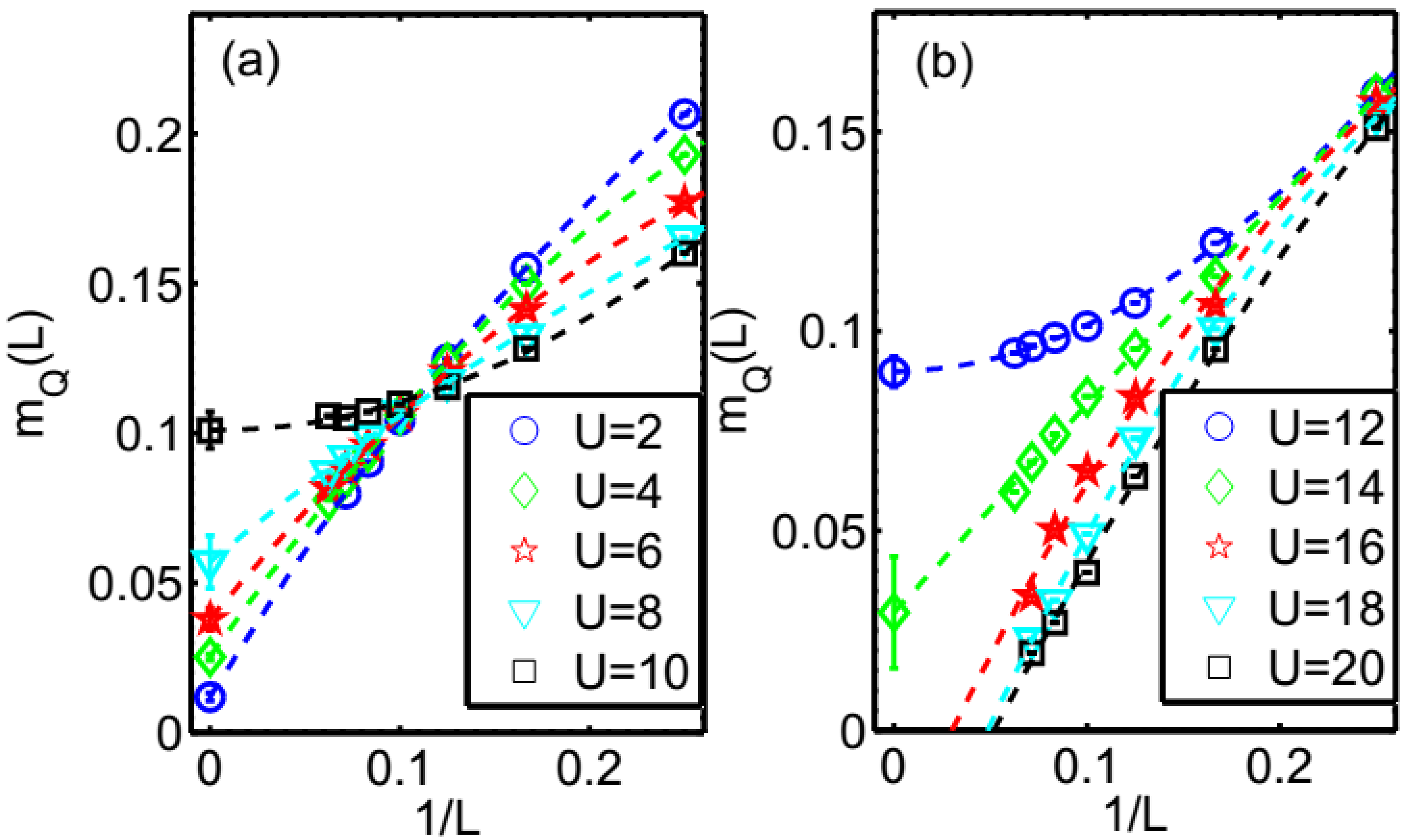
Finite size scaling: QMC with the pinning field SU(4)

Non-monotonic behavior



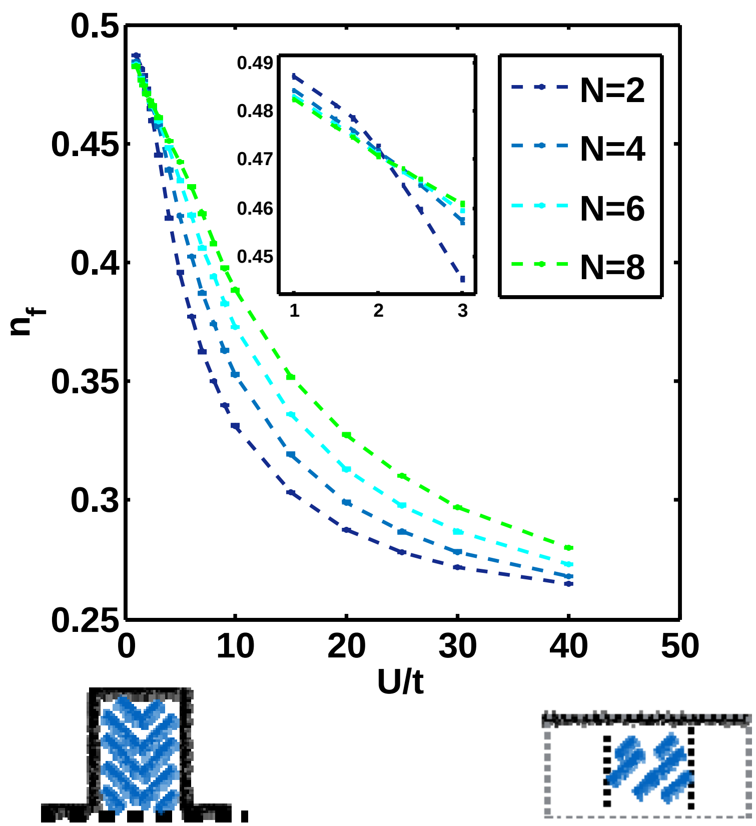
Finite size scaling: QMC with the pinning field SU(6)

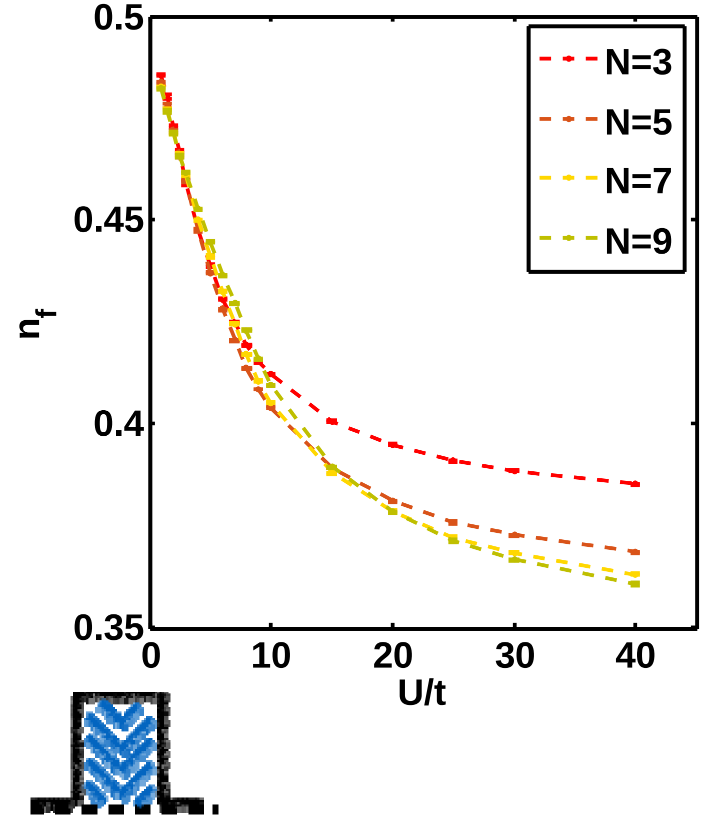
AF even disappear at large U!



density of particles within fermi surface as a probe of effect of U and N

$$n_f = \frac{1}{2\pi}\int_{-k_f}^{k_f} n_k$$



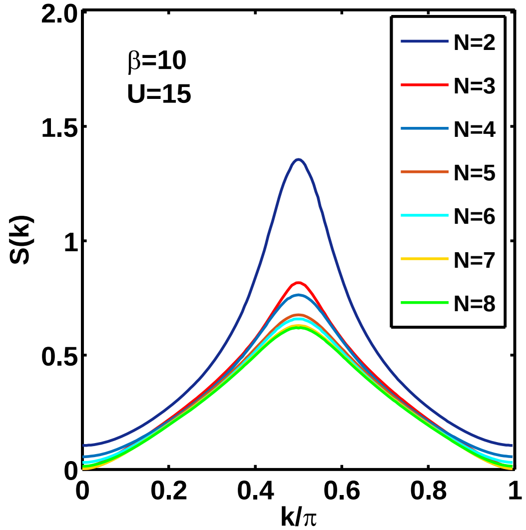


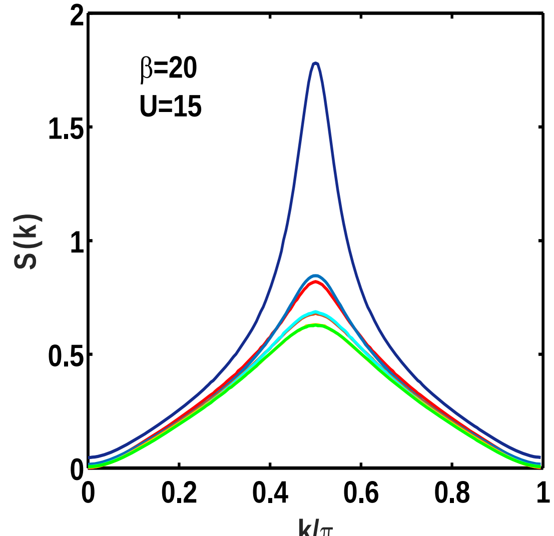
- Same saturated value at infinite U
- all curves cross over from weak coupling to strong coupling at same point

- Different saturated values at infinite U

Result: Spin Channel

$$S(k)=\frac{1}{L}\sum_{r,r'}(n_{\alpha,r}-n_{\beta,r})(n_{a,r'}-n_{\beta,r'})e^{ik(r-r')}$$



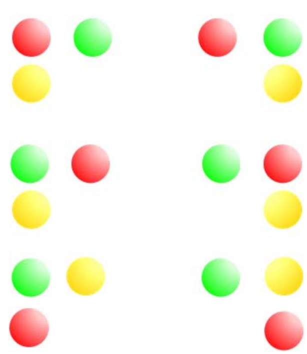


quasi-long range
AFM correlation

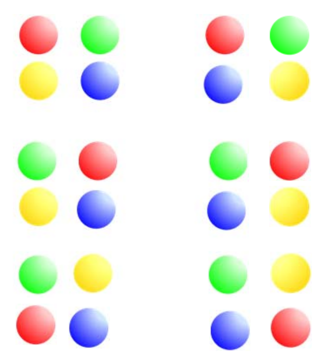
structure factor for N=2n-1 and N=2n are very similar

same number of resonating configurations

N=3



N=4

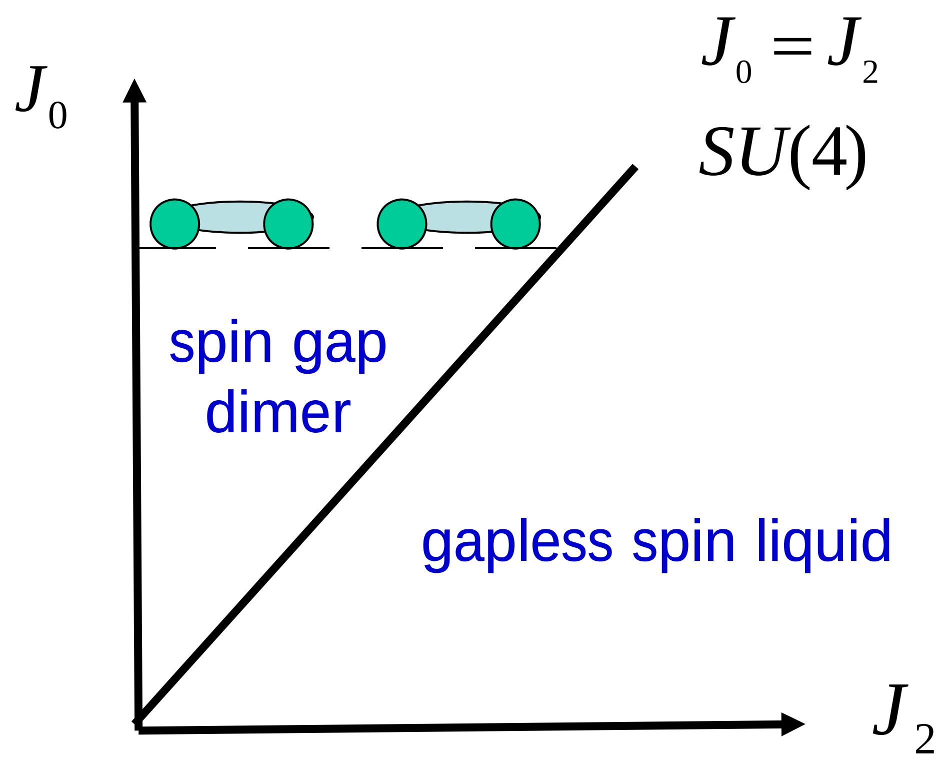


1D lattice (one particle per site)

• Phase diagram is obtained from bosonization analysis and confirmed from DMRG calculations.

• Gapped spin dimer phase at $J_0 > J_2$; bond spin singlet.

• Gapless spin liquid phase at $J_0 \le J_2$. Spin correlation exhibits 4-site periodicity of oscillations.



C. Wu, Phys. Rev. Lett. 95, 266404 (2005); Hung, Wang, and Wu, PRB 05446, (2011)

Unsolved difficulty: 2D phase diagram

- J_2=0, Neel ordering obtained by QMC.

K. Harada et. al. PRL 90, 117203, (2003).

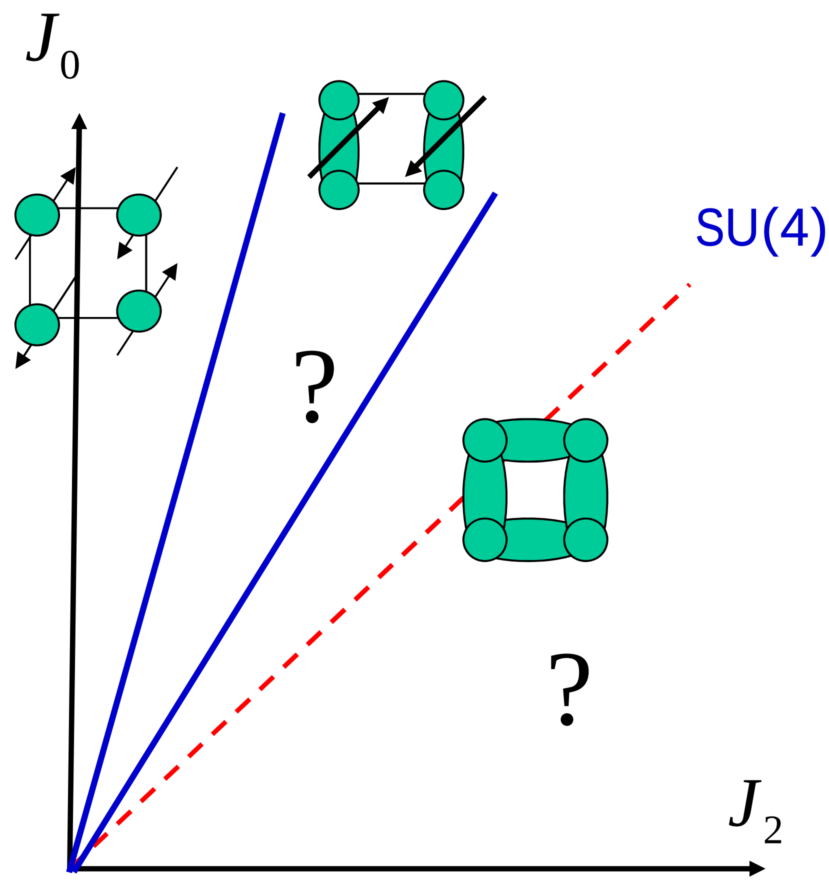
- J_2>0, no conclusive results! Difficult both analytically and numerically.

2D Plaquette ordering at the SU(4) point?

Exact diagonalization on a 4*4 lattice

Bossche et. al., Eur. Phys. J. B 17, 367 (2000).

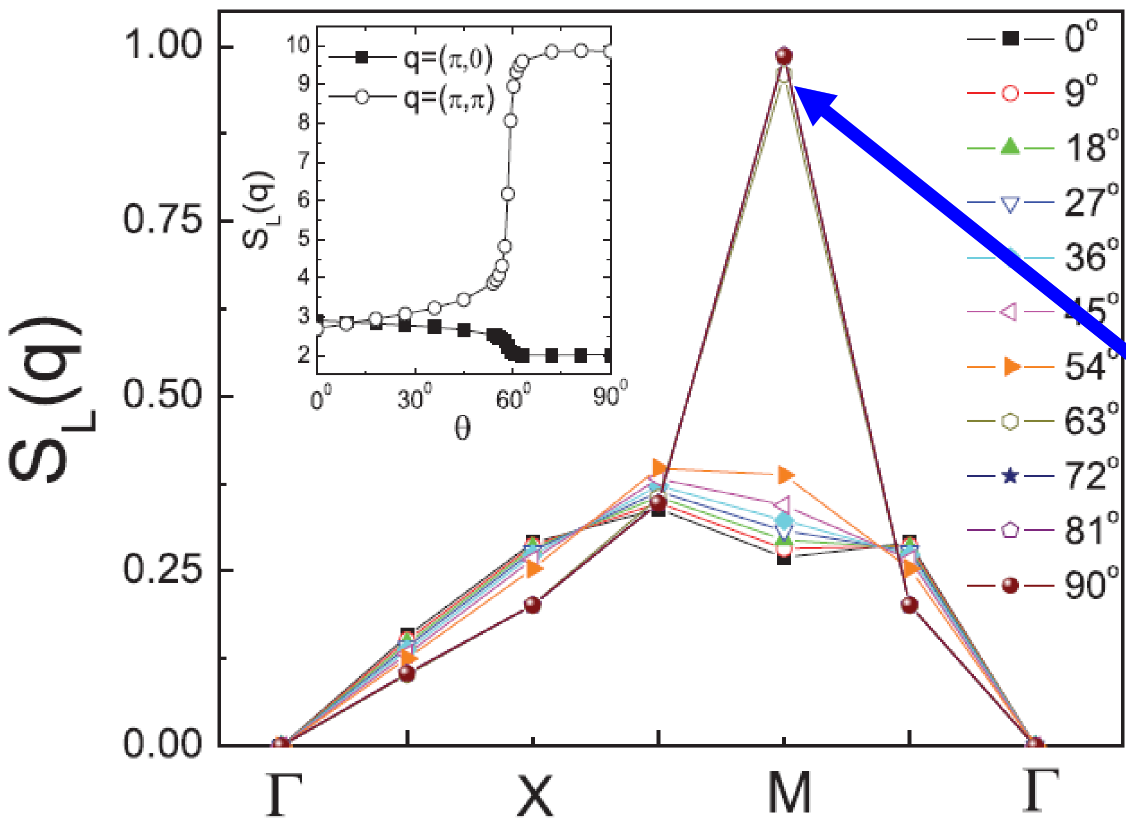
- Phase transitions as J_0/J_2? Dimer phases? Singlet or magnetic dimers?

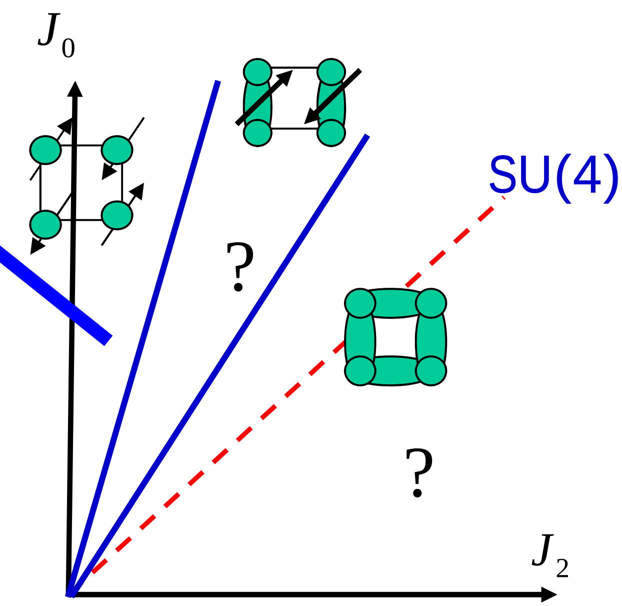


4x4 Exact diag. (I): Neel correlation

• Spin structure form factor peaks at (π,π) at $\theta > 60^\circ$, indicating strong Neel correlation.

$$S_L(q) \sim \sum_{1\le a<b\le 5} \sum_{i,j} \langle L_{ab}(i) L_{ab}(j) \rangle e^{i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)}$$





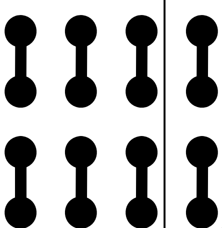
H. H. Hung, Y. P. Wang, C. Wu, Phys. Rev. B 84, 054406, (2011).

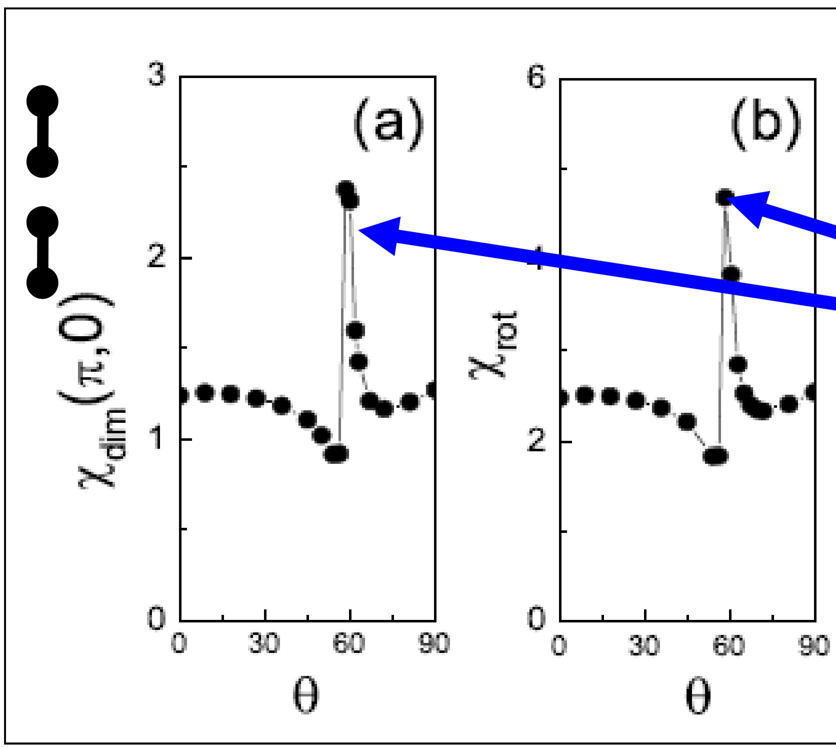
4x4 Exact Diag. (II): Dimer correlation

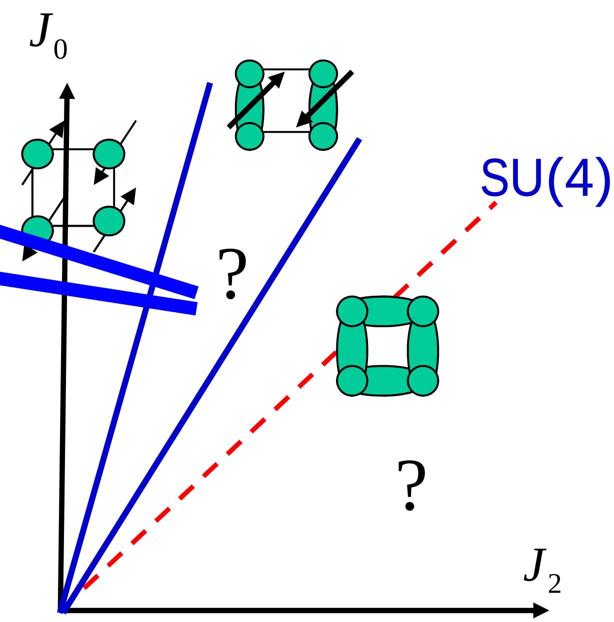
- Susceptibility: $H(\delta) = H_{exc} + \delta * H_{perp}$ $E(\delta) = E(0) - \frac{1}{2}\chi\delta^2,$
- a) Break translational symm:
- b) Break rotational symm:

$$H_{pert} = \sum_i \cos(\vec{Q}\cdot\vec{r}_i)\ H_{ex}(i,i+x),$$

$$H_{pert} = \sum_i H_{ex}(i,i+x) - H_{ex}(i,i+y)$$





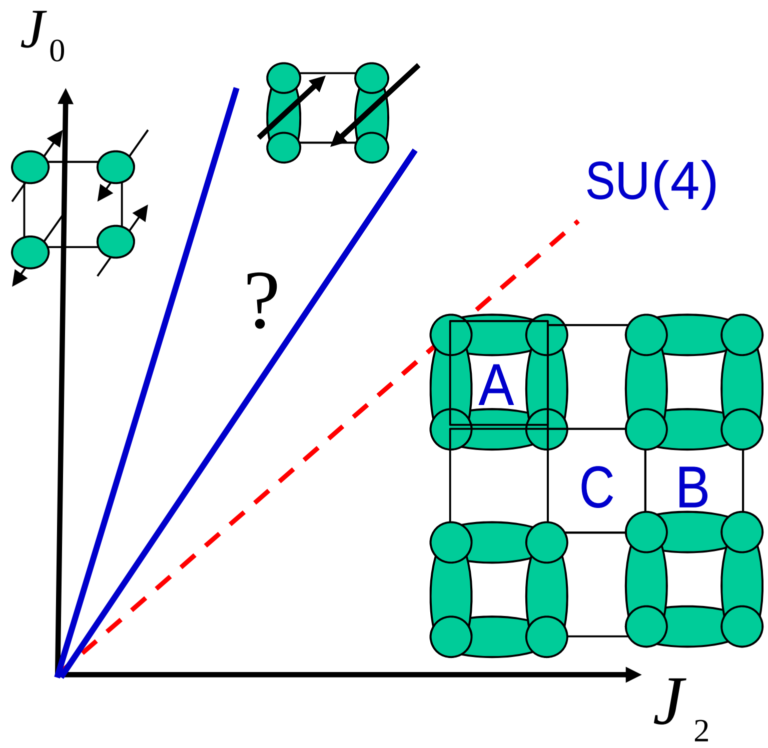


4x4 Exact Diag. (III): Plaquette formation?

- Local Casimir; analogy to total spin of SU(2).

$$C(r) \sim \left\langle \sum_{1\le a<b\le 5} \{ \sum_{i\in plaquette\, r} L_{ab}(i)\}^2 \right\rangle$$

C(r) →0: singlet



Open boundary condition