# Unconventional superfluid states in two-leg ladder systems

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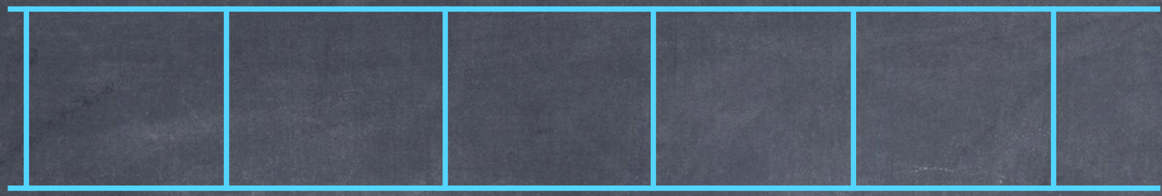


1. Two-leg Fermionic ladder w/ state-dependent hopping

Parity-mixed superfluid

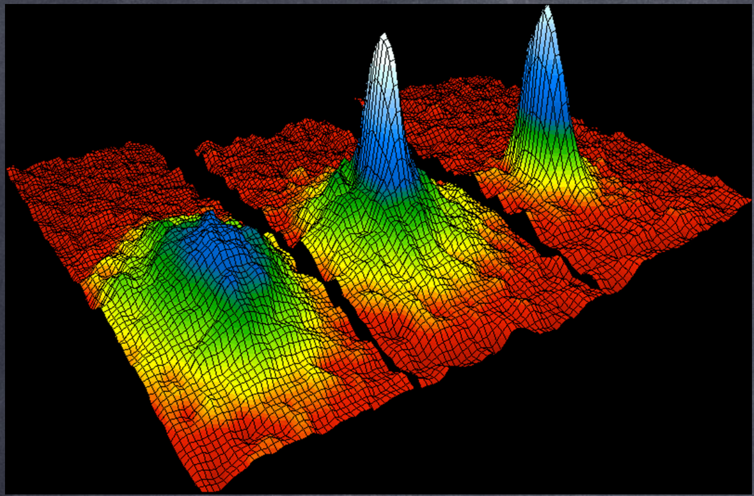
Spin-triplet superfluid

2. Two-leg Bosonic ladder w/ magnetic flux

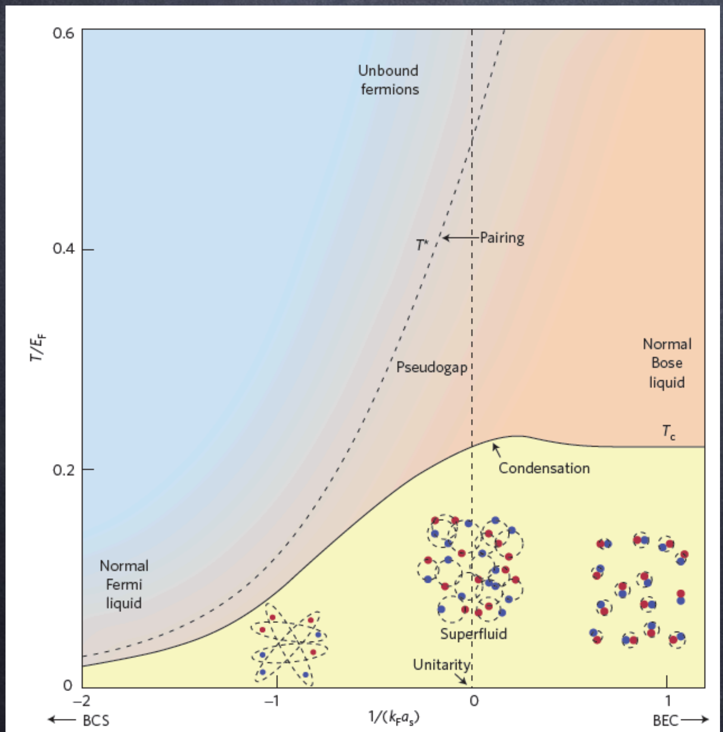
Population imbalance

Similarity with ferromagnetic XXZ model

# Superfluids with cold atoms



Bose-Einstein condensate (BEC)
with cold bosons




BCS-BEC crossover
with cold fermions

# 1D superfluid

- BEC does not occur in (interacting) 1D

Coleman, Mermin and Wagner, Hohenberg,...

- Superfluid can occur even in 1D

$$\left\langle \psi^{+}(r)\psi(0) \right\rangle \sim \frac{1}{r^{K}}$$

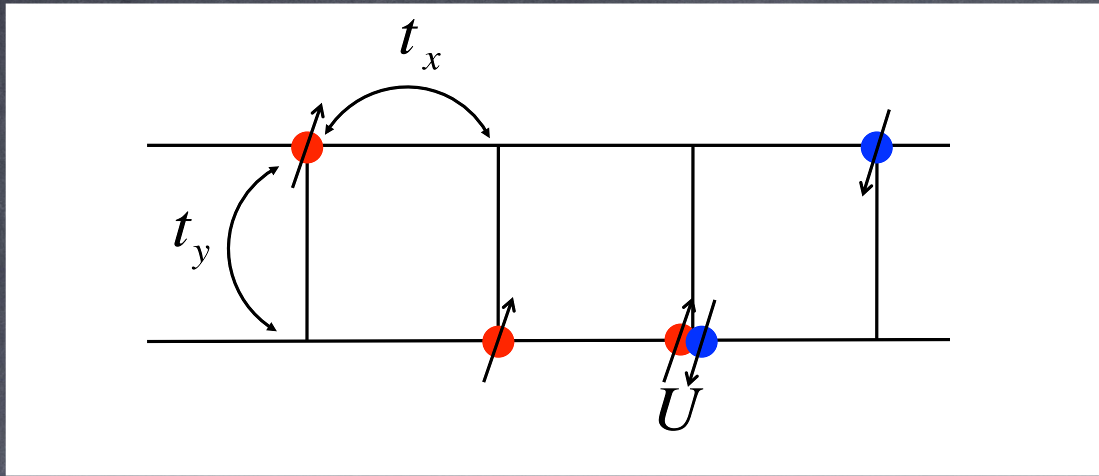
Quasi-Long-Range-Order

e.g., 1D Bose-Hubbard, 1D Fermi-Hubbard with attractive interaction,....

# Two-leg Fermionic ladder w/ state-dependent hopping

SU, A. Tokuno, T. Giamarchi, Phys. Rev. A**89**, 023623 (2014)
SU, T. Giamarchi, Phys. Rev. A**91**, 033605 (2015)

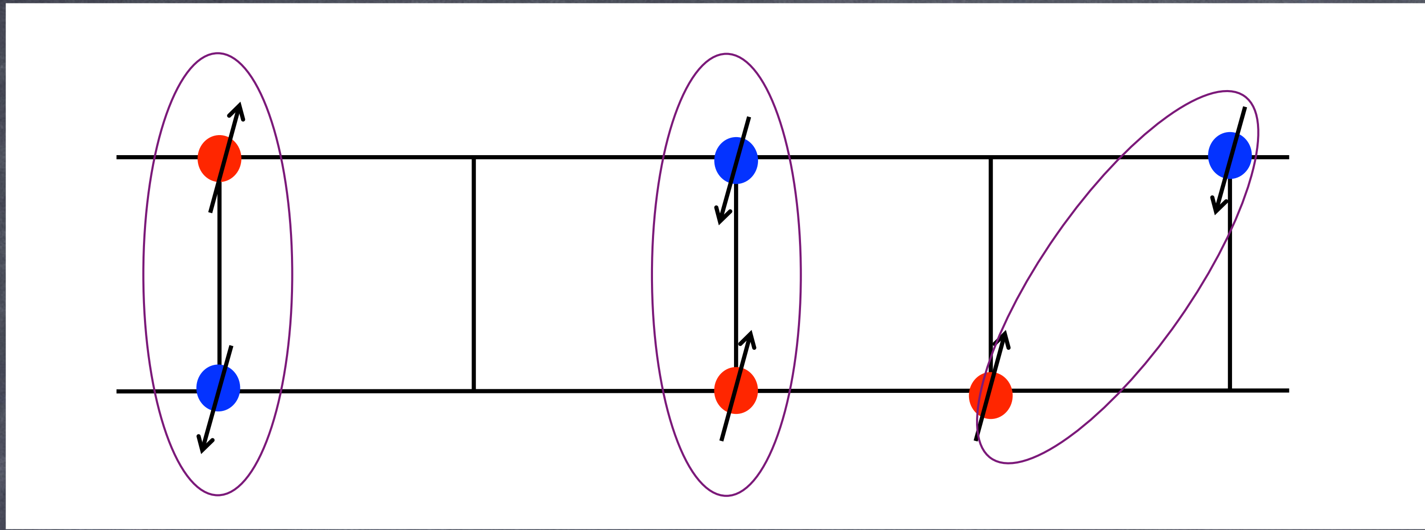
# Two-leg fermionic ladder




$$H = -t_x \sum_{j,p} c^{+}_{j,p,\sigma} c_{j+1,p,\sigma} - t_y \sum_{j} c^{+}_{j,1,\sigma} c_{j,2,\sigma}$$

$$+ U \sum_{j,p} n_{j,p,\uparrow} n_{j,p,\downarrow} + h.c.$$

# Two-leg fermionic ladder

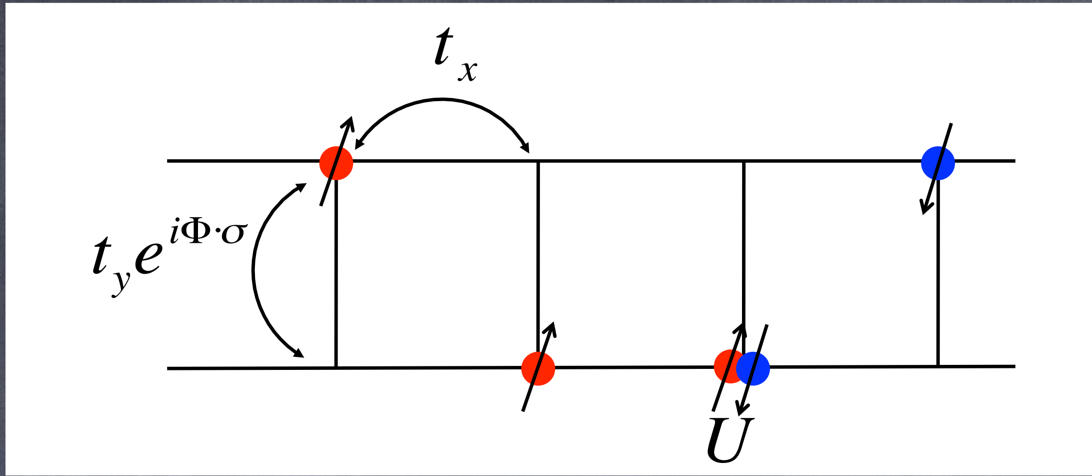


- This model had been explored in condensed matter
- For U>0 at incommensurate filling, the ground state is spin-singlet superfluid

⇒ analogy with d-wave superfluid

See e.g., T. Giamarchi, Quantum Physics in one dimension

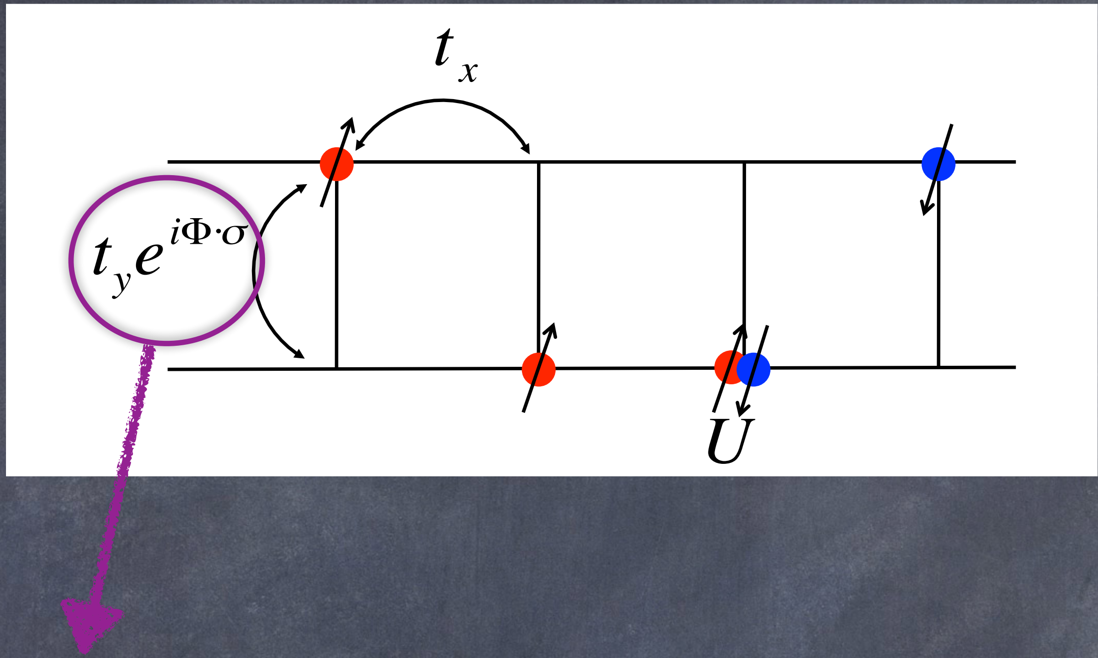
# Our model




$$H = -t_x \sum_{j,p} c^{+}_{j,p,\sigma} c_{j+1,p,\sigma} - t_y \sum_{j} (e^{i\vec{\Phi}\cdot\vec{\sigma}})_{\sigma\sigma'} c^{+}_{j,1,\sigma} c_{j,2,\sigma'}$$

$$+ U \sum_{j,p} n_{j,p,\uparrow} n_{j,p,\downarrow} + h.c.$$

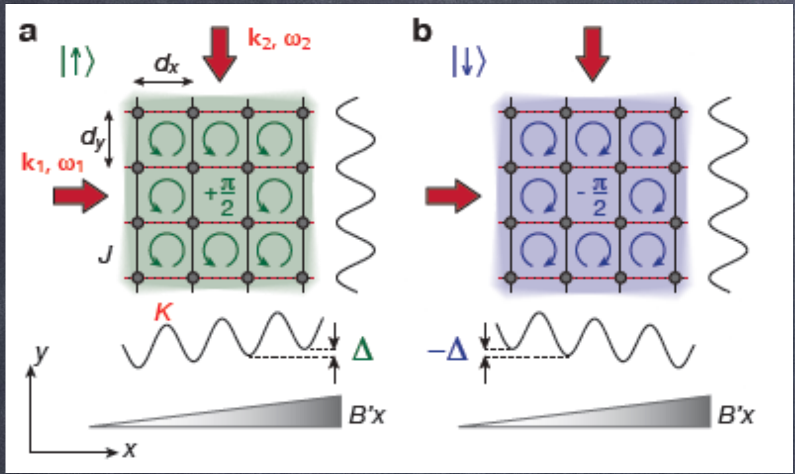
# Our model




State-dependent rung hopping
(spin-orbit coupling)

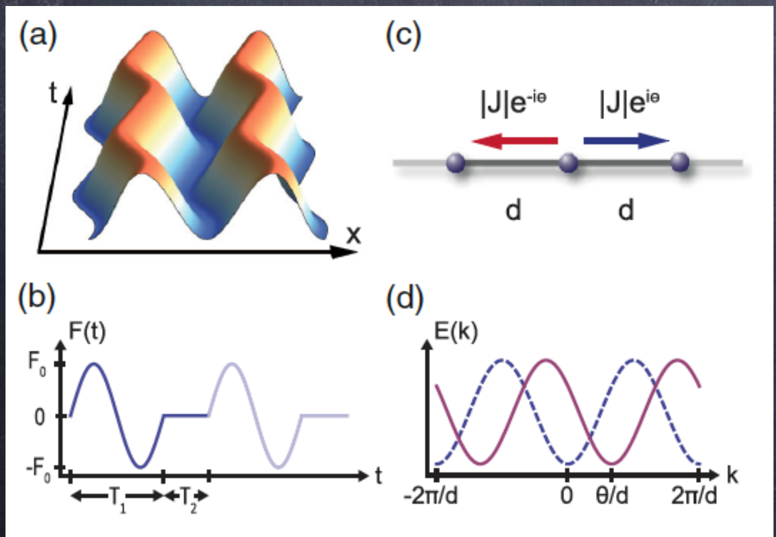
# Synthetic gauge fields

## Laser assisted tunneling




M. Aidelsburger et al., PRL **107**, 255301 (2011);
M. Aidelsburger et al., PRL **111**, 185301 (2013);
H. Miyake et al., PRL **111**, 185302 (2013).

## Lattice shaking




J. Struck et al., PRL **108**, 255304 (2012);
J. Struck et al., Nature Physics, **9**, 738 (2013).

# How to determine a ground state

e.g., spin-orbit coupling along z axis

$$-t_y \sum_j (e^{i\Phi\sigma_z})_{\sigma\sigma'} c^+_{j,1,\sigma} c_{j,2,\sigma'} + h.c.$$

Canonical transformation

$$c_{j,1\sigma} \leftrightarrow d_{j,1\sigma}$$

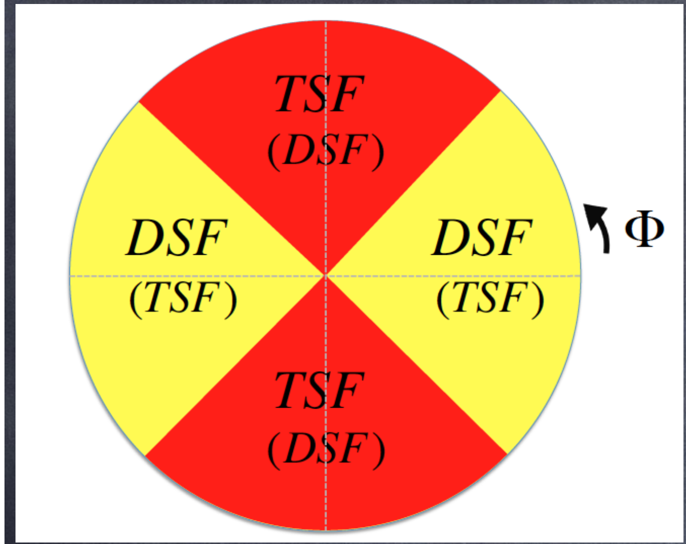
$$c_{j,2\sigma} \leftrightarrow e^{-i\Phi\sigma_z} d_{j,2\sigma}$$

Two-leg Hubbard model w/o spin-orbit coupling!

$$\left\langle O^{(c)}_{DSF}{}^{+}(r) O^{(c)}_{DSF}(0) \right\rangle_{(c)} = \cos^2\Phi \left\langle O^{(d)}_{DSF}{}^{+}(r) O^{(d)}_{DSF}(0) \right\rangle_{(d)} + \sin^2\Phi \cancel{\left\langle O^{(d)}_{TSC^z}{}^{+}(r) O^{(d)}_{TSC^z}(0) \right\rangle_{(d)} + \ldots}$$

$$\left\langle O^{(c)}_{TSC^z}{}^{+}(r) O^{(c)}_{TSC^z}(0) \right\rangle_{(c)} = \sin^2\Phi \left\langle O^{(d)}_{DSF}{}^{+}(r) O^{(d)}_{DSF}(0) \right\rangle_{(d)} + \cos^2\Phi \cancel{\left\langle O^{(d)}_{TSC^z}{}^{+}(r) O^{(d)}_{TSC^z}(0) \right\rangle_{(d)} + \ldots}$$

# Phase diagram




DSF: Interchain spin-singlet superfluid
(d-wave superfluid)

TSF; Interchain spin-triplet superfluid

- In the presence of spin-orbit coupling, admixture between spin-singlet and triplet pairings occurs
- Admixture is fully controllable with spin-orbit coupling
- Pure spin-triplet superfluid can also be obtained

# Temperature for realization

$$T \leq t, U, \Delta$$

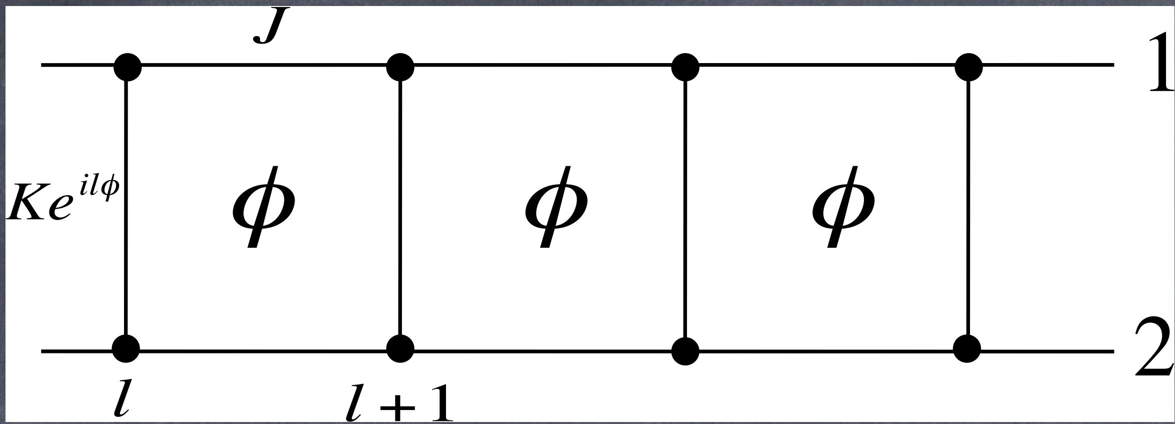
$$\Delta \sim t^2/U \text{ for } U/t \gg 0$$

C. Hayward and D. Poilblanc, PRB **53,** 11721 (1996);
R. Noack, S. White, and D. Scalapino, Physica C **270**, 281(1996)**;**
S. White, I. Affleck, and D. J. Scalapino, PRB **65**,165122 (2002);
G. Roux et al., PRB **75**, 245119 (2007).

# Two-leg Bosonic ladder w/ magnetic flux

SU and A. Tokuno, arXiv:1504.06159

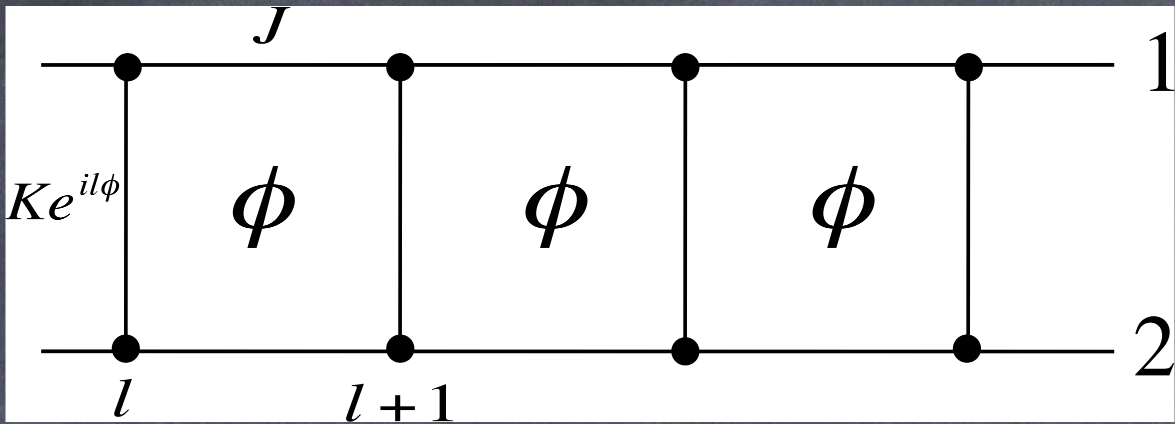
# The system

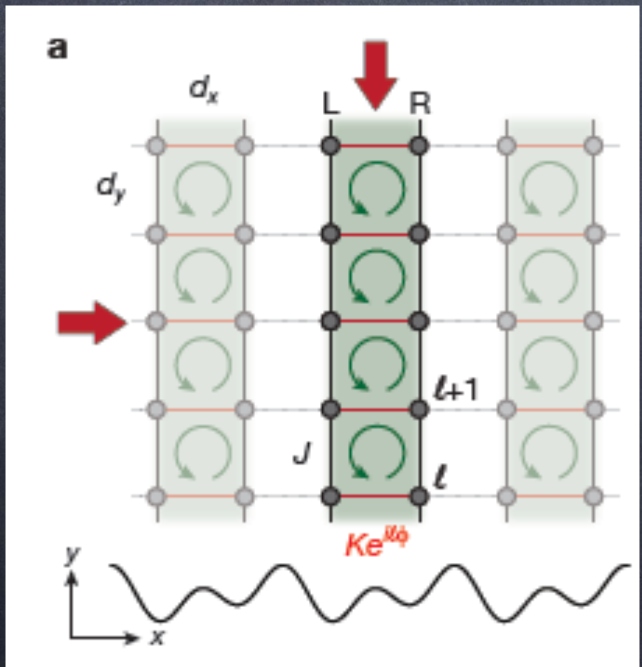


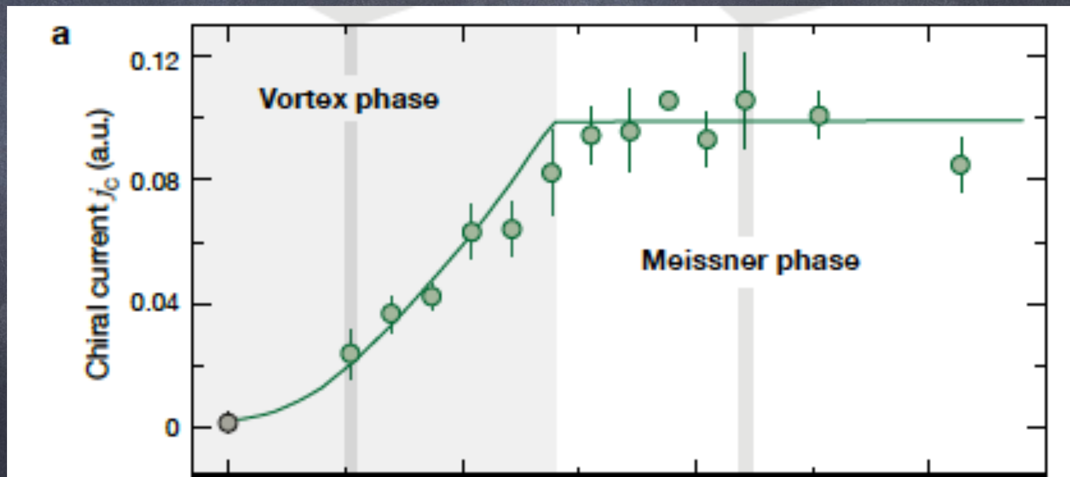

$$H = -J \sum_{l=1}^{L} \sum_{p=1,2} b^{\dagger}_{l+1,p} b_{l,p} - K \sum_{l} b^{\dagger}_{l,1} b_{l,2} e^{il\phi} + U \sum_{l,p} n_{l,p} n_{l,p} + h.c.$$

E. Originac and T. Giamarchi, PRB**64**, 144515 (2001).

# The system

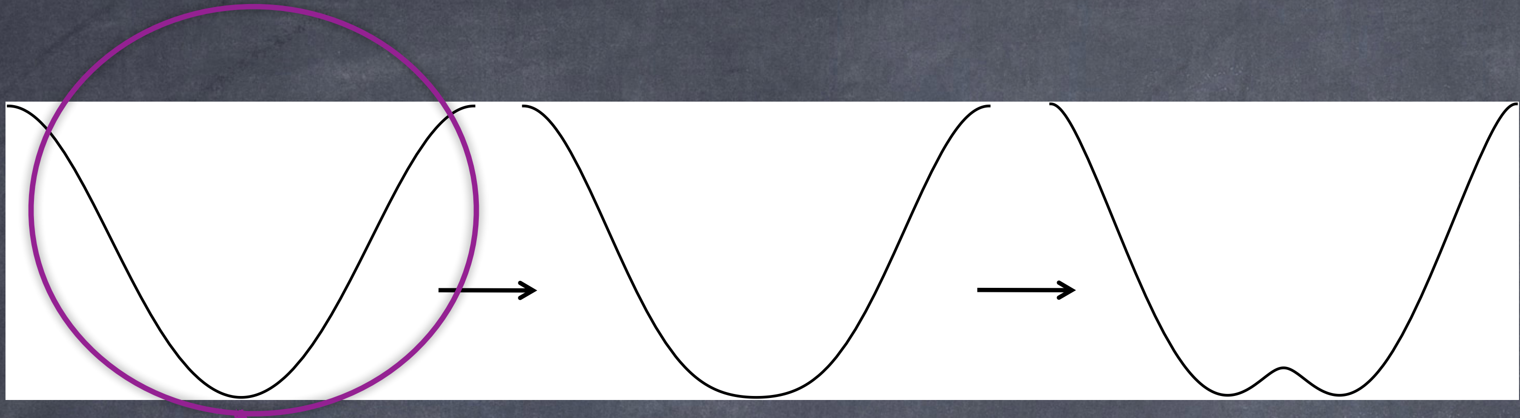


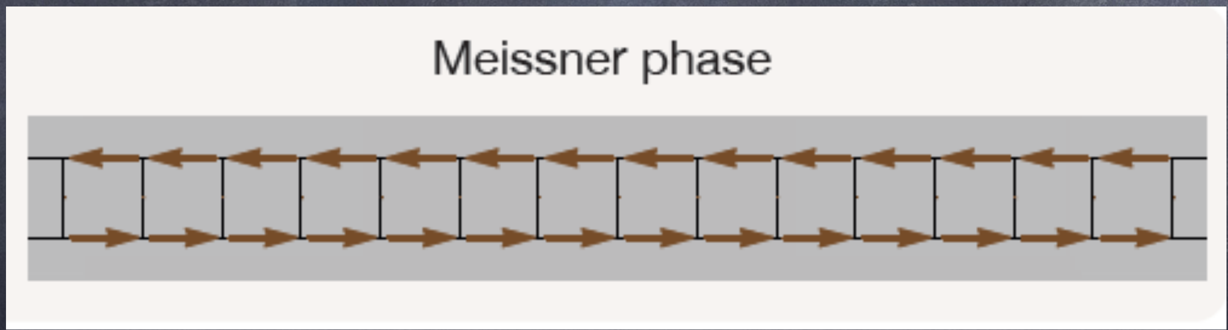




M. Atala et al., Nature Phys. **10,** 588 (2014).

# Change of the band structure



$\phi \nearrow$ with fixed $K/J$




E. Originac and T. Giamarchi, PRB**64**, 144515 (2001).

M. Atala et al., Nature Phys. **10**, 588 (2014).

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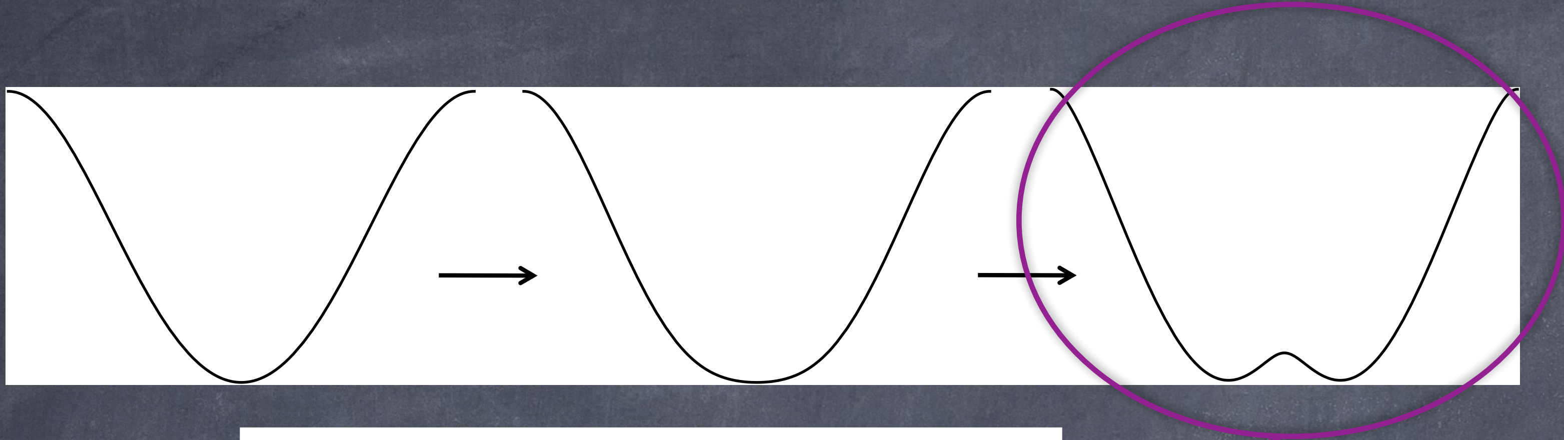
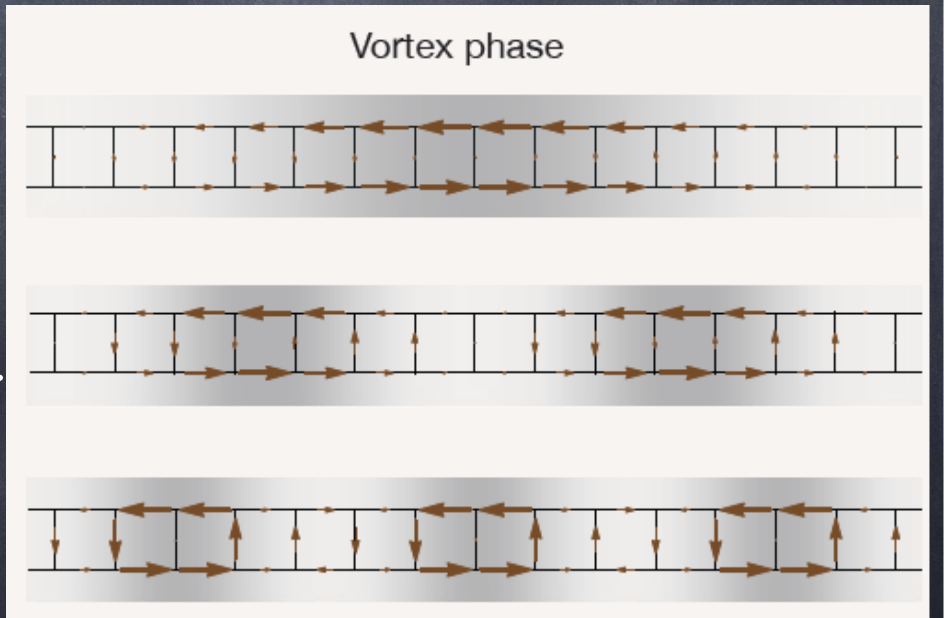


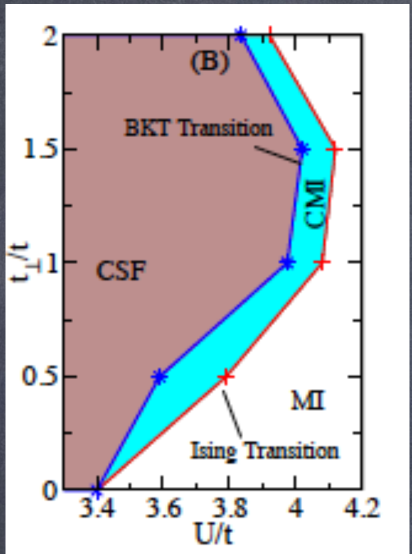
$\phi \nearrow$ with fixed $K/J$




E. Originac and T. Giamarchi, PRB**64**, 144515 (2001).
M. Atala et al., Nature Phys. **10**, 588 (2014).

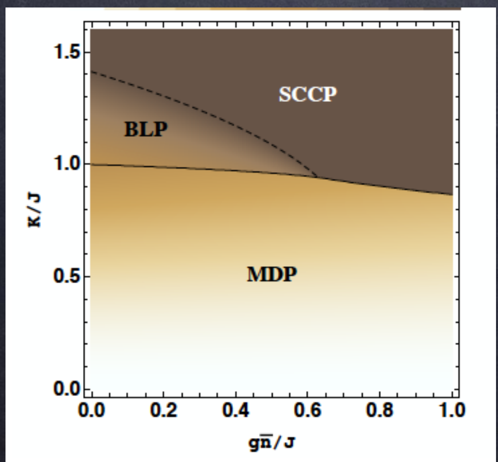
# Do we understand the phase diagram completely?

- Mott-insurator with chirality (strong coupling)



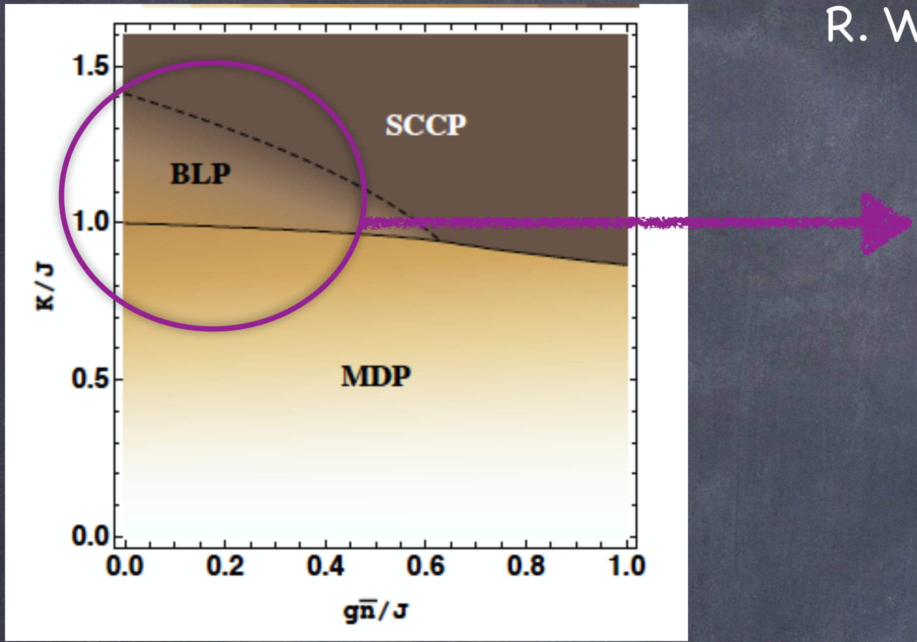

A. Dhar et al., PRA**85**, 041602 (2012)
A. Dhar et al., PRB**87**, 174501 (2013)
A. Petrescu and K. Le Hur, PRL**111**, 150601 (2013)
A. Tokuno and A. George, NJP**16**, 073005 (2014)

- $Z_2$ symmetry breaking phase (weak coupling)




R. Wei and E. Mueller, PRA**89**, 063617 (2014).

# $Z_2$ Symmetry breaking phase?




R. Wei and E. Mueller, PRA**89**, 063617 (2014).

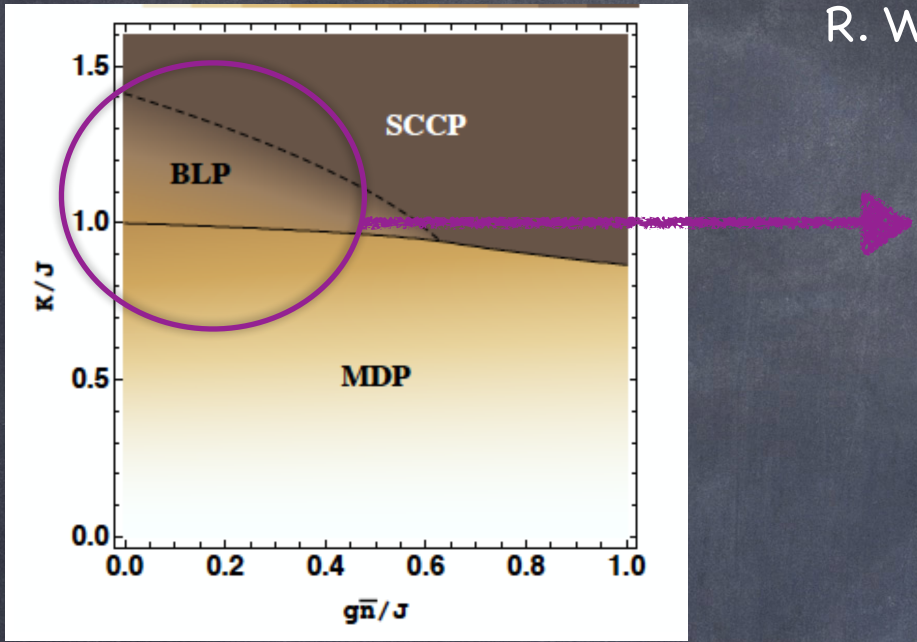
Phase with population-imbalance between two legs emerges

One of the wells is spontaneously selected

Similar transition occurs in shaken optical lattice recently observed in

L. Ha et al., PRL**114**, 055301 (2015)

# $Z_2$ Symmetry breaking phase?




R. Wei and E. Mueller, PRA**89**, 063617 (2014).

Phase with population-imbalance between two legs emerges

- Gross-Pitaevskii approach is used to obtain phase diagram

$$|GP\rangle = \frac{1}{\sqrt{N!}}(e^{i\theta_+}\cos\gamma b_+^\dagger + e^{i\theta_-}\sin\gamma b_-^\dagger)^N|0\rangle$$

- What`s a role of quantum fluctuation or commensurability of flux?

# Our solutions with bosonization

- recipe

1. determine mean density with GP approach

2. incorporate quantum fluctuations with bosonization

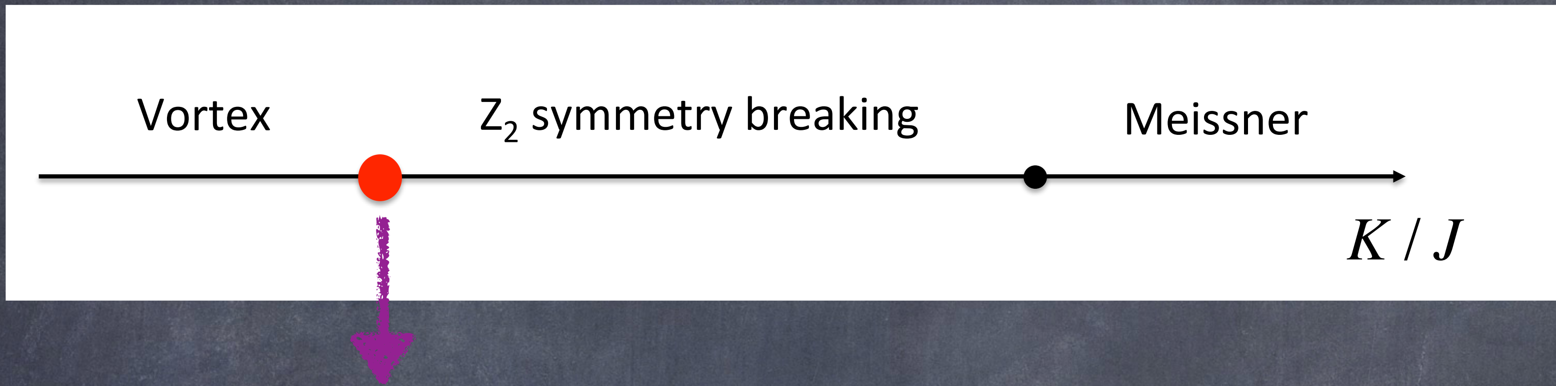
$$\beta(x) \sim [n - \nabla\phi(x)/\pi]^{1/2} \sum_m e^{2im[\pi n x - \phi(x)]} e^{-i\theta(x)}$$

FDM Haldane, J Phys. C**14**, 2585 (1981)
T. Giamarchi, Quantum Physics in One dimensions (2003)
M. Cazalilla, J Phys. B**37** S1 (2004)

3. RG method is implemented to examine deviation from Luttinger liquid

# Our solutions with bosonization

- Phase w/ $Z_2$ symmetry breaking robust against fluctuation

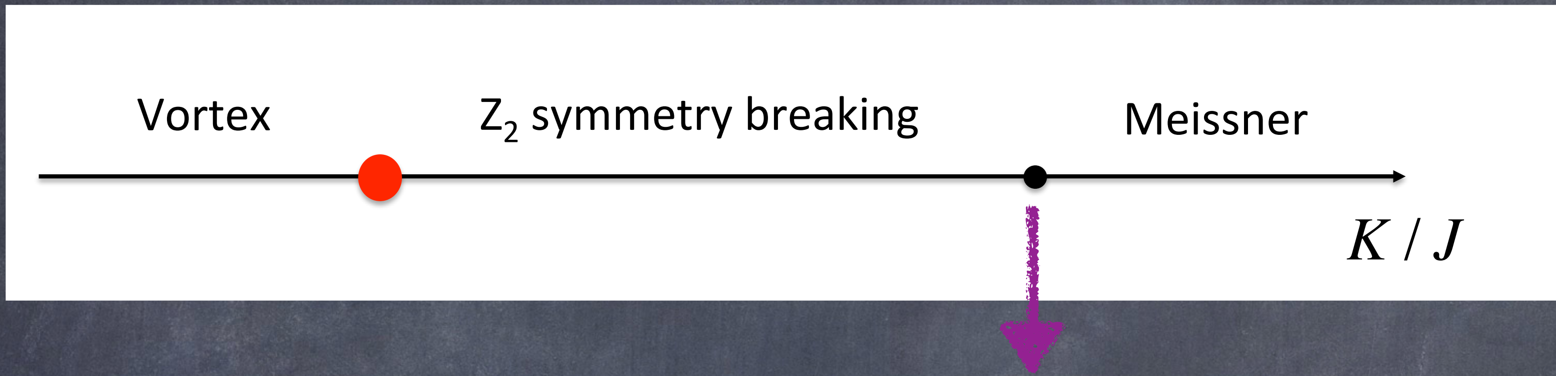



SU(2) point (Heisenberg)

Similarity with ferromagnetic XXZ model

# Our solutions with bosonization

- Phase w/ $Z_2$ symmetry breaking robust against fluctuation




Non-Luttinger liquid

$$\epsilon(k) \sim k^2$$

Absence of ODLRO

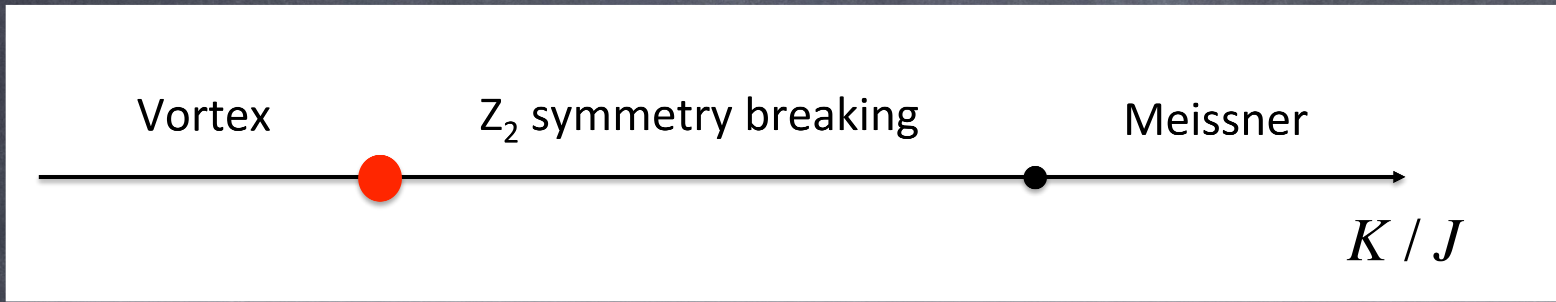
$$\langle b^\dagger(r) b(0) \rangle \sim e^{-r/\xi}$$

Similar behavior emerges in 1D spin-orbit coupling system

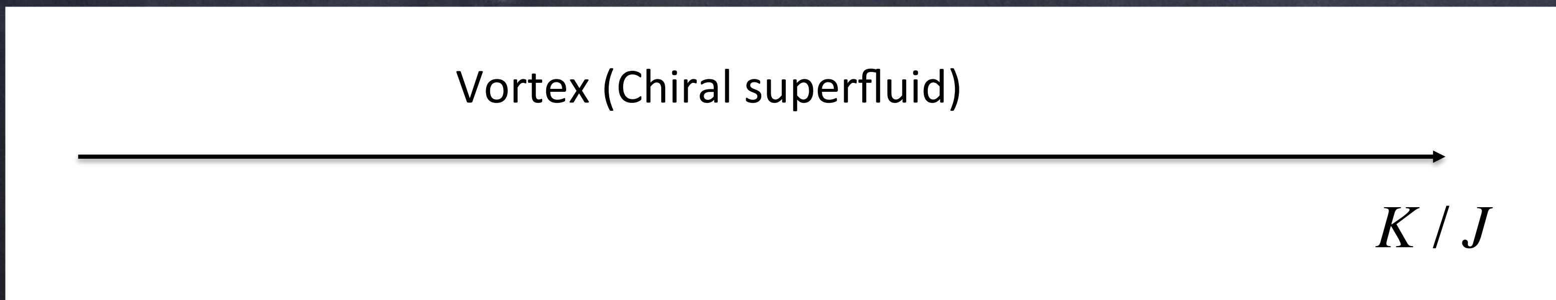
H. Po et al, PRA**90**, 011602 (2014).

# Our solutions with bosonization

- Phase w/ Z2 symmetry breaking robust against fluctuation




- Umklapp process from commensurability of flux changes the phase diagram $(\phi = \pi)$



# Summary

Quasi-one dimensional system with cold atoms
can be a playground to look at interesting superfluid states

Parity-mixed superfluid

Spin-triplet superfluid

Superfluid with population imbalance

Chiral Superfluid