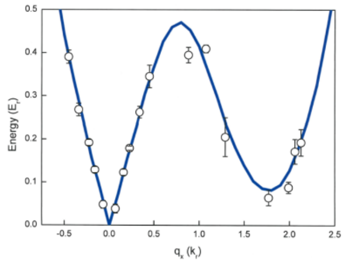
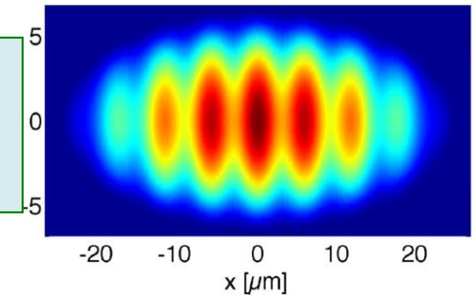


INT Seattle
25 March 2015



ROTONS AND STRIPES
IN SPIN-ORBIT COUPLED BECs

Yun Li, Giovanni Martone, Lev Pitaevskii and Sandro Stringari

University of Trento

Now in Swinburne

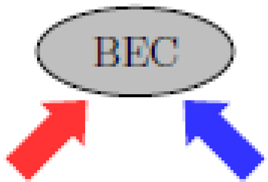
Now in Bari

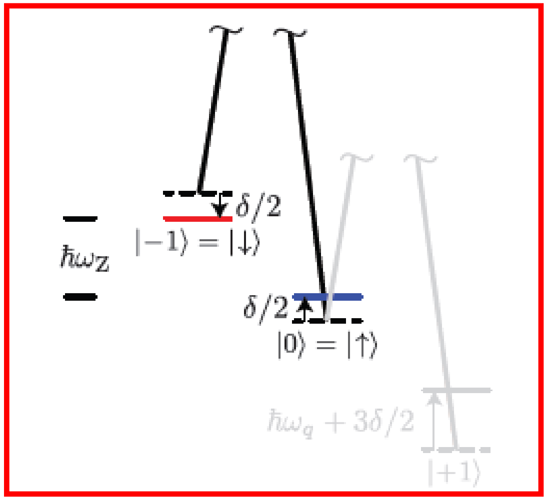
BEC

CNR-INO

Stimulating discussions with:
Jean Dalibard, Gabiele Ferrari, Giacomo Lamporesi

Simplest realization of (1D) spin-orbit coupling in s=1/2 Bose-Einstein condensates (Spielman team at Nist, 2009)

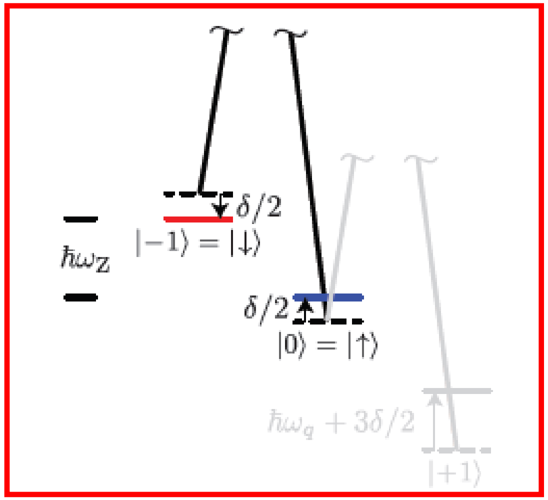




Two detuned ($\Delta\omega_L$) and **polarized** laser beams + non linear Zeeman field (ω_Z) provide Raman transitions between **two** spin states, giving rise to new single particle physics

New single particle Hamiltonian

$$h_0 = \frac{1}{2}[(p_x - k_0\sigma_z)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x + \frac{1}{2}\delta\sigma_z$$



p_x is canonical momentum

k_0 is laser wave vector difference

Ω is strength of Raman coupling

$\delta = \Delta\omega_L - \omega_Z$ is effective Zeeman field

physical velocity equal to

$$v_x = (p_x - k_0\sigma_z)/m$$

Hamiltonian h_0

- is **translationally invariant** despite the presence of the laser fields $[h_0, p_x] = 0$

- **Breaks parity**, **time reversal** and **Galilean** invariance

Symmetry properties of the spin-orbit Hamiltonian

$$h_0 = \frac{1}{2}[(p_x - k_0\sigma_z)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x + \frac{1}{2}\delta\sigma_z$$

- **Translational** invariance: ⇒ uniform ground state configuration, unless crystalline order is formed spontaneously (**stripes, supersolidity**)

- **Violation** of **parity** and **time** reversal symmetry ⇒ breaking of symmetry $\omega(q) = \omega(-q)$ in excitation spectrum (exp: Si-Cong Ji et al. PRL 2015; theory: Martone et al. PRA 2012)

- **Violation** of **Galilean** invariance: ⇒ breakdown of Landau criterion for superfluid velocity and emergence of dynamical instabilities in uniform configurations (exp: Zhang et. al. PRL 2012, theory: Ozawa et al. (PRA 2013))

Different strategies to realize novel quantum phases

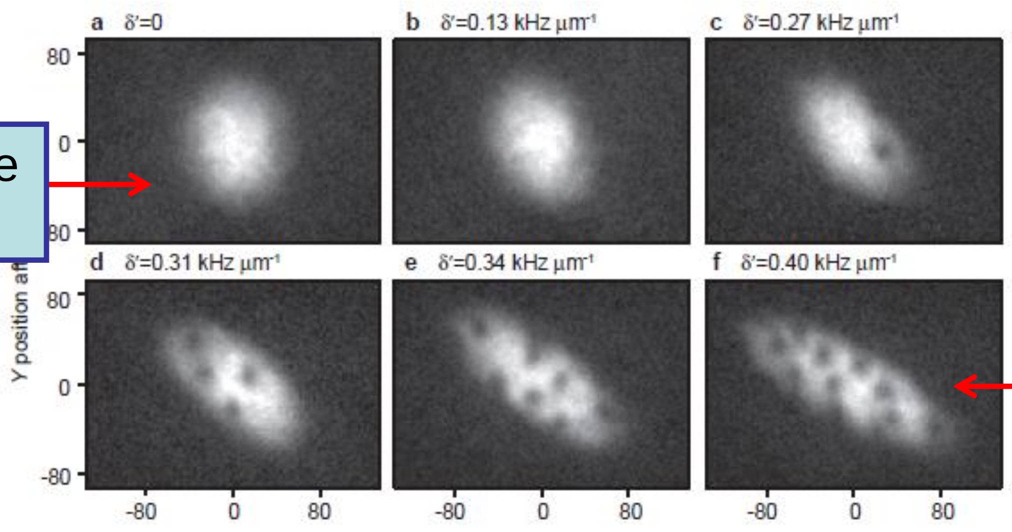
- **First strategy** (Lin et al., Nature 2009). Spatially dependent detuning ($\delta(y)$) in strong Raman coupling ($\Omega >> k_0^2$) regime yields position dependent vector potential

$$h_0 = \frac{1}{2m^*}\left(p_x - A_x(y)\right)^2$$

and **effective Lorentz force in neutral atoms.**

This causes the appearence of quantized vortices.

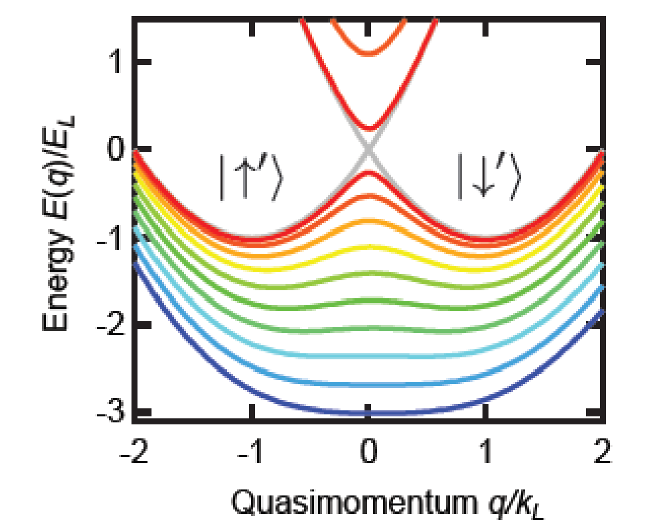
No y-dependence
In detuning

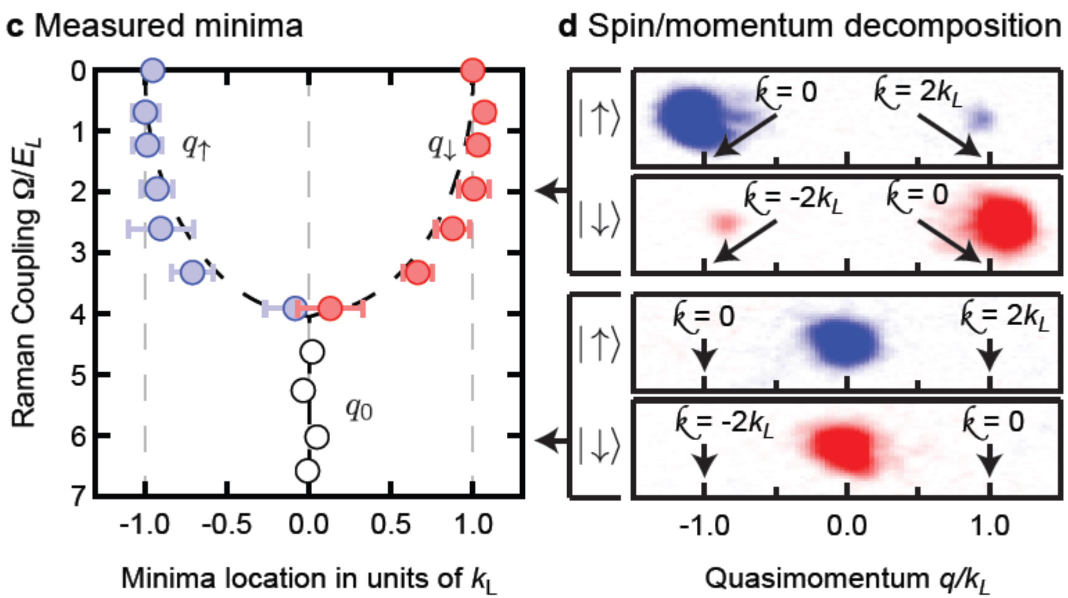


strong
y-dependence
In detuning

Second strategy (Lin et al. Nature 2011)

- Small detuning ($\delta \approx 0$) and smaller Raman coupling ($\Omega < 2k_0^2$) give rise to the appearence of **two degenerate minima** which can host a Bose-Einstein condensate.





Key question:

Role of **interactions** in the presence of the novel spin-orbit single particle Hamiltonian.

Rich scenario with **new quantum phases**

Theory of quantum phases in 1D SO coupled s=1/2 BECs (T=0)

Ho and Zhang (PRL 2011), ……., Yun Li, Pitaevskii, Stringari (PRL 2012)

$$H = \sum_i h_0(i) + \sum_{\alpha,\beta} \frac{1}{2} \int d\vec{r}\, g_{\alpha\beta} n_\alpha n_\beta$$

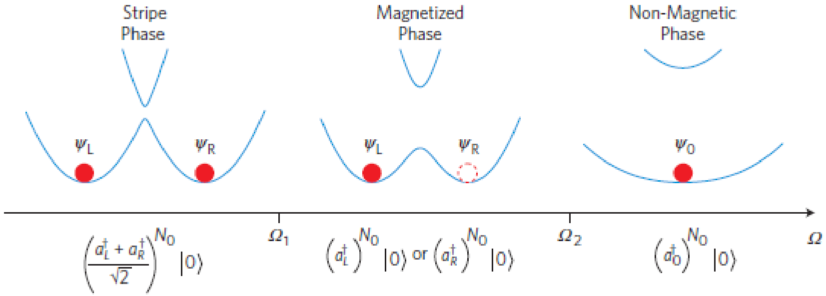
- With $$h_0 = \frac{1}{2}[(p_x - k_0\sigma_z)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x + \frac{1}{2}\delta\sigma_z$$

- We assume $g_{\uparrow\uparrow} g_{\downarrow\downarrow} > g_{\uparrow\downarrow}^2$ ⇨ **phase mixing**
  in the absence of Raman coupling

- Interactions are treated within mean field approximation
  (s=1/2 coupled Gross-Pitaevskii equations)

For small values of Ω two sp states can host BEC with canonical momentum

$$|k_1| = k_0\sqrt{1-\Omega^2/[2k_0^2+\mathrm{int}]^2} \neq k_0$$



Order parameter in the new phases

I) Stripe phase

$$\Psi = \sqrt{\frac{N}{2V}}\left[\begin{pmatrix}\cos\theta \\ -\sin\theta\end{pmatrix} e^{ik_1x} + \begin{pmatrix}\sin\theta \\ -\cos\theta\end{pmatrix} e^{-ik_1x}\right]$$

+ higher harmonics

$$\cos\frac{\theta}{2} = \frac{k_1}{k_0}$$

$$n(x) = n\left(1+\frac{\Omega}{2k_0^2}\cos 2k_1x\right)$$

← density fringes fixed by k_1

I) Plane wave phase

$$\Psi = \sqrt{\frac{N}{2V}}\begin{pmatrix}\sin\theta \\ -\cos\theta\end{pmatrix} e^{-ik_1x}$$

$$<\sigma_z> = \frac{k_1}{k_0}$$

II) Zero momentum phase

$$\Psi = \sqrt{\frac{N}{V}}\begin{pmatrix}1 \\ -1\end{pmatrix}$$

New quantum phases (T=0)

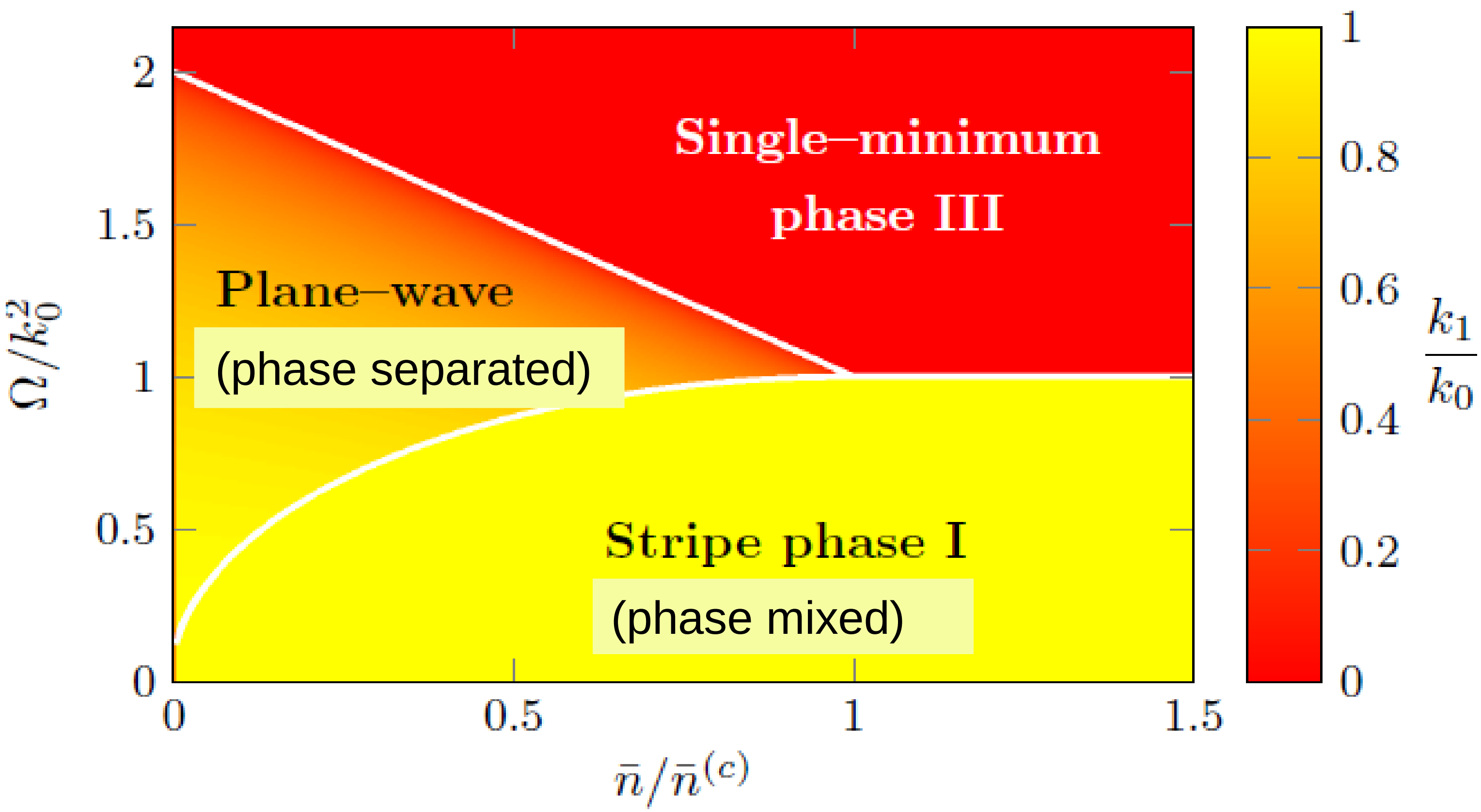
^{87}Rb

$|\uparrow\rangle = |F=1, m_F=0\rangle$

$|\uparrow\rangle = |F=1, m_F=-1\rangle$

$a_{\uparrow\uparrow} = 101.41 a_B$

$a_{\downarrow\downarrow} = a_{\uparrow\updownarrow} = 100.94 a_B$



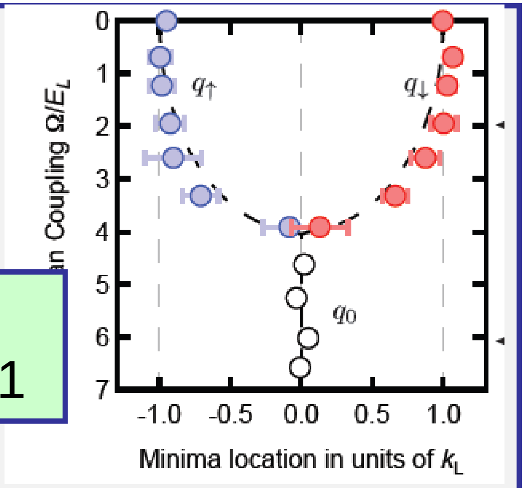
Plane wave-single minimum phase transition

Transition is **second order**.
It has been observed
at the predicted value

$$\Omega \approx 4E_L = 2k_0^2$$

of the Raman coupling

Lin et al., Nature 2011



Phase transition is driven by single-particle Hamiltonian.
does not require two-body interactions.

Spin polarizability diverges at the transition
(G. Martone, Yun Li, S.S. EPL 2012)

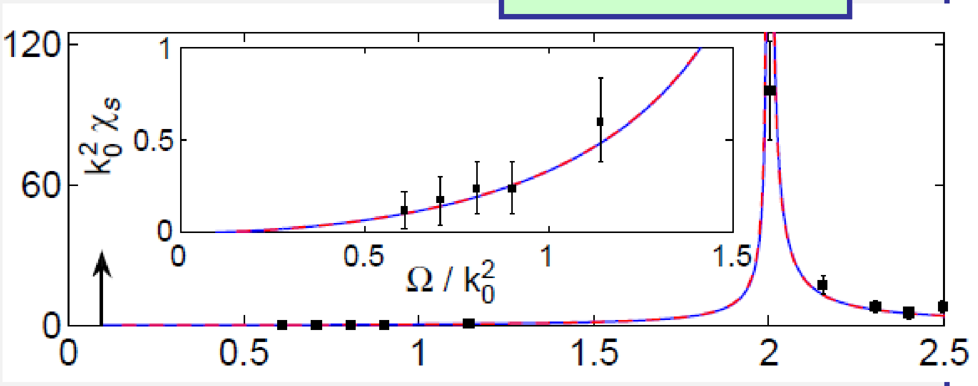
Zhang et al. PRL 2012

$$\Omega < 2k_0^2$$

$$\chi(\sigma_z) = \frac{\Omega^2}{k_0^2(4k_0^2 - \Omega^2)}$$

$$\Omega > 2k_0^2$$

$$\chi(\sigma_z) = \frac{2}{\Omega - 2k_0^2}$$



Striped-plane wave phase transitions

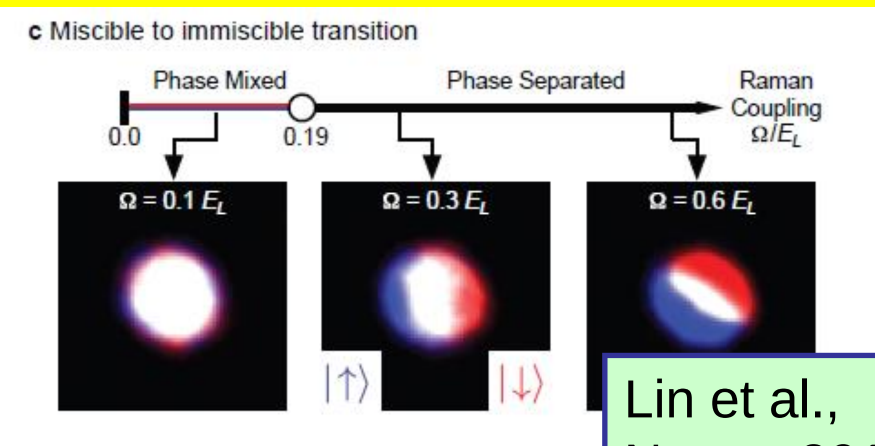
Transition is **first order**.
Critical frequency is
(Ho and Zhang PRL 2011)

$$\Omega_{cr} = 2k_0^2\sqrt{\frac{2\gamma}{1+2\gamma}}$$

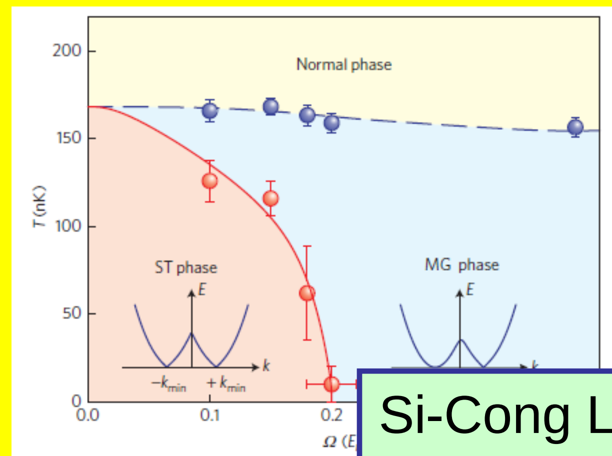
where

$$\gamma = \frac{g_{\uparrow\uparrow} + g_{\downarrow\downarrow} - 2g_{\uparrow\downarrow}}{g_{\uparrow\uparrow} + g_{\downarrow\downarrow} + 2g_{\uparrow\downarrow}}$$

A phase transition between a spin mixed and a spin separated phase has been observed at the predicted value of Ω



Lin et al., Nature 2011



Si-Cong Li et al. Nat. Phys.,2014

Density modulations are not however visible in the spin mixed phase (too small contrast and too small fringe separation)
Proof of coherence is provided by **fringes**, not by mixing.

Spin-orbit coupling

has **important** consequences on the **dynamic behavior** of BECs

Experiments already **available**

Quenching of **Center of mass** frequency in harmonic trap

(violation of Kohn's theorem)

(exp: Zhang et al.2012; theory: Yun Li et al. EPL 2012)

Emergence of **Roton** and softed **Phonon** Modes

(exp: Si-Cong Ji et al.; PRL 2015;

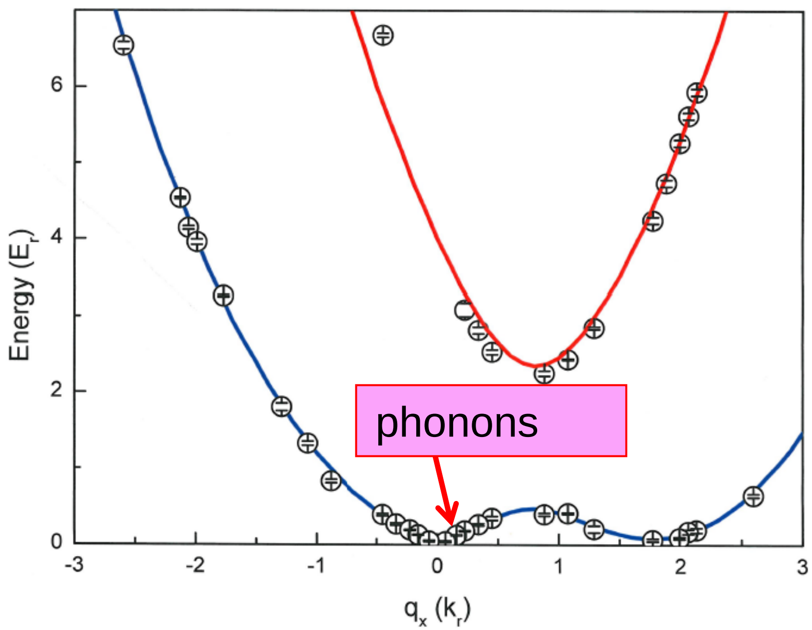
theory Yun Li et al. Martone et al. PRA 2012)

Two Goldstone modes in striped Phase

Theory: Yun Li et al. PRL 2013

Emergence of Rotons in Plane Wave phase

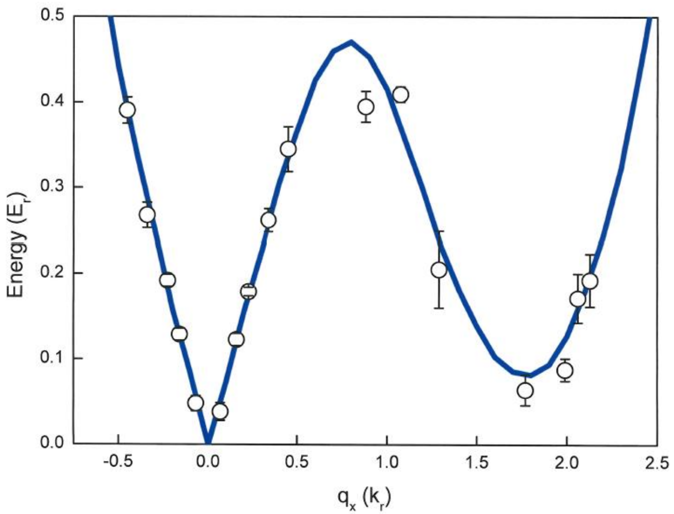
Excitation spectrum exhibits two branches.
Due to Raman coupling **only one branch is gapless**
and exhibits a phonon behavior at small q



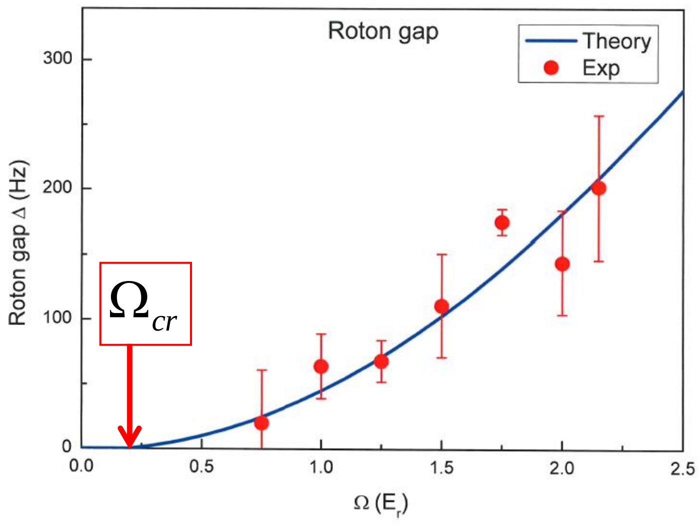
Exp: Si-Cong Ji et al., PRL 2015
Theory: Martone et al., PRA 2012

At small Raman coupling, a **roton** structure emerges in the lower branch

Exp: Si-Cong Ji et al., PRL 114, 105301 (2015)
Theory: Martone et al., PRA 86, 063621 (2012)

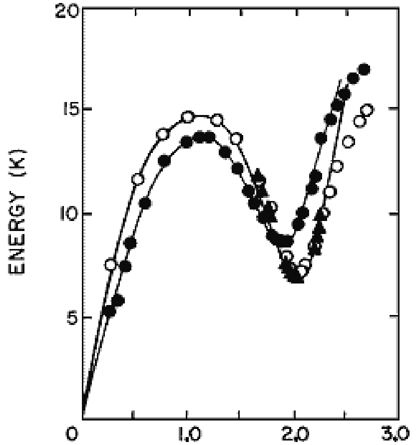


$\omega(q) \neq \omega(-q)$ consequence of violation of **parity** and **time reversal** symmetry

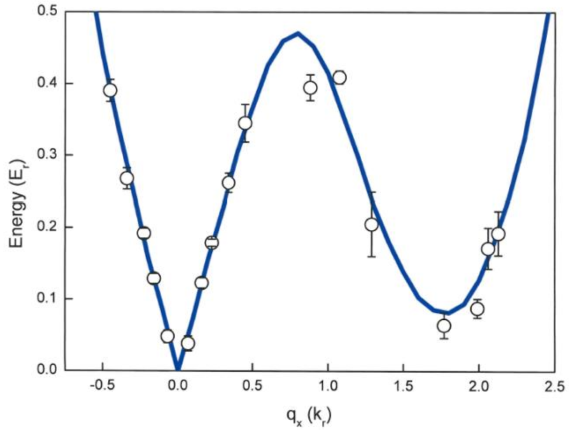


Roton gap decreases as Raman coupling is lowered: **onset of crystallization** (**striped** phase)

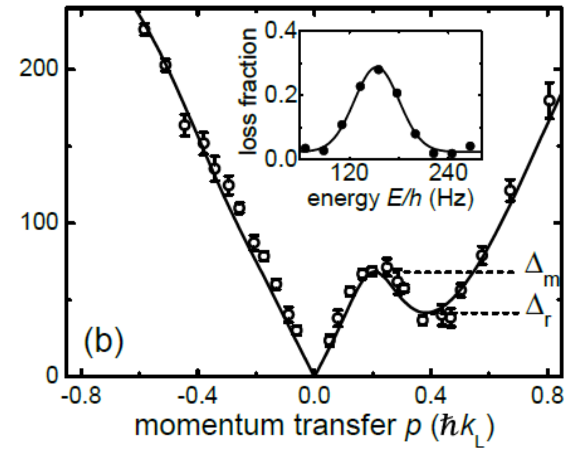
Roton Gallery



Liquid Helium (historical)

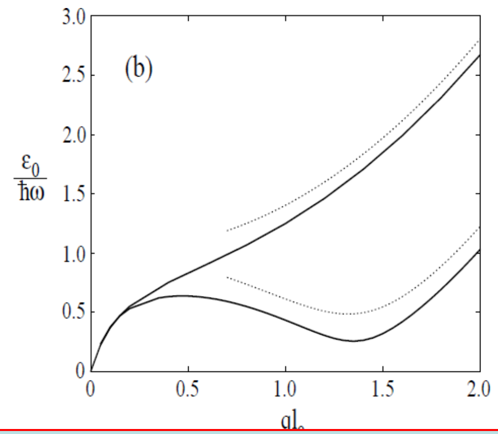


Spin-orbit BEC's (2014)

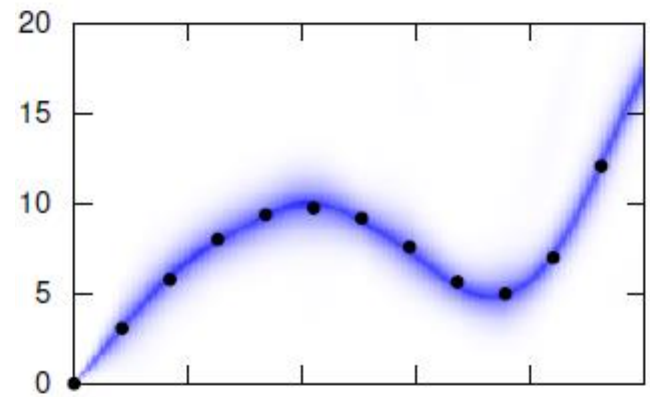


BEC's in shaken lattices (2014)

Other theoretical proposals



Pancake dipolar BEC's



BEC's with soft core repulsive potentials

THE STRIPED PHASE

Why is the **striped phase** of a Bose-Einstein condensate interesting ?

Can we make **stripes visible** and **stable** ?

Striped phase is the result of **interaction** effects

Competition between **spin** and **density** dependent interactions (for simplicity we assume $g_{\uparrow\uparrow} \approx g_{\downarrow\downarrow} \equiv g$)

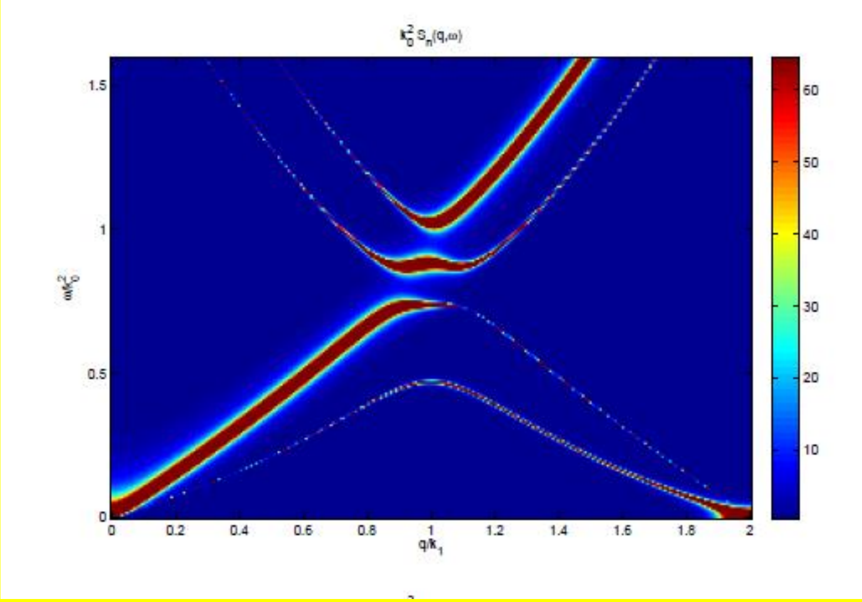
- **Spin** dependent term (proportional to $(g - g_{\uparrow\downarrow})\int d\vec{r}(n_\uparrow - n_\downarrow)^2$) favours spin mixing and hence, in the presence of SO term and Raman coupling, favours the emergence of density modulations (**stripes**)

- **Density** dependent term (proportional to $(g + g_{\uparrow\downarrow})\int d\vec{r}(n_\uparrow + n_\downarrow)^2$) favours uniformity (**plane wave phase**).

At small Raman coupling (for $\Omega < \Omega_{cr}$) stripes are energetically favoured. Value of Ω_{cr} depends only on the ratio $(g - g_{\uparrow\downarrow})/(g + g_{\uparrow\downarrow})$

Striped phase results from **spontaneous** breaking of two continuous symmetries.
gauge and **translational** symmetries

Two Goldstone modes:



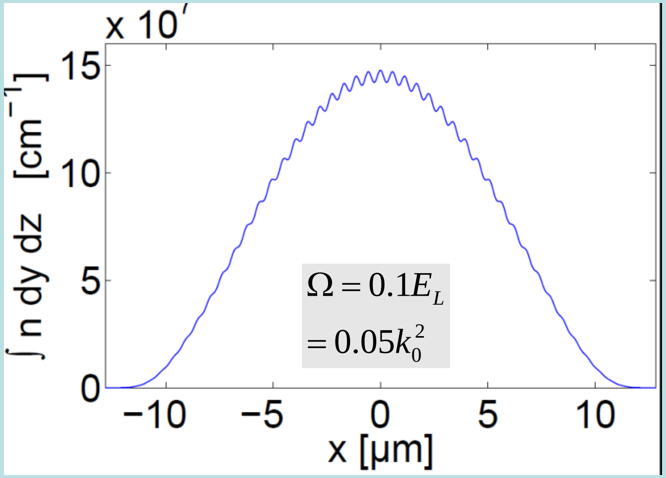
- Double band structure in the striped phase of a SO coupled Bose-Einstein condensate
- Lower phonon branch better excited by spin operator (Yun Li et al. PRL 2013)

Improving visibility and stability of superstripes

(Martone, Yun Li and Stringari, Phys. Rev. A 90, 041604(R) (2014)

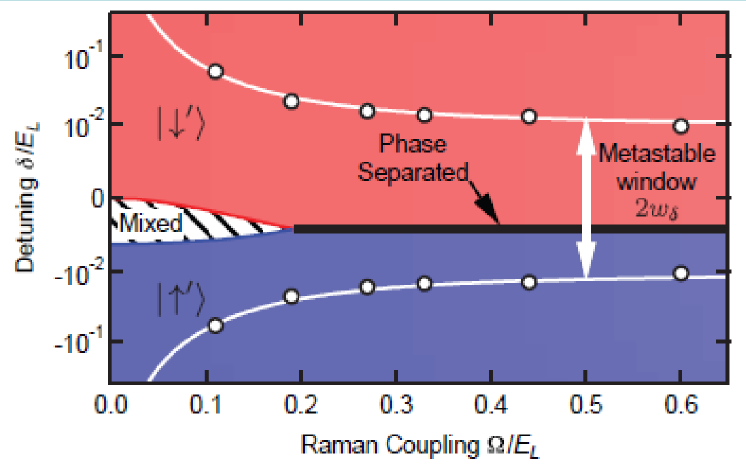
In Nist (Lin et al., Nature 2011) and Shanghai (Si-Cong Li Nat. Phys. 2014) experiments **stripes** are **not** visible:

- **contrast** is too **small**.
  Doubly integrated density from Gross-Pitaevskii simulation in the same condition of 87Rb Nist exp. exhibits small contrast.



- **Separation** between fringes is too **small** (fraction of micron)

- Stripes are **fragile** against magnetic fluctuations.
  Tiny magnetic field (corresponding to detuning of 3-5 Hertz) destabilizes the stripes.



How to increase the contrast of density modulations ?

$$n(x) = n[1 + \frac{\Omega}{2k_0^2}\cos(2k_1 x + \varphi)]$$

Maximum value of Raman coupling compatible with the striped phase is given by $\Omega_{cr} = 2k_0^2\sqrt{\frac{2\gamma}{(1+2\gamma)}}$ where $\gamma = \frac{g_{\uparrow\uparrow} + g_{\downarrow\downarrow} - 2g_{\uparrow\downarrow}}{g_{\uparrow\uparrow} + g_{\downarrow\downarrow} + 2g_{\uparrow\downarrow}}$

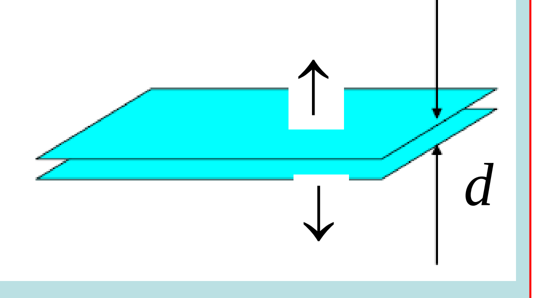
In 87Rb $\Omega_{cr} = 0.1\, k_0^2$ and hence achievable contrast is very small.

In order to increase Ω_{cr} one should **reduce** $g_{\uparrow\downarrow}$. HOW ?

- Feshbach tuning of interspecies scattering length $a_{\uparrow\downarrow}$ preserving condition $a_{\uparrow\uparrow} \approx a_{\downarrow\downarrow}$

- We propose 2D geometry based on two spin layers separated by distance d. Separation reduces $g_{\uparrow\downarrow}$ by factor $\exp(-d^2/2a_z^2)$ with respect to $g_{\uparrow\uparrow}$ and $g_{\downarrow\downarrow}$

$$V_{ext}(z) = \frac{1}{2}\omega_z^2 (z - \frac{d}{2}\sigma_z)^2$$



3D Gross-Pitaevskii simulation for $4\times10^4\ ^{87}Rb$ atoms in harmonic trap

$$V_{ext}(\vec{r}) = \frac{1}{2}\omega_x^2 x^2 + \frac{1}{2}\omega_y^2 y^2 + \frac{1}{2}\omega_z^2 (z - \frac{d}{2}\sigma_z)^2$$

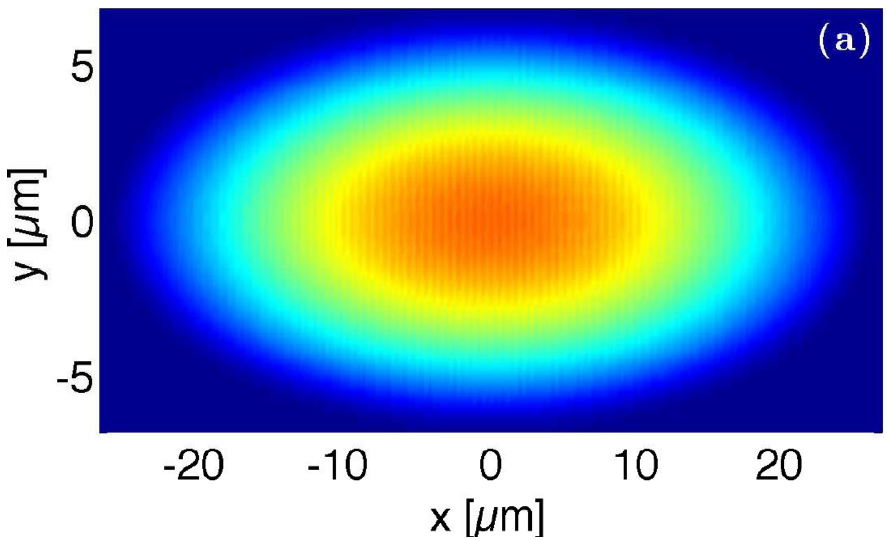
Parameters:

$$(\omega_x, \omega_y, \omega_z) = 2\pi\times(25, 100, 2500)Hz$$

$$a_{\uparrow\uparrow} = 101.41 a_B;\ a_{\downarrow\downarrow} = a_{\uparrow\downarrow} = 100.94\ a_B$$

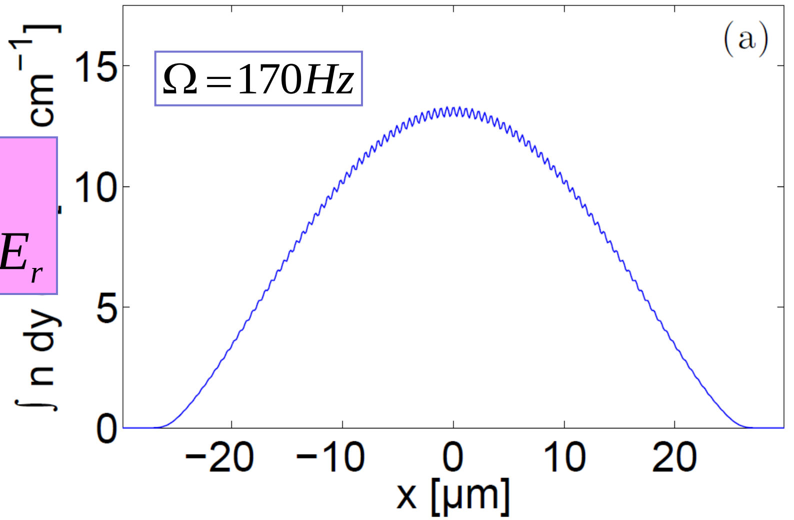
$$E_r = k_0^2/2 = 2\pi\times1.8\,KHz$$

$$\Omega = \frac{1}{2}\Omega_{cr}$$



$d = 0$

$\Omega = 0.1 E_r$



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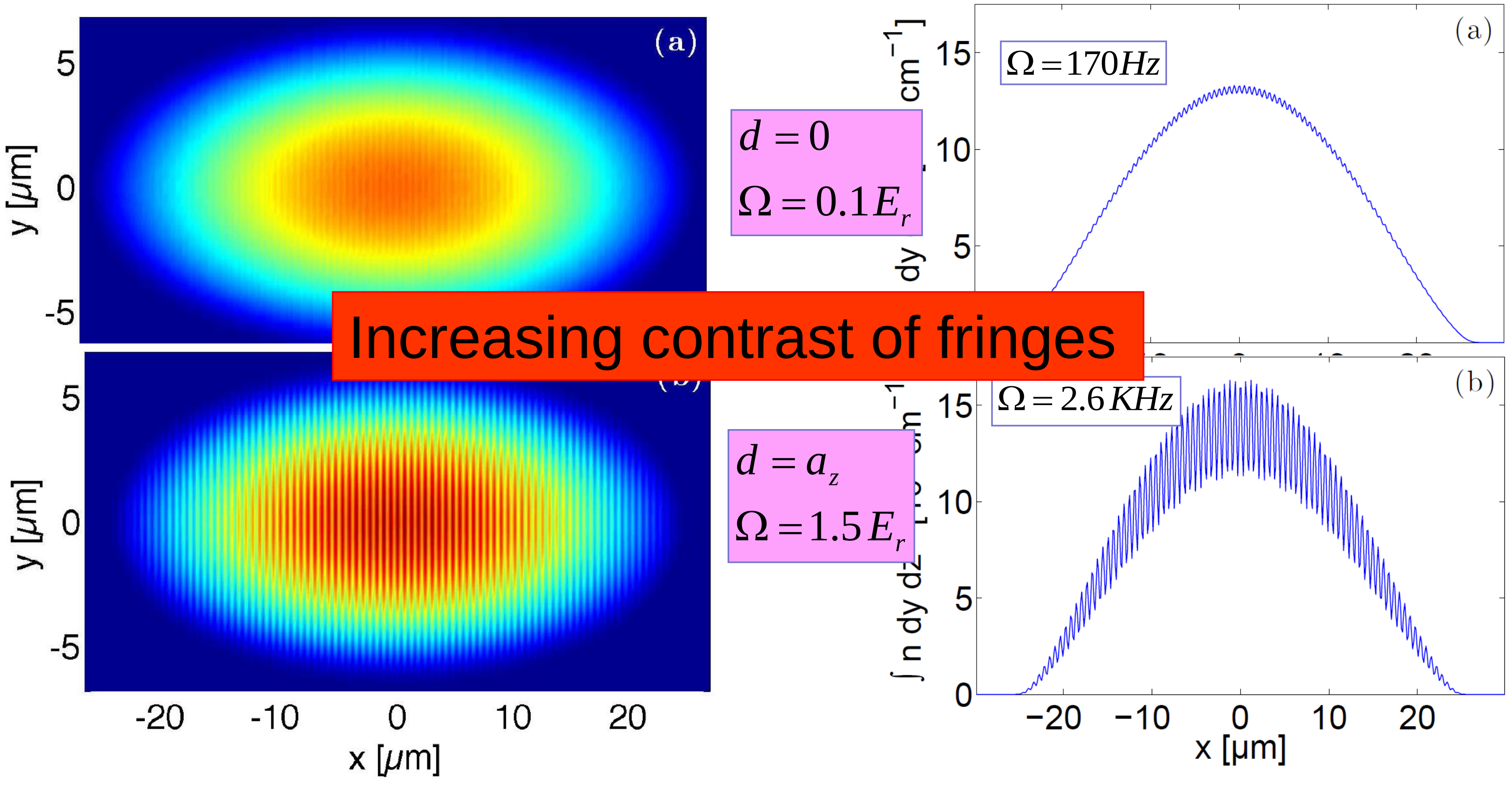
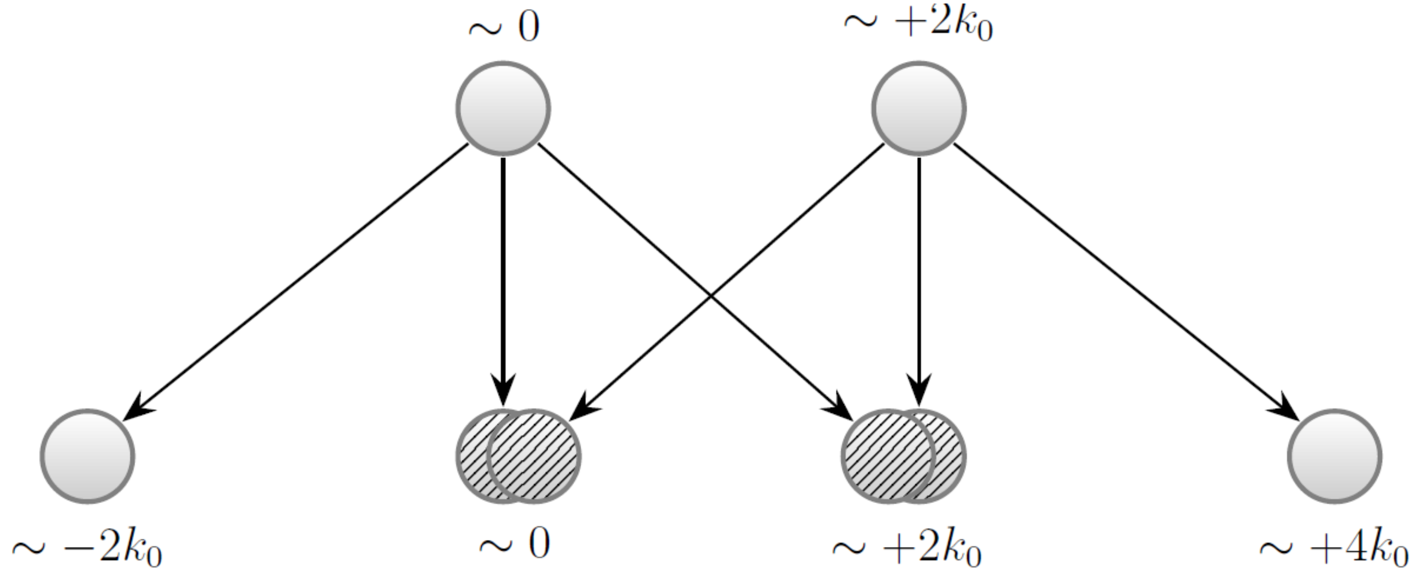


Increasing wave length of fringes

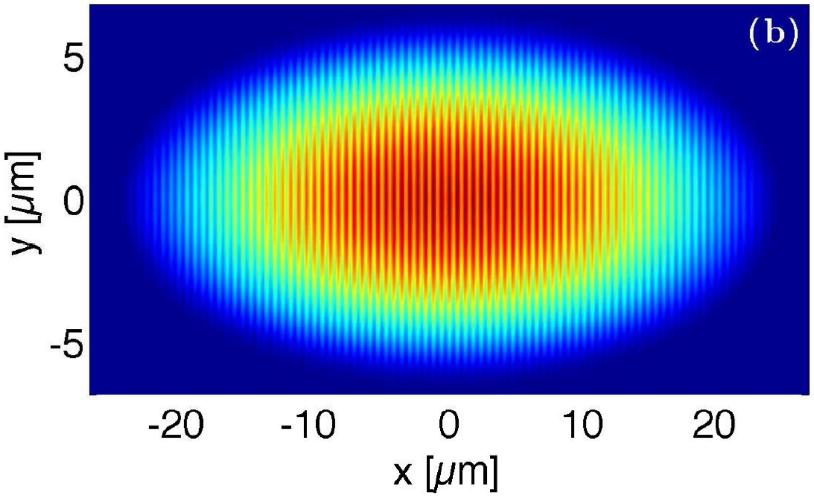
Apply $\pi/2$ Bragg pulse transferring momentum $p_B = 2k_1 - \varepsilon$ to the atomic cloud (with $\varepsilon << k_1$). Each component of the condensate function in the striped phase

$$\Psi_\downarrow \propto \underbrace{e^{-i(k_1-k_0)x}}_{(a)} + \underbrace{e^{i(k_1+k_0)x}}_{(b)}$$

will be fragmented in 3 terms. Components (a) and (b) , after $\pi/2$ pulse, will be able to interfere with wave length $2\pi/\varepsilon$

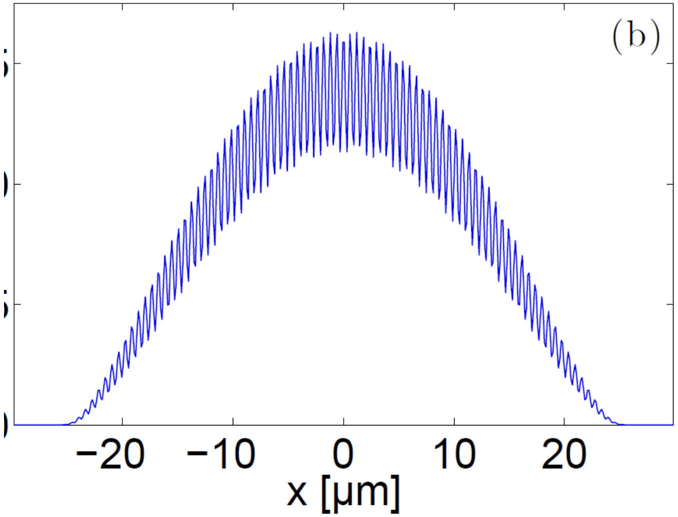


Density fringes of striped phase after $\pi/2$ Bragg pulse with momentum $p_B = 2k_1 - \varepsilon$ and $\varepsilon = 0.2\,k_1$



$d = a_z$

Without Bragg



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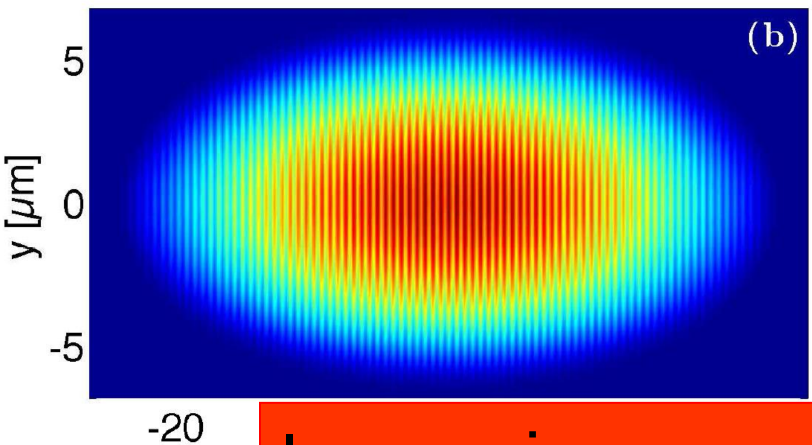

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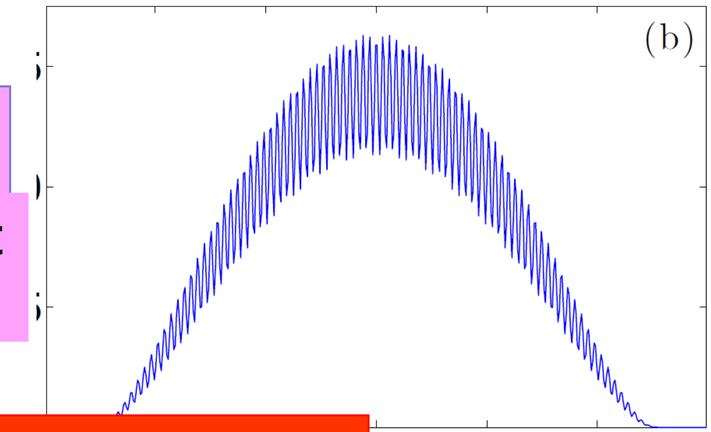
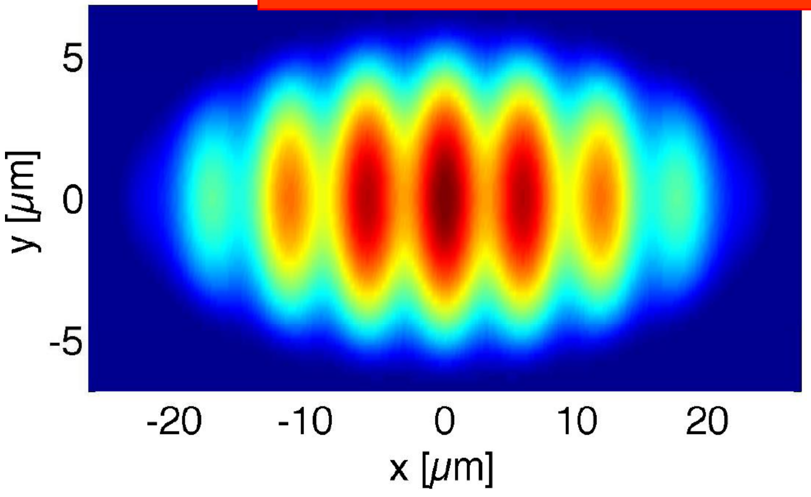
Without Bragg



Increasing wave length of fringes



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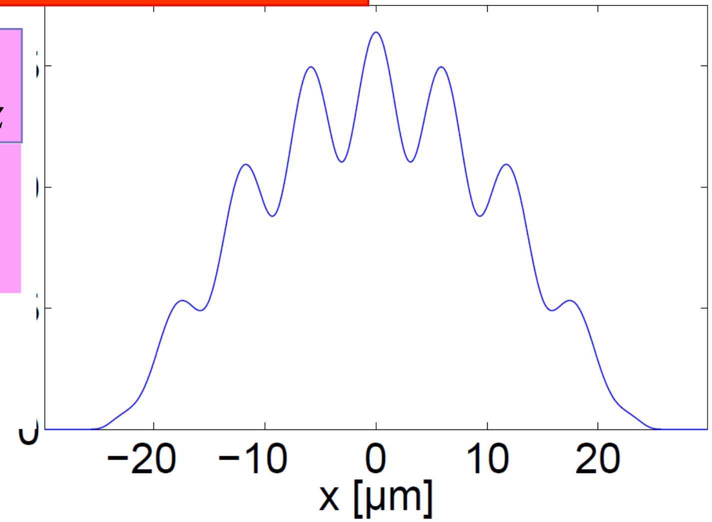
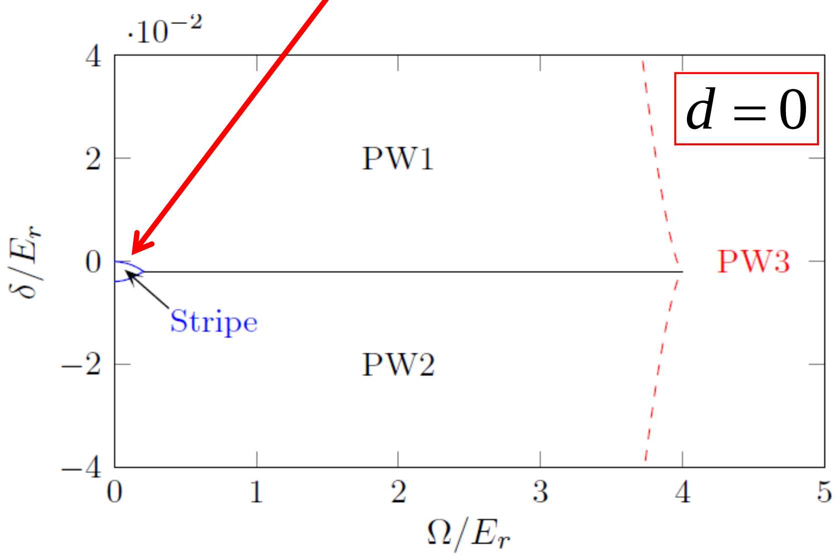
With Bragg



Stability of the striped phase

Reducing interspecies coupling constant **enhances robustness** of the striped phase.
Chemical potential difference between mixed and demixed phase at $\Omega = 0$ is given by $\Delta\mu = n(g - g_{\uparrow\downarrow})/2$ with $g_{\uparrow\uparrow} \approx g_{\downarrow\downarrow} \equiv g$

Choosing $d = 0$ ($g_{\uparrow\updownarrow} \approx g$) critical detuning corresponds to tiny fraction of recoil energy (**few Hertz**)



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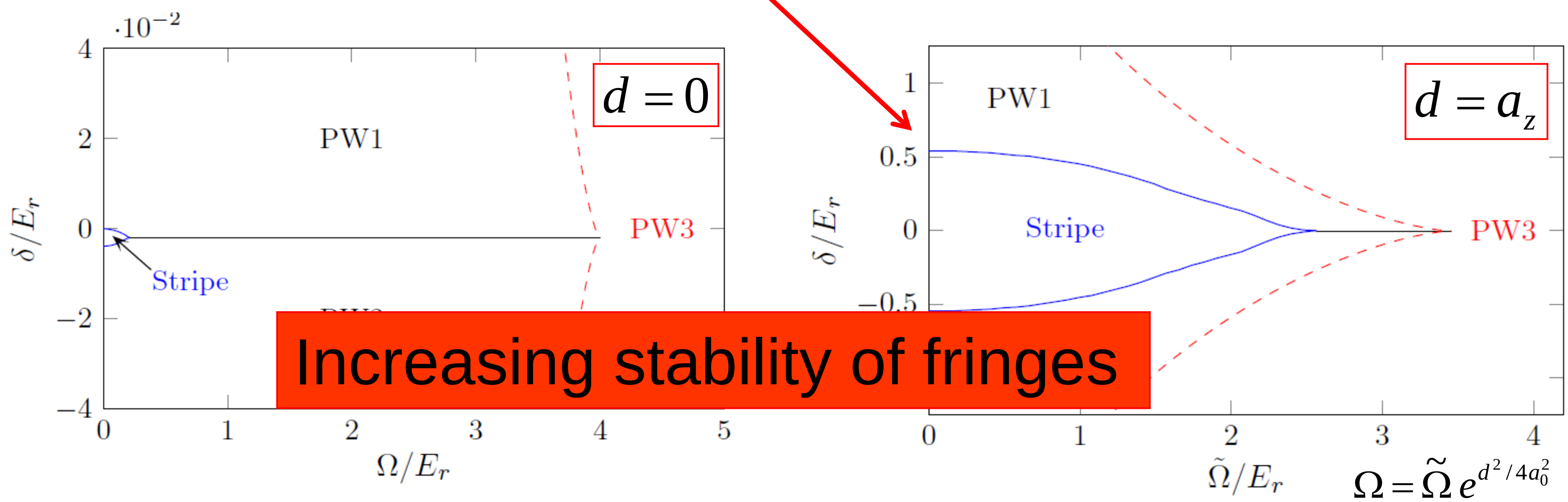
Choosing $d = 0$ ($g_{\uparrow\updownarrow} \approx g$) critical detuning corresponds to tiny fraction of recoil energy (**few Hertz**)
Choosing $d = a_z$ ($g_{\uparrow\updownarrow} \approx 0.6g$) critical detuning corresponds to $\approx 0.6 E_r$ (**a few hundred Hertz**)



Increasing stability of fringes

Rotons and stripes

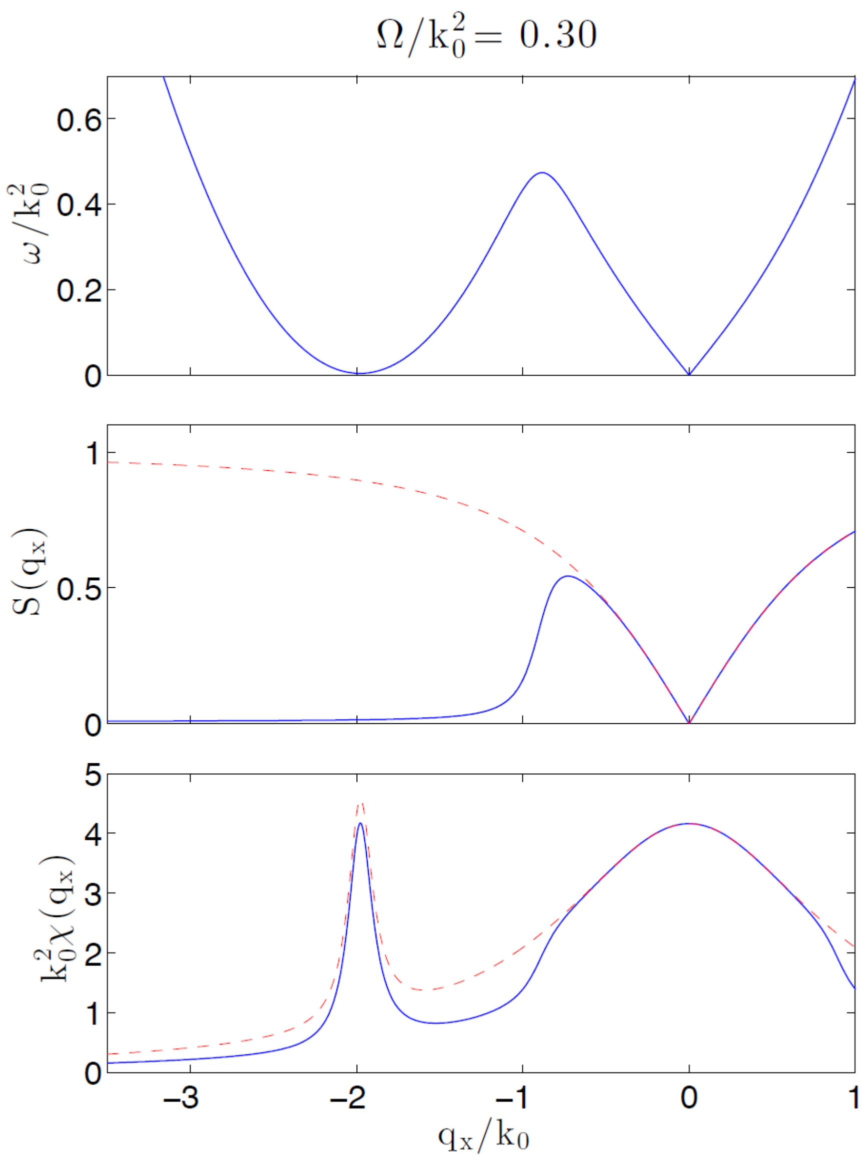
Rotonic structure,
static structure factor and
static response function
in plane wave phase

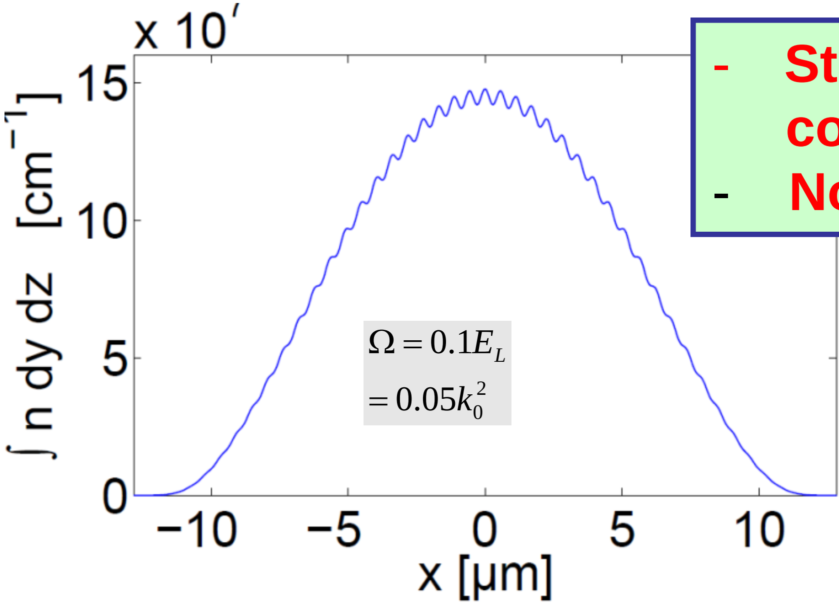
$$\Psi = \sqrt{\frac{N}{V}} \begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix} e^{ik_1 x}$$

Apply static perturbation

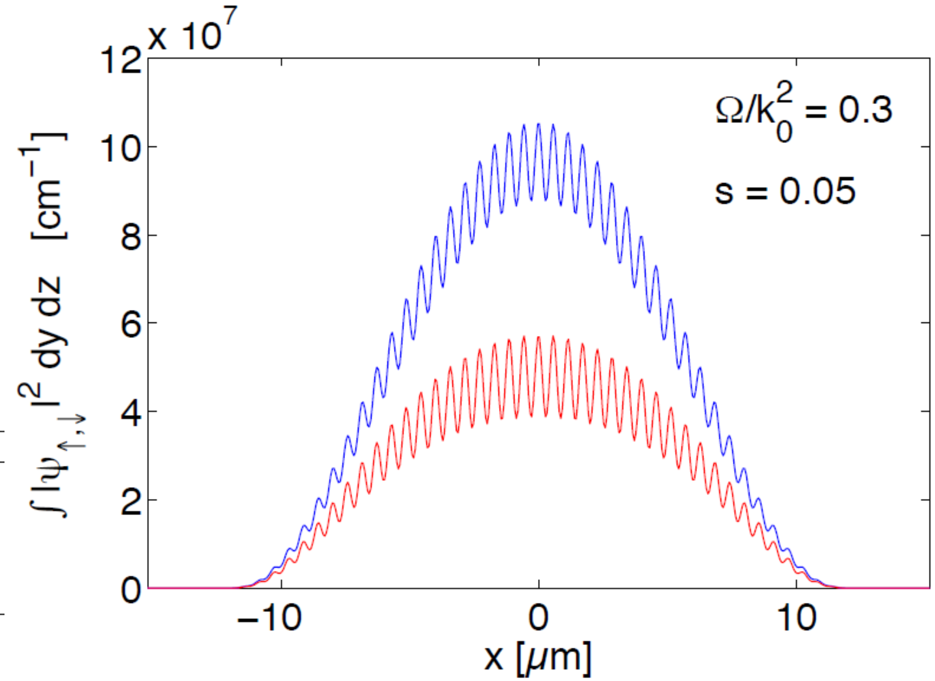
$$\delta V_{ext} = s E_R (2k_1) e^{i2k_1 x}$$

Strong non linear effects
caused by large value
of static response:
Emergence of stripes

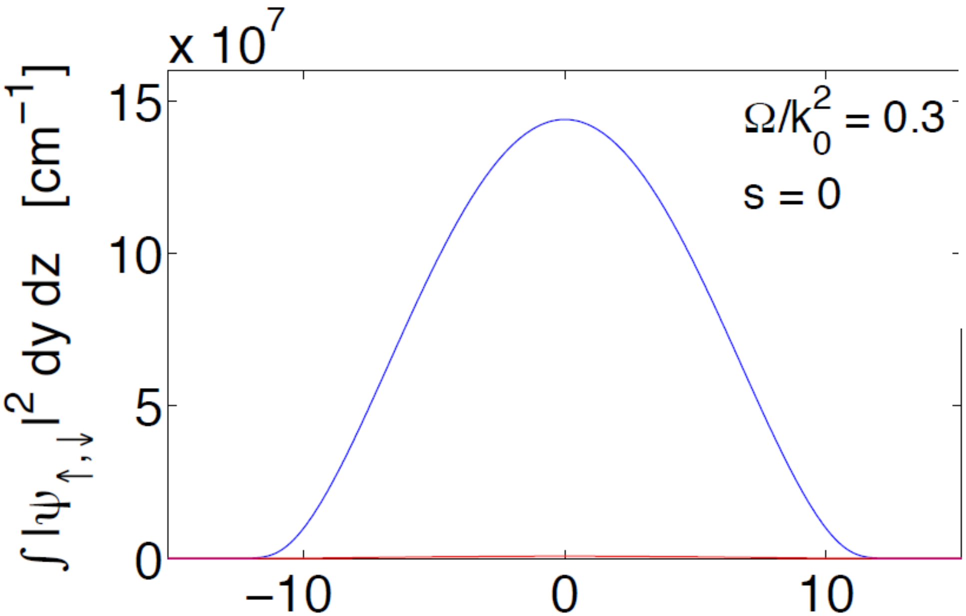




- Stripes have small contrast in stripe phase
- No polarization



- stripes have large contrast in PW phase with s=0.05
- spin polarization is strongly reduced



- No stripes in PW phase
- Almost full spin polarization

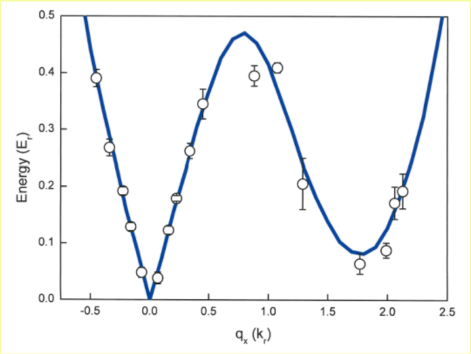
Main conclusions :

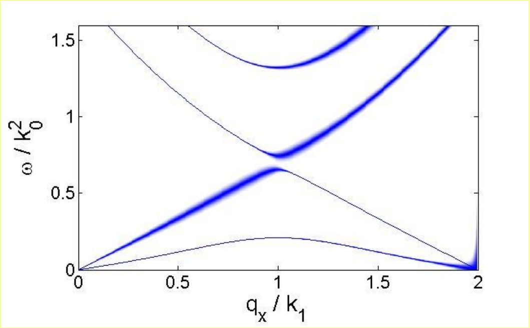
- **Rotonic excitation** in plane wave phase is onset of **crystallization** exhibited by striped phase

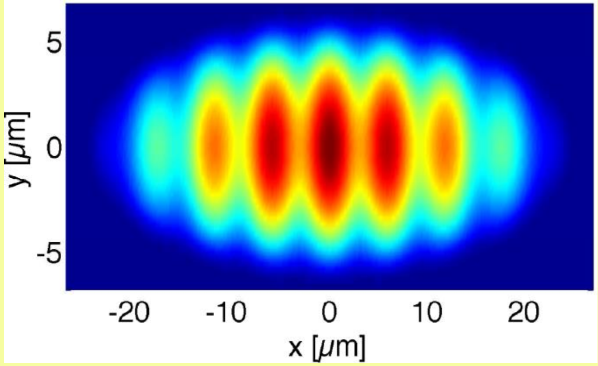
- **Two Goldstone** modes in striped phase

- **Contrast** and **stability** of stripes can be significantly **enhanced** creating two separated spin layers.
- **Wave length** of density fringes can be **increased** by applying $\pi/2$ Bragg or rf pulse

- **Stripes** can be produced in **plane wave** phase by adding small **static perturbation**







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ECT*

EUROPEAN CENTRE FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS
TRENTO, ITALY
Institutional Member of the ESF Expert Committee NuPECC

Castello di Trento ("Trifus"), watercolour, 19.8x 27.7, painted by A. Dürer on his way back from Venice (1495) British Museum, London.

COLD ATOMS MEET HIGH ENERGY PHYSICS

Trento, June 22-25, 2015

Main Topics

Spontaneously broken symmetries, abelian and non abelian gauge fields, supersymmetries, Fulde-Ferrel-Larchin-Ochinokov phase, Superfluidity in strongly interacting Fermi systems, High density QCD and bosonic superfluidity, quantum hydrodynamics, Kibble-Zurek mechanism, SU(N) configurations, quantum simulation of quark confinement, magnetic monopoles, Majorana Fermions, role of extra dimensions, lattice QCD, Black holes, Hawking radiation, Higgs excitations in cold atoms, AdS/CFT correspondence, Efimov states, instantons

Key Participants

Roberto Balbinot *(Bologna)*, Michael Baranov *(Innsbruck)*, Andrea Cappelli *(Firenze)*, Iacopo Carusotto *(Trento)*, Roberto Casalbuoni *(Firenze)*, Leonardo Fallani *(Firenze)*, Francesca Ferlaino *(Innsbruck)*, Gabriele Ferrari *(Trento)*, Margarita Garcia Perez *(Madrid)*, Jason Ho *(Columbus, Ohio)*, Kenichi Konishi *(Pisa)*, Manuel Endres *(Harvard)*, Simone Montangero *(Ulm)*, Muneto Nitta *(Keio University)*, Giorgio Parisi *(Roma)*, Saverio Pascazio *(Bari)*, Christophe Salomon *(LKB-ENS Paris)*, Augusto Smerzi *(Firenze)*, Luca Tagliacozzo *(ICFO Barcelona)*, Andrea Trombettoni *(SISSA Trieste)*, Ettore Vicari *(Pisa)*, Erez Zohar *(Munich)*, Peter Zoller *(Innsbruck)*, Willi Zwerger *(Munich)*, Martin Zwierlein *(MIT)*

Organizers

Massimo Inguscio *(LENS Firenze and INRIM Torino)*, Guido Martinelli *(SISSA Trieste)*, Sandro Stringari *(Trento)*

Director of the ECT*: Professor Wolfram Weise (ECT*)

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For local organization please contact: Ines Campo - ECT* Secretariat - Villa Tambosi - Strada delle Tabarelle 286 - 38123 Villazzano (Trento) - Italy
Tel.:(+39-0461) 314721 Fax:(+39-0461) 314750, E-mail: **ect@ectstar.eu** or visit **http://www.ectstar.eu**