Inducing Resonant Interactions with a Modulated Magnetic Field

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Outline

1. Motivation and Background
2. Tuning interactions: MFR, OFR, rf/mwFR
3. Modulated Magnetic FR
    A. Toy model
    B. Matching to a physical system
4. Universal results
5. Experimental application
6. Conclusion

Motivation and background

- Control the interaction strength between particles
- Desire access to all regimes
  - Attractive / Repulsive
  - Strong (UFG, UBG?) / Weak (BCS, BEC)
- Parametrize interactions by s-wave scattering length a

$$\sigma \propto a^2$$

- Related to s-wave phase shift:

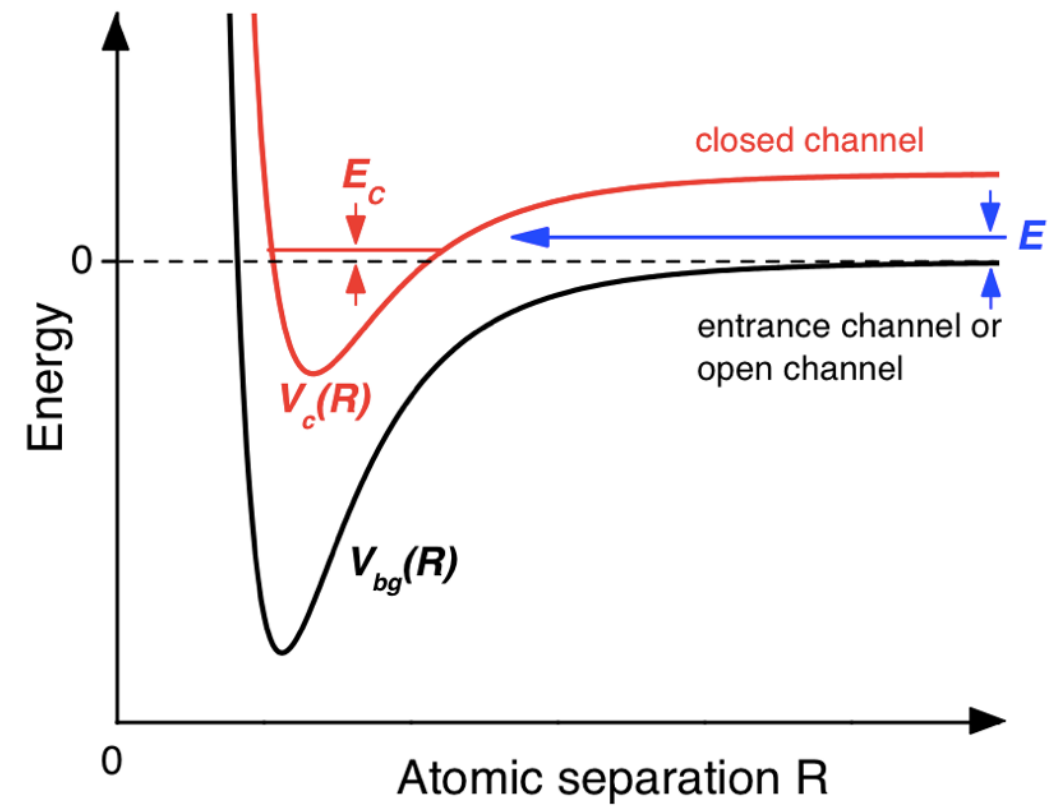
$$1/a = -k \cot \delta_0$$

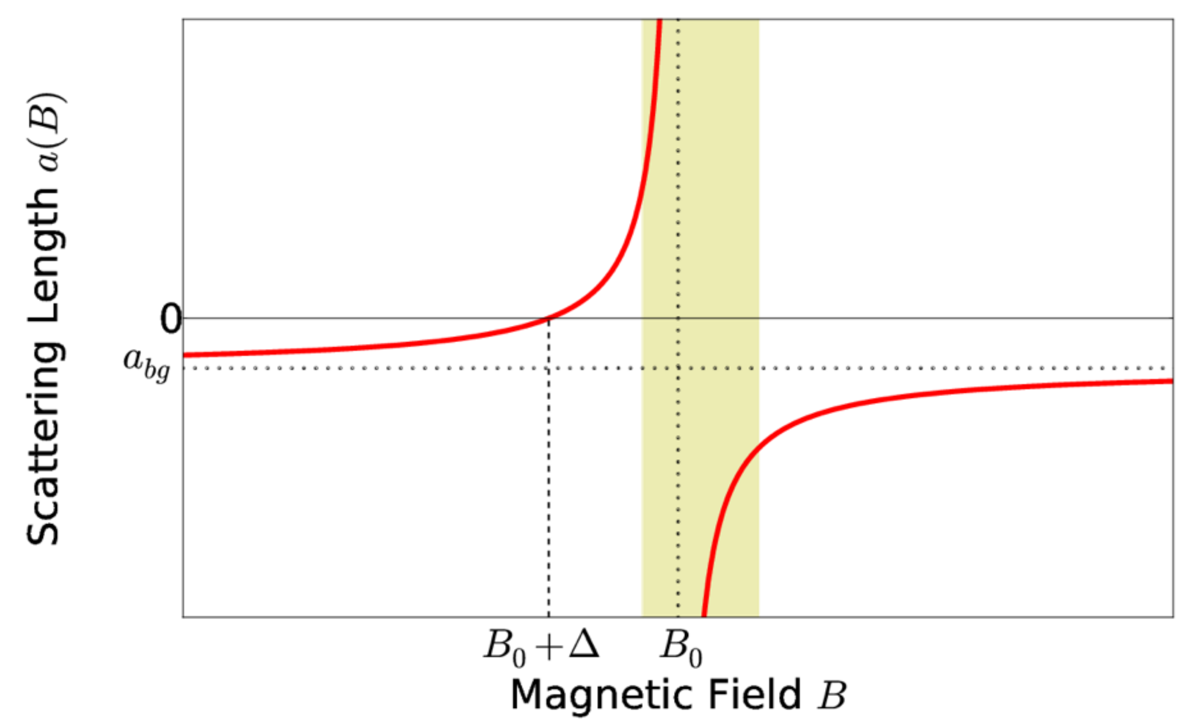
- Control a by resonantly coupling scattering state to bound state.

Magnetic Feshbach resonance (MFR)

- Resonantly couple to molecule in closed hyperfine channel
- Tune relative energy with DC magnetic field
- Some limitations:
  - No control over resonance properties
  - 3-body losses

$$\frac{1}{a(B)} = \frac{1}{a_{bg}} \frac{B - B_0}{B - B_0 - \Delta} + i\gamma$$

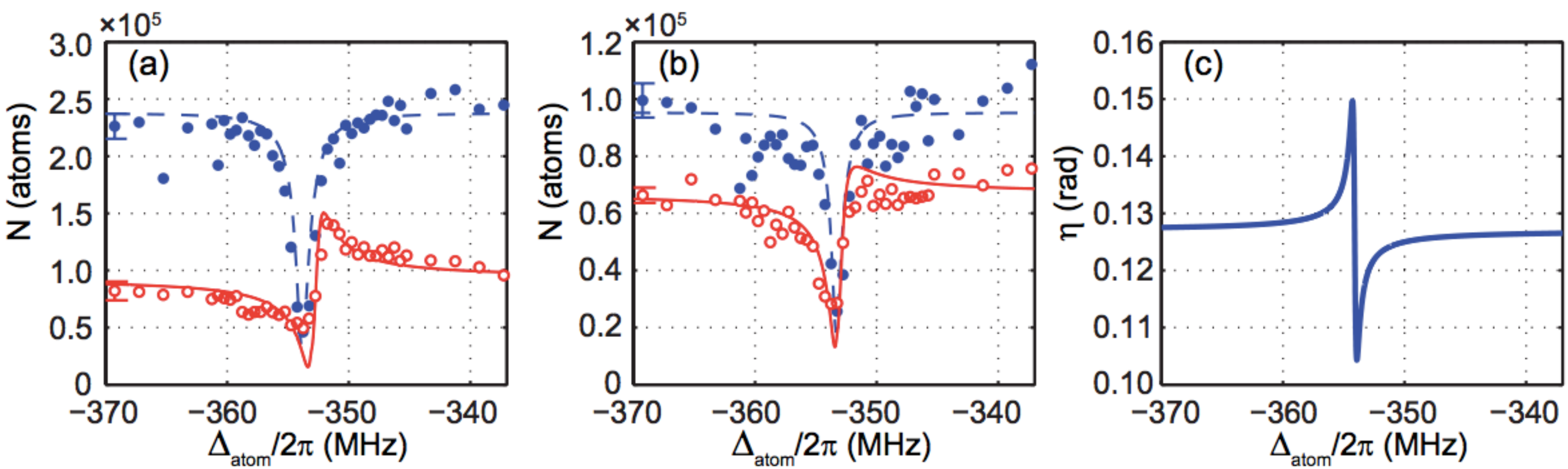




Optical Feshbach resonance (OFR)

- Use laser to couple to electronically excited p-wave molecule
- **Good:** tune interactions with laser detuning and intensity
- **Bad:** spontaneous decay leads to losses

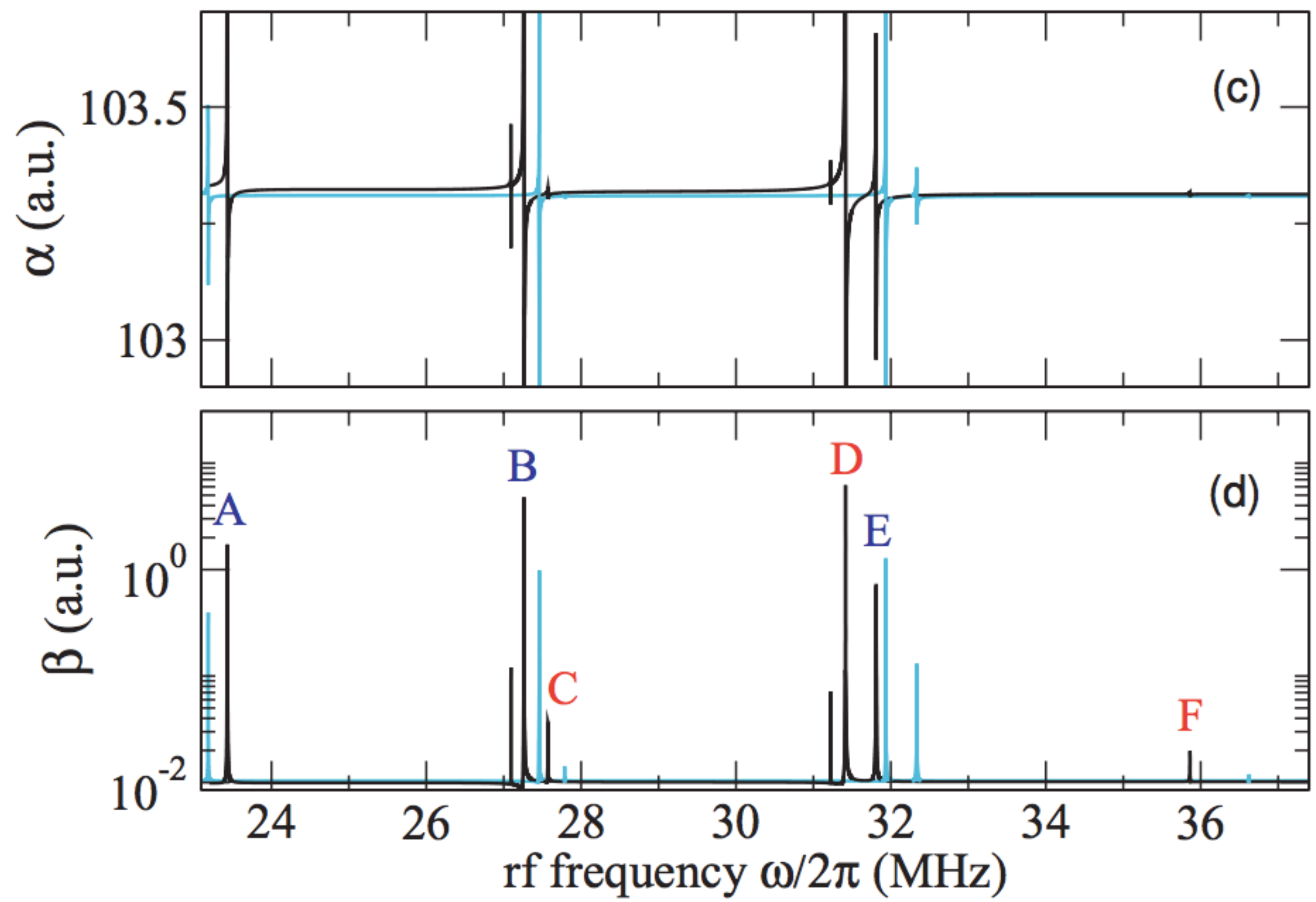
OFR in Yt: Yamazaki et al., Phys. Rev. A 87, 010704(R), (2013)



η: scattering phase shift

mw/rf Feshbach resonance (mw/rfFR)

- Use rf/mw fields to couple to molecule in closed hyperfine channel
- **Good:**
  - tune interactions with detuning and intensity
  - controllable losses
- **Bad:** Induced coupling is weak, making it difficult to significantly enhance a.



Theory: Tscherbul et al. Phys. Rev. A 81, 050701(R) (2010)

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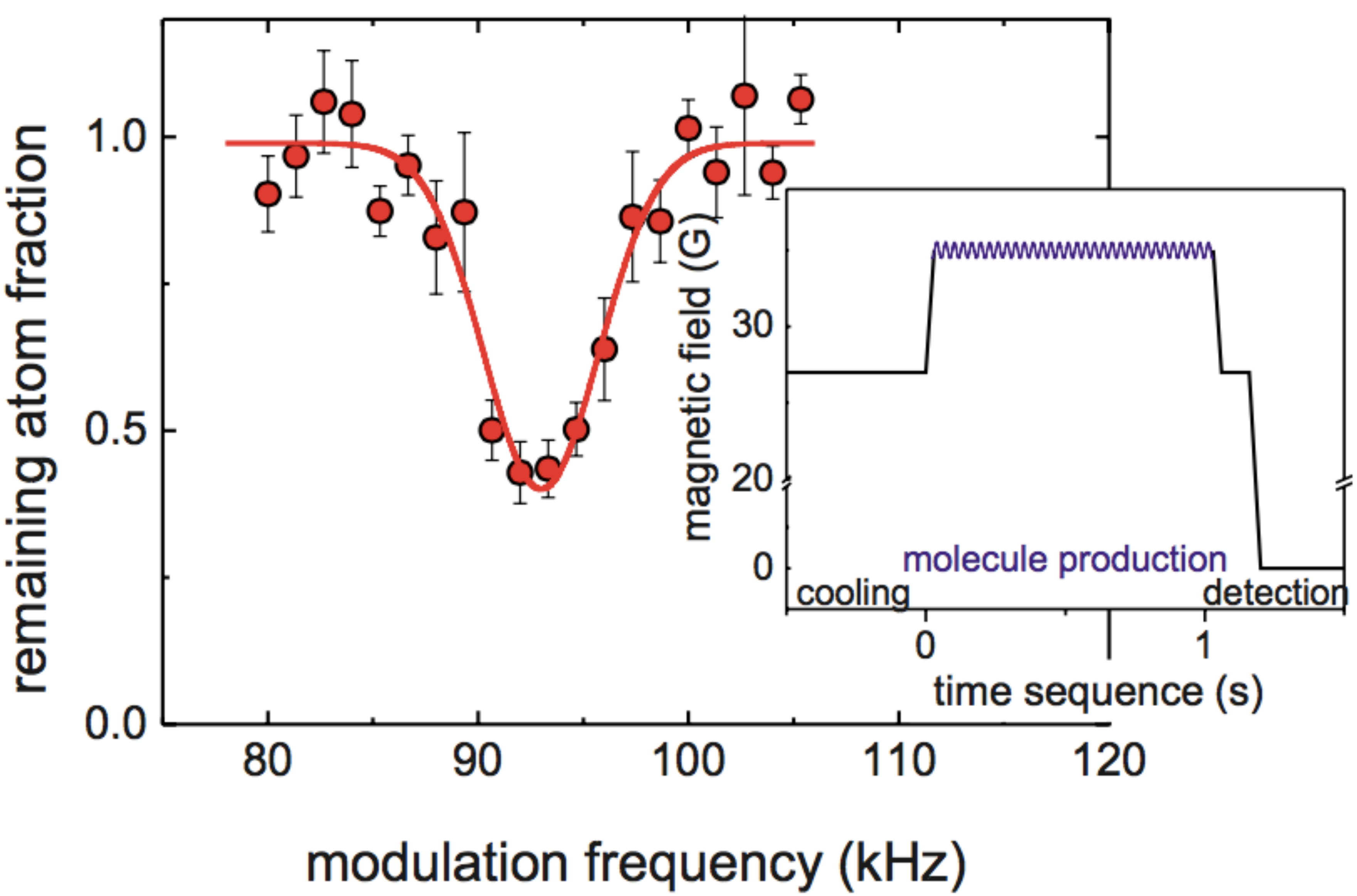
Modulated-magnetic Feshbach resonance (MMFR)

Wiggle the magnetic field: $B(t) = \bar{B} + \tilde{B}\cos(\omega t)$

Similar to wiggle spectroscopy:



Loss spectrum for Cs.

Lange et al., PRA 79, 013622 (2009)

MMFR

Simple parametrization of the scattering length:

$$\frac{1}{a} = \frac{1}{\bar{a}} \frac{\omega - \omega_0}{\omega - \omega_0 - \delta} + i\gamma$$

If bound state is a shallow dimer in scattering channel

dimensionless resonance parameters

$$\frac{\Delta\omega_0}{\omega_{\rm B}} = \frac{\omega_0 - \omega_{\rm B}}{\omega_{\rm B}}, \ \ \frac{\delta}{\omega_{\rm B}}, \ \ \gamma\bar{a}$$

are universal numbers multiplied by

$$\left[\frac{a'(\bar{B})}{a(\bar{B})}\tilde{B}\right]^2 \sim \left[\frac{\tilde{B}}{\bar{B} - B_0}\right]^2$$

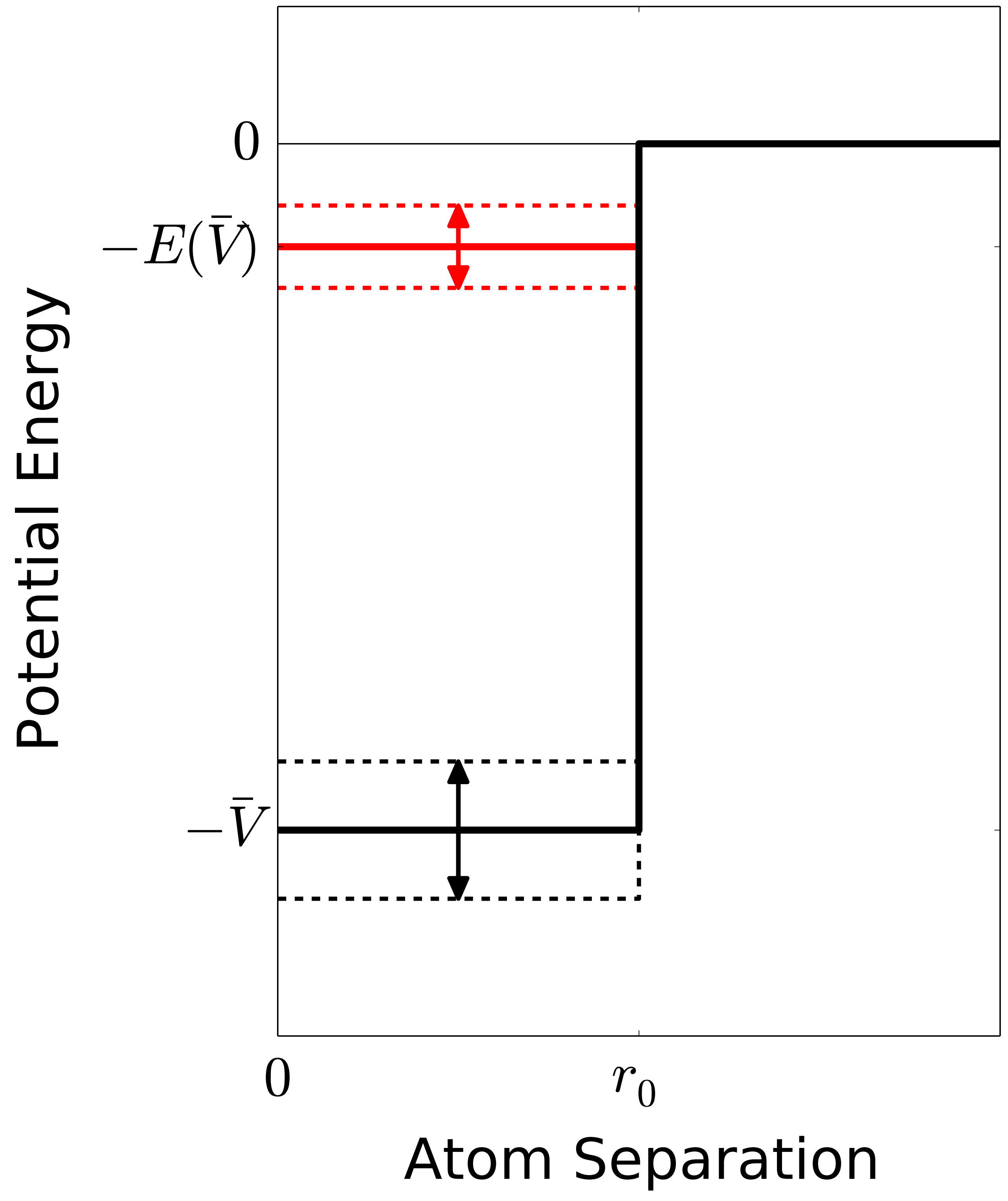
In this regime, a can be tuned without introducing dramatic loss.

Toy model

Square well with oscillating depth

$$V(r,t) = -\left[\bar{V} + \tilde{V}\cos(\omega t)\right] \times \Theta(r_0 - r)$$

Wiggle depth → wiggle binding energy



Toy model

Scattering described by the time-dependent Schrödinger equation:

$$i\frac{d}{dt}u(r,t) = -\frac{1}{m}\frac{\partial^2}{\partial r^2}u(r,t) - \left[\bar{V} + \widetilde{V}\cos(\omega t)\right]\theta(r_0 - r)u(r,t).$$

Solved analytically using Floquet's theorem:

$$u(r,t) = e^{iE_{\mathrm{F}}t}\phi(r,t) \qquad \phi(r,t) = \phi(r,t+2\pi/\omega)$$

E_{F}, the "Floquet eigenvalue", is determined by the asymptotic boundary condition.

The full solution is:

$$u(r,t) = \sum_{n=-\infty}^{\infty}\begin{cases} 2ia_n\sin(q_n r)\exp\left[-i(k_n^2/m)t + i\widetilde{V}\sin(\omega t)/\omega\right] & r < r_0, \\ (A_n^{\mathrm{out}}e^{ik_n r} + A_n^{\mathrm{in}}e^{-ik_n r})\exp\left[-i(k_n^2/m)t\right] & r \geq r_0, \end{cases}$$

$$k_n = [k^2 + mn\omega]^{1/2} \qquad q_n = [k^2 + m(\widetilde{V} + n\omega)]^{1/2}$$

Toy model

The S-matrix relates the amplitudes of incoming and outgoing states:

$$A_n^{\text{out}} = \sum_j S_{nj} A_j^{\text{in}}$$

$$S_{nj} = \sum_l (M_-)_{nl} (M_+)^{-1}_{lj}$$

$$(M_\pm)_{jn} = \frac{e^{\pm i k_j r_0}}{k_j} \left[(k_j \mp q_n) e^{i q_n r_0} - (k_j \pm q_n) e^{-i q_n r_0} \right] J_{j-n}(\widetilde{V}/\omega)$$

The inverse scattering length is:

$$\frac{1}{a} = - \lim_{k \to 0} \, k \cot \left(-\frac{i}{2} \ln S_{00} \right)$$

Matching to physical system

- Goal: use toy model to make quantitative predictions for MMFR where the molecule is a shallow dimer from MFR.
- Logic: for shallow dimer, physics insensitive to details of potential.
- Matching conditions:
  1. Determine $\bar{V}$: $\bar{a}/r_0 \gg 1$
  2. Determine (small) $\tilde{V}$:

$$E(\bar{V}+\tilde{V}) - E(\bar{V}) = \omega_{\rm B}(\bar{B}+\tilde{B}) - \omega_{\rm B}(\bar{B})$$

$$\tilde{V} = -2\omega_{\rm B}(\bar{B})\frac{1}{E'(\bar{V})}\underbrace{\frac{a'(\bar{B})}{a(\bar{B})}\tilde{B}}_{\tilde{b}}$$

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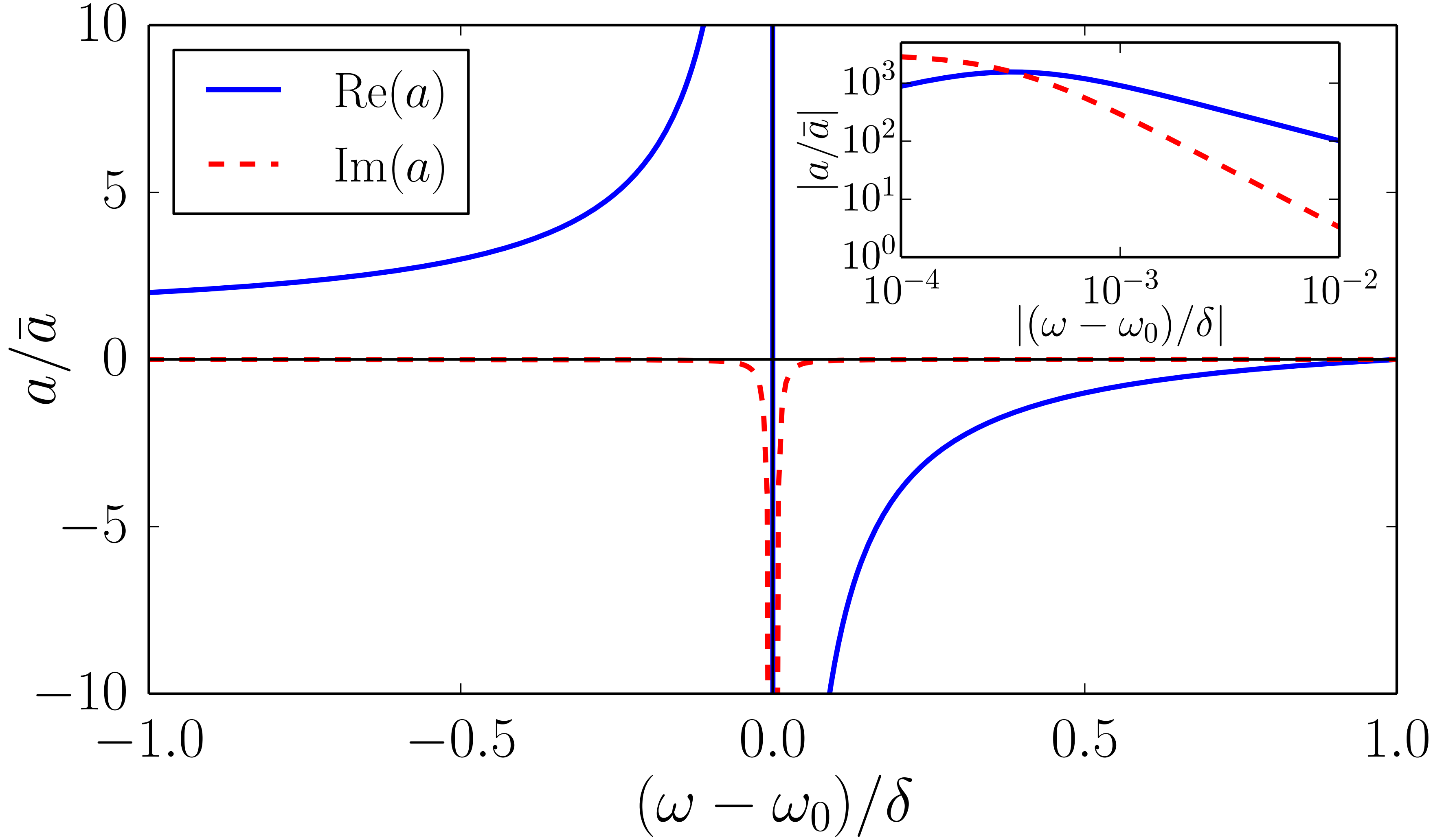
Universal results

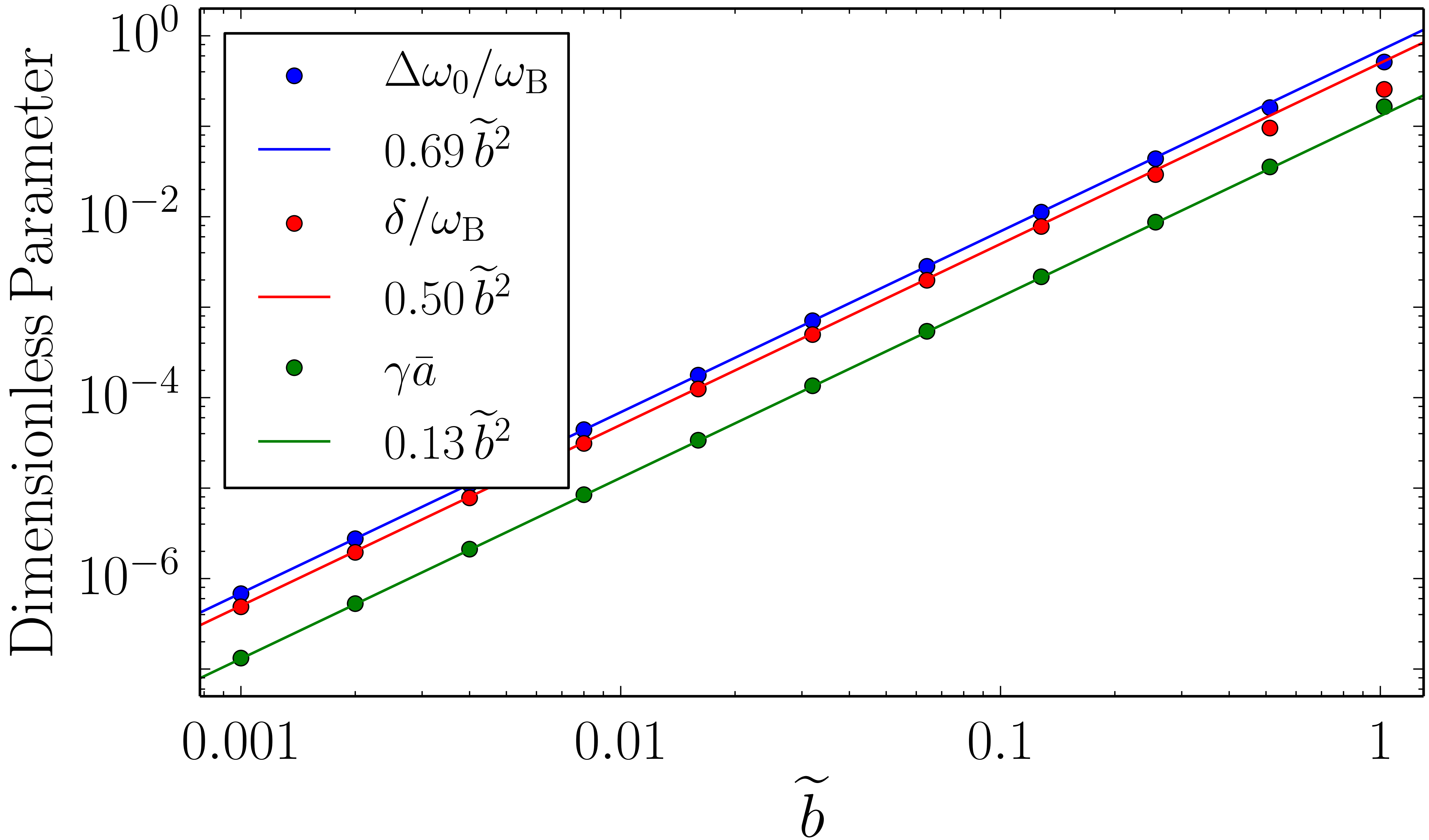
Scattering length as function of frequency for $\tilde{b} = 0.05$



Fit with $\dfrac{1}{a} = \dfrac{1}{\bar{a}}\dfrac{\omega - \omega_0}{\omega - \omega_0 - \delta} + i\gamma$ for $\dfrac{\Delta\omega_0}{\omega_B} = \dfrac{\omega_0 - \omega_B}{\omega_B},\ \dfrac{\delta}{\omega_B},\ \gamma\bar{a}$

Scaling behavior

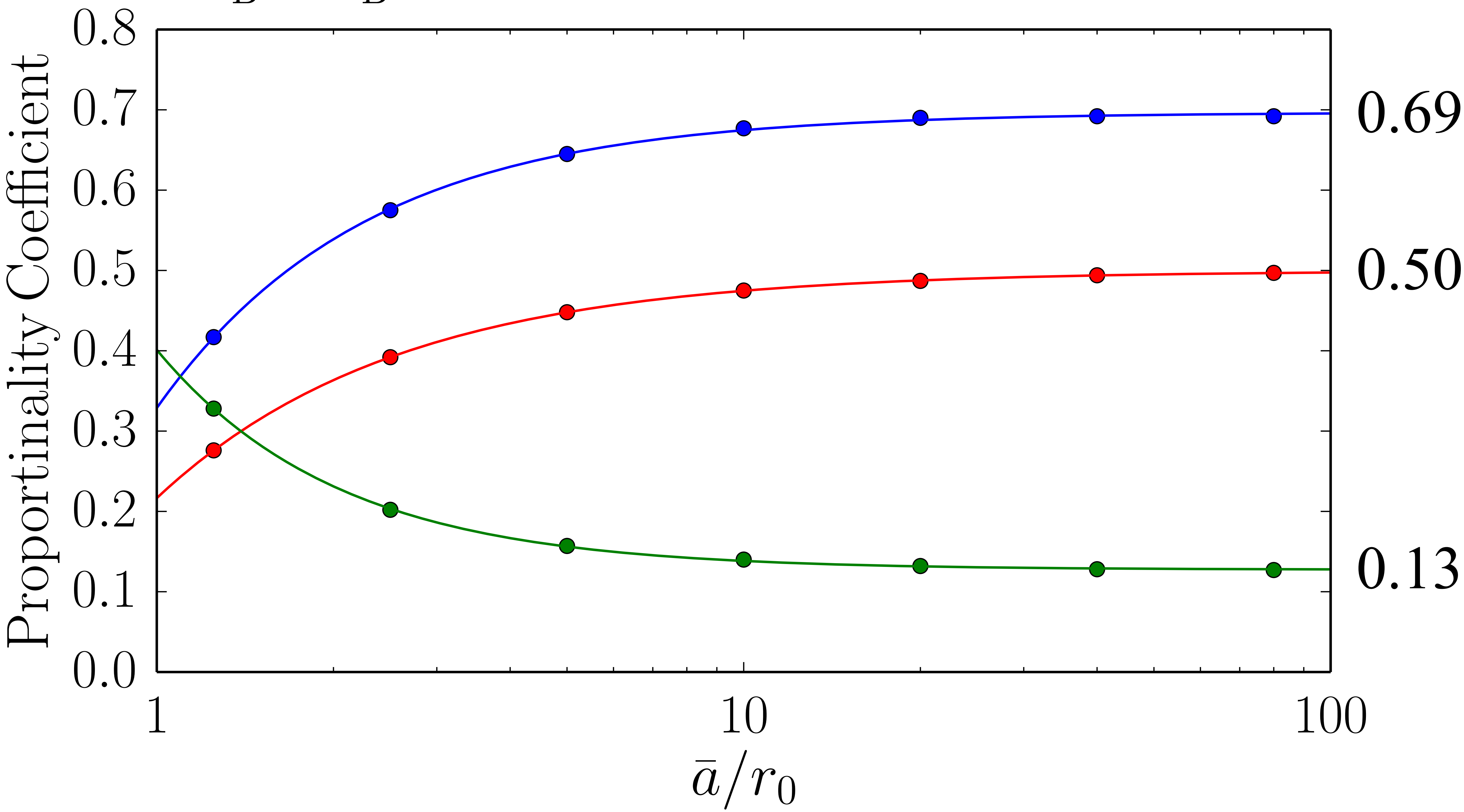
For small amplitudes: $$\frac{\Delta\omega_0}{\omega_{\rm B}},\ \frac{\delta}{\omega_{\rm B}},\ \gamma\bar{a} \propto \tilde{b}^2$$



Convergence to universality

$$\frac{\Delta\omega_0}{\omega_\mathrm{B}},\ \frac{\delta}{\omega_\mathrm{B}},\ \gamma\bar{a} \propto \tilde{b}^2$$

Plot the proportionality constants.



Summary of universal results

Scattering length near resonance:

$$\frac{1}{a} = \frac{1}{\bar{a}} \frac{\omega - \omega_0}{\omega - \omega_0 - \delta} + i\gamma$$

Resonance parameters are controllable
and have universal form:

$$(\omega_0 - \omega_{\mathrm{B}})/\omega_{\mathrm{B}} = 0.69\,\tilde{b}^2$$

$$\delta/\omega_{\mathrm{B}} = 0.50\,\tilde{b}^2$$

$$\gamma\bar{a} = 0.13\,\tilde{b}^2$$

$$\tilde{b} = \frac{a'(\bar{B})}{a(\bar{B})}\tilde{B}$$

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Experimental application

Can the scattering length be tuned over a significant range without introducing dramatic loss in the universal regime?

Scaling behavior is favorable:

$$\frac{\mathrm{Re}\,a}{\mathrm{Im}\,a} = \frac{1}{\gamma \bar{a}} \frac{\omega_0 - \omega}{\omega_0 - \omega + \delta} \propto \frac{1}{\tilde{b}^2}$$

Decreasing amplitude helps and hurts, but it helps more!

Look at wiggle spectroscopy experiment with ^{7}Li near resonance at 738 G: Dyke et al., PRA **88,** 023625 (2013).

Experimental application

$$\tilde{B} = 0.57 \text{ G}$$

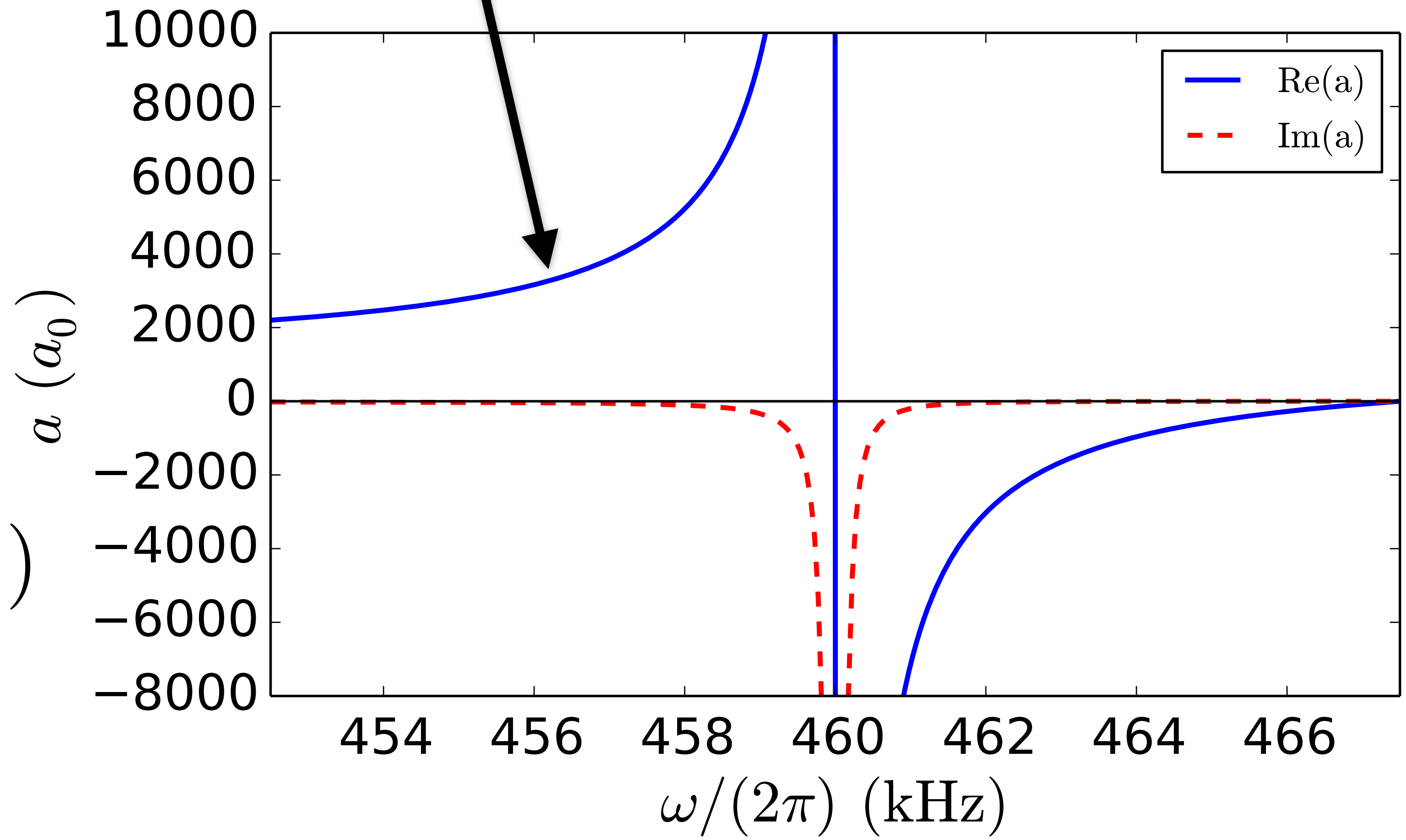
$$\mathrm{Re}\,a \approx 3\bar{a} \quad (\mathrm{Im}\,a = -0.04\bar{a})$$

From universal relations:

$$\delta = 7.5 \text{ kHz}$$

$$\Delta\omega_0 = 10 \text{ kHz}$$

$$\gamma = 1/(2.6 \times 10^5\ a_0)$$



Conclusion

- S-wave interactions can be resonantly enhanced by applying an oscillating magnetic field with frequency near the transition to a molecular state.
- Molecule formation leads to atom loss.
- For a shallow dimer in the scattering channel, the dimensionless resonance parameters have universal forms.
- The scattering length can be significantly enhanced without introducing dramatic atom loss.