Fermions in Synthetic Gauge Potentials and Synthetic Dimensions

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Frontiers in Quantum Simulation with Cold Atoms
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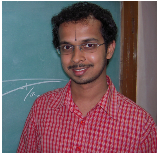
Acknowledgements

- Generous research funding: DST, DAE

- Key contributors:



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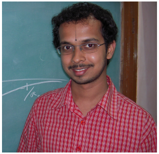

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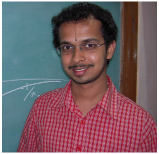

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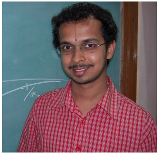

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Umesh Yadav

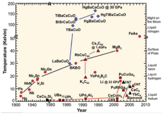
Overview

- Background and questions
- Fermions in synthetic gauge potentials and dimensions
  - Interacting fermions in synthetic Rashba (with JPV)
  - Feshbach resonances
  - Finite momentum pairing
  - Kondo effect (with AA)
  - Route to novel Hamiltonians (with SG/JPV)
  - Few body physics in synthetic dimensions (with SG/UY)

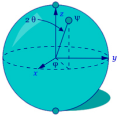
Based on `cond-mat:1101.0411, 1104.5633, 1108.4872, 1109.5279, 1201.5332, 1211.1831, 1212.2858, Unpublished (coming soon)`

(Related work: Gong et al., 1105.1796, Yu and Zhai 1105.2250, Hu et al., 1105.2408, Subasi and Iskin 1106.0473, Han and Sá de Melo 1106.3613..and many others; see also, Chaplik and Magrill (2006))

- What is all this good for...



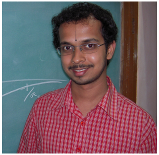
Superfluids with high T_c



Topological phases/Quantum computing

...quantum simulation of many body physics.

Fermions in SU(2) Gauge Potentials



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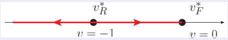
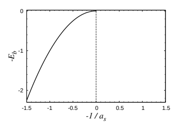
Interacting Spin-$\frac{1}{2}$ Fermions in 3D ("Free Vacuum")

2-Body

- Need critical attraction v for bound state





(RG picture: Sachdev (1999), Sauli and Kopeitz (2006), Nikolić and Sachdev (2007), Nishida and Son (2007))

- $v_F^* = 0$: "free vacuum" fixed point, $v_R^* = -1$: *resonant* fixed point
- Binding energy $E_b = 1/a_s^2$ for scattering length $a_s > 0$

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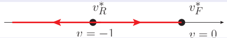

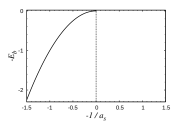

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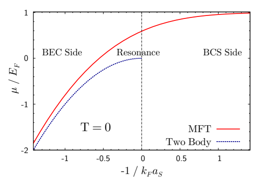
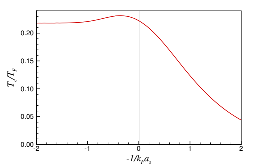
Finite fermion density $\rho \sim k_F^3$: BCS-BEC Crossover

(Eagles (1969), Leggett (1980), Norziéres and Schmitt-Rink (1985), Randeria et al., (1990s), Zwerger (ed.) (2011))

Ground state



Transition temperature



- BCS: $k_F|a_s| \ll 1$, $\mu \approx E_F$, large pairs, $T_c \sim exp(-1/k_F|a_s|)$
- BEC: $k_F a_s \ll 1$, $\mu \approx -1/2a_s^2$, tight bosonic fermion-pairs, boson-boson scattering length $\approx 2a_s$, $T_c \approx 0.218 T_F$

Spin-$\frac{1}{2}$ Fermions in Non-Abelian Gauge Potentials

- Spin $\frac{1}{2}$ particles in a non-Abelian SU(2) gauge potentials

$$\mathcal{H}_{GF} = \int \mathrm{d}^3\boldsymbol{r}\,\Psi^\dagger(\boldsymbol{r})\left[\frac{1}{2}(p_i\mathbf{1} - A_i^\mu\boldsymbol{\tau}^\mu)(p_i\mathbf{1} - A_i^\nu\boldsymbol{\tau}^\nu) + \Omega^\mu\boldsymbol{\tau}^\mu\right]\Psi(\boldsymbol{r})$$

$\Psi(\boldsymbol{r}) = \{\psi_\sigma(\boldsymbol{r})\}, \sigma = \uparrow, \downarrow$, p_i–momentum operator $\boldsymbol{\tau}^\mu$ – Pauli spin operators

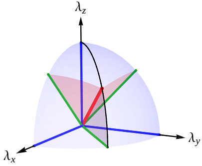
- Uniform gauge fields– A_i^ν, spin potentials Ω^μ (Zeeman fields)
- $A_i^\nu = \lambda_i\delta_i^\nu$ (experimentally relevant)

- Hamiltonian for $A_i^\nu = \lambda_i\delta_i^\nu$ $(\Omega^\mu = 0)$

$$\mathcal{H}_R = \int \mathrm{d}^3\boldsymbol{r}\,\Psi^\dagger(\boldsymbol{r})\left[\frac{p^2}{2}\mathbf{1} - \boldsymbol{p}_\lambda\cdot\boldsymbol{\tau}\right]\Psi(\boldsymbol{r}),$$

$$\boldsymbol{p}_\lambda\cdot\boldsymbol{\tau} = \lambda_x p_x\boldsymbol{\tau}^x + \lambda_y p_y\boldsymbol{\tau}^y + \lambda_z p_z\boldsymbol{\tau}^z$$

...generalized Rashba spin-orbit coupling

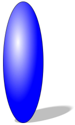


Rashba SOC described by $\boldsymbol{\lambda} = \lambda\hat{\boldsymbol{\lambda}}$,
λ SOC strength $\equiv$ gauge coupling

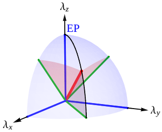
“High-Symmetry” Rashba Gauge Fields

(Vyasanakere and VBS, arXiv:1101.0411)

Prolate



$\lambda_x = \lambda_y < \lambda_z$



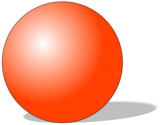
Extreme Prolate

(Equal Rashba-Dresselhaus)

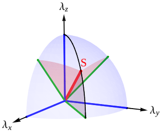
$\lambda_x = \lambda_y = 0, \lambda_z = \lambda$

(XJLiu et al. 0808.4137, Shanxi (2012), MIT (2012), NIST(2013))

Spherical



$\lambda_x = \lambda_y = \lambda_z$

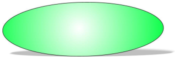


$\lambda_x = \lambda_y = \lambda_z = \frac{\lambda}{\sqrt{3}}$

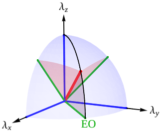
(Proposal: Anderson et al., 1112.6022)

Oblate

(Rashba)



$\lambda_x = \lambda_y > \lambda_z$



Extreme Oblate (Rashba)

$\lambda_x = \lambda_y = \frac{\lambda}{\sqrt{2}}, \lambda_z = 0$

(Proposal: Campbell et al., 1102.3945; Anderson et al., 1306.2606, Xu et al., 1306.2829)

Spin-$\frac{1}{2}$ Fermions in Rashba Gauge Fields

- One particle states

$$|\boldsymbol{k}\alpha\rangle = |\boldsymbol{k}\rangle \otimes |\alpha\hat{\boldsymbol{k}}_\lambda\rangle, \qquad \varepsilon_{\boldsymbol{k}\alpha} = \frac{k^2}{2} - \alpha|\boldsymbol{k}_\lambda|$$

$\boldsymbol{k}_\lambda = \lambda_x k_x \boldsymbol{e}_x + \lambda_y k_y \boldsymbol{e}_y + \lambda_z k_z \boldsymbol{e}_z$,

$\alpha = \pm 1$ *helicity*

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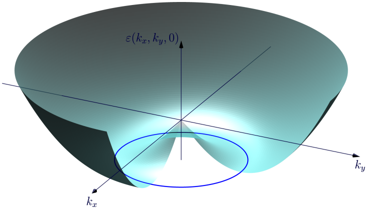

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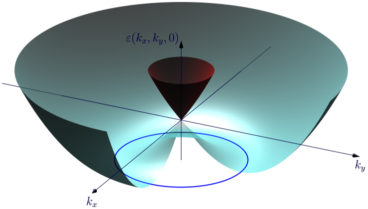

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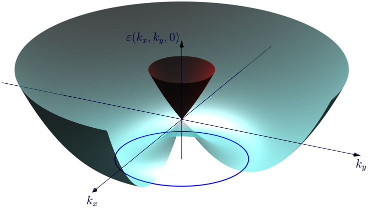


- Interaction between fermions...attraction in the *singlet channel*

$$\mathcal{H}_v = \frac{v}{2}\int \mathrm{d}^3\boldsymbol{r}\,\psi^\dagger_\uparrow(\boldsymbol{r})\psi^\dagger_\downarrow(\boldsymbol{r})\psi_\downarrow(\boldsymbol{r})\psi_\uparrow(\boldsymbol{r})$$

  with scattering length a_s

- Natural formulation in terms of *singlet amplitudes*,

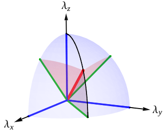
$$A_{\alpha\beta}(\boldsymbol{k},\boldsymbol{q}) = \langle \boldsymbol{q},\boldsymbol{k},s|\boldsymbol{q},\boldsymbol{k},\alpha\beta\rangle, \qquad |\boldsymbol{q},\boldsymbol{k},\alpha\beta\rangle = C^\dagger_{\frac{\boldsymbol{q}}{2}+\boldsymbol{k},\alpha} C^\dagger_{\frac{\boldsymbol{q}}{2}-\boldsymbol{k},\beta}|0\rangle$$

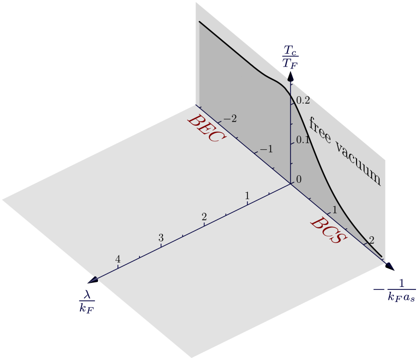
$$H_v = \frac{v}{2V}\sum_{\boldsymbol{q}}\sum_{\boldsymbol{k},\boldsymbol{k}'} \underbrace{A_{\alpha\beta}(\boldsymbol{q},\boldsymbol{k})A^*_{\alpha'\beta'}(\boldsymbol{q},\boldsymbol{k}')}_{U_{\alpha\beta,\beta'\alpha'}(\boldsymbol{q},\boldsymbol{k},\boldsymbol{k}')} C^\dagger_{(\frac{\boldsymbol{q}}{2}+\boldsymbol{k})\alpha} C^\dagger_{(\frac{\boldsymbol{q}}{2}-\boldsymbol{k})\beta} C_{(\frac{\boldsymbol{q}}{2}-\boldsymbol{k}')\beta'} C_{(\frac{\boldsymbol{q}}{2}+\boldsymbol{k}')\alpha'}$$

The Question

- Given: Fermions at density $\rho \sim k_F^3$ with scattering length a_s

- Given: Rashba spin-orbit coupling $\boldsymbol{\lambda} = \lambda \hat{\boldsymbol{\lambda}}$





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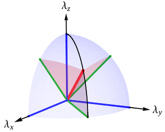
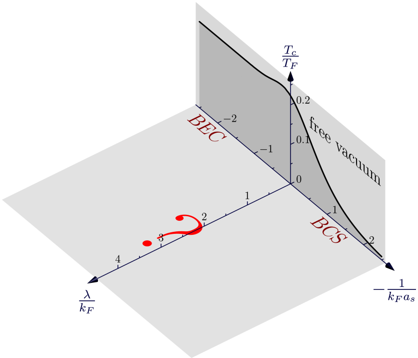


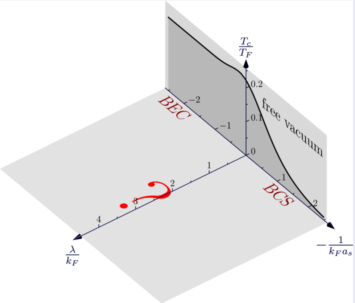


- Question: What happens to BCS-BEC in a Rashba gauge field $\lambda\hat{\boldsymbol{\lambda}}$?

Fermions in Rashba Gauge Fields

Organization

- Ground State
  - Two-body
  - Many body
- Excitations
  - Two-body
  - Many body
- Finite temperatures
- Phase diagram



2-Body Ground State

2-Body Ground State – High Symmetry Gauge Fields

(Vyasanakere and VBS, arXiv:1101.0411)

- For EP ($\lambda_x = \lambda, \lambda_y = \lambda_z = 0$) binding energy $E_b = \frac{\Theta(a_s)}{a_s^2}$ – same energetics as in free vacuum

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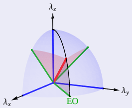
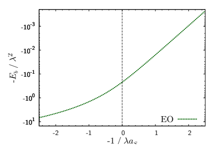
Extreme Oblate: $\lambda_x = \lambda_y = \frac{\lambda}{\sqrt{2}}, \lambda_z = 0$

- Critical scattering length a_{sc} *vanishes*: bound state for *any* scattering length!
- Binding energy

$a_s > 0$	$1/a_s = 0$	$a_s < 0$
$\frac{1}{(\lambda a_s)^2} + \frac{\log 2}{\sqrt{2}\lambda a_s}$	$0.22\ \lambda^2$	$\frac{4\lambda^2}{e^2} e^{\frac{-2\sqrt{2}}{\lambda \lvert a_s \rvert}}$





- Wavefunction (singlet + triplet)
  $|\Psi\rangle \propto \psi_s(\boldsymbol{r}) |\uparrow\downarrow - \downarrow\uparrow\rangle + \psi_a(\boldsymbol{r}) |\uparrow\uparrow\rangle + \psi_a^*(\boldsymbol{r}) |\downarrow\downarrow\rangle$
  ...uniaxial spin nematic (ABM of ^{3}He!)

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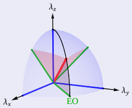

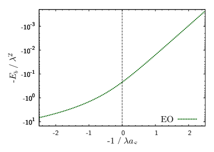

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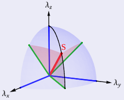
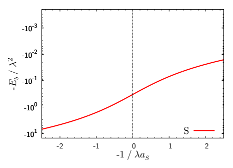
Spherical: $\lambda_x = \lambda_y = \lambda_z = \frac{\lambda}{\sqrt{3}}$

- Bound state for *any* scattering length
- Binding energy
  $$E_b = \frac{1}{4}\left(\frac{1}{a_s} + \sqrt{\frac{1}{a_s^2} + \frac{4\lambda^2}{3}}\right)^2$$
- ..."algebraic" in the BCS side: $E_b = \left(\frac{\lambda a_s}{3}\right)^2$

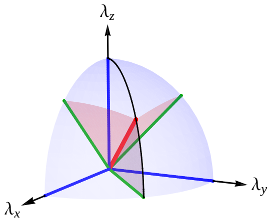


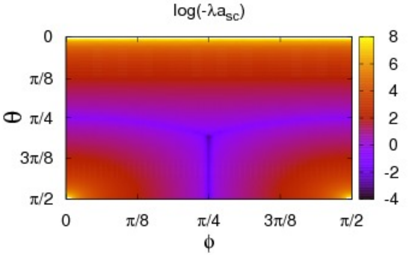


Generic Rashba SOC

- Critical scattering length

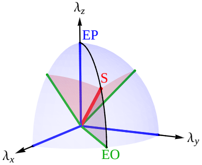
$$\lambda a_{sc} = \mathcal{F}(\hat{\boldsymbol{\lambda}})$$





- Critical scattering length $a_{sc} < 0$; *for a generic SOC bound state appears at a weaker attraction* (negative scattering length, do not need a resonance scattering length)
- SOC – *attractive interaction amplifier!*

State at Resonance Scattering Length



		$a_s < 0$, $\lambda\|a_s\| \ll 1$			Resonance $1/(\lambda a_s) = 0$			$a_s > 0$, $\lambda a_s \ll 1$		
GFC	a_{sc}	E_b	η_t	spin structure	E_b	η_t	spin structure	E_b	η_t	spin structure
EP	$-\infty$	no bound state			0	$\frac{1}{2}$	biaxial nematic (BW)	$\frac{1}{a_s^2}$	0	singlet
S	0^-	$\frac{\lambda^4 a_s^2}{3}$	$\frac{1}{2}$	spherical	$\frac{\lambda^2}{3}$	$\frac{1}{4}$	spherical	$\frac{1}{a_s^2} + \frac{2\lambda^2}{3}$	0	singlet
EO	0^-	$\frac{2\lambda^2}{e^2} e^{-2\sqrt{2/\lambda\|a_s\|}}$	$\frac{1}{2}$	uniaxial nematic (ABM)	$0.22\lambda^2$	0.28	uniaxial nematic (ABM)	$\frac{1}{a_s^2} + \frac{\lambda^2}{2}$	0	singlet

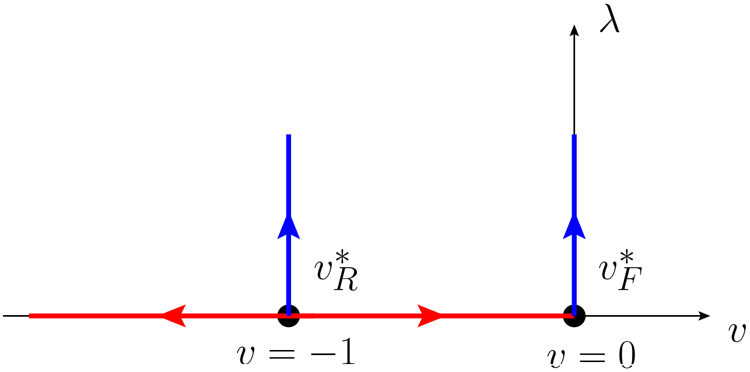
- Characteristic triplet content η_t
- Size of bound state wavefunction is λ^{-1}
- For S-SOC,

$$|\Psi_b\rangle \propto \frac{e^{-\lambda r/\sqrt{3}}}{r}\left(\sin\frac{\lambda r}{\sqrt{3}} + \cos\frac{\lambda r}{\sqrt{3}}\right)|\uparrow\downarrow - \downarrow\uparrow\rangle$$

$$+i\left(\left(\frac{\lambda}{\sqrt{3}} + \frac{1}{r}\right)\sin\frac{\lambda r}{\sqrt{3}} - \frac{\lambda}{\sqrt{3}}\cos\frac{\lambda r}{\sqrt{3}}\right)\frac{e^{-\lambda r/\sqrt{3}}}{\lambda r/\sqrt{3}}|\uparrow\downarrow + \downarrow\uparrow\rangle_{\hat{r}}$$

Physics of Enhanced Binding: RG Picture

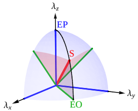
- Relevance-irrelevance of Rashba term

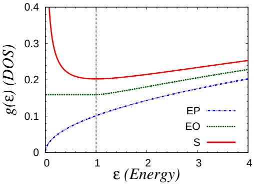


- Spin orbit interaction is a relevant operator at v_R^* and v_F^*

The "Physical" Picture

- Singlet density of states of high symmetry SOCs





$$g(\varepsilon) \sim \begin{cases} \sqrt{\varepsilon} & \text{for EP} \\ \lambda(\text{constant}) & \text{for EO} \\ \frac{1}{\sqrt{\varepsilon}} & \text{for S} \end{cases}$$

$$g(\varepsilon) \to \sqrt{\varepsilon}, \quad \varepsilon \to \infty$$

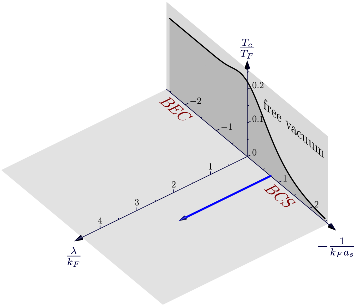
SOC induces large infrared degeneracies! DOS determined by the *co-dimension* of the one-particle ground state manifold

- Simple model ($\varepsilon_0 \sim \lambda^2$)

$$g(\varepsilon) = \begin{cases} \frac{\sqrt{2\varepsilon_0}}{\pi^2}\left(\frac{\varepsilon}{\varepsilon_0}\right)^\gamma \Theta(\varepsilon) & \text{if } \varepsilon < \varepsilon_0, \\ \frac{\sqrt{2\varepsilon}}{\pi^2} & \text{if } \varepsilon \geq \varepsilon_0 \end{cases} \Bigg| \implies \sqrt{2\varepsilon_0}a_{sc} = \frac{\pi\gamma}{2\gamma - 1}\Theta(\gamma)$$

- Highly symmetric SOCs strongly modify the infrared density of states...promotes bound state formation

Many-Body Ground State

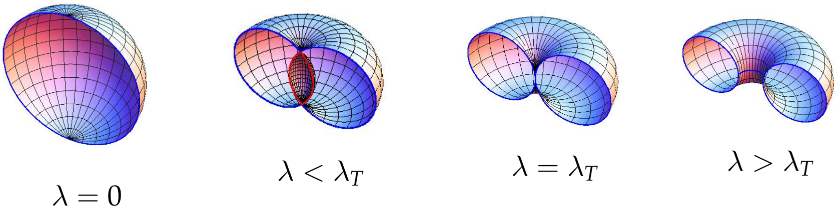


Finite Density of Non-Interacting Fermions with SOC

- Density $\rho \sim k_F^3$
- Chemical potential changes with λ...e.g. EP(Rashba)-SOC

$$\frac{\mu_{NI}(\lambda)}{E_F} \approx 1 - \left(\frac{\lambda}{k_F}\right)^2 \ (\lambda \ll k_F) \quad \text{and} \quad \frac{\mu_{\mathrm{NI}}(\lambda)}{\mathrm{E_F}} \approx \frac{\mathrm{k_F}}{\lambda}\ (\lambda \gg \mathrm{k_F})$$

- *Change in the topology* of the non-interacting Fermi surface with increasing gauge coupling λ



- The topology of the fermi surface changes at $\lambda = \lambda_T \approx k_F$
- For $\lambda > \lambda_T$ the occupied states are only of + helicity

Interacting Fermions in Rashba Gauge Fields

- For $\lambda = 0$, small negative ve a_s ($k_F|a_s| \ll 1$): BCS superfluid
- What happens if λ is increased at *fixed* a_s?

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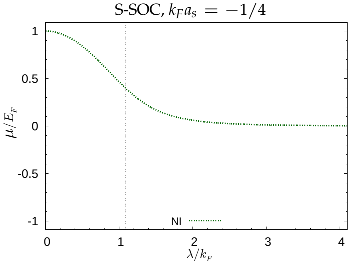
- For $\lambda \gtrsim \lambda_T$ the pair wavefunction is same as that of the two body wavefunction



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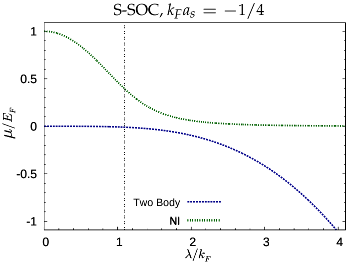

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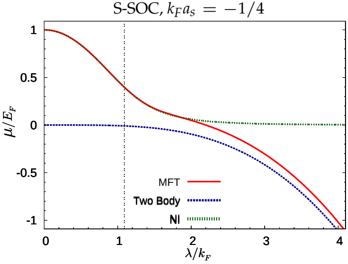

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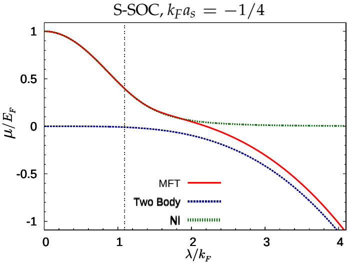

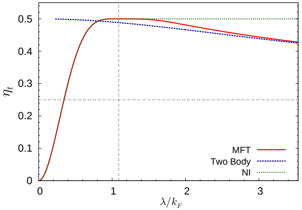

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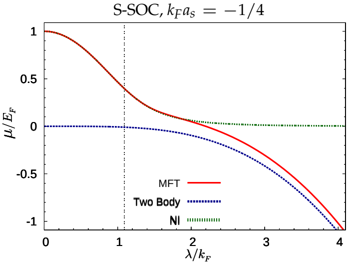
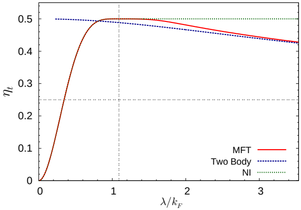
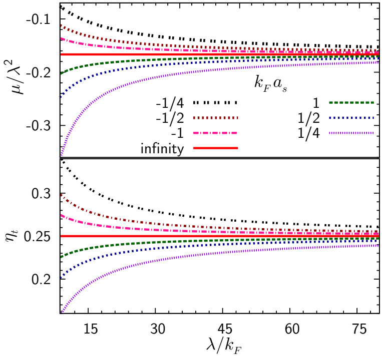




- Rashba gauge field (spin-orbit coupling) induces BCS-BEC crossover for a fixed attraction (a_s)! (Vyasanakere et al., arXiv:1104.5633)

BEC Induced by Rashba Gauge Fields

- Crossover to BEC occurs in the regime $\lambda \gtrsim \lambda_T$
- What is the nature of the BEC for $\lambda \gg \lambda_T$?



- The BEC for $\lambda \gg \lambda_T$ is a condensate of bosons whose property is *solely* determined by the gauge field (and not by scattering length a_s)
- These bosons are named rashbons
- Rashbon: the bound bosonic state of two fermions at *resonance scattering length* in the Rashba gauge field

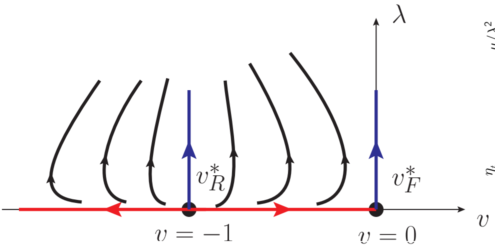
- The gauge field induces a crossover from a BCS like state (even for small negative a_s) to a Rashbon-BEC (RBEC) state

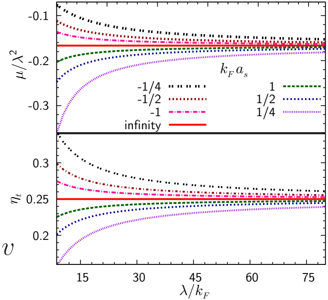
Whence Rashbons?

- For large λ, the dimensionless gap equation (all energies measured in units of λ^2)

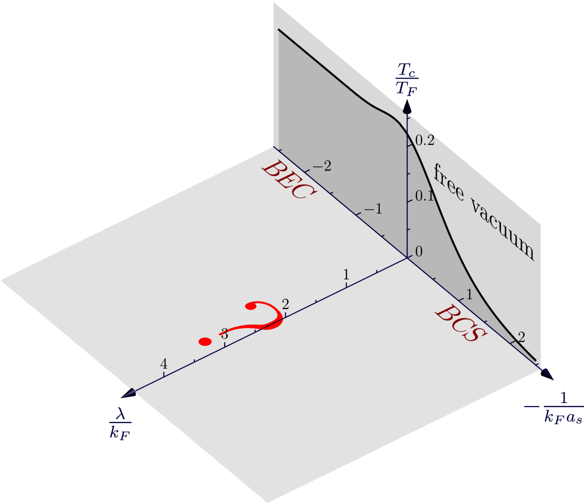
$$\frac{1}{4\pi\lambda a_s} = \frac{1}{2V}\sum_{\boldsymbol{k}\alpha}\left(\frac{1}{E - 2\varepsilon_{\boldsymbol{k}\alpha}} + \frac{1}{k^2}\right)$$

- As $\lambda \to \infty$, $\lambda a_s \to \infty$, equivalent to fixing λ and $a_s \to \infty$
- The binding energy/spin structure becomes *independent* of a_s and depends only on the Rashba gauge field!





Ground State: Summary



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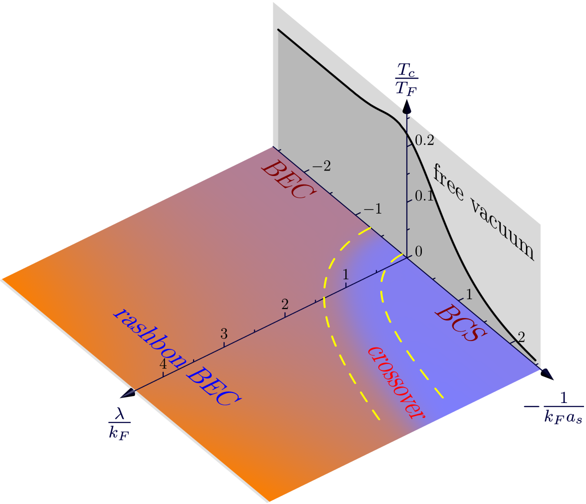
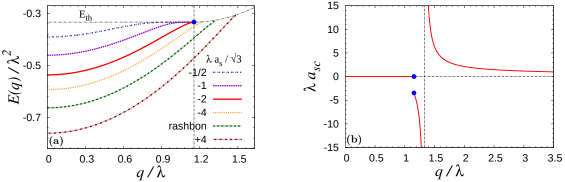


Next Question: Transition temperature T_c?

Excitations: 2-Body

2-Body Excited States

- Bound state dispersion (q : centre of mass momentum)



- For $q \ll \lambda$, $E(q) \approx -E_R + \sum_i \frac{q_i^2}{2m_i}$

2-Body Excited States

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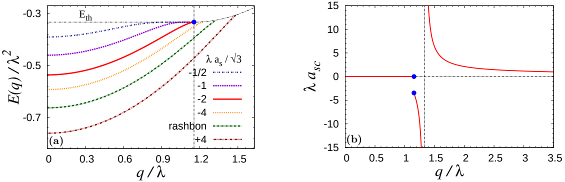


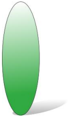
- For $q \ll \lambda$, $E(q) \approx -E_R + \sum_i \frac{q_i^2}{2m_i}$
- There is *no bound state at a finite center-of-mass momentum* $q \sim \lambda$!
- For large q, a *positive* scattering length is necessary to obtain a bound sate! Rashba gauge field *inhibits* formation of bound state at centre of mass momenta $q \gtrsim \lambda$!
- Physics: Lack of Galilean invariance- in the Galilean boosted (by $\boldsymbol{q}$) frame, the kinetic energy operator (for S-SOC)

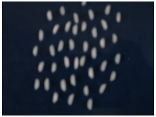
$$H_{GF}^{boosted} = \frac{(\boldsymbol{P} - \boldsymbol{q})^2}{2} - \lambda \boldsymbol{P} \cdot \boldsymbol{\tau} + \underbrace{\lambda \boldsymbol{q} \cdot \boldsymbol{\tau}}_{\text{Zeeman field!}}$$

...this additional Zeeman field inhibits bound state formation at finite COM!

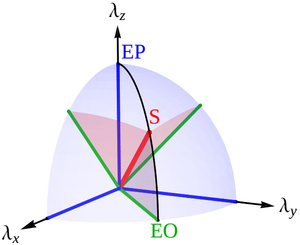
Properties of Rashbons

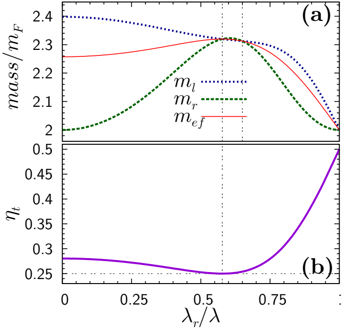
- Rashbons are anisotropic particles with a nematic spin structure, anisotropic mass, e. g., for EO gauge field





- For $\boldsymbol{\lambda} = (\lambda_l, \lambda_l, \lambda_r)$

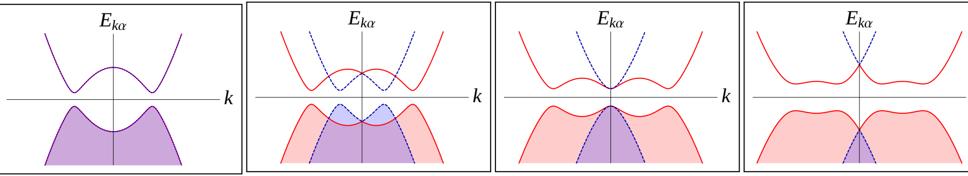




- T_c of RBEC is determined by properties of rashbons

Excitations: Many Body

Bogoliubov Quasipartilces

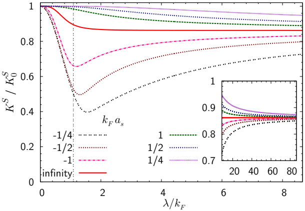


$\lambda = 0$ $\quad 0 < \lambda < \lambda_B$ $\quad \lambda = \lambda_B$ $\quad \lambda > \lambda_B$

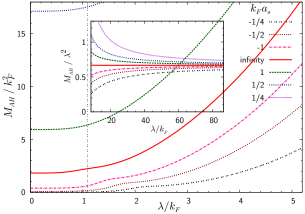
- Bogoliubov qasiparticle dispersion also mimick the topology transition of the bare Fermi surface
- For $\lambda \gtrsim \lambda_B$, low energy quasiparticle excitations are only of one helicity
- λ_B depends on scattering length; $\lambda_B \approx k_F$ for small negative scattering length

Collective Excitations

- Gaussian fluctuation theory (c.f. Engelbrecht et al. (1996))
- Two modes: Gapless phase mode, gapped amplitude mode (results for any $\boldsymbol{\lambda}$)



Phase stiffness ($K_s^0 = \frac{\rho}{4m}$)



Anderson-Higgs Mass

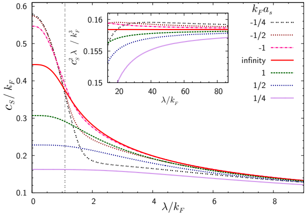
- Non-monotonic dependence of K^s on λ
- Lack of Galilean invariance (Zhou and Zhang, arXiv:1110.3565)
- New result: *Emergent Galilean invariance*

$$K^s(\lambda \to \infty) = \frac{\rho}{2m^R} \qquad \left(\frac{\rho}{2}\frac{1}{m_i^R}\delta_{ij}\right)$$

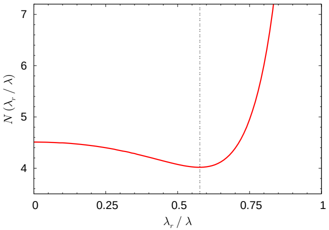
...consistent with Leggett's theorem

- ...*rashbons must be interacting*!

Rashbon-Rashbon Interaction



Sound speed



- For large λ, R-BEC is described by a Bogoliubov theory of anisotropic bosons interacting via a contact potential
- Effective rashbon-rashbon scattering length $\frac{N(\hat{\boldsymbol{\lambda}})}{\lambda}$...for S-SOC

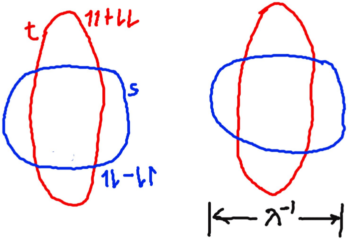
$$a_R = \frac{3\sqrt{3}(4+\sqrt{2})}{7}\frac{1}{\lambda}$$

independent of the scattering length a_s of fermions!

- Remarkable state...the interaction between emergent bosons is determined by a parameter λ that enters the *kinetic energy* of the constituent fermions!

Physics of Rashbon-Rashbon Interactions

- Recall: Size of rashbons (extent of bound state wavefunction at large λ) is λ^{-1}

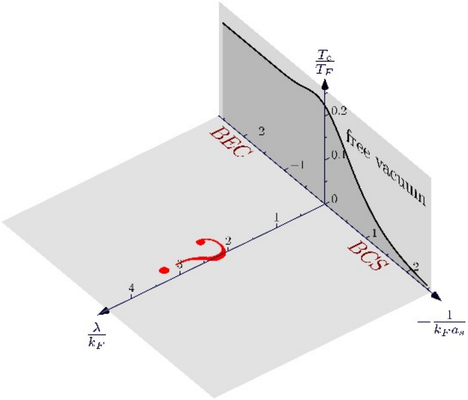


- Crude argument: Pauli exclusion between like fermions keeps the rashbons apart over a distance of λ^{-1}

Transition Temperature and Phase Diagram

Phase Diagram of Interacting Rashba Fermions

- Beyond Gaussian fluctuation effects are *crucial*! (coming soon on arXiv)



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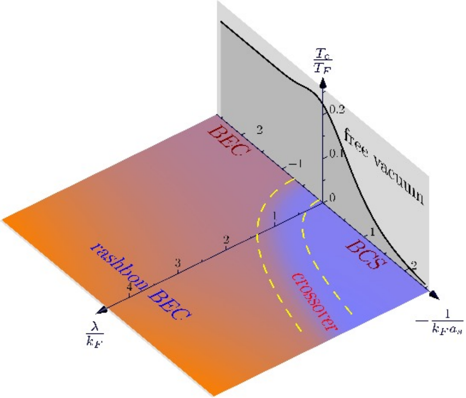

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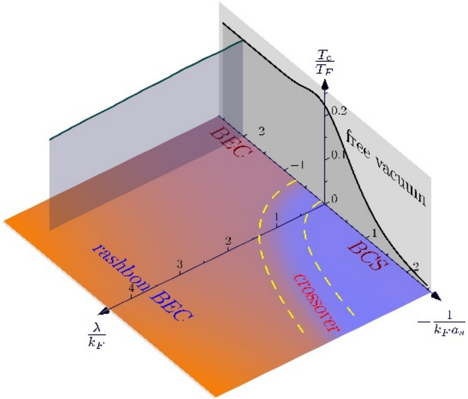

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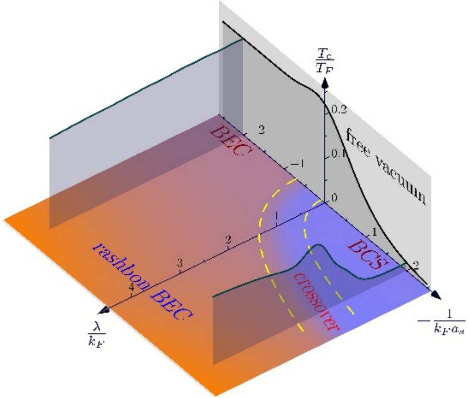

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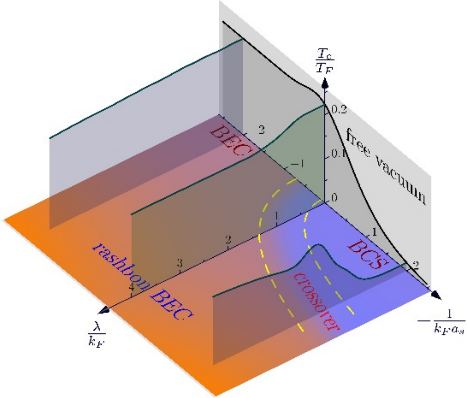

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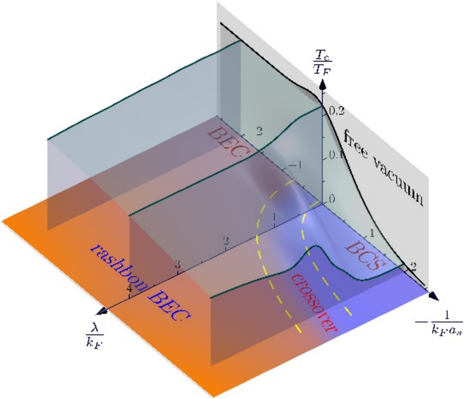

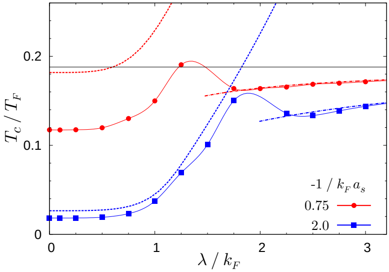

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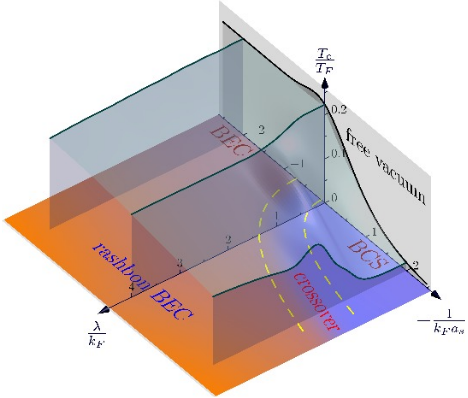
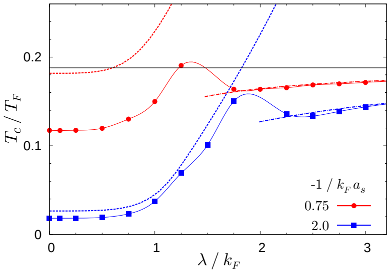
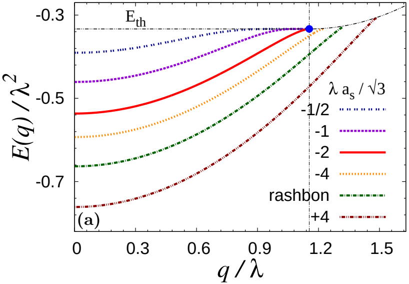


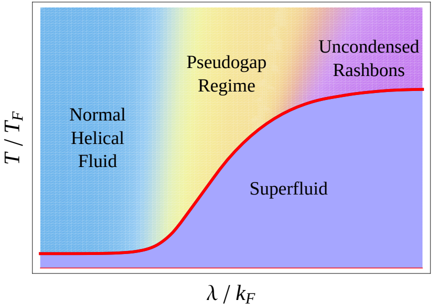


- The Rashba gauge field *enhances the exponentially small transition temperature by orders of magnitude to the order of the Fermi temperature* even for weak attraction!
- For $\lambda \gg k_F, 1/a_s$, physics is independent of a_s (need only $a_s \neq 0$)...transition temperature determined by rashbon mass m_R

Pseudogap Physics

- Finite temperature properties determined by rashbon dispersion





- In the regime $k_F \approx \lambda \approx T$, the normal state will be a "dynamical mixture" of rashbons and helical fermions...strong pseudogap effects over a large regime (Review of pseudogap physics: Randeria, INT Symposium (2011))
- Need to go beyond Gaussian fluctuations in the pseudogap regime

Kondo Effect in a Synthetic Gauge Field

(Coming soon on the arXiv)

Adhip Agarwala

Kondo Effect: Background

- Anderson impurity model - impurity at the origin

$$\mathcal{H} = \int \mathrm{d}^d \boldsymbol{r}\, c_\sigma^\dagger(\boldsymbol{r}) \left(-\frac{\boldsymbol{\nabla}^2}{2} - E_F \right) c_\sigma(\boldsymbol{r}) + \varepsilon_d d_\sigma^\dagger d_\sigma + U d_\uparrow^\dagger d_\downarrow^\dagger d_\downarrow d_\uparrow$$
$$+ V \left(d_\sigma^\dagger c_\sigma(\boldsymbol{0})_\sigma + \text{h. c.} \right)$$

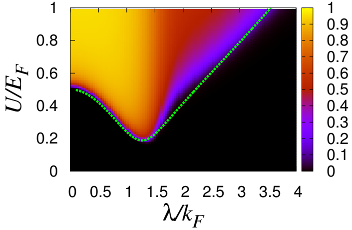
- For appropriate conditions, impurity has no-charge fluctuations and becomes a “local moment” with an AF exachange with the fermi gas
- Ground state: Kondo singlet with an energy scale $T_K \sim E_F e^{-1/JE_F}$, $J \sim V^2/U$

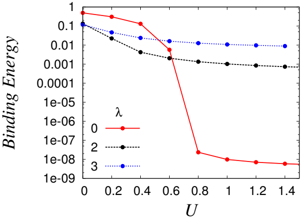
Kondo Effect with Rashbha Spin Orbit Coupling

- Anderson impurity model - impurity at the origin

$$\mathcal{H} = \int \mathrm{d}^d \boldsymbol{r}\, c^\dagger_\sigma(\boldsymbol{r}) \left(H_{\sigma\sigma'}(-i\boldsymbol{\nabla}, \boldsymbol{\lambda}) - E_F \delta_{\sigma,\sigma'}\right) c_{\sigma'}(\boldsymbol{r}) + \varepsilon_d d^\dagger_\sigma d_\sigma + U d^\dagger_\uparrow d^\dagger_\downarrow d_\downarrow d_\uparrow$$

$$+ V \left(d^\dagger_\sigma c_\sigma(\mathbf{0})_\sigma + \text{h. c.}\right)$$





- For $\lambda \gtrsim k_F$, a new kind of Kondo state with 2/3 of the d-moment!
- Enhancement of Kondo scale by spin-orbit coupling!

Fermions in Synthetic Dimensions

Baryon "Squishing"

(1503.02301)

Sudeep K. Ghosh

Umesh Yadav

$SU(M)$ Symmetric Interactions

- Atoms with M hyperfine states and $SU(M)$ symmetric interactions $[K, M = 8], [Yb, M = 6], [Dy, M = 22]$ etc. (eg. Takhashi group, Inguscio group, Bloch group)
- $SU(M)$ (fermions) atoms in a 1D optical lattice with attraction U

$$H = -t \sum_{j,\gamma} \left(C^\dagger_{(j+1)\gamma} C_{j\gamma} + \text{h.c.} \right) - \frac{U}{2} \sum_{j,\gamma,\gamma'} C^\dagger_{j\gamma} C^\dagger_{j\gamma'} C_{j\gamma'} C_{j\gamma}$$

- M-fermion ground state is a $SU(M)$-singlet – baryon (e.g. Hofsetter group (2013))

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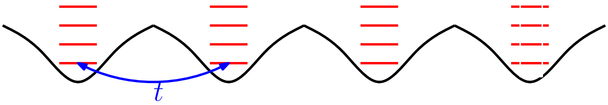

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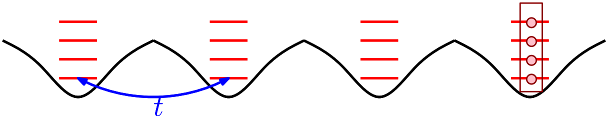

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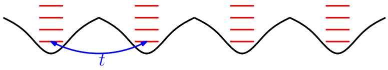

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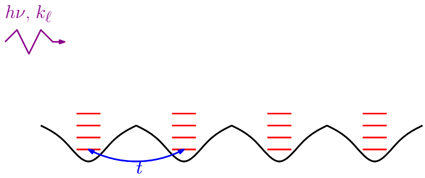

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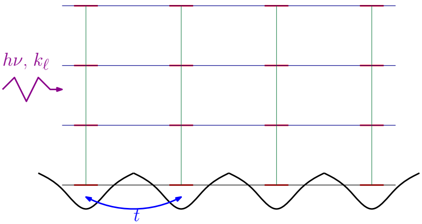

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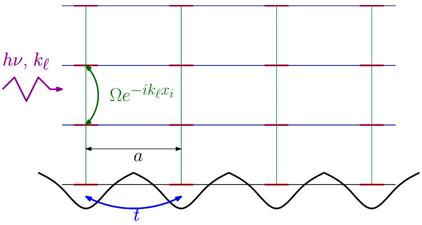

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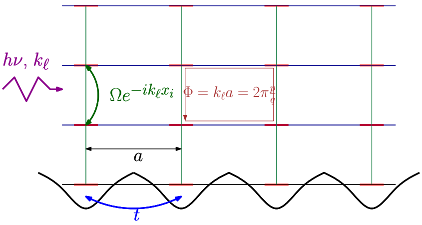

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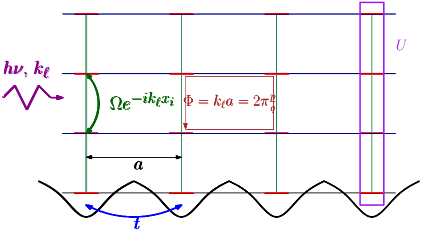

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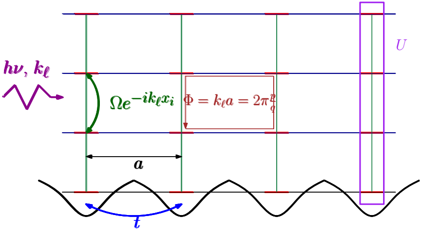

- Hofstadter model with a $2\pi\frac{p}{q}$ flux

$$H = -t\sum_{j,\gamma}\left(C^\dagger_{j\gamma}C_{j\gamma} + \text{h.c.}\right) + \sum_{j\gamma}\left(\Omega_\gamma e^{ik_\ell x_j} C^\dagger_{j(\gamma+1)}C_{j\gamma} + \text{h.c.}\right)$$
$$- \frac{U}{2}\sum_{j,\gamma,\gamma'} C^\dagger_{j\gamma}C^\dagger_{j\gamma'}C_{j\gamma'}C_{j\gamma}$$

- Interactions are nonlocal along synthetic dimension, and local along real dimension
- Experimentally realized! (Fallani et al., Spielman et al. (this meeting))

Our Perspective

- By a gauge transformation, problem can be recast as

$$H = -t\sum_j \mathbb{B}^\dagger_{j+1}\mathbb{U}^\dagger_{j+1}\mathbb{U}_j\mathbb{B}_j + \sum_j \mathbb{B}^\dagger_j \mathbf{\Omega}\mathbb{B}_j - H_U^{SU(M)}$$

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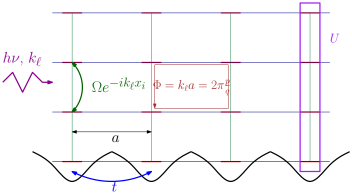

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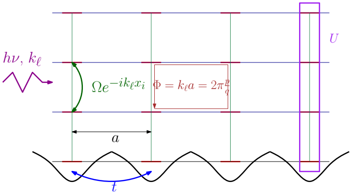
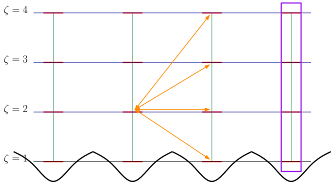




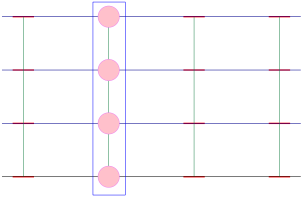
Our Perspective

What we learn

- Gauge field (FOC) mitigates the effect of the (usually) baryon-breaking Zeeman field $\mathbf{\Omega}$!
- Possible to produce *non-local interactions in real space*
- Outcome
  - Few Body: **Suqished** baryon!
  - Many Body: Superfluid by application of a magnetic field!

Few Body Physics

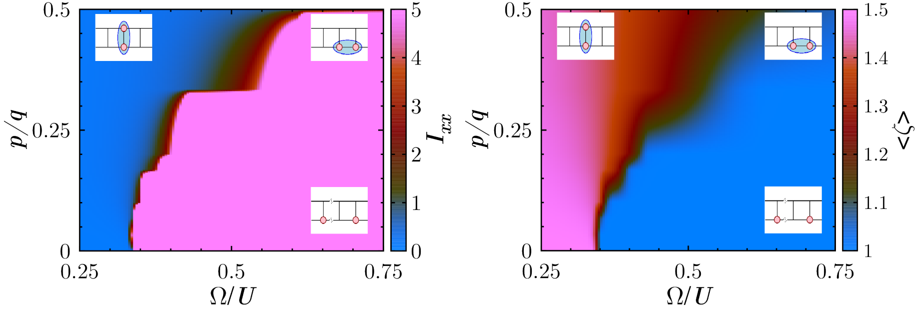
- For $\Omega = 0$, ground state of M fermions is a “baryon” (Hofsetter group(2013))



- Exact diagonalization studies (with finite size scaling)
- Characterize states by
  - $\langle I_{xx} \rangle$ – Mean square size in the x direction
  - $\langle \zeta \rangle$ – Mean position along synthetic dimension
- Question: Do we see baryon squishing ($\Omega \neq 0, p/q \neq 0$)?

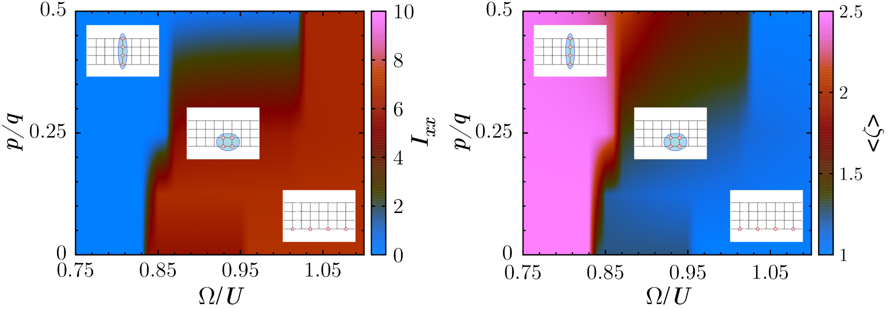
Bound States with $\Omega \neq 0, p/q \neq 0$

$M = 2$



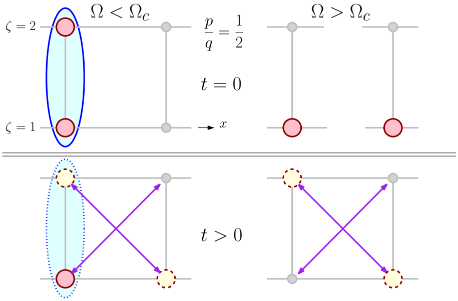
New type of "non-local" (in real space) baryon is stabilized!..."squished" baryon

$M = 4$



Numerical demonstration of “baryon squishing”

Induced Nonlocal Interactions

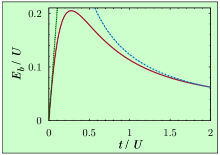


$$\Omega_c = \frac{U}{2}$$

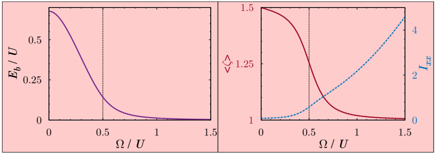
- Energy gain when two particles are on neighbouring $\zeta = 1$ state – $\frac{t^2 U}{\Omega^2}$
- Interactions depend on the details of Ω_γ – for a given M and Ω_γ there are special p/q that gives best nonlocal interactions

More on Squished Baryons

- Can obtain analytic results in appropriate limits



- Pair braking effects of Zeeman fields mitigated!



- For a give M there are special fluxes that give rise to strong squishing
- Rich many body phase diagram being constructed

Realizing Novel Hamiltonians
“Gauge Fields in Momentum Space”

Gauge Fields from Gauge Fields!

- Hamiltonian : $\mathcal{H} = \frac{p^2}{2} - \boldsymbol{p}_\lambda \cdot \boldsymbol{\tau} - \frac{{\omega_0}^2}{2}\frac{\partial^2}{\partial \boldsymbol{p}^2}, \qquad \boldsymbol{r} = i\frac{\partial}{\partial \boldsymbol{p}}$
- For $\lambda^2 \gg \omega_0$, spin degrees of freedom are "fast" – helicity is a good quantum number to a very good approximation...motivates the ansatz

$$|\psi\rangle = \int \mathrm{d}\boldsymbol{p}\, \psi(\boldsymbol{p})\, |\boldsymbol{p}\rangle \otimes |\chi_+(\boldsymbol{p})\rangle$$

- Wave function $\psi(\boldsymbol{p})$ satisfies $\mathcal{H}_{\text{eff}}\psi(\boldsymbol{p}) = \varepsilon\psi(\boldsymbol{p})$

$$\mathcal{H}_{\text{eff}} = \frac{{\omega_0}^2}{2}\left(i\frac{\partial}{\partial \boldsymbol{p}} - \boldsymbol{A}\right)^2 + \varepsilon_+(\boldsymbol{p}) + V_{BO}(\boldsymbol{p}), \qquad \boldsymbol{A} = -i\langle\chi_+(\boldsymbol{p})|\frac{\partial\chi_+(\boldsymbol{p})}{\partial \boldsymbol{p}}\rangle$$

$$V_{BO}(\boldsymbol{p}) = \frac{{\omega_0}^2}{2}\left(\langle\frac{\partial\chi_+(\boldsymbol{p})}{\partial p_i}|\frac{\partial\chi_+(\boldsymbol{p})}{\partial p_i}\rangle - \langle\frac{\partial\chi_+(\boldsymbol{p})}{\partial p_i}|\chi_+(\boldsymbol{p})\rangle\langle\chi_+(\boldsymbol{p})|\frac{\partial\chi_+(\boldsymbol{p})}{\partial p_i}\rangle\right)$$

Gauge field begets gauge field!

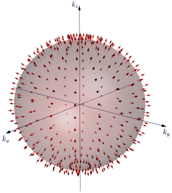
Realization of New Hamiltonians with Non-Abeilian Gauge Fields and a Potential

- Spherical gauge field with harmonic trapping potential

$$\mathcal{H} = \frac{p^2}{2} - \frac{\lambda}{\sqrt{3}}\boldsymbol{p}\cdot\boldsymbol{\tau} + \frac{\omega_0^2}{2}r^2, \qquad \boldsymbol{r} = i\frac{\partial}{\partial \boldsymbol{p}}$$

- Adiabatic hamiltonian including Pancharatnam-Berry phase effects

$$\mathcal{H}_{eff} = -\frac{\omega_0^2}{2}\left(\frac{1}{p^2}\frac{\partial}{\partial p}p^2\frac{\partial}{\partial p}\right) + \frac{\omega_0^2}{2p^2}\left[-\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \left(Q\cot\theta + \frac{i}{\sin\theta}\frac{\partial}{\partial\phi}\right)^2\right] + \frac{\omega_0^2}{4p^2} + \left(\frac{p^2}{2} - \frac{\lambda}{\sqrt{3}}p\right)$$



- *Realization of a monopole in momentum space*
- Opens possibility to generate interesting Hamiltonians by designing additional potential $V(\boldsymbol{r})$ (also for bosons!)

Summary Highlights

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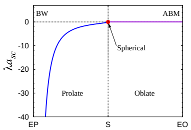
2-body



- Bound state for any attraction

(JPV/VBS, arXiv:1101.0411)

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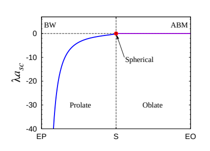

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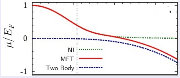

- Rashba driven BCS-BEC
- Rashbon BEC
- Emergent Galilean Invariance
- Rashbon-rashbon interactions

(JPV et al., arXiv:1104.5633, 1108.4872,1201.5332)

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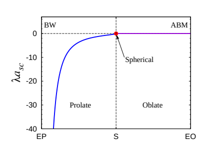

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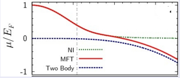

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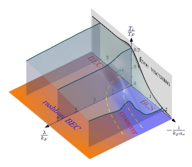

- Phase diagram, High T_c
- Pseudogap regime

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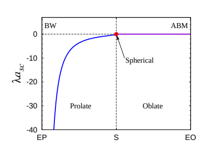

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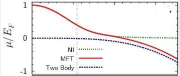

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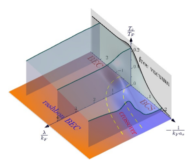

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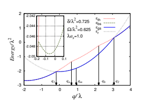

- Finite momentum pairing!
- SO coupled Fermi liquids

(VBS, arXiv:1211.1831)

Summary Highlights

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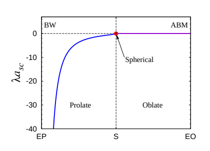

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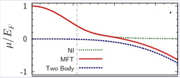

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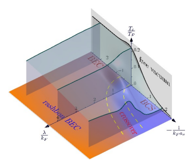

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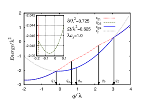

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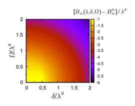
Feshbach resonance



- Shift of resonance
- CM dependent interaction

(VBS, arXiv:1212.2858)

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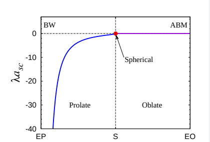

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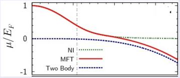

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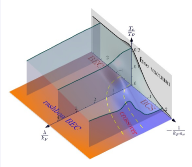

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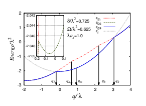

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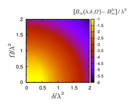
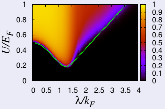


- Shift of resonance
- CM dependent interaction

(VBS, arXiv:1212.2858)

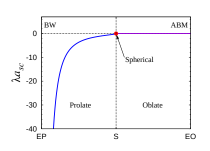
Kondo effect



(AA/VBS: Coming soon)

Summary Highlights

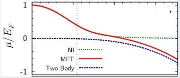
2-body



- Bound state for any attraction

(JPV/VBS, arXiv:1101.0411)

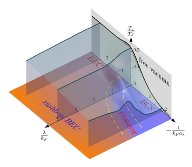
Many body



- Rashba driven BCS-BEC
- Rashbon BEC
- Emergent Galilean Invariance
- Rashbon-rashbon interactions

(JPV et al., arXiv:1104.5633, 1108.4872,1201.5332)

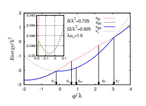
Phase diagram



- Phase diagram, High T_c
- Pseudogap regime

(JPV/VBS: Coming soon)

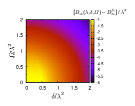
FF pairing



- Finite momentum pairing!
- SO coupled Fermi liquids

(VBS, arXiv:1211.1831)

Feshbach resonance



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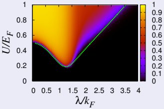
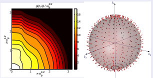


(AA/VBS: Coming soon)

Gauge fields in p-space



- Novel Hamiltonians

(SKG et al., arXiv:1109.5279)

Summary Highlights

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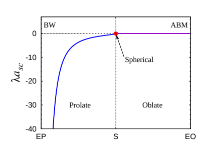

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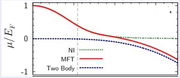

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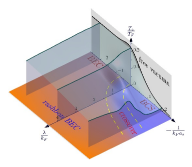

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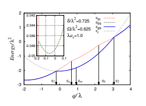

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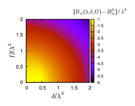

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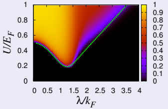
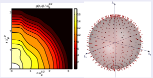


(AA/VBS: Coming soon)

Gauge fields in *p*-space



- Novel Hamiltonians

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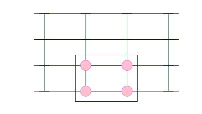
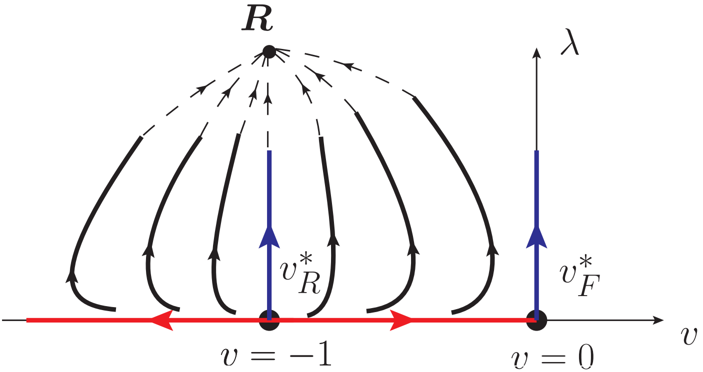
Synthetic dimensions



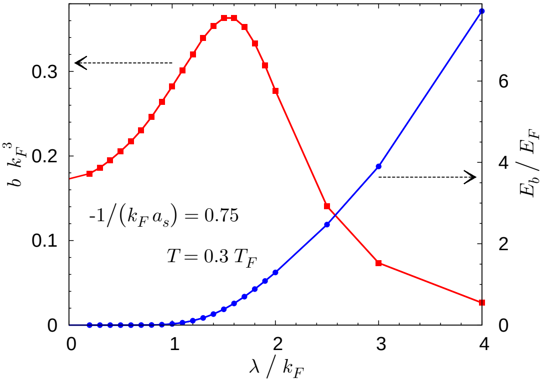
- New kind of "nonlocal" baryons

(SKG/UY/VBS, arXiv:1503.02301)

Overall RG Picture



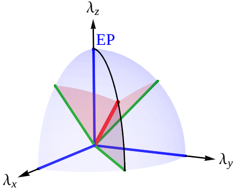
Non-Gaussian Effects

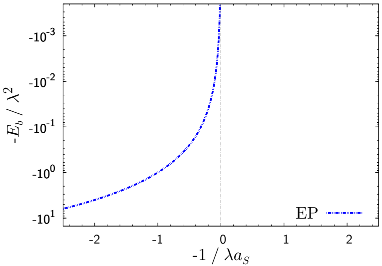


Extreme Prolate: $\lambda_x = \lambda_y = 0, \lambda_z = \lambda$

(Vyasanakere and VBS, arXiv:1101.0411)

- Critical scattering length a_{sc}: $\frac{1}{a_{sc}} = \infty$ – just as in free vacuum
- Binding energy $E_b = \frac{1}{a_s^2}$, as in free vacuum





- Bound state wavefunction

$$|\Psi_b\rangle \propto \psi_s(\boldsymbol{r})|\uparrow\downarrow - \downarrow\uparrow\rangle + \psi_a(\boldsymbol{r})|\uparrow\downarrow + \downarrow\uparrow\rangle$$

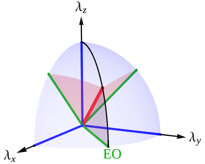
(ψ_s–symmetric, ψ_a–antisymmetric) with *biaxial spin nematic* structure (similar to BW state of ^{3}He)

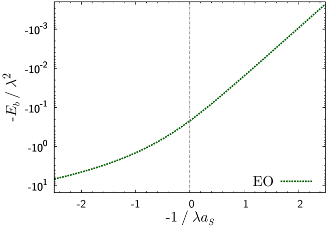
Extreme Oblate: $\lambda_x = \lambda_y = \frac{\lambda}{\sqrt{2}}, \lambda_z = 0$

(Vyasanakere and VBS, arXiv:1101.0411)

- Critical scattering length a_{sc} *vanishes*! $a_{sc} = 0^-$...bound state for *any* scattering length!
- Binding energy

$a_s > 0$	$1/a_s = 0$	$a_s < 0$
$\frac{1}{(\lambda a_s)^2} + \frac{\log 2}{\sqrt{2}\lambda a_s}$	$0.22\,\lambda^2$	$\frac{4\lambda^2}{e^2} e^{\frac{-2\sqrt{2}}{\lambda \lvert a_s \rvert}}$





- Bound state wavefunction

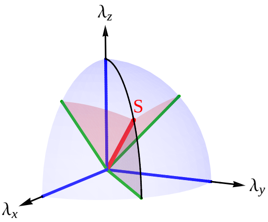
$$|\Psi_b\rangle \propto \psi_s(\boldsymbol{r})|\uparrow\downarrow - \downarrow\uparrow\rangle + \psi_a(\boldsymbol{r})|\uparrow\uparrow\rangle + \psi_a^*(\boldsymbol{r})|\downarrow\downarrow\rangle$$

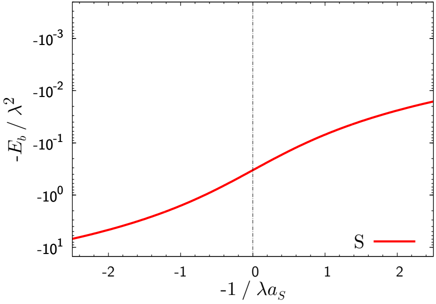
...uniaxial spin nematic (ABM state of ^{3}He!)

Spherical SOC: $\lambda_x = \lambda_y = \lambda_z = \frac{\lambda}{\sqrt{3}}$

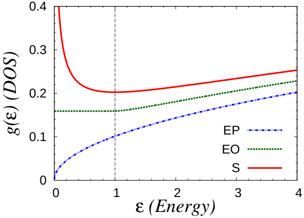
(Vyasanakere and VBS, arXiv:1101.0411)

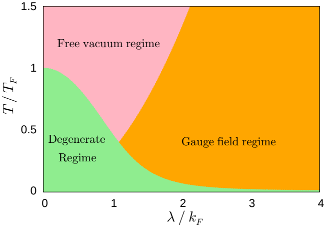
- Bound state for *any* scattering length: $a_{sc} = 0^-$
- Binding energy $E_b = \frac{1}{4}\left(\frac{1}{a_s} + \sqrt{\frac{1}{a_s^2} + \frac{4\lambda^2}{3}}\right)^2$
- ..."algebraic" in the BCS side: $E_b = \left(\frac{\lambda a_s}{3}\right)^2$





Temperature Regimes of the Non-interacting System





- Three temperature regimes
- Degenerate regime: Scale set by density
- Gauge field regime: non-degenerate, scale set by λ
- Free vacuum regime: Above both these scales
- Chemical potential in non-degenerate regime (S-SOC)

$$\mu_{NI}(T) \simeq -T \log \left(\frac{\sqrt{T}(3T + \lambda^2)}{3\sqrt{2}\pi^{3/2}\rho} \right)$$