TU WIEN

Quantum fields <-> Correlation functions

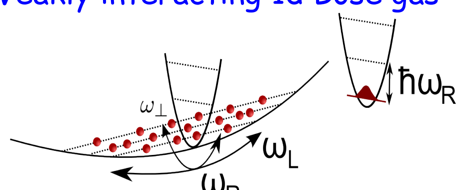
VCQ Vienna Center for Quantum Science and Technology

On the Green's functions of quantized fields
J. Schwinger PNAS (1951)

- ✧ Solving a quantum many-body problem is equivalent to knowing **all** its correlation functions.
- ✧ In practice, an observer can only measure a **finite** number of correlations describing the propagation and scattering of excitations.
- ✧ To solve a problem one need to **find degrees of freedom** where only few (low order) correlation functions are relevant.
- ✧ If one finds the degrees of freedom (basis) where the **correlation functions factorize**, this is equivalent to **diagonalization of the many body Hamiltonian.**

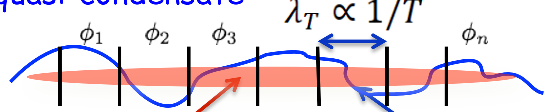
System under investigation

Weakly interacting 1d Bose gas



All energies $\mu,\ k_B T << \hbar\omega_\perp$

quasi-condensate



uniform density fluctuating phase

$$\hat{\psi}(x) = e^{i\hat{\Phi}_1(x)}\sqrt{\rho + \hat{n}_1(x)}$$

thermally populated

Lieb-Liniger model

- Exactly solvable integrable theory

low energy effective field theory:

Luttinger-liquid

$$H = \frac{c}{2}\int dx \left[\frac{K}{\pi}(\nabla\hat{\varphi})^2 + \frac{\pi}{K}\hat{n}^2\right]$$

- excitations are soundwaves (phonons)
- linear dispersion relation

coupled 1d systems:

Sine-Gordon model

$$\hat{H}_{SG} = \frac{\hbar c}{2}\int_{-\mathcal{L}/2}^{\mathcal{L}/2} dz \left[\frac{\pi}{K}\hat{n}^2(z) + \frac{K}{\pi}\left(\frac{\partial}{\partial z}\hat{\theta}(z)\right)^2\right] - 2n_{1D}J\int_{-\mathcal{L}/2}^{\mathcal{L}/2} dz \cos[\sqrt{2}\,\hat{\theta}(z)]$$

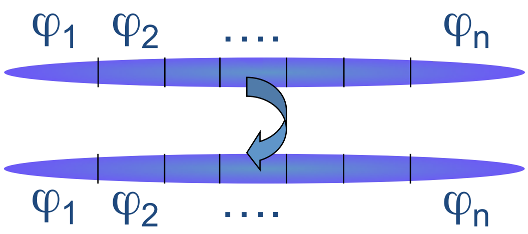
Model for interacting many body systems which can be described by a field theory with long lived excitations.

The longitudinal phase fluctuations are key for our experiments

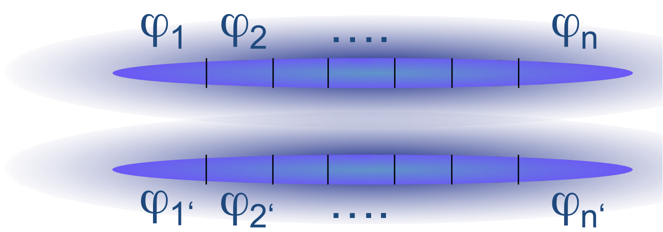
interference of phase fluctuating 1D condensates

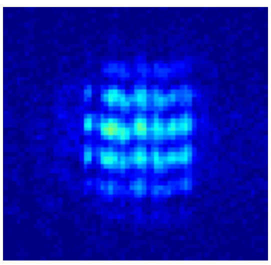
Study the dynamics of excitations on a quantum field

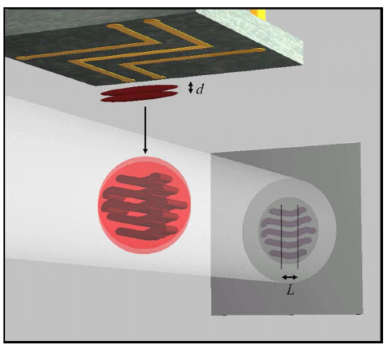
create a copy by splitting
quantum connected

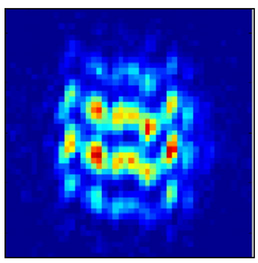


create two independet samples
classically separated









Outline

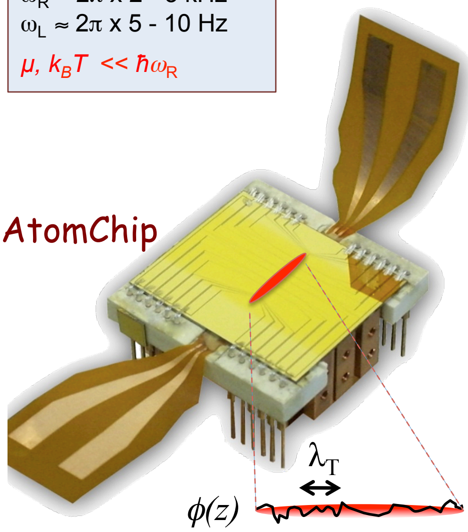
Correlation functions
- fields <-> phase <-> excitations

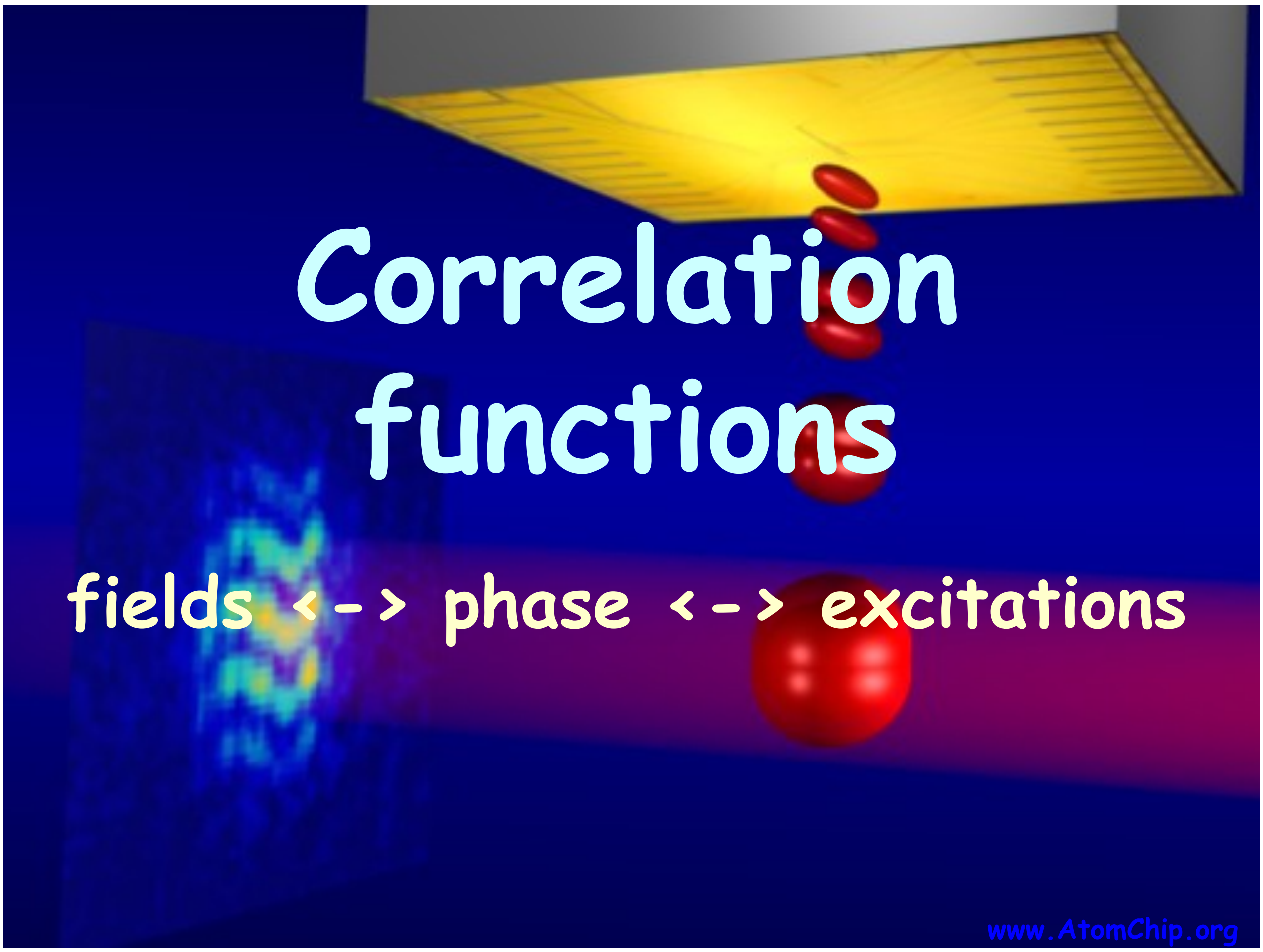
Characterizing the pre-thermalized state
- Generalized Gibbs ensemble

High order correlation functions
- Quantifying factorization
- Sine-Gordon model
- Quench to a free system

Outlook
- entanglement and spin squeezing
- quantum state tomography
- relaxation in SG model

1000-10000 Rb atoms
T = 10-100 nK
$\omega_R \approx 2\pi$ x 2 - 3 kHz
$\omega_L \approx 2\pi$ x 5 - 10 Hz
$\mu, k_B T << \hbar\omega_R$





Correlation functions
fields <-> phase

experiments in a trap
-> non translation invariant correlation functions

$$C(z_1, z_2) = \frac{\langle \Psi_1(z_1)\Psi_2^\dagger(z_1)\Psi_1^\dagger(z_2)\Psi_2(z_2)\rangle}{\langle |\Psi_1(z_1)|^2\rangle\langle |\Psi_2(z_2)|^2\rangle}$$

with
$$\Psi(z) = e^{i\theta(z)}\sqrt{\rho_0(z) + \delta\hat{n}(z)}$$
$$\varphi(z) = \theta_1(z) - \theta_2(z)$$

neglecting $\delta\hat{n}(z)$ ⟹ $C(z_1, z_2) \approx \langle \exp[i\varphi(z_1) - i\varphi(z_2)]\rangle$

4th order:
$$C(z_1, z_2, z_3, z_4) = \frac{\langle \Psi_1(z_1)\Psi_2^\dagger(z_1)\Psi_1^\dagger(z_2)\Psi_2(z_2)\Psi_1(z_3)\Psi_2^\dagger(z_3)\Psi_1^\dagger(z_4)\Psi_2(z_4)\rangle}{\langle |\Psi_1(z_1)|^2\rangle\langle |\Psi_1(z_2)|^2\rangle\langle |\Psi_2(z_3)|^2\rangle\langle |\Psi_2(z_4)|^2\rangle}$$
$$C(z_1, z_2, z_3, z_4) \approx \langle \exp[i\varphi(z_1) - i\varphi(z_2) + i\varphi(z_3) - i\varphi(z_4)]\rangle$$

Correlation functions
excitations <-> phase

in experiment we measure the phase $\varphi(z)$ directly
-> look at phase correlators

$$C^{(2)}(z_1, z_2) = \langle [\varphi(z_1) - \varphi(z_2)]^2\rangle = \langle [\Delta\varphi(z_1,z_2)]^2\rangle$$

with $\Delta\varphi(z_1, z_2) = \varphi(z_1) - \varphi(z_2)$ Note: $\Delta\varphi$ is NOT restricted to 2π

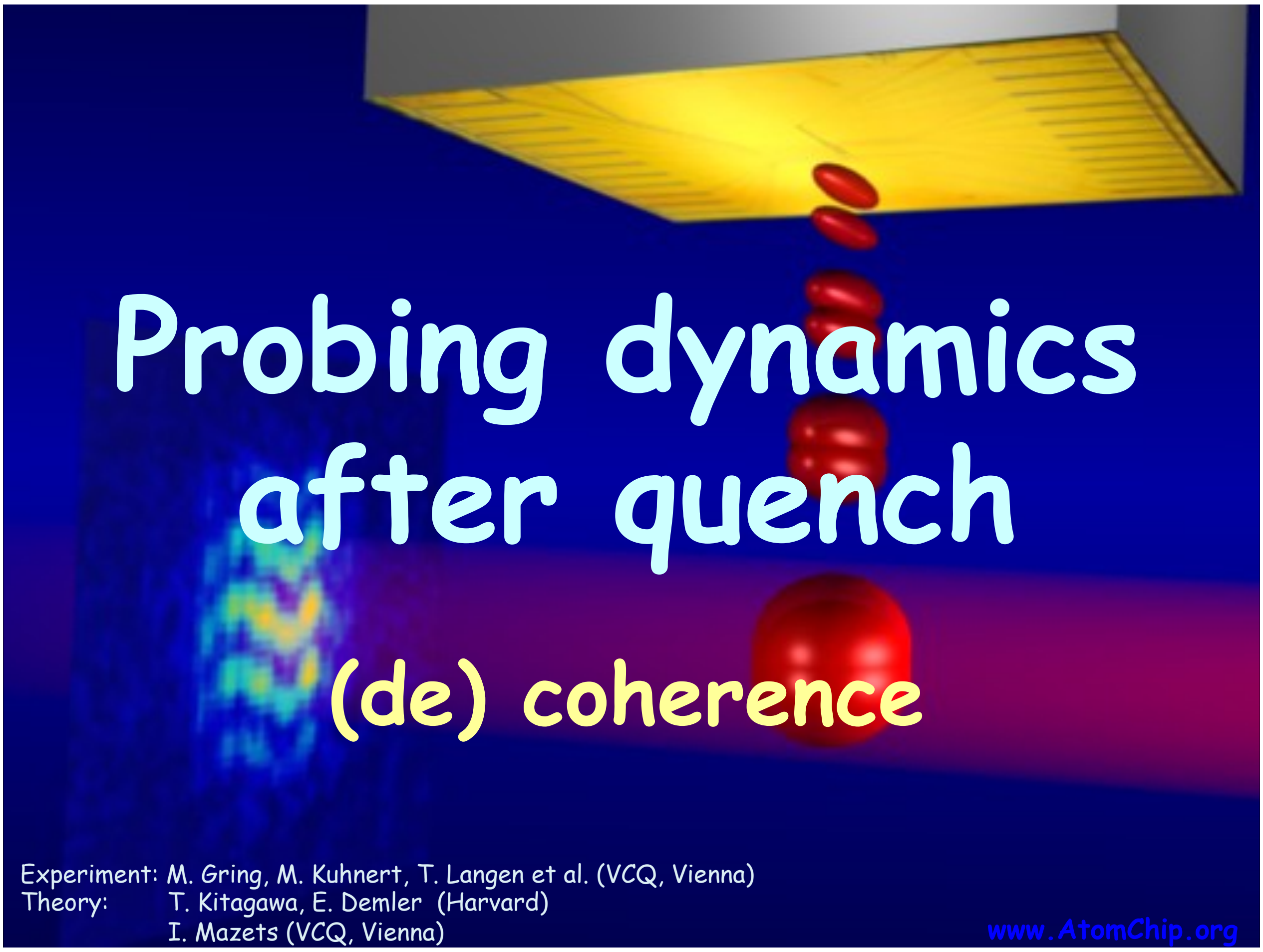
using
$$\varphi(z) = \frac{1}{\sqrt{L}}\sum_{k\neq 0}\left[(-i)\sqrt{\frac{\pi}{|k|K}}(b_k^\dagger - b_{-k})e^{ikz}\right]$$

$$\Longrightarrow \langle [\varphi(z_1) - \varphi(z_2)]^2\rangle = \sum_{k_1,k_2}\frac{\pi}{K\sqrt{|k_1k_2|}}b_{k_1}^\dagger b_{-k_2}e^{ik_1z_1+ik_2z_2} + \ldots$$

-> phase correlators are related to the **quasi particles**

4th order
$$C^{(4)}(z_1, z_2, z_3, z_4) = \langle [\varphi(z_1) - \varphi(z_2)]^2 [\varphi(z_3) - \varphi(z_4)]^2\rangle$$
$$\propto b_{k_1}^\dagger b_{k_2}^\dagger b_{-k_3} b_{-k_4} + \ldots$$

-> quasi particle scattering



TU WIEN

Relaxation in a nearly integrable quantum system

VCQ Vienna Center for Quantum Science and Technology

Quench:

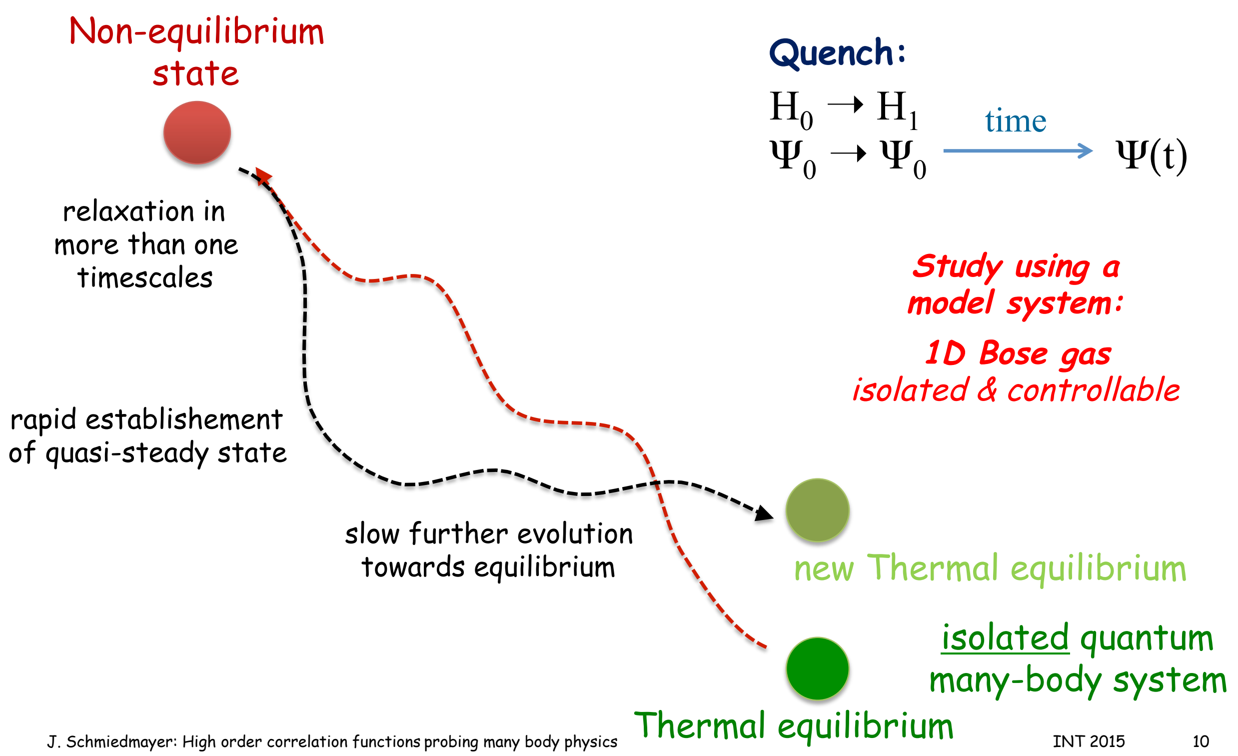
$H_0 \rightarrow H_1$

$\Psi_0 \rightarrow \Psi_0 \xrightarrow{\text{time}} \Psi(t)$

Study using a model system:

1D Bose gas

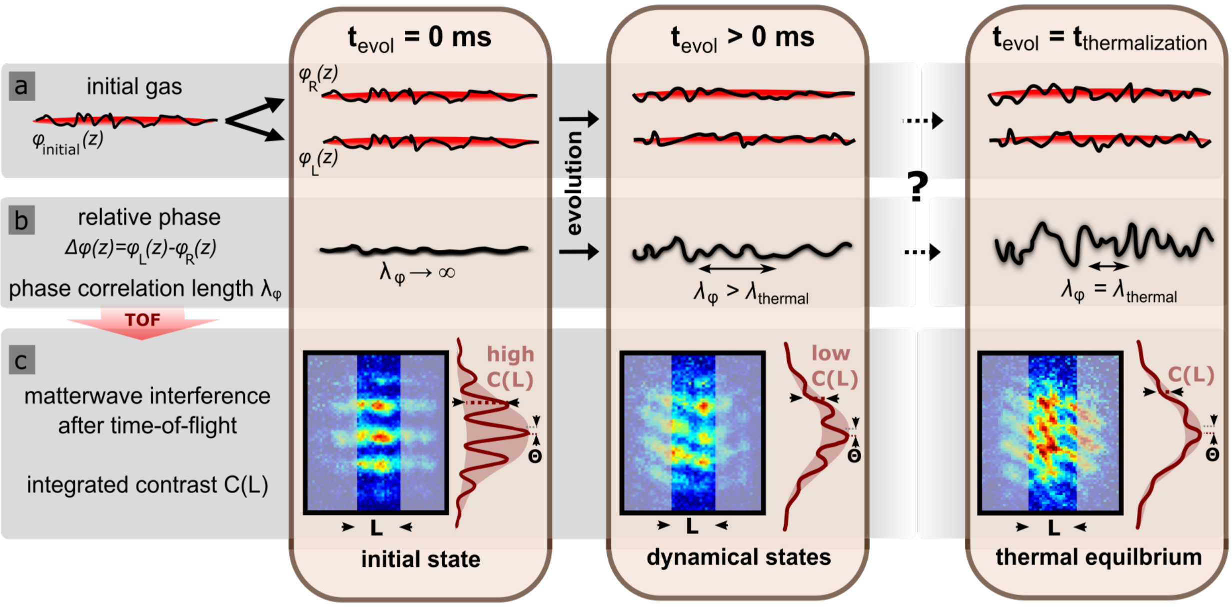
isolated & controllable



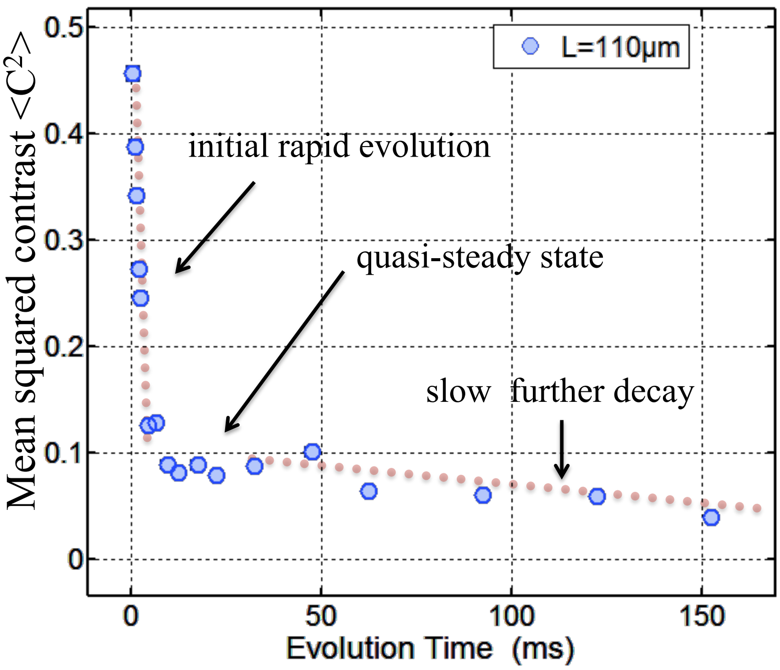
<u>isolated</u> quantum many-body system

Experimental Procedure

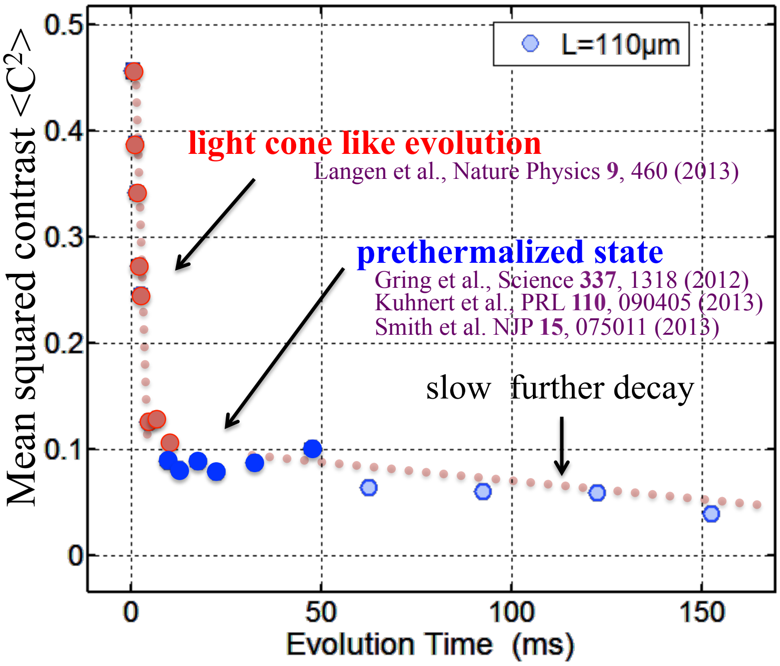
Gring et al., Science **337**, 1318 (2012)



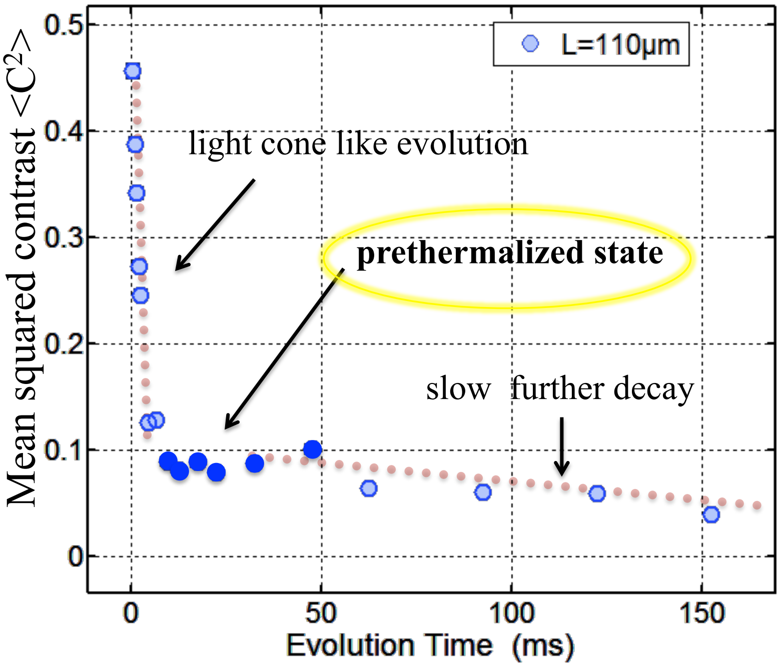
Decay of the mean contrast



Decay of the mean contrast



Decay of the mean contrast



The generalized Gibbs ensemble

1D Bose gas is a (nearly) integrable system

→ many conserved quantities inhibit thermalization

$$\hat{H}_{\text{eff}} = \sum_{\alpha=1}^{L} \epsilon_\alpha \hat{\mathcal{I}}_\alpha,$$

Conjecture:
Quantum system to **relax to maximum entropy state** decribed by a **Generalized Gibbs Ensemble**:

$$\hat{\rho} = \frac{1}{Z} \exp\left(- \sum_m \lambda_m \hat{\mathcal{I}}_m \right)$$

partition function

Lagrange multiplier $\lambda_m \rightarrow \beta_m = 1/k_B T_m$

conserved quantities: mode occupations

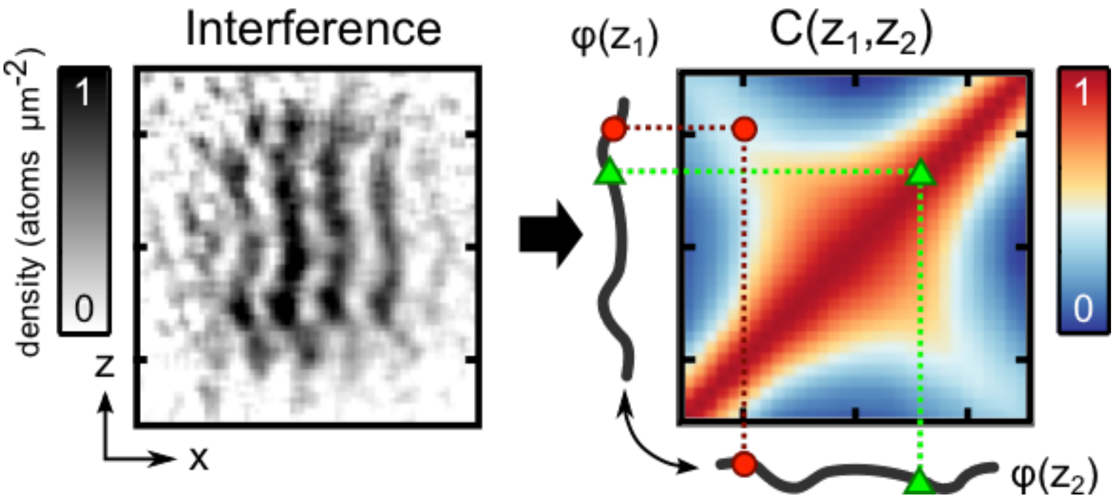
striking feature: a temperature for every mode!

E. T. Jaynes, Phys. Rev. **106**, 620 (1957); Phys. Rev. **108**, 171 (1957)
M. A. Cazalilla, Phys. Rev. Lett. **97**, 156403 (2006)
M. Rigol, et al, Phys. Rev. Lett. **98**, 050405 (2007)
C. Cramer et al. Phys. Rev. Lett. **100**, 030602 (2008)

Non-Translation Invariant Correlation Functions

T. Langen et al. Science 2015, arXiv:1411.7185

$$C(z_1, z_2) = \left\langle e^{i(\varphi(z_1) - \varphi(z_2))} \right\rangle$$



2d phase correlation function for 'Light Cone'

T. Langen et al. Science 2015, arXiv:1411.7185

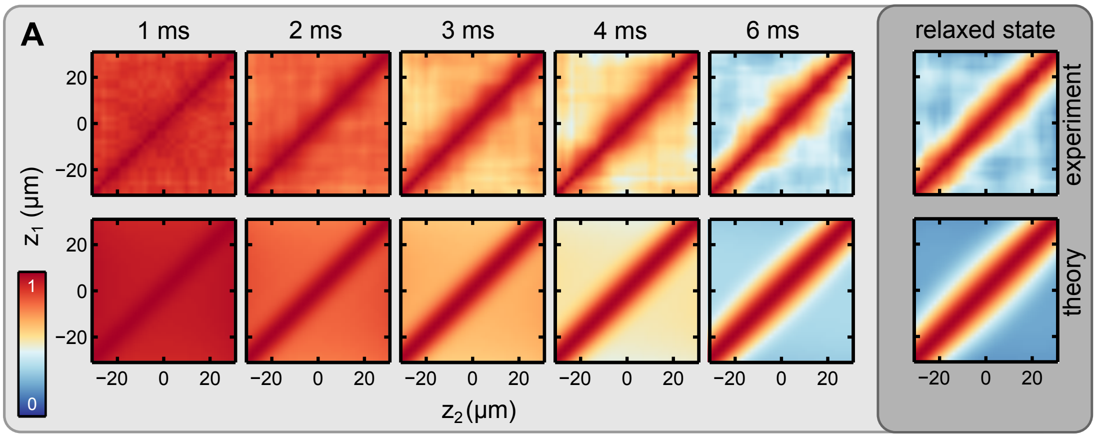
instantaneous Quench

Choose different starting points to evaluate the phase correlation function $C(z_1, z_2)$

$$C(z_1, z_2) = \left\langle e^{i(\varphi(z_1) - \varphi(z_2))} \right\rangle$$

Observation: the decay of phase correlation function is independent on starting point z_1

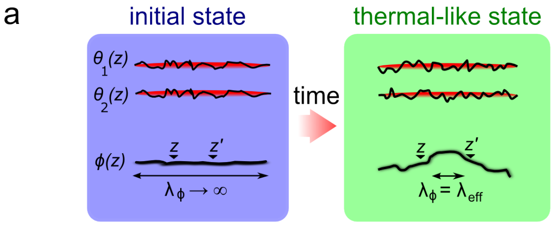
Data is described by a model with a single temperatures for ponon modes in the *anti symmetric* state.



Light cone evolution: T. Langen et al NatPhys **9**, 460 (2013)

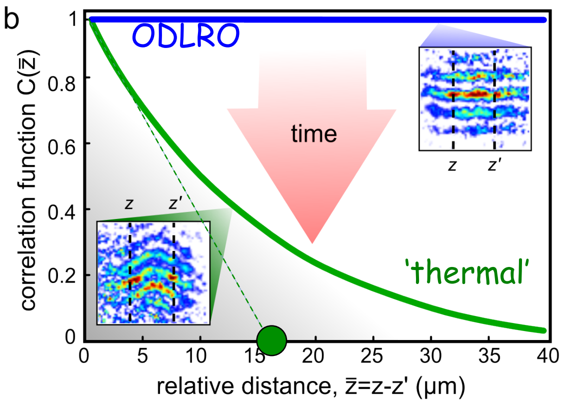
Light-Cone dynamics in the decay of coherence

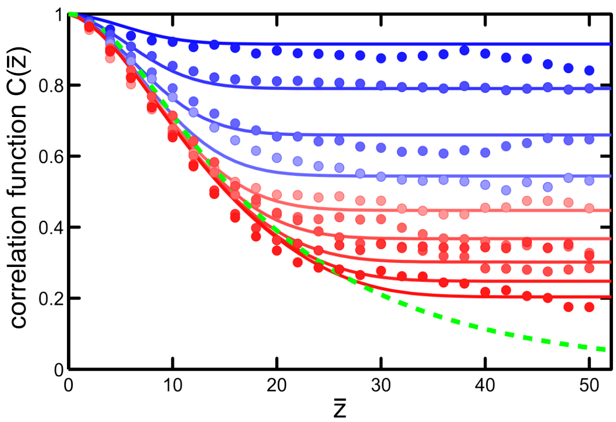
T. Langen et al NatPhys **9**, 460 (2013)
LL theory in trap: R. Geiger et al. arXiv:1312.7568



Time evolution of the phase correlation function

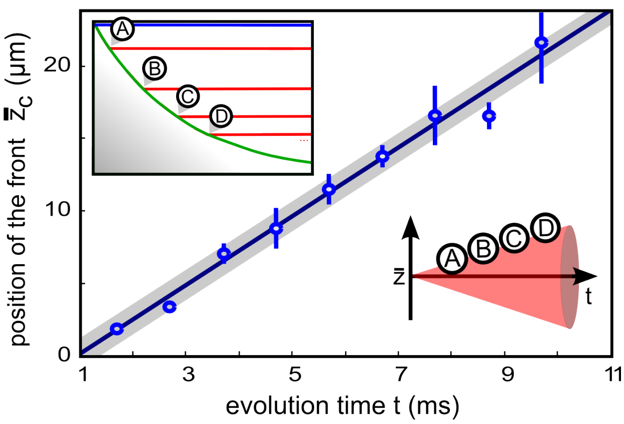
$$C(\bar{z} = z - z') = \left\langle e^{i(\varphi(z) - \varphi(z'))} \right\rangle$$

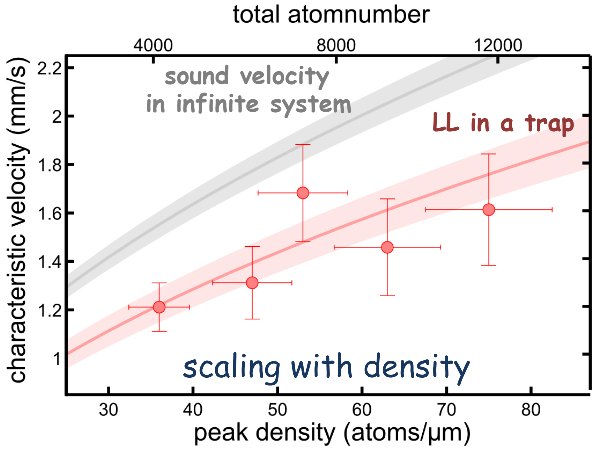




Light-Cone dynamics in the decay of coherence

T. Langen et al NatPhys **9**, 460 (2013)
LL theory in trap: R. Geiger et al. NJP **16**, 053034 (2014)





Linear dispersion relation -> Light-Cone dynamics

The region with the final form of the phase correlation function expands with **sound velocity**

Linear disperison relation of the phonons relates to the questions asked in:

CFT: Calabrese, P. & Cardy, J. Phys. Rev. Lett. **96**, 011368 (2006)
Lattice model: Cramer, M., et al. Phys. Rev. Lett. **100**, 030602 (2008).

Generalized Gibbs Ensemble

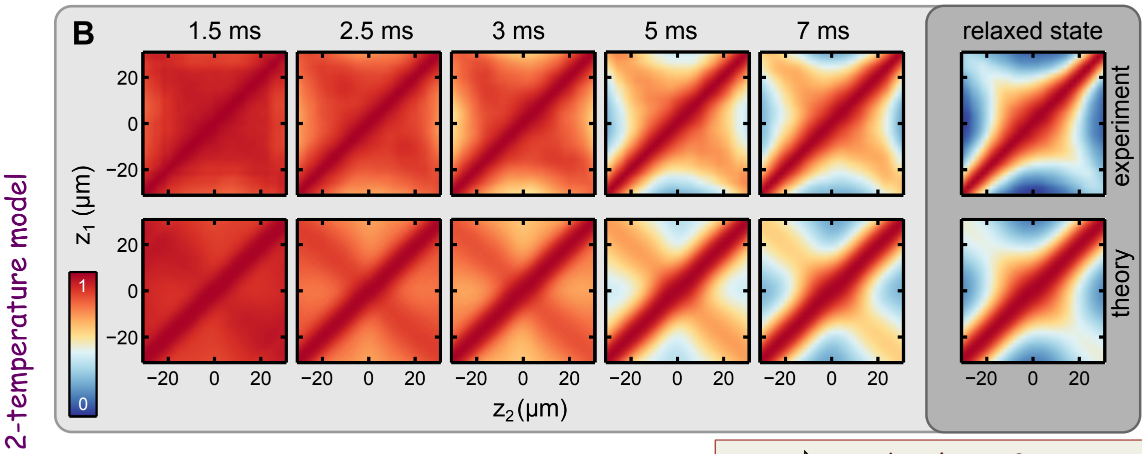
T. Langen et al. Science 2015, arXiv:1411.7185

slow-fast Quench

Observation: For specific splitting procedures the decay of phase correlation function depends on starting point z_1 and shows ‚revivals' of coherence

$$C(z_1,z_2)=\left\langle e^{i(\varphi(z_1)-\varphi(z_2))}\right\rangle$$

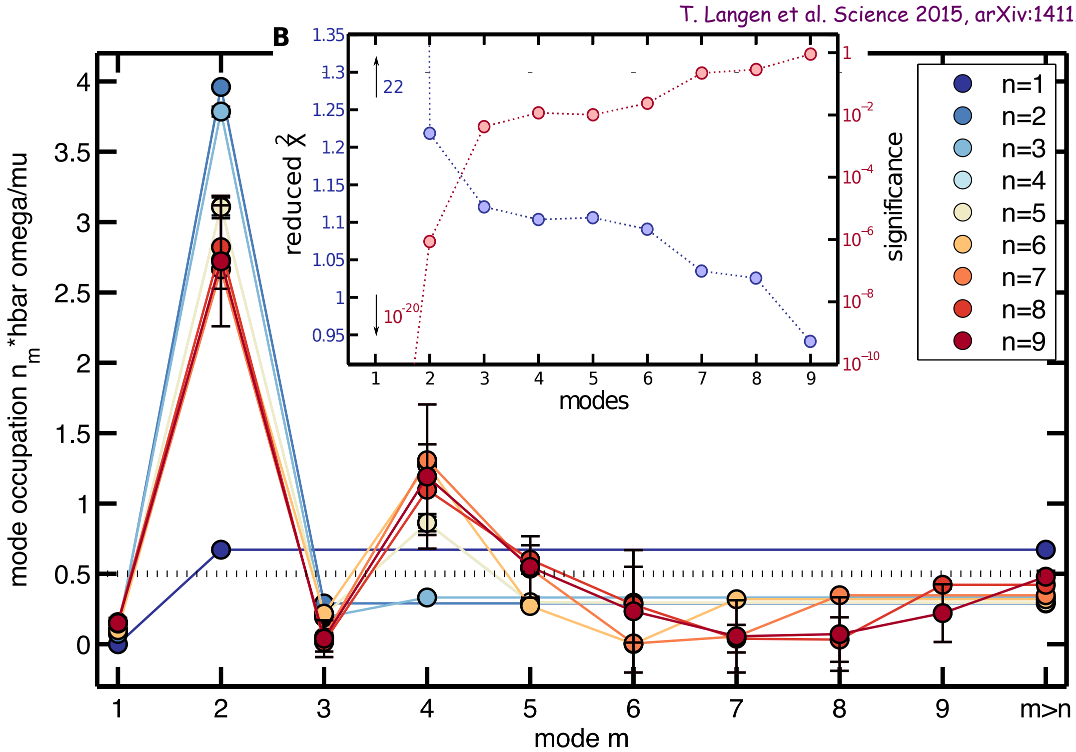
Data is better described by a model with different temperatures for *even* phonon modes and *odd* phonon modes in the *anti symmetric* state.



need at least 2 temperatures

How many Parameters are needed for the GGE

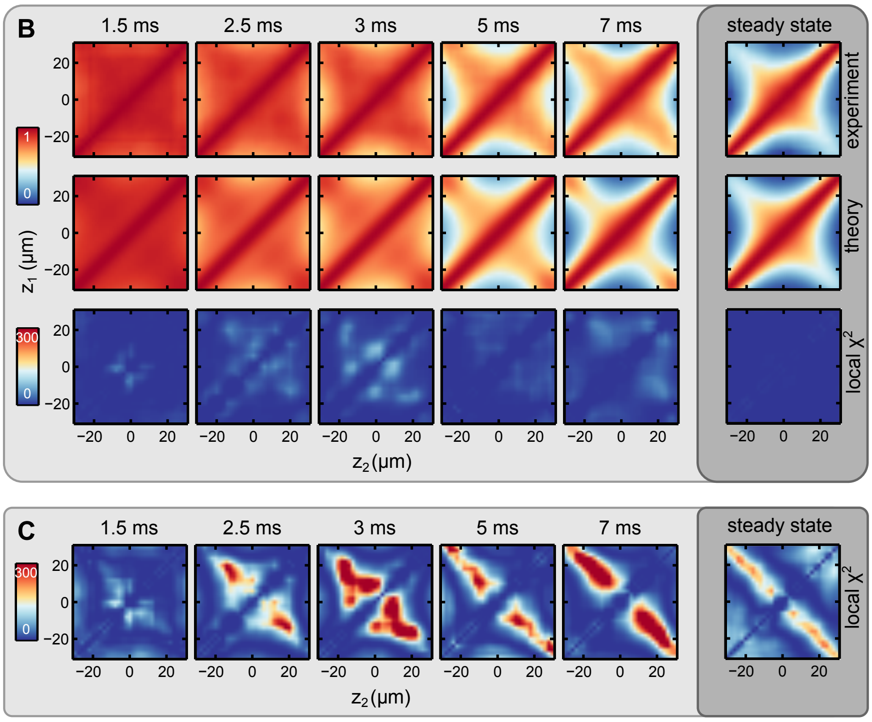
T. Langen et al. Science 2015, arXiv:1411.7185



Generalized Gibbs Ensemble

T. Langen et al. Science 2015, arXiv:1411.7185

8 temperature model

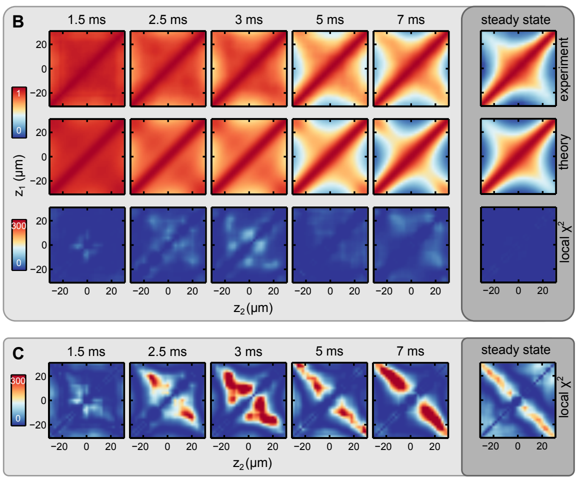


Generalized Gibbs Ensemble

T. Langen et al. Science 2015, arXiv:1411.7185

- Correlations outside the 'Light-cone', imprinted by the quench
- 8 temperature model describes the relaxed state
- number of parameters limited by experimental resolution + occupation numbers
- a single temperature model (Gibbs ensemble) shows very large deviations
- 8 temperature model describes approximately the evolution to the state
- Conjecture: Differences due to the initial phase of the excitations (in the model we assumed zero phase, as in prethermlisation)
- -> path to reconstruct the initial state

8 temperature model



Higher order phase correlation functions

T. Langen et al. Science 2015, arXiv:1411.7185

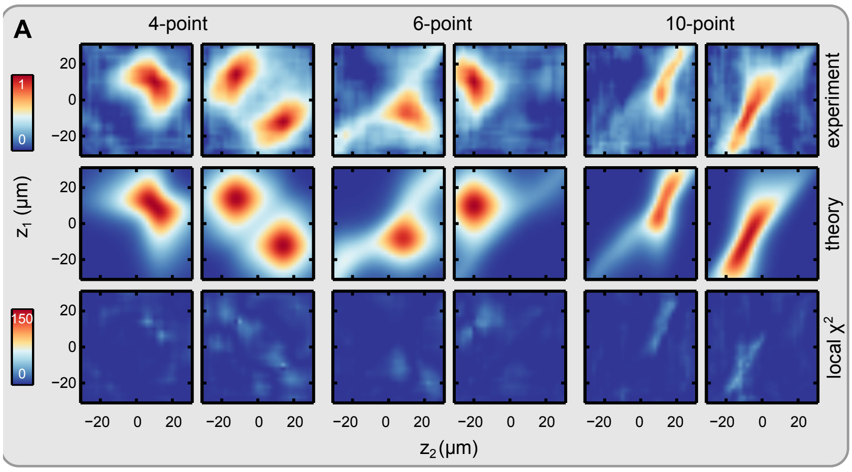
$$C(z_1, z_2) \approx \langle \exp[i\varphi(z_1) - i\varphi(z_2)] \rangle,$$

$$C(z_1, z_2, z_3, z_4) \approx \langle \exp[i\varphi(z_1) - i\varphi(z_2) + i\varphi(z_3) - i\varphi(z_4)] \rangle,$$

$$\vdots$$

Regular fast Quench (1 temperature)

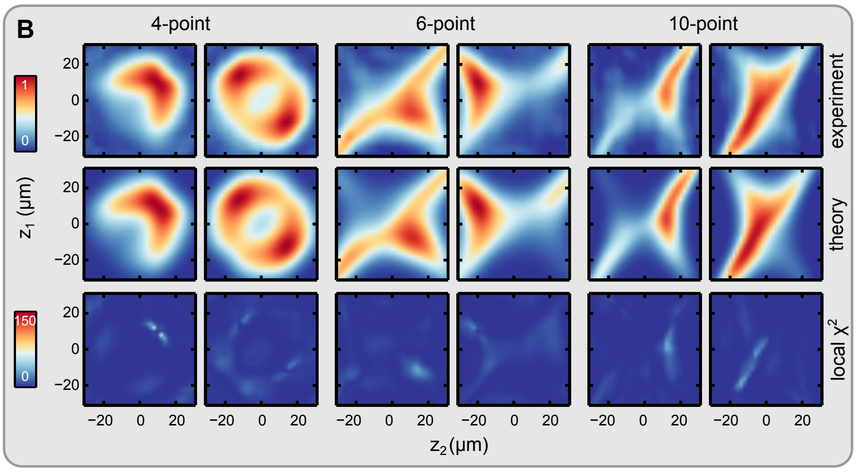
Data well described by Gibbs Ensemble



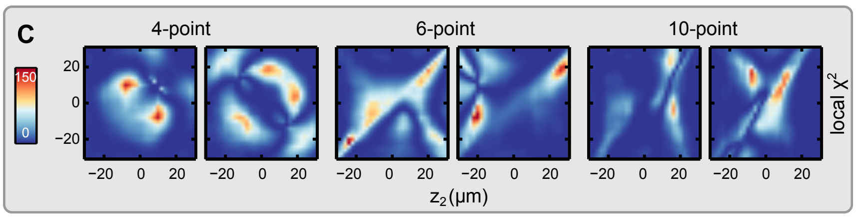
Higher order phase correlation functions

T. Langen et al. Science 2015, arXiv:1411.7185

Data well described by Generalized Gibbs Ensemble



Gibbs Ensemble

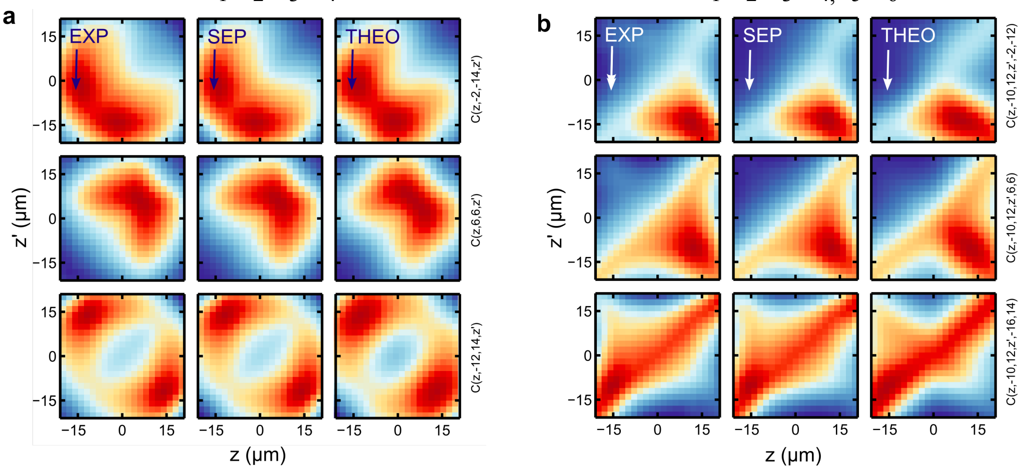


Do higher-order correlation functions factorize?

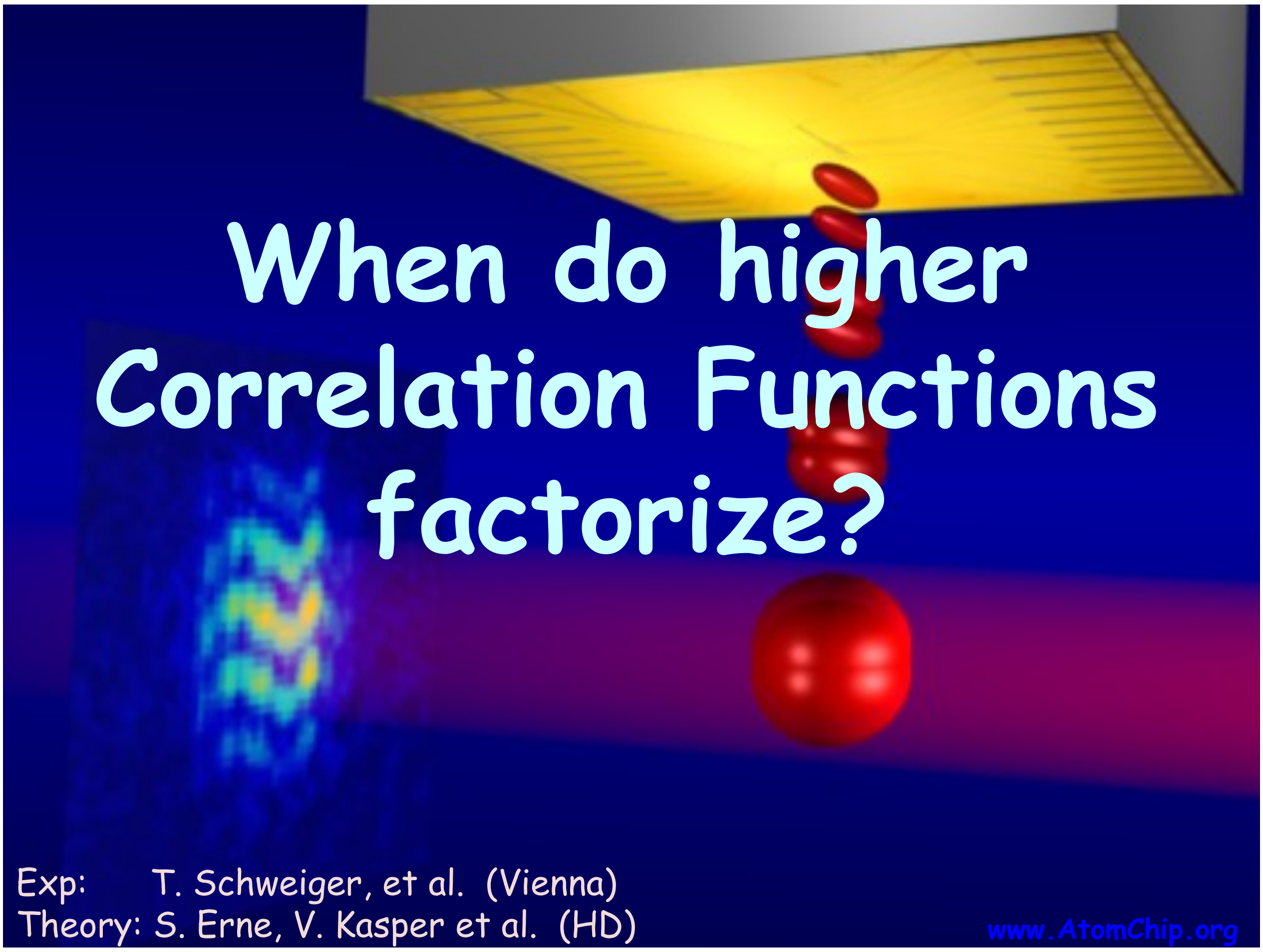
The Luttinger Liquid Hamiltonian is quadratic:
Correlations factorize into 2-point functions

4th order $C(z_1,z_2,z_3,z_4)$

6th order $C(z_1,z_2,z_3,z_4,z_5,z_6)$

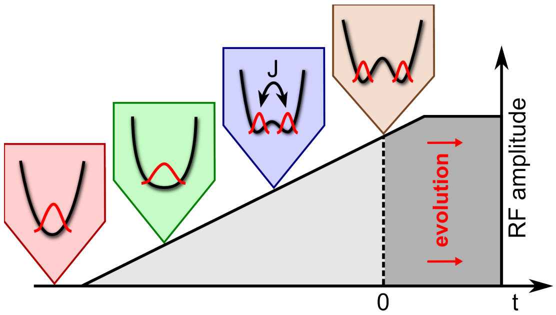


collaboration with Berges & Gasenzer groups, Heidelberg



Sine-Gordon physics
tunable tunnel coupling J in double-well

experiment: T. Schweigler et al.
theory: V. Kasper, S. Erne
T Gasenzer, J. Berges



Quantum Sine-Gordon model:

$$\hat{H}_{\rm SG} = \frac{\hbar c}{2}\int_{-\mathcal{L}/2}^{\mathcal{L}/2} dz \left[\frac{\pi}{K}\hat{n}^2(z) + \frac{K}{\pi}\left(\frac{\partial}{\partial z}\hat{\theta}(z)\right)^2\right] - 2n_{\rm 1D}J\int_{-\mathcal{L}/2}^{\mathcal{L}/2} dz \cos[\sqrt{2}\,\hat{\theta}(z)]$$

that's what we have seen so far …
"uncoupled harmonic oscillators"

anharmonic, non-gaussian,
gapped, universality?

Characterising the factorisation

experiments probe the phase

-> look at the '**connected part**' of the phase correlation function

Gaussian fluctuations

Variance

$$\langle(\Delta\varphi)^2\rangle_c = \langle(\Delta\varphi)^2\rangle$$

$$\langle(\Delta\varphi)^4\rangle_c = \langle(\Delta\varphi)^4\rangle - 3\,\langle(\Delta\varphi)^2\rangle^2 \quad = 0$$

$$\langle(\Delta\varphi)^6\rangle_c = \langle(\Delta\varphi)^6\rangle - 15\,\langle(\Delta\varphi)^4\rangle\,\langle(\Delta\varphi)^2\rangle^2 + 30\,\langle(\Delta\varphi)^2\rangle^3 \quad = 0$$

$$\langle(\Delta\varphi)^8\rangle_c = \langle(\Delta\varphi)^8\rangle + 420\,\langle(\Delta\varphi)^4\rangle\,\langle(\Delta\varphi)^2\rangle - 630\,\langle(\Delta\varphi)^2\rangle^4 - 35\,\langle(\Delta\varphi)^4\rangle^2 - 28\,\langle(\Delta\varphi)^6\rangle\,\langle(\Delta\varphi)^2\rangle \quad = 0$$

characterized by '**Kurtosis**'

Gaussian fluctuations

$$\gamma_2 = \frac{\langle(\Delta\varphi)^4\rangle}{3\,\langle(\Delta\varphi)^2\rangle^2} - 1 \quad = 0$$

$$\gamma_3 = \frac{\langle(\Delta\varphi)^6\rangle}{15\,\langle(\Delta\varphi)^4\rangle\,\langle(\Delta\varphi)^2\rangle^2 - 30\,\langle(\Delta\varphi)^2\rangle^3} - 1 \quad = 0$$

$$\gamma_4 = \frac{\langle(\Delta\varphi)^8\rangle}{630\,\langle(\Delta\varphi)^2\rangle^4 + 35\,\langle(\Delta\varphi)^4\rangle^2 + 28\,\langle(\Delta\varphi)^6\rangle\,\langle(\Delta\varphi)^2\rangle - 420\,\langle(\Delta\varphi)^4\rangle\,\langle(\Delta\varphi)^2\rangle} - 1 \quad = 0$$

Correlation functions
fields <-> phase

correlation functions for the fields:

$$C(z_1, z_2) = \frac{\langle \Psi_1(z_1)\Psi_2^\dagger(z_1)\Psi_1^\dagger(z_2)\Psi_2(z_2)\rangle}{\langle|\Psi_1(z_1)|^2\rangle\langle|\Psi_2(z_2)|^2\rangle}$$

$$C(z_1, z_2) \approx \langle \exp[i\varphi(z_1) - i\varphi(z_2)]\rangle$$

$C(z_1,z_2)$ contains **all orders of connected** parts

$$C(z_1, z_2) = \exp\left[\sum_{k=1}^{\infty}(-1)^k \frac{\langle(\Delta\varphi)^{2k}\rangle_c}{(2k)!}\right]$$

for Gaussian fluctuations

$$C(z_1, z_2) = \exp\left[-\frac{1}{2}\langle(\Delta\varphi)\rangle^2\right]$$

Observable and non-gauss measure

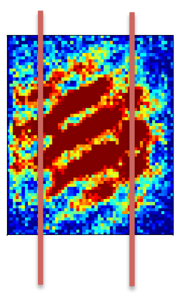
to study factorization of correlation functions we look at

$$C^{(2)}(z_1, z_2) = \langle[\varphi(z_1) - \varphi(z_2)]^2\rangle = \langle[\Delta\varphi(z_1,z_2)]^2\rangle$$

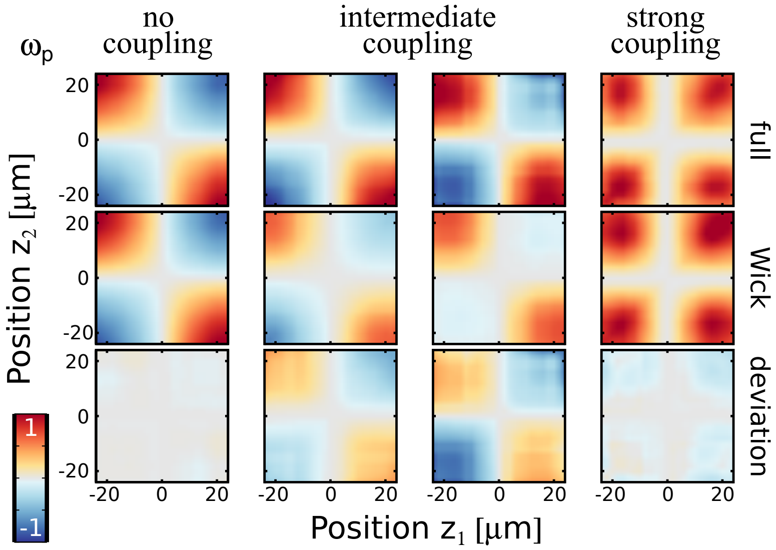
$$C^{(4)}(z_1, z_2, z_3, z_4) = \langle[\varphi(z_1) - \varphi(z_2)]^2 [\varphi(z_3) - \varphi(z_4)]^2\rangle = \langle[\Delta\varphi(z_1,z_2)]^2 [\Delta\varphi(z_3,z_4)]^2\rangle,$$

$\Delta\varphi(z_1, z_2) = \varphi(z_1) - \varphi(z_2)$

$\Delta\varphi$ *is* NOT restricted to 2π



$C^{(4)}(z_1, z_2, -15, 15)$



experiment: T. Schweigler et al.
theory: V. Kasper, S. Erne

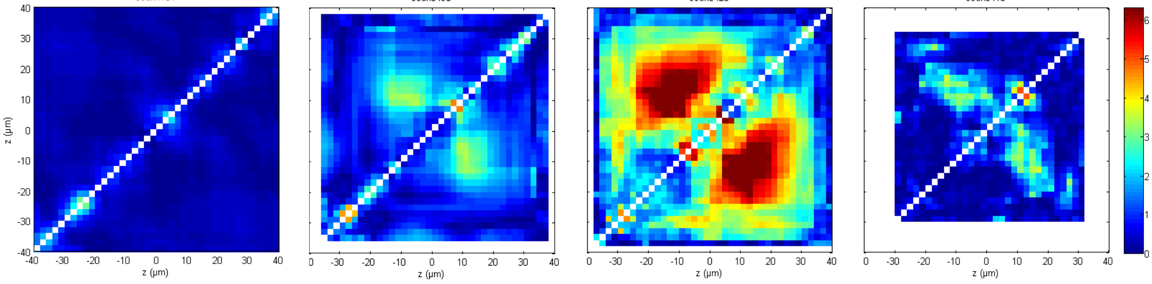
Characterising non-Gaussian phase fluctuations

experiment: T. Schweigler et al.
theory: V. Kasper, S. Erne

Characterising the factorisation by the connected part: $\langle(\Delta\varphi)^4\rangle_c = \langle(\Delta\varphi)^4\rangle - 3\,\langle(\Delta\varphi)^2\rangle^2$

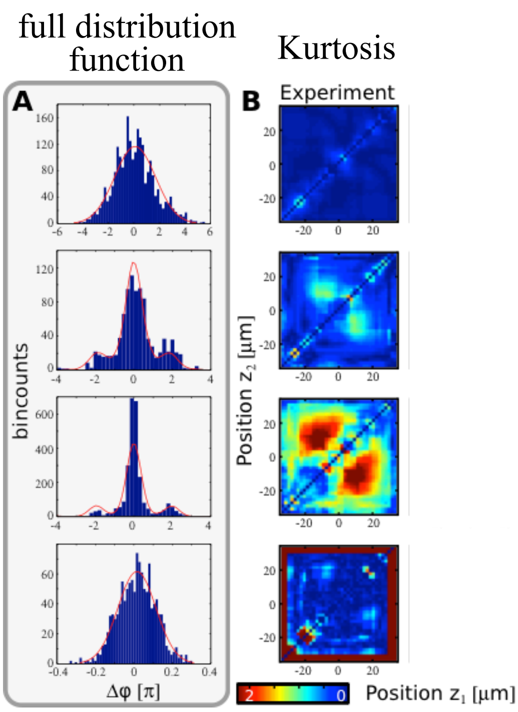
excess Kurtosis $\gamma_2 = \dfrac{\langle(\Delta\varphi)^4\rangle}{3\,\langle(\Delta\varphi)^2\rangle^2} - 1$

Experimental data, thermal state in a double well

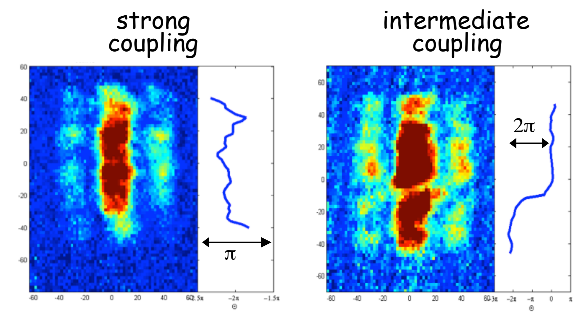


Quantifying factorization of correlation functions

experiment: T. Schweigler et al.
theory: V. Kasper, S. Erne



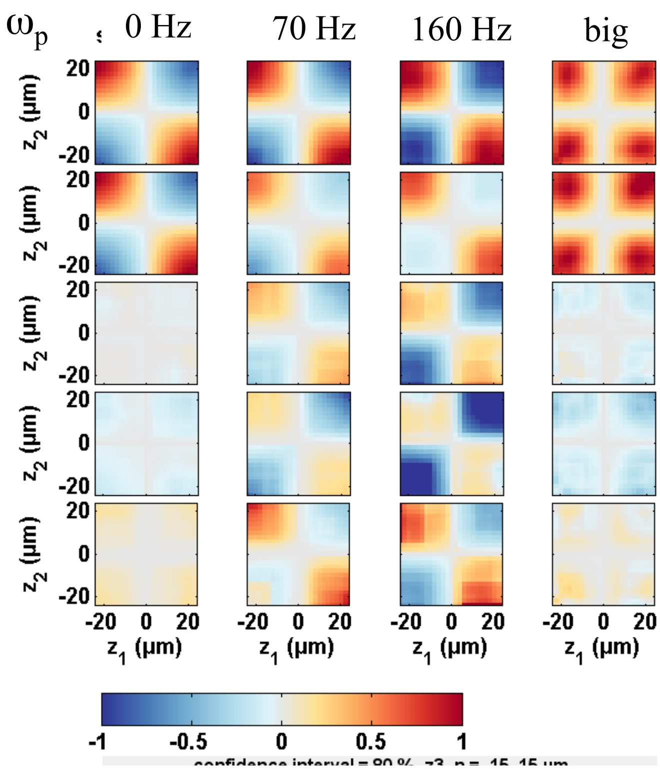
- the breakdown of factorization is evident in the **full distribution functions** of the phase by new peaks at multiples of 2π
- caused by the 2π ***periodic*** SG Hamiltonian -> 2π phase jumps, 'kinks', SG solitons



- ***SG Solitons*** are topological excitations
- Phase fluctuations around *topologically different Vaccua*

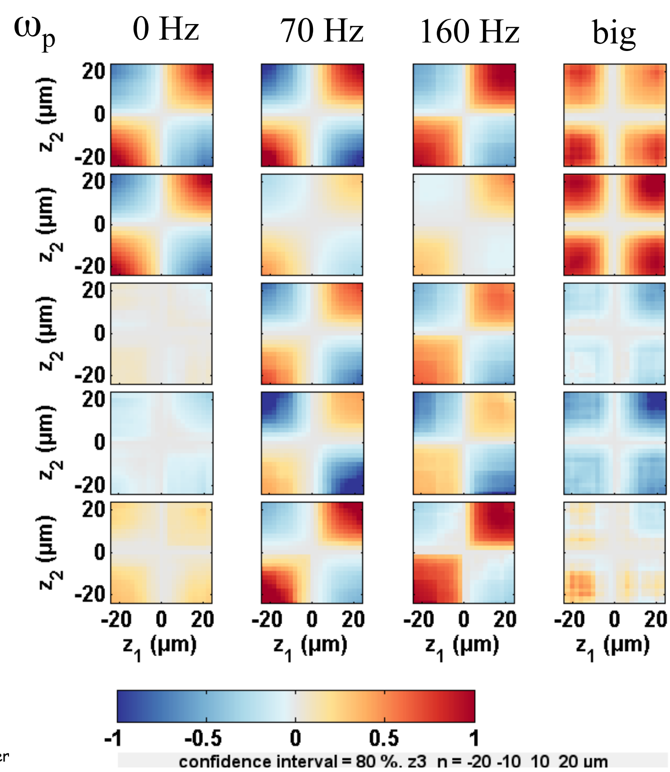
4-point phase correlators

experiment: T. Schweigler et al.
theory: V. Kasper, S. Erne



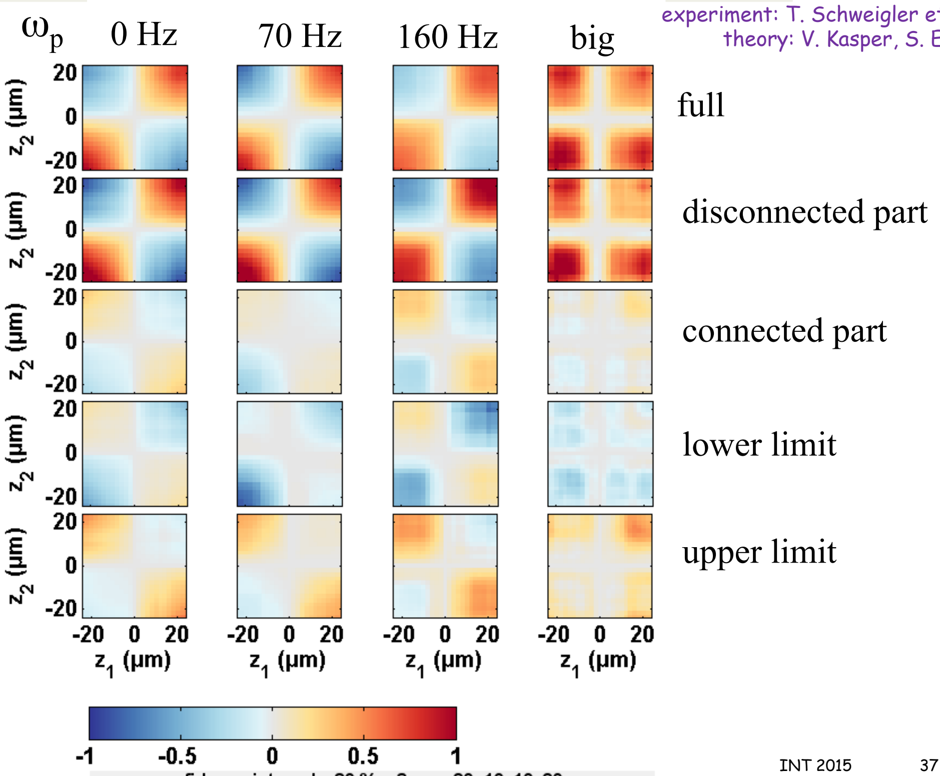
6-point Phase correlators

experiment: T. Schweigler et al.
theory: V. Kasper, S. Erne



6-point phase correlators, connected part

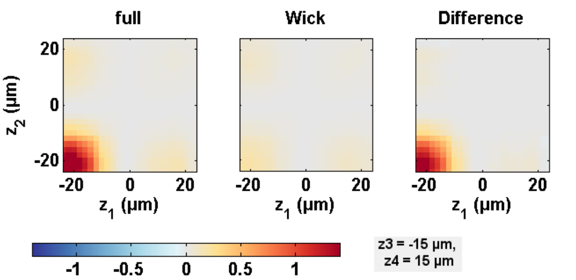
experiment: T. Schweigler et al.
theory: V. Kasper, S. Erne



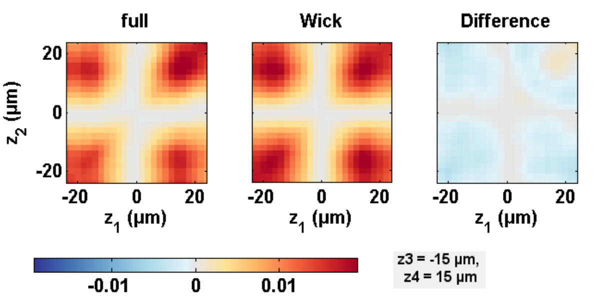
Remove Solitons
Strongly coupled ω_p > 500 Hz

experiment: T. Schweigler et al.
theory: V. Kasper, S. Erne

4-point correlator does not factorize:



without Solitons:



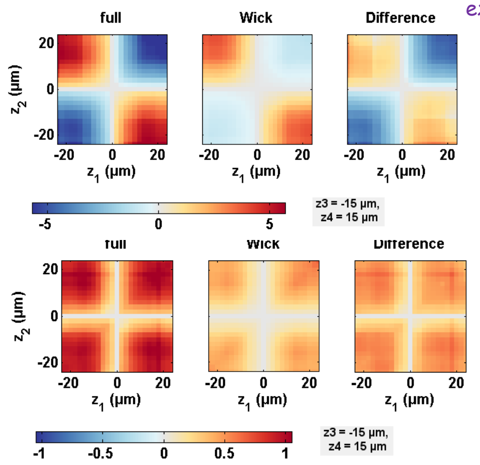
Remove Solitons

intermediate coupling ω_p=160 Hz

experiment: T. Schweigler et al.
theory: V. Kasper, S. Erne

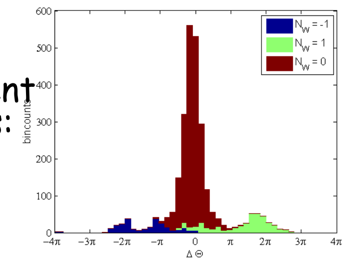
4-point correlator does not factorize:

without Solitons:

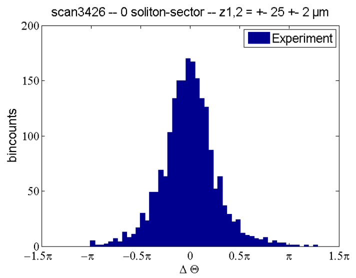


phase distribution:

different sectors:



without solitons:

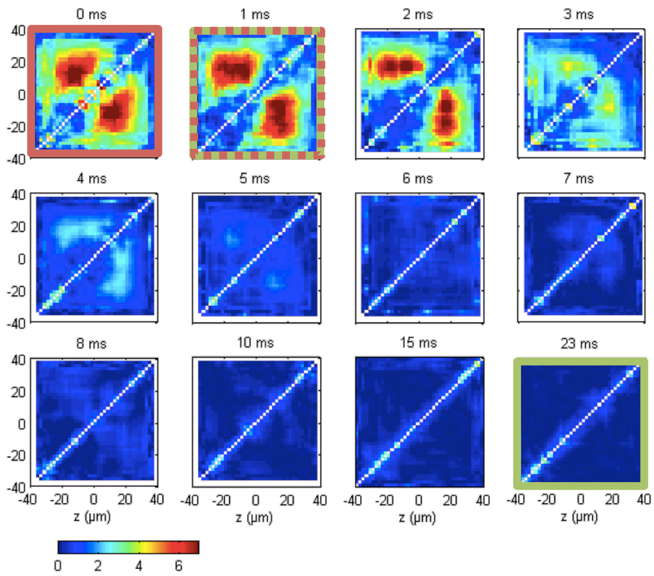


Quench from J>0 to J=0

very preliminary

experiment: T. Schweigler et al.
theory: V. Kasper, S. Erne

Initial state non-Gaussian, dynamics Gaussian

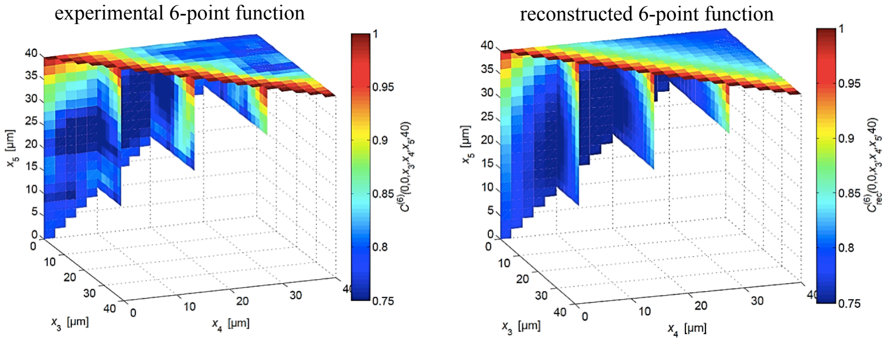


collaboration with Berges & Gasenzer groups, Heidelberg

Quantum state tomography

A. Steffens et al. arxiv:1406.3632

Use 2- and 4-point functions to reconstruct higher-order functions via continuous matrix product states with bond length 2



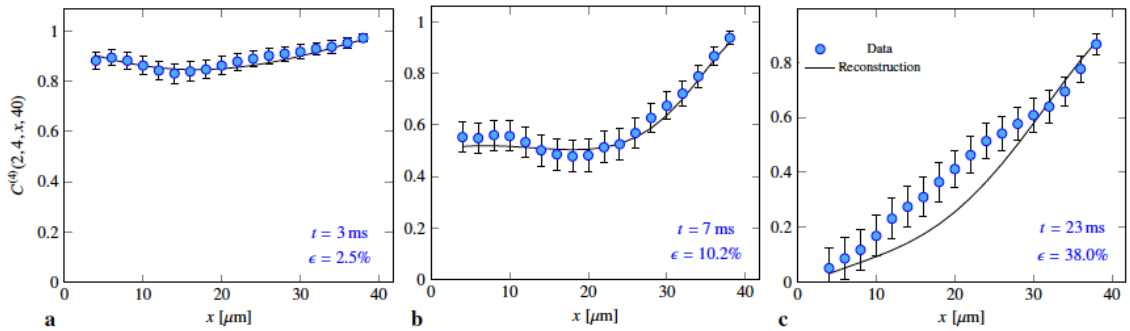
State reconstruction with very weak assumptions

Theory:
A. Steffens, C. Riofrıo, R. Hubener, and J. Eisert, "Quantum field tomography," NJP **16** (2014) 123010.

Quantum state tomography

A. Steffens et al. arxiv:1406.3632

Use 2- and 4-point functions to reconstruct higher-order functions via continuous matrix product states with bond length 2



State reconstruction gets worse with time

C-MPS with bond length 2 have finite entanglement

Question: Can one build a measure for entanglement growth after the quench?

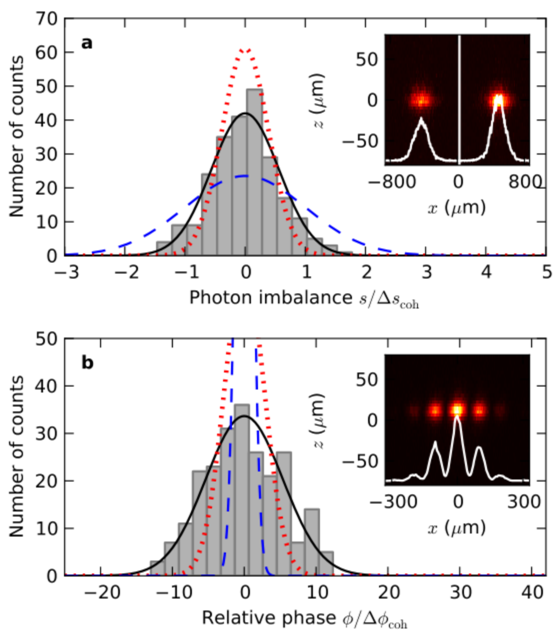
Squeezing

$N = 1200$ atoms, $\mu \simeq 0.5\,\text{kHz}$, $T \simeq 25\,\text{nK}\ (0.5\ \text{kHz})$

T. Berrada, et al., Nat. Comm **4**, 2077 (2013)

number and phase distribution

(black: measured, blue: binomial, red: detection noise)



RMS fluctuations of the **number difference**

$$n \equiv N_L - N_R$$

$$\Delta n = 14(3)\ \text{atoms}$$

Whereas $\sqrt{N} = 35$

Spin squeezing: $\xi_S^2 \equiv \dfrac{\xi_N^2}{\langle \cos\phi \rangle^2} = -7.7\,\text{dB}$

Implies that ≈ 150 atoms are entangled!

RMS fluctuations of the **phase**

$$\Delta\phi = 0.168(8)\ \text{rad}$$

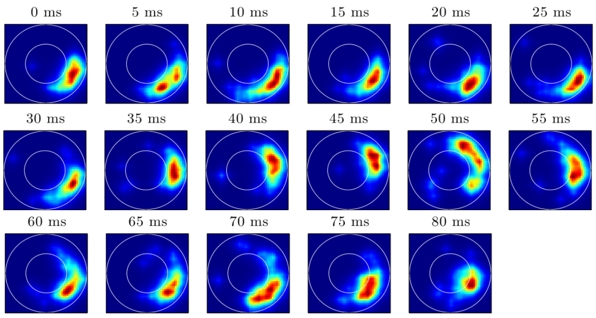
Whereas $1/\sqrt{N} = 0.03$

$$\Delta n\ \Delta\phi\ =\ 2.3\ (7)$$

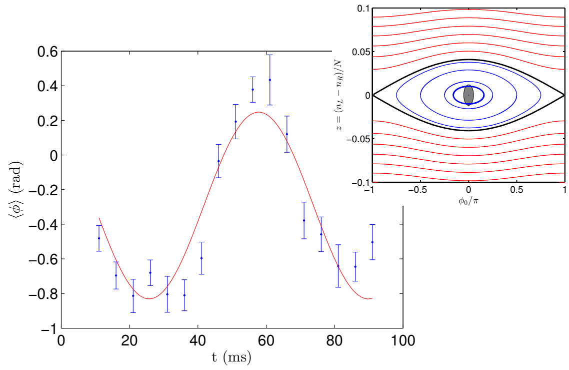
when correcting for measurement noise: $\Delta n\ \Delta\phi\ \sim\ 1$

Evolution of ξ^2~-8dB 1d gas

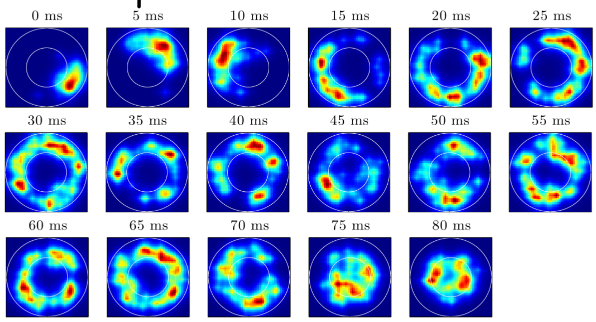
Tunnel Coupled ω_p=14Hz

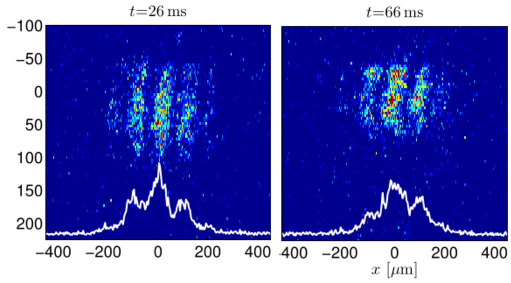


T. Berrada prelininary



Separated

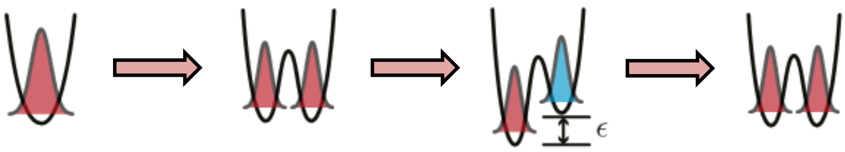


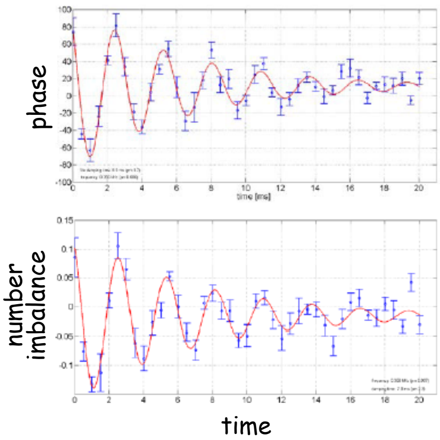


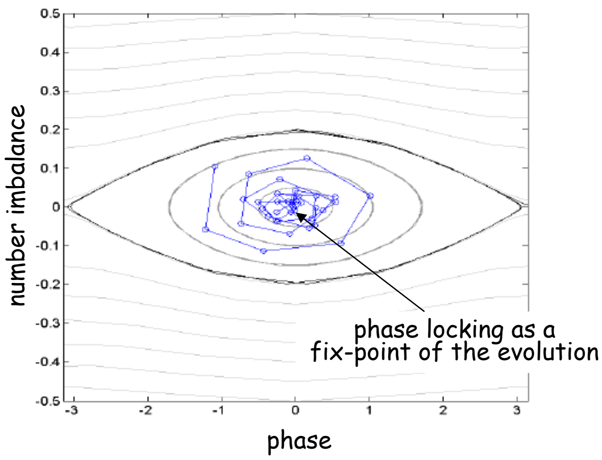
Relaxation in coupled superfluids

re-coupling starts SG model with a specific phase
-> study phase locking

experiment: M. Pigeur
theory: E. DelaTorre, E. Demler

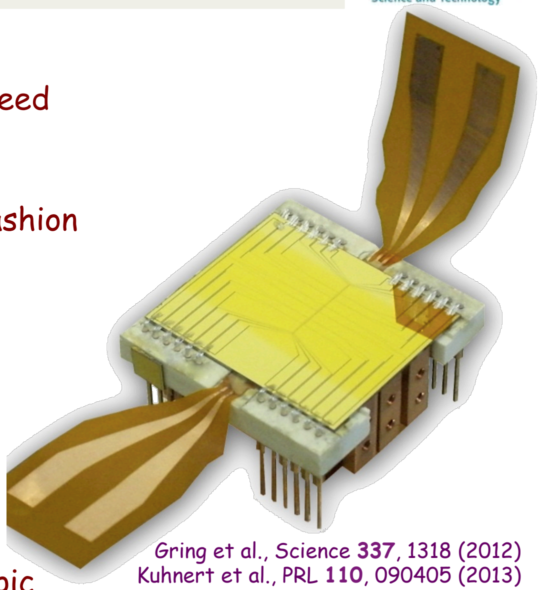






What have we learned

- Relaxation in quantum systems does not proceed through a simple path: '**prethermalization**'
- Relaxed state emerges localy and spreads throughout the system in a **light cone** like fashion
- Prethermalized state is associated with a ***Generalized Gibbs Ensemble***
- **Higher order correlation functions** and the question if they factorize (**full distribution functions**) gives insight in the effective theories describing the many body system
- Experiments allow to probe how classical statistical properties emerge from microscopic quantum evolution through de-phasing of many body eigenstates.



Gring et al., Science **337**, 1318 (2012)
Kuhnert et al., PRL **110**, 090405 (2013)
Smith et al. NJP **15**, 075011 (2013)
Langen et al., Nature Physics **9**, 460 (2013)
R. Geiger et al. NJP **16** 053034 (2014)
Langen et al. Science (2015) arXiv:1411.7185

T. Berrada, et al., Nat. Comm **4**, 2077 (2013)
S. Van Frank, et al., Nat. Comm **5**, 4009 (2014)

Atom Chip Experiment

S. Manz, T. Betz, R. Bücker, **T. Berrada**, S. vanFrank, **M. Pigeur**, A. Perrin, T Schumm, JF Schaff, R. Wu, **M. Bonneau**

M. Kuhnert, M. Gring, **B. Rauer, Th. Schweigler**
D. Smith, Remi Geiger, **T. Langen**

Atom Chip Fabricati
D. Fischer, M. Trinker, M. Sc
S. Groth (HD), Israel B

Theor

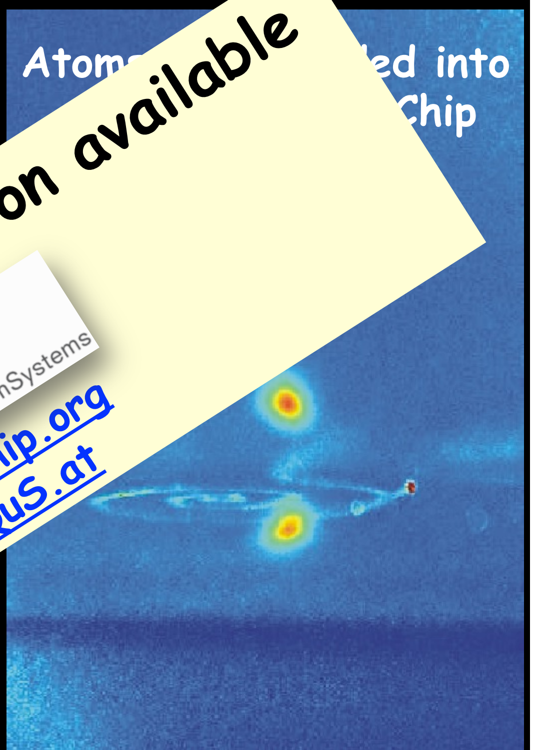
T. Gasen

PhD and PostDoc position available

CoQuS ComplexQuantumSystems

www.atomchip.org
www.CoQuS.at

Wien



www.AtomChip.org