QUANTUM SIMULATION OF LATTICE GAUGE THEORIES WITH COLD ATOMS

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In collaboration with

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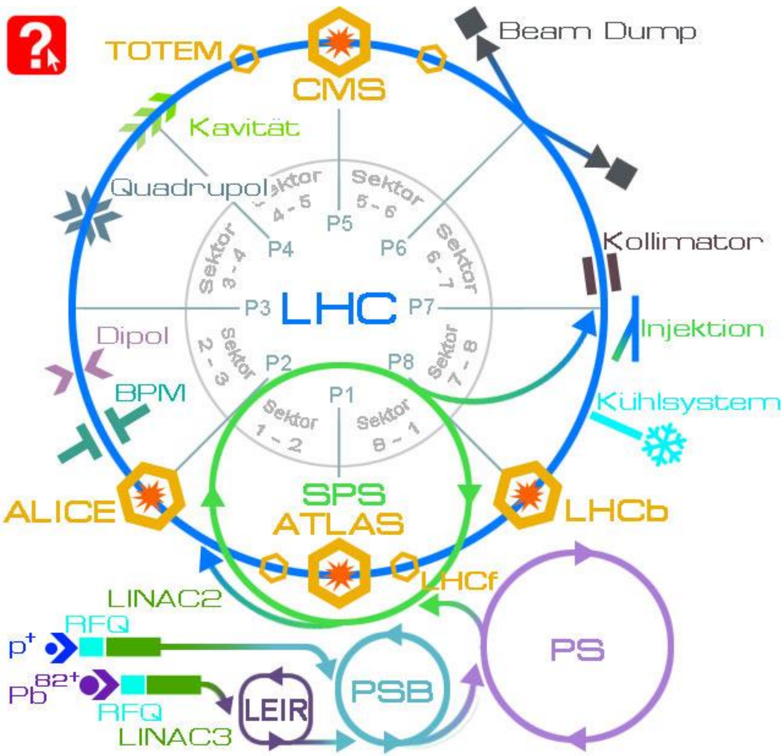
INT Conference: Frontiers in QS with Cold Atoms, March 30th 2015

16 < orders of magnitude



TALK OUTLINE

- On the general Nature of HEP, SM models.
- Fundamental set of Requirements.
- Local gauge invariance in Lattice Gauge Theory
- QS: Compact QED, U(1) symmetry
- QS: Non-Abelian, Yang Mills theory.

- Several comments

LONG RANGE FORCES?

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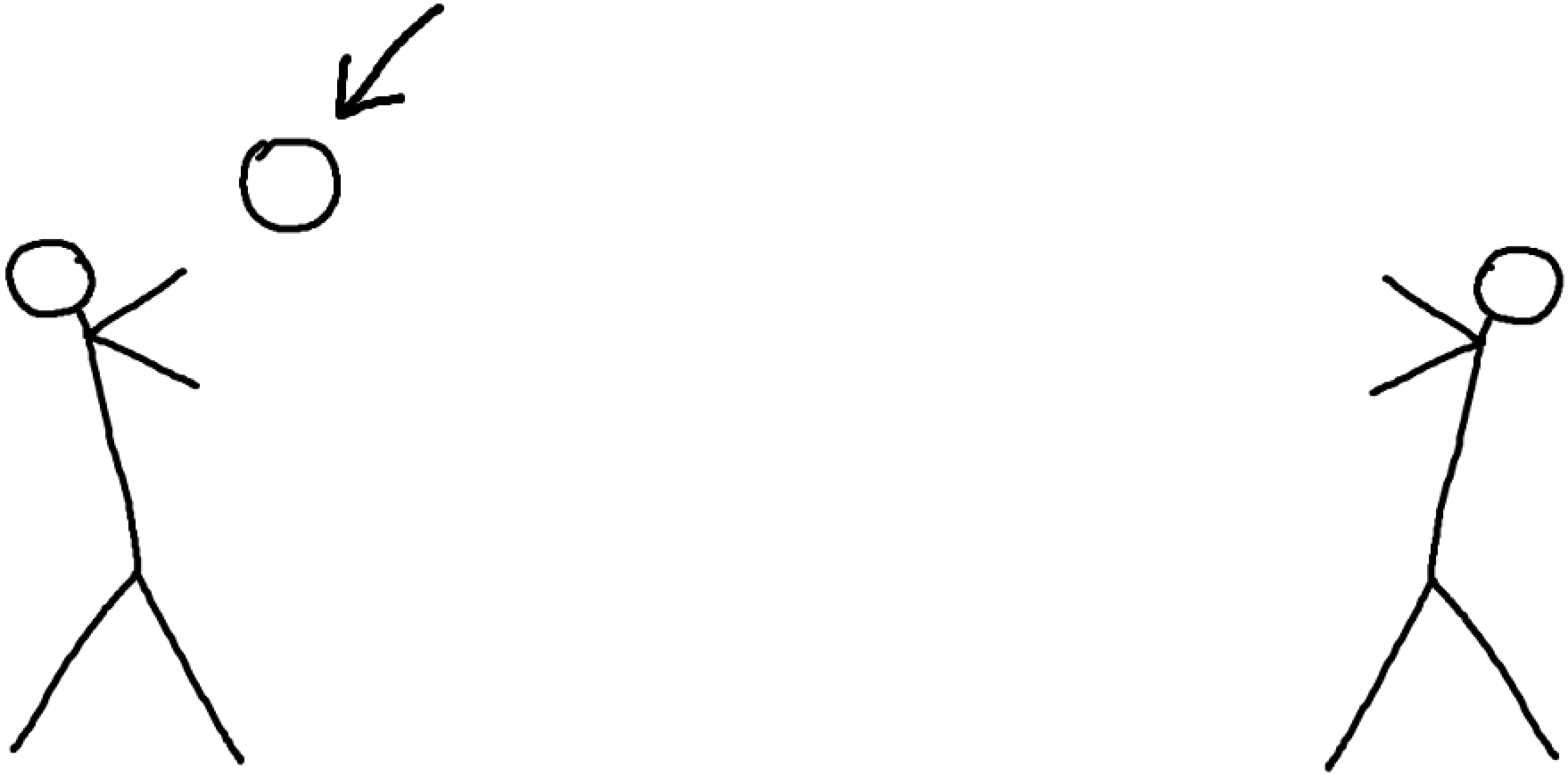
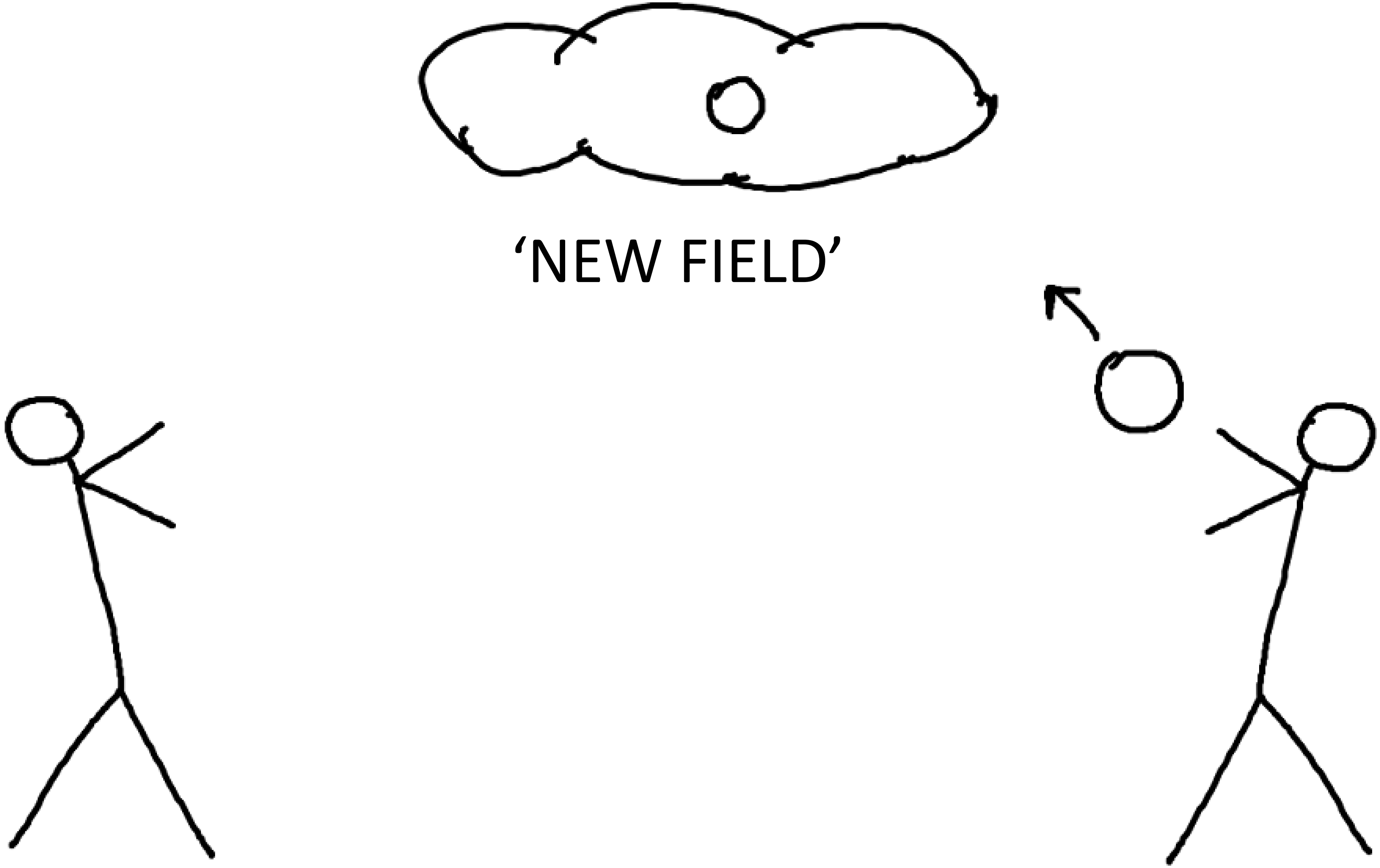


REQUIRE FORCE CARRIER



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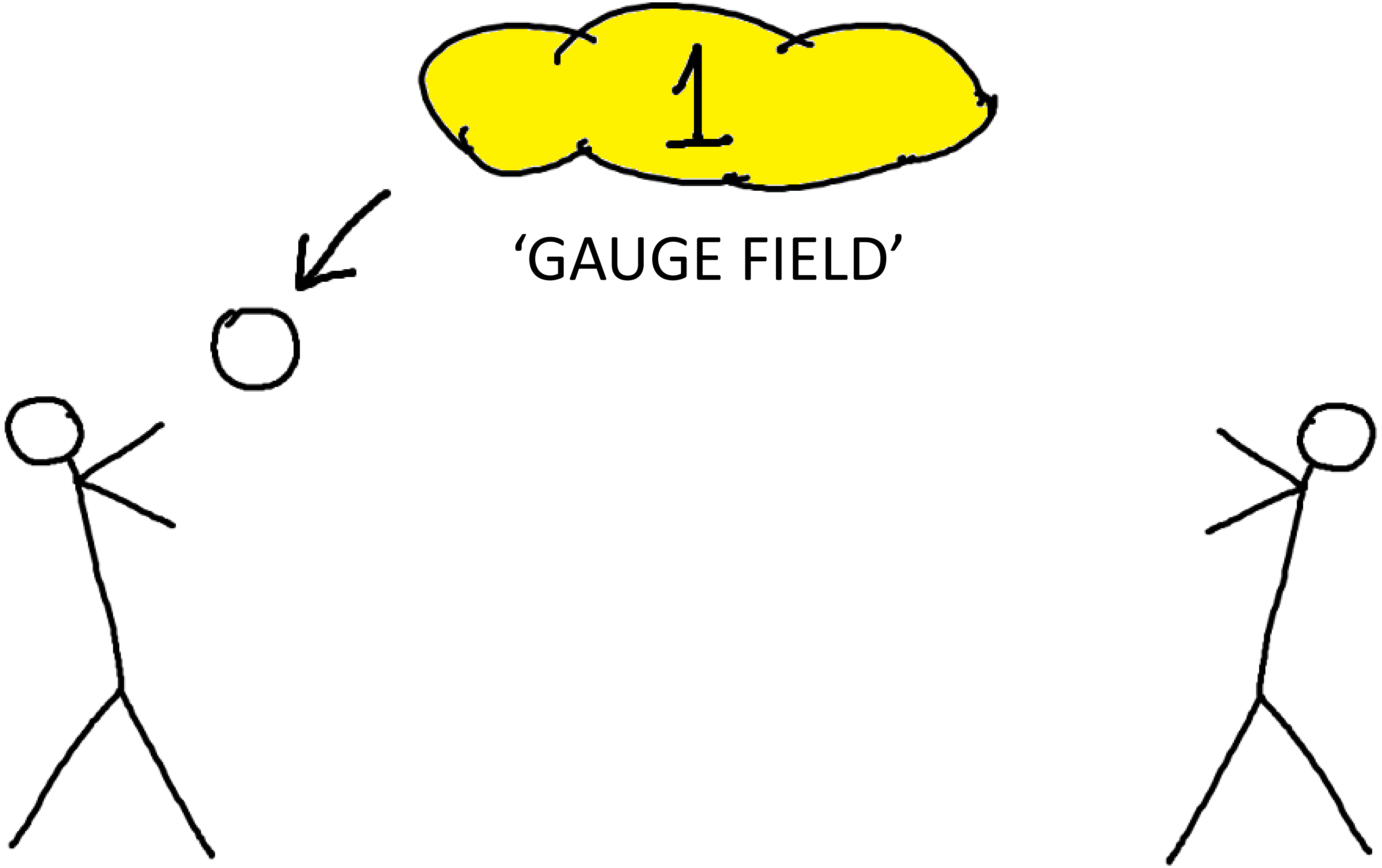


THE STANDARD MODEL

Matter Particles= Fermions

Quarks and Leptons:

Mass, Spin, Flavor

Force Carriers = Spin 1 Bosons

local gauge symmetries:

Massless, chargeless photon (1): Electromagnetic, U(1)

Massive, charged Z, W's (3): Weak interactions, SU(2)

Massless, charged Gluons (8): Strong interactions, SU(3)

GAUGE FIELDS

Abelian Fields **Maxwell theory**	**Non-Abelian fields** **Yang-Mills theory**
Massless	Massless
Long-range forces	**<u>Confinement</u>**
Chargeless	Carry charge
Linear dynamics	Self interacting & NL

QED: THE CONVENIENCE OF BEING ABELIAN

$$\alpha_{QED} \ll 1, \quad V_{QED}(r) \propto \frac{1}{r}$$

We (ordinarily) don't need QFT quantum field theory to understand the structure of atoms:

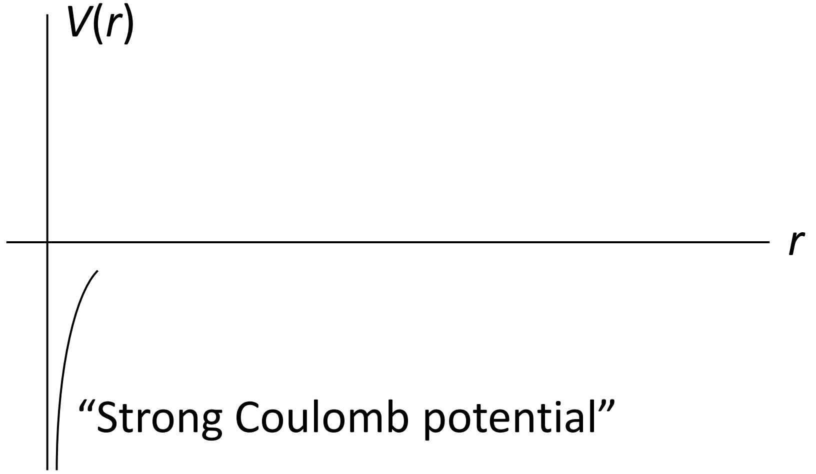
$$m_e c^2 \gg E_{Rydberg} \simeq \alpha_{QED}^2 \, m_e c^2$$

But also higher energies effects are well described using perturbation theory - (Feynman diagrams) works well.

QCD: AT HIGH ENERGY
ASYMPTOTIC FREEDOM

- Quantum Chromodynamics asymptotic freedom:
  at high energies, coupling constant ‘goes’ to zero.

- The nucleus, are seen
  as built of ‘free’ point-like
  particles= quarks.



QCD: AT LOW ENERGIES
THE DARK SIDE OF ASYMPTOTIC FREEDOM

$$\alpha_{QCD} > 1 \, , V_{QCD}(r) \propto r$$

non-perturbative confinement effect!

No free quarks! they construct Hadrons:

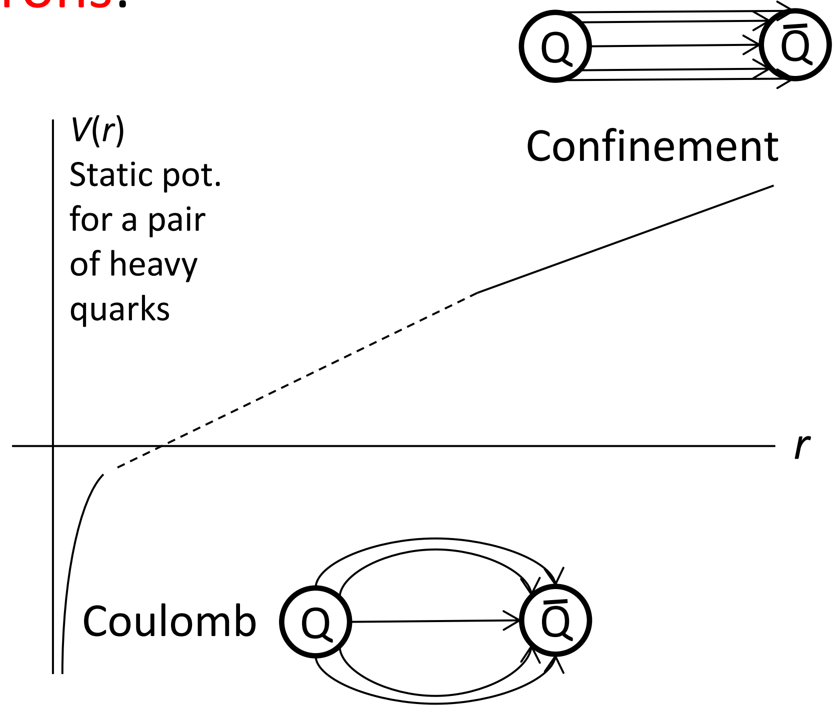
Mesons (two quarks),

Baryons (three quarks),

...

Color Electric flux-tubes:

“a non-abelian Meissner effect”.



Fundamental properties of HEP models

1. <u>Fields</u>

   Fermion Matter fields

   Bosonic gauge fields

2. <u>Relativistic invariance</u>

   Causal structure, in the continuum limit

3. <u>Local gauge invariance</u>

   Exact, or low energy, effective

REQUIREMENT 1.

Fermion fields : = Matter

Bosonic, Gauge fields:= Interaction mediators

One needs both bosons and fermions

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Fermion fields : = Matter

Bosonic, Gauge fields:= Interaction mediators

One needs both bosons and fermions

✓ Trapped ultracold atoms can have both bosons and fermions.

REQUIREMENT 2.

The theory has to be relativistic => i.e. have a causal structure.

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Atoms are governed by a non-relativistic Hamiltonian.

Can we use atoms on a lattice?

REQUIREMENT 2.

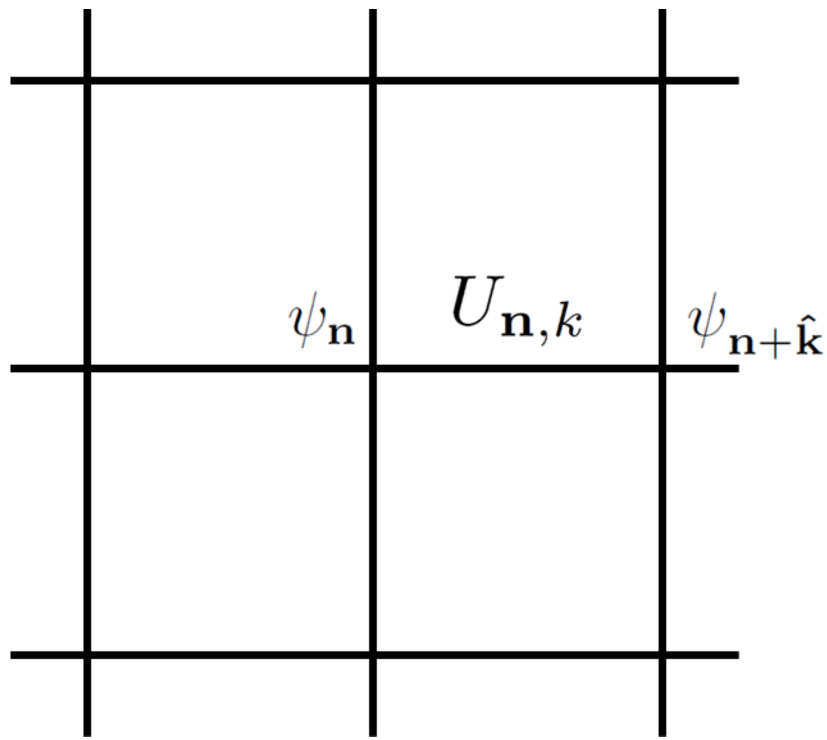
The theory has to be relativistic => have a causal structure.

The atomic dynamics (and Hamiltonian) is non-relativistic.

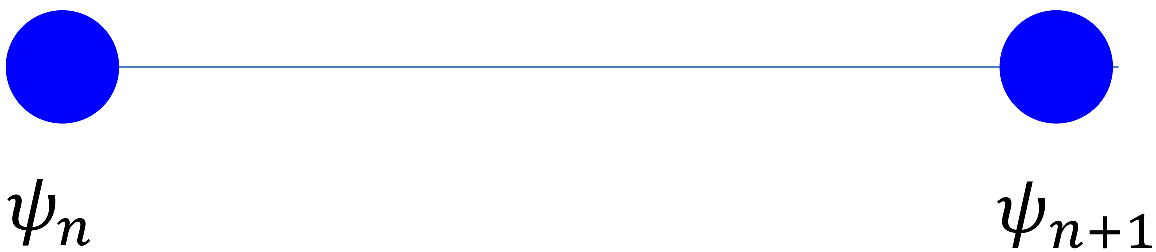
Can we use atoms trapped on a lattice?

✓ If our model on the lattice has
the correct continuum limit !

LATTICE GAUGE THEORY



Toy Example: U(1)



$$H = \sum_n M_n \psi_n^\dagger \psi_n + \epsilon \sum_n \left(\psi_n^\dagger \psi_{n+1} + h.c. \right)$$

Toy Example: U(1)

$$H = \sum_n M_n \psi_n^\dagger \psi_n + \epsilon \sum_n \left(\psi_n^\dagger \psi_{n+1} + h.c. \right)$$

H is invariant under global transformations:

$$\psi_n \longrightarrow e^{-i\Lambda} \psi_n \quad ; \quad \psi_n^\dagger \longrightarrow \psi_n^\dagger e^{i\Lambda}$$

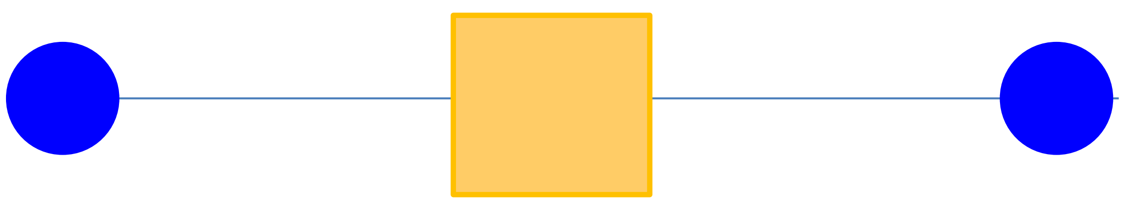
Toy Example: U(1)

Promote the transformation to be local:

$$\psi_n \longrightarrow e^{-i\Lambda_n}\psi_n \quad ; \quad \psi_n^\dagger \longrightarrow \psi_n^\dagger e^{i\Lambda_n}$$

Add a **new field** on the links:

$$U_n = e^{i\phi_n}$$



Toy Example: U(1)

$$H = \sum_n M_n \psi_n^\dagger \psi_n + \epsilon \sum_n \left(\psi_n^\dagger U_n \psi_{n+1} + h.c.\right)$$

Invariance under a **local** gauge transformations:

$$\psi_n \longrightarrow e^{-i\Lambda_n} \psi_n \quad ; \quad \psi_n^\dagger \longrightarrow \psi_n^\dagger e^{i\Lambda_n}$$

$$\phi_n \longrightarrow \phi_n + \Lambda_{n+1} - \Lambda_n$$

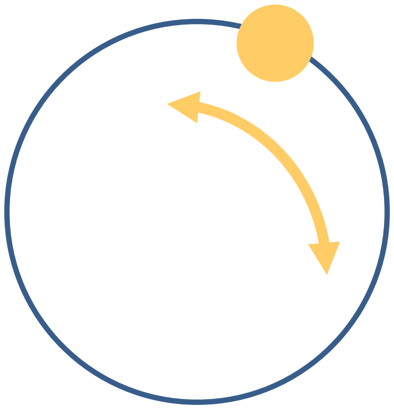
Toy Example U(1): ON LINKS

Gauge field kinetic energy:

$$H_E = \frac{g^2}{2}\sum_n L_n^2$$

$$L\left|m\right\rangle = m\left|m\right\rangle$$

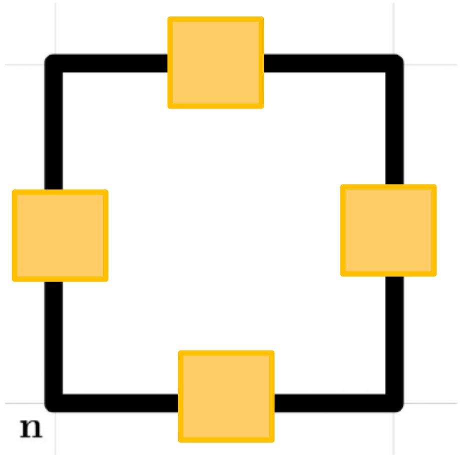
$$u_m\left(\phi\right) = \langle\phi|m\rangle = \frac{1}{\sqrt{2\pi}}e^{im\phi}$$



Mechanical Analog

Toy Example: U(1): D>1: PLAQUETTES

Gauge field potential energy:



$$H_B = -\frac{1}{2g^2} \sum_{plaquettes} U_1 U_2 U_3^\dagger U_4^\dagger + h.c. =$$

$$-\frac{1}{g^2} \sum_{plaquettes} \cos\left(\phi_1 + \phi_2 - \phi_3 - \phi_4\right)$$

In the continuum limit, this REDUCES to $(\nabla \times \boldsymbol{A})^2$: the magnetic energy density.

3.

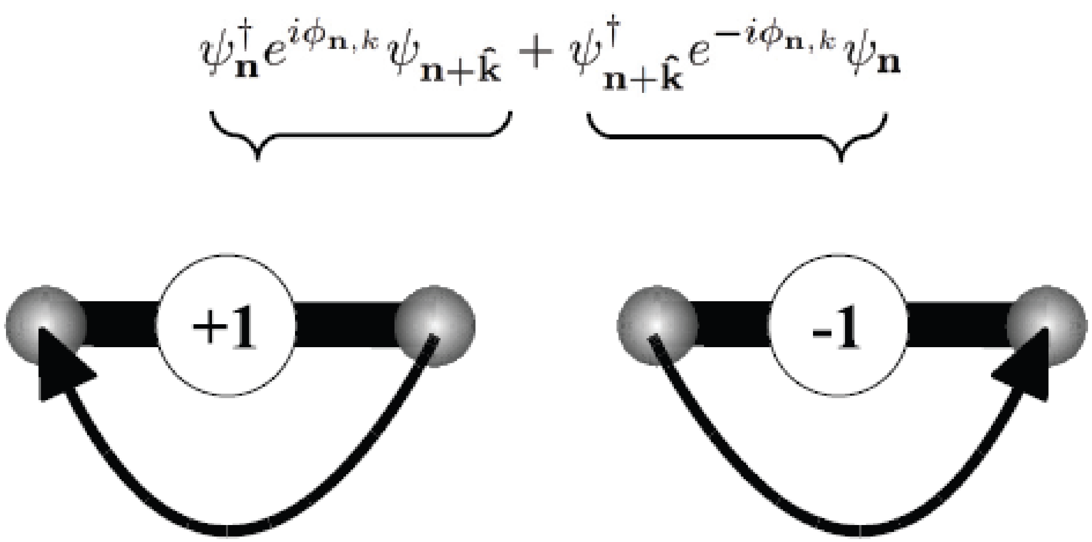
Local gauge invariance:

IN ATOMIC SYSTEMS???

REQUIREMENT 3.

The theory has to be local gauge invariant.

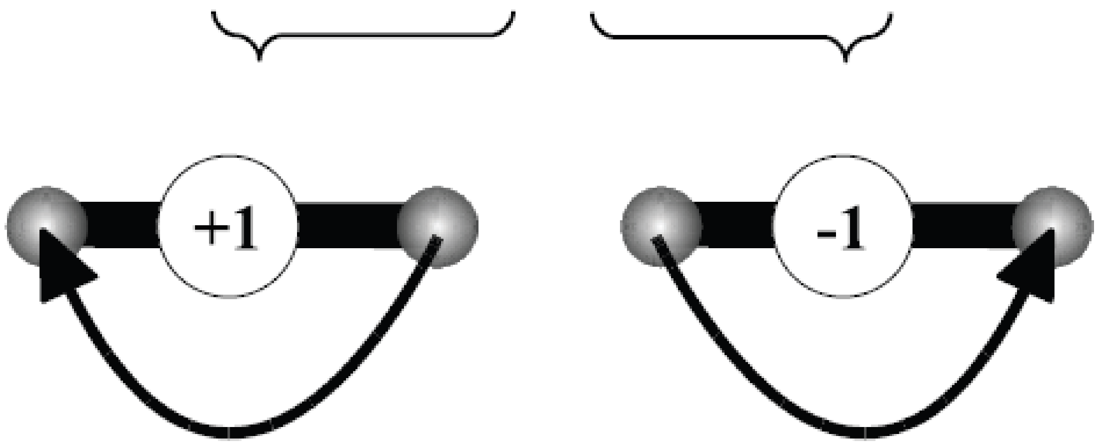
local gauge invariance = "charge" conservation



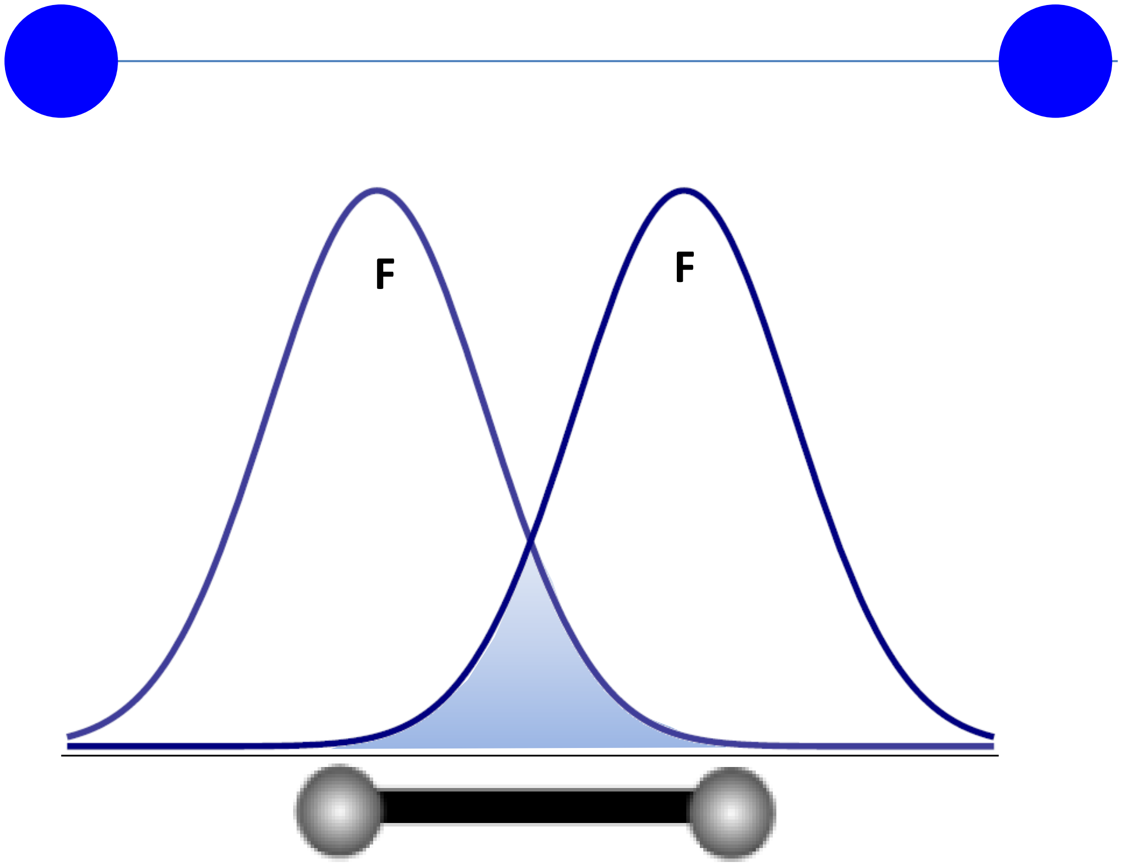
REQUIREMENT 3.

The atomic Hamiltonian conserves total number – ***only global symmtery***!

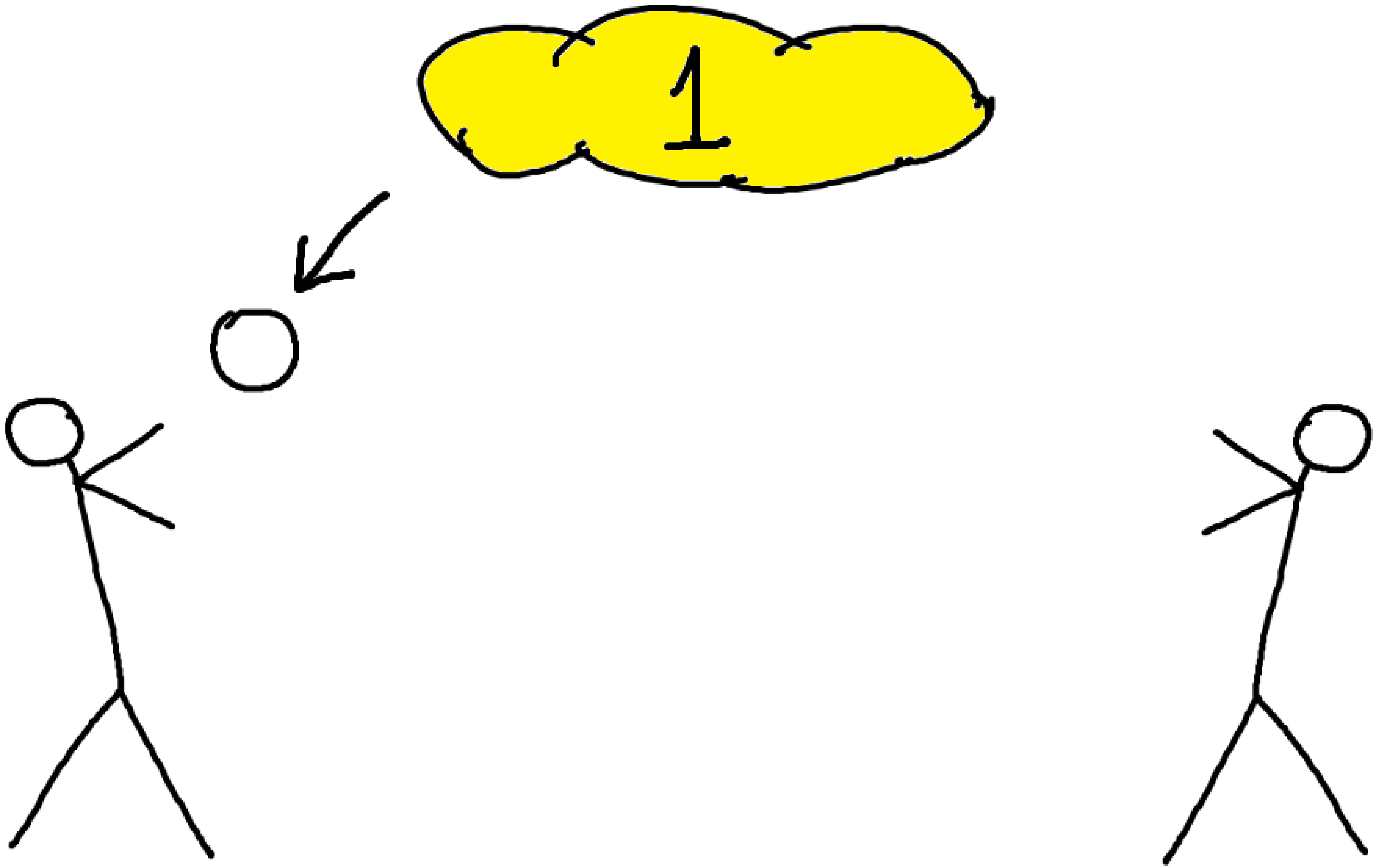
$$\psi^\dagger_{\mathbf{n}} e^{i\phi_{\mathbf{n},k}} \psi_{\mathbf{n}+\hat{\mathbf{k}}} + \psi^\dagger_{\mathbf{n}+\hat{\mathbf{k}}} e^{-i\phi_{\mathbf{n},k}} \psi_{\mathbf{n}}$$



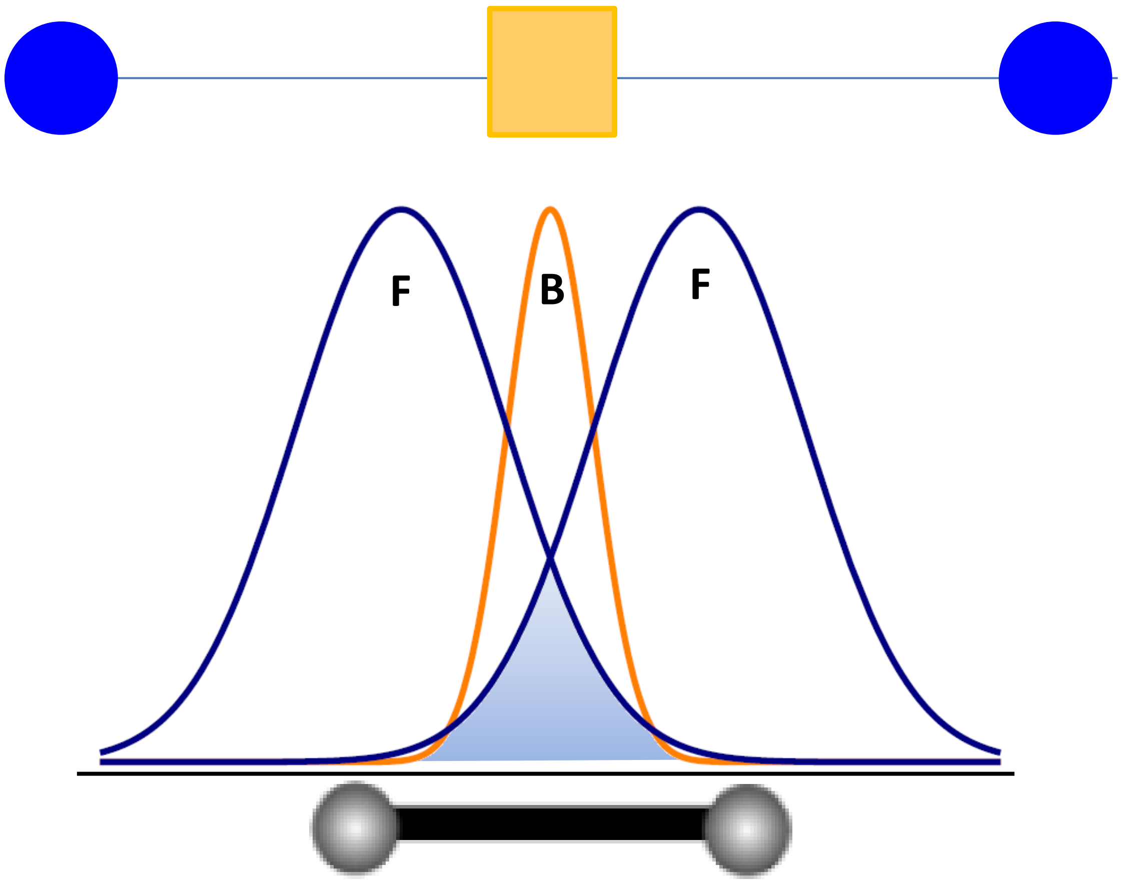
TUNNELING OF FERMIONS



LOCAL GAUGE INVARIANCE => MEDIATOR!



TUNNELING W. BOSONS ON THE LINK



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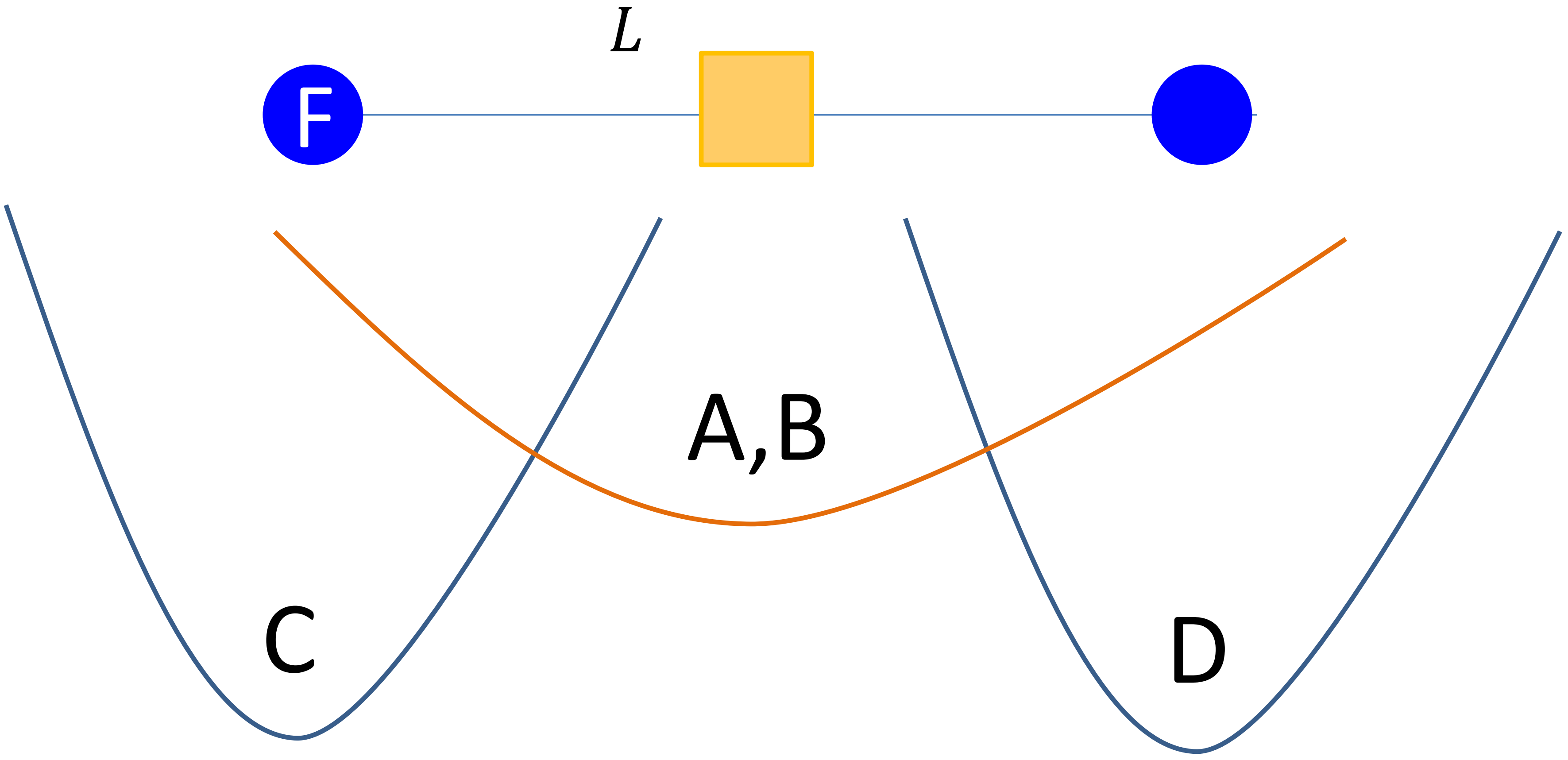

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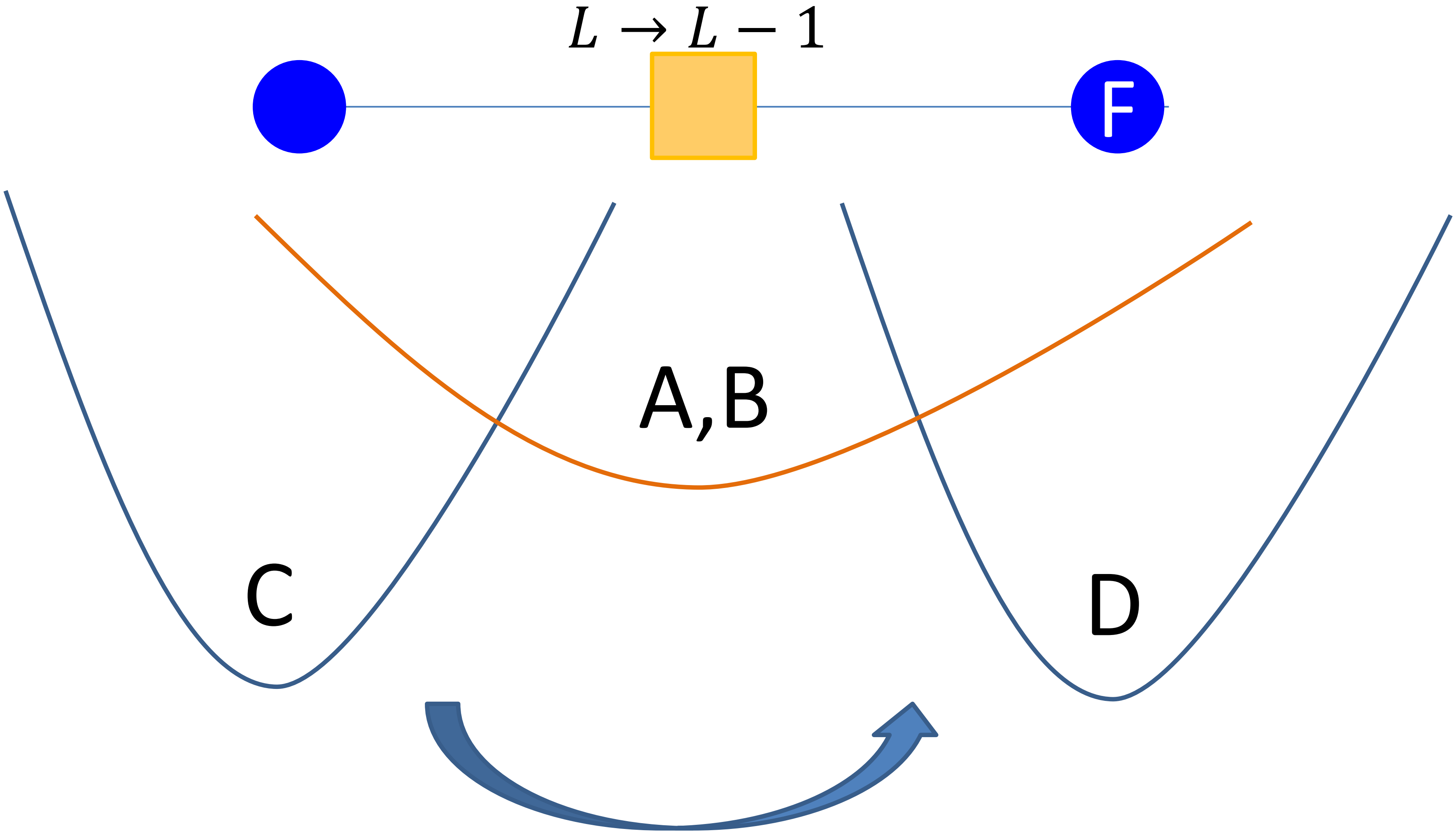
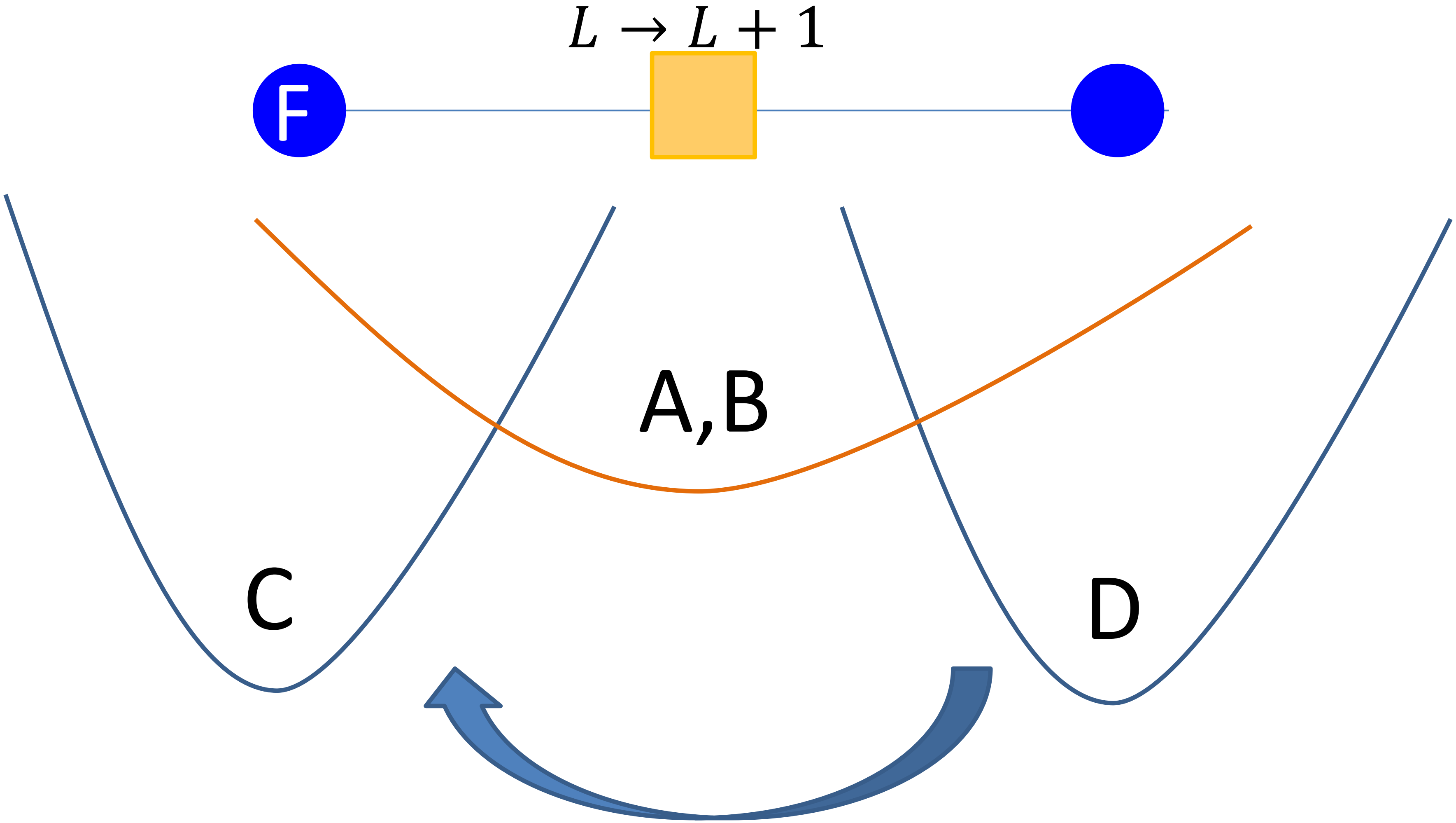
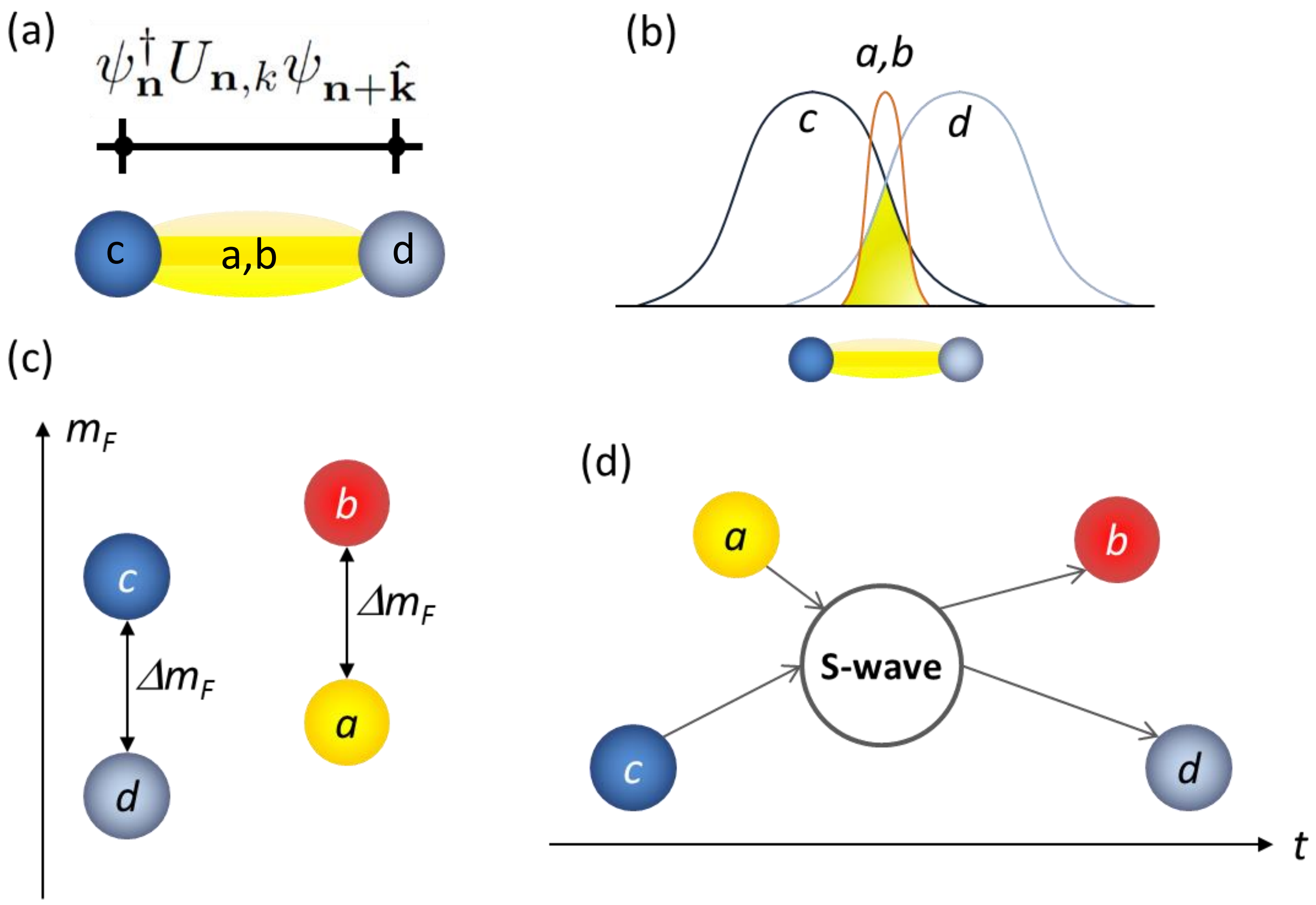


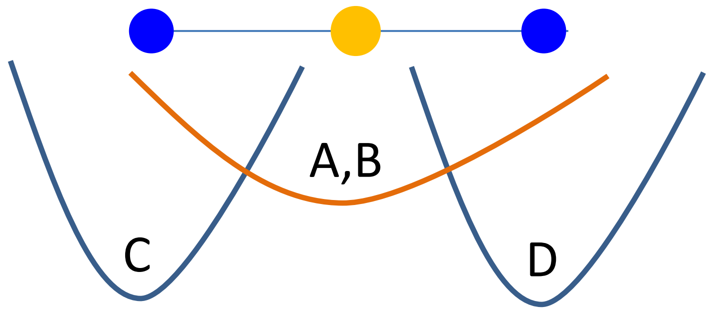
TUNNELING W. BOSONS ON THE LINK



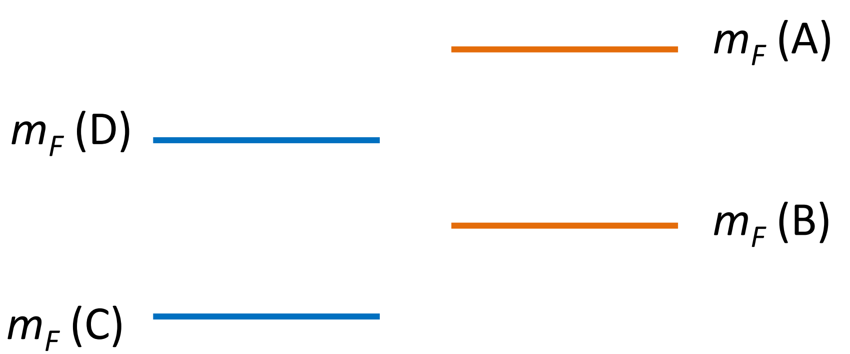
REALIZING A LINK



ANG. MOM. CONSERVATION ⇔ LOCAL GAUGE INVARIANCE



$$c^\dagger a^\dagger b d + d^\dagger b^\dagger a c$$



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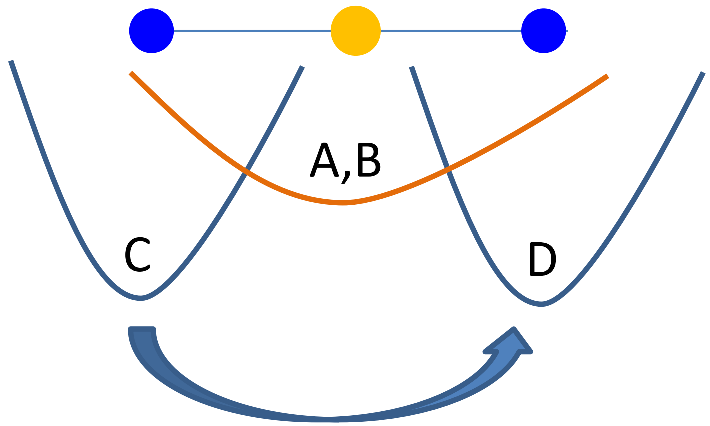

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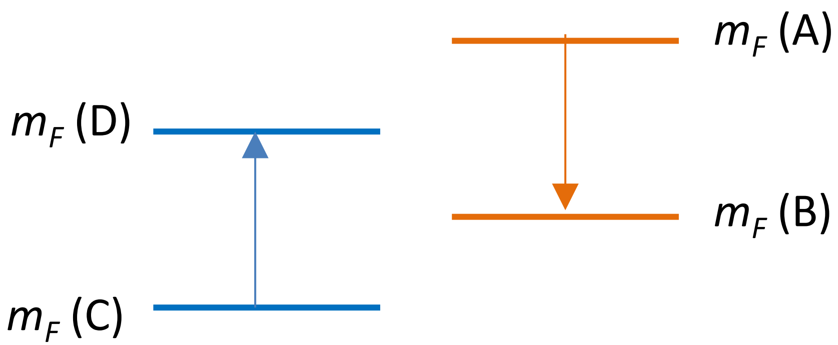

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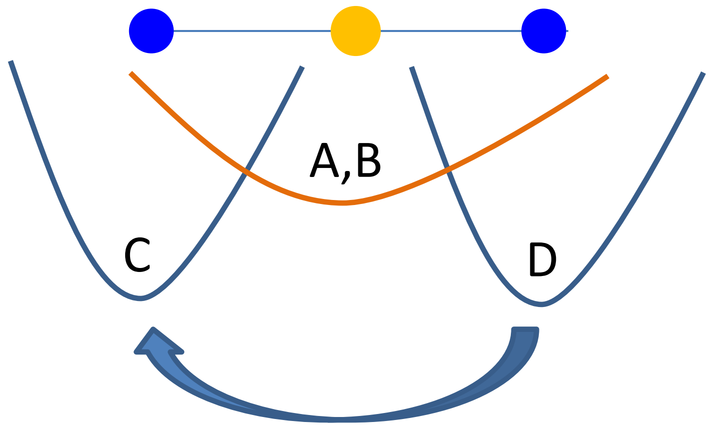

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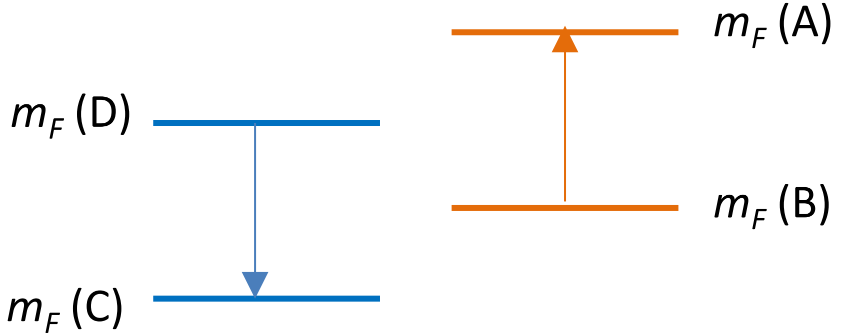

GAUGE BOSONS AND SCHWINGER'S ALGEBRA

$$L_+ = a^\dagger b \qquad L_- = b^\dagger a$$

$$L_z = \frac{1}{2}\left(a^\dagger a - b^\dagger b\right) \qquad \ell = \frac{1}{2}\left(a^\dagger a + b^\dagger b\right)$$

GAUGE BOSONS AND SCHWINGER'S ALGEBRA

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$$L_z = \frac{1}{2}\left(a^\dagger a - b^\dagger b\right) \qquad \ell = \frac{1}{2}\left(a^\dagger a + b^\dagger b\right)$$

and thus what we have is

$$c^\dagger a^\dagger b d + d^\dagger b^\dagger a c$$

$$\Downarrow$$

$$\psi_L^\dagger L_+ \psi_R + \psi_R^\dagger L_- \psi_L$$

LOCAL GAUGE INVARIANCE: ON LINKS

$$\psi_L^\dagger L_+ \psi_R + \psi_R^\dagger L_- \psi_L$$

For large ℓ , $m \ll \ell$

$$L_+ = a^\dagger b \sim e^{i(\phi_a - \phi_b)} \equiv e^{i\phi} = U$$

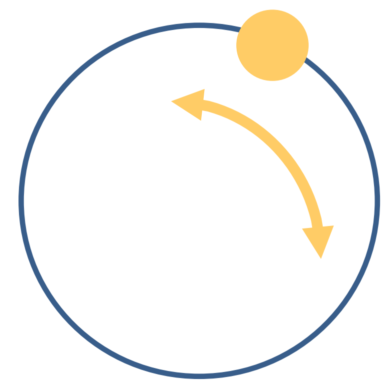
$$\boxed{\psi_L^\dagger U \psi_R + \psi_R^\dagger U^\dagger \psi_L} \checkmark$$

Qualitatively similar results can be obtained with just two bosons on the link, as the U(1) gauge symmetry is ℓ-independent.

KINETIC TERM ⇔ BOSONIC SCATTERING

$$E^2 = L_z^2 = \frac{1}{4}(N_a - N_b)^2$$

$$= \frac{1}{4}(N_a^2 + N_b^2 - 2N_aN_b)$$



Mechanical Analog

$$H_E = \frac{g^2}{2}\sum_n L_{z,n}^2$$ ✓

$L_z = \frac{1}{2}(N_a - N_b)$ *conjugate to* $\phi \equiv \phi_a - \phi_b$

SCHWINGER MODEL: cQED D=1

Quantum Simulation of The Schwinger model (with staggered fermions):

$$H = M\sum_n (-1)^n \psi_n^\dagger \psi_n + \alpha\left(\psi_n^\dagger U_n \psi_{n+1} + H.c.\right)$$

F-B scattering: link interaction

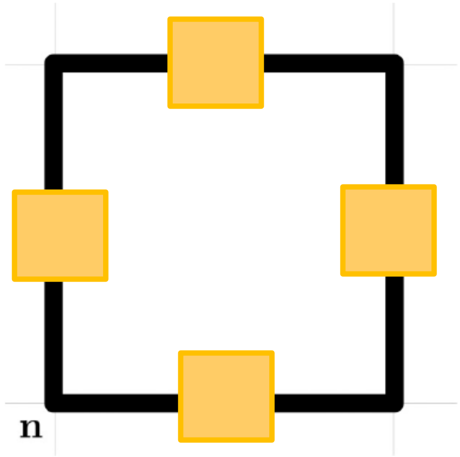
$$+\frac{g^2}{2}\sum_n L_{nz}^2$$

B-B Scattering: electric energy

D>1 REQUIRES PLAQUETTES

$$H_B = -\frac{1}{2g^2} \sum_{plaquettes} U_1 U_2 U_3^\dagger U_4^\dagger + h.c. =$$

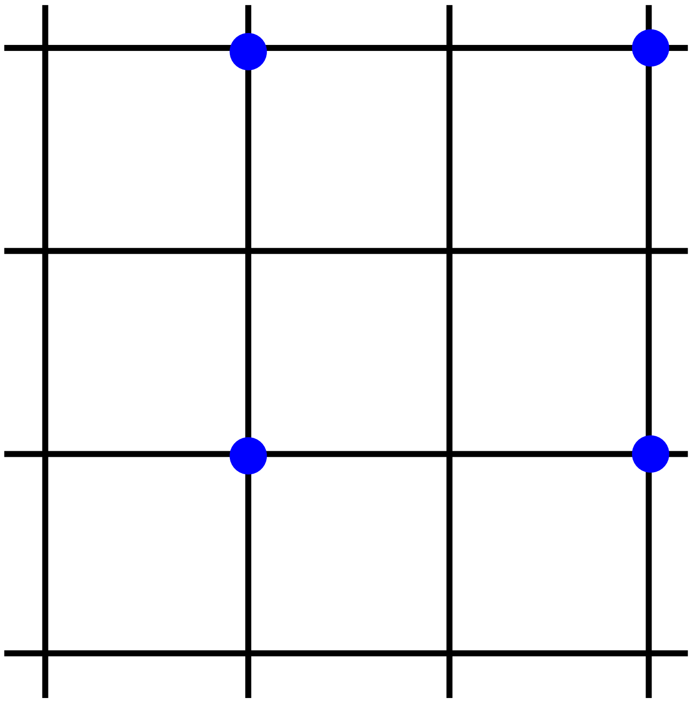
$$-\frac{1}{g^2} \sum_{plaquettes} \cos\left(\phi_1 + \phi_2 - \phi_3 - \phi_4\right)$$



In the continuum limit, this REDUCES to $(\nabla \times \boldsymbol{A})^2$: the magnetic energy density.

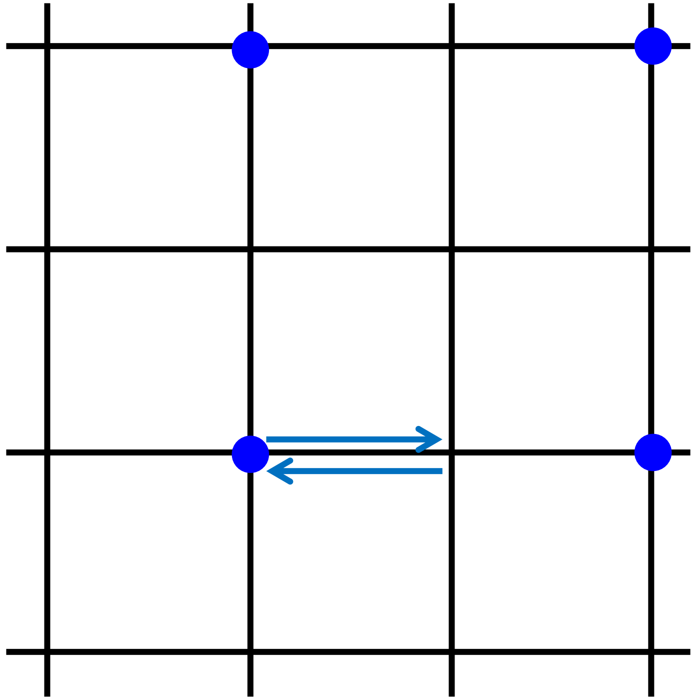
QS PLAQUETTES

Auxiliary fermions



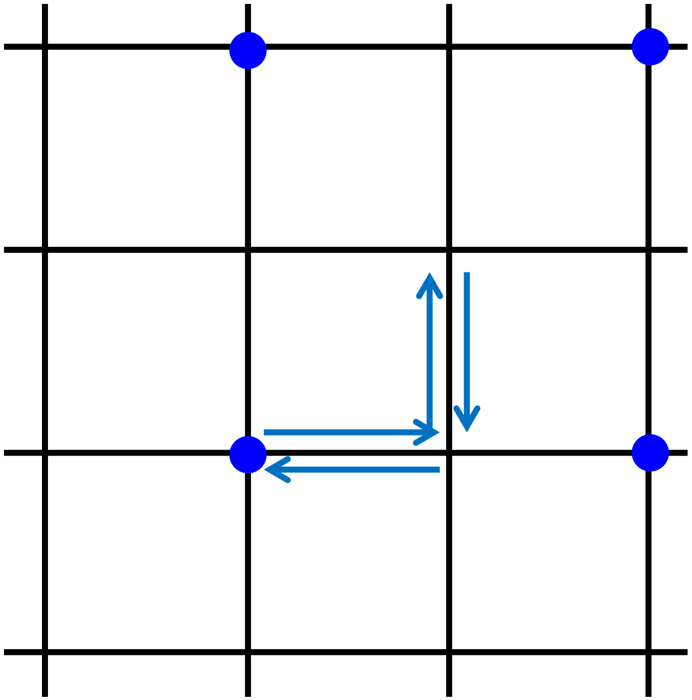
QS PLAQUETTES

Virtual process



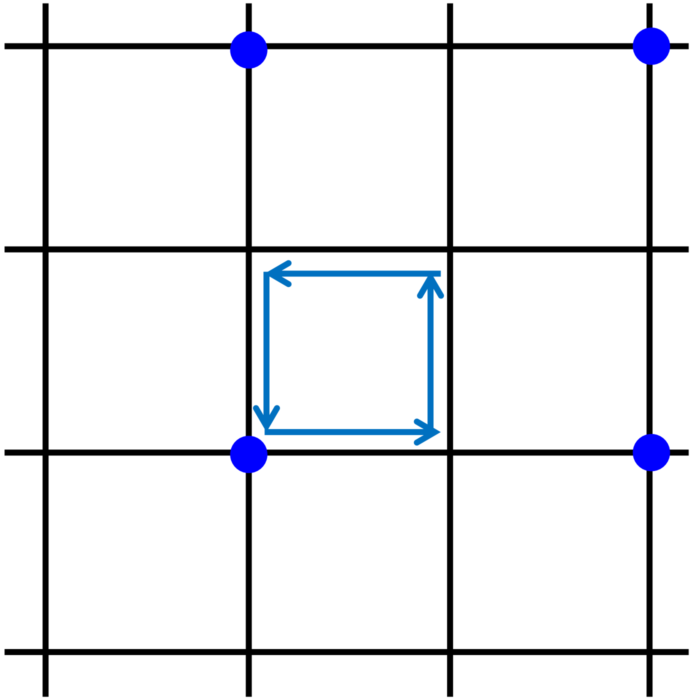
QS PLAQUETTES

Virtual process



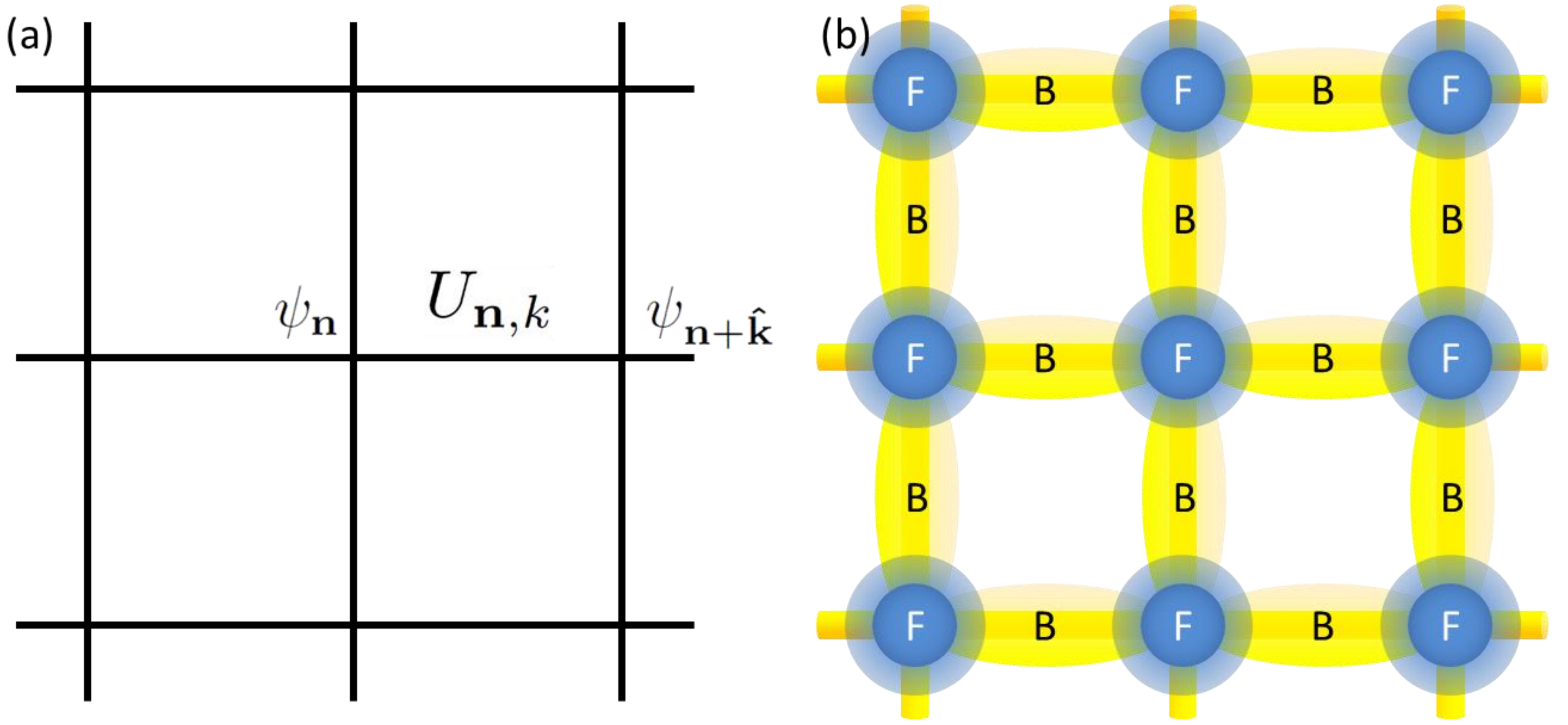
QS PLAQUETTES

Virtual process => $\sum_{\text{plaquettes}} \left(\mathrm{Tr}\left(U_1 U_2 U_3^\dagger U_4^\dagger \right) + h.c. \right)$



discrete groups
abelian groups
non-abelian

QS: U(1) KOGUT-SUSKIND



NON ABELIAN Yang-Mills

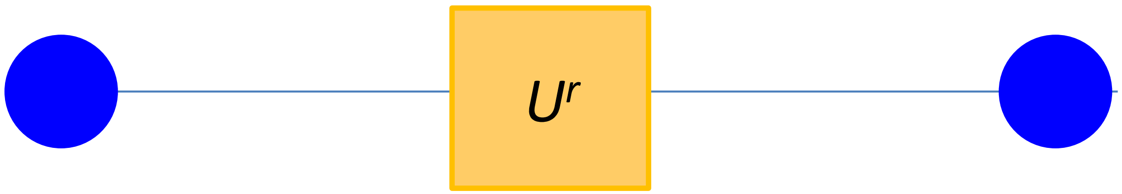
NON ABELIAN Yang-Mills

The STANDARD MODEL is built of particular non-abelian theories, that are Yang-Mills QFTs.

(Celebrating this year 60 since their discovery).

Renormalization (‘t Hooft) , and asymptotic Freedom (Wiltczek, Gross, Polizer), have been proved for Yang-Mills theories.

NON-ABELIAN LINKS



$$\psi_{\mathbf{n}} = (\psi_{\mathbf{n},a}) = \begin{pmatrix} \psi_{\mathbf{n},1} \\ \psi_{\mathbf{n},2} \\ \cdots \end{pmatrix}$$

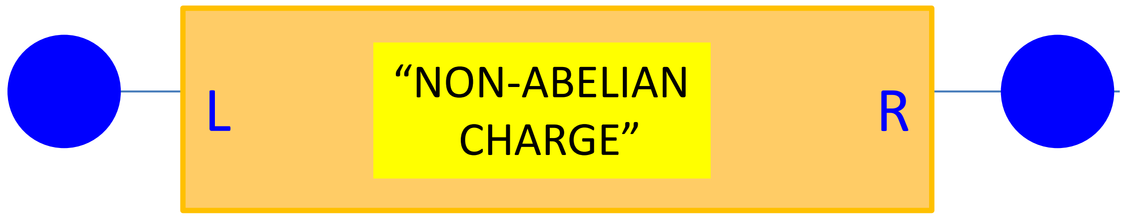
$$\psi_{\mathbf{n}} \to V^r_{\mathbf{n}} \psi_{\mathbf{n}}$$

U^r = element of the gauge group

$$U^r_{\mathbf{n},k} \to V^r_{\mathbf{n}} U^r_{\mathbf{n},k} V^{\dagger r}_{\mathbf{n}+\hat{\mathbf{k}}}$$

$$H_{int} = \epsilon \sum_{\mathbf{n},k} \left(\psi^\dagger_{\mathbf{n}} U^r_{\mathbf{n},k} \psi_{\mathbf{n}+\hat{\mathbf{k}}} + h.c. \right)$$

LEFT AND RIGHT SIDES OF THE LINK



$$[L_a, R_b] = 0$$

$$(G_{\mathbf{n}})_a = \mathrm{div}_{\mathbf{n}} E_a = \sum_k \left((L_{\mathbf{n},k})_a - \left(R_{\mathbf{n}-\hat{\mathbf{k}},k} \right)_a \right)$$

$$[L_a, U^r] = T_a^r U^r \quad ; \quad [R_a, U^r] = U^r T_a^r$$

$$[L_a, L_b] = -i f_{abc} L_c \quad ; \quad [R_a, R_b] = i f_{abc} R_c \quad ;$$

$$\sum_a L_a L_a = \sum_a R_a R_a \equiv \sum_a E_a E_a$$

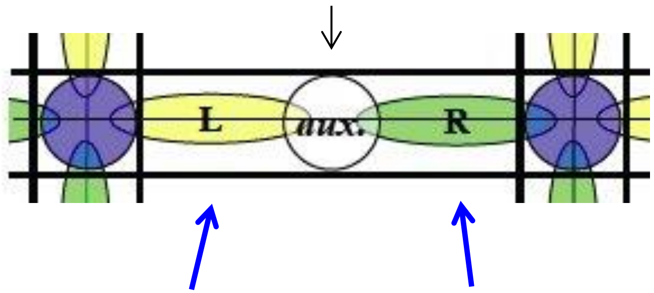
MECHANICAL ANALOG:

BODY AND LABORATORY REPRESENTATIONS FOR A TOP’S ANGULAR MOMENTUM

QS: SU(2)

Ancillary “constraint” Fermion

“color” fermions

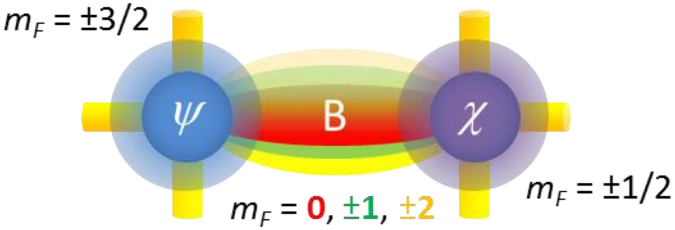


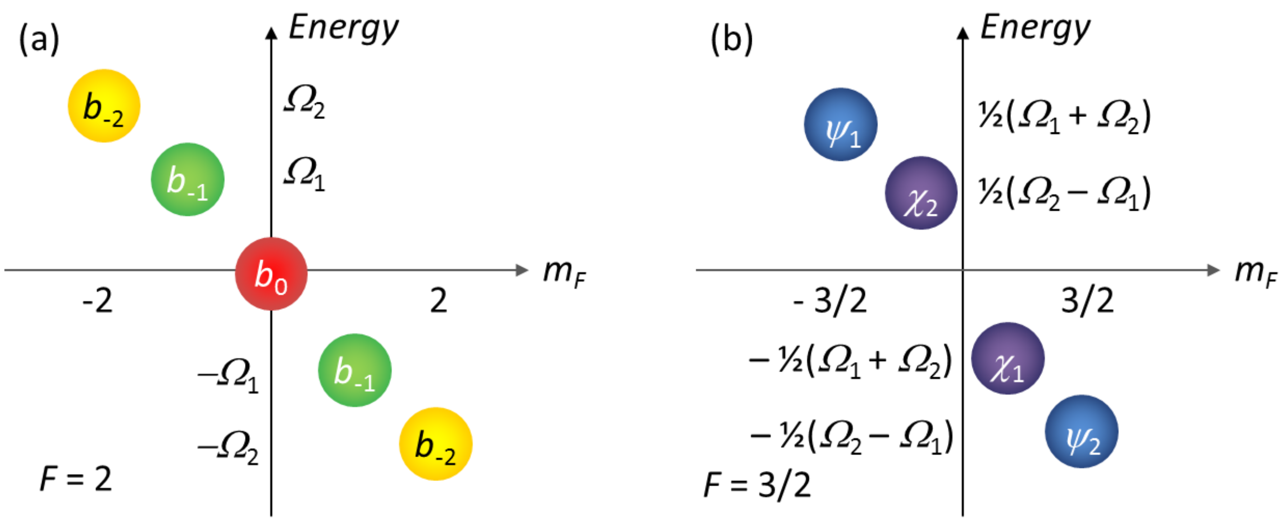
$$U_L = \frac{1}{\sqrt{N_L+1}} \begin{pmatrix} a_1^\dagger & -a_2 \\ a_2^\dagger & a_1 \end{pmatrix} \; ; \; U_R = \begin{pmatrix} b_1^\dagger & b_2^\dagger \\ -b_2 & b_1 \end{pmatrix} \frac{1}{\sqrt{N_R+1}}$$

$$U = U_L U_R$$

On each link – $a_{1,2}$ bosons on the left, $b_{1,2}$ bosons on the right

QS: SU(2) W. ANGULAR MOMENUM CONSERVATION





SOME COMMENTS:

THE DARK SIDE OF ASYMPTOTIC FREEDOM

- QCD becomes highly non-perturbative at low energies.
- EXACT methods have been successful for certain toy models. (e.g. 2+1 cQED Polyakov). But not to QCD in 3+1 dimensions, or with dynamical matter.
- Proof of confinement (or Mass Gap) in Yang Mills theory = one Clay institute's Millenium problems.

CLASSICAL SIMULATION: MONTE CARLO & TN

- WILSON'S APPROACH: Monte Carlo methods + discretized Euclidean spacetime. Has been very successful.

  PROBLEMS:

- Many quarks (quark-gluon plasma, color superconductivity): Grassman integration for fermions gives rise to a "sign problem"
- Cannot be used to calculate real time dynamics.

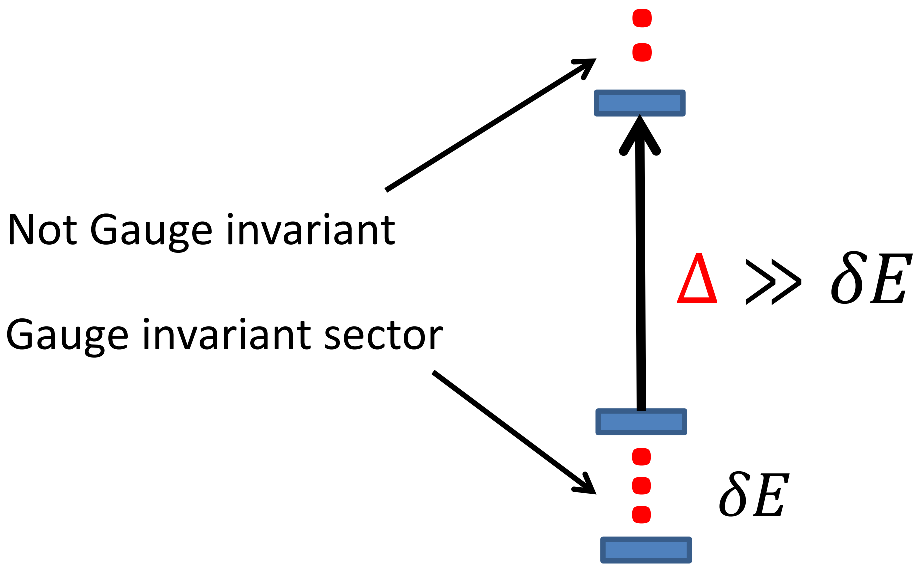
  Provides only correlations.

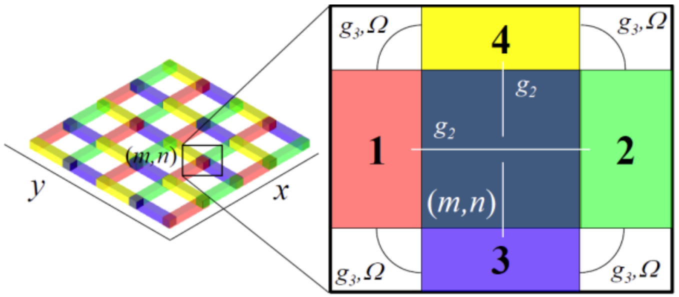
- TENSOR NETWORKS: CURRENTLY STILL RESTRICTED TO D=1.

"**Emerging**" Local Gauge Invariance at low enough energies

<u>Gauss's law</u> is added as a constraint.

<u>Low energy effective gauge invariant KS Hamiltonian</u>.



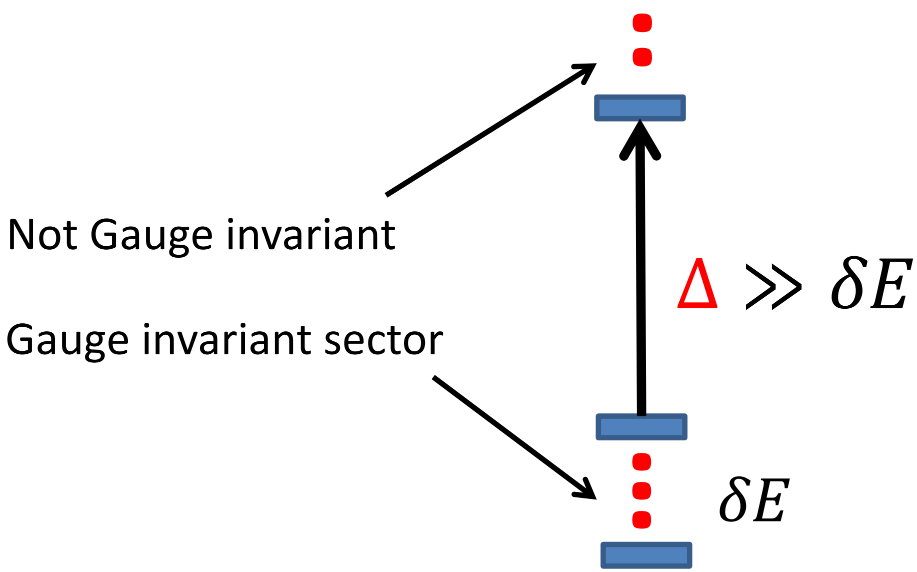


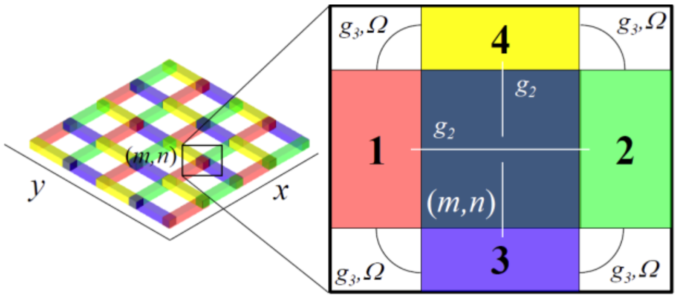
E. Zohar & BR, PRL. (2011)

“**Emerging**“ Local Gauge Invariance at low enough energies

<u>Gauss’s law</u> is added as a constraint.

<u>Low energy effective gauge invariant KS Hamiltonian</u>.





E. Zohar & BR, PRL. (2011)

ROBUSTNESS w. imprefections

=> static Higgs

Kasamatsu et. al. PRL 2013, 2014.

TOY MODELS

- Confinement in Abelian lattice models

- Toy models with "QCD-like properties" that capture the essential physics of confinement.

QED IN 1+1d : SCHWINGER'S MODEL

- No magnetic fields: EM has no dynamics of its own. Non trivial dynamics obtained by coupling to dynamical charge sources.

- Schwinger: e^+e^- form bound states. (analytic and lattice results available.)

- Non-abelian extension: in 1+1: QCD_2 version, not completely solved. Only in the large-N limit ('t Hooft).

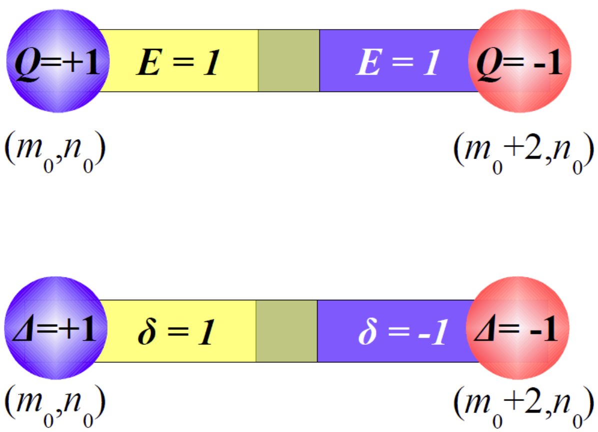
CONFINEMENT IN LATTICE CQED MODELS

- 1+1D: "Schwinger model" manifests confinement. (analytic and lattice results available).

- 2+1D: confinement for all values of the coupling constant, a non-perturbative mechanism (Polyakov).
  (For T > 0: there is a phase transition also in 2+1 D.)

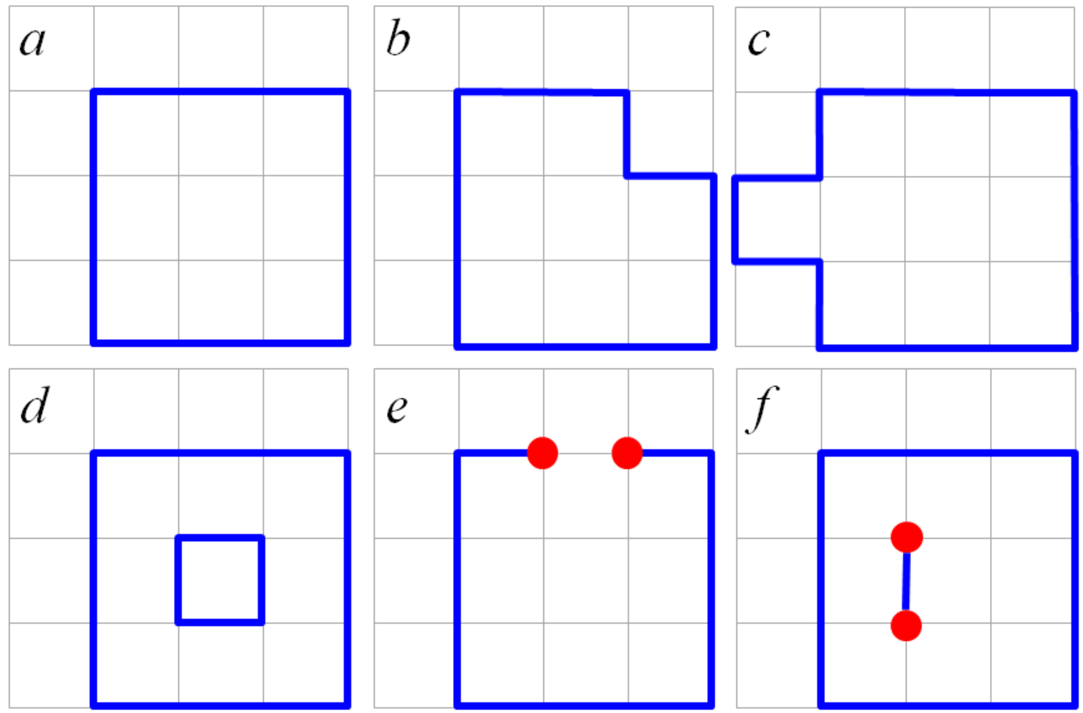
- 3+1D: phase transition between strong coupling confinement phase, and weak coupling coulomb phase.

DIRECTLY OBSERVE CONFINEMENT



Electric flux tubes

E. Zohar & B. R,
Phys. Rev. Lett. (2011).



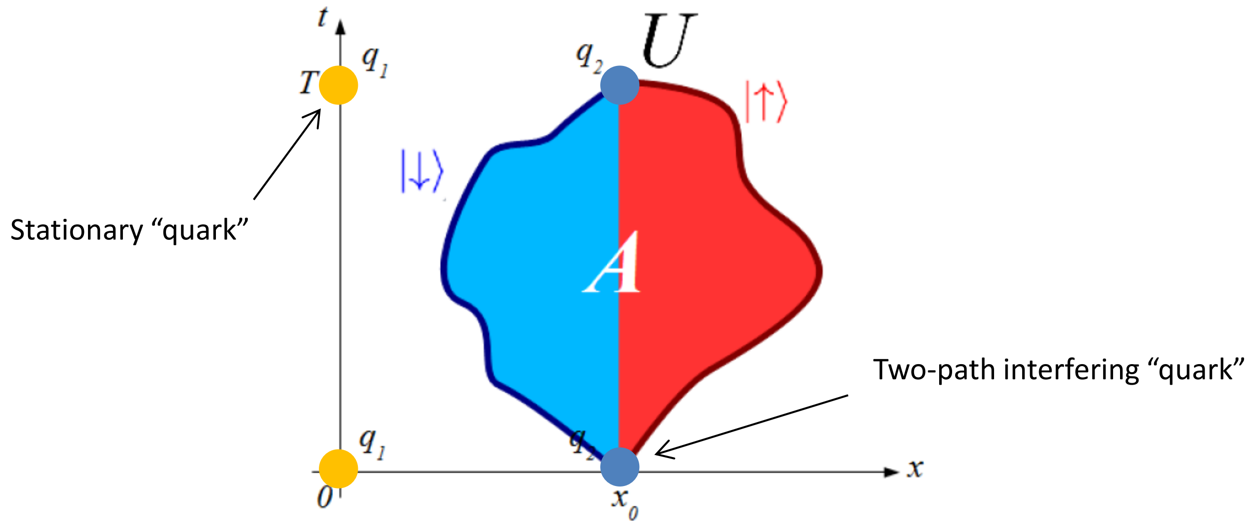
Flux loops deforming and breaking effects

E. Zohar, J. I. Cirac, & B. R,
Phys. Rev. Lett. (2013)

WILSON LOOPS

$$W(C) = P\left(e^{i\oint_C A_\mu dx^\mu}\right)$$

Confining phase → "Area law"



Detecting Wilson's area law is equivalent

to Ramsey Spectroscopy in quantum optics!

E. Zohar & B. R, New J. Phys. 15 (2013) 043041

QS: MODELS

	1+1 with matter	2+1 Pure	2+1 with matter
$U(1)$	Full KS + trunc.	Full KS + trunc.	Full KS + trunc.
$\mathbb{Z}_3$	Full	Full	Full
$SU(2)$	YM + trunc.	YM + trunc. (st. c.)	YM + trunc. (st. c.)

KS = Kogut Susskind
YM = Yang Mills theory
st. c= Strong coupling limit

that’s a wonderful problem, because it doesn’t look so easy…

"…nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly that's a wonderful problem, because it doesn't look so easy."

Richard Feynman, **Simulating physics with computers**, 1982

REFERECES

QS: Wilson’s /Kogut –Susskind Hamiltonian.

For a recent review:

E. Zohar, I. J. Cirac, and B. R. arXiv:1503.02312 (Sub. Rep. Prog. Phys.)

QS: Link/Magnet/Rishons models: Horn (81), Rohrlich& Orland (88), Wiese (96).

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U.-J. Wiese, Annalen der Physik, 525(10-11):777796, 2013.

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THANK YOU!