Cold Atoms in Cavities: a New Frontier in Cavity QED

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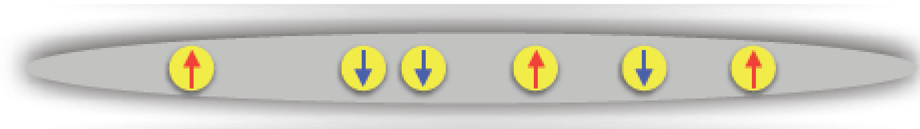
Houston, Texas, USA

INT Program on Frontiers in Quantum Simulation with Cold Atoms, Seattle, May, 2015

Original title

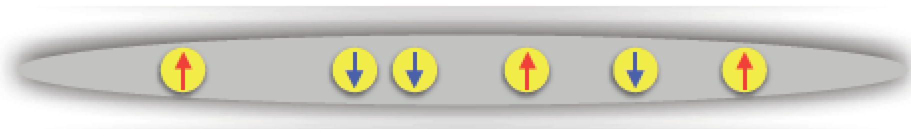
Strongly Interacting Quantum Gases in 1D Traps



Li Yang, Liming Guan, HP, PRA **91**, 043634 (2015)

Goal: construct an effective model

Strongly Interacting Quantum Gases in 1D Traps



$$H = \underbrace{\sum_{i=1}^{N}\left[-\frac{1}{2}\frac{\partial^2}{\partial x_i^2} + V(x_i)\right]}_{H_f} + \underbrace{g\sum_{i<j}\delta(x_i - x_j)}_{H_{\text{int}}}$$

For large $g \rightarrow \infty$:

H_{int} : unperturbed Hamiltonian

H_f : perturbation

Unperturbed system

$$H = \underbrace{g \sum_{i<j} \delta(x_i - x_j)}_{H_{\text{int}}}$$

Ground state manifold: $\{\mathcal{P}_0 : \forall i, j \;\; \Psi(x_i = x_j) = 0\}$

An anti-symmetric wavefunction can be constructed

$$\Psi(x_1 \cdots x_N) = \sum_P (-1)^P P(\varphi(x_1 \cdots x_N)\chi(\sigma_1 \cdots \sigma_N))$$

F. Deuretzbacher et.al. Phys. Rev. Lett. 100,16040 (2008).
Liming Guan et.al. Phys. Rev. Lett. 102, 160402 (2009).

First-order perturbation

$$H = \underbrace{\sum_{i=1}^{N}\left[-\frac{1}{2}\frac{\partial^2}{\partial x_i^2} + V(x_i)\right]}_{H_f} + \underbrace{g\sum_{i<j}\delta(x_i - x_j)}_{H_{\text{int}}}$$

$$\Psi(x_1 \cdots x_N) = \sum_P (-1)^P P(\varphi(x_1 \cdots x_N)\chi(\sigma_1 \cdots \sigma_N))$$

ground state of H_f
(slater determinant)

Second-order perturbation

$$H = \underbrace{\sum_{i=1}^{N}\left[-\frac{1}{2}\frac{\partial^2}{\partial x_i^2} + V(x_i)\right]}_{H_f} + \underbrace{g\sum_{i<j}\delta(x_i - x_j)}_{H_{\text{int}}}$$

$$\Psi(x_1\cdots x_N) = \sum_P (-1)^P P(\varphi(x_1\cdots x_N)\chi(\sigma_1\cdots\sigma_N))$$

ground state of H_f
(slater determinant)

Determined by
super-exchange interaction

$\chi(\sigma_1\cdots\sigma_N)$ are eigenstates of $H_{eff} = -\frac{1}{g}\sum_{i=1}^{N-1} C_i(1-\mathcal{E}_{i,i+1})$

Effective spin-chain model

$$H = \underbrace{\sum_{i=1}^{N}\left[-\frac{1}{2}\frac{\partial^2}{\partial x_i^2} + V(x_i)\right]}_{H_f} + \underbrace{g\sum_{i<j}\delta(x_i - x_j)}_{H_{\text{int}}}$$

$$H_{eff} = -\frac{1}{g}\sum_{i=1}^{N-1} C_i(1 - \mathcal{E}_{i,i+1})$$

$$C_i = 2\cdot\mathcal{S}\int dx_1\cdots dx_N\,|\partial_i\varphi|^2\,\delta(x_{i+1} - x_i)$$

$\mathcal{E}_{i,i+1}$ are exchange operators

$$\mathcal{E}_{i,i+1}|\cdots\sigma_i\sigma_{i+1}\cdots\rangle = |\cdots\sigma_{i+1}\sigma_i\cdots\rangle$$

C_i only depends on V(x) and N

Effective spin-chain model

$$H_{eff} = -\frac{1}{g}\sum_{i=1}^{N-1} C_i(1-\mathcal{E}_{i,i+1})$$

$$E = E^* - \frac{K}{g} + O(\frac{1}{g^2})$$

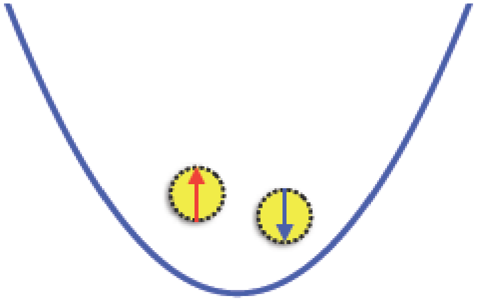
tan contact

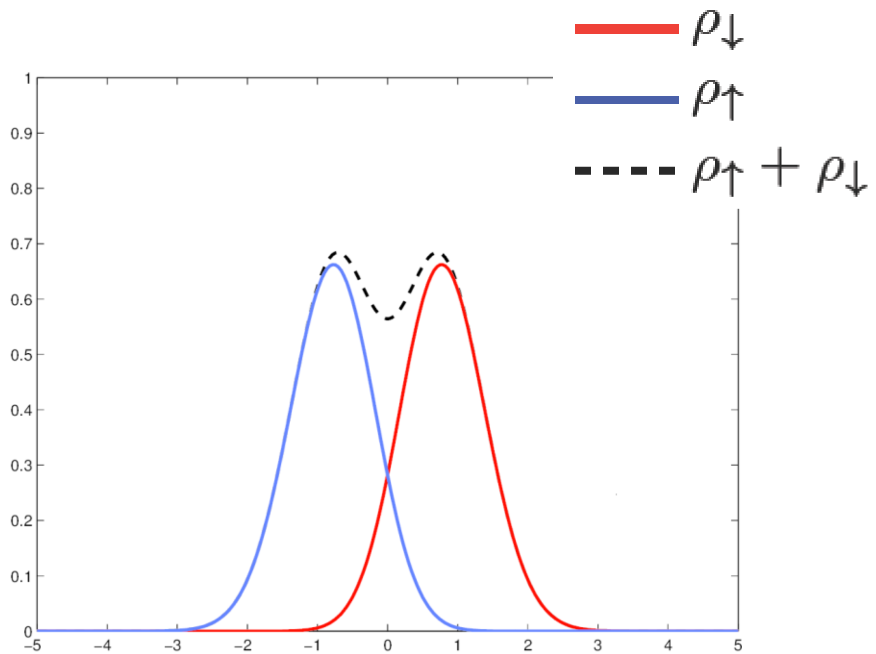
A. G. Volosniev et.al. Nature Communications 5, 5300 (2014)

F. Deuretzbacher, et.al. Phys. Rev. A 90, 013611 (2014)

2 particles in a trap

$$H_{eff} = -\frac{1}{g}\sqrt{\frac{2}{\pi}}(1-\mathcal{E}_{12})$$



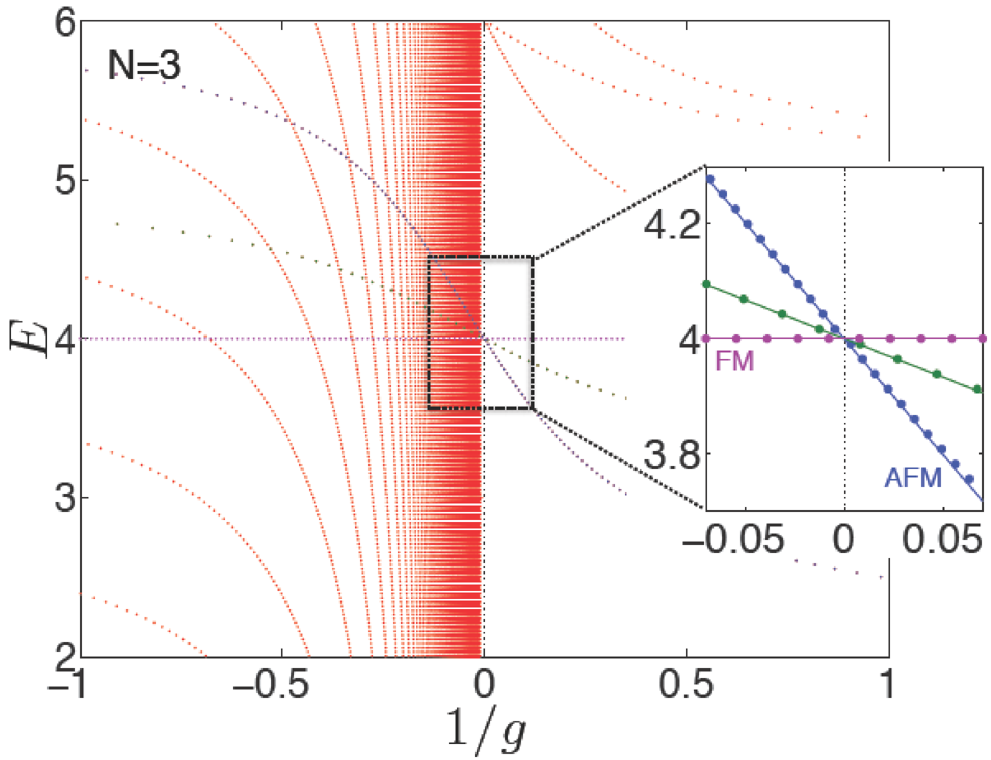


Spin-1/2 fermions

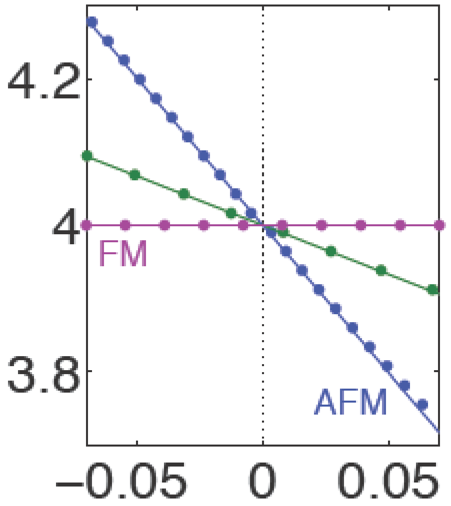
$$H_{eff} = -\frac{1}{g}\sum_{i=1}^{N-1} C_i(1-\mathcal{E}_{i,i+1})$$

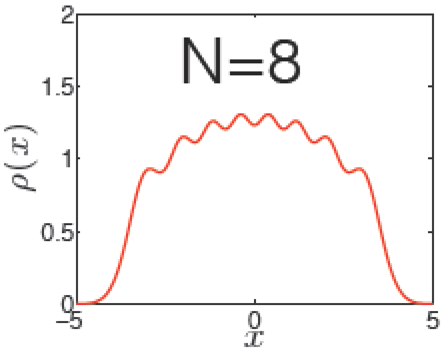
For spin 1/2 fermions, the spin chain models are

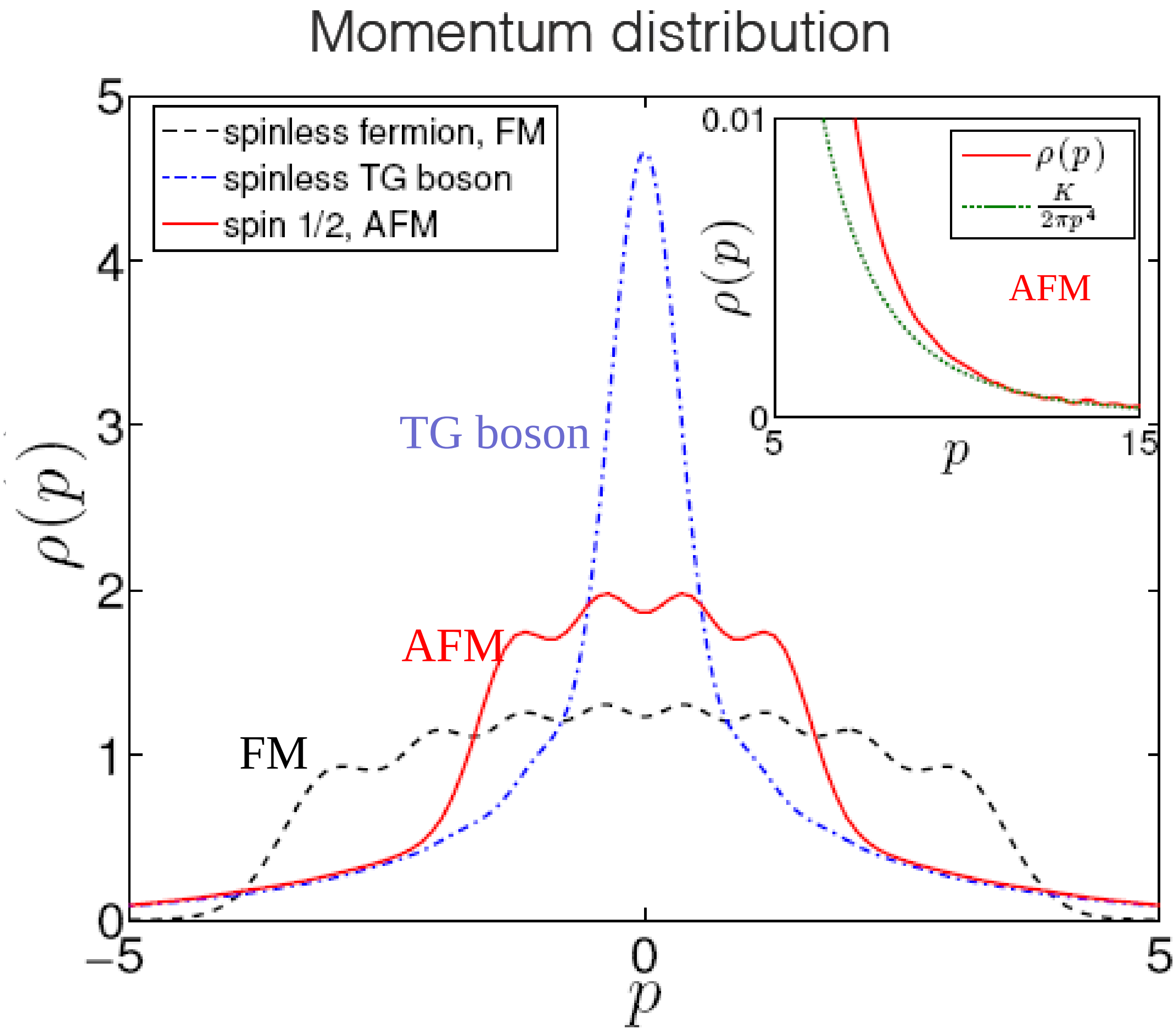
Heisenberg models $H_{\text{eff}} = -\frac{1}{g}\sum_{i=1}^{N-1} C_i(1-\vec{\sigma}_i\cdot\vec{\sigma}_{i+1})/2$



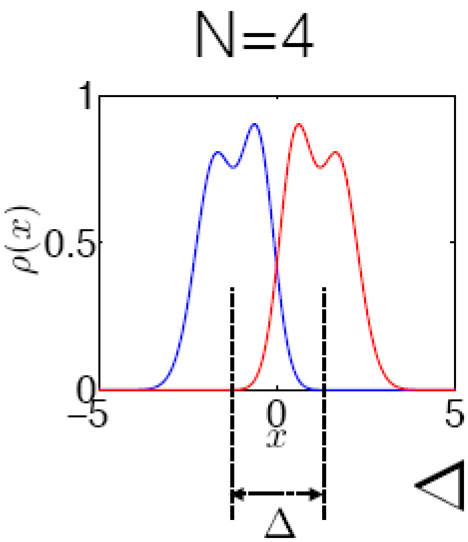
Spin-1/2 fermions: FM vs. AFM

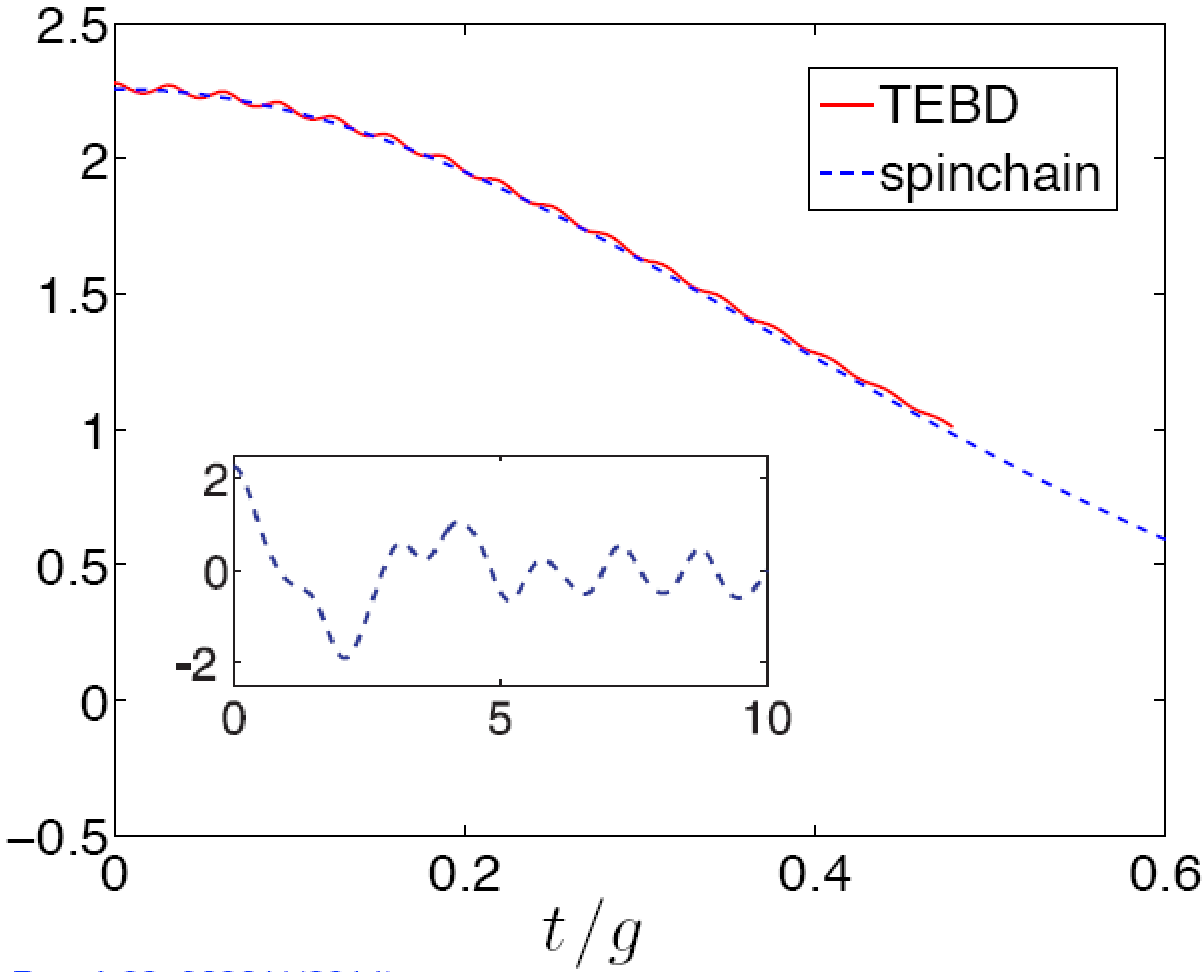






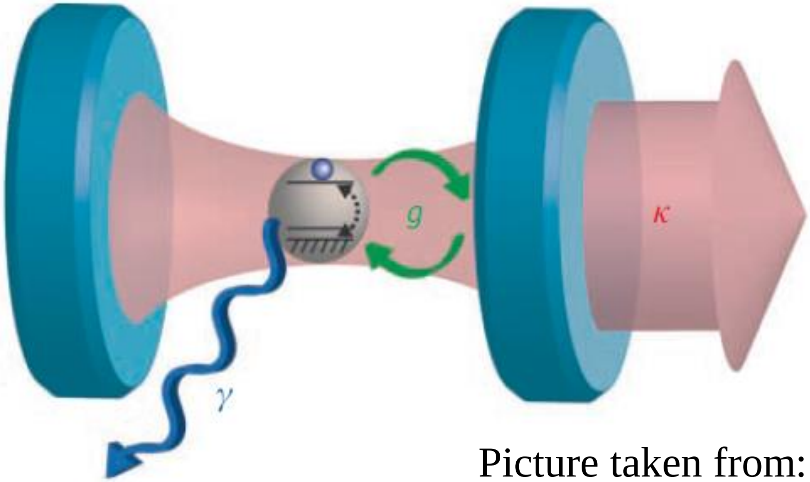
Spin-1/2 fermions: simulating dynamics





X. Cui, and T.-L. Ho, Phys. Rev. A 89, 023611(2014)

Cold atoms in cavity

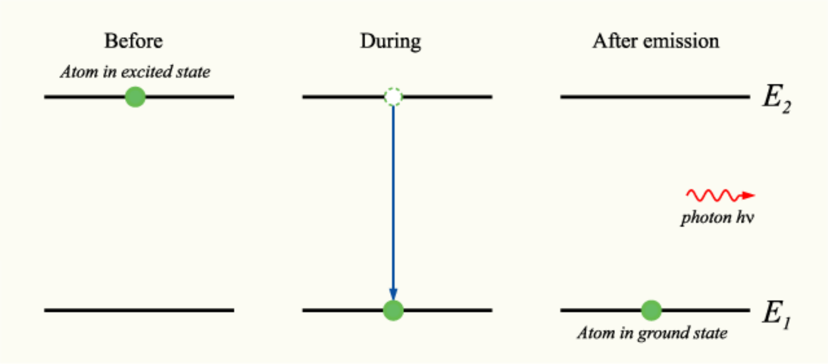


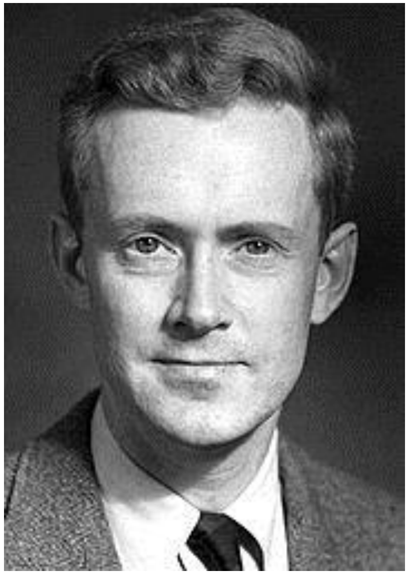
Picture taken from:
Kimble, Nature **453**, 1023 (2008)

Purcell effect: the birth of CQED

Phys. Rev. **69**, 681 (1946)

B10. Spontaneous Emission Probabilities at Radio Frequencies. E. M. PURCELL, *Harvard University.*—For





Spontaneous emission of an excited atom can be controlled.

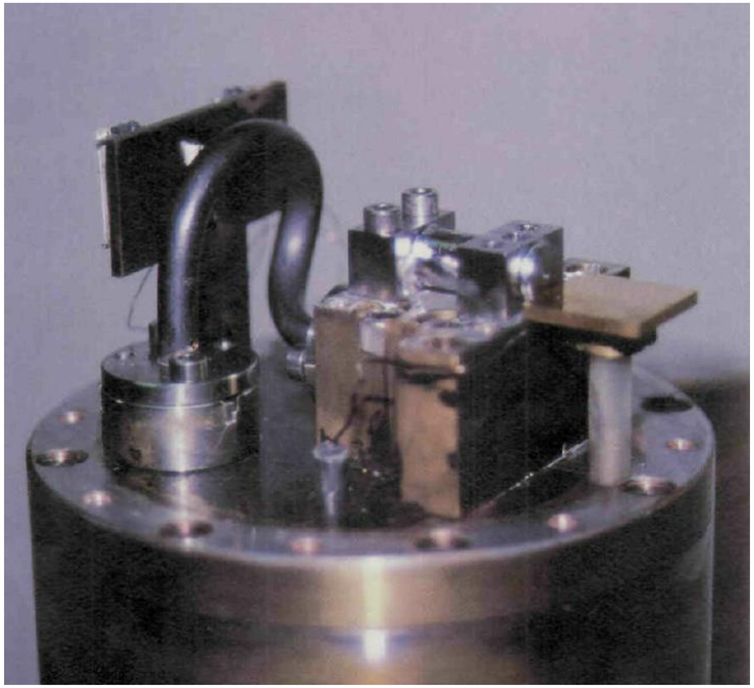
$$\Gamma = \frac{2\pi}{3}\left|\Omega_{eg}\right|^2 \rho\left(\omega_0\right)$$

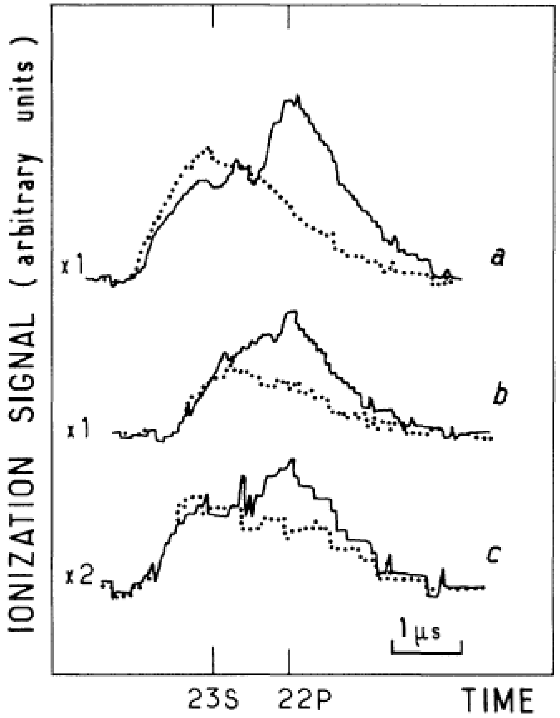
$$\Omega_{eg} = d_{eg}E_{vac} / \hbar, \qquad E_{vac} = \sqrt{\hbar\omega_0 / (2\varepsilon_0 V)}$$

$\rho\left(\omega_0\right)$: density of photon modes at ω_0

Modifying spontaneous emission rate

Enhancement of spontaneous emission.





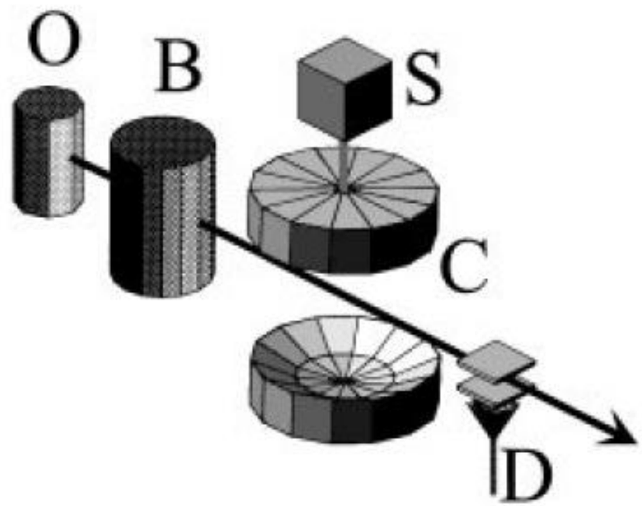
Goy, Raimond, Gross, Haroche, PRL **50**, 1903 (1983)

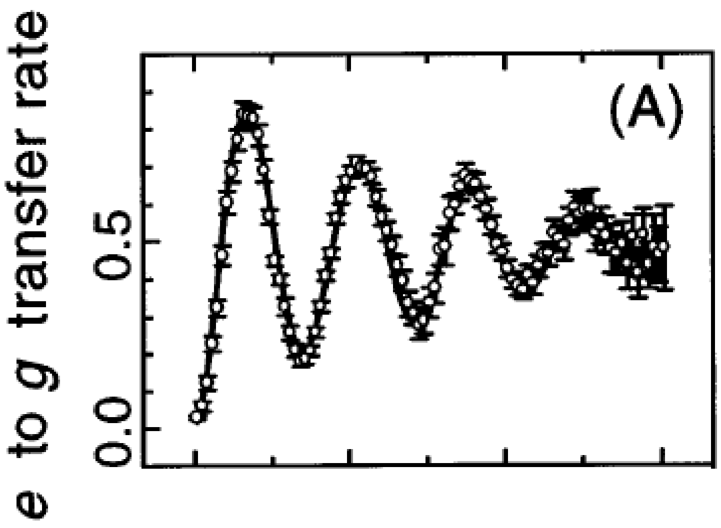
$$\Gamma_{cav} = \eta \Gamma_0$$

$$\eta = \frac{3}{4\pi^2} \frac{Q\lambda^3}{V}: \text{ Purcell factor}$$

Strong coupling regime: Reversible sp. emission

Vacuum Rabi oscillation in high-Q cavity





Brune *et al*., PRL **76**, 1800 (1996)

$$|\Psi(t)\rangle = \cos(gt)|e,0\rangle + \sin(gt)|g,1\rangle$$

Jaynes-Cummings model:

$$H = \frac{\hbar\omega_0}{2}\sigma_z + \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar g\left(\hat{a}\sigma_+ + \sigma_- \hat{a}^\dagger\right), \qquad \sigma_+ = \sigma_-^\dagger = |e\rangle\langle g|$$

$$g \gg \gamma,\ \kappa$$

From 1 atom to N atoms

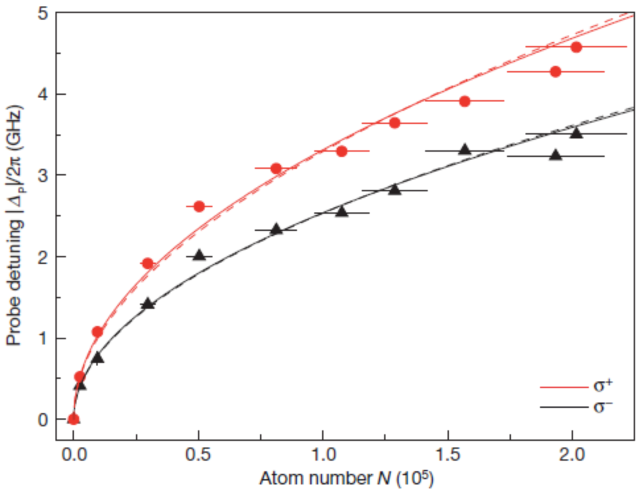
One-atom Jaynes-Cummings model:

$$H = \frac{\hbar\omega_0}{2}\sigma_z + \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar g\left(\hat{a}\sigma_+ + \sigma_- \hat{a}^\dagger\right)$$

N-atom Tavis-Cummings model:

$$H = \hbar\omega_0 J_z + \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar g\left(\hat{a}J_+ + J_- \hat{a}^\dagger\right), \quad J_z = \frac{1}{2}\sum_{i=1}^{N}\sigma_{z,i}, \; J_\pm = \sum_{i=1}^{N}\sigma_{\pm,i}$$

Effective atom-cavity coupling strength: $g' \approx \sqrt{N} g$



Brennecke *et al.*, Nature **450**, 268 (2007)
Colombe *et al.*, Nature **450**, 272 (2007)

From 1 atom to N atoms

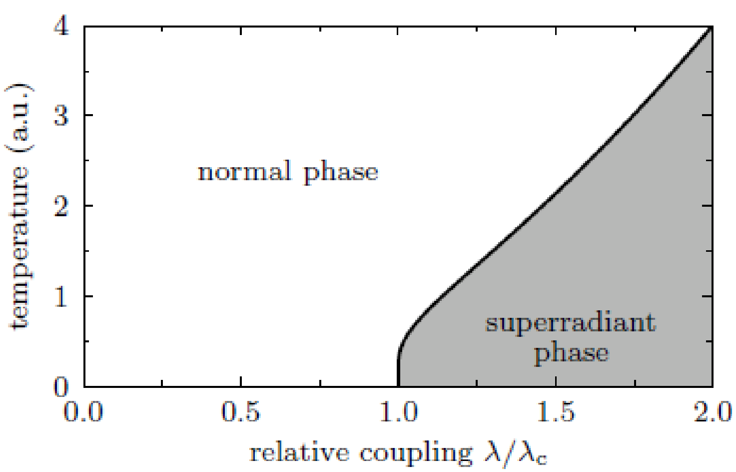
One-atom Rabi model:

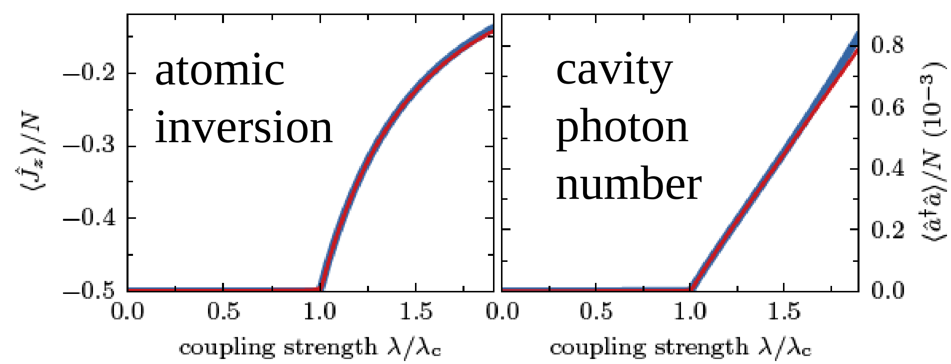
$$H = \frac{\hbar\omega_0}{2}\sigma_z + \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar g\left(\hat{a} + \hat{a}^\dagger\right)\left(\sigma_+ + \sigma_-\right)$$

N-atom Dicke model:

$$H = \hbar\omega_0 J_z + \hbar\omega_c \hat{a}^\dagger \hat{a} + \frac{\hbar\lambda}{\sqrt{N}}\left(\hat{a} + \hat{a}^\dagger\right)\left(J_+ + J_-\right), \quad J_z = \frac{1}{2}\sum_{i=1}^{N}\sigma_{z,i}, \; J_\pm = \sum_{i=1}^{N}\sigma_{\pm,i}$$

Dicke phase transition: $\lambda_c = \sqrt{\omega_c\omega_0}/2 \quad \left(N \gg 1,\ \kappa \text{ negligible}\right)$





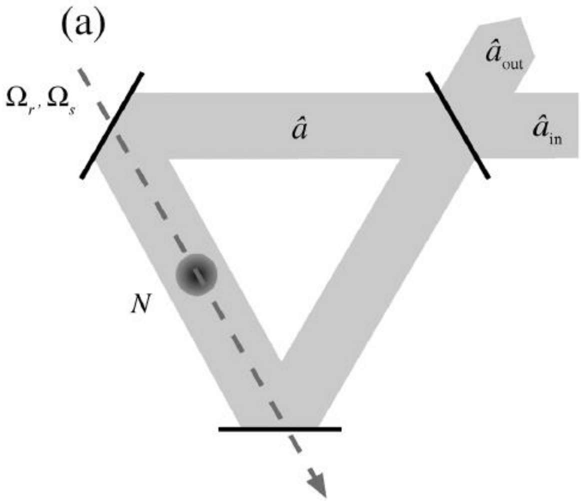
Hepp and Lieb, PRA **8**, 2517 (1973)
Wang and Hioe, PRA **7**, 831 (1973)
Hioe, PRA **8**, 1440 (1973)

From 1 atom to *N* atoms: Dicke model

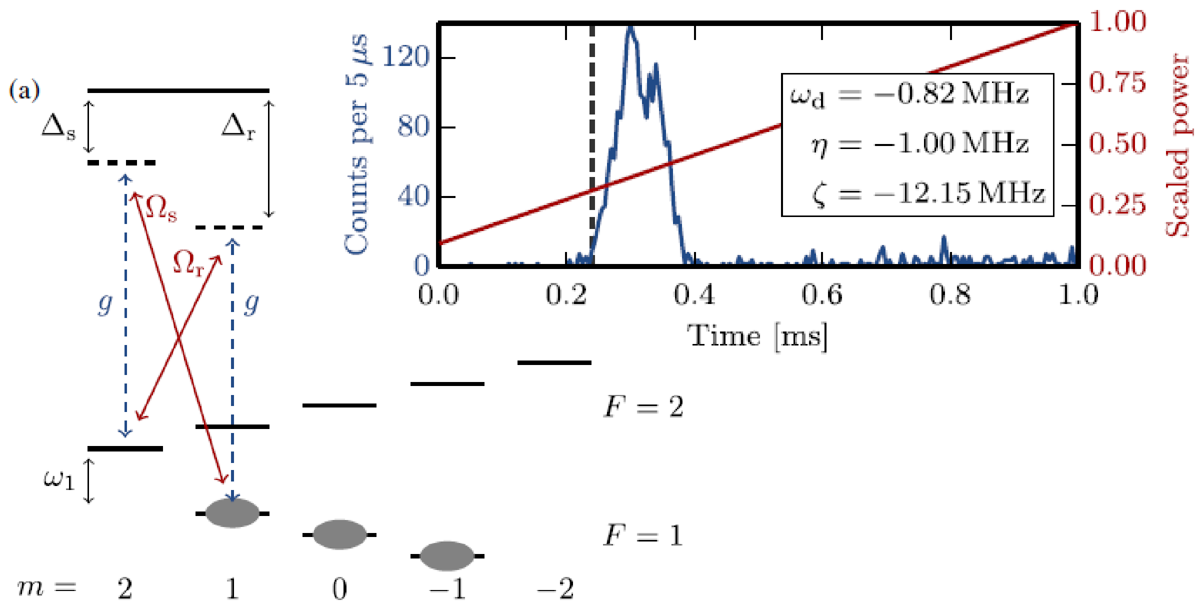
N-atom Dicke model:

$$H = \hbar\omega_0 J_z + \hbar\omega_c \hat{a}^\dagger \hat{a} + \frac{\hbar\lambda}{\sqrt{N}}\left(\hat{a}+\hat{a}^\dagger\right)\left(J_+ + J_-\right), \quad J_z = \frac{1}{2}\sum_{i=1}^{N}\sigma_{z,i}, \; J_\pm = \sum_{i=1}^{N}\sigma_{\pm,i}$$

Dicke phase transition: $\lambda_c = \sqrt{\omega_c\omega_0}/2 \quad \left(N \gg 1, \; \kappa \text{ negligible}\right)$

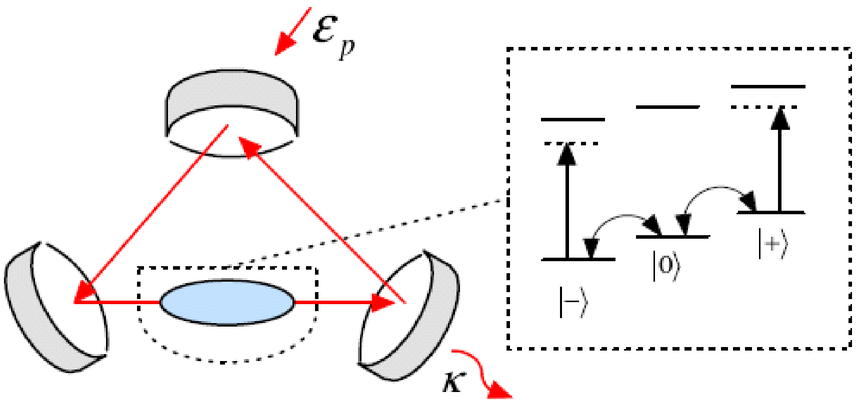


Dimer *et al.*, PRA **75**, 013804 (2007)



Baden *et al.*, PRL **113**, 020408 (2014)

Spinor BEC in ring cavity: cavity optomechanics



Zhou, Pu, Ling, Zhang, PRL **103**, 160403 (2009)
PRA **81**, 063641 (2010)

$$\hat{H} = \hat{H}_0 + \left[U_0\left(\hat{c}_+^\dagger \hat{c}_+ + \hat{c}_-^\dagger \hat{c}_-\right) - \delta_c\right]\hat{a}^\dagger a + i\varepsilon_p\left(\hat{a}^\dagger - \hat{a}\right),$$

Spinor BEC — Dispersive coupling — Pump of cavity

Photons adiabatically follow atomic dynamics:

$$\hat{a} = \frac{\varepsilon_p}{\kappa - i\left[\delta_c - U_0\left(\hat{c}_+^\dagger \hat{c}_+ + \hat{c}_-^\dagger \hat{c}_-\right)\right]}$$

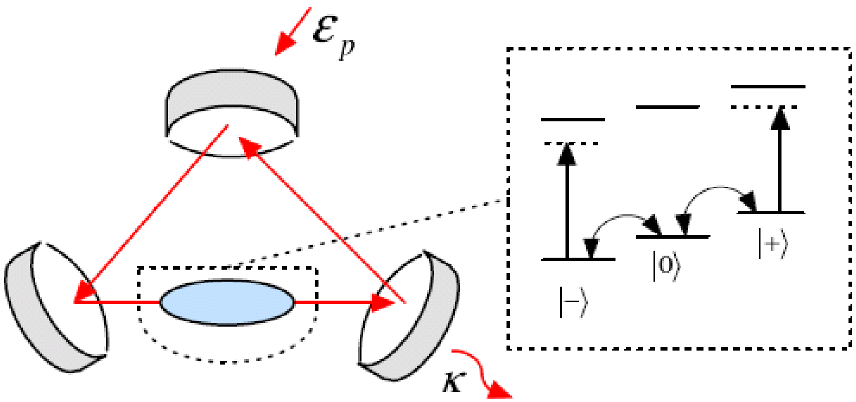
Atom-induced cavity shift

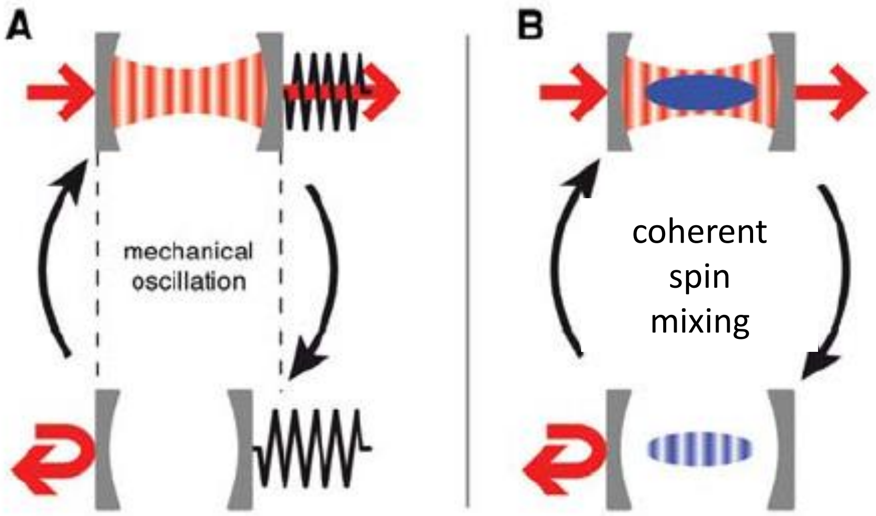
Effective Hamiltonian:

$$\hat{H}_{eff} = H_0 - \frac{\varepsilon_p^2}{\kappa}\tan^{-1}\left[\frac{\delta_c - U_0\left(\hat{c}_+^\dagger \hat{c}_+ + \hat{c}_-^\dagger \hat{c}_-\right)}{\kappa}\right]$$

Cavity-mediated atom-atom interaction

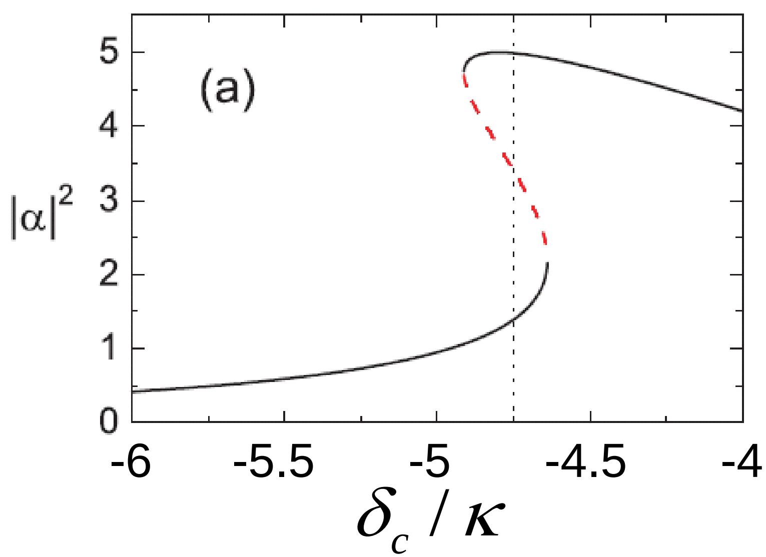
Spinor BEC in ring cavity : cavity optomechanics



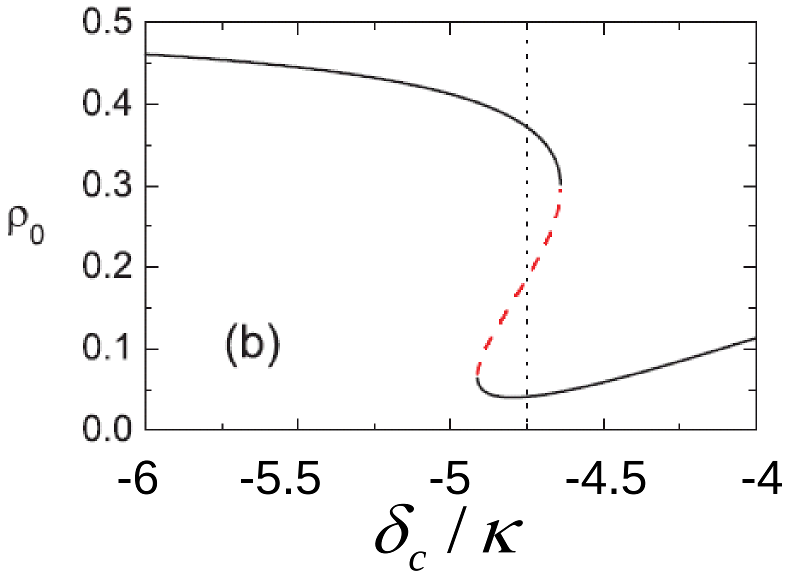


BEC as a mechanical oscillator
Bistability in matter wave and cavity field

Cavity photon number



Atomic population in spin-0



Effects of atomic center-of-mass motion

"Traditional" CQED:

cavity photon ⟷ atomic internal states

The advent of cold atoms makes the atomic COM motion no longer negligible.

cavity photon ⟷ atomic external states

Effects of atomic motion: atom-cavity microscope

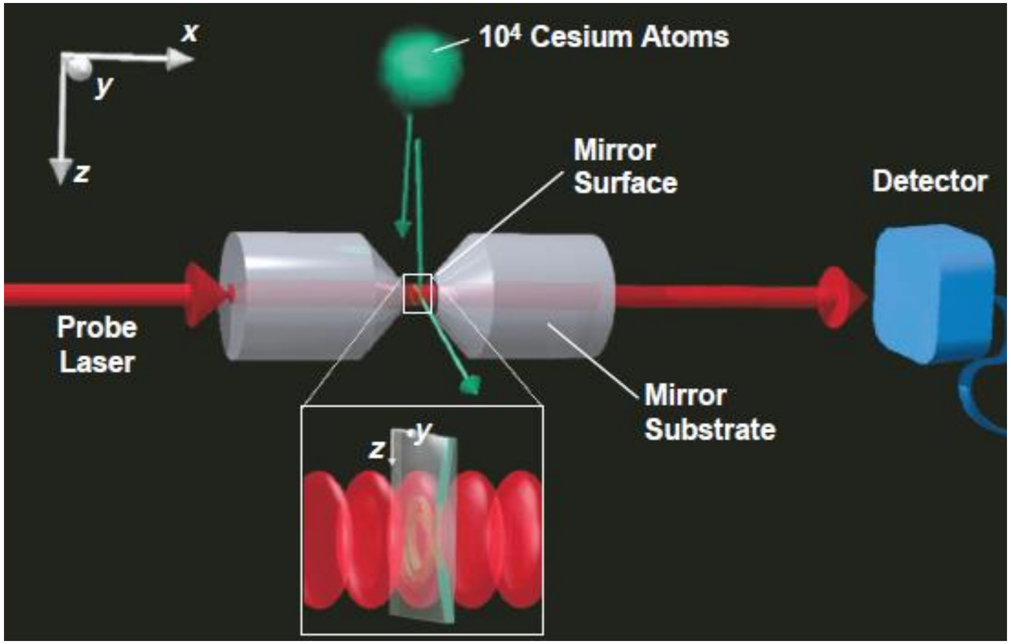
inhomogeneous cavity mode profile

↓

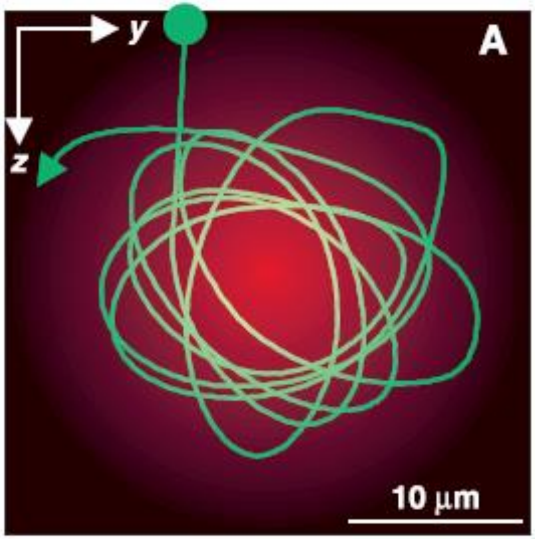
position-dependent atom-cavity coupling

↓

position-dependent atomic back-action

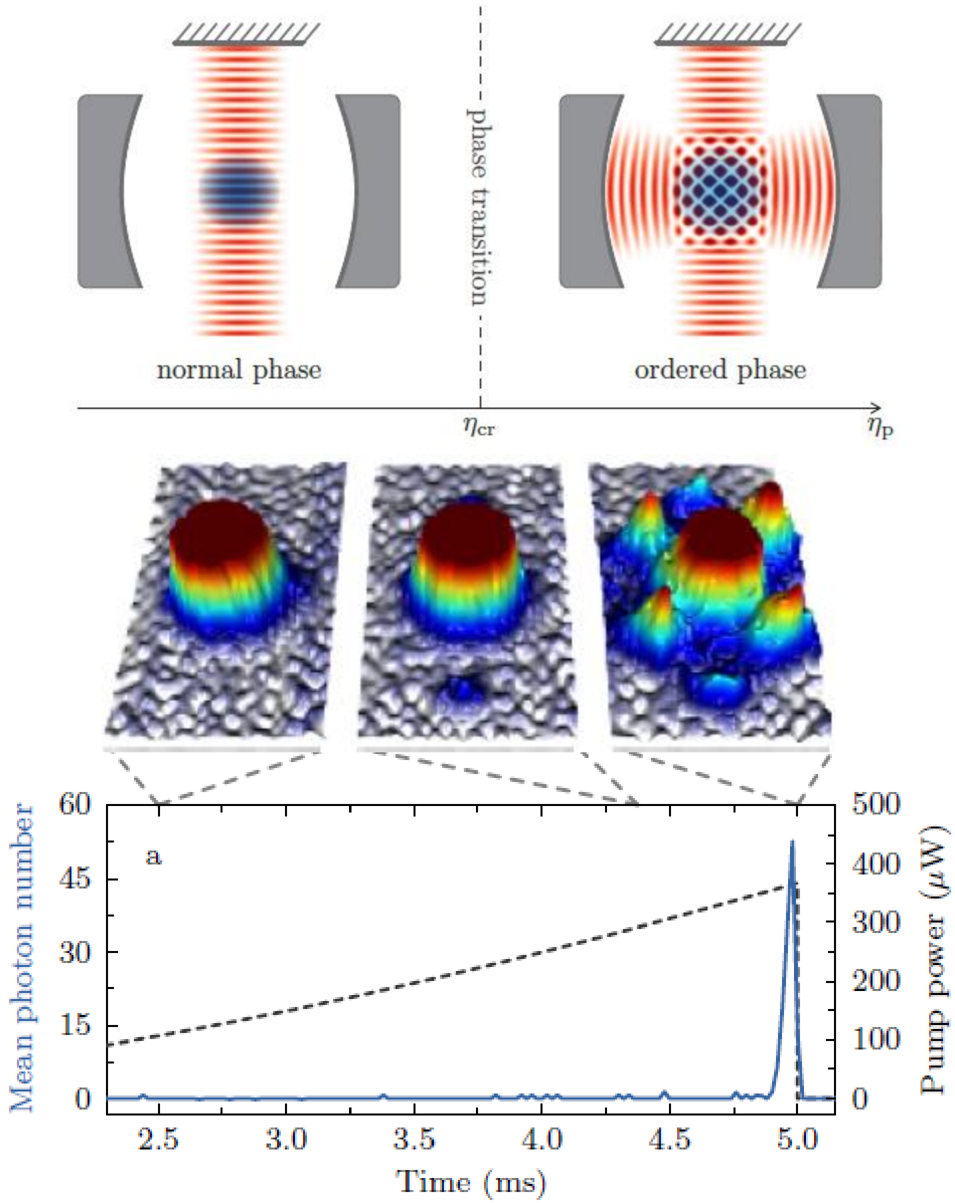


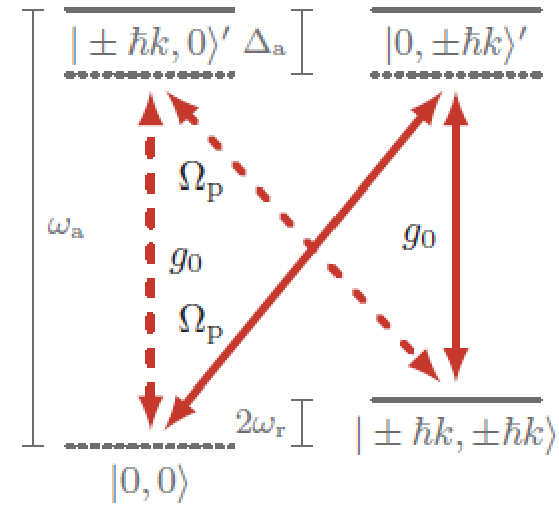
atomic intra-cavity trajectory



Hood *et al.*, Science **287**, 1447 (2000)

Dicke model revisited





Two-level system formed by motional states

Baumann, Guerlin, Brennecke, Esslinger, Nature **464**, 1301 (2010)

Superradiance with spinless fermions

PRL **112,** 143002 (2014) PHYSICAL REVIEW LETTERS week ending 11 APRIL 2014

Fermionic Superradiance in a Transversely Pumped Optical Cavity

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(Received 10 September 2013; published 8 April 2014)

PRL **112,** 143003 (2014) PHYSICAL REVIEW LETTERS week ending 11 APRIL 2014

Umklapp Superradiance with a Collisionless Quantum Degenerate Fermi Gas

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(Received 11 September 2013; published 8 April 2014)

PRL **112,** 143004 (2014) PHYSICAL REVIEW LETTERS week ending 11 APRIL 2014

Superradiance of Degenerate Fermi Gases in a Cavity

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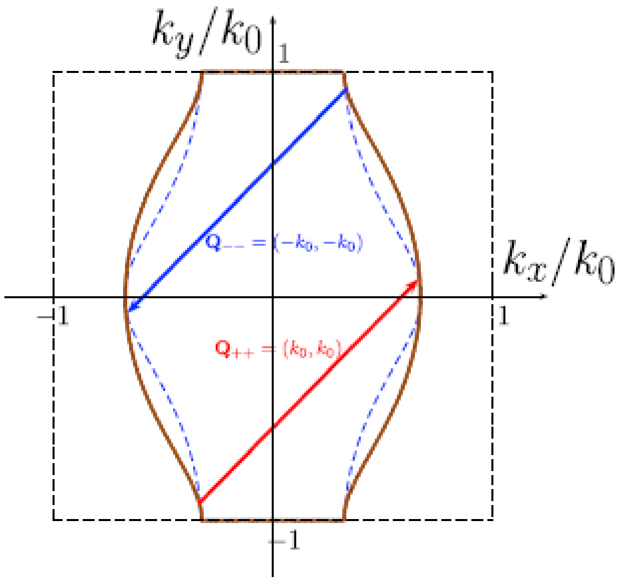
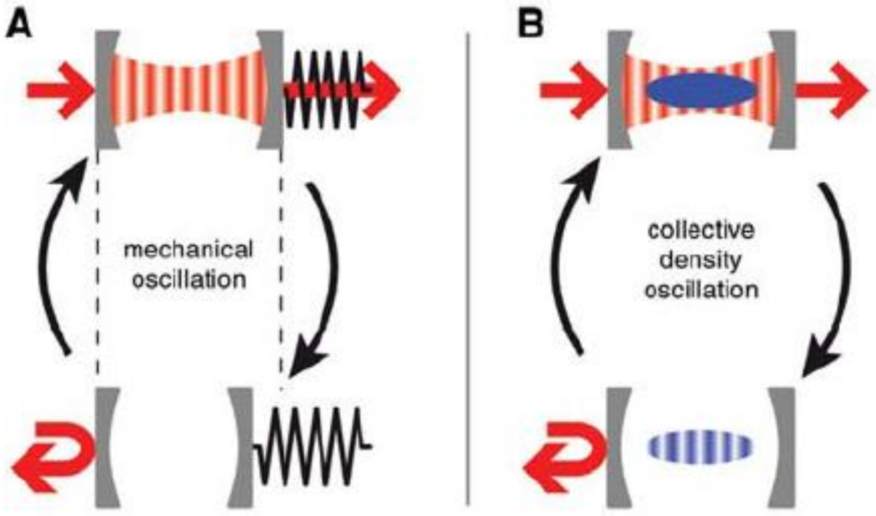
Institute for Advanced Study, Tsinghua University, Beijing 100084, China
(Received 25 September 2013; published 8 April 2014)



Cavity optomechanics with motional states



BEC as a mechanical oscillator

Brennecke, Ritter, Donner, Esslinger, Science **322**, 235 (2008)

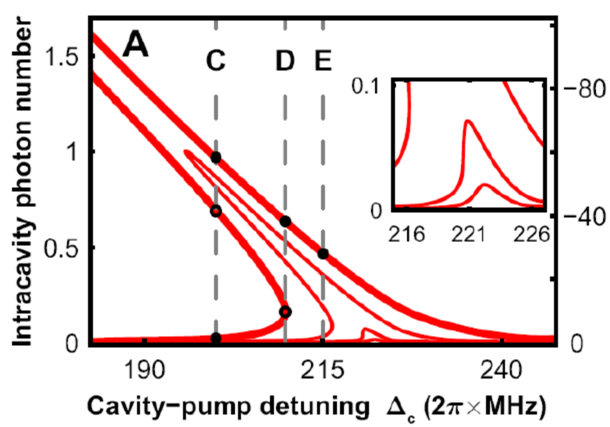
Similar work by Gupta *et al.*, PRL **99**, 213601 (2007)

$$\hat{H} = \int \hat{\Psi}^\dagger(x)\left(\frac{-\hbar^2}{2m}\frac{d^2}{dx^2} + \hbar U_0 \cos^2(kx)\hat{a}^\dagger\hat{a}\right)\hat{\Psi}(x)\,dx - \hbar\Delta_c\hat{a}^\dagger\hat{a} - i\hbar\eta(\hat{a} - \hat{a}^\dagger)$$

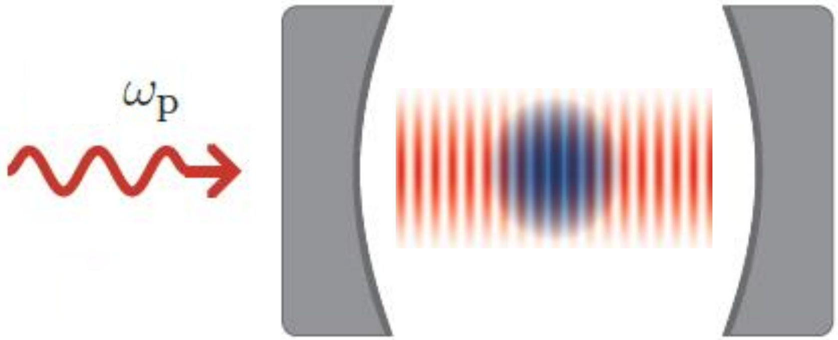
Mean field description

$$i\dot{\psi}(x,t) = \left(\frac{-\hbar}{2m}\frac{d^2}{dx^2} + |\alpha(t)|^2 U_0 \cos^2(kx)\right)\psi(x,t)$$

$$\alpha(t) = \frac{\eta}{\kappa - i(\Delta_c - NU_0\langle\cos^2(kx)\rangle)}.$$



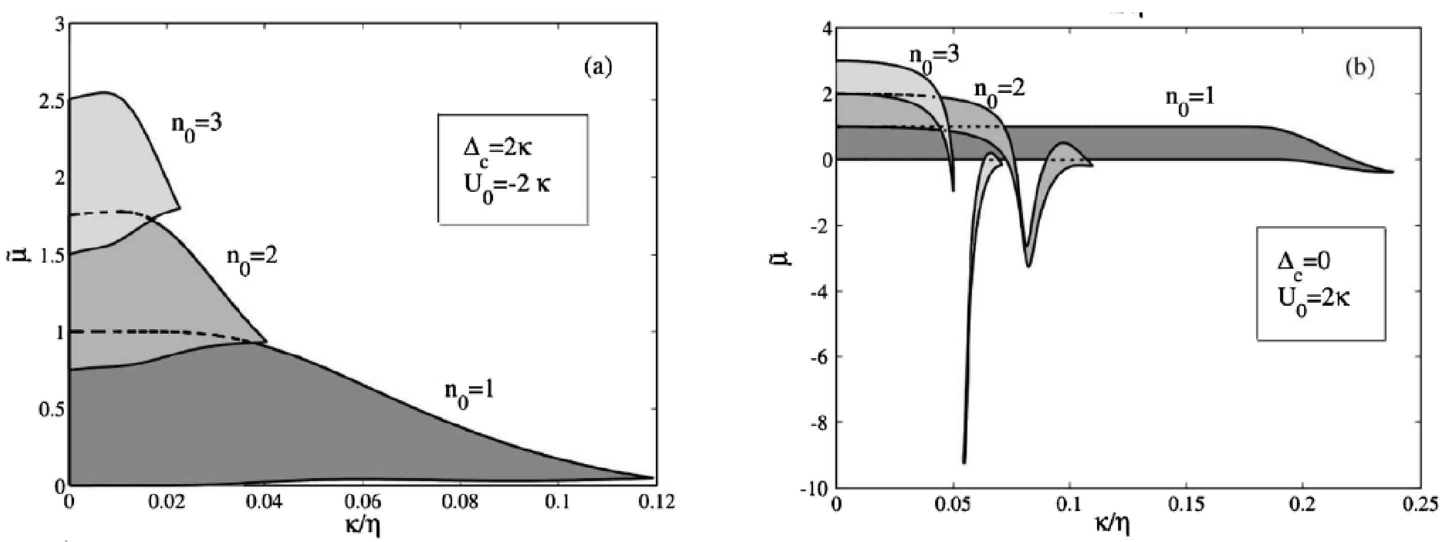
Dynamic optical lattice: effect on many-body physics



"Cold atoms in cavity-generated dynamical optical potentials"
Ritsch *et al.*, RMP **85**, 553 (2013)

$$\hat{H}_0 = \frac{\hat{p}^2}{2m} + \hbar[U_0\cos^2(k\hat{x}) - \Delta_c]\hat{n}_{\text{ph}} - i\hbar\eta(\hat{a} - \hat{a}^\dagger).$$

Mott-SF boundary for a spinless boson gas



Larson, Damski, Morigi, Lewenstein, PRL **100**, 050401 (2008)

Effects of atomic center-of-mass motion

"Traditional" CQED:

cavity photon ⟷ atomic internal states

The advent of cold atoms makes the atomic COM motion no longer negligible.

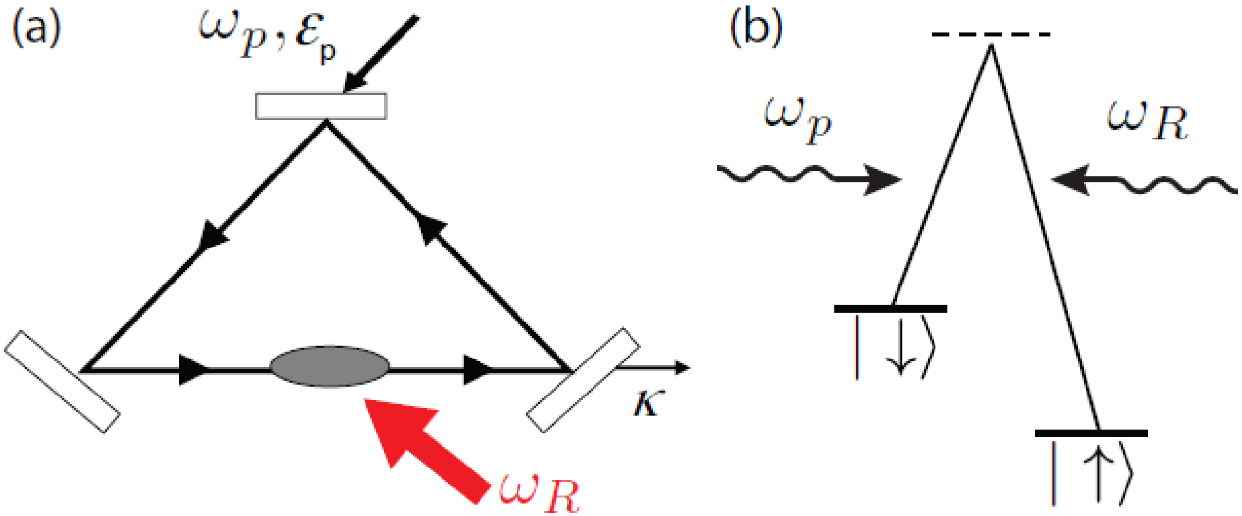
cavity photon ⟷ atomic external states

Cavity field couples directly to both internal and external atomic states.

cavity photon

atomic internal states ⟷ atomic external states

Cavity-induced spin-orbit coupling



Dong, Zhou, Wu, Ramachandhran, Pu, PRA **89**, 011602(R) (2014)
Related work: Mivehvar, Feder, PRA **89**, 013803 (2014)

Atomic back-action to cavity photon → "dynamic" spin-orbit coupling

$$\mathcal{H}_{\text{eff}} = \sum_{\sigma=\uparrow,\downarrow} \int dz \left[\hat{\psi}_\sigma^\dagger(z) \left(\frac{k^2 + 2\alpha_\sigma q_r k}{2m} + \alpha_\sigma \delta \right) \hat{\psi}_\sigma(z) \right] + \frac{\Omega}{2} \int dz \left[\hat{\psi}_\uparrow^\dagger(z) \hat{\psi}_\downarrow(z) \hat{c} + h.c. \right]$$
$$+ \; i\varepsilon_p(\hat{c}^\dagger - \hat{c}) - \delta_c \hat{c}^\dagger \hat{c}\,, \qquad \alpha_{\uparrow,\downarrow} = \pm 1$$

If $q_r = 0$, this model reduces to the JC/TC model

Mean-field approach: nonlinear SOC

$$\hat{c} \rightarrow c \equiv \langle \hat{c} \rangle = \frac{\varepsilon_p - i\frac{\Omega}{2}\varphi_\downarrow^*\varphi_\uparrow}{\kappa - i\delta_c}$$

$$i\dot{\varphi}_\uparrow = \left(\frac{k^2}{2m} + q_r k + \delta\right)\varphi_\uparrow + \frac{\Omega_{\text{eff}}}{2}\varphi_\downarrow$$

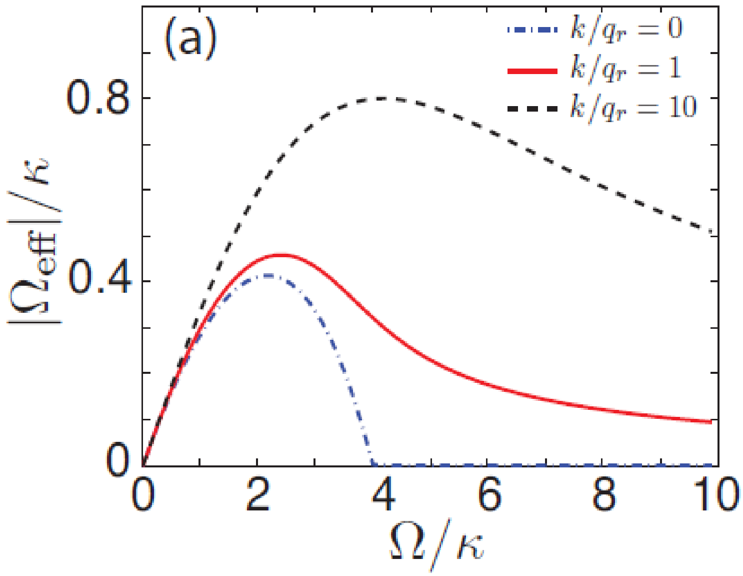
$$i\dot{\varphi}_\downarrow = \left(\frac{k^2}{2m} - q_r k - \delta\right)\varphi_\downarrow + \frac{\Omega_{\text{eff}}^*}{2}\varphi_\uparrow$$

$$\Omega_{\text{eff}} \equiv \Omega c = \Omega\frac{\varepsilon_p - i\frac{\Omega}{2}\varphi_\downarrow^*\varphi_\uparrow}{\kappa - i\delta_c}$$

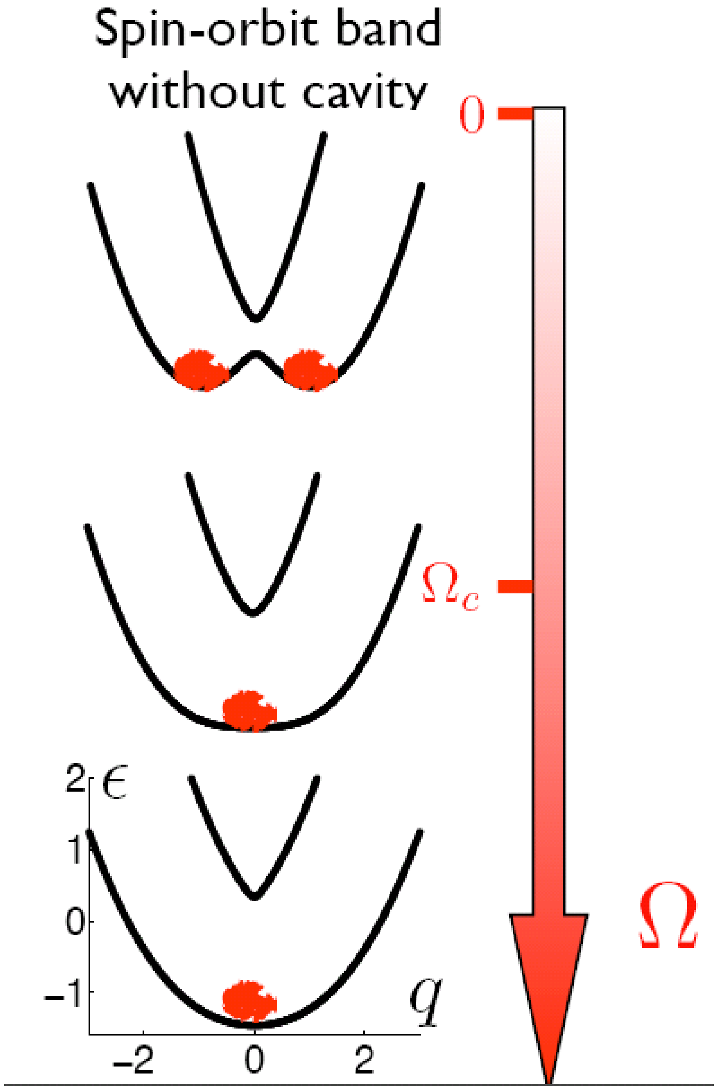
energy dispersion $\varepsilon(k)$ satisfies a quartic equaion:

$$4\epsilon^4 + B\epsilon^3 + C\epsilon^2 + D\epsilon + E = 0$$

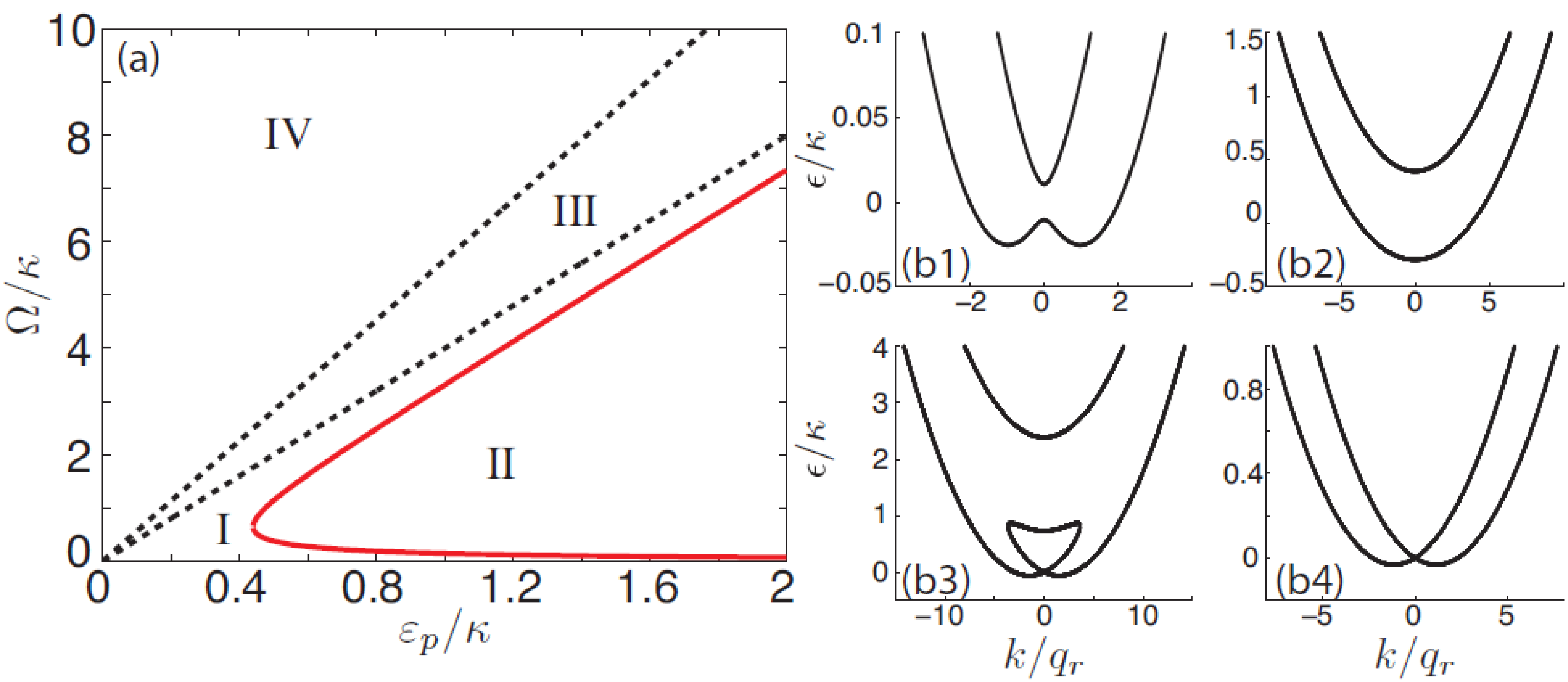
<u>Validity of MF approach:</u>
Negligible atom-photon correlation
Cavity field: coherent state



Dispersion without cavity



Dispersion with cavity

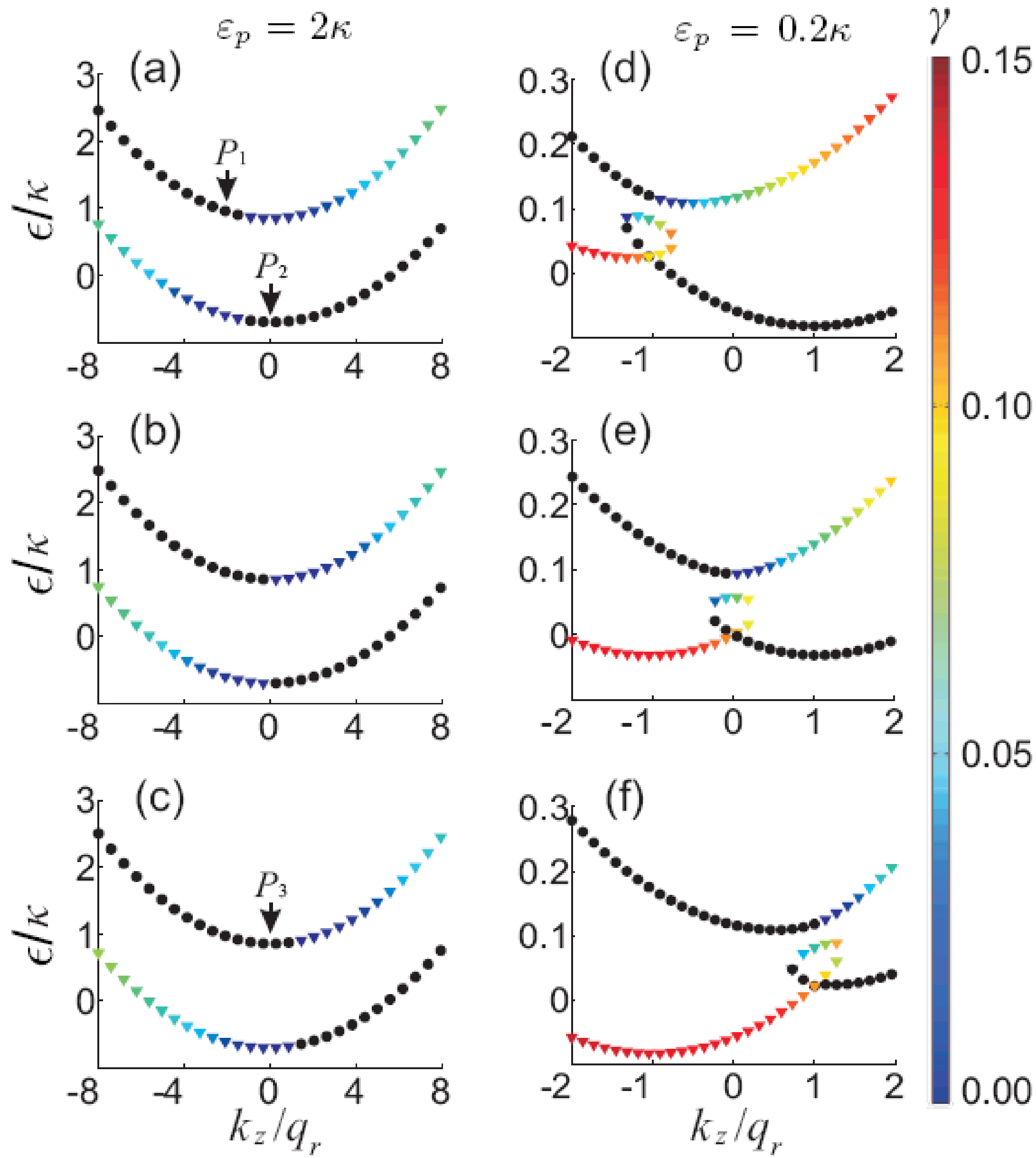


Stability analysis

$\tilde{\delta} = 0.05\kappa$

$\tilde{\delta} = 0$

$\tilde{\delta} = -0.05\kappa$

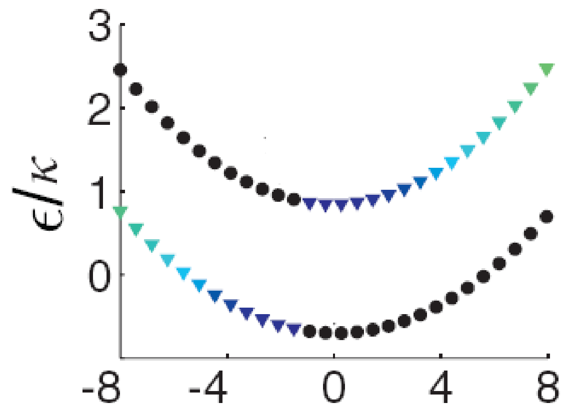


Beyond mean-field

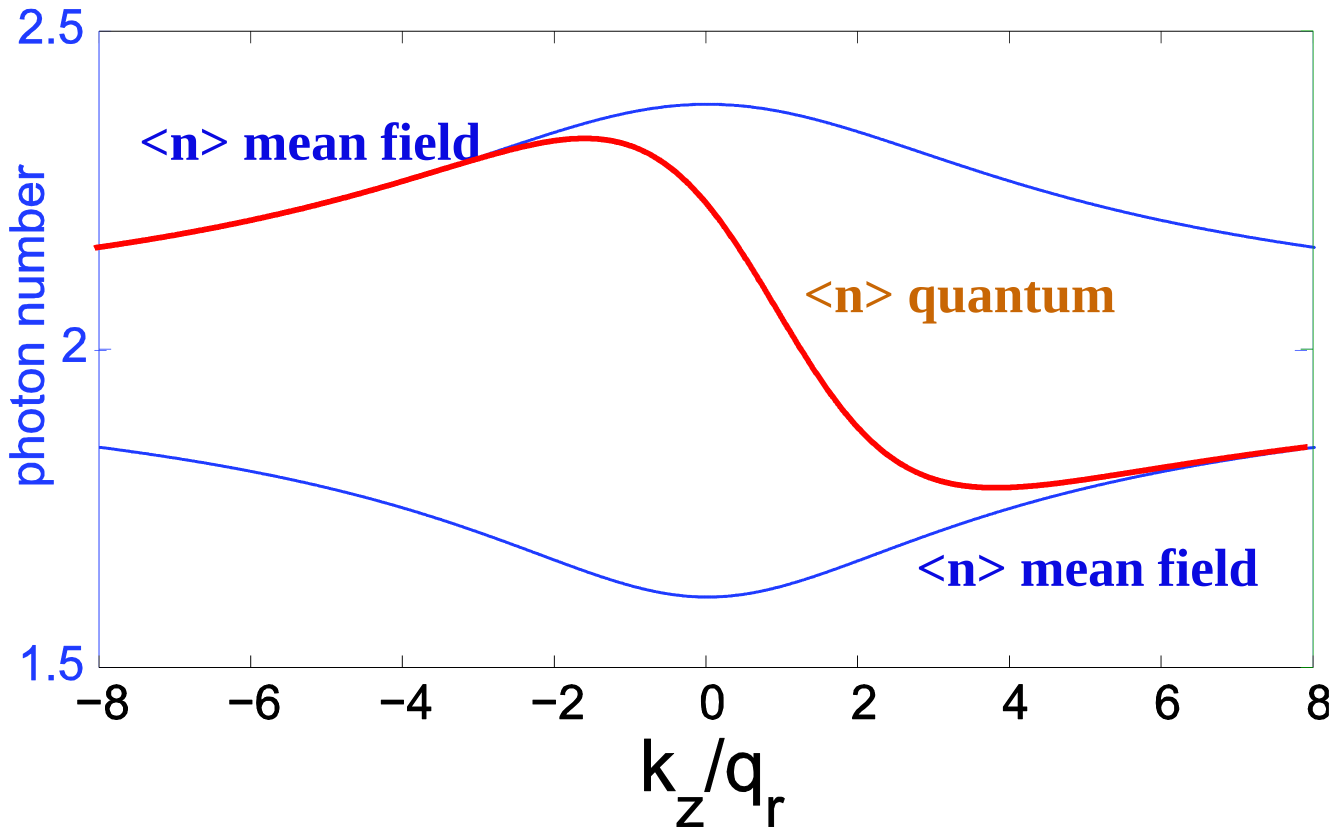
Master equation:

$$\dot{\rho} = \frac{1}{i\hbar}[\mathcal{H}_{\text{eff}}, \rho] + \mathcal{L}[\rho]$$

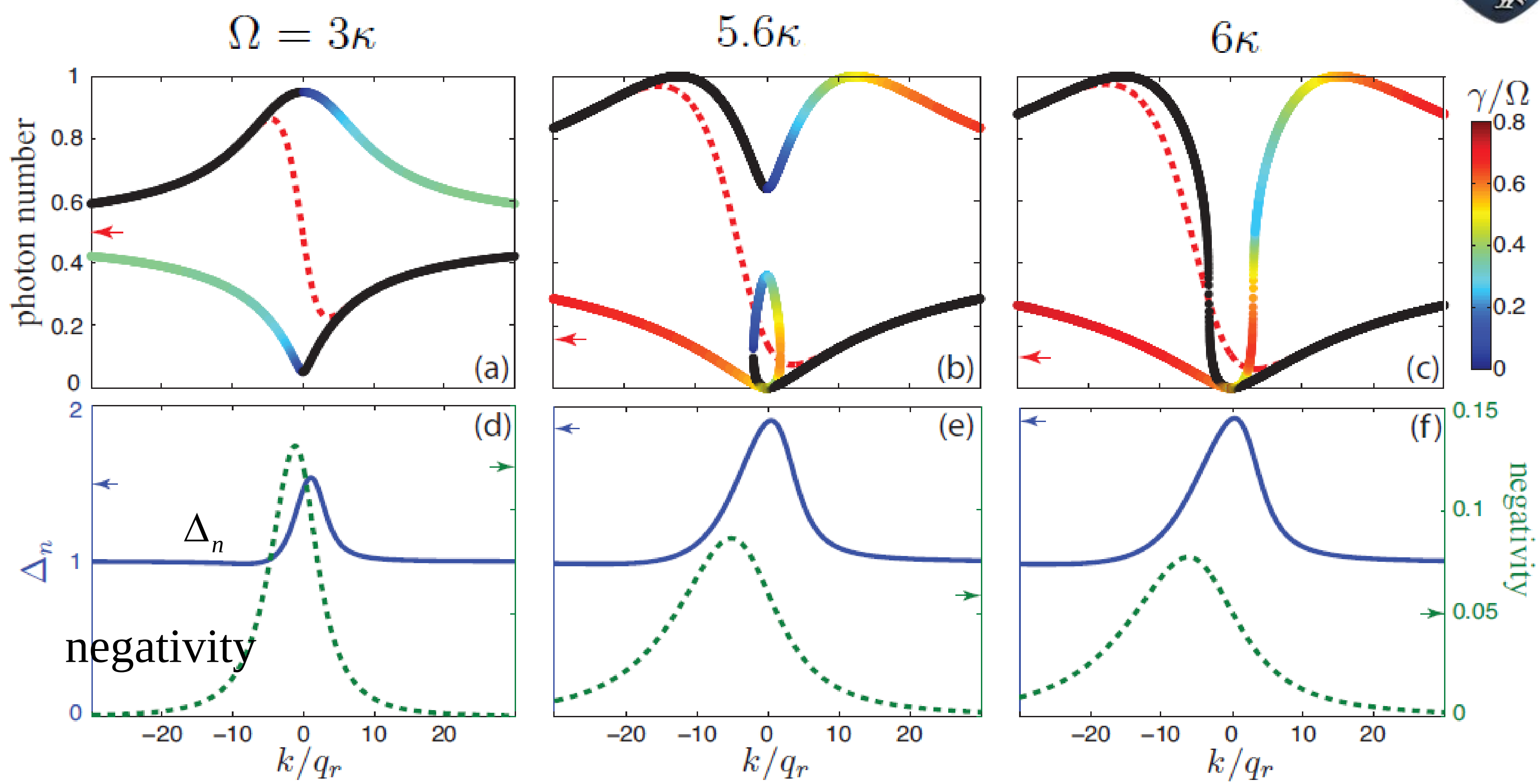
$$\mathcal{L}[\rho] = \kappa(2c\rho\hat{c}^\dagger - \hat{c}^\dagger\hat{c}\rho - \rho\hat{c}^\dagger\hat{c})$$



quantum vs. mean-field



Quantum vs. Mean-field

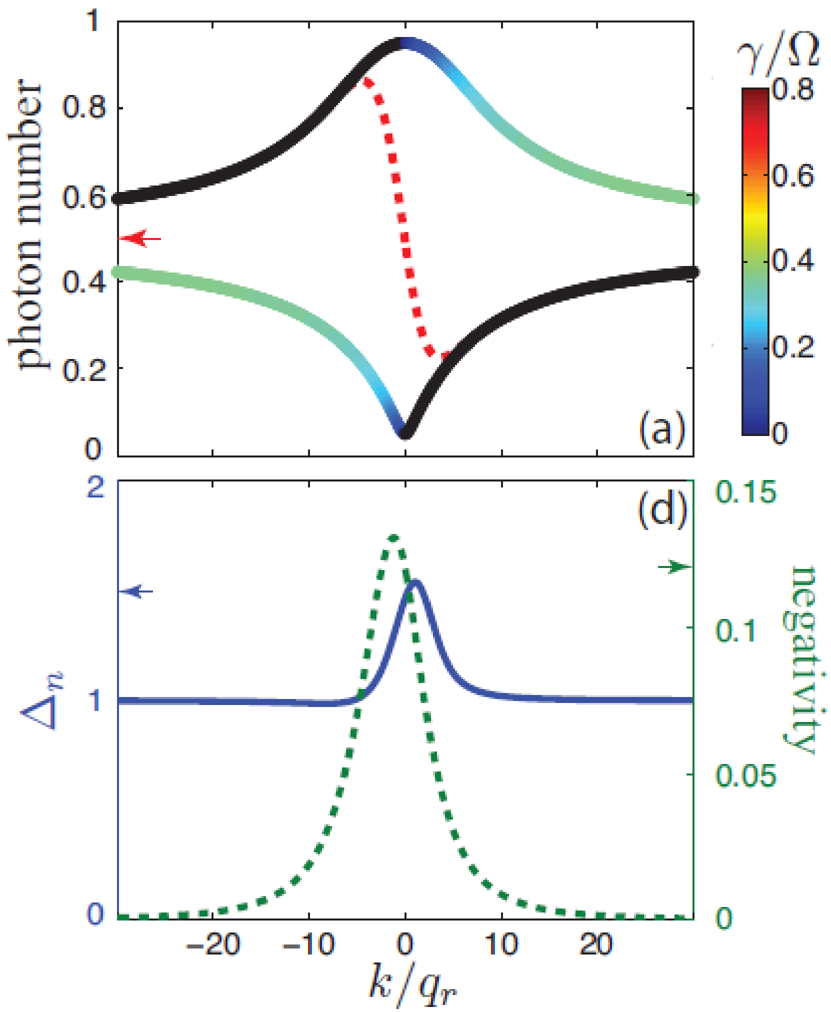


$$\Delta_n = \frac{\langle(\Delta n)^2\rangle}{\langle\hat{n}\rangle} = \frac{\langle\hat{n}^2\rangle - \langle\hat{n}\rangle^2}{\langle n\rangle},$$

$\Delta_n = 1$: Poisson distribution

negativity: measures atom-cavity entanglement

Quantum vs. Mean-field



Dong, Zhu, Pu, arXiv:1504.01729

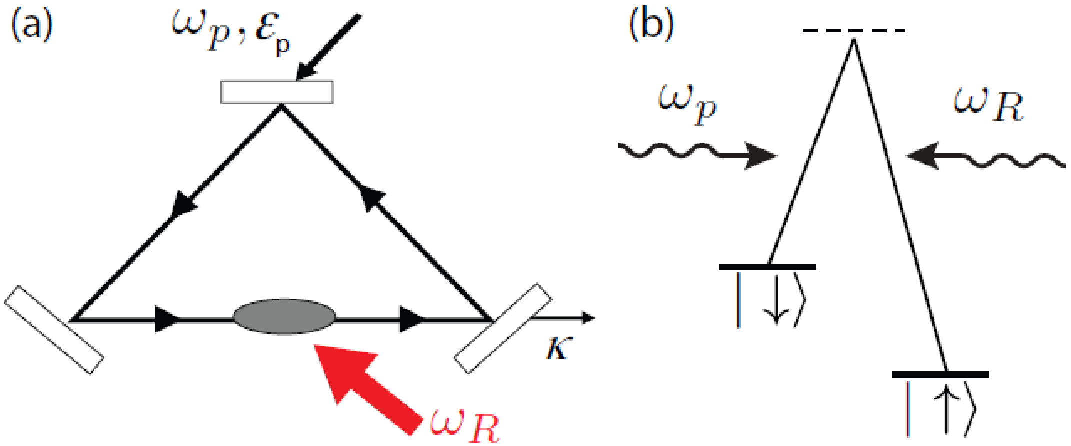
At large $|k|$,

- quantum and MF results agree w/ each other
- photon distribution becomes Poissonian
- atom-cavity entanglement is negligible

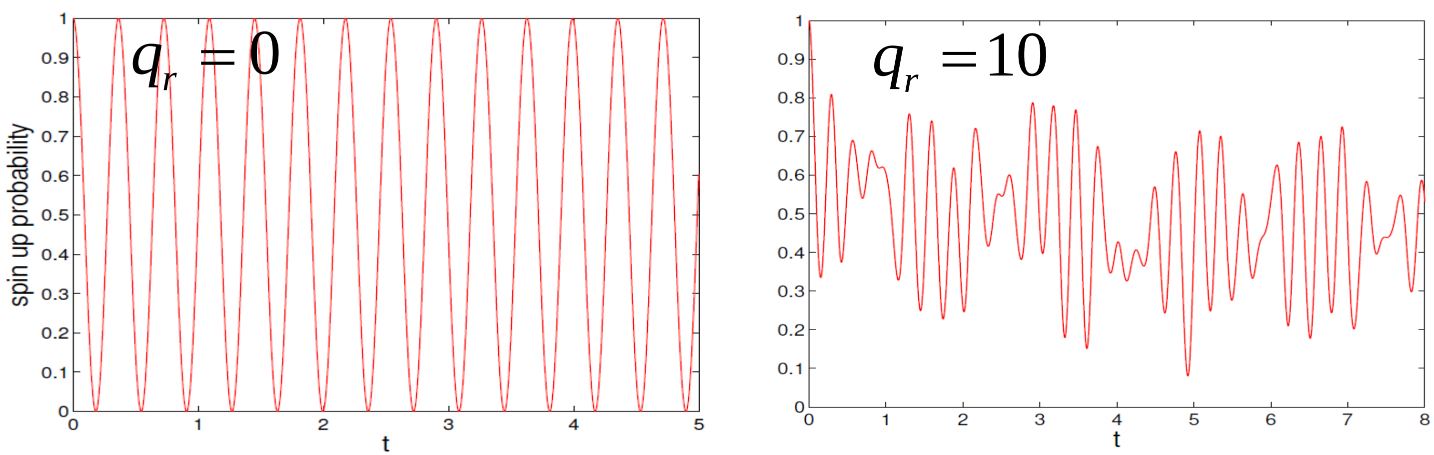
At given k, the kinetic energy mismatch between the two states is $2q_r k / m$

At large $|k|$, the Raman transition becomes far off-resonant.

Adding a harmonic trap

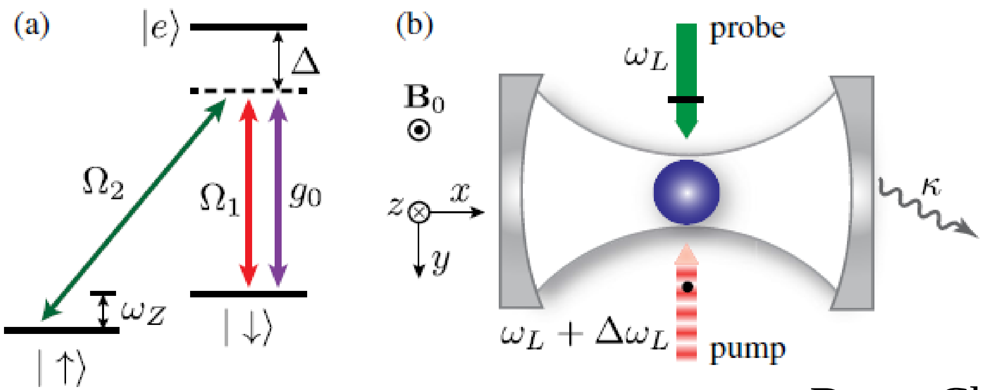


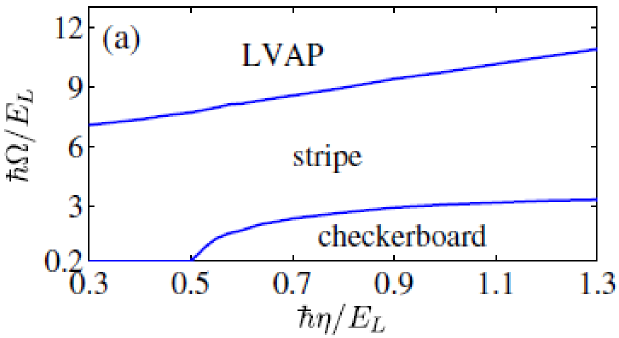
$$H = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 z^2 + \frac{q_r}{m}\hat{p}\sigma_z + \frac{\Omega}{2}\left(\hat{c}\sigma_+ + \sigma_-\hat{c}^\dagger\right) + \hbar\delta\hat{c}^\dagger\hat{c}$$



Cavity-assisted SOC in many-body systems

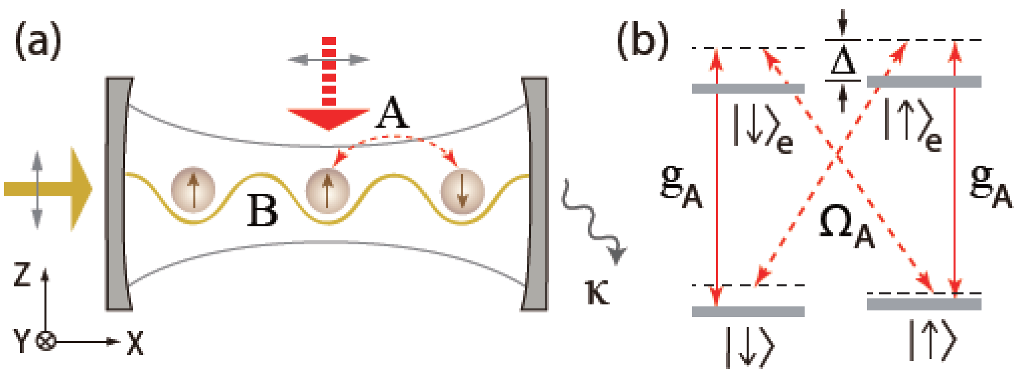
BEC in cavity

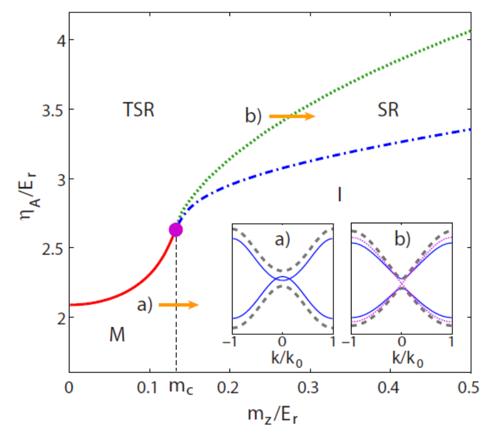




Deng, Cheng, Jing, Yi, PRL **112**, 143007 (2014)

Spin-1/2 Fermi gas in cavity

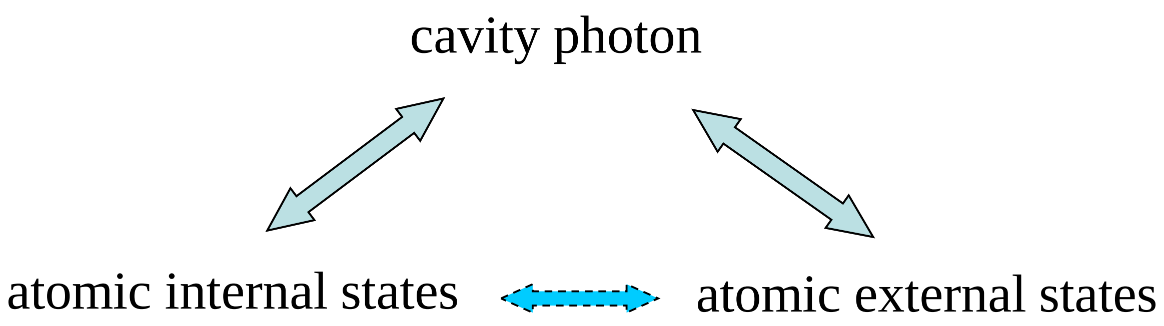




Pan, Liu, Zhang, Yi, Guo, arXiv:1410.8431

Cold atoms in cavity: a new frontier

Cavity field couples directly to both internal and external atomic states.



Quantum optics meets few/many-body physics